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## **RESEARCH ARTICLE**

# A Self-Sovereign Identity Based on Zero-Knowledge Proof and Blockchain

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**ABSTRACT** Systems for generating and managing digital identities are in the process of being transformed to improve data sharing security and increase decentralization. Addressing both issues, a theoretical solution to create and manage Self-Sovereign Identities (SSI) is proposed using two Zero-Knowledge Proof (ZKP) protocols based on the discrete logarithm difficulty. Automorphism group properties are introduced to link several identities, their identifiers and attributes to produce a proof. The proposed SSI protocol does not encounter the problem of reusing the same secret key as in the case of the initial ZKP Schnorr protocol. The designed protocol ensures minimal disclosure of information to a single trusted third party. In addition, it allows zero disclosure of information to service providers requiring proof of authentication or identification. Such a SSI protocol is compliant with Electronic IDentification And Trust Services (eIDAS) as well as General Data Protection Regulation (GDPR) regulations.

**INDEX TERMS** Decentralized identity (DID), self-sovereign identity (SSI), zero-knowledge proof (ZKP), higher education.

*g*

### **NOMENCLATURE**



*x<sup>i</sup>* Sequence of product (*a*; *b*).  $-a$  Inverse of *a*, such  $q - a = -a$  in  $\mathbb{Z}_q^{(*,+)}$ .  $-b$  Inverse of *b*, such  $q - b = -b$  in  $\mathbb{Z}_q^{(*,+)}$ .  $2^{-1}$ Inverse of 2, such  $2 \times 2^{-1} = 1$  in  $\mathbb{Z}_q^*$ .  $\sigma_i \in \mathbb{Z}_q^*$ , signature of the prover. *a* A secret element  $\in \mathbb{Z}_q^*$ .  $a_i$  A secret sequences  $\in \mathbb{Z}_q^*$ . *b* A secret element  $\in \mathbb{Z}_q^*$ . *b*<sup>*i*</sup> A secret sequences  $\in \mathbb{Z}_q^*$ . *c*  $\therefore$  Constraint on *m*, *k* and  $\circ$ ,  $\in \mathbb{Z}_q^*$ . *c*<sub>*i*</sub> Sequence of hashed value of  $(g, LH_i, m_iu^{-1} + (k_i \cdot$  $(o_i)^{u^{-1}}$ ), simulated challenge. *e* A secret element ∈  $\mathbb{Z}_q^*$ .

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- *f* A secret element  $\in \mathbb{Z}_q^*$ .
- $k \in \mathbb{G}$ , response of the prover.

*<sup>x</sup> g* raised to the power of *x*. *g<sup>i</sup>* Sequence of group generator. *t* A known element  $\in \mathbb{G}$ .

- $k_i \in \mathbb{G}$ , response of the prover.
- $m \in \mathbb{G}$ , response of the prover.
- $m_i \in \mathbb{G}$ , response of the prover.
- *n* A secret element  $\in \mathbb{Z}_q^*$ .
- $o \in \mathbb{G}$ , response of the prover.
- $o_i \in \mathbb{G}$ , response of the prover.
- *p*<sup>*i*</sup> A secret sequence element  $\in \mathbb{Z}_q^*$ .
- *Q* A known element  $\in \mathbb{G}$ .
- *q* Order of the group G.
- $r \in \mathbb{Z}_q^*$ , response of the prover.
- *s* A secret element  $\in \mathbb{Z}_q^*$ .
- $t^{-1}$ Inverse of *t*, such  $t \cdot t^{-1} = 1 \in \mathbb{G}$ .
- $u \in \mathbb{Z}_q^*$ .
- *w* Integer such as  $0 \lt w \lt q$ , number of proofs  $(m, k, o)$ .
- *x* A secret element  $\in \mathbb{Z}_q^*$ .
- *y* A known element  $\in \mathbb{G}$ .
- *Y* A known element  $\in \mathbb{G}$ .
- *y* A known element  $\in \mathbb{G}$ .
- *Z* <sup>*′′*</sup> Constraint on *m*, *k* and  $o, \in \mathbb{Z}_q^*$ .
- *Z Constraint on <i>m*, *k* and  $o \in \mathbb{Z}_q^*$ .
- *z ′* Sum of *a* and *b* ∈  $\mathbb{Z}_q^*$ .
- *Z* Product of *m*, *k* and  $o \in \mathbb{Z}_q^*$ .
- *z* Produce of *a* and  $b \in \mathbb{Z}_q^*$ .

### **I. INTRODUCTION**

In an increasingly connected world, the management and control of users' digital identities are becoming crucial elements of our numerical lives [\[1\] wh](#page-9-0)ere we are more and more led to share sensitive information. To address this issue, the Electronic Identification and Trust Services (eIDAS) has established standards. A service provider offering an identification (or authentication) system must comply with these standards to be certified [\[2\]. H](#page-9-1)owever, eIDAS compliant service providers for identity such as the ones proposed by GAFAM [\[3\] req](#page-9-2)uire the user to transfer the rights of sensitive data in order to benefit from their services.

<span id="page-1-3"></span><span id="page-1-2"></span>As an alternative identity service, a self sovereign identity (SSI) would improve user's experience by giving them power and control over their digital identifiers [\[4\], \[](#page-9-3)[5\]. S](#page-9-4)uch an SSI could help address the metaverses' need for scalable interoperability and security highlighted in [\[6\]. Se](#page-9-5)lf sovereign identity systems have been inspired by the developments of digital identity management systems for distributed architectures [\[4\], \[5](#page-9-3)[\]. By](#page-9-4) relying on the properties of distributed architectures, such as public distributed blockchains, providing immutability on all information written in the blockchain, the majority of SSI implemented with a blockchain comply with C.Alen's 11 criteria defining self-sovereign identities [\[3\], \[](#page-9-2)[7\],](#page-9-6) [\[8\]. H](#page-9-7)owever, a trusted third party certifying the identity is still required in order to uniquely identify a person [\[5\].](#page-9-4)

<span id="page-1-7"></span><span id="page-1-6"></span>Following European Union's General Data Protection Regulation principles, data related to a person on the blockchain must remain absolutely confidential. By default data on a blockchain is plain text  $[9]$ . Then, data must be encrypted  $[10]$  and the amount of data transferred while exchanging information should be minimized. The two technologies minimizing the amount of data transferred while exchanging information are selective disclosure of identity attributes and verifiable claims [\[5\]. Go](#page-9-4)ing beyond these technologies, Zero-Knowledge Proof (ZKP) enables writing only a proof of knowledge on the blockchain instead of the information itself [\[11\].](#page-9-10)

<span id="page-1-9"></span>The challenge is to implement a decentralized SSI using blockchain and ZKP technology allowing access to a service without disclosing sensitive information.

