IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 20 March 2023, accepted 31 March 2023, date of publication 14 April 2023, date of current version 25 April 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3267125

RESEARCH ARTICLE

A Data-Driven Hybrid Methodology Using Randomized Low-Rank DMD Approximation and Flat-Top FIR Filter for Voltage Fluctuations Monitoring in Grid-Connected Distributed Generation Systems

NEETHU MOHAN[1], S. SACHIN KUMAR[1], K. P. SOMAN[1], V. G. SUJADEVI[2], PRABAHARAN POORNACHANDRAN[2], AND U. RAJENDRA ACHARYA[3,4,5]

[1]Center for Computational Engineering and Networking (CEN), Amrita Vishwa Vidyapeetham, Coimbatore 641112, India
[2]Centre for Internet Studies and Artificial Intelligence, Amrita Vishwa Vidyapeetham, Amritapuri, Kerala 690525, India
[3]School of Mathematics, Physics and Computing, University of Southern Queensland, Springfield, QLD 4300, Australia
[4]Department of Biomedical Engineering, School of Science and Technology, Singapore University of Social Sciences, Singapore 599494
[5]International Research Organization for Advanced Science and Technology (IROAST), Kumamoto University, Kumamoto 860-0862, Japan

Corresponding author: S. Sachin Kumar (s_sachinkumar@cb.amrita.edu)

ABSTRACT The voltage fluctuations are caused by variations in renewable energy source outputs, increased usage of nonlinear loads, high reactive power consumption of loads, etc. This results in damage to electric components and enormous economic losses. It is necessary to measure the parameters of voltage fluctuations and flicker levels to achieve a secure supply of power. However, higher-order harmonics and noises in voltage waveform make such assessment a burdensome task. The traditional techniques fail to provide exact monitoring due to the uncharacteristic variations in the input voltage signal. Nowadays, data-driven methods have become a prominent choice for non-stationary, nonlinear signal analysis as it efficiently tackles atypical changes by its capability to identify spatio-temporal information from given data. The present paper proposes a data-driven hybrid methodology for monitoring voltage fluctuations using the low-rank approximation of dynamic mode decomposition (DMD) and flat-top finite impulse response (FIR) filter. The low-rank approximation of DMD enables to identify the spatio-temporal coherent structures from the data by mapping the data into a lower-dimensional space. Adaptive flat-top FIR filter tracks the fundamental dynamic phasor by identifying the instantaneous frequency, magnitude and phase variations. The performance of the proposed hybrid methodology is assessed using different test cases, and is compared with various existing approaches. The promising results suggest that the proposed methodology can be used for accurate parameter identification and monitoring voltage fluctuations in smart grid scenarios. It can also be used as an accurate tool in distribution grids to detect higher-order harmonics.

INDEX TERMS Dynamic mode decomposition, flat-top FIR filter, power quality, Hankelization, smart grid, voltage fluctuations.

I. INTRODUCTION

Voltage fluctuations are defined as random or systematic changes in the voltage envelope. According to the Information, Education & Communication (IEC) standards the voltage fluctuations are 10% of the rated voltage [1]. The major sources of voltage fluctuations are (1) the presence of nonlinear loads such as electric arc furnaces (EAF), arc welders, and large motors, (2) variations in renewable energy resources such as photovoltaic and wind power generator

The associate editor coordinating the review of this manuscript and approving it for publication was Li He.

output, (3) increased reactive power consumption of loads, (4) switching of power factor capacitors, etc. The voltage fluctuations can result in undesired illumination effects known as flickers in power distribution systems. The voltage flicker induces an impression of unsteadiness of visual sensation. The adverse effects of voltage fluctuations can cause damage of electric power components, malfunctioning end user equipment, and enormous economic losses [2]. The concept of microgrid (MG) and distributed generation (DG) systems import new concerns in voltage fluctuations monitoring [3]. Due to the associated detrimental, technical and economic aspects, voltage fluctuations are considered as a major hazardous power quality (PQ) issue. A total of 83% of the PQ problems are due to voltage fluctuations and flickers in distribution systems [4]. Thus, proper monitoring of these fluctuations is paramount for the distribution system operator to establish a secure, reliable and quality supply of power [5], [6].

The voltage flickers are mainly divided into two categories: (1) cyclic (periodic) flicker - it is caused by slow periodic voltage magnitude changes. It is repetitive and mainly due to the operation of nonlinear loads. (2) non-cyclic (aperiodic) flicker - it is caused due to the occasional voltage magnitude changes. The main sources of non-cyclic flicker is starting of large motors. The two main indices recommended by IEC Std 61000-4-15 to measure the voltage flicker severity are, (1) short-term flicker severity (Pst)- it is measured for ten-minute intervals based on voltage variations, (2) long-term flicker severity (Plt)- it is measured for two-hour interval by 12 consecutive Pst values [2]. To measure the voltage flicker severity and monitor the flicker level, parameter estimation of voltage fluctuations is required. The presence of higher-order harmonics, noises and other uncharacteristic variations in the input voltage waveform makes the voltage fluctuation assessment difficult. A robust voltage fluctuation monitoring system should efficiently deal with the non-stationary, nonlinear waveform characteristics. Further, the smart grid architecture imports more complexities to the distribution grid [7]. Therefore, developing an accurate voltage fluctuations monitoring system by considering all these uncertainties is troublesome.

A. REVIEW OF RELATED WORKS AND MOTIVATION

In the recent past different methods have been introduced for voltage fluctuations monitoring and flicker severity assessment in power distribution systems. Fourier-based analysis is the most widely used conventional technique for voltage fluctuation monitoring, in which the popular approach is fast Fourier transform (FFT)-based techniques [8], [9]. However, FFT-based techniques are more appropriate for stationary signal analysis. In general, the Fourier-based techniques suffer from spectral leakage and picket fence effect in asynchronous sampling conditions. This in turn leads to faulty analysis in the presence of harmonics and nominal frequency deviations. Even though Fourier-based techniques are easier to implement, due to the above limitations, it is not a suitable choice for voltage fluctuations monitoring.

