IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 3 April 2023, accepted 11 April 2023, date of publication 17 April 2023, date of current version 21 April 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3267434

RESEARCH ARTICLE

Fitness Dependent Optimizer Based Computational Technique for Solving Optimal Control Problems of Nonlinear Dynamical Systems

GHULAM FAREED LAGHARI[1], SUHEEL ABDULLAH MALIK[1], IRFAN AHMED KHAN[2], AMIL DARAZ[3], SALMAN A. ALQAHTANI[4], AND HAYAT ULLAH[5]

[1]Department of Electrical and Computer Engineering, Faculty of Engineering and Technology, International Islamic University Islamabad (IIUI), Islamabad 44000, Pakistan
[2]Department of Electrical Engineering, Faculty of Engineering, Universiti Malaya, Kuala Lumpur 50603, Malaysia
[3]School of Information Science and Engineering, NingboTech University, Ningbo 315100, China
[4]Department of Computer Engineering, College of Computer and Information Sciences, King Saud University, Riyadh 11543, Saudi Arabia
[5]Department of Electrical and Computer Engineering, Tandon School of Engineering, New York University, New York, NY 11201, USA

Corresponding author: Ghulam Fareed Laghari (ghulam.phdee1@iiu.edu.pk)

This work was supported by King Saud University, Riyadh, Saudi Arabia, through the Research Supporting Project, under Project RSPD2023R585.

ABSTRACT This paper presents a pragmatic approach established on the hybridization of nature-inspired optimization algorithms and Bernstein Polynomials (BPs), achieving the optimum numeric solution for Nonlinear Optimal Control Problems (NOCPs) of dynamical systems. The approximated solution for NOCPs is obtained by the linear combination of BPs with unknown parameters. The unknown parameters are evaluated by the conversion of NOCP to an error minimization problem and the formulation of an objective function. The Fitness Dependent Optimizer (FDO) and Genetic Algorithm (GA) are used to solve the objective function, and subsequently the optimal values of unknown parameters and the optimum solution of NOCP are attained. The efficacy of the proposed technique is assessed on three real-world NOCPs, including Van der Pol (VDP) oscillator problem, Chemical Reactor Problem (CRP), and Continuous Stirred-Tank Chemical Reactor Problem (CSTCRP). The final results and statistical outcomes suggest that the proposed technique generates a better solution and surpasses the recently represented methods in the literature, which eventually verifies the efficiency and credibility of the recommended approach.

INDEX TERMS Bernstein polynomials, dynamical systems, fitness dependent optimizer, genetic algorithm, nonlinear optimal control problems, optimization problem, optimization techniques.

I. INTRODUCTION

The dynamical systems can be represented by the mathematical model designed in the form of an optimization problem, for instance, Optimal Control Problems (OCPs). The OCPs include dynamic optimization problems which demand complex mathematical operations and contain enormous practical significance and industrial applications in nearly all branches of science, i.e., robotics, aeronautics, plasma physics, chemical engineering, etc. Due to the sophisticated nature of OCPs, achieving the optimal solution numerically could be relatively tedious [1], [2].

Several numerical methods in the literature were proposed by distinguished researchers with considerable attention toward OCPs containing nonlinear and dynamical characteristics to determine their optimal solution and improve the quality of existing techniques [2], [3]. The following papers highlight a few of such OCPs with applicable numerical schemes where Ratković [4] identified for OCPs certain limitations of indirect and direct methods

The associate editor coordinating the review of this manuscript and approving it for publication was Frederico Guimarães.

by applying them to two endogenous growth models. Stryk and Bulirsch [5] presented a precise list of generally applied direct and indirect approaches to achieve the numerical solution of different OCPs. The authors suggested a hybrid approach to adequately overcome two deficiencies, i.e., less accuracy and convergence areas, for problems like the Brachistochrone Problem and the Apollo Reentry Problem. Wu et al. [6] devised a computational approach using the control parametrization scheme and gradient-based optimization techniques for solving OCPs having diverse time delays and canonical inequality and equality constraints. The authors ultimately yielded more effective approximate numerical results than conventional techniques. Alipour et al. [7] tailored a hybrid scheme using a combination of the Homotopy Analysis Method (HAM) and discretization techniques. The authors applied this technique to various NOCPs, comparing the obtained approximate solution with the traditional methods and verifying the applicability of the proposed hybrid approach. Jia et al. [8] employed Optimal HAM (OHAM), solving varied linear OCPs with quadratic performance index, for instance, a linear scalar time-invariant system. The authors attained the approximated solution and represented the verification for the validity of this scheme by the results of comparison with other techniques. S. Ganjefar and S. Rezaei [9] suggested a hybrid scheme, consolidating the Homotopy Perturbation Method (HPM) and the Padé technique (HPM-Padé method). The authors enhanced the accuracy and broadened the convergence domain of the approximate analytical solution for the concerned OCPs. Nazemi et al. [10] utilized a robust and efficient approximate method, namely the Differential Transform Method (DTM), constructively solving a class of NOCPs. The authors acquired the approximate solutions of several nonlinear differential equations without the requirement for linearization or discretization processes. Xuesong et al. [11] presented a spectral method based on Galerkin Approximation and Chebyshev Polynomials (GACP) for NOCPs and demonstrated its effectiveness via some numerical experiments, which provided a more stable approximate solution than the compared techniques, whereas Dehghan [12] designed an iterative numerical procedure that is dependent on State Parametrization (SP) and Cardan Polynomials (CP) for competent approximation of numerous OCPs. Mirinejad and Inanc [13] introduced a direct numerical technique for OCPs which is based on Radial Basis Functions (RBFs), namely, the RBF collocation method, discretized the OCPs, and transcribed them into some Nonlinear Programming (NLP) problems. The authors applied the proposed scheme to the Brachistochrone Problem and the Unmanned Aerial Vehicles (UAVs) navigation problem, achieving better approximate results than the Legendre pseudospectral method and B-spline technique, whereas Moghaddam et al. [14] recommended a direct numerical approach to accurately deal with OCPs via Genocchi polynomial basis, transforming the OCP into an NLP problem. The authors examined the performance and accuracy of the proposed methodology and implemented it on five different OCPs, including the Breakwell Problem. El-Kady [15] exploited divergent numerical techniques, e.g., Legendre approximation, Penalty Partial Quadratic Interpolation (PPQI), transforming OCPs represented as Ordinary Differential Equation (ODE) into equivalent constrained optimization problems and eventually ascertaining the numerical results executed. Edrisi-Tabri et al. [16] generated the approximate analytical solution for diverse nonlinear constrained quadratic OCPs by adopting linear B-spline functions. Cichella et al. [17] implemented BPs on various nonlinear constrained OCPs after rigorous analysis of their peculiarities and rendered the numerically accurate and computationally appropriate approximate results, whereas Ahmed and Ouda [18] elaborated on the effectiveness of Boubaker Polynomials with the aid of theoretically noticed rapid convergence rates for Quadratic OCPs (QOCPs) and compared the optimum solution achieved with some other approximation techniques.

The NOCPs demand high-quality approximate solutions, which could not be achieved solely by available numerical methods because of their limitations and drawbacks. Recently, nature-inspired optimization techniques have emerged as a promising approach to operating NOCPs substantially better than conventional numerical schemes. These optimization algorithms exhibit tremendous applications in the industrial and scientific domains [19]. The nature-inspired optimization techniques encompass a plethora of advantages, including simple conceptualization, lucid mathematical calculations, derivative-free, and the ability to deftly grapple with the intricacies of genuine engineering problems and continuous-time systems par excellence [20].

To name just a few such techniques, GA proficiently tackled the trajectory tracking problem for some nonlinear systems [21]. Particle Swarm Optimization (PSO) meticulously handled the tuning of the Artificial Neural Network (ANN) controller for nonlinear systems [22]. Differential Evolution (DE) tactfully addressed the OCPs utilized for the power flow in a complex structure of Microgrids (MGs) [23]. Artificial Bee Colony Optimization (ABCO) splendidly achieved an optimal solution for a nonlinear inverted pendulum and ingeniously produced a linear quadratic optimal controller design [24]. Cuckoo Search Algorithm (CSA) demonstrated its diverse characteristics by providing justifiable accuracy measures and convergence rates when applied to several engineering design optimization problems, e.g., pressure vessel design and compression/tension spring design [25].Firefly Algorithm (FA) proved its significantly powerful optimization capability by solving the nonlinear dispatch problem and providing the optimum solution for multi-generation systems [26], [27], [28]. Teaching Learning Based Optimization Algorithm (TLBOA) outperformed conventional numerical methods by competently solving various complex optimization problems, including the Optimal Power Flow (OPF) problem for the minimization of dual objective functions [29].

FDO demonstrated its capability by favorably dealing with the multi-source Interconnected Power System (IPS) via the implementation of a prolific Automatic Generation Control (AGC) [30], [31]. Further, FDO has also been utilized for the optimization of various NOCPs, and the solutions generated were validated by comparative and statistical analyses [32]. Other equivalent algorithms are sufficient to unravel the prevailing strength of multifaceted nature-inspired optimization techniques, yielding global optimal results with swift convergence rates compared to miscellaneous formerly designed schemes practiced on distinctly authentic optimization problems.

Despite verifying the capabilities of individual nature-inspired optimization algorithms at each stage for determining the optimal solution of several nonlinear problems, it is worth mentioning that such techniques occasionally require the incorporation of other methods to enhance the quality of the solution. An advanced technique, entitled the hybridization approach, originated for resolving numerous complex real-world optimization problems. The hybrid approaches sometimes apply local search techniques, optimizing the performance of various algorithms reliant on the population, ameliorating the quality of previously obtained results, and further dampening the processing time. Eventually, such immaculately designed hybrid techniques proved to be a remarkable strategy for the refinement of the solution accomplished solely by evolutionary optimization algorithms.

The incredible problem-solving attributes pertaining to hybridized optimization techniques could be verified by investigating the below mentioned citations where Ma [33] introduced an optimal control technique for a Whole Network Control System (WNCS) and utilized the eclectic ensemble of GA, Neural Network (NN), and fuzzy control, incorporating the benefits of excellent self-learning capacity of NN with powerful global search capability of GA. Haghighi et al. [34] proposed a hybrid architecture by integrating GA and PSO for optimal path planning problems of diversified UAVs amid coverage missions. Malik et al. [35] recommended a heuristic scheme based on GA for numerically solving the nonlinear dynamical system of the generalized Burgers'-Fisher equation. Mahfoud et al. [36] presented a hybrid structure with a fusion of controller, i.e., Proportional Integral Derivative (PID) and Ant Colony Optimization (ACO) algorithm optimizing PID controller gains pertaining to Direct Torque Control (DTC) for Doubly Fed Induction Motor (DFIM). Stodola et al. [37] executed a progressive hybrid approach with a unique combination of ACO and Simulated Annealing (SA) to significantly outperform other existing metaheuristic algorithms applied to the Dynamic Traveling Salesman Problem (DTSP). Khadanga et al. [38] consolidated Modified Grey Wolf Optimization (MGWO) and CSA (MGWO-CSA) for Load Frequency Control (LFC). The authors finally applied the hybrid algorithm-based load frequency controller on a hybrid power system with multiple areas and sources for its robustness and effectiveness verification. Taeib et al. [39] formulated an innovative Model Predictive Control (MPC) technique for nonlinear systems by combining Takagi-Sugeno (TS) fuzzy models and constrained CSA. The authors validated the efficacy of the hybridization scheme by executing it on a Three Tank System (TTS). Alghamdi [40] implemented a hybrid evolutionary technique entailing FA and Jaya Algorithm (JA), i.e., HFAJAYA, providing the optimal solution for single and multiple objective functions OPF problems of power systems. El-Shorbagy et al. [41] offered a hybrid scheme combining the strengths of GA and FA metaheuristic algorithms, i.e., HGAFA. The authors utilized the HGAFA technique to solve multitudinous Engineering Design Problems (EDPs) with better convergence rates and reduced computational intricacies. Dastan et al. [42] featured a distinctive compilation of optimization algorithms, i.e., Hybrid TLBOA and Charged System Search (HTC) algorithm, providing the optimal solution for numerous complex engineering and mathematical optimization problems, e.g., 72, 120, 244, and 942-bar truss structure design optimization problems. Ali et al. [43] employed the Dandelion Optimizer (DO) for the optimization of LFC of the IPS. Abbas et al. [44] achieved the optimum values of the weights and biases of NN, reduced the Mean Square Error (MSE) for relevant optimal problems, and effectively employed a hybridization scheme merging FDO and Multi-Layer Perceptron (FDOMLP). Chiu et al. [45] rendered an innovative hybridization approach by amalgamating FDO and the Sine Cosine Algorithm (FDO-SCA). The authors efficiently provided improved performances and rapid convergence speed for numerous Benchmark Functions (BFs).