<span id="page-1-10"></span>The remainder of the paper is organized as follows. Section [II](#page-1-0) presents a literature review of works addressing this challenge. Section [III](#page-2-0) summarizes the main contributions of this work. In section [IV,](#page-2-1) an original SSI solution is detailed with an emphasis on its compliance with eIDAS, GDPR as well as criteria introduced by C. Allen and complemented by Q. Stokkink and J. Pouwelse with provability [\[3\], \[](#page-9-2)[12\].](#page-9-11) Section [V](#page-6-0) illustrates the proposed solution with an SSI-Protocol example. In section [VI,](#page-8-0) a qualitative comparison and an evaluation based on simulations for the introduced SSI- $ZKP_{a\cdot b}$  is proposed. Finally, section  $VII$  concludes the paper.

### <span id="page-1-0"></span>**II. STATE OF THE ART**

Several SSI solutions have been implemented with blockchain [\[3\], \[](#page-9-2)[4\], \[](#page-9-3)[5\]. Su](#page-9-4)ch SSI blockchain implementations can be further improved adding ZKP layers [\[4\], \[](#page-9-3)[5\],](#page-9-4) bearing in mind that different ZKP protocol implementations will have different properties such as their ZKP size, proof size, time duration for creation and verification [\[13\], \[](#page-9-13)[14\],](#page-9-14) [\[15\].](#page-9-15)

<span id="page-1-12"></span><span id="page-1-11"></span><span id="page-1-1"></span>In practice, as stated in the report for the French ministry of interior on Blockchain and digital identification working group [\[5\], m](#page-9-4)ost SSI implementation solutions provide three common tools, namely:

- **Cryptographic and hash functions.** The user recovers or creates a unique digital identity, generates his decentralized identifiers and links his certified documents or attributes to it (by a trusted third party, another user or a self-certificate).
- <span id="page-1-5"></span><span id="page-1-4"></span>• **Decentralized IDentifiers (DID).** The user uses his decentralized identifier once connected to access a good or a service. He declares to be ''who he is'', and proves that he has the attributes required to access service through verifiable assertions relating to attributes of his identity and/or authentic documents.
- **Verifiable assertions.** The verifier checks the certificates and proofs provided before granting access to the service.

<span id="page-1-8"></span>A SSI solution can be tailored for specific purposes. Different organizations have made proposals addressing different civil, economic and social problems. For example, the Secure Identity Alliance (SIA) consortium proposed interfaces (API) in Open Standard for the interoperability of operations on civil, identity and administrative registers. The ID2020 Alliance

aimed to fund the technology development of secure digital certificates and establish standards to facilitate interoperability and multi-party collaboration [\[16\]. C](#page-9-16)urrently the French company Archipel offers an identity management solution based on the Blockchain which tends towards an SSI compliant with eIDAS and GDPR.

SSIs can be evaluated with the following C.Allen [\[3\], \[](#page-9-2)[12\]](#page-9-11) criteria, which gives an overview of their performance and level of security:

- 1) **Existence:** users must have independence and autonomy in the creation of their digital identity. No collision of identifiers, which must be attached to a unique identity in real life.
- 2) **Control:** the user must have control over the creation and management of their identities.
- 3) **Access:** users should have full access to their own data without any restrictions.
- 4) **Transparency:** the proposed SSI implementation modalities and algorithms must be shared, open-source, well-known, and as independent as possible
- 5) **Persistence:** the identity must have a durable lifespan, and any update related to this identity must be made only by the user.
- 6) **Portability:** the storage and durability of the identity must not rely entirely on a trusted third party, at the risk that its disappearance implies the disappearance of the identity.
- 7) **Interoperability:** The number of niche does not impact the proper functioning of the proposed protocol.
- 8) **Consent:** any manipulation of the identity or claim must be preceded by a consent of the user attributed to this identity.
- 9) **Minimization:** only information relevant and required to serve claims is disclosed and shared.
- 10) **Protection:** all the legitimate rights of users must be preserved and respected.
- 11) **Provable:** claims must be verifiable.

Existing SSI blockchain implementations can be further improved toward better checking of the above mentioned criteria by adding ZKP layers [\[3\], \[](#page-9-2)[4\], \[](#page-9-3)[5\], \[](#page-9-4)[7\], \[](#page-9-6)[8\]. Z](#page-9-7)ero-Kowledge Proof protocols pave the way toward technologies embedding privacy-first principles for data transmission. The user is given the possibility to remain anonymous while being able to prove he possesses his verifiable credential (anonymous credentials) [\[17\].](#page-9-17)

<span id="page-2-3"></span>Different ZKP protocols are based on different cryptographic assumptions. Different ZKP protocols are also designed or implemented for different use cases with ad-hoc computational problems. ZKP differences induce limited comparisons between them due to heterogeneous design parameters. Among these parameters one could note the size of proofs as well as exchange protocols for generation and verification keys [\[13\], \[](#page-9-13)[14\], \[](#page-9-14)[15\], \[](#page-9-15)[18\].](#page-9-18)

Two classes of protocols are distinguished, Interactive ZKP (IZKP) and Non-Interactive ZKP (NIZKP), depending on

<span id="page-2-2"></span>whether the protocol requires (IZKP), or does not require (NIZKP), the simultaneous connection of the prover and the verifier(s). NIZKP protocols have a strong user experience design advantage. The three non-interactive ZKP protocols which have been reported to remain unbroken at the time of writing are zk-SNARK, BulletProofs and zk-STARK [\[18\], \[](#page-9-18)[19\].](#page-9-19)

<span id="page-2-5"></span>Different SSI protocols fulfilling each and every one of the above listed criteria have been published. For example, Sora identity [\[1\] ha](#page-9-0)s been proposed as a secure digital identity based on the Blockchain. Sora presents itself as a mobile application that allows the creation of verifiable credentials while guaranteeing pseudonymity.

<span id="page-2-6"></span>Another SSI protocol example, fulfilling the previously enumerated criteria, is SelfKey [\[20\]. S](#page-9-20)elfKey is proposed as a blockchain-based self-sovereign digital identity network using ZKP. In SelfKey, users can completely control and reveal their identity to a third party. ZKP is used to minimize shared information while approving third-party access to the revealed data. LifeID [\[21\] is](#page-10-0) similarly proposed as a self-sovereign digital identity platform that allows users to create independent online identities. LifeID uses ZKP to ensure only necessary information is revealed during identity verification.

<span id="page-2-7"></span>Although the above-cited SSI protocols claim using ZKP techniques, they still reveal information to a third party. Using existing ZKP techniques has limitations that could be overcome by designing ZK-native SSI based on a tailored NIZKP protocol.