Wavelet transform is preferred over Fourier-based techniques for voltage fluctuation assessment due to its potential for non-stationary signal analysis [10], [11]. The shifting and scaling property of wavelet transform is suitable to detect the high and low frequency components in the measured voltage signal. The usage of wavelets for voltage flicker measurement is initially proposed in [10]. This method proposes a scheme combining low-pass demodulation filter with wavelet decomposition to measure the voltage flicker level. Huang et al. [11] applied continuous wavelet transform (CWT) to enhance the voltage flicker measurements by employing the Gaussian wavelet basis function. The time-frequency relationship extracted by CWT provides a flexible tool for voltage flicker analysis. The selection of suitable wavelets, and complexity in implementation are the two important liabilities of wavelet transform-based techniques.

Another widely exploited choice for voltage fluctuation monitoring is adaptive filtering-based methods [12], [13], [14], [15]. The ability to adapt the filter parameters according to the measured waveform characteristics makes it a good choice for voltage fluctuation analysis. Hamadi et al. [12] developed a voltage fluctuation estimation system using a Kalman filter by adopting an extended state space model. This system has utilized a fuzzy rule-based logic to tune the system parameters. Kose et al. [13] proposed an approach combining an extended Kalman filter (EKF) and linear Kalman filter (LKF) for voltage flicker evaluation. In this approach, EKF provides the voltage envelope and LKF provides the magnitudes around the fundamental frequency. The major advantage of this approach is its less dependency to fundamental frequency deviations. Sadinezhad et al. [14] applied hybrid adaptive least-squares-Kalman (LSK) to measure the instantaneous voltage flicker from amplitude modulated grid voltage signal. To adaptively estimate the fundamental frequency of the signal, least-square optimization is employed. Elnady and Salama [15] developed a flicker mitigation approach using the Kalman filter and its derivatives by reducing the instantaneous flicker level to 60%. The adaptive filtering methods require more time to converge and thus have limitations in real-time monitoring.

Hilbert transform (HT) based techniques are commonly used to track the fluctuated voltage waveform [1], [16], [17]. Marei et al. [1] proposed HT-based control algorithm for voltage fluctuation mitigation. In this method, the easiness of HT is utilized to track the fluctuated voltage envelope and to measure the flicker level in distribution systems. Even though the method offers a fast response time, the minimum error in tracking the voltage envelope is achieved through HT realized with a long filter length. This will increase the computation cost and delay time than the short filter length. Galil et al. [16] identified the voltage flicker parameters by combining the Teager-energy operator (TEO) and HT. The TEO-HT method has achieved online tracking of voltage flicker with less than a 3% margin of error. However, the TEO-HT method suffers from instability issues in the presence of harmonics and noises. A filtering technique is proposed as an essential preprocessing stage to reduce this limitation. Feila et al. [17] introduced a Prony-HT (PA-HT) combined method to detect instantaneous flicker levels. The main advantage of the PA-HT method is its easy implementation.

Various energy operator based techniques are developed for voltage flicker assessment in power systems [18], [19], [20]. Eldery et al. [18] proposed an algorithm to track the voltage flicker by calculating the Teager-energy operator (TEO) of the sinusoidal waveform. The key advantage of the algorithm is its independency from prior estimations and hectic optimization procedures. Li et al. [19] developed an improved voltage flicker parameter estimation algorithm using TEO and chirp z-transform (CZT). In this method, the extraction errors of the voltage waveform are minimized using the TEO error correction factor, then the frequency and magnitude are extracted using the CZT. A computationally efficient TEO for power system applications is developed in [20]. The major advantages of this model are low-cost implementation and faster response under dynamic scenarios.

Optimization-based techniques are proposed for voltage fluctuations monitoring [21], [22], [23]. Hasawi et al. [21] proposed a genetic-algorithm based optimization technique for voltage flicker monitoring. The main disadvantage of GA-based technique is the selection of proper parameters. Sun et al. [22] developed a sparse representation-based algorithm for voltage fluctuation assessment. In this algorithm, voltage fluctuation detection is formulated as a signal inpainting problem, where ℓ_0-norm-based regularization is used to import sparsity. Further, an ℓ_2-norm regularization is developed to extract the fluctuated voltage component. Al-Othman et al. [23] utilized a particle swarm optimization (PSO) algorithm for voltage flicker measurement. The performance of the PSO algorithm hinges on the choice of suitable parameters. In general, the possibility of converging to the local optimum locations and high computational complexity limits the performance of optimization based techniques.

S-transform (ST) based techniques offer improved time-frequency visualization for voltage flicker monitoring [24], [25]. Eghtedarpour et al. [24] exploited the potential of ST in combination with the moving window approach to calculate the voltage flicker level in power systems. In this method, the moving window is used to extract the voltage envelope, followed by ST to identify the flicker characteristics. Even though the method has a good identification rate, the computation is high due to the two-level implementation. Yao et al. [25] proposed a fast s-transform (FST) technique for voltage flicker analysis and monitoring. The FST method offers less computational complexity than ST for flicker analysis.

Decomposition algorithms such as empirical mode decomposition (EMD), synchrosqueezing transform (SST), variational mode decomposition (VMD) are applied to voltage flicker analysis. Onal et al. [26] utilized EMD for estimating the short-term flicker severity index by decomposing the measured voltage signal into modes. However, EMD is prone to noise and other uncharacteristic variations in voltage signals. Chang et al. [2] proposed a hybrid method for voltage flicker monitoring using SST. In this method, SST combined with the mean-shift algorithm is employed for detecting the number of frequency components. Further, band-pass filters are implemented to measure the amplitude of frequency components. Achlerkar et al. proposed a VMD-based approach for flicker identification and PQ detection in the power grid [27].