In this paper, a hybrid scheme is presented, which is composed of a contemporary nature-inspired optimization algorithm, i.e., FDO, and an eminent approximation technique, namely, BPs, to procure precisely the optimal solution of various real-world NOCPs. The proposed approach approximates divergent NOCPs by utilizing a linear combination of Bernstein basis polynomials containing unrecognized parameters. An objective function is devised by an adequate modification of a dynamical system under consideration to some essentially equivalent optimization problem. The best values of unknown parameters are yielded by exploiting GA and FDO, which effectually solve the nonlinear optimization problem(s). Besides, the efficacy of the recommended hybrid scheme is quantified by comparison with the formerly designed techniques and significantly minimized Absolute Error (AE) values. Finally, the statistical analysis is presented to establish the robustness and proficiency of the proposed hybrid approach.

The main contributions of the proposed hybrid scheme are mentioned as follows:

1. A stochastic hybrid computational technique designed for numerically solving distinct real-world OCPs of nonlinear dynamical systems.
2. The hybridization of BPs with metaheuristic algorithms, i.e., FDO and GA, to effectively attain the optimal solution.

3. The approximation of NOCPs by a simple and efficient approach hybridizing BPs with evolutionary algorithms, i.e., FDO-BP and GA-BP.
4. The formulation of FDO-BP and GA-BP based fitness functions for obtaining the optimum results.
5. The performance assessment of the suggested hybrid methodology via extensive comparison with previously designed techniques.
6. The validation of the robustness of the proposed method by evaluating a detailed statistical analysis.

This article is organized into several crucial sections, as follows: Section II elaborates on a general mathematical description of OCPs. Section III discusses the working mechanism of the nature-inspired optimization algorithms and BPs utilized for the current study. Section IV introduces the recommended technique. Section V illustrates the results accomplished by implementing the proposed approach. Section VI presents the statistical analysis conducted for the suggested method, followed by the conclusion and future directions concerning such research fields in Section VII.

II. OPTIMAL CONTROL PROBLEMS

The dynamical system represents a mathematical model defining the temporal evolution of a particular engineered system over time. The mathematical model of such control systems can be designed by utilization of manifold optimization problems, e.g., OCPs. The OCP denotes the generalized mathematical form containing a set of differential equations which describe the control variable(s) that optimize the cost function over a specific period of time.

The pivotal constituents of an OCP statement are as follows: (i) A mathematical model of a particular system necessary to be optimized; (ii) An appropriate outcome of the concerned mechanism; (iii) A group comprising permitted input values; (iv) A cost function/performance index required for efficacy measurement of the control system in contemplation.

Optimal control theory deals with the problem of encountering a control function for a system under consideration during predetermined time limitation $[t_i, t_f]$. A set containing nonlinear differential equations represents the control law for a specific system, as mentioned below:

$$u(t) = f(t, x(t), \dot{x}(t)) \tag{1}$$

here $u(t)$ symbolizes the mathematical model of a specified dynamical system that needs the application of various control processes for optimum performance and is typically presented by an assemblage of first-order differential equations. Moreover, f exemplifies a real-valued function that is continuously differentiable.

For a dynamical system required to be controlled, the state function consists of a set of boundary conditions implemented at the initial and final time, i.e., t_i and t_f, respectively, as follows:

$$x(t_i) = \boldsymbol{x_0}, x\left(t_f\right) = \boldsymbol{x_f} \tag{2}$$

where $u(.) : [t_i, t_f] \to \mathbb{R}$ and $x(.) : [t_i, t_f] \to \mathbb{R}$ demonstrate the control and state variables, respectively. Additionally, $\boldsymbol{x_0}$ and $\boldsymbol{x_f}$ represent some vector elaborating $x(.)$ subjected to boundary condition(s).

Throughout the limit $t \in [t_i, t_f] \to \mathbb{R}$, it is presumed that $u(.)$ equals to piecewise continuous function. Therefore, it could be implied that modifications due to the mathematical operation(s) enforced on $u(.)$ directly affect the final result of the related differential equation(s).

The performance index J is presented in a mathematical form by using some scalar function that defines the necessary requirements. The minimization of J assists in determining the optimum solution for a given OCP. Generally, the mathematical formulation of J could be expressed as below [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], and [20]:

$$J = \int_{t_i}^{t_f} L(t, x(t), u(t))dt \tag{3}$$

here, it is assumed that L equals to the nonnegative scalar function, which is tentatively differentiable in all cases, e.g., $L(t, 0, 0) = 0$. In addition, the Lagrange problem is an alternative term for the optimization problem containing J, as exhibited in (3).

Subsequently, OCP established the fundamental objectives of determining an optimal value for control $u(\cdot)$, transmitting $u(t)$ from condition $x(t_i) = \boldsymbol{x_0}$ to $x\left(t_f\right) = \boldsymbol{x_f}$ in the duration $\left(t_f - t_i\right)$, and executing best-minimized solution for J [1], [2], [3], and [12]. Besides, for a comprehensive study of the OCPs discussed in this research work, i.e., (1) - (3), avid readers should refer to [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], and [45] and the citations therein.

III. NATURE-INSPIRED OPTIMIZATION TECHNIQUES AND BERNSTEIN POLYNOMIALS

This section introduces concisely BPs, a versatile approximation method, and two nature-inspired optimization algorithms utilized in this research work, i.e., GA, a renowned algorithm, and FDO, the latest algorithm, providing a gist of their actual working mechanism.

A. GENETIC ALGORITHM

The GA is a powerful probabilistic heuristic optimization technique that mimics the process of evolution and utilizes a global search strategy to find the best possible solution to an optimization problem. The profound notion behind GA is established on the survival of the fittest candidate, which warrants merely the competent and fit chromosome(s) to execute the successive iteration(s). A stepwise description of the working of GA is presented below [32], [33], [34], [35], [41], and [46]:

Step 1:Initialization GA scrutinizes for an optimum solution by initiating with a randomly created population of individuals (i.e., not only a solitary individual) known as chromosomes. These chromosomes offer a probable solution for the specified problem. Therefore, an arbitrarily produced fitness-based value is allocated to every chromosome to help identify the quality of the solution.

Step 2: Evaluation The fitness of each individual in the population is evaluated using a fitness function that is designed to either maximize or minimize an objective function. Individuals with better fitness values are preferred for reproduction in the succeeding step.

Step 3: Selection Based on the fitness level of individuals, a subset is selected from the overall population to be the parents of the upcoming generation. There are several methods for selection, e.g., roulette wheel selection, tournament selection, and rank selection.

Step 4: Crossover The goal of crossover is to originate child chromosomes via reincorporation of parent chromosomes, rendering the updated generation which exclusively contains the robust individuals.

Step 5: Mutation If the desired fitness level is not achieved by the reformed generation or there is rapid convergence, then mutation could offer some solution by inaugurating diversity into the population and exploring new dimensions of the search space. However, the probability of mutation is typically low, i.e., ranging from 0.1% to 1%, contingent upon the problem.

Step 6: Termination The aforementioned steps are reiterated during consecutive generations, assessing the most appropriate solution for a particular problem by utilizing the genetic operators (e.g., selection, crossover, and mutation), where required, until the termination is attained. The stopping criteria encompass the following necessary condition(s): (i) an adequate fitness value accomplished; (ii) the maximal count of iterations achieved.

Algorithm 1 exhibits the pseudocode of GA.

Algorithm 1 The Pseudocode representing Genetic Algorithm [20]

```
Initiate t → 0
Produce P_i randomized %P_i =Population
Estimate P_i
  while (standardized cessation unfulfilled)
    Reiterate
      for i = 1 till total P_i amount
      CHOOSE M candidates
       Search least fit candidate
       Eliminate least fit candidate
      CROSSOVER produce advanced candidates
       Determine recent candidates
      MUTATION employ (elective)
       Assess reformed candidates
    end for
  end while
```

B. FITNESS DEPENDENT OPTIMIZER

FDO is a contemporary optimization algorithm which provides numerous effective real-life applications and assists in efficiently determining the optimum solution for intricate optimization systems, e.g., task planning in robotics, engineering design, and economic problems. FDO contains a few similar traits as PSO but also exhibits diversified and significantly unique features. It emulates the procreation conduct of the bee swarms when the bees are exploring appropriate hives. However, the fundamental conceptualization behind this metaheuristic algorithm is acquired from the practice of scout bees choosing the most pertinent hive among the potential ones. Further, all the scout bees seeking desired hives indicate a prospective solution. Therefore, the selection of the finest hive raises the possibility of securing the optimal solution with better convergence.

FDO initializes the randomly executed population of scout bees inside the realm of the search area. The identification of an appropriate hive location is vital, having a dependency on the position of scout bee and the relevant fitness function(s). Consequently, searching for the fittest hive is the ultimate motive of scout bee which when fulfilled and an advanced solution is discovered successfully then the formerly allocated solution value is eluded. Conversely, the inability of scout bee to attain an exceptional result would be considered as a futile attempt and the previously achieved result would be preferred to vary the scout bee position [44], [45], [47], [48], [49].

The scout bee could be expressed as below:

$$S_b (b = 1, 2, \ldots, n) \tag{4}$$

Scout bees utilize random walk techniques and fitness weight fw to randomly search within the search area. The movement of scout bee relies upon the pace p maintained by the current scout for modifying position, as mentioned in (5).

The inclusion of p assists in determining new and high-quality solutions [47], [48].

$$S_{b,t+1} = S_{b,t} + p \tag{5}$$

here b refers to the current scout bee, which serves the purpose of search agent. t denotes a recent generation, whereas S demonstrates artificial scout bee. Further, p, which is based upon fw, describes the direction and movement rate of artificial scout bee. Nonetheless, the direction of p totally counts upon the randomization process. Accordingly, fw could be estimated for some optimization problems by (6) and lies inside the bound of [0, 1] as formulated below [47] and [48]:

$$fw = \left| \frac{s^*_{b,t\ fitness}}{s_{b,t\ fitness}} \right| - wf \tag{6}$$

here $s_{b,t\ fitness}$ exhibits the optimal result concerning recent scout bee, $s^*_{b,tfitness}$ represents some desirable result obtained through scout bee until now, whereas wf (having the numerical value of either 1 or 0) indicates the weight factor.

