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RESEARCH ARTICLE

# Membership Score of an Interval-Valued Pythagorean Fuzzy Numbers and Its Applications

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**ABSTRACT** The main aim of the paper is to define a new Membership score function on the class of interval-valued Pythagorean fuzzy numbers, which can overcome the drawbacks of existing familiar ranking methods. In this paper, firstly, we show the limitations of various ranking functions in ordering/ comparing any two arbitrary interval-valued Pythagorean fuzzy numbers in detail. Secondly, we define a new Membership score function on the class of interval-valued Pythagorean fuzzy numbers and study their properties. Then we compare our proposed method with many other different existing methods to show the efficacy of the proposed method. Finally, we show the applicability of the proposed Membership score function in solving interval-valued Pythagorean fuzzy multi-criteria decision-making problems using a numerical example.

**INDEX TERMS** Interval-valued Pythagorean fuzzy numbers, improved accuracy score function, membership score function, MCDM.

## I. INTRODUCTION

Zadeh [19] introduced the idea of fuzzy sets as a generalization of classical sets. The need for the fuzzy set arose because the problem of incorrect information could not be resolved with his crisp sets without loss of information. The main idea of fuzzy sets is that an object's degrees of membership and non-membership are not always 0 or 1, which can take any value between 0 and 1. Furthermore, the main characteristic of fuzzy sets is that the sum of the membership and non-membership values of each object (element) in the non-empty fuzzy subset must be exactly one. That is the information about the fuzzy object to be complete (100 %). In a real problem/scenario, we may not get 100 % information about the object. This problem leads researchers to generalize the fuzzy set further. As a result, Atanassov [2] proposed the intuitionistic fuzzy set (IFS) as a generalization of the concept of fuzzy sets (FS). Intuitionistic fuzzy sets, introduced by Atanassov, are more efficient than fuzzy sets in modelling problems with incomplete information. This is because, for IFS, the sum of membership and non-membership for each element in the IFS is flexible. The efficiency obtained using IFS was further improved by using the Pythagorean fuzzy set (PFS) introduced by Yager [16], [17], [18]. The PFS was obtained from the IFS by relaxing the IFS condition to "the sum of the squares of the membership and non-membership degrees of the elements included in the PFS is less than or equal to 1". Peng and Yang [14], [15] were strongly motivated by the idea of Pythagorean fuzzy sets and proposed the concept of interval-valued Pythagorean fuzzy sets (IVPFS) as a generalization of Pythagorean fuzzy sets. In real-world scenarios, many real-world problems contain incomplete and ambiguous information. This kind of problem is better modelled using interval-valued Pythagorean fuzzy sets than Pythagorean fuzzy sets, intuitionistic fuzzy sets, and fuzzy sets. Also, it may not be possible to give an exact intuitionistic fuzzy value or interval-valued intuitionistic fuzzy value for every problem for which the information is incomplete in the actual problem. For example, if a decision maker (DM) defines alternative membership (support) value as [0.5, 0.6] and alternative non-membership value as [0.55, 0.65]. where the sum of the upper bound of the membership value (0.6) and upper bound of the non-membership value (0.65) is greater than 1, and the declared value is neither IFS nor IVIFS, but $0.6^2 + 0.65^2 = 0.7825 \leq 1$.

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Therefore, it can be considered as IVPFS. That is, the interval-valued Pythagorean fuzzy set is more effective in handling incomplete and inaccurate information than the interval-valued intuitionistic fuzzy set and the intuitionistic fuzzy set. The main advantage of IVPFS is that it gives better modelling of Decision problems involving imprecise and incomplete information than interval-valued intuitionistic fuzzy numbers. Decision problems with interval-valued Pythagorean fuzzy information require a ranking principle for making a better model, which makes the ranking issues of interval-valued Pythagorean fuzzy numbers (IVPFNs) more important in the literature. After introducing IVPFNs, many researchers worldwide tried to discriminate any two arbitrary IVPFNs by defining various ranking functions. However, unfortunately, every time, the available methods have some drawbacks, which are rectified by the new method, which is where the research gap exists [10]. A few researchers have achieved total ordering on the set of intuitionistic fuzzy numbers [12], [13]. Peng and Yang [15] introduce the idea of score and accuracy function on the set of IVPFNs to compare arbitrary IVPFNs. They have also discussed various properties of the proposed score functions and studied their applicability in solving an MCDM problem. Garg [4], [5] has tried to give a better ranking method for comparing arbitrary IVPFNs, which can overcome the drawbacks of previously available methods. Unfortunately, his method also fails to satisfy some important properties of the ranking functions. Hence the main aim of this work is to point out the limitations of a few familiar existing methods and develop a new Membership score for discriminating arbitrary IVPFNs, which can result in the drawbacks of familiar existing methods [5], [15]. Jeevaraj [9] has introduced a new distance-based similarity measure on the set of IVIFNs by using a non-hesitance score function defined on the set of IVIFNs and solved a pattern recognition problem using the proposed similarity measure. Jeevaraj and Abhijit [8] have proposed a new score function on the set of IVIFNs to compare arbitrary IVIFNs. Also, they have shown the applicability of the proposed score function in solving a multi-criteria decision-making problem. This method overcomes many drawbacks of the previous method. However, Jeevraj and Abhijit [8] ranking principle does not define a total ordering on the set of IVIFNs. Abhijit et al. [1] have used trapezoidal intuitionistic fuzzy numbers to model a real-life problem related to the selection of resilient suppliers in manufacturing industries. Different ranking methods are available on the set of intuitionistic fuzzy numbers [8], [9], [11] and their applications in decision-making. However, all the ranking methods discussed above do not define a total ordering on the set IVIFNs and IVPFNs.