Despite the above mentioned techniques, various other approaches have also been proposed for voltage fluctuation monitoring and flicker assessment. Alkandari et al. [28] introduced a two-level forward-backward algorithm to detect the voltage fluctuation parameters and nominal frequency of the waveform. However, the two-level implementation increases the complexity of the approach. Dejamkhooy et al. [29] developed a grey system-based model for voltage fluctuation modeling. This model has utilized the rolling Grey model and GM(1,1) to model and predict the envelope of the fluctuated voltage waveform. Marei et al. [30] proposed a control algorithm for flicker mitigation using 1) adaptive linear neuron (ADALINE) and 2) recursive least-squares (RLS). The RLS algorithm outperformed ADALINE by its robust flicker tracking performance. Geiger et al. [31] introduced a voltage flicker assessment model using RMS voltage for off-line applications like planning of grid. Mazadi et al. [32] utilized a state estimation algorithm for measuring voltage flicker. A summary of different benchmark techniques for voltage fluctuation monitoring and flicker assessment is given in Table 1. The description of each method and its limitation are briefly summarized in Table 1.

Although the aforementioned methods have been applied for voltage fluctuation monitoring in distribution systems, they also have several limitations. The Fourier-based approaches fail to give exact monitoring of time-varying flicker waveform due to spectral leakage and picket fence effect. The selection of proper input parameters and implementation complexities are the two important constraints in optimization-based techniques. Even though the HT approach gives a good estimation of fluctuated voltage envelope, its performance is not satisfactory in the presence of noises and higher-order harmonics. The mathematical complexity in Kalman filter-based techniques is higher that limits its practical applications. Therefore, there is a requirement for simple, reliable and fast methodology to monitor the voltage fluctuations, which must be robust against noise contamination and other atypical variations that may occur in power signals.

TABLE 1. Summary of different benchmark methods for voltage fluctuations monitoring and flicker assessment.

Category	Technique	Description	Limitations
Fourier transform (FT)	FFT [8]	FFT is used to detect the voltage flicker.	- Spectral leakage - Picket fence effect
	SDFT [9]	Smart DFT is used to detect the magnitude and frequency of fluctuated voltage waveform.	
Wavelet transform (WT)	LPF-WT [10]	Low-pass filtered signal is decomposed using wavelets to detect voltage flicker level.	- Selection of mother wavelets - Complex implementation
	CWT [11]	Gaussian wavelet basis function is used with CWT to enhance fluctuation measurement.	
Adaptive filtering	Kalman filter (KF)[12]	Kalman filter with extended state-space model is used. Adaptive tuning of parameters is done by fuzzy logic.	- Suitable parameter tuning - Mathematical complexity
	EKF-LKF[13]	Voltage envelope extraction using EKF. Magnitude and frequency estimation using LKF.	
	LSK [14]	Least squares (LS) is used to estimate fundamental freq. Voltage flicker level is measured by LS-KF.	
	KF [15]	Kalman filter and its derivatives are used to measure instantaneous flicker level.	
Hilbert transform (HT)	HT-control [1]	Voltage flicker mitigation is performed using Hilbert transform (HT) based control algorithm.	- Prone to noises - Instability issues
	TEO-HT [16]	Online voltage flicker tracking using TEO-HT approach.	
	Prony-HT [17]	Voltage signal is decomposed using Prony into different components. Further HT is used to measure flicker level.	
Energy operators	TEO [18]	Teager energy operator (TEO) is used to measure instantaneous voltage flicker level.	- Prone to higher-order harmonics
	TEO-CZT [19]	TEO error correction factor reduces the extraction error. CZT is used to measure frequency and magnitude.	
Optimization algorithms	GA [21]	Genetic algorithm (GA) based optimization is used to measure voltage flicker level.	- Convergence to local optimum - Computational complexity
	Sparse [22]	ℓ_0-norm based sparse representation is used for fluctuation detection. ℓ_2-norm is used to extract components.	
	PSO [23]	Particle swarm optimization (PSO) is used for voltage flicker measurement.	
S-transform (ST)	ST-MW [24]	Voltage envelope is detected using moving window(MW). ST is used to identify flicker level.	- Prone to the uncharacteristic variations in voltage signal
	FST [25]	Fast ST is used to detect voltage fluctuation level.	
Decomposititon algorithms	EMD [26]	EMD decomposes the signal into modes. Pst and flicker level is measured from EMD modes.	- EMD is prone to noise - Complex implementation - Failed to detect all flicker frequency components
	Hybrid SST [2]	SST combined with mean-shift algorithm is used to detect freq.componets. Bandpass filter measured the magnitude and frequency.	
	VMD [27]	VMD is used for flicker identification.	
Other algorithms	Fwd-Bwd [28]	Two-level forward-backward algorithm is used to detect flicker parameters and nominal frequency.	- Complex implementation - Prone to the uncharacteristic variations in voltage signal
	Grey system [29]	Voltage flicker is detected using rolling Grey model and GM(1,1).	
	ADALINE [30]	Adaptive linear neuron (ADALINE) system is used to measure the voltage components.	
	RMS [31]	Cycle-by-cycle rms value is used to measure short-term flicker severity index.	
	State-est.[32]	Measured the voltage flicker using state-estimation algo.	

B. OBJECTIVE AND KEY CONTRIBUTIONS

The key objective of this work is to develop a simple system for voltage fluctuations monitoring using data-driven techniques. The data-driven models provide the leverage of identifying the hidden dynamic features with no prior assumption of the physical model of the system. It can determine the temporal and spatial behavior of the data by discovering the underlying dynamicity.