The challenge addressed in this article is then to design a non-interactive SSI data exchange protocol using blockchain technology respecting the above mentioned criteria and inherently ensuring zero information disclosure (NIZKP) while communicating sensitive information.

### <span id="page-2-0"></span>**III. CONTRIBUTIONS**

The main contributions of this work can be summarized as follows:

- An original Self-Sovereign Identity solution embedding Zero Knowledge Proof based on blockchain technologies is proposed.
- A ZKP protocol enabling to prove that *a* and *b* such that  $a \cdot b = n$  are known without revealing *a* and *b* has been proposed.
- Automorphism sequence linking different identities, identifiers and/or attributes as well as different blockchain in formation has been designed linking the ZKP Schnorr and an original ZKP protocol named  $ZKP_{a\cdot b}$ .

### <span id="page-2-1"></span>**IV. METHODS**

<span id="page-2-4"></span>Would a theoretical implementation for a privacy-first SSI solution based on ZKP and blockchain be able to carry out the objective of giving to the user the ability to manage his digital identity and associated wallet? Depending on what a user needs to authenticate, identify, justify or prove, such an

implementation would enable the user to select and share a proof of his identity attributes without disclosing any other sensitive information.

The proposed SSI solution corresponds to the challenges, demands and objectives raised in [5] *[i.e.](#page-9-4)* selective disclosure of attributes/identifiers, confidentiality and security. In order to implement the envisaged SSI solution, a decentralized protocol scheme, cryptographic techniques based on the structure of algebraic groups and blockchain technology have been assembled.

<span id="page-3-5"></span>Two ZKPs have been employed, a ZKP Schnorr [\(IV-A\)](#page-3-0) and an original one [\(IV-B\)](#page-3-1) tailored to meet C. Allen criteria when exploited on a blockchain. In order to reduce the risk of usurpation [\[22\], a](#page-10-1) trusted third party referred to as ''The State'' has been introduced. The State will allow to have a basic identity created by the user, which can guarantee its uniqueness and veracity. Starting from this State approved digital identity, other digital identities can be built. In the proposed SSI, identities and its attributes can be gathered and linked to produce a single proof.

### <span id="page-3-0"></span>A. SCHNORR's ZERO KNOWLEDGE-PROOF

A ZKP Schnorr transformed with the Fiat-Shamir method [\[23\]](#page-10-2) has been chosen for its straightforward crytprographic architecture, its compatibility with the original ZKP protocol and its robustness induced by the discrete logarithm problem. This protocol has the advantage of being generalized [\[24\]](#page-10-3) with homomorphisms instead of the automorphism which forms  $g \to g^w$ , *g* being a group generator and *w* an integer such as  $0 < w < q$ , *q* being the order of the group.

Let's consider the following problem: Alice wishes to prove to Bob that she knows an element *x*, without disclosing it, using a Schnorr ZKP. To do so, let *q* be a prime number and G a group generated by *g* of order *q*.

Alice wishes to prove Bob that she knows an element *x* such that  $y = g^x$ . Let's also define the following steps of a Schnorr ZKP protocol:

- Creation step of the interactive Schnorr environment where Alice generates the proof to be sent to Bob:
	- **-** Commitment: Alice randomly draws an integer *s* from the set  $\mathbb{Z}_q^*$  and keeps it secret. Alice will then calculate  $t = g^3$  and send it to Bob.
	- **-** Challenge: Bob randomly draws an integer  $c \in \mathbb{Z}_q^*$ and sends it to Alice.
	- **-** Answer: Alice calculates  $r = s c \cdot x$  and sends it to Bob.
- Verification step where Bob validates the received answer *r* from Alice:

Using  $r = s - c \cdot x$  and  $y = g^x$ , Bob verifies that Alice knows and possesses the secret *x* if  $g^r \cdot y^c$  corresponds to *t* as in equation [\(1\)](#page-3-2).

$$
g^{s-c \cdot x} \cdot g^{c \cdot x} = g^s = t \tag{1}
$$

Adding the Fiat-Shamir transformations to the Schnorr protocol, the signature scheme becomes non-interactive using a hash function. The creation and verification steps of the Schnorr ZKP protocol are then modified as follows:

- Creation step where Alice still wants to prove that she knows *x* such that  $y = g^x$  with  $y \in G$  a group generated by *g* of order *q* (*g* and *q* being public):
	- **-** Keys generation: Alice will randomly draw an integer  $s \in \mathbb{Z}_q^*$ , and calculate  $t = g^s$ . *s* is Alice's private key and *t* is Alice's public key.
	- **-** Challenge and answer: Alice computes *c*  $H(g, y, t)$  where *H* is a public hash function and calculates  $r = s - c \cdot x$ . Alice sends to Bob the tuple  $(c, r)$  which corresponds to Alice's signature.
- Verification step: Bob or anyone having access to the public variables  $(g, q, t, H, c, r)$  can verify that *t* is equal to  $g^r \cdot y^c$ , proving that Alice knows and possesses the secret *s*. Bob will authenticate that the secret belongs to Alice when receiving Alice's signature (*c*,*r*). The Fiat-Shamir transformation therefore removes the need of interaction between the provider of a proof and the verifier.

<span id="page-3-6"></span>Both the interactive and non-interactive protocols satisfy all three ZKP fundamental properties:

- Consistency: if anyone using the proposed protocol and verifying the relation *t* is equal to  $g^r \cdot y^c$ , he can be certain that an another claiming to know *x* (like Alice) is true.
- <span id="page-3-7"></span>• Robustness: First, it is not possible for Alice to generate a proof for Bob without possessing her private key (required to produce *s*). Second, it is not possible for Alice to generate a proof for Bob without knowing the secret *x*.
- Zero-knowledge disclosure: Alice's secret *x* is protected as it is not possible for Bob to calculate or access Alice's private key from the public data.

This protocol will be used in [\(IV-C\)](#page-5-0) alongside an original protocol described in the following section.