We organize the entire paper in the following manner. After the introduction, we give a few basic Mathematical definitions in Section II. Section III points out the limitations of the improved accuracy score function and discusses the drawbacks of various ranking functions defined on the class of IVPFNs. Section IV discusses the new membership score function and its Mathematical properties. In section IV, We show the efficacy of a membership score function in ranking arbitrary IVPFNs by numerical examples, which can be done by showing the places in which few other existing ranking functions fail. We develop a multi-criteria decision-making (MCDM) algorithm for solving MCDM problems with Pythagorean Fuzzy information in Section V, and also we show the execution part using a numerical example. Finally, we give conclusions in Section VI.

## II. PRELIMINARIES

In this section, we give some basic definitions.

*Definition 1 (Atanassov, [2]):* Let $X$ be a nonempty set. An intuitionistic fuzzy set $A$ in $X$ is defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\}$, where $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1], x \in X$ with the conditions $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x), \nu_A(x) \in [0, 1]$ denote the degree of membership and non-membership of $x$ to lie in $A$ respectively. For each intuitionistic fuzzy subset $A$ in $X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy degree of $x$ to lie in $A$.

*Definition 2 (Yager [16]):* Let $X$ be a nonempty set. A Pythagorean fuzzy set $P$ in $X$ is defined by $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle \,|x \in X\}$, where $\mu_P(x) : X \to [0, 1]$ and $\nu_P(x) : X \to [0, 1], x \in X$ with the conditions $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1, \forall x \in X$. The numbers $\mu_P(x), \nu_P(x) \in [0, 1]$ denote the degree of membership and non-membership of $x$ to lie in $P$ respectively. For each Pythagorean fuzzy set $P$ in $X$, $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 + (\nu_P(x))^2}$ is called the indeterminacy degree of $x$ to lie in $A$. For convenience Peng and Yang [15] called $(\mu_P(x), \nu_P(x))$ a Pythagorean Fuzzy Number, denoted by $p = (\mu_P, \nu_P)$

*Definition 3 (Atanassov & Gargov, [3]):* Let $D[0, 1]$ be the set of all closed sub-intervals of the interval $[0, 1]$. An interval-valued intuitionistic fuzzy set on a set $X \neq \phi$ is an expression given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where $\mu_A : X \to D[0, 1], \nu_A : X \to D[0, 1]$ with the condition $0 < sup_x \mu_A(x) + sup_x \nu_A(x) \leq 1$.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the membership and non-membership degree of the element $x$ to present in the set $A$. Thus for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals whose lower and upper endpoints are, respectively, denoted by $\mu_{A_L}(x)$, $\mu_{A_U}(x)$ and $\nu_{A_L}(x)$, $\nu_{A_U}(x)$. We denote

$$A = \left\{\left\langle x, [\mu_{A_L}(x), \mu_{A_U}(x)], [\nu_{A_L}(x), \nu_{A_U}(x)]\right\rangle : x \in X\right\}$$

where $0 < \mu_A(x) + \nu_A(x) \leq 1$

For each element $x \in X$, we can compute the unknown degree (hesitance degree) of belongingness $\pi_A(x)$ to $A$ as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = [1 - \mu_{A_U}(x) - \nu_{A_U}(x), 1 - \mu_{A_L}(x) - \nu_{A_L}(x)]$. We denote the set of all IVIFSs in $X$ by IVIFS$(X)$. $A = ([a, b], [c, d])$ denotes an interval-valued intuitionistic fuzzy number is denoted by for convenience.

*Definition 4 (Peng and Yang, [15]):* Let $D[0,1]$ be the set of all closed sub-intervals of the interval $[0,1]$. An interval-valued Pythagorean fuzzy set on a set $X \neq \phi$ is an expression given by

$$P = \{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$$

where $\mu_P : X \to D[0,1], \nu_P : X \to D[0,1]$ with the condition $0 < sup_x \mu_P(x) + sup_x \nu_P(x) \leq 1$.

The intervals $\mu_P(x)$ and $\nu_P(x)$ denote the membership and non-membership degree of the element $x$ to present in the set $P$. Thus for each $x \in X$, $\mu_P(x)$ and $\nu_P(x)$ are closed intervals whose lower and upper endpoints are, respectively, denoted by $\mu_{P_L}(x)$, $\mu_{P_U}(x)$ and $\nu_{P_L}(x)$, $\nu_{P_U}(x)$. We denote

$$P = \left\{\left\langle x, [\mu_{P_L}(x), \mu_{P_U}(x)], [\nu_{P_L}(x), \nu_{P_U}(x)]\right\rangle : x \in X\right\}$$

where $0 < \mu_P(x) + \nu_P(x) \leq 1$.

For each element $x \in X$, we can compute the unknown degree (hesitance degree) of belongingness $\pi_P(x)$ to $P$ as

$$\pi_P(x) = 1 - \mu_P(x) - \nu_P(x)$$
$$= [\sqrt{1-(\mu_{P_U}(x))^2-(\nu_{P_U}(x))^2}, \sqrt{1-(\mu_{P_L}(x))^2-(\nu_{P_L}(x))^2}].$$

We denote the set of all IVPFSs in $X$ by IVPFS($X$). $A = ([a,b],[c,d])$ denotes an interval-valued Pythagorean fuzzy number for convenience.