FDO possesses R random value enclosed by purview [-1, 1]. Besides, the Levy flight operation is prioritized over other random walk approaches since it contains a suitable distribution curve and facilitates attaining highly stable movements. Thereupon, it is established that FDO needs straightforward computations, i.e., R and fw, for accurately evaluating the fitness function [47], [48].

Additionally, the global solution is achieved by random initialization of scout bee inside the scope of the inspection area, and utilizing the related lower and upper limits. Henceforth, fw is gauged depending upon the imperative requirements mentioned below [47], [48].

For $s_{b,tfitness} = 0$ or $fw = 0$ or $fw = 1$, p can be found by the following [47], [48], [49]:

$$p = s_{b,t} \times R \tag{7}$$

Similarly, R is evaluated while $fw > 0$ and $fw < 1$ by the considerations defined as follows:

In-case $R < 0$, (8) can be utilized otherwise when $R \geq 0$ applying (9) can provide the measure of p as follows [47], [48], [49]:

$$p = distance_{best_{bee}} \times fw \times (-1) \tag{8}$$

$$p = distance_{best_{bee}} \times fw \tag{9}$$

For the evaluation of $distance_{best_{bee}}$ calculate the following:

$$distance_{best_{bee}} = s_{b,t} - s^*_{b,t} \tag{10}$$

Algorithm 2 demonstrates the pseudocode of FDO.

C. BERNSTEIN POLYNOMIALS

The BPs provide a resourceful approximation technique that is endowed with several essential properties, making it an indispensable method for refining approximated solutions [17], [50], [51].

Algorithm 2 The Pseudocode representing Fitness Dependent Optimizer [47]

```
Initiate S_{b,t} Population    %S_{b,t} = Scout bee and b = 1, 2, 3, .., n
while maximum generation (t) unaccomplished
for S_{b,t}
    examine best s*_{b,t}
    perform randomization process R∈[−1, 1]
                % R = random number
if S_{b,tfitness} == 0
    fw = 0          %fw = fitness weight
else
utilize fw = |S*_{b,tfitness} / S_{b,tfitness}| − wf     % wf = weight factor
end if
 if fw == 0 OR fw == 1
    p = (s_{b,t} × R)     %p = pace
 else
 if R ≥ 0
      p = (s_{b,t} − s*_{b,t}) × fw
 else
      p = (s_{b,t} − s*_{b,t}) × fw × (−1)
 end if
 end if
 evaluate S_{b,t+1} = S_{b,t} + p
 if S_{b,t+1fitness} < S_{b,tfitness}
    acknowledge S_{b,t+1fitness}
    preserve p
 else
    execute S_{b,t+1} = S_{b,t} + p     % p contains prior value
 if S_{b,t+1fitness} < S_{b,tfitness}
    acknowledge S_{b,t+1fitness}
    preserve p
 else
    retain present position
 end if
 end if
end for
end while
```

The general form of BPs with degree k on the interval $[0, T]$ can be expressed as below [17], [50]:

$$B_{i,k}(t) = \binom{k}{i} \frac{t^i (T-t)^{k-i}}{T^k} \tag{11}$$

here

$$\binom{k}{i} = \frac{k!}{i!\,(k-i)!} \tag{12}$$

For $i > k$ or $i < 0$ we get $B_{i,k}(t) = 0$, and for $i = k = 0$ we obtain $B_{0,0}(t) = 1$.

The BPs are positive on the interval $[0, T]$, which is a critical factor in numerous applications where quantities could not be negative, i.e., $B_{i,k}(t) > 0$. Also, the sum of all BPs equals to unity for each real-valued t, as mentioned below [51]:

$$\sum_{i=0}^{k} B_{i,k}(t) = 1 \tag{13}$$

Such polynomials could be executed recursively using the equation as follows [17], [50], [51]:

$$B_{i,k}(t) = \frac{(T-t)}{T} B_{i,k-1}(t) + \frac{t}{T} B_{i-1,k-1}(t) \tag{14}$$

Some derivatives of BPs within the limits of $[0, T]$ are provided below [51]:

$$\dot{B_{i,k}}(t) = \frac{k}{T}(B_{i-1,k-1}(t) - B_{i,k-1}(t)) \tag{15}$$

$$\ddot{B_{i,k}}(t) = \frac{k(k-1)}{T^2}(B_{i-2,k-2}(t) - 2B_{i-1,k-2}(t) + B_{i,k-2}(t)) \tag{16}$$

Besides, for further details interested readers should refer to [17], [32], [46], [50], and [51].

IV. PROPOSED METHODOLOGY

This section expounds on the investigation methodology considered in this research study, which involves combining various Nature-Inspired Computation (NIC) algorithms and BPs through hybridization techniques. The ensuing discussion provides a comprehensive overview of these techniques and their application in the research.

This research methodology assumes that the optimal numeric solution for distinct real-world NOCPs can be derived by expressing the approximate solution as a linear combination of BPs accompanying unknown coefficients. The approximations for the state variable(s) $x(t)$, control variable $u(t)$, and performance index J are then formulated by (17)-(19) as follows:

The accuracy of the approximation for $x(t)$ is assessed through the following evaluation:

$$x(t) = \sum_{i=0}^{k} \alpha_i B_{i,k}(t) \tag{17}$$

The estimation of the approximation for $u(t)$ is determined by considering (1) which involves measuring $u(t)$ as a function of the unknown coefficients of $x(t)$, as illustrated in (18):

$$u(t) = f(t, \sum_{i=0}^{k} \alpha_i B_{i,k}(t), \sum_{i=0}^{k} \alpha_i \dot{B_{i,k}}(t)) \tag{18}$$

The approximation for J is ascertained by substituting in (3) the approximated estimates for $x(t)$ and $u(t)$, as computed above through (17) and (18), respectively.

Find J according to the following formulation:

$$J = \int_{t_i}^{t_f} L(t, \sum_{i=0}^{k} \alpha_i B_{i,k}(t), f(t, \sum_{i=0}^{k} \alpha_i B_{i,k}(t), \sum_{i=0}^{k} \alpha_i \dot{B_{i,k}}(t))dt \tag{19}$$

The set of $(\alpha_0, \alpha_{1,\ldots,}\alpha_k)$ consists of parameters/coefficients whose values are not initially known and need to be determined and tuned through computational operations. Here, the value of k represents the degree of the BPs.

A. PROPOSED METHODOLOGY BASED ON HYBRID APPROACH OF EVOLUTIONARY ALGORITHMS AND BERNSTEIN POLYNOMIALS

This subsection particularly presents the investigation approach followed herein, which involves hybridizing NIC methods with BPs to address and overcome the inherent complexities of distinctive real-world NOCPs. The proposed approach has the potential to yield an effective and reliable solution for the concerned problems under consideration, as described below:

The assumption is that the control signal $u(t)$ represents an approximated numeric solution obtained by a linear combination of Bernstein basis functions with unknown coefficients, i.e., denoted by $k = 8$, as formulated in (21):

$$x(t) = \sum_{i=0}^{8} \alpha_i B_{i,8}(t) \tag{20}$$

$$u(t) = f(t, \sum_{i=0}^{8} \alpha_i B_{i,8}(t), \sum_{i=0}^{8} \alpha_i \dot{B_{i,8}}(t)) \tag{21}$$

The necessary optimum values of the unknown parameters are yielded by transforming the relevant NOCPs into an adequate error minimization problem, as elaborated by (22)-(25) for Problem 01 of this work.

$$\varepsilon_1 = \frac{1}{2}((x_1(0) - 1)^2 + (x_2(0) - 0)^2) \tag{22}$$

$$\varepsilon_2 = \frac{1}{N+1} \sum_{i=0}^{N} (\dot{x_1}(t) - x_2(t))^2 \tag{23}$$

$$\varepsilon_3 = \frac{1}{N+1} \sum_{i=0}^{N} \left(\dot{x_2}(t) - u(t) + x_2(t) - (1 - x_1^2(t))x_2(t)\right)^2 \tag{24}$$

$$\varepsilon_j = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{25}$$

here the symbol N denotes the count of steps exploited within the specified range, while ε_1 signifies the mean for the sum of square error of the initial condition(s). The symbols ε_2 and ε_3 define the mean for the sum of square errors of the system state equations, and j symbolizes the count of total generations executed.

Moreover, the fitness function ε_j value is significantly minimized and improved by employing the evolutionary algorithms, for instance, FDO and GA. Ultimately, the approximated numerical solution is accomplished by using the optimum values of $(\alpha_0, \alpha_1, \ldots, \alpha_8)$, which help achieve minimal error. Likewise, this research methodology is implemented on other NOCPs mentioned in this work to acquire their desirable approximate outcomes.

V. SIMULATION AND RESULTS

The optimization method represented in Section IV is implemented on different real-world NOCPs to establish that the proposed scheme is valid and outperforms previously applied numerical techniques. The simulations of the concerned NOCPs are executed by accurately utilizing the MATLAB tool. Additionally, the comparative analysis between the presented technique and other existing methods is evaluated, accrediting the efficacy of the suggested hybrid scheme.

The parameter settings for the computation of GA involve a population size equivalent to 500 and iterations to 300, together with some other ones. Likewise, FDO settings encompass a scout bee count equal to 15 and total generations up to 200, accompanying the other ones. Further, the degree of BPs is adjusted to $k = 8$, executing the optimal values of unknown coefficients. Moreover, the parameter settings of the algorithms are the same for the entire NOCPs examined in this study.

The numerical behavior of GA-BP and FDO-BP is studied by considering the parameters as follows:

The Relative Error E_J of J defined as:

$$E_J = \left| \frac{J - J^*}{J^*} \right| \tag{26}$$

here J^* denotes the best achieved solution amidst all evaluations.

Norm for the final state constraint(s) $\boldsymbol{\psi}$, i.e., φ_f, formulated as:

$$\varphi_f = \|\boldsymbol{\psi}\|_2 \tag{27}$$

here $\boldsymbol{\psi} = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots, \boldsymbol{\psi}_n]^T$ represents the vector of final state constraints.