### <span id="page-3-1"></span>B. ZKP<sub>(a·b)</sub>: CONTEXT AND PROTOCOL DEFINITION 1) 1- CONTEXT

The purpose of the proposed  $ZKP_{(a\cdot b)}$  protocol is for someone to demonstrate that he knows and possesses two secrets, the numbers *a* and *b*. Let *z* be the product of *a* and *b*:

<span id="page-3-3"></span>
$$
a \cdot b = z \tag{2}
$$

<span id="page-3-2"></span>If *a* and *b* are not prime numbers, several pairs satisfying this challenge can be found. As a trivial example, for  $a \cdot b = 12$ , three pairs of possible positive solutions are (2; 6), (3; 4), (1; 12). Let's add a constraint on *a* and *b* that guarantees uniqueness for  $(a,b)$ , and let it be:

<span id="page-3-4"></span>
$$
a + b = z'
$$
 (3)

Let's introduce the integers *e* and *f* as solutions of equations [\(2](#page-3-3)[,3\)](#page-3-4):

$$
\begin{cases}\n a \cdot b = e \cdot f = z \\
 a + b = e + f = z'\n\end{cases}
$$
\n(4)

<span id="page-4-0"></span>As deductions from the constraint in equations  $(4)$  and  $(5)$ :

$$
a + b = e + f \text{ thus } (a + b)^2 = (e + f)^2
$$
  
=  $a^2 + b^2 + 2 \cdot a \cdot b$   
=  $e^2 + f^2 + 2 \cdot e \cdot f$  (6)

$$
a^2 + b^2 = e^2 + f^2 \tag{7}
$$

Considering equation [\(4\)](#page-4-0):

$$
a^{2}-f^{2}+b^{2}-e^{2} = (a+f)(a-f) - (b+e)(e-b)
$$
  
= (a-f)[a+f-b-e]  
= 0 (8)

<span id="page-4-1"></span>Two solution are obtained:

$$
\begin{cases}\n a = f \text{ or } (9) \\
 a + f = b + e \n\end{cases}
$$
\n(9)

Again combining equation  $(5)$  with equation  $(10)$ :

$$
a+f - (a+b) = b+e - (e+f)
$$
  

$$
f - b = b - f
$$
  

$$
b = f
$$
 (11)

Consequently, a unique couple is obtained from equations  $(2,3)$  $(2,3)$  since equations  $(9,11)$  $(9,11)$  give:

$$
\begin{cases}\n a = f \text{ and } b = e \\
 \text{or} \\
 a = e \text{ and } b = f\n\end{cases}
$$
\n(12)

Since *a* and *b* are the secrets that someone wants to prove he knows, let's introduce *m*, *k* and *o* as images of *a* and *b* in order to be able to exchange information without disclosing *a* and *b*. In that spirit, let *m* be an image of a linear combination of *a* and *b*, let *k* be an image of *a* and let *o* be an image of *b*. Similarly as done with equation [\(2\)](#page-3-3) for *a* and *b*, let *Z* be the product of the images *m*, *k* and *o*:

$$
m \cdot k \cdot o = Z, Z \in \mathbb{Z}_q^*
$$
 (13)

<span id="page-4-4"></span>Let's add the following constraints for the triplet (*m*; *k*; *o*):

$$
\begin{cases}\nm + k \cdot o = Z', Z' \in \mathbb{Z}_q^* \\
k + o = Z'', Z'' \in \mathbb{Z}_q^*\n\end{cases} \tag{14}
$$

*Z*, *Z* ′ and *Z* ′′ will be defined as known variables in the protocol. Let's define the images *m*, *k* and *o* as follows:

$$
\int_{L} m = g^{(a+b)^2}
$$
 (16)

$$
\left\{k = g^{-a^2} \tag{17}\right\}
$$

$$
\begin{cases} \rho = g^{-b^2} \end{cases} \tag{18}
$$

<span id="page-4-7"></span>Using equations  $(13, 14, 15)$  $(13, 14, 15)$  $(13, 14, 15)$  $(13, 14, 15)$  $(13, 14, 15)$ , a triplet  $(m; k; o)$  is the image of a couples of two secrets, the couple  $(a, b) \in \mathbb{Z}_q^{(*, +)} \times \mathbb{Z}_q^{(*, +)}$ 

and its inverse. Let's demonstrate this affirmation starting by defining the bijective application  $\Gamma$  as well as the function  $\Lambda$ :

<span id="page-4-9"></span><span id="page-4-8"></span>
$$
\Gamma: \mathbb{G} \to \mathbb{G}, x \to y = g^x \tag{19}
$$

$$
\Lambda: \mathbb{G} \to \mathbb{G}, x \to y = x^2 = (-x)^2 \tag{20}
$$

Let's calculate  $g^{a \cdot b}$  for later use in the protocol. First, using the remarkable identity eq. [\(21\)](#page-4-5),  $g^{2 \tcdot a \tcdot b}$  is obtained in eq. [\(22\)](#page-4-6).

$$
(a+b)^2 = a^2 + b^2 + 2ab
$$
  
\n
$$
\iff 2 \cdot a \cdot b = (a+b)^2 - a^2 - b^2 \qquad (21)
$$
  
\n
$$
g^{2 \cdot a \cdot b} = g^{(a+b)^2 - a^2 - b^2} \qquad (22)
$$

Secondly, choosing  $u = 2^{-1} \in \mathbb{Z}_q^*$  and using equations  $(16,17,18)$  in equation [\(22\)](#page-4-6),  $(m \cdot k \cdot o)$  and  $g^{a \cdot b}$  are related by the following equation:

<span id="page-4-10"></span><span id="page-4-6"></span><span id="page-4-5"></span>
$$
(m \cdot k \cdot o)^u = g^{a \cdot b} \tag{23}
$$

Finally, applying the constraints from eq.[\(13](#page-4-3)[,14,15](#page-4-4)[,19](#page-4-8)[,20\)](#page-4-9) on the triplet  $(m; o; k)$ , the unique solutions of eq.  $(23)$  are the couples  $(a, b) \in \mathbb{Z}_q^{(*,+)} \times \mathbb{Z}_q^{(*,+)}$  and  $(-a, -b) = (q-a, q-b)$  $\in \mathbb{Z}_q^{(*,+)} \times \mathbb{Z}_q^{(*,+)}.$ 

### <span id="page-4-2"></span>2) 2- ZKP(*a*·*b*) PROTOCOL

Based on the above background, let us now define the  $ZKP_{(a\cdot b)}$  protocol itself and start setting up the following variables:

$$
\begin{cases}\ng^{a \cdot b} = \Gamma(a \cdot b) & (24) \\
t = g^s, \text{ with } s \text{ a secretly and randomly drawn integer } (25) \\
m = \Gamma(s + u \cdot \Lambda(a + b)) & (26) \\
k = \Gamma(-\Lambda(a) \cdot u) & (27)\n\end{cases}
$$

$$
t = gs
$$
, with s a secretly and randomly drawn integer (25)

$$
m = \Gamma(s + u \cdot \Lambda(a + b))
$$
 (26)  
\n
$$
k = \Gamma(-\Lambda(a) \cdot u)
$$
 (27)

$$
x = \Gamma(-\Lambda(a) \cdot u) \tag{27}
$$

$$
\begin{cases}\n o = \Gamma(-\Lambda(b) \cdot u) & (28) \\
 c' = H(g, y, (t^{-1} \cdot m)^{u^{-1}} + (k \cdot o)^{u^{-1}}), & (29) \\
 H \text{ a hash function and } y \text{ a given number.} \\
 u = c'/2, \text{ if } c' \text{ is even or } u = (c' + 1)/2 \text{ otherwise} & (30) \\
 c = 2u, c \text{ being the challenge} & (31)\n\end{cases}
$$

$$
c' = H(g, y, (t^{-1} \cdot m)^{u^{-1}} + (k \cdot o)^{u^{-1}}),
$$
\n(29)