*Definition 5 (Peng and Yang, [15]):* Let $P_1 = ([a_1,b_1],[c_1,d_1])$, $P_2 = ([a_2,b_2],[c_2,d_2])$ be two IVPFNs. Then their relations are defined as follows,

1) $P_1 = P_2$ *iff* $a_1 = a_2, b_1 = b_2, c_1 = c_2,$ *and* $d_1 = d_2$
2) $P_1 \prec P_2$ *iff* $a_1 \leq a_2, b_1 \leq b_2, c_1 \geq c_2,$ *and* $d_1 \geq d_2$

*Definition 6 (Peng and Yang, [15]):* For any IVPFN $P = ([a,b],[c,d])$, the score function of $P$ is defined as follows,

$$s(P) = \frac{a^2+b^2-c^2-d^2}{2}, \quad s(P) \in [-1,1].$$

*Definition 7 (Peng and Yang, [15]):* For any IVPFN $P = ([a,b],[c,d])$, the accuracy function of $P$ is defined as follows,

$$a(P) = \frac{a^2+b^2+c^2+d^2}{2}, \quad a(P) \in [0,1].$$

*Definition 8 (Peng and Yang, [15]):* For any two IVPFNs $P_1 = ([a_1,b_1],[c_1,d_1])$, $P_2 = ([a_2,b_2],[c_2,d_2])$,

1) if $s(P_1) > s(P_2)$, then $P_1 \succ P_2$
2) if $s(P_1) = s(P_2)$, then
   - if $a(P_1) > a(P_2)$, then $P_1 \succ P_2$
   - if $a(P_1) = a(P_2)$, then $P_1 = P_2$.

*Definition 9 (Garg 2016, [4]):* Let $A_i = ([a_i,b_i],[c_i,d_i])$ be the collection of IVPFN. Then the aggregated value by using an interval-valued Pythagorean fuzzy weighted averaging average (IPFWA) operator is defined as

$$IPFWA(A_i) = \prod_{i=1}^{n} w_i A_i$$
$$= \langle [\sqrt{1-\prod_{i=1}^{n}(1-a_i^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-b_i^2)^{w_i}}], [\prod_{i=1}^{n}(c_i)^{w_i}, \prod_{i=1}^{n}(d_i)^{w_i}]\rangle,$$

where $w_i$ is the weight of $A_i (i = 1,2,\ldots,n)$ such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

*Definition 10 (Garg 2016, [4]):* Let $A_i = ([a_i,b_i],[c_i,d_i])$ be the collection of IVPFN. Then the aggregated value by using an interval-valued Pythagorean fuzzy weighted geometric average (IPFWG) operator is defined as,

$$IPFWG(A_i) = \prod_{i=1}^{n} A_i^{w_i}$$
$$= \langle [\prod_{i=1}^{n}(a_i)^{w_i}, \prod_{i=1}^{n}(b_i)^{w_i}], [\sqrt{1-\prod_{i=1}^{n}(1-c_i{}^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-d_i{}^2)^{w_i}}]\rangle,$$

where $w_i$ is the weight of $A_i (i = 1,2,\ldots,n)$ such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

## III. LIMITATIONS OF SCORE AND ACCURACY FUNCTION

In this section, we investigate the limitations of the Score and Accuracy function ( [15]) defined on the class of IVPFNs.

*Definition 11 (Peng and Yang, [15]):* For any IVPFN $P = ([a,b],[c,d])$, the score function of $P$ is defined as follows,

$$s(P) = \frac{a^2+b^2-c^2-d^2}{2}, \quad s(P) \in [-1,1].$$

*Definition 12 (Peng and Yang, [15]):* For any IVPFN $P = ([a,b],[c,d])$, the accuracy function of $P$ is defined as follows,

$$a(P) = \frac{a^2+b^2+c^2+d^2}{2}, \quad a(P) \in [0,1].$$

*Definition 13 (Peng and Yang, [15]):* For any two IVPFNs $P_1 = ([a_1,b_1],[c_1,d_1])$, $P_2 = ([a_2,b_2],[c_2,d_2])$,

1) if $s(P_1) > s(P_2)$, then $P_1 \succ P_2$
2) if $s(P_1) = s(P_2)$, then
   - if $a(P_1) > a(P_2)$, then $P_1 \succ P_2$
   - if $a(P_1) = a(P_2)$, then $P_1 = P_2$.

The ranking principle given in Definition 13 has the following Limitations.

1) If $P_1 = ([a_1,b_1],[0,0])$, then both $s(P_1) = a(P_1)$. That is, the accuracy function does not play any role in the ranking. In this case, Garg's improved accuracy function [5] can perform well.

2) If $P_1 = ([a_1, b_1], [0, 0]), P_2 = ([\sqrt{a_1^2 - \epsilon}, \sqrt{b_1^2 + \epsilon}], [0, 0])$, where $0 \leq \epsilon \leq a_1^2$ and $b_1^2 + \epsilon \leq 1$ are any two IVPFNs, then $s(P_1) = s(P_2) = a(P_1) = a(P_2) = \frac{{a_1}^2 + {b_1}^2}{2}$ which implies $P_1 = P_2$. However, it is very clear that $P_1 \neq P_2$.
Note: For different values of $a_1, b_1$ and $\epsilon$, we have infinite IVPFNs, which are not differentiated using Peng and Yang's ranking function. This shows the inefficiency of Peng and Yang's ranking principle. In these cases, the improved accuracy function performs better.
3) If $P_1 = ([\sqrt{a_1}, \sqrt{b_1}], [\sqrt{a_1 - \epsilon_1}, \sqrt{b_1 + \epsilon_1}])$, $P_2 = ([\sqrt{a_1 - \epsilon_2}, \sqrt{b_1 + \epsilon_2}], [\sqrt{a_1 - \epsilon_3}, \sqrt{b_1 + \epsilon_3}])$ are any two IVPFN then both $s(P_1) = s(P_2) = 0, a(P_1) = a(P_2) = a_1 + b_1 \Rightarrow P_1 = P_2$. But $P_1 \neq P_2$.
Note: For different values of $a_1, b_1, \epsilon_1, \epsilon_2$ and $\epsilon_3$, we have infinite IVPFNs which are not discriminated using Peng and Yang's ranking function. This shows the inefficiency of Peng and Yang's ranking principle. In these cases, the improved accuracy function [5] performs better.
4) If $P_1 = ([0, 0], [c_1, d_1]), P_2 = ([0, 0], [\sqrt{c_1^2 - \epsilon_1}, \sqrt{d_1^2 + \epsilon_1}])$, $P_3 = ([0, 0], [\sqrt{c_1^2 + \epsilon_2}, \sqrt{d_1^2 - \epsilon_2}])$ where $0 \leq \epsilon_1 \leq c_1^2, 0 \leq \epsilon_2 \leq d_1^2, \sqrt{d_1^2 + \epsilon_1} \leq 1$ and $\sqrt{d_1^2 - \epsilon_2} \geq \sqrt{c_1^2 + \epsilon_2}$ are any three IVPFNs, then $s(P_1) = s(P_2) = s(P_3) = \frac{-{c_1}^2 - {d_1}^2}{2}$, $a(P_1) = a(P_2) = a(P_3) = \frac{{c_1}^2 + {d_1}^2}{2}$ which implies $P_1 = P_2 = P_3$. But it is very clear that $P_1 \neq P_2 \neq P_3$. For different values of $c_1, d_1, \epsilon_1, \epsilon_2$ and $\epsilon_3$, we have infinite IVPFNs that are not discriminated properly. In these places, both Peng and Yang's ranking function and Garg's improved accuracy function [5] failed to rank different IVPFNs.