The factor K_ψ depicts the addition of two main errors for quality assessment, as mentioned below:

$$K_\psi = E_J + \varphi_f \tag{28}$$

A. PROBLEM 01: VAN DER POL OSCILLATOR PROBLEM

The first NOCP discussed in this article is the VDP oscillator problem [20], [52] containing one control variable $u(t)$ and two state variables $x_1(t)$, $x_2(t)$. The VDP problem includes a final state constraint that must be satisfied, i.e., $\boldsymbol{\psi} = x_1(t_f) - x_2(t_f) + 1 = 0$. The minimization of the below mentioned cost function is required:

$$J = \frac{1}{2} \int_0^5 \left(x_1^2 + x_2^2 + u^2 \right) dt \tag{29}$$

subject to the system state equations:

$$\dot{x_1} = x_2 \tag{30}$$

$$\dot{x_2} = -x_2 + \left(1 - x_1^2\right) x_2 + u \tag{31}$$

with initial conditions as:

$$x_1(0) = 1, \text{ and } x_2(0) = 0 \tag{32}$$

over the period $t \in [0, 5]$ the final state constraint is:

$$\boldsymbol{\psi} = x_1(t_f) - x_2(t_f) + 1 = 0 \tag{33}$$

The optimal values of $(\alpha_0, \alpha_1, \ldots, \alpha_8)$ accomplished by the represented hybrid approach are provided in Table 1, whereas the approximate values of $x_1(t)$, $x_2(t)$, and $u(t)$ at different points in t are shown in Tables 2 and 3, accordingly. Likewise, a comprehensive comparison between J of the proposed technique and formerly contemplated methods, e.g., Sequential Unconstrained Minimization Technique (SUMT), Continuous GA (CGA), Linear Interpolation (LI), and Spline Interpolation (SI), is mentioned in Table 4. Moreover, the approximate numeric values of $x_1(t)$, $x_2(t)$,

TABLE 1. Unknown parameters generated from FDO-BP and GA-BP regarding Problem 01.

Parameters	GA-BP	FDO-BP
α_0	0.860525758	0.868228189
α_1	0.788196895	0.807178958
α_2	0.711270903	0.733306180
α_3	0.628203345	0.652606285
α_4	0.544684453	0.568985727
α_5	0.467558267	0.484979466
α_6	0.402119780	0.402230106
α_7	0.351274955	0.321807843
α_8	0.315642953	0.244422950

TABLE 2. Approximated numeric value of state variables for Problem 01.

t	$x_1(t)$ GA-BP	$x_1(t)$ FDO-BP	$x_2(t)$ GA-BP	$x_2(t)$ FDO-BP
0.0	1.000525758	1.000000000	0.000000000	0.000000488
0.5	0.860479351	0.900568255	-0.375496824	-0.356573670
1.0	0.690615018	0.702468230	-0.484982560	-0.456190791
1.5	0.455642953	0.450401299	-0.502845718	-0.495136011
2.0	0.241670530	0.202657540	-0.503637949	-0.503863294
2.5	-0.059363261	-0.050423723	-0.454893671	-0.452423779
3.0	-0.253612075	-0.250504576	-0.375465913	-0.350793610
3.5	-0.375485915	-0.401497798	-0.221356139	-0.221063984
4.0	-0.504366269	-0.473977153	-0.130697051	-0.072907343
4.5	-0.483612075	-0.452149921	0.255056012	0.247254186
5.0	-0.236465429	-0.241935711	0.763534571	0.758064289

TABLE 3. Approximated numeric value of control variable for Problem 01.

t	$u(t)$ GA-BP	$u(t)$ FDO-BP
0.0	0.048385458	0.050862798
0.5	0.631857910	0.527304179
1.0	0.823517489	0.881536073
1.5	0.803546426	0.823763082
2.0	0.704910973	0.682476125
2.5	0.517386250	0.552643874
3.0	0.374083525	0.353829351
3.5	0.000000090	0.000000004
4.0	-0.041938975	-0.053882154
4.5	0.320835248	0.351825090
5.0	0.414910973	0.423103318

TABLE 4. Comparison of results for Van der Pol Oscillator Problem.

Technique	J	E_J	φ_f	K_ψ	$x_1(t_f)$	$x_2(t_f)$
SUMT [52]	1.7672	–	2.02E-2	–	-0.2693	0.7105
Gradient Algorithm [52]	3.2677	–	1.1E-3	–	-1.2009	-0.1998
Shooting Algorithm [52]	Failed	–	Failed	–	Failed	Failed
CGA [52]	1.7404	0.0912	2.67E-11	0.0912	-0.2258	0.7741
LI [20]	1.5949	0	1.08E-13	1.08E-13	–	–
SI [20]	1.5950	6.27E-05	9.99E-14	6.27E-05	–	–
GA-BP	**1.4587**	**0**	**1.199E-14**	**1.199E-14**	**-0.23646**	**0.76353**
FDO-BP	**1.1656**	**0**	**9.992E-16**	**9.992E-16**	**-0.24194**	**0.75806**

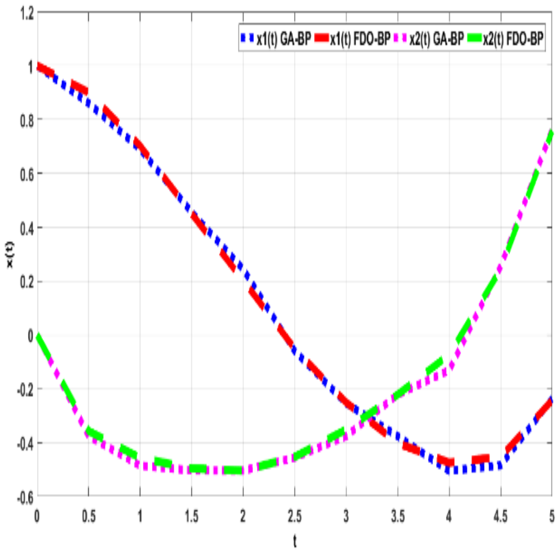

FIGURE 1. $x_1(t)$, $x_2(t)$ approximation for Problem 01.

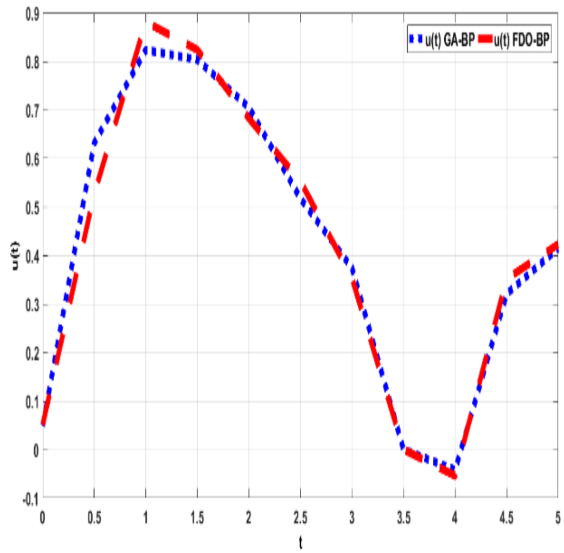

FIGURE 2. $u(t)$ approximation for Problem 01.

and $u(t)$, determined by FDO-BP and GA-BP, are illustrated in Figures 1 and 2, respectively.

It is shown evidently in Table 4 that results achieved by the proposed hybrid technique offer a better solution and reduced values of parameters J, E_J, φ_f, and K_ψ. The best value for J, i.e., 1.1656, is achieved by FDO-BP, the good J value, i.e., 1.4587, is provided by GA-BP, and the highest J value, i.e., 3.2677, is offered by Gradient Algorithm. The Shooting Algorithm proved abortive for the convergence of this problem. Likewise, the final equality constraints executed by FDO-BP and GA-BP give the best solution compared to other techniques. Also, it is apparent from Figures 1 and 2 that the approximate values depicted are more appropriate and help yield an optimal solution.

B. PROBLEM 02: CHEMICAL REACTOR PROBLEM

The second NOCP scrutinized in this study is the CRP [20], [52], which possesses one control variable, i.e., $u(t)$, and two state variables, i.e., $x_1(t)$, $x_2(t)$. The CRP contains two final state constraints which are mandatory to be fulfilled, i.e., $\boldsymbol{\psi} = [x_1, x_2]^T$. The objective function of CRP demands the maintenance of temperature and concentration, e.g., keeping their values approximately near the steady-state values without ample control usage. The performance index to be minimized is as follows:

$$J = \frac{1}{2}\int_0^{0.78} \left(x_1^2 + x_2^2 + 0.1u^2\right) dt \tag{34}$$

subject to the nonlinear state equations:

$$\dot{x_1} = x_1 - 2\,(x_1 + 0.25) + (x_2 + 0.5)\, e^{\left(\frac{25x_1}{(x_1+2)}\right)} - (x_1 + 0.25)\, u \tag{35}$$

$$\dot{x_2} = 0.5 - x_2 - (x_2 + 0.5)e^{\left(\frac{25x_1}{(x_1+2)}\right)} \tag{36}$$

with respect to the initial conditions:

$$x_1(0) = 0.05, \text{ and } x_2(0) = 0 \tag{37}$$

over the period $t \in [0, 0.78]$

the final state constraint is:

$$\boldsymbol{\psi} = [x_1, x_2]^T \tag{38}$$

Table 5 exhibits the optimum values for $(\alpha_0, \alpha_1, \ldots, \alpha_8)$ attained by applying FDO-BP and GA-BP, whereas Table 6

TABLE 5. Unknown parameters generated from FDO-BP and GA-BP regarding Problem 02.

Parameters	GA-BP	FDO-BP
α_0	0.046112139	0.046012134
α_1	0.040132757	0.036131159
α_2	0.030212476	0.029211307
α_3	0.027014204	0.023011718
α_4	0.032134512	0.030134505
α_5	0.028012587	0.025012133
α_6	0.024401148	0.024401329
α_7	0.020140733	0.020142085
α_8	0.014031507	0.018031161

TABLE 6. Approximated numeric value of state variables for Problem 02.

t	$x_1(t)$ GA-BP	$x_1(t)$ FDO-BP	$x_2(t)$ GA-BP	$x_2(t)$ FDO-BP
0.0	0.051349579	0.050249980	0.000000070	0.000000067
0.1	0.028615342	0.028271101	-0.026751601	-0.026746593
0.2	0.013473295	0.013258494	-0.036594728	-0.033289413
0.3	0.006583128	0.006135921	-0.032475078	-0.032136892
0.4	-0.002146357	-0.002061371	-0.028310351	-0.028387160
0.5	-0.010254817	-0.010213616	-0.020892515	-0.020632137
0.6	-0.013853298	-0.013426841	-0.010461397	-0.010328196
0.7	-0.008354726	-0.008135692	-0.002439763	-0.002123045
0.78	1.436E-10	3.251E-11	-7.271E-10	-8.382E-11

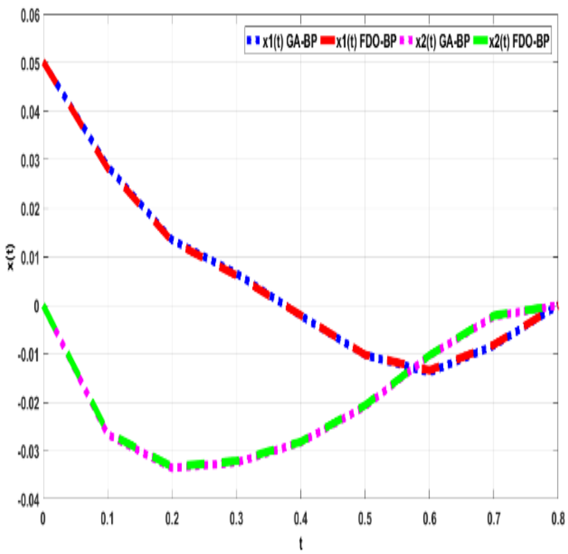

FIGURE 3. $x_1(t)$, $x_2(t)$ approximation for Problem 02.

presents the approximate values for $x_1(t)$ and $x_2(t)$, and Table 7 demonstrates the optimal values of $u(t)$.

Correspondingly, Table 8 compares the numerical solution of J exploited by the suggested scheme and some previously designed methods. In addition, Figure 3 compares the approximate results of $x_1(t)$ and $x_2(t)$, whereas Figure 4 depicts the approximate count of $u(t)$.