Dynamic mode decomposition (DMD) is an emerging, versatile data-driven technique developed using the power of singular value decomposition (SVD) [33], [34]. It is a matrix decomposition technique with an outstanding capability to extract the temporal and spatial features of the associated system. The application of DMD to various domains such as fluid dynamics, video processing, neuroscience, time-series analysis, etc, is well investigated [33], [34], [35], [36]. In the recent past, DMD algorithm is applied to power systems for stability analysis and situational awareness [37], [38]. The wide application of DMD in power system domain is supported by its high computational speed [37], [38]. In [37], DMD is utilized to assess the global instability of power system. Situational awareness and monitoring of the electric grids using DMD is described in [38]. Harmonic monitoring using total-DMD algorithm are discussed in [39]. The present paper proposes a data-driven hybrid methodology for voltage fluctuations monitoring in power systems using the DMD algorithm and flat-top finite impulse response (FIR) filter. The proposed methodology utilizes the low-rank approximation of DMD using the randomized singular value decomposition (rSVD) algorithm. The rSVD implementation enables a low-rank SVD computation. It thus helps to map the data from a high-dimensional space into lower-dimensional space based on the intrinsic rank of the data matrix. The ability of DMD to identify the time-series characterization is explored for the effective monitoring and assessment of voltage fluctuations. The proposed hybrid methodology converts the measured voltage waveforms into multi-dimensional voltage data through matrix Hankelization. To compute the dynamic mode matrix, the randomized, low-rank dynamic mode decomposition (DMD) algorithm is applied to the multi-dimensional Hankel matrices. Further, the associated frequency and magnitude components are estimated based on the extracted dynamic modes. Finally, a flat-top FIR filter tuned to the estimated fundamental frequency is implemented to obtain the fundamental dynamic phasor of the voltage waveform. The major contributions of the present paper are,

- Development of a simple and robust hybrid methodology for voltage fluctuations monitoring in power systems using a randomized low-rank approximation of dynamic mode decomposition (DMD) and adaptive flat-top FIR filter by identifying the underlying dynamical characteristics of voltage data.
- The randomized low-rank DMD approximation maps the measurement data into a low-dimensional space based on the data matrix rank. This enables a computationally efficient and accurate estimation of data characteristics.
- The proposed methodology is free from tedious training tasks and automatically identifies the number of frequency components, associated magnitudes and fundamental dynamic phasor.

Several experiments are performed to verify the effectiveness of the proposed hybrid methodology in noisy and noiseless conditions using simulated and real-time field measurements. The rest of this paper is organized as follows. Section II describes the major concepts used in this work and its mathematical details and Section III explains the proposed hybrid approach. Section IV describes the results and evaluations and finally, the paper concludes in Section V.

II. MATERIALS AND METHODS

The major concepts employed in the proposed voltage fluctuations monitoring system and its mathematical background are described in this section.

A. VOLTAGE FLUCTUATION MODEL

The voltage fluctuation can be modeled as an AM waveform, with a sinusoidal modulated signal of random amplitude and frequency. The voltage fluctuation model can be mathematically defined as,

$$y(t) = (A_0 + \sum_{i=1}^{n} A_i \cos(\omega_i t + \phi_i)) \cos(\omega_0 t + \phi_0) \quad (1)$$

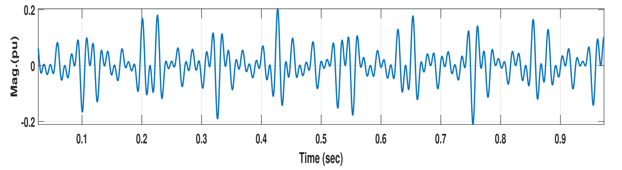

FIGURE 1. Representation of voltage fluctuation waveform.

where A_0, ω_0, ϕ_0 denotes the magnitude, frequency, and phase of the fundamental signal, ω_i denotes the flicker frequency, A_i denotes the magnitude of the voltage flicker at ω_i, and ϕ_i denotes the phase of the modulation signal. A robust and efficient voltage fluctuation monitoring system will identify the magnitude, frequency and phase of the flicker components precisely. However, the presence of noises, harmonics and other uncertainties makes this identification a difficult task. A sample voltage waveform is given in Fig. 1.

B. RANDOMIZED SINGULAR VALUE DECOMPOSITION (rSVD)

Singular value decomposition (SVD) is a matrix factorization methodology that decomposes the input matrix into three different matrices [40]. It is a dimensionality reduction technique that gives best approximation of the data points through lower dimension matrices [41]. In general, the SVD factorization of the input data matrix (A) can be represented as,

$$A_{P\times Q} = U_{P\times P}\Sigma_{P\times Q}V^T_{Q\times Q} \quad (2)$$

Here, $U_{P\times P}$, $V_{Q\times Q}$ are orthogonal matrices, and $\Sigma_{P\times Q}$ is a diagonal matrix with singular values as diagonal entries. In Σ the elements are in descending order. In the proposed methodology, the input matrix (A) in Eqn. 2 is equivalent to the measurement matrix Y_1 created from the measured voltage waveform. The details of Y_1 is explained in Section III-A. Fig. 2 illustrates the pictorial demonstration of SVD from a linear algebraic point of view. However for many practical systems, only few singular values are necessary to capture the data approximation; the rest can be discarded. This can be achieved through randomization concept [42]. The randomization algorithms provides fast and effective low-rank matrix approximations. One such algorithm is randomized SVD.

The randomized SVD (rSVD) is a truncated SVD approximation that computes the singular values through randomization and thus achieves a low-rank approximation of the given matrix [42], [43]. The rSVD factorization is performed in two steps:

1) Construct an approximate basis or low-dimensional subspace corresponding to the given input matrix.
2) Restrict the input matrix to this subspace and find the standard factorization using the constructed basis.