### A. LIMITATIONS OF IMPROVED ACCURACY FUNCTION OF [5])

In this subsection, we discuss the limitations of the improved accuracy function ( [5]) defined on the class of IVPFNs.

The ranking principle in Definition 13 has the following Limitations.

1) If $P_1 = ([a_1, a_1], [\sqrt{1 - a_1^2}, \sqrt{1 - a_1^2}])$, $P_2 = ([0, a_1], [0, 0])$ are any two IVPFNs, then the improved accuracy function $K(P_1) = a_1^2 = K(P_2) \Rightarrow P_1 = P_2$. However, it is evident that $P_1 \neq P_2$. Hence we need a better method to overcome the limitation of improved accuracy function [5].
2) If $P_1 = ([0, 0], [c_1, d_1]), P_2 = ([0, 0], [c_2, d_2])$ are any two IVPFNs, then $K(P_1) = K(P_2) = 0$ which implies $P_1 = P_2$. But it is very clear that $P_1 \neq P_2$.
Note: For different values of $a_1, b_1$, we have infinite IVPFNs which are not differentiated using the improved accuracy function [5]. This shows the inefficiency of the improved accuracy score [5].
3) If $P_1 = ([a_1, a_1], [\sqrt{1 - a_1^2}, \sqrt{1 - a_1^2}])$, $P_2 = ([0, a_1], [0, \sqrt{1 - a_1^2}])$ are any two IVPFNs, then the improved accuracy function $K(P_1) = a_1^2 = K(P_2) \Rightarrow P_1 = P_2$. But $P_1 \neq P_2$. This shows the inefficiency of the improved accuracy function.
4) Let $M_1 = ([0, 1], [0, 0]), M_2 = ([1, 1], [0, 0])$ be any two IVPFNs, then the improved accuracy function $K(M_1) = K(M_2) = 1$ which implies $M_1 = M_2$. However, $M_1 \neq M_2$. Actually, $M_1$ must be ranked better (Since there is no non-membership and hesitant degree). This shows the anti-intuitive case of the improved accuracy function.
5) Let $A = ([0, b], [0, d]), B = ([0, b], [0, 0])$ be any two IVPFNs, then the improved accuracy function $K(A) = K(B) = b^2$ which implies $A = B$. However, $M_1 \neq M_2$. This shows the illogicality of the improved accuracy function [5].

Hence from the above examples, we conclude that Garg's method [5] outperforms Yang and Peng's ranking principle but which cannot perform well for all the cases; therefore, there is a need for introducing a new score function which can improve the ranking scenario better. By keeping all the limitations (which we have identified above) in mind, in the next section, we introduce a new Membership Score function which overcomes all the identified drawbacks.

## IV. MEMBERSHIP SCORE OF AN INTERVAL-VALUED PYTHAGOREAN FUZZY NUMBERS

In this section, firstly, we introduce a new membership score function and the ranking principle for comparing arbitrary IVPFNs and studying some of their properties. Then we discuss the efficacy of the proposed membership score function in satisfying the limitations identified in Section III.

*Definition 14:* Let $A = ([a_1, b_1], [c_1, d_1]), B = ([a_2, b_2], [c_2, d_2]) \in IVIPN. A \prec B$ if $a_1^2 \leq a_2^2, b_1^2 \leq b_2^2, c_1^2 \geq c_2^2$ and $d_1^2 \geq d_2^2$.

*Definition 15:* Let $P = \langle [a_1, b_1], [c_1, d_1] \rangle \in IVPFN$. Then the membership score function for $P$ is defined as $J(P) = \frac{a_1^2 + b_1^2 - c_1^2 - d_1^2 + a_1^2 b_1^2 + c_1^2 d_1^2}{3}$.

*Definition 16:* Let $A, B \in IVPFN$.

- If $J(A) > J(B)$ then $A > B$
- If $J(A) < J(B)$ then $A < B$
- If $J(A) = J(B)$ then $A \approx B$ ($A$ and $B$ are considered equal)

Proofs of the following propositions are immediate from definition 15. Hence they are omitted.

*Proposition 1:* For any real number $r = ([r, r], [r, r]) \in [0, 0.7], J(r) = \frac{2r^4}{3}$.

*Proposition 2:* If $P = (a, 1 - a) = ([a, a], [1 - a, 1 - a])$ is any fuzzy number, then $J(A) = \frac{2a^2 - 2(1-a)^2 + a^4 + (1-a)^4}{3}$.

*Proposition 3:* 1) Let $A = ([1, 1], [0, 0])$ be an IVPFN. Then $J(A) = 1$.
2) Let $A = ([0, 0], [1, 1])$ be an IVPFN. Then $J(A) = -\frac{1}{3}$.