It is clearly noticeable from Table 8 that the recommended technique generates improved results than the prior ones. Also, the J, E_J, φ_f, and K_ψ values are decreased to a considerable level, indicating the efficacy of FDO-BP and GA-BP.

TABLE 7. Approximated numeric value of control variable for Problem 02.

t	$u(t)$ GA-BP	$u(t)$ FDO-BP
0.0	1.678535921	1.675024996
0.1	1.129643716	1.125183235
0.2	0.389735192	0.375146155
0.3	0.258967985	0.250314863
0.4	0.220837415	0.201453298
0.5	0.056241730	0.050158473
0.6	-0.321682584	-0.325249714
0.7	-0.443927862	-0.475103956
0.78	-0.354873618	-0.375046157

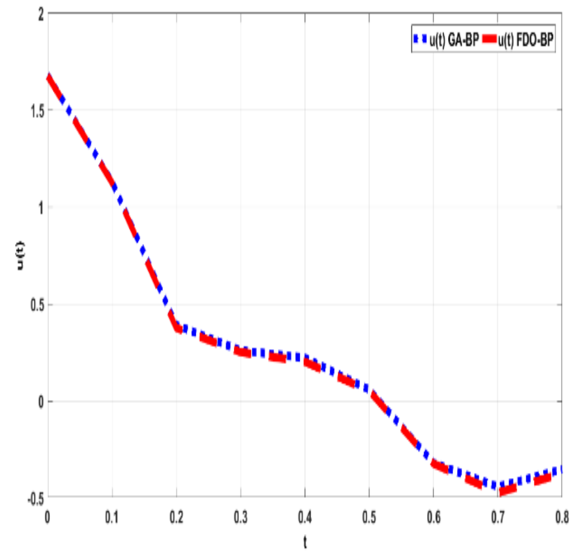

FIGURE 4. $u(t)$ approximation for Problem 02.

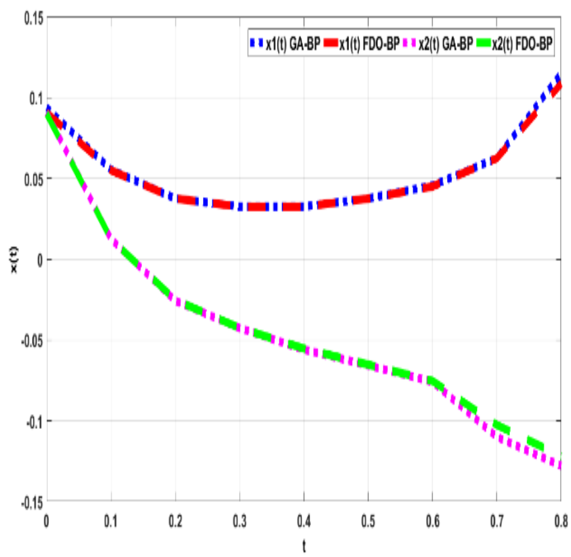

FIGURE 5. $x_1(t)$, $x_2(t)$ approximation for Problem 03.

TABLE 8. Comparison of results for Chemical Reactor Problem.

Technique	J	E_J	φ_f	K_ψ	$x_1(t_f)$	$x_2(t_f)$
SUMT [52]	2.54E-02	–	1.11E-03	–	4.07E-04	-1.04E-03
Gradient Algorithm [52]	1.80E-02	–	9.46E-07	–	-9.25E-07	-2.01E-07
Shooting Algorithm [52]	1.78E-02	–	6.85E-09	–	4.79E-09	4.90E-09
CGA [52]	1.632E-02	0.2835	7.5790E-10	0.2835	7.479E-10	-1.228E-10
LI [20]	1.27E-02	0	1.15E-09	1.15E-09	–	–
SI [20]	1.27E-02	0	5.99E-09	5.99E-09	–	–
GA-BP	**1.00E-02**	**0**	**7.411E-10**	**7.411E-10**	**1.436E-10**	**-7.271E-10**
FDO-BP	**9.672E-03**	**0**	**8.990E-11**	**8.990E-11**	**3.251E-11**	**-8.382E-11**

TABLE 9. Unknown parameters generated from FDO-BP and GA-BP regarding Problem 03.

Parameters	GA-BP	FDO-BP
α_0	0.093858191	0.090760513
α_1	0.086312978	0.083371855
α_2	0.080156985	0.079191279
α_3	0.077126585	0.073220696
α_4	0.070285741	0.068757985
α_5	0.065241944	0.060694602
α_6	0.059371238	0.053938215
α_7	0.054879897	0.050801483
α_8	0.050655327	0.046891641

TABLE 10. Approximated numeric value of state variables for Problem 03.

t	$x_1(t)$ GA-BP	$x_1(t)$ FDO-BP	$x_2(t)$ GA-BP	$x_2(t)$ FDO-BP
0.0	0.093857291	0.090763514	0.090873156	0.090148375
0.1	0.055483167	0.055019598	0.012561327	0.012518639
0.2	0.037562384	0.037548253	-0.025714825	-0.025297845
0.3	0.032573961	0.032531726	-0.042562180	-0.042587396
0.4	0.032684514	0.032518497	-0.055973698	-0.055238147
0.5	0.037561492	0.037526942	-0.065718216	-0.065193769
0.6	0.045732675	0.045028174	-0.075921483	-0.075329678
0.7	0.062517238	0.062513890	-0.109853862	-0.102591804
0.78	0.115043862	0.108735146	-0.127845239	-0.121853725

The best value for J, i.e., 9.672E-03, is achieved by FDO-BP, the good J value, i.e., 1.00E-02, is provided by GA-BP, and the highest J value, i.e., 2.54E-02, is offered by SUMT method. Similarly, the final equality constraints computed by FDO-BP and GA-BP for this system prove that the deviation of temperature and concentration from their steady-state is the lowest. Additionally, Figures 3 and 4 disclose that the acquired $x_1(t)$, $x_2(t)$, and $u(t)$ values are adequate and assist in estimating the optimum solution.

C. PROBLEM 03: CONTINUOUS STIRRED-TANK CHEMICAL REACTOR PROBLEM

The CSTCRP [19], [20] is a benchmark NOCP utilized by numerous researchers for the evaluation of their respective techniques. In CSTCRP, the state variable $x_1(t)$ defines the variation from steady-state temperature, and $x_2(t)$ indicates the aberration from the steady-state concentration, while control variable $u(t)$ stipulates the outcome of the flow rate of coolant on the concerned chemical reactor. There are no final state constraints since it is an unconstrained CSTCRP. The minimization of the following quadratic performance index

TABLE 11. Approximated numeric value of control variable for Problem 03.

t	$u(t)$ GA-BP	$u(t)$ FDO-BP
0.0	3.554871796	3.550134172
0.1	2.238657304	2.202451349
0.2	1.058134176	1.050387621
0.3	0.627682458	0.604358174
0.4	0.438973125	0.402965371
0.5	0.251945621	0.250481564
0.6	0.000095974	0.000003240
0.7	0.148956194	0.101354983
0.78	0.000009287	0.000000108

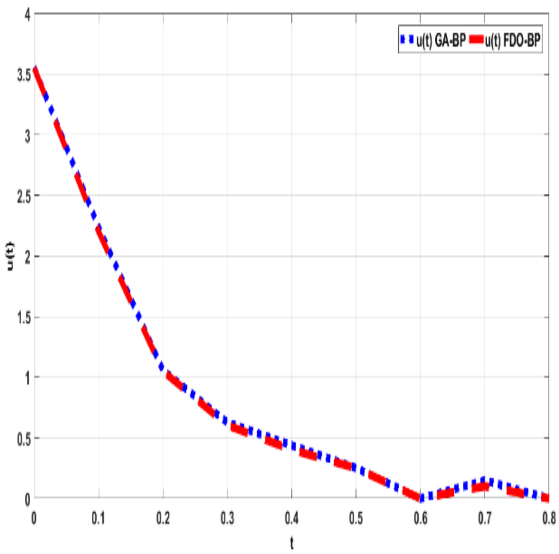

FIGURE 6. $u(t)$ approximation for Problem 03.

TABLE 12. Comparison of results for Continuous Stirred-Tank Chemical Reactor Problem.

Technique	J	E_J	φ_f	K_ψ	$x_1(t_f)$	$x_2(t_f)$
PSO-LDW [19]	0.1381	–	–	–	–	–
PSO-NDW [19]	0.1377	–	–	–	–	–
IPSO [19]	0.1365	–	–	–	–	–
IPSO-SQP [19]	0.1355	0.2098	–	0.2098	–	–
LI [20]	0.1120	0	–	0	–	–
SI [20]	0.1120	0	–	0	–	–
GA-BP	**0.1000**	**0**	–	**0**	**0.115043862**	**-0.127845239**
FDO-BP	**0.0953**	**0**	–	**0**	**0.108735146**	**-0.121853725**

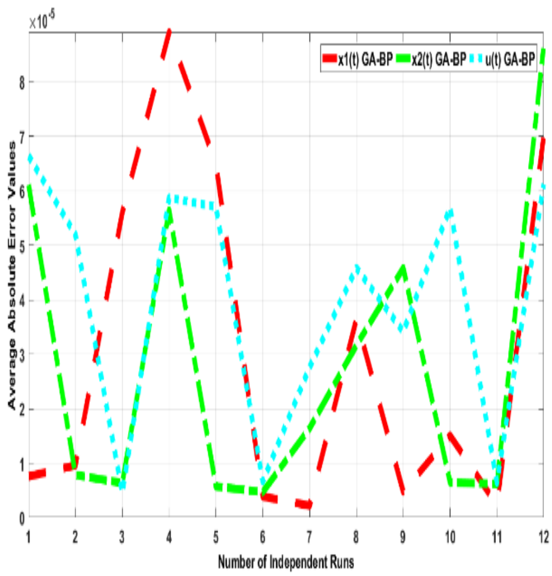

FIGURE 7. Average absolute error generated by GA-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 01.