*H* a hash function and *y* a given number.

$$
u = c'/2
$$
, if c' is even or  $u = (c' + 1)/2$  otherwise (30)

 $c = 2u$ , *c* being the challenge (31)

<span id="page-4-3"></span>In order for one person, let her be Alice, to demonstrate to another person, let him be Bob, that she knows and possesses two secrets (the numbers *a* and *b*) using the protocol  $ZKP_{(a\cdot b)},$ the below successive steps should be followed:

- Proof generation
	- **–** Objective: Alice wants to prove that she knows *a* and *b* both in the set  $Z_q^*$  such that  $a \cdot b = z$ .
	- **–** Generation of keys: Alice will randomly draw an integer *s* and calculate  $t = g<sup>s</sup>$ . Let *s* the secret and Alice's private key. Alice publishes *t*, her public key.
	- **–** Generation of the challenge and of the answer to the proof to be verified: Alice computes  $c' = H(g, y, (t^{-1} \cdot m)^{u^{-1}} + (k \cdot o)^{u^{-1}})$ , *H* being a public hash function. Alice also calculates the

set  $r = (m, k, o)$ . The couple  $(c, r)$  will represent Alice's signature. Alice sends to Bob  $(c, r)$ . Let's note that Alice cannot only send the variable *r* since Bob will not be able to reconstruct  $c'$  from public variables and *r*.

- **–** Alice publishes  $\sigma_r = k + o$ , the constraint from equation. [\(15\)](#page-4-4).
- Response verification
	- **–** Bob calculates  $\sigma_v = k + o$  from the received set *r* and compares  $\sigma_v$  to the published  $Z''$ .
	- **–** Bob will then verifies with *r* that the product  $m \cdot k \cdot o$ corresponds to  $t \cdot g^{c \cdot a \cdot b}$  which Bob will calculates from public variables *t*, *c*, *g* and the *z* (the product of *a* and *b*).

The  $ZKP_{(a\cdot b)}$  protocol verifies the robustness, consistence and zero knowledge properties of a ZKP protocol:

- consistency: if anyone using the proposed protocol and verifying the relation  $t \cdot g^{a \cdot b \cdot c}$  is equal to  $m \cdot k \cdot o$  and  $k + o$ is equal to  $\sigma_r$ , he can be certain that an another person claiming to know (*a*, *b*) is sincere.
- robustness: First, it is not possible for Alice to generate a proof for Bob without knowing two elements from the list  $[a^2, b^2, (a+b)^2]$ . Second, it is not possible for Alice to calculate *c* without knowing *u* which is required to calculate *c* ′ .
- zero-knowledge disclosure: Alice's secret *x* is protected as it is not possible for Bob to calculate or access Alice's private key from public data.

The  $ZKP_{(a\cdot b)}$  protocol ensures that it is impossible for Bob, knowing  $r = (m, k, o)$  and *c*, to retrieve Alice's secrets *a* and *b* with currently available algorithms since:

- <span id="page-5-7"></span>• Bob cannot find the secret resolving  $y = g^{x^2}$ , indeed
	- **-** finding *x* <sup>2</sup> has the complexity of the discrete logarithm problem [\[25\].](#page-10-4)
	- **--** finding *x* the modular root of  $x^2$  has nonpolynomial complexity. [\[26\]](#page-10-5)
- Bob cannot find the secret *a* and *b* resolving  $y = g^{a \cdot b}$ , indeed
	- **-** finding *a* · *b* has the complexity of the discrete logarithm problem.
	- **-** finding *a* and *b* knowing  $z = a \cdot b$  is difficult [\[27\].](#page-10-6)

### <span id="page-5-0"></span>C. PROOF OF KNOWLEDGE AND AUTOMORPHISMS 1) 1- CONTEXT

Automorphisms properties can be exploited to create a chain of (proofs, verifications) containing several assertions.

To this aim, let's first define a group *G* generated by *g* of order *q*, *q* being a prime number:

$$
\forall y \in G, \exists i, 0 \le i \le q - 1 \text{ such that } y = g^i \tag{32}
$$

Let *Aut*(*G*) be the set of automorphisms of *G*. From group theory,  $Aut(G)$  will:

- be isomorphic to  $Z_q^*$ .
- have, if  $\zeta \in Aut(G)$ ,  $\zeta(g)$  also a generator of *G*.

• have exactly  $q - 1$  generators, *G* is of order *q*.

*Aut*(*G*) having  $q - 1$  generators,  $q - 1$  assertions can be linked together using  $H_i$  and  $g_i$  defined as follows:

$$
\mathbb{G} \to \mathbb{G} \ g \to H_i(g) = g_i = g^{p_i}
$$
  
with  $0 \le i \le q - 1$  and  $0 \le p_i \le q - 1$  (33)

Consequently, a successive composition of  $w$  ( $1 \le i \le w$ ) automorphisms  $H_i$  can be written as:

$$
LH_w(g) = H_w \circ H_{w-1} \circ \cdots \circ H_1(g) = g^{p_1 \cdot p_2 \cdots \cdot p_w} \qquad (34)
$$

Assuming  $y = g^n$ , equation [\(34\)](#page-5-1) becomes:

<span id="page-5-4"></span><span id="page-5-3"></span><span id="page-5-2"></span><span id="page-5-1"></span>
$$
LH_w(y) = g^{n \cdot p_1 \cdot p_2 \cdots p_w} \tag{35}
$$

With  $0 \le i \le w$ ,  $a = p_1 \cdot p_2 \cdot \cdots \cdot p_w \mod q$  and  $b = n$ , equations [\(33,](#page-5-2)[35\)](#page-5-3) become:

$$
LH_w(y) = g^{a \cdot b}, \exists i \in [1, q] \text{ such that, } g^b = g_i. \tag{36}
$$

To put it in a nutshell, automorphisms offer the possibility to use equation  $(36)$  similarly as in:

• a ZKP Schnorr knowing one secret *a*:

$$
(36)LH_w = LH_w(g) = g_i^a \tag{37}
$$

•  $ZKP_{a,b}$  knowing two secrets *a* and *b*:

$$
(36)LH_w^n = g^{a \cdot b} = LH_w(y) \tag{38}
$$

### <span id="page-5-6"></span>2) 2- MULTI-STEP PROOF OF KNOWLEDGE

Combining a ZKP Schnorr, the proposed ZKP*a*·*<sup>b</sup>* and making use of automorphisms, let's build a multi-step proof of knowledge enabling to prove a set of  $w$  secrets  $p_i$  and an other secret *n* (which can be delivered for example by a trusted third party).