*Proposition 4:* For any subset, $P = ([a, b], [a, b]), \forall a, b \in [0, 0.7]$, the membership score $J$ of $A$ is, $J(A) = \frac{2a^2b^2}{3}$.

*Proposition 5:* If $M : IVPFN \rightarrow [0, 1]$, then

- the inverse image of 0 is $([0, 0], [0, 0])$.
- the inverse image of 1 is $([1, 1], [0, 0])$.

Note: Here, IVPFN is the set of all interval-valued Pythagorean Fuzzy numbers. i.e., $IVPFN \subset [0, 1] \times [0, 1]$ with the sum of squares of the upper bound of membership and non-membership interval must be less than or equal to 1.

*Theorem 1:* Let $A, B \in IVPFN$. If $A \prec B$, then $J(A) \leq J(B)$.

*Proof:* Let $A, B \in IVPFN$.

$$\text{Assume: } A \prec B \Rightarrow a_1^2 \leq a_2^2, b_1^2 \leq b_2^2, c_1^2 \geq c_2^2, d_1^2 \geq d_2^2 \text{ (From definition 14)} \tag{1}$$

We claim that $J(A) \leq J(B) \Rightarrow J(B) - J(A) \geq 0$.

$$\begin{aligned} &3(J(B) - J(A)) \\ &= (a_2^2 + b_2^2 - c_2^2 - d_2^2 + a_2^2 b_2^2 + c_2^2 d_2^2) \\ &\quad - (a_1^2 + b_1^2 - c_1^2 - d_1^2 + a_1^2 b_1^2 + c_1^2 d_1^2) \\ &= (a_2^2 - a_1^2) + (b_2^2 - b_1^2) + (c_1^2 - c_2^2) + (d_1^2 - d_2^2) \\ &\quad + (a_2^2 b_2^2 - a_1^2 b_1^2) + (c_2^2 d_2^2 - c_1^2 d_1^2). \end{aligned} \tag{2}$$

Add and subtract $a_1^2 b_2^2$ from (2), we get, $3(J(A) - J(B)) = (a_2^2 - a_1^2) + (b_2^2 - b_1^2) + (c_1^2 - c_2^2) + (d_1^2 - d_2^2) + (a_2^2 b_2^2 - a_1^2 b_2^2 + a_1^2 b_2^2 - a_1^2 b_1^2) + (c_2^2 d_2^2 - c_1^2 d_1^2) \Rightarrow 3(J(A) - J(B)) = (a_2^2 - a_1^2) + (b_2^2 - b_1^2) + (c_1^2 - c_2^2) + (d_1^2 - d_2^2) + b_2^2(a_2^2 - a_1^2) + a_1^2(b_2^2 - b_1^2) + (c_2^2 d_2^2 - c_1^2 d_1^2)$ Since from Equation (1), we know that every term of the above sum is $\geq 0$. Hence the entire sum is greater than or equal to 0.

Hence $(J(B) - J(A)) \geq 0$. Hence the proof.

In the following example, we show how the proposed membership score function is better in comparing arbitrary IVPFNs. Limitations 1 and 2 discussed in subsection III are rectified by the proposed membership score function, which can be seen from example 1.

*Example 1:* Let us consider $A_1 = ([0.4, 0.5], [0, 0])$, $A_2 = ([\sqrt{0.4^2 - 0.14}, \sqrt{0.5^2 + 0.14}], [0, 0])$ be any two IVPFNs. If we apply Peng and Yang's ranking function (Definition 13) to the considered IVPFNs $A_1$ and $A_2$, then we get $s(A_1) = a(A_1) = 0.205$ and $s(A_2) = a(A_2) = 0.1392$ which implies that $A_1$ and $A_2$ are equal. But $A_1 \neq A_2$. If we apply our Membership score $J$ to $A_1$ and $A_2$, then we get $J(A_1) = 0.15 > J(A_2) = 0.1392$ which implies that $A_1$ is grater than $A_2$ ($A_1 > A_2$). This result favours human intuition. Hence, Limitation 1 and Limitation 2 of subsection III are overcome by the proposed membership score.

Limitation 3 of subsection III can be improved by the proposed membership score, which we can see from the example 2.

*Example 2:* Let $A = ([\sqrt{0.35}, \sqrt{0.45}], [\sqrt{0.25}, \sqrt{0.55}])$, $B = ([\sqrt{0.2}, \sqrt{0.6}], [\sqrt{0.4}, \sqrt{0.4}])$ be two IVPFN. By using Peng and Yang's ranking function (Definition 13), we get $s(A) = s(B) = 0$ and $a(A) = a(B) = 0.8$, which implies that $A$ and $B$ are incomparable using Peng and Yang's ranking principle. If we apply Membership score $J$ to $A$ and $B$, then we get $J(A) = 0.0983 > J(B) = 0.0933$. This implies that $A$ is greater than $B$, favouring human intuition. Hence, the proposed membership score is better than Peng and Yang's ranking functions.

Limitation 4 of subsection III and Limitation 2 of subsection III-A have been rectified by the Membership score of an IVPFN, which is shown in Example 3.

*Example 3:* Let $A = ([0, 0], [0.8, 0.9])$, $B = ([0, 0], [\sqrt{0.54}, \sqrt{0.91}])$ be two IVPFN. Then by using Peng and Yang's ranking function (Definition 13), we get $s(A) = s(B) = -0.725$, $a(A) = a(B) = 0.725$ which implies that $A$ and $B$ are equal. Also, if we apply the improved accuracy function, then we get $K(A) = K(B) = 0 \Rightarrow A = B$. Hence both methods failed to rank $A$ and $B$. However, if we apply our proposed Membership score to these $A$ and $B$, then we get $J(A) = -0.3105$ and $J(B) = -0.3195$, which implies that $J(A) > J(B)$. Therefore, by using Definition 16, we get $A > B$, which is logical. Hence we conclude that limitation 4 is also rectified by our proposed membership score.