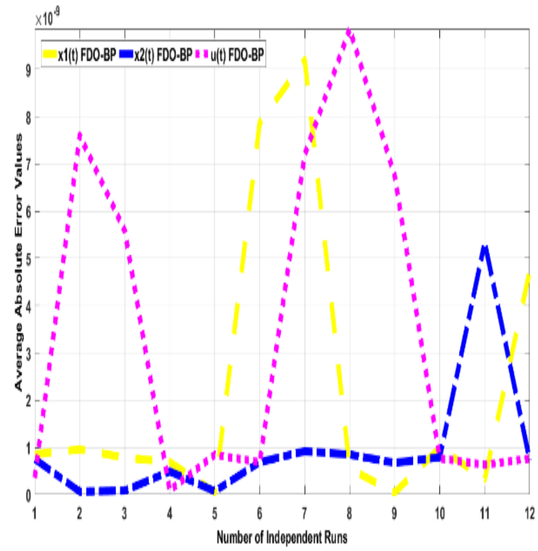

FIGURE 8. Average absolute error generated by FDO-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 01.

is statutory:

$$J = \int_0^{0.78} \left(x_1^2 + x_2^2 + 0.1u^2\right) dt \tag{39}$$

subject to the nonlinear system state equations:

$$\dot{x_1} = -(2+u)(x_1+0.25) + (x_2+0.5)\, e^{\left(\frac{25x_1}{(x_1+2)}\right)} \tag{40}$$

$$\dot{x_2} = 0.5 - x_2 - (x_2+0.5)e^{\left(\frac{25x_1}{(x_1+2)}\right)} \tag{41}$$

with respect to the initial conditions:

$$x_1(0) = 0.09, \text{ and } x_2(0) = 0.09 \tag{42}$$

over the period $t \in [0, 0.78]$

Table 9 displays the best values of $(\alpha_0, \alpha_1, \ldots, \alpha_8)$ yielded by successfully manipulating the proposed method. Besides, Table 10 articulates the optimum values achieved for $x_1(t)$ and $x_2(t)$, and Table 11 manifests the adequately estimated values of $u(t)$, highlighting the excellence of the proposed scheme. Similarly, Table 12 compares the numerical values of J evaluated by implementing GA-BP, FDO-BP, and other formerly existing techniques, e.g., Improved PSO with

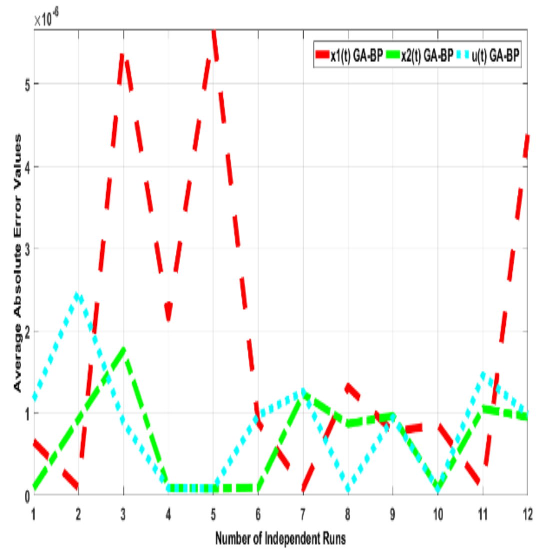

FIGURE 9. Average absolute error generated by GA-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 02.

Successive Quadratic Programming (IPSO-SQP), PSO with Linearly Decreasing inertia Weight (PSO-LDW), PSO

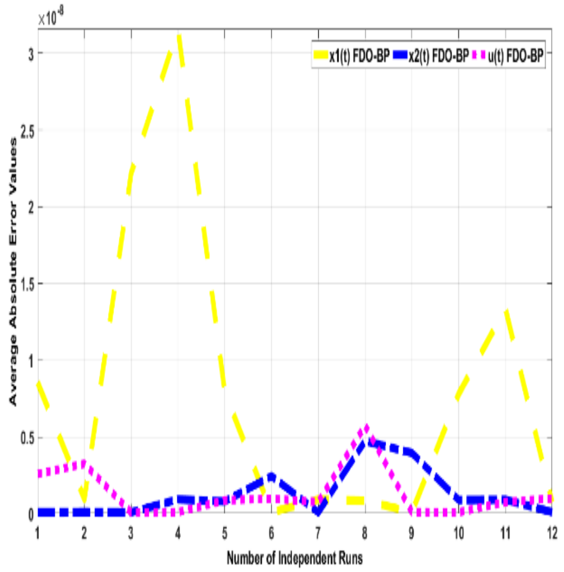

FIGURE 10. Average absolute error generated by FDO-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 02.

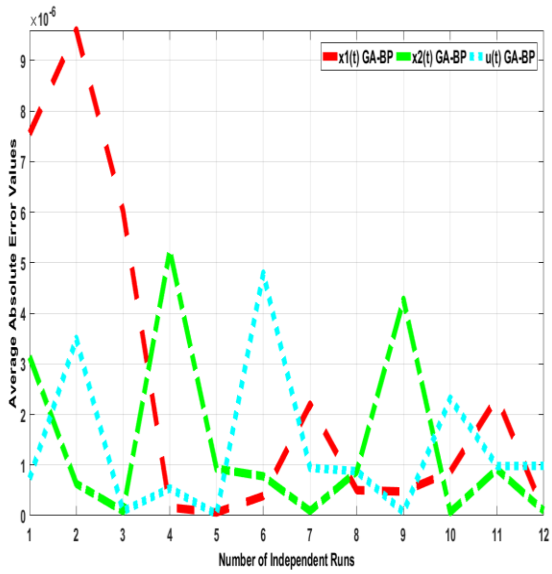

FIGURE 11. Average absolute error generated by GA-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 03.

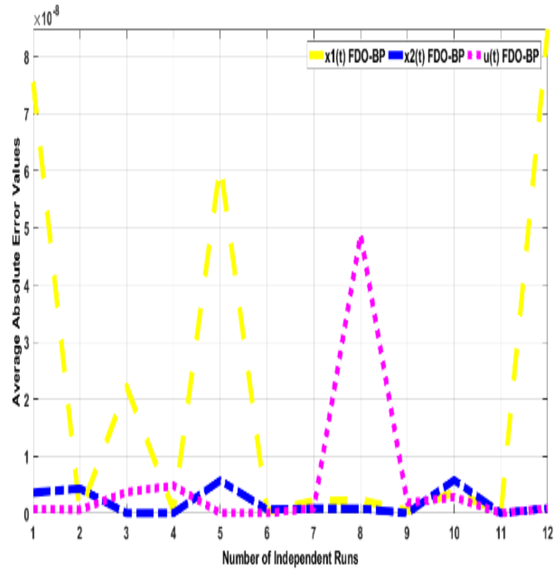

FIGURE 12. Average absolute error generated by FDO-BP for $x_1(t)$, $x_2(t)$, and $u(t)$ of Problem 03.

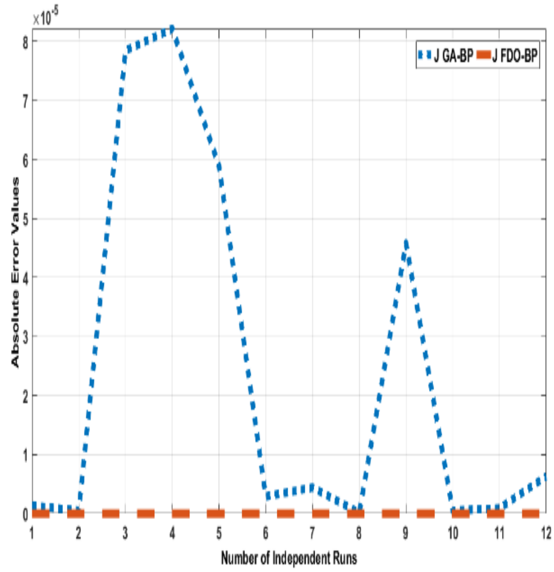

FIGURE 13. Absolute Error demonstrated pictorially for J of Problem 01.

with Nonlinearly Decreasing inertia Weight (PSO-NDW). Moreover, Figures 5 and 6 provide a graphical representation of approximated values obtained for $x_1(t)$, $x_2(t)$, and $u(t)$, respectively.

As markedly summarized in Table 12, the J, E_J, and K_ψ are reduced sufficiently, which is substantial to establish the significance of FDO-BP and GD-BP. The best value for J, i.e., 0.0953, is attained by FDO-BP, the good J value, i.e., 0.1000, is generated by GA-BP, and the highest J value, i.e., 0.1381, is executed by PSO-LDW algorithm. Furthermore, the attained approximate solution, plotted in Figures 5 and 6, reveals that the proposed technique renders an optimal outcome for $x_1(t)$, $x_2(t)$, and $u(t)$.

VI. STATISTICAL ANALYSIS OF THE RECOMMENDED COMPUTATIONAL METHOD

This section elucidates the statistical analysis performed thoroughly on every NOCP aforementioned, i.e., Problems 01–03, acknowledging the stability and efficiency of the proposed technique. For the implementation of this objective, 12 autonomous runs of FDO-BP and GA-BP are effectuated, retaining parameter settings to their predefined features. The pictorial description of average absolute error for $x_1(t)$, $x_2(t)$, and $u(t)$ of relevant NOCPs is presented in Figures 7–12.

TABLE 13. Statistical analysis for $x_1(t)$, $x_2(t)$, and $u(t)$ of NOCPs.

Problem	Technique	state variable $x_1(t)$				state variable $x_2(t)$				control variable $u(t)$			
		MIN	MAX	MEAN	SD	MIN	MAX	MEAN	SD	MIN	MAX	MEAN	SD
Problem 01 [20]	GA-BP	2.19E-06	8.92E-05	3.02E-05	3.04E-05	4.69E-06	8.67E-05	2.80E-05	2.69E-05	4.56E-06	6.65E-05	3.98E-05	2.23E-05
	FDO-BP	4.74E-11	9.19E-09	2.25E-09	3.05E-09	6.60E-11	5.34E-09	9.55E-10	1.35E-09	9.22E-11	9.85E-09	3.43E-09	3.48E-09
Problem 02 [20]	GA-BP	9.19E-08	5.66E-06	1.88E-06	2.01E-06	8.68E-08	1.76E-06	6.86E-07	5.48E-07	8.13E-08	2.48E-06	8.76E-07	6.86E-07
	FDO-BP	8.87E-11	3.16E-08	7.92E-09	9.64E-09	8.43E-11	4.73E-09	1.26E-09	1.53E-09	7.70E-11	5.74E-09	1.35E-09	1.63E-09
Problem 03 [20]	GA-BP	6.11E-08	9.60E-06	2.52E-06	3.17E-06	6.86E-08	5.25E-06	1.43E-06	1.70E-06	2.68E-08	4.79E-06	1.32E-06	1.42E-06
	FDO-BP	9.60E-11	8.47E-08	2.11E-08	3.13E-08	8.37E-11	5.86E-09	1.96E-09	2.19E-09	8.34E-11	4.88E-08	5.49E-09	1.31E-08

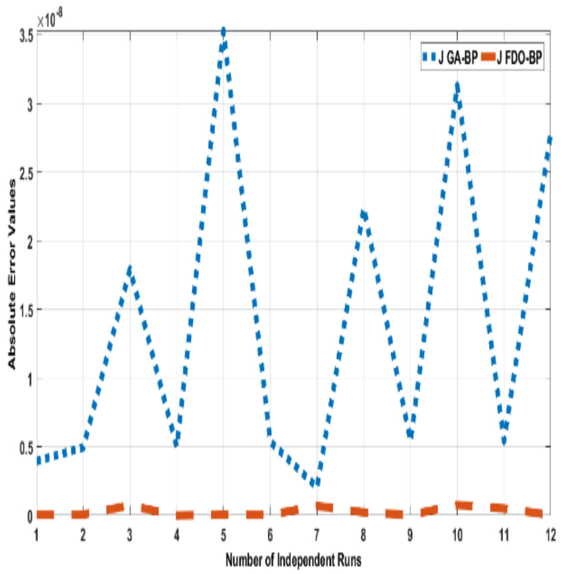

FIGURE 14. Absolute Error demonstrated pictorially for J of Problem 02.