Let's define the following sequence  $x_i$  based on the secrets *p<sup>i</sup>* and initialised with *n*:

$$
x_i = x_{i-1} \cdot p_i, \ x_0 = n, LH_i(y) = g^{x_i} \tag{39}
$$

<span id="page-5-8"></span>Using the  $ZKP_{a\cdot b}$  protocol when proving each secrets  $p_i$  and *n*, *a* and *b* will be identified as:

<span id="page-5-5"></span>
$$
a_i = x_{i-1} \text{ and } b_i = p_i. \tag{40}
$$

<span id="page-5-9"></span>However, it is not possible for  $a_i$  and  $b_i$  to remain secrets while using the  $ZKP_{a_i \cdot b_i}$  protocol successively to prove each secrets  $p_i$  and  $n$ . Indeed, let's consider the following secrets verification for *n*,  $p_1$  and  $p_2$ :

Step (1) - ZKP<sub>a<sub>1</sub></sub>,<sub>b<sub>1</sub></sub>: 
$$
x_1 = a_1 \cdot b_1 = n \cdot p_1
$$
  
\nStep (2) - ZKP<sub>a<sub>2</sub></sub>,<sub>b<sub>2</sub></sub>:  $x_2 = a_2 \cdot b_2 = n \cdot p_1 \cdot p_2$   
\n $\implies p_2 = x_2/x_1$  (41)

From equation [\(41\)](#page-5-5) for successive steps  $ZKP_{a_i \cdot b_i}$ , a verifier will be able to guess  $p_2$  (using  $x_2$  which is public), invalidating the Zero Knowledge Protocol.

In order to resolve this issue and keep  $p_i$  as secrets during verification, let's redefine the second step in equation [\(41\)](#page-5-5):

Step (1) - ZKP<sub>a1·b1</sub> : 
$$
x_1 = n \cdot p_1
$$

Step (2) - ZKP<sub>a3</sub>.<sub>b3</sub>: 
$$
x_3 = x_2 \cdot p_3
$$
  
 $\implies p_3 = x_3/x_2$  (42)

Using the procedure in equation  $(42)$  where  $x_2$  is kept as a secret, a verifier cannot find the secret *p*3.

Considering the following ZKP verification steps:

Step (1) -  $ZKP_{a_1 \cdot b_1}: a_1 = x_0; b_1 = p_1$ Step (2) -  $ZKP_{a_2 \cdot b_2}$ :  $a_2 = x_1$ ;  $b_2 = p_2$  with  $x_1 = x_0 \cdot p_1$ Step (3) -  $ZKP_{a_3 \cdot b_3}$ :  $a_3 = x_2$ ;  $b_3 = p_3$  with  $x_2 = x_1 \cdot p_2$ (43)

From equations  $(41, 42)$  $(41, 42)$  $(41, 42)$ , verifying Step  $(i - 1)$  and Step  $(i+1)$  to prove knowing the secrets  $(x_{i-2}; p_{i-1})$  and  $(x_i; p_{i+1})$ , is enough to avoid a  $ZKP_{a_i \cdot b_i}$  verification at Step (*i*) to prove the secret  $p_i$ . Indeed, looking at equations [\(43\)](#page-6-2), after  $ZKP_{a_1 \cdot b_1}$ and  $ZKP_{a_3\cdot b_3}$  verification for Steps [\(1\)](#page-3-2) and [\(3\)](#page-3-4),  $ZKP_{a_2\cdot b_2}$ verification for Step [\(2\)](#page-3-3) will be already completed since:

Step (1) 
$$
\implies
$$
  $(x_0; p_1)$  proved: $a_2 = x_1 = x_0 \cdot p_1$   
Step (3)  $\implies$   $x_2$  proved: $b_2 = p_2 = x_2/x_1$  (44)

As a consequence, after verifying  $ZKP_{a_{i-1}\cdot b_{i-1}}$  for  $p_{i-1}$ ,  $ZKP_{a_i \cdot b_i}$  verification for  $p_i$  can be skipped by verifying ZKP<sub>*a*<sub>*i*+1</sub></sub> *b*<sub>*i*+1</sub> for  $p_{i+1}$  which will also provide a proof for  $p_i$ .

Assuming  $x_i = x_{i-1} \cdot p_i$ ,  $x_0 = n$  and  $LH_i(y) = g^{x_i}$ , with  $a_i = x_{i-1}$  and  $b_i = p_i$  for each step, the verification step for ZKP*a*·*<sup>b</sup>* becomes:

$$
m_i \cdot k_i \cdot o_i = t \cdot LH_i(y)^c \tag{45}
$$

Let's note that a procedure could disclose  $LH_i$  for the verification step instead of  $z = a \cdot b$ . Such a disclosure would avoid the successive  $ZKP_{a\cdot b}$  problem shown in equation [\(41\)](#page-5-5). An example for the proposed SSI using a ZKP Schnorr and a  $ZKP_{a\cdot b}$  is presented in the next section.

### <span id="page-6-0"></span>**V. ILLUSTRATIVE EXAMPLE OF THE SSI-PROTOCOL**

### A. SCENARIO

Alice wishes to present proof of existence of a set of documents without revealing their content to Bob. As an illustration for a SSI-Protocol, a first document of the set of documents would be a proof of identity. One could think of numerous applications for proofs of existence of documents to be linked to an identity such as for example a proof of social security or a proof of solvency with a bank. In the application described in [\[28\] m](#page-10-7)otivating this research, a student would link a proofs of existence for his student identity, working permit or social security number to proofs he would have aggregated in his wallet for his university degrees or verifiable credentials certifying his skills.

<span id="page-6-4"></span>For such applications linking a proof of existence of an identity to proofs of existence of other documents, two steps are required. In the first step presented in the next section, a protocol generating a Self-Sovereign Identity using a ZKP Schnorr is proposed. In the following section, the introduced  $ZKP_{a\cdot b}$  protocol is exploited to link proofs of existence of other documents to the generated Self-Sovereign Identity.