Now, we show the efficacy of our proposed membership score function in improving the limitations discussed in subsection III-A by using Example 4 to Example 8. Limitation 1 to Limitation 5 of subsection III-A has been improved by applying the proposed Membership score (Definition 15) and the ranking principle (Definition 16), which is shown below.

*Example 4:* Let $A = ([0.7, 0.7], [\sqrt{0.51}, \sqrt{0.51}])$, $B = ([0, 0.7], [0, 0])$ be any two IVPFN. If we apply the improved accuracy function ( [5]), then we get $K(A) = K(B) = 0.49$, which implies that $A$ and $B$ are equal. However, $A \neq B$. Suppose, if we apply our proposed Membership score, then we get $J(A) = 0.1534$ and $J(B) = 0.1633 \Rightarrow J(B) > J(A)$. Thus by using Definition 16, we get $B > A$.

*Example 5:* Let $A_1 = ([0, 0], [0.8, 0.9])$, $B_1 = ([0, 0], [0.1, 0.2]) \in IVPFN$. If we apply the improved accuracy function ( [5]) to $A_1$ and $B_1$, then we get $K(A_1) = K(B_1) = 0 \Rightarrow A_1 = B_1$ which shows that $A_1$ and $B_1$ are equal. If we apply Membership score to these $A_1$ and $B_1$, then we get $J(A_1) = -0.3105$ and $J(B_1) = -0.0165 \Rightarrow J(B_1) > J(A_1)$. Hence by applying our proposed ranking principle (Definition 16), we get $B_1 > A_1$, which favours human intuition.

*Example 6:* Let $P_1 = ([0.4, 0.4], [\sqrt{0.84}, \sqrt{0.84}])$, $P_2 = ([0, 0.4], [0, \sqrt{0.84}])$ be two IVPFN. If we apply the improved accuracy function ( [5]), then we get $K(P_1) = K(P_2) = 0.16 \Rightarrow P_1 = P_2$. However, $P_1$ and $P_2$ are two different IVPFNs. If we apply our Membership score, then we get $J(P_1) = -0.2096$ and $J(P_2) = -0.2267 \Rightarrow J(P_1) > J(P_2)$. Hence by using our ranking principle (Definition 16), we get $P_1 > P_2$.

*Example 7:* Let $A = ([0, 1], [0, 0])$, $B = ([1, 1], [0, 0]) \in IVPFN$. If we apply the improved accuracy function ( [5]), we get $K(A) = K(B) = 1 \Rightarrow A = B$, which is anti-intuitive. If we apply our Membership score to these $A$ and $B$, then we get $J(A) = 0.3333$ and $J(B) = 1 \Rightarrow J(B) > J(A)$. Therefore, by applying our ranking principle (Definition 16) we get $B > A$, which favours human intuition.

*Example 8:* Let $A_1 = ([0, 0.5], [0, 0.7])$, $A_2 = ([0, 0.5], [0, 0]) \in IVPFN$. If we apply improved accuracy function [5], then we get $K(A_1) = K(A_2) = 0.25 \Rightarrow A_1 = A_2$ which is wrong. If we apply Membership score to these $A_1$ and $A_2$, then we get $J(A_1) = -0.08$ and $J(A_2) = 0.0833 \Rightarrow J(A_1) < J(A_2)$. Hence $A_1 < A_2$.

## V. MULTI CRITERIA DECISION MAKING METHOD BASED ON MEMBERSHIP SCORE

In this section, we develop a new algorithm for solving multi-criteria decision-making (MCDM) problems under the Pythagorean fuzzy environment that uses the proposed Membership score for comparing two or more interval-valued Pythagorean fuzzy numbers. We assume that there are $p$ alternatives $A = \{A_1, A_2, \ldots, A_p\}$, which are evaluated with respect to $q$ criteria $C = \{C_1, C_2, \ldots, C_q\}$. We consider that the Decision Maker evaluates $p$ alternatives based on $q$ criteria and give the preferences in terms of interval-valued Pythagorean fuzzy numbers represented by $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$ with $b_{ij}+d_{ij} \le 1$, where $[a_{ij}, b_{ij}]$ represent the degree of satisfaction of the alternative $A_i$ with respect to criteria $C_j$ and $[c_{ij}, d_{ij}]$ represents the degree of dissatisfaction of the alternative $A_i$ with respect to criteria $C_j$. The matrix $P = (r_{ij})_{p\times q}$ is called the interval-valued Pythagorean fuzzy Decision Matrix, and we represent it as follows, as shown in the equation at the bottom of the page.

Let $w_j$ be the weight of $j$-th criteria $C_j$ given by the Decision Maker. Where $w_j \in [0, 1]$ and $\sum_1^q w_j = 1$. The aggregated performance of each alternative $A_i$ with respect to all the criteria $C_j$ is calculated using either a weighted averaging operator or a weighted geometric operator and represented by $R_i = ([a_i, b_i], [c_i, d_i]), i = 1, 2, \ldots, p$. Finally, we use Definition 15 for calculating the membership score of an aggregated performance $J(R_i)$ of each alternative $A_i$ and rank the alternatives based on the Membership score $J(R_i)$.

We summarize the decision-making algorithm that uses the membership score as follows in Algorithm 1.

### A. APPLICABILITY OF THE PROPOSED RANKING FUNCTION IN SOLVING MCDM PROBLEM

In this subsection, we apply Algorithm 1 for solving multi-criteria decision-making problems with interval-valued Pythagorean fuzzy information. We show the applicability of the proposed method in solving the MCDM problem using a numerical example. Here we consider the investment problem, which is adapted from Garg [5].