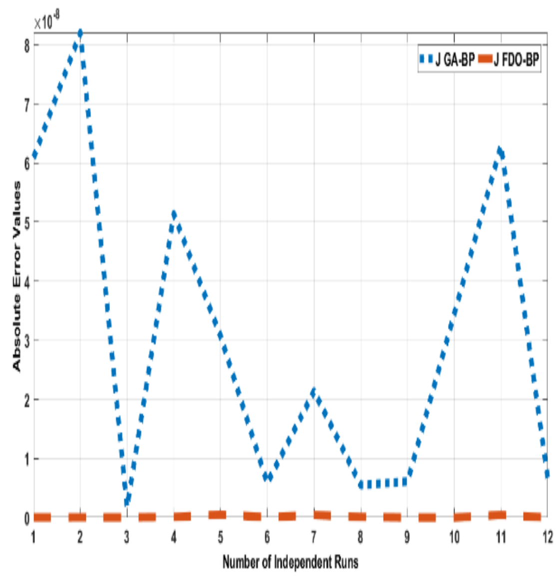

FIGURE 15. Absolute Error demonstrated pictorially for J of Problem 03.

It can be ascertained from Figures 7–12, for $x_1(t)$, $x_2(t)$, and $u(t)$, the average absolute error values are lessened significantly by suggested hybrid-based approach, which corroborates its productivity and accuracy.

The visual demonstration of the AE for J of related NOCPs is given in Figures 13–15 for at least 12 separate runs of recommended technique.

The numeric solutions illustrated in Figures 13–15 for Problem 01–03 indicate that the represented technique renders optimal results for J by adequate minimization of AE values, which eventually ratifies its efficiency.

The statistical analysis of previously mentioned NOCPs is based on the following decisive parameters: minimum error count (MIN), maximum error value (MAX), MEAN of error values, and Standard Deviation (SD). Moreover, the correlation between MIN and MAX values of error is analogous to that of the best and worst values. Likewise, MEAN and SD parameters help detect the central tendency and gauge the extent of variation in final results determined by FDO-BP and GA-BP, authenticating their robustness [46], [53], [54]. The numerical solutions evaluated by statistical analysis are compiled in the following Tables 13 and 14.

It is certainly apparent from Tables 13 and 14, for Problem 01–03, the MEAN of $x_1(t)$, $x_2(t)$, $u(t)$, and J are approximately 10^{-05} to 10^{-09}, 10^{-05} to 10^{-10}, 10^{-05} to 10^{-09}, and 10^{-05} to 10^{-10}, respectively. Similarly, for Problem 01–03, the SD for $x_1(t)$, $x_2(t)$, $u(t)$, and J ranges

TABLE 14. Statistical analysis for J of Nonlinear Optimal Control Problems.

Problem	Technique	performance index J			
		MIN	MAX	MEAN	SD
Problem 01 [20]	GA-BP	1.56E-07	8.21E-05	2.35E-05	3.15E-05
	FDO-BP	1.12E-11	2.85E-09	4.05E-10	7.59E-10
Problem 02 [20]	GA-BP	2.06E-09	3.53E-08	1.39E-08	1.18E-08
	FDO-BP	3.33E-12	7.69E-10	2.81E-10	2.96E-10
Problem 03 [20]	GA-BP	1.74E-09	8.20E-08	3.08E-08	2.64E-08
	FDO-BP	2.39E-12	5.09E-10	1.51E-10	1.82E-10

from 10^{-05} to 10^{-09}, 10^{-05} to 10^{-10}, 10^{-05} to 10^{-09}, and 10^{-05} to 10^{-10}, respectively. Subsequently, Tables 13 and 14 evidently manifest that MEAN and SD are adjacent to one another, i.e., implying only slight deviation from the obtained result and validating the superiority of represented hybrid-based scheme over other methods regarding reliability and efficacy [46], [53], [54].

VII. CONCLUSION

This research work demonstrates a hybrid computational technique that is based on variants of the evolutionary optimization algorithms combining the beneficial attributes of FDO and GA with BPs. The recommended hybrid technique is applied to several real-world NOCPs as an alternative for obtaining the optimum solution. The final experimental outcomes attained verify the capability of the presented hybrid scheme for determining superior quality solutions than the previous computational methods reported recently in the literature. The suggested approach effectively optimizes various real-world NOCP systems by the minimization of J, E_J, φ_f, and K_ψ for all three problems, including the VDP oscillator problem, CRP, and CSTCRP, respectively. Moreover, our findings were validated by statistical analysis that the proposed technique is adequate for the optimization of distinct nonlinear dynamical systems.

In the near future, we intend to utilize the presented approach for optimally solving various real-world OCPs, for instance, Free Floating Robot (FFR), time-delayed problems, and problems containing disturbances, including exogenous disturbances. Besides, the hybridization of GA and FDO with local search methods, e.g., Interior Point Algorithm (IPA) and Active Set Algorithm (ASA), will be considered. In addition, various basis functions, for example, Boubaker Polynomials, would be employed with contemporary heuristic optimization algorithms evaluating approximate outcomes for benchmark NOCPs.

ACKNOWLEDGMENT

This research work is funded by Research Supporting Project Number RSPD2023R585, King Saud University, Riyadh, Saudi Arabia.

REFERENCES

[1] K. Djendel and Z. Zhou, "On the uniqueness and stability of Mayer optimal control problem with quasi-convexity functional," *Int. J. Control, Autom. Syst.*, vol. 21, no. 1, pp. 31–39, Jan. 2023, doi: 10.1007/s12555-021-0570-6.

[2] G. Rigatos, M. Abbaszadeh, K. Busawon, Z. Gao, and J. Pomares, "A nonlinear optimal control approach for multi-DOF brachiation robots," *Int. J. Humanoid Robot.*, vol. 18, no. 5, Oct. 2021, Art. no. 2150015.

[3] Y. Liu, Y. Li, Y. Qiu, and J. Lang, "Indirect hp-pseudospectral method for nonlinear quadratic optimal control problem," in *Proc. 40th Chin. Control Conf. (CCC)*, Jul. 2021, pp. 1575–1580.

[4] K. Ratkovic, "Limitations in direct and indirect methods for solving optimal control problems in growth theory," *Industrija*, vol. 44, no. 4, pp. 19–46, 2016.

[5] O. von Stryk and R. Bulirsch, "Direct and indirect methods for trajectory optimization," *Ann. Oper. Res.*, vol. 37, no. 1, pp. 357–373, 1992.

[6] D. Wu, Y. Bai, and C. Yu, "A new computational approach for optimal control problems with multiple time-delay," *Automatica*, vol. 101, pp. 388–395, Mar. 2019.

[7] M. Alipour, M. A. Vali, and A. H. Borzabadi, "A hybrid parametrization approach for a class of nonlinear optimal control problems," *Numer. Algebra, Control Optim.*, vol. 9, no. 4, pp. 493–506, 2019.

[8] W. Jia, X. He, and L. Guo, "The optimal homotopy analysis method for solving linear optimal control problems," *Appl. Math. Model.*, vol. 45, pp. 865–880, May 2017.

[9] S. Ganjefar and S. Rezaei, "Modified homotopy perturbation method for optimal control problems using the Padé approximant," *Appl. Math. Model.*, vol. 40, nos. 15–16, pp. 7062–7081, Aug. 2016.

[10] A. Nazemi, S. Hesam, and A. Haghbin, "An application of differential transform method for solving nonlinear optimal control problems," *Comput. Methods Differ. Equc.*, vol. 3, pp. 200–217, Jan. 2015.

[11] C. Xuesong, L. Xingke, Z. Lili, and C. Shuting, "A new spectral method for the nonlinear optimal control systems," in *Proc. 36th Chin. Control Conf. (CCC)*, Jul. 2017, pp. 2495–2500, doi: 10.23919/ChiCC.2017.8027735.

[12] R. Dehghan, "A numerical method based on Cardan polynomials for solving optimal control problems," *Afrika Matematika*, vol. 29, nos. 5–6, pp. 677–687, Sep. 2018.

[13] H. Mirinejad and T. Inanc, "An RBF collocation method for solving optimal control problems," *Robot. Auto. Syst.*, vol. 87, pp. 219–225, Jan. 2017.

[14] M. Moghaddam, Y. Edrisi, and M. Lakestani, *Numerical Solution of Optimal Control Problems Using Genocchi Polynomials*. Hoboken, NJ, USA: Authorea, Jul. 2020, doi: 10.22541/au.159386613.30405839.

[15] M. El-Kady, "Legendre approximations for solving optimal control problems governed by ordinary differential equations," *Int. J. Control Sci. Eng.*, vol. 2, no. 4, pp. 54–59, Aug. 2012.

[16] Y. Edrisi-Tabri, M. Lakestani, and A. Heydari, "Two numerical methods for nonlinear constrained quadratic optimal control problems using linear B-spline functions," *Iranian J. Numer. Anal. Optim.*, vol. 6, no. 2, pp. 17–38, 2016.

[17] V. Cichella, I. Kaminer, C. Walton, N. Hovakimyan, and A. M. Pascoal, "Consistent approximation of optimal control problems using Bernstein polynomials," in *Proc. IEEE 58th Conf. Decis. Control (CDC)*, Dec. 2019, pp. 4292–4297, doi: 10.1109/CDC40024.2019.9029677.

[18] I. N. Ahmed and E. H. Ouda, "An iterative method for solving quadratic optimal control problem using scaling Boubaker polynomials," *Open Sci. J.*, vol. 5, no. 2, pp. 1–10, Jun. 2020.

[19] H. Modares and M.-B. N. Sistani, "Solving nonlinear optimal control problems using a hybrid IPSO–SQP algorithm," *Eng. Appl. Artif. Intell.*, vol. 24, no. 3, pp. 476–484, Apr. 2011.

[20] S. Nezhadhosein, A. Heydari, and R. Ghanbari, ''A modified hybrid genetic algorithm for solving nonlinear optimal control problems,'' *Math. Problems Eng.*, vol. 2015, Jan. 2015, Art. no. 139036.

[21] A. Benamor, W. Boukadida, and H. Messaoud, ''Genetic algorithm-based multi-objective design of optimal discrete sliding mode approach for trajectory tracking of nonlinear systems,'' *Proc. Inst. Mech. Eng., C, J. Mech. Eng. Sci.*, vol. 233, no. 15, pp. 5237–5252, Aug. 2019.

[22] S. Slama, A. Errachdi, and M. Benrejeb, ''Tuning artificial neural network controller using particle swarm optimization technique for nonlinear system,'' in *Deep Learning Applications*. London, U.K.: IntechOpen, 2021, doi: 10.5772/intechopen.96424.

[23] A. Shariq and K. Vaisakh, ''Optimal control of microgrid networks using gradient descent and differential evolution methods,'' *Int. Res. J. Eng. Technol.*, vol. 3, no. 7, pp. 1–10, 2016.

[24] B. Ata and R. Coban, ''Artificial bee colony algorithm based linear quadratic optimal controller design for a nonlinear inverted pendulum,'' *Int. J. Intell. Syst. Appl. Eng.*, vol. 3, no. 1, p. 1, Jan. 2015.

[25] T. Cuong-Le, H.-L. Minh, S. Khatir, M. A. Wahab, M. T. Tran, and S. Mirjalili, ''A novel version of Cuckoo search algorithm for solving optimization problems,'' *Expert Syst. Appl.*, vol. 186, Dec. 2021, Art. no. 115669.

[26] S. Liaquat, M. S. Fakhar, S. A. R. Kashif, A. Rasool, O. Saleem, M. F. Zia, and S. Padmanaban, ''Application of dynamically search space squeezed modified firefly algorithm to a novel short term economic dispatch of multi-generation systems,'' *IEEE Access*, vol. 9, pp. 1918–1939, 2021, doi: 10.1109/ACCESS.2020.3046910.