The low-dimensional subspace is identified through random sampling and it is assumed that it captures most of the actions of the given matrix. The input matrix is reduced to this low-dimensional subspace using explicit or implicit methods [44].

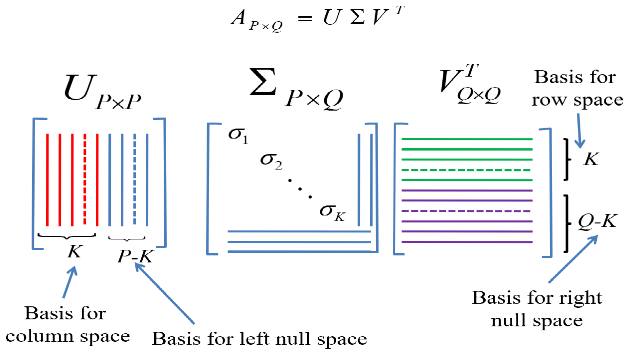

FIGURE 2. Illustration of the pictorial view of SVD. Here, *r* indicates the rank of matrix *A*.

Finally, the low-rank matrix approximation is obtained deterministically. The rSVD factorization offers better accuracy and speed than its non-randomized versions. Thus, in the proposed methodology, a fast and simple implementation of SVD is ensured using rSVD algorithm. The steps involved in the rSVD algorithm are described in Table 2.

TABLE 2. Randomized SVD algorithm.

Input:
$A \in R^{P\times Q} \leftarrow$ Data matrix
$K \leftarrow$ Rank of data matrix
Output:
$U \in R^{P\times K}, S \in R^{K\times K}, V \in R^{Q\times K}$
Procedure
1. $\Omega = randn(Q, K) \leftarrow$ Compute $Q \times K$ random matrix (Ω) drawn from the standard normal distribution
2. $D = A * \Omega \leftarrow$ Compute the matrix-matrix product (D) based on random matrix (Ω)
3. $W = orth(D) \leftarrow$ Compute the orthonormal basis for the range of D
4. $B = W^* * D \leftarrow$ Compute projection of the data matrix onto lower dimensional space
5. $[\tilde{U}, S, V] = svd(B) \leftarrow$ Compute the SVD based on the obtained projection matrix
6. $U = W * \tilde{U} \leftarrow$ Compute the restricted column-space basis vectors of data matrix (A)

C. DYNAMIC MODE DECOMPOSITION (DMD)

Dynamic mode decomposition (DMD) is a powerful, data-driven algorithm capable of discover the nonlinear dynamical structures from high-dimensional data. It is a matrix factorization methodology integrating proper orthogonal decomposition (POD) and Fourier transform [33]. DMD methodology provides a spatio-temporal decomposition of data into a set of dynamic modes from time-resolved measurements. The extracted dynamic modes are capable of identifying the hidden dynamic structures of the voltage waveform. DMD algorithm approximates the data collected over time, separated by Δt interval to a set of dynamic modes. The collected data measurements of dimension P are grouped into two measurement matrices, Y_1 and Y_2.

$$Y_1 = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_{Q-1} \end{bmatrix} \in R^{P\times(Q-1)} \quad (3)$$
$$Y_2 = \begin{bmatrix} y_2 & y_3 & y_4 & \cdots & y_Q \end{bmatrix} \in R^{P\times(Q-1)} \quad (4)$$

The measurement matrices are also known, as snapshot matrices as it is created using the snapshot vectors over time. As DMD assumes that the system is evolving slowly, it is possible to express the Q^{th} measurement vector as a linear combination of previous $Q-1$ measurements.

$$y_Q = a_1 y_1 + a_2 y_2 + \cdots + a_{Q-1} y_{Q-1} + \eta \quad (5)$$

Here, η denotes the residual error. DMD algorithm computes the leading eigendecomposition of the best-fit linear operator A relating to the system data [34].

$$AY_1 \approx Y_2 \Rightarrow A = Y_2 Y_1^{\dagger} \quad (6)$$

where $\dagger$ represents pseudo-inverse. The eigenvectors of operator matrix A corresponds to DMD modes or dynamic modes with associated unique eigenvalue. In DMD, the dynamic modes are approximated using the linear Koopman operator. Therefore, the DMD algorithm can be interpreted as a computation of low-dimensional dynamic modes of a finite-dimensional system using the infinite dimensional Koopman operator. The Koopman operator forms the basic foundation of DMD. However, due to the huge dimension of A for many practical applications, its analysis is a computational hardship. To overcome this difficulty, a rank-reduced matrix, $\tilde{A}$ is defined. The matrix $\tilde{A}$ defines a lower-dimensional linear model of the given system and have similar same set of non-zero eigenvalues with A. The $\tilde{A}$ is obtained through the following relation.

$$Y_2 \approx Y_1 \tilde{A} \quad (7)$$

The low-rank $\tilde{A}$ is defined as follows.

$$\tilde{A} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 & a_1 \\ 1 & 0 & \dots & 0 & 0 & a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & a_{Q-2} \\ 0 & 0 & \dots & 0 & 1 & a_{Q-1} \end{bmatrix} \in R^{(Q-1)\times(Q-1)} \quad (8)$$

Here, $[a_1, a_2, \dots, a_{Q-1}]$ represents the unknown coefficients. The steps involved in the DMD algorithm is described in Table 3.

TABLE 3. Dynamic mode decomposition algorithm.