**Algorithm 1** MCDM Algorithm

1) **Formation of Decision Matrix $P$**
Decision Matrix $P$ is formed by asking the decision maker, the performance of each alternative $A_i(i = 1, 2, \ldots, p)$ with respect to criteria $C_j(j = 1, 2, \ldots, q)$ in terms of interval-valued Pythagorean fuzzy numbers and represent it as $P = (r_{ij})_{p\times q}$, where $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$.
2) **Aggregated performance of Alternative $A_i$**
Aggregated performance of Alternative $A_i$ with respect to Criteria $C_j$ is denoted by $R_i = ([a_i, b_i], [c_i, d_i]), i = 1, 2, \ldots, p$ and obtained by using either weighted averaging operator or weighted geometric operator.
3) **Ranking of Alternative $A_i$**
Here, we calculate the membership score $J$ of aggregated performance of alternative $R_i$ (using Definition 15) and rank the alternatives based on $J(R_i)$ using Definition 16. The alternative with a high value of $J(R_i)$ ranks high.

*Example 9:* Adopted from Garg [5]. Assume the panel with four possible alternatives $A_1, A_2, A_3, A_4$, namely, car, food, computer and an arms company in which an investor wants to invest money. The investment company decides according to three criteria given by the Risk analysis $C_1$, the growth analysis $C_2$, and the environmental impact analysis $C_3$. Assume that the weight of these criteria is set to be $0.35, 0.25, 0.40$.

### B. SOLVING EXAMPLE V-A.1 USING ALGORITHM 1 WITH WEIGHTED ARITHMETIC OPERATOR AS THE AGGREGATION OPERATOR

Here, we apply Algorithm 1 to the numerical example 9 by using Weighted Arithmetic Operator (Definition 9) as the aggregation operator.

**Step 1:** Formation of Decision Matrix $P$

Decision Matrix $P$ is formed by asking the decision maker, the performance of each alternative $A_i(i = 1, 2, 3, 4)$ with respect to criteria $C_j(j = 1, 2, 3)$ in terms of interval-valued Pythagorean fuzzy numbers and represent the matrix $P = (r_{ij})_{4\times 3}$ as follow (where $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$.), as shown in the equation at the bottom of the next page.

**Step 2:** Aggregated performance of Alternative $A_i$

Aggregated performance of Alternative $A_i$ with respect to Criteria $C_j$ is denoted by $R_i = ([a_i, b_i], [c_i, d_i]), i = 1, 2, 3, 4$. It is obtained by using the weighted averaging operator as

$$P = (r_{ij})_{p\times q} = \begin{bmatrix} ([a_{11}, b_{11}], [c_{11}, d_{11}]) & ([a_{12}, b_{12}], [c_{12}, d_{12}]) & \cdots & ([a_{1q}, b_{1q}], [c_{1q}, d_{1q}]) \\ ([a_{21}, b_{21}], [c_{21}, d_{21}]) & ([a_{22}, b_{22}], [c_{22}, d_{22}]) & \cdots & ([a_{2q}, b_{2q}], [c_{2q}, d_{2q}]) \\ \vdots & \vdots & \ddots & \vdots \\ ([a_{p1}, b_{p1}], [c_{p1}, d_{p1}]) & ([a_{p2}, b_{p2}], [c_{p2}, d_{p2}]) & \cdots & ([a_{pq}, b_{pq}], [c_{pq}, d_{pq}]) \end{bmatrix}$$

**TABLE 1.** Incorrectness in calculating the values of Aggregated performance $R_i$.

| Incorrect values obtained by Garg [5] | Correct Values obtained by us |
|---|---|
| $R_1 = ([0.2297, 0.4266], [0.3826, 0.4966])$ | $R_1 = ([0.2297, 0.4266], [0.3826, 0.6215])$ |
| $R_2 = ([0.5102, 0.6581], [0.1677, 0.2652])$ | $R_2 = ([0.5102, 0.6581], [0.1677, 0.2893])$ |
| $R_3 = ([0.34090.6000], [0.2425, 0.3642])$ | $R_3 = ([0.34090.6000], [0.2425, 0.4249])$ |
| $R_4 = ([0.47990.5864], [0.1000, 0.2297])$ | $R_4 = ([0.47990.5864], [0.1000, 0.2577])$ |

given in Definition 9, and the values of $R_i$ are given below.

$$\begin{aligned} R_1 &= ([0.3208, 0.4703], [0.3325, 0.4704]),\\ R_2 &= ([0.5352, 0.6645], [0.1516, 0.2551]),\\ R_3 &= ([0.3646, 0.6000], [0.3000, 0.3565]),\\ R_4 &= ([0.5653, 0.6699], [0.1000, 0.2213]) \end{aligned}$$

**Step 3:** Ranking of Alternative $A_i$

Here, we have calculated the membership score $J$ of aggregated performance of alternative $R_i$ using Definition 15 and the values are listed below, $J(R_1) = 0.0132, J(R_2) = 0.2559, J(R_3) = 0.1270, J(R_4) = 0.2844$. Then by using Definition 16, we get $A_4 > A_2 > A_3 > A_1$ and hence $A_4$ is the best alternative among others. Suppose if we apply Garg's algorithm to the same problem; then we get the ranking of alternatives as $A_4 > A_2 > A_3 > A_1$. For this example, our proposed algorithm results coincide with Garg's [5] work. This happens because IVPFNs are present in the matrix $P$. That is, all the IVPFNs present in $P$ can be discriminated accurately using the improved accuracy function; hence, we have got accurate results.

### C. SOLVING EXAMPLE V-A.1 USING ALGORITHM 1 WITH WEIGHTED GEOMETRIC OPERATOR AS THE AGGREGATION OPERATOR

If we apply Algorithm 1 to the numerical example 9 by using Weighted Geometric Operator as the aggregation operator, then the steps and results are given below.

**Step 1:** Formation of Decision Matrix $P$

Decision Matrix $P$ is formed by asking the decision maker, the performance of each alternative $A_i (i = 1, 2, 3, 4)$ with respect to criteria $C_j (j = 1, 2, 3)$ in terms of interval-valued Pythagorean fuzzy numbers and represent the matrix $P = (r_{ij})_{4\times3}$ as given in subsection V-B (where $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$.)