[27] S. Liaquat, M. S. Fakhar, S. A. R. Kashif, and O. Saleem, ''Statistical analysis of accelerated PSO, firefly and enhanced firefly for economic dispatch problem,'' in *Proc. 6th Int. Conf. Renew. Energy, Gener. Appl. (ICREGA)*, Al Ain, United Arab Emirates, Feb. 2021, pp. 106–111, doi: 10.1109/ICREGA50506.2021.9388303.

[28] S. Liaquat, M. S. Fakhar, S. A. R. Kashif, A. Rasool, O. Saleem, and S. Padmanaban, ''Performance analysis of APSO and firefly algorithm for short term optimal scheduling of multi-generation hybrid energy system,'' *IEEE Access*, vol. 8, pp. 177549–177569, 2020, doi: 10.1109/ACCESS.2020.3027436.

[29] M. H. Sulaiman, Z. Mustaffa, and M. I. M. Rashid, ''An application of teaching-learning-based optimization for solving the optimal power flow problem with stochastic wind and solar power generators,'' *Results Control Optim.*, vol. 10, Jan. 2023, Art. no. 100187.

[30] A. Daraz, S. Abdullah, H. Mokhlis, I. U. Haq, G. Fareed, and N. N. Mansor, ''Fitness dependent optimizer-based automatic generation control of multi-source interconnected power system with non-linearities,'' *IEEE Access*, vol. 8, pp. 100989–101003, 2020.

[31] A. Daraz, S. A. Malik, A. Waseem, A. T. Azar, I. U. Haq, Z. Ullah, and S. Aslam, ''Automatic generation control of multi-source interconnected power system using FOI-TD controller,'' *Energies*, vol. 14, no. 18, p. 5867, Sep. 2021.

[32] G. F. Laghari, S. A. Malik, A. Daraz, A. Ullah, T. Alkhalifah, and S. Aslam, ''A numerical approach for solving nonlinear optimal control problems using the hybrid scheme of fitness dependent optimizer and Bernstein polynomials,'' *IEEE Access*, vol. 10, pp. 50298–50313, 2022, doi: 10.1109/ACCESS.2022.3173285.

[33] X. Ma, ''Optimal control of whole network control system using improved genetic algorithm and information integrity scale,'' *Comput. Intell. Neurosci.*, vol. 2022, Jan. 2022, Art. no. 9897894.

[34] H. Haghighi, S. H. Sadati, S. M. M. Dehghan, and J. Karimi, ''Hybrid form of particle swarm optimization and genetic algorithm for optimal path planning in coverage mission by cooperated unmanned aerial vehicles,'' *J. Aerosp. Technol. Manage.*, vol. 12, pp. 1–10, Sep. 2020.

[35] S. A. Malik, I. M. Qureshi, M. Amir, A. N. Malik, and I. Haq, ''Numerical solution to generalized Burgers'-Fisher equation using exp-function method hybridized with heuristic computation,'' *PLoS ONE*, vol. 10, no. 3, Mar. 2015, Art. no. e0121728.

[36] S. Mahfoud, A. Derouich, N. El Ouanjli, N. V. Quynh, and M. A. Mossa, ''A new hybrid ant colony optimization based PID of the direct torque control for a doubly fed induction motor,'' *World Electr. Vehicle J.*, vol. 13, no. 5, p. 78, Apr. 2022.

[37] P. Stodola, K. Michenka, J. Nohel, and M. Rybanský, ''Hybrid algorithm based on ant colony optimization and simulated annealing applied to the dynamic traveling salesman problem,'' *Entropy*, vol. 22, no. 8, p. 884, Aug. 2020.

[38] R. K. Khadanga, A. Kumar, and S. Panda, ''A modified grey wolf optimization with cuckoo search algorithm for load frequency controller design of hybrid power system,'' *Appl. Soft Comput.*, vol. 124, Jul. 2022, Art. no. 109011.

[39] A. Taeib, H. Salhi, and A. Chaari, ''Constrained cuckoo search algorithm and Takagi–Sugeno fuzzy models for predictive control,'' *Trans. Inst. Meas. Control*, vol. 43, no. 5, pp. 1088–1101, Mar. 2021.

[40] A. S. Alghamdi, ''A hybrid firefly–JAYA algorithm for the optimal power flow problem considering wind and solar power generations,'' *Appl. Sci.*, vol. 12, no. 14, p. 7193, Jul. 2022.

[41] M. A. El-Shorbagy and A. M. El-Refaey, ''A hybrid genetic–firefly algorithm for engineering design problems,'' *J. Comput. Design Eng.*, vol. 9, no. 2, pp. 706–730, Apr. 2022.

[42] M. Dastan, S. Shojaee, S. Hamzehei-Javaran, and V. Goodarzimehr, ''Hybrid teaching-learning-based optimization for solving engineering and mathematical problems,'' *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 44, no. 9, p. 431, 2022.

[43] T. Ali, S. A. Malik, A. Daraz, S. Aslam, and T. Alkhalifah, ''Dandelion optimizer-based combined automatic voltage regulation and load frequency control in a multi-area, multi-source interconnected power system with nonlinearities,'' *Energies*, vol. 15, no. 22, p. 8499, Nov. 2022.

[44] D. K. Abbas, T. A. Rashid, K. H. Bacanin, and A. Alsadoon, ''Using fitness dependent optimizer for training multi-layer perceptron,'' *J. Internet Technol.*, vol. 22, no. 7, pp. 1575–1585, Dec. 2021.

[45] P. C. Chiu, A. Selamat, O. Krejcar, and K. K. Kuok, ''Hybrid sine cosine and fitness dependent optimizer for global optimization,'' *IEEE Access*, vol. 9, pp. 128601–128622, 2021.

[46] A. Ullah, S. A. Malik, and K. S. Alimgeer, ''Evolutionary algorithm based heuristic scheme for nonlinear heat transfer equations,'' *PLoS ONE*, vol. 13, no. 1, Jan. 2018, Art. no. e0191103.

[47] J. M. Abdullah and T. Ahmed, ''Fitness dependent optimizer: Inspired by the bee swarming reproductive process,'' *IEEE Access*, vol. 7, pp. 43473–43486, 2019.

[48] H. M. Mohammed and T. A. Rashid, ''Chaotic fitness-dependent optimizer for planning and engineering design,'' *Soft Comput.*, vol. 25, no. 22, pp. 14281–14295, Nov. 2021.

[49] D. S. Abdul-Minaam, W. M. E. S. Al-Mutairi, M. A. Awad, and W. H. El-Ashmawi, ''An adaptive fitness-dependent optimizer for the one-dimensional bin packing problem,'' *IEEE Access*, vol. 8, pp. 97959–97974, 2020.

[50] C. Kielas-Jensen, V. Cichella, T. Berry, I. Kaminer, C. Walton, and A. Pascoal, ''Bernstein polynomial-based method for solving optimal trajectory generation problems,'' *Sensors*, vol. 22, no. 5, p. 1869, Feb. 2022, doi: 10.3390/s22051869.

[51] F. Ghomanjani, S. Noeiaghdam, and S. Micula, ''Application of transcendental Bernstein polynomials for solving two-dimensional fractional optimal control problems,'' *Complexity*, vol. 2022, Apr. 2022, Art. no. 4303775, doi: 10.1155/2022/4303775.

[52] Z. S. Abo-Hammour, A. G. Asasfeh, A. M. Al-Smadi, and O. M. K. Alsmadi, ''A novel continuous genetic algorithm for the solution of optimal control problems,'' *Optim. Control Appl. Methods*, vol. 32, no. 4, pp. 414–432, Jul. 2011.

[53] C. Andrade, ''Understanding the difference between standard deviation and standard error of the mean, and knowing when to use which,'' *Indian J. Psychol. Med.*, vol. 42, no. 4, pp. 409–410, Jul. 2020.

[54] H. Darling, ''Do you have a standard way of interpreting the standard deviation? A narrative review,'' *Cancer Res., Statist., Treat.*, vol. 5, no. 4, p. 728, 2022.

GHULAM FAREED LAGHARI was born in Pakistan, in 1990. He received the B.S. and M.S. degrees in electronic engineering from International Islamic University Islamabad (IIUI), Pakistan, in 2012 and 2016, respectively, where he is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering (DECE). His research interests include power electronics, renewable energy technologies, fuzzy control systems, robust control systems, optimization problems, and nature-inspired metaheuristic algorithms to solve emerging nonlinear problems in the engineering domain.

SUHEEL ABDULLAH MALIK received the B.E. degree in electrical and electronics from Bangalore University (BU), India, in 1997, the M.S. degree in electronic engineering from Muhammad Ali Jinnah University (MAJU), Pakistan, and the Ph.D. degree in electronic engineering from International Islamic University Islamabad (IIUI), Pakistan. Since 2007, he has been with IIUI, where he is currently an Associate Professor with the Department of Electrical and Computer Engineering (DECE). He has authored more than 50 journals and conference papers to his credit. His research interests include nature-inspired computational techniques, control systems, power system operation and control, and the numerical investigation of nonlinear dynamical systems. He was a member/chair of many international conferences and a reviewer of several research journals.

IRFAN AHMED KHAN received the B.S. degree in electronics from the Sir Syed University of Engineering and Technology, Pakistan, in 2007, and the M.S. degree in electrical engineering from the King Fahd University of Petroleum and Minerals, Saudi Arabia, in 2012. He is currently pursuing the Ph.D. degree with Universiti Malaya. His research interests include load frequency control, power system modeling and optimization, control systems, hybrid energy storage systems, wide-area measurements, FACTS, and renewable energy.

AMIL DARAZ was born in Pakistan, in 1989. He received the B.E. degree in electronics engineering from COMSATS University Islamabad, Pakistan, in 2012, and the M.S. degree in power and control engineering and the Ph.D. degree in electrical engineering from International Islamic University Islamabad (IIUI), Pakistan. His research interests include control systems, power system operation and control, and the optimization of nonlinear problems.

SALMAN A. ALQAHTANI is currently a Full Professor with the Department of Computer Engineering, College of Computer and Information Sciences, King Saud University. His main research interests include radio resource management for wireless and cellular networks (4G, 5G, the IoT, Industry 4.0, LTE, LTE-advanced, femtocell, cognitive radio, and cyber sovereignty) with a focus on call admission control, packet scheduling, radio resource sharing, and the quality of service guarantees for data services.

HAYAT ULLAH received the B.S. and M.S. degrees in electronic engineering from International Islamic University Islamabad (IIUI), Pakistan, and the Ph.D. degree from the Department of Electrical and Computer Engineering (DECE), IIUI. He was a Visiting Ph.D. Research Scholar with Prof. Ivan Selesnick with the Department of Electrical and Computer Engineering, Tandon School of Engineering, New York University, New York, NY, USA, from December 2019 to December 2020. He is currently a Visiting Lecturer with the Department of Electrical Engineering, IIUI. His research interests include optimization, compressed sensing, signal and image processing (e.g., sparse signal processing techniques, biomedical signal and image processing, and source separation), time-frequency analysis, computer vision, and machine learning. He is an Indigenous Ph.D. Fellow of the Higher Education Commission of Pakistan. He received the ICT Research and Development Scholarship from IIUI.