<span id="page-6-3"></span><span id="page-6-2"></span><span id="page-6-1"></span>

**FIGURE 1.** SSI generation protocol using a ZKP Schnorr: generation (1,2), registration [\(3\)](#page-3-4) and verification (4,5,6) of a Self-Sovereign Identity.

### B. INITIALISATION STEP

In order for Alice (A) to generate a Self-Sovereign Identity, register it on a blockchain through the State (S) and for Bob (B) to verify Alice's identity, the following steps (illustrated by figure  $(1)$ ) are proposed:

- generation of a unique and robust couple  $(x, y)$  representing Alice's identity:
	- **-** A chooses an *x* that she will keep as a secret in the group  $\mathbb{Z}_q^*$  and calculates  $y = g^x$  using *g* a generator of the group  $G$  of order  $q$ ,  $q$  being a prime number. It is assumed here that *g* and *q* have been already defined, that they are public variables and therefore known by all protocol stakeholders.
	- **-** A sends the couple (*x*, *y*) to S. It is assumed that any communication in the proposed protocol is encrypted.
	- **-** S checks if the received *x* has not already been taken and that it is difficult to find *x* knowing *y* and *g*. *S* should ensure that both sizes of *x* and *y* must respect the minimum size to resist attacks and the maximum size to be supported by the verification algorithms.
	- S then sends back the couple  $(y', H_S)$  to A as confirmation,  $H<sub>S</sub>$  being the hash function used to calculate  $y' = g^{H_S(x)}$ .
- registration of information related to Alice's generated by the state identity on the blockchain:
	- **-** using a ZKP Schnorr, S generates *H* and *r* which are information related to Alice's generated identity as well as *t*, the state public key, and *T* which combines information from Alice's public key, the state public key and  $y'$  (an image of  $H_S(x)$ ).
	- **-** through a smart contract and signing with *t*, S registers  $(H, r, g, q, T)$  on a blockchain.

<span id="page-7-0"></span>

**FIGURE 2.** Different blockchain services from a single blockchain or multiple blockchains, Derived Step: Top (several steps); Bottom (single step).

- verification of Alice's claim by Bob
	- **-** A proves to B that she knows *x* by providing B with the couple (*y*;*y* ′ ), related to the existence of Alice's identity, as well as the set  $(H', r', g, q, t')$  related to the proof of knowledge by A of *x* using a Schnorr's ZKP.
	- **-** Bob, following the verification step of the Schnorr protocol, goes on the blockchain and checks whether *T* is such that  $T \cdot y'^{-1} = t \cdot t'$  (*t* · *t'* being the multiplication of the public keys of *S* and *A*).

Bob has then checked whether Alice's unique couple  $(x,y)$ is indeed a couple approved and assigned by *S* to *A* without knowing *x*.

### C. ZKP<sub>a</sub>.<sub>b</sub> PROTOCOL: LINKING PROOFS OF EXISTENCE OF DOCUMENTS

In order to link a proof of existence of the previously generated Self-Sovereign Identity using a ZKP Schnorr to proofs of existence of other documents registered in separated blocks and on different blockchains services, the proposed ZKP*a*·*<sup>b</sup>* protocol can be employed.

Let's assume that Alice wants to link  $\kappa$  proofs on separated blocks belonging to different blockchains (or chains) using the  $ZKP_{a\cdot b}$  protocol and that a first step for generating a Self-Sovereign Identity with a ZKP Schnorr has been completed. Alice initialises the ZKP*a*·*<sup>b</sup>* protocol randomly generating  $(p_1, p_2, \dots, p_k)$  secrets. Alice then calculates the following variables:

$$
g_i=g^{p_i}
$$

$$
f_{\rm{max}}
$$

$$
x_0 = n = H_S(x)
$$
  
\n
$$
x_i = x_{i-1} \cdot p_i
$$
  
\n
$$
y' = g^n
$$
  
\n
$$
LH_i = LH_i(y') = g^{x_i}
$$

Finalising the  $ZKP_{a,b}$  initialisation step described in the previous section, Alice will save  $g_i$  and  $LH_i(y')$  on the blockchain $_{\#i}$  (or on the chain $_{\#i}$ ). As illustrated by figure [2,](#page-7-0) Alice can now generate proofs for  $p_i$  either in several steps or in one single step.

1) *Several steps proof for p<sup>i</sup>*

For each step *i*, Alice sets up a ZKP<sub>*a*</sub>·*b* with ( $a = x_{i-1}$ ,  $b = p_i$ ) and saves  $\sigma_i$  in a block<sub>*i*</sub>.

Let's note that the challenge for Alice is  $c_i = 2u$ ; this challenge enables the  $ZKP_{a\cdot b}$  protocol to be noninteractive.

Alice then sends to Bob:

$$
\begin{cases}\n[(r_1, c_1); (r_2, c_2); \dots; (r_k, c_k)]\n\\
r_i = (m_i, k_i, o_i)\n\\
c'_i = H(g, LH_i, (t^{-1} \cdot m_i)^{u^{-1}} + (k_i \cdot o_i)^{u^{-1}})\n\end{cases}
$$

Bob can now calculate *u*:

$$
\begin{cases}\n u = c'_i/2, \text{ if } c'_i \text{ is even, OR} \\
 u = (c'_i + 1)/2 \text{ otherwise.} \n\end{cases}
$$

For verification, Bob checks whether both following conditions are met:

$$
\begin{cases} k_i + o_i = \sigma_i \\ m_i \cdot k_i \cdot o_i = t' \cdot LH_i^{c_i} \end{cases}
$$

Using the  $ZKP_{a\cdot b}$  properties mentioned in [IV-C2,](#page-5-6) Bob can perform verification every two steps.

2) *Single step proof for p<sup>i</sup>*

From a ZKP Schnorr where she proved she knows *n*, Alice obtained  $(H, r', g, q, t')$ . From a ZKP<sub>a</sub><sub>*b*</sub> where she proves she knows  $a = n \cdot m_{k-1} \cdot \cdot \cdot \cdot m_1$  and  $b = m_k$ , Alice obtained  $(m_k, k_k, o_k)$ .

Alice then calculates and registers on a blockchain the values of  $y' = g^n$ ,  $\sigma_k$  and  $LH_k$ .