**Step 2:** Aggregated performance of Alternative $A_i$

Aggregated performance of Alternative $A_i$ with respect to Criteria $C_j$ is denoted by $R_i = ([a_i, b_i], [c_i, d_i]), i = 1, 2, 3, 4$. It is obtained using the weighted geometric operator as given in Definition 10, and the values of $R_i$ are given in Table 1.

**Step 3:** Ranking of Alternative $A_i$

Here, we have calculated the membership score $J$ of aggregated performance of alternative $R_i$ using Definition 15 and the values are listed below, $J(R_1) = -0.0772, J(R_2) = 0.2322, J(R_3) = 0.0964, J(R_4) = 0.1925$. Then by using Definition 16, we get $A_2 > A_4 > A_3 > A_1$ and hence $A_2$ is the best alternative among others. Suppose we apply Garg's algorithm to the same problem; then we get $K(R_1) = 0.216, K(R_2) = 0.6198, K(R_3) = 0.4409, K(R_4) = 0.5254$ and the ranking of alternatives we get it as $A_2 > A_4 > A_3 > A_1$. In this case, our proposed algorithm results agree with Garg's [5] work. This happens because of IVPFNs present in the matrix $P$. That is, all the IVPFNs present in $P$ can be discriminated properly using the improved accuracy function ( [5]); hence, we have got proper results.

### D. SOLVING EXAMPLE V-A.1 USING ALGORITHM 1 WITH WEIGHTED ARITHMETIC OPERATOR AS THE AGGREGATION OPERATOR

The main aim of this subsection is to show the inefficiency of the improved accuracy function using the numerical example. Here, we apply Algorithm 1 to the numerical example 9 by using Weighted Arithmetic Operator as the aggregation operator.

**Step 1:** Formation of Decision Matrix $P$

Decision Matrix $P$ is formed by asking the decision maker, the performance of each alternative $A_i (i = 1, 2, 3, 4)$ with respect to criteria $C_j (j = 1, 2, 3)$ in terms of interval-valued Pythagorean fuzzy numbers and represent the matrix $P = (r_{ij})_{4\times3}$ as follow (where $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$.), as shown in the equation at the bottom of the page.

**Step 2:** Aggregated performance of Alternative $A_i$

$$P = (r_{ij})_{4\times3} = \begin{bmatrix} ([0.4, 0.5], [0.3, 0.4]) & ([0.4, 0.6], [0.2, 0.4]) & ([0.1, 0.3], [0.5, 0.6]) \\ ([0.6, 0.7], [0.2, 0.3]) & ([0.6, 0.7], [0.2, 0.3]) & ([0.4, 0.6], [0.1, 0.2]) \\ ([0.3, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.3, 0.4]) & ([0.3, 0.6], [0.1, 0.3]) \\ ([0.7, 0.8], [0.1, 0.2]) & ([0.6, 0.7], [0.1, 0.3]) & ([0.3, 0.4], [0.1, 0.2]) \end{bmatrix}$$

$$P = (r_{ij})_{4\times3} = \begin{bmatrix} ([0, 0], [0.3, 0.4]) & ([0, 0], [0.2, 0.4]) & ([0, 0], [0.5, 0.6]) \\ ([0, 0], [0.2, 0.3]) & ([0, 0], [0.2, 0.3]) & ([0, 0], [0.1, 0.2]) \\ ([0, 0], [0.3, 0.4]) & ([0, 0], [0.3, 0.4]) & ([0, 0], [0.1, 0.3]) \\ ([0, 0], [0.1, 0.2]) & ([0, 0], [0.1, 0.3]) & ([0, 0], [0.1, 0.2]) \end{bmatrix}$$

Aggregated performance of Alternative $A_i$ with respect to Criteria $C_j$ is denoted by $R_i = ([a_i, b_i], [c_i, d_i]), i = 1, 2, 3, 4$. It is obtained using the weighted averaging operator as given in Definition 9, and the values of $R_i$ are given below.

$$R_1 = ([0, 0], [0.3325, 0.4704]),$$
$$R_2 = ([0, 0], [0.1516, 0.2551]),$$
$$R_3 = ([0, 0], [0.1933, 0.3565]),$$
$$R_4 = ([0, 0], [0.1000, 0.2213])$$

**Step 3:** Ranking of Alternative $A_i$

Suppose if we apply Garg's algorithm to the same problem, then we get $K(R_1) = 0, K(R_2) = 0, K(R_3) = 0, K(R_4) = 0$, and the ranking of alternatives as $A_1 = A_2 = A_3 = A_4$. All the alternatives are equally ranked, but it is not true. If we calculate the membership score $J$ of aggregated performance of alternative $R_i$ using Definition 15, then we get $J(R_1) = -0.1025, J(R_2) = -0.0289, J(R_3) = -0.0532, J(R_4) = -0.0195$. Then by using Definition 16, we get $A_4 > A_2 > A_3 > A_1$. i.e., Arm company is considered the most desirable alternative among others. This example shows that our proposed membership score function outperforms the improved accuracy function.

## VI. CONCLUSION

In this paper, firstly, we have discussed the limitations of the improved accuracy score function and other similar ranking functions in ordering/ comparing any two arbitrary interval-valued Pythagorean fuzzy numbers in detail. Secondly, we have introduced a new Membership score function on the class of IVPFNs and studied their properties. Thirdly, we have compared our proposed method with two other familiar methods for the superiority of the proposed method. Finally, a numerical example has been solved to discuss the application of the proposed membership score function in solving interval-valued Pythagorean fuzzy multi-criteria decision-making problems. This work is an initial attempt to define a total ordering principle on the class of IVPFNs, and the total ordering will be achieved in the near future as similar to the total ordering on the set of interval-valued Fermatean fuzzy numbers introduced by Jeevaraj [7].

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