Input:
$[Y_1, Y_2] \in R^{P\times(Q-1)} \leftarrow$ Measurement matrices
Output:
$\Phi \leftarrow$ Dynamic mode matrix
Procedure
1. $Y_1 = U\Sigma V^H \leftarrow$ Compute SVD of Y_1, where $U \in C^{P\times K}, \Sigma \in C^{K\times K}, V \in C^{(Q-1)\times K}$, K is rank of reduced SVD approximation to Y_1
2. $A = Y_2 V \Sigma^{\dagger} U^H \leftarrow$ Compute A based on Eqn. 6
3. $\tilde{A} = U^H Y_2 V \Sigma^{\dagger} \leftarrow$ Compute rank-reduced matrix $\tilde{A}$ based on Eqn. 7
4. $\tilde{A}W = W\Omega \leftarrow$ Compute eigendecomposition of $\tilde{A}$, where W is eigenvector matrix, Ω is diagonal matrix of eigenvalues
5. $\Phi = Y_2 V \Sigma^{\dagger} W \leftarrow$ Reconstruct eigendecomposition of A using W and Ω, where Φ is the dynamic mode matrix

In the current work, randomized SVD is employed instead of deterministic SVD as it is computationally efficient for large dimensional data matrices [43]. Thus a randomized, low-rank approximation of DMD is utilized to monitor the voltage fluctuations possible to occur in the grid.

D. FLAT-TOP FIR FILTER

To diminish the effect of spectral leakage phenomena in DFT based techniques, windowing concepts have been introduced [45], [46]. The characteristics of window functions are depending on properties such as 1) flatness and width of the main lobe, 2) highest side lobe level and 3) rate of falloff of side lobes. The windowing performance for various applications is improved through different design specifications with definite window properties. Flat-top windows are window functions with flat main lobe and have the advantage of a higher detection rate for harmonic components by reducing discontinuities. In the literature, various methodologies have been discussed for flat-top window design for highly accurate estimation [46], [47]. In [45], Duda et al. introduced a novel design criteria for perfectly flat-top and equiripple flat-top cosine windows. The perfectly flat-top cosine windows have perfect flat main lobe with monotonically decaying nature and asymptotically decaying side lobes. At the same time, the equiripple flat-top cosine windows have a main lobe with small ripples of the same amplitude. The perfect flatness is achieved by imposing two conditions on the frequency response of window functions:

1) At the baseband, the frequency derivative of the frequency response is kept as zero similar to the IIR Butterworth filter. This condition will ensure perfect flatness at the main lobe.
2) Time derivatives of the frequency response is kept as zero, at the boundaries. This condition will ensure the rapid decaying of the side lobes.

The perfect flat-top cosine window function of the K^{th} order is defined as,

$$w_K[n] = \sum_{k=0}^{K} b_K[k] \cos\left(k\frac{\pi}{N}n\right), n = -N, \cdots, N \tag{9}$$

where, $b_K[k]$ denotes the coefficients of K^{th} order cosine window and window length is $W_L = 2N + 1$. For a K^{th} order window, there are $K + 1$ coefficients ($b_K[k]$). Fig. 3 represents the flat-top FIR filter and its frequency response. The passband flatness can be improved by increasing the passband width through the window order, K. As the passband flatness improves, the stopband attenuation also improves.

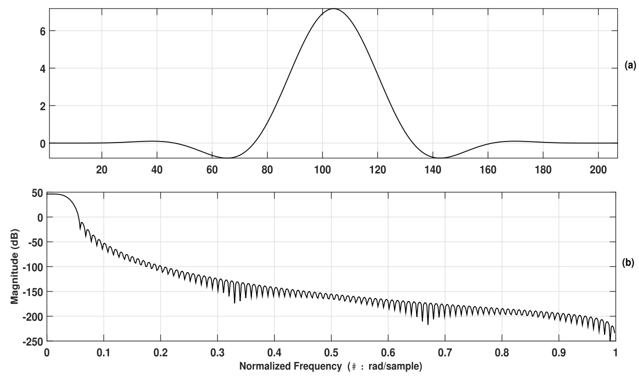

FIGURE 3. Visualization of (a) flat-top FIR filter, (b) its frequency spectrum.

III. PROPOSED DATA-DRIVEN HYBRID METHODOLOGY FOR VOLTAGE FLUCTUATIONS MONITORING

The data-driven algorithms have the implicit intelligence to learn the data dynamics to characterize the input data. In the present work, a novel methodology is developed using DMD and flat-top FIR filter for voltage fluctuations monitoring. The main steps of the proposed data-driven hybrid method are,

1) Multi-dimensional data matrix creation through Hankelization
2) Estimation of dynamic mode matrix through randomized low-rank DMD approximation
3) Identification of frequency components and associated amplitudes
4) Fundamental dynamic phasor estimation using flat-top FIR filter

The following subsection explains the steps of the proposed hybrid methodology in detail.

A. MULTI-DIMENSIONAL DATA MATRIX CREATION THROUGH HANKELIZATION

To capture the non-periodic and non-stationary characteristics of the signal, the measured voltage samples, $y \in R^Q$, $y_i \in R, i = 1, 2, \ldots, Q$ is converted into a multi-dimensional data matrix, $Y = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_k, \ldots, \bar{y}_H] \in R^{G\times H}$, by appending multiple time-shifted copies of input voltage (each row of Y). The Hankel matrix, Y is defined as,

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_H \\ y_2 & y_3 & \cdots & y_{H+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_G & y_{G+1} & \cdots & y_{G+H-1} \end{bmatrix} \in R^{G\times H} \tag{10}$$

Here, $\bar{y}_k$ is defined as, $\bar{y}_k = [y_k, y_{k+1}, \ldots, y_{G+k-1}]^T \in R^G$, $H = Q - G + 1$ denotes the length of the segment and G denotes the number of overlapping segments. An illustration of a part of the appended time-shifted copies of input voltage waveform is shown in Fig. 4. The need for Hankelization and fixing the number of overlapping segments (G) are explained in Section IV-A and IV-B respectively. Then, the modified observation matrices Y_1 and Y_2 is defined as follows.

$$Y_1 = [\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_{G-1}] \in R^{G\times(H-1)} \tag{11}$$

$$Y_2 = [\bar{y}_2, \bar{y}_3, \cdots, \bar{y}_G] \in R^{G\times(H-1)} \tag{12}$$