Let's note that the challenge for Alice is  $c = c_k$  = *H*(*g*, *y*',  $(t^{-1} \cdot m_i)^{u^{-1}} + (k_i \cdot o_i)^{u^{-1}}$ ); this challenge enables the ZKP*a*·*<sup>b</sup>* protocol to be non-interactive. Alice then sends to Bob:

$$
\begin{cases}\n(m_K, k_K, o_K) \\
(H, r', g, q, t')\n\end{cases}
$$

For verification, Bob checks whether both following conditions are met:

$$
\begin{cases} k + o = \sigma_k \\ m_k \cdot k_k \cdot o_k = g^{r'} \cdot (y' \cdot LH_k)^{c_k}, t' = g^{r'} \cdot y'^{c} \end{cases}
$$

Let's note that for the application described in [\[28\] m](#page-10-7)otivating this research, the objective is to contribute to a matchmaking process enabling recruitment based on certified skills

<span id="page-8-1"></span>

**FIGURE 3.** Impact of the number of proofs on generation and verification time.

regardless of other personal information in order to be as inclusive as possible. Using ZKP*a*·*<sup>b</sup>* to combine proofs for student identity, working permit or social security number with credentials for skills or diploma would help fulfilling such an objective, provided that skills or diploma data which would have to be disclosed for matchmaking purposes will be stripped from personal data.

### <span id="page-8-0"></span>**VI. EVALUATION**

### A. SSI-ZKP<sub>a</sub>.b COMPARISON

A protocol for an SSI embedding the proposed ZKP*a*·*<sup>b</sup>* being an original contribution, a technical analysis emphasizing its design advantages is given below. The proposed SSI design's performance will be compared using the same methodology and criterias (C. Allen's & provability) as well as against the same list of existing SSI as in [\[3\] and](#page-9-2) [\[12\].](#page-9-11)

- Portability: Information and services about identity should be transportable. Our solution implements this criteria using ZKP since information and services related to identity are condensed into a proof key and a verification key. The two keys can have sizes between 600 bits and 1024 bits depending on the level of security desired. Each user can keep his identity without needing a trusted third party validating the viability of his identity.
- Interoperability: Several aspects may reduce interoperability. However, scalability is the only relevant criteria for the proposed protocol. The two proposed ZKP in this article are scalable as the generation and/or verification of proofs is executed in a decentralized way. As a consequence, scalability depends only on the number of proofs that a user will generate. The time necessary for the generation and/or verification steps is polynomial. The proposed SSI ZKP is a generic protocol and it is not limited to particular use cases.
- Minimalization: In the proposed SSI using ZKP protocols, shared information is limited to the ones related to the verification of the prover's response. The State is the only trusted (and legitimate) third party introduced in the primary step while certificating the sovereign identity. For any other identity/identifier/attribute, no

information other than that of the proof/verification key is shared.

- Protection: The user has full control over his Zero Knowledge Proofs generation using his private key thereby protecting his full ownership rights over his identities.
- Provable: The two ZKP are verifiable within a finite and reasonable time (cf. figure [3\)](#page-8-1).

The proposed SSI-ZKP*a*·*<sup>b</sup>* checks all 11 criteria (C. Allen's and provablity) and differs from other SSIs checking the 11 criteria by using the two proposed ZKP, thereby addressing the proposed challenge of this article.

### B. ZKP<sub>a</sub>.<sub>b</sub> SIMULATIONS AND COMPARISON

<span id="page-8-2"></span>An feasibility study is supplied based on ZKP*a*·*b*'s protocol simulation results obtained from a python implementation. The implementation source code is published alongside this article and referenced here as ZKP*a*·*<sup>b</sup>* python implementation repository [\[29\]. S](#page-10-8)imulations have been run for a cyclic group  $\mathbb{Z}_q^{(*,+)}$ , *q* being a prime number chosen such that  $q = 2 \cdot q' +$  $1$  with  $q'$  a prime number. In order to comply with current security requirements, simulations have been carried out with a size for *q* of 1024 bits.

<span id="page-8-3"></span>Considering that studies similar to [\[30\] fo](#page-10-9)r ZKP Schnorr protocol evaluation are available in the literature, the feasibility study has been delimited to the  $ZKP_{a\cdot b}$  protocol. More specifically, the feasibility study has been delimited to  $(m, k, o)$  generation and verification both for the multi-step (a) and the single-step (b) proofs. As summarized in Figure [3,](#page-8-1) an evaluation of the SSI-ZKP $_{a,b}$  is provided for the protocol's time complexity as a function of the number of proofs to be generated and verified. In Figure [3 \(a\)](#page-8-1) concerned with multistep proofs, time complexity is shown to increase linearly with the number of proofs to be generated and to be constant with the number of proofs to be verified. In Figure  $3$  (b) concerned with single-step proofs, time complexity is shown to be constant with both the number of proofs to be generated and validated.

As highlighted in [\[18\], d](#page-9-18)ifferent ZK-proof systems are based on different cryptographic assumptions and designed



#### <span id="page-9-21"></span>**TABLE 1.** Qualitative comparison of the proposed ZKP<sub>a</sub>.b protocol against other ZK-proof algorithms as proposed in [\[18\].](#page-9-18)

\* hPKC : homomorphic Public-Key Cryptography ; \*\* DLP : Discrete Logarithm Problem ; \*\*\* IP : Interactive Proofs based ; \*\*\*\* ; MPC : secure Multi-Party Computation ; \*\*\*\*\* ; IVC : Incrementally Verifiable Computation [18].

for different computational systems. Acknowledging such limitations, a qualitative comparison for the proposed ZKP*a*·*<sup>b</sup>* protocol against other ZK-proof algorithms is provided in Table [1.](#page-9-21) Simulations of  $ZKP_{a\cdot b}$ 's scalability for the prover,  $(m, k, o)$  generation, as well as for the verifier,  $(m, k, o)$  verification, have been reported in Figure [3](#page-8-1) while investigating time complexity as a function of the number of proofs to be emitted and verified. Concerning ZKP*a*·*b*'s transparency, this criteria has been implemented within the proposed exchange protocol of private keys and use of hash functions.

### <span id="page-9-12"></span>**VII. CONCLUSION**

An original Self-Sovereign Identity solution with Zero Knowledge Proof based on blockchain technologies is proposed. This solution is compliant with eIDAS, GDPR as well as criteria introduced by C. Allen and complemented by Q. Stokkink and J. Pouwelse with provability. A ZKP protocol enabling to prove that *a* and *b* such that  $a \cdot b = n$ are known without revealing *a* and *b* has been proposed. Automorphism sequence linking different identities, identifiers and/or attributes as well as different blockchain information has been designed linking the ZKP Schnorr and an original ZKP protocol named ZKP*a*·*b*. The properties of group automorphisms have been exploited to create a link between these two ZKP protocols in order to link multiple SSI in a single SSI. In this proposal the creation/acquisition of identity and the management of identifiers/attributes are entirely managed by the user and at most one trusted third party. Once implemented, such a method will minimize the amount of information shared and facilitate the regulation of identification/authentication by allowing only relevant actors to access relevant information.

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