B. ESTIMATION OF DYNAMIC MODE MATRIX THROUGH RANDOMIZED LOW-RANK DMD APPROXIMATION

The dynamic mode matrix Φ is estimated by utilizing the rSVD factorization of Y_1 matrix. The random matrix is generated based on the standard normal distribution and the size

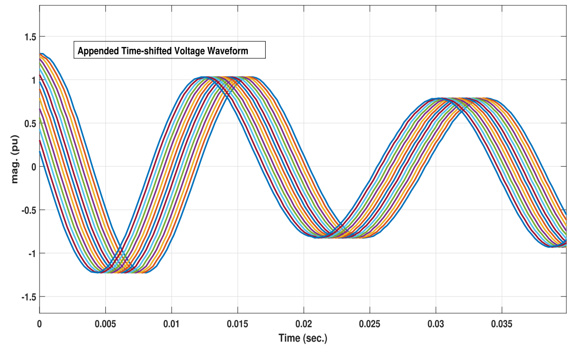

FIGURE 4. Representation of a portion of the appended time-shifted copies of measured voltage waveform.

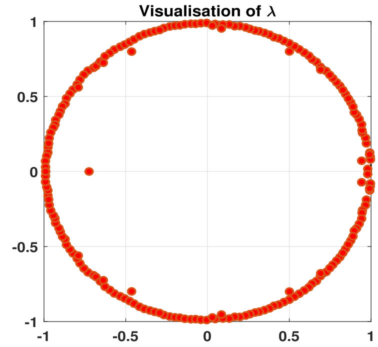

FIGURE 5. Visualization of λ in complex plane.

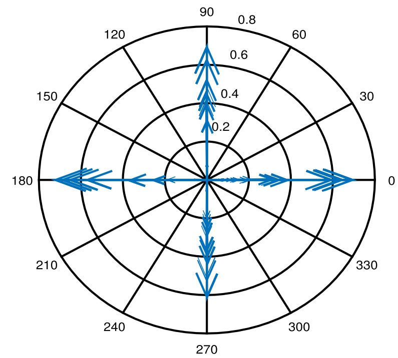

FIGURE 6. Visualization of eigenvectors using compass plot.

is decided based on the rank of the data matrix [43]. The usage of rSVD instead of SVD is supported by the fact that it provides a fast and efficient matrix decomposition. From a real-time implementation perspective, the computational cost of rSVD is much lesser than deterministic SVD as it only involves matrix-matrix multiplications. The rSVD of Y_1 matrix is computed using Eqn. 13.

$$Y_1 \approx U \Sigma V^H \tag{13}$$

Further, the rank-reduced $\tilde{A}$ matrix is computed as,

$$\tilde{A} = U^H Y_2 V \Sigma^\dagger \tag{14}$$

Then, the eigendecomposition of $\tilde{A}$ is performed to compute the dynamic mode matrix Φ.

$$\tilde{A}W = W\Omega \Rightarrow \Phi = Y_2 V \Sigma^\dagger W \tag{15}$$

Here, W is the eigenvectors of the linear operator matrix in DMD, and $\dagger$ denotes the pseudo-inverse operation. The dynamic modes, ϕ_i are denoted by columns of Φ and the associated eigenvalues are denoted by λ_i. The dynamics of the underlying system are identified by visualizing the eigendecomposition in a complex plane. Fig. 5 and Fig. 6 represents the visualization of eigenvalues and eigenvectors, respectively. The eigenvalues over the unit circle explicitly indicate the stable dynamic modes and these modes characterize the spatio-temporal features of the data. The DMD algorithm captures the inherent dynamics of the underlying system through the eigenvalues and corresponding DMD modes. The eigenvectors are represented by arrows from the origin, as shown in Fig. 6. The spread of the eigenvectors corresponding to detected DMD modes is indicated by the plot.

C. IDENTIFICATION OF FREQUENCY COMPONENTS AND ASSOCIATED AMPLITUDES

The extracted eigenvalues indicate the number of frequency components (K) present in the fluctuated voltage waveform. Further, the associated frequency and magnitude of each component is estimated using the dynamic mode matrix. The frequency is estimated by the logarithmic mapping of the eigenvalues as shown in Eqn. 16.

$$f_{est} = log(\lambda)/2\pi\Delta t \tag{16}$$

The magnitude of each component is estimated through the pseudo-inverse calculation of Φ using Eqn. 17.

$$A_{est} = \left(\Phi^T\Phi\right)^\dagger \Phi^T y_1 \tag{17}$$

Here, $y_1 \in R^Q$ represents the initial measurement vector.

D. FUNDAMENTAL DYNAMIC PHASOR ESTIMATION USING FLAT-TOP FIR FILTER

To estimate the fundamental dynamic phasor, a flat-top FIR filter is implemented. The flat-top FIR filter is adaptively tuned to the estimated fundamental frequency in the previous step and computed the fundamental dynamic phasor. This gives an accurate estimation of instantaneous frequency, magnitude and phase corresponding to the fundamental component. The window function of flat-top FIR filter is designed as discussed in Eqn. 9. The entire procedure of the proposed hybrid voltage fluctuations monitoring approach is given in Table 4. Fig. 7 represents the simplified block diagram of the proposed data-driven hybrid method.

IV. EXPERIMENTS AND ANALYSIS

This section explains the experimental analysis performed to evaluate the proposed hybrid methodology for voltage

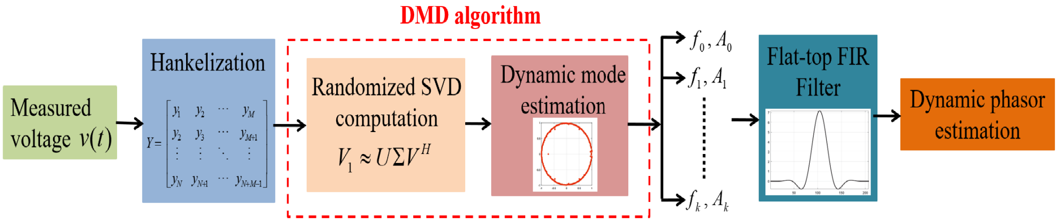

FIGURE 7. Simplified block diagram of the proposed data-driven hybrid methodology for voltage fluctuations monitoring.

TABLE 4. Steps involved in the proposed data-driven hybrid methodology for voltage fluctuations monitoring.

Input:
y ← Input voltage waveform
Outputs:
f_{est} ← Estimated frequencies
A_{est} ← Estimated amplitudes
Fundamental dynamic phasors
Steps
1. Create observation matrices using Eqn.10 to Eqn.12
2. Compute rSVD of observation matrix using algorithm described in Table 2
3. Compute low-rank approximated dynamic modes (ϕ_i) using Eqn.13 to Eqn.15
4. Compute frequency (f_{est}) and amplitude (A_{est}) using Eqn.16 and Eqn.17
5. Compute fundamental dynamic phasors using adaptively tuned flat-top FIR filter

fluctuation monitoring. The experiments are performed using simulated and real-time voltage waveforms in the noisy and noise-less environments. The extensive experiments conducted highlights the generality and potentiality of the proposed voltage fluctuation monitoring approach.

A. NEED FOR HANKELIZATION

This section explains how Hankelization naturally leads to the estimation of the frequency components present in the data. To do so, consider the differential equation corresponding to simple harmonic motion, namely $\frac{d^2y(t)}{dt^2} + \omega^2 y(t) = 0$. The general solution to this equation is a monotone oscillatory wave of the form $\sin(\omega t + \phi)$. By converting this differential equation into two coupled first-order equations, we obtain,

$$\underbrace{\begin{bmatrix} \frac{dy(t)}{dt} \\ \frac{d^2y(t)}{dt^2} \end{bmatrix}}_{\dot{Y}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y(t) \\ \frac{dy(t)}{dt} \end{bmatrix}}_{Y} \tag{18}$$

$$\underbrace{\begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix}}_{\dot{Y}} \tag{19}$$

The solution to this homogeneous equation is given by,

$$e^{At}Y(0) = Y(t) \tag{20}$$

Here, e^{At} is a 2-by-2 matrix. The spectral decomposition of e^{At} yields,

$$S \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} S^{-1} \begin{bmatrix} y(0) \\ \dot{y}(0) \end{bmatrix} = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} \tag{21}$$

The eigenvalues of e^{At} matrix are $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ with $\lambda_1 = 0 + \omega i$ and $\lambda_2 = 0 - \omega i$. The angular frequency of the oscillation appears in the imaginary part of the eigenvalue of the e^{At} matrix. Further on discretizing Eqn. 20 by initializing $t = 1\Delta t$ gives:

$$e^{A\Delta t} \begin{bmatrix} y(0) \\ \frac{y(1)-y(0)}{\Delta t} \end{bmatrix} = \begin{bmatrix} y(1) \\ \frac{y(2)-y(1)}{\Delta t} \end{bmatrix} \tag{22}$$

where, $y(k) \equiv y(k\Delta t)$. Now, let $B = e^{A\Delta t}$. From Eqn. 22, it can be inferred that B maps $y(0)$ to $y(1)$ and $\frac{y(1)-y(0)}{\Delta t}$ to $\frac{y(2)-y(1)}{\Delta t}$ or equivalently maps $y(1) - y(0)$ to $y(2) - y(1)$. In other words,

$$B \begin{bmatrix} y(0) \\ y(1) \end{bmatrix} = \begin{bmatrix} y(1) \\ y(2) \end{bmatrix} \tag{23}$$

Generally, by setting $t = k\Delta t$, Eqn. 23 can be expressed as,

$$B \begin{bmatrix} y(k) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} y(k+1) \\ y(k+2) \end{bmatrix} \tag{24}$$

In the frequency estimation process, it is assumed that matrix B is unknown and can be estimated from the data itself. This approach can be extended to a higher dimension by considering the sampled version of the data denoted as $y(1), y(2), \ldots\ldots.., y(N)$. Using Eqn. 24, the relation between Y_1 and Y_2 can be written as follows:

$$BY_1 = Y_2 \tag{25}$$

where,

$$Y_1 = \begin{bmatrix} y(1) & y(2) & \cdots & y(N-3) & y(N-2) \\ y(2) & y(3) & \cdots & y(N-2) & y(N-1) \end{bmatrix} \tag{26}$$

$$Y_2 = \begin{bmatrix} y(2) & y(3) & \cdots & y(N-2) & y(N-1) \\ y(3) & y(4) & \cdots & y(N-1) & y(N) \end{bmatrix} \tag{27}$$

Furthermore, the least-squares estimation of B can be obtained as follows:

$$B = Y_2(Y_1)^{\dagger} \tag{28}$$

where, † stands for the pseudo-inverse. This forms the origin of the Hankelization of the observed data. The angular frequency of the measured signal can be extracted from the eigenvalues of B. Further, this concept of single-frequency oscillation can be extended to multiple-frequency oscillations.

B. PARAMETERS FOR HANKELIZATION

In DMD, the conjugate pair of complex eigenvalues are necessary to capture the oscillatory behavior or frequency information present in the input data [34], [48]. By appending multiple time-shifted copies of the input waveform into the Hankel matrix (Y), a larger data matrix that can capture more information about the system's dynamics is created. If the input waveform contains K distinct frequencies, then snapshot matrices Y_1 and Y_2 must contain a minimum of $2K$ stacked rows to effectively capture the frequency content of the data. This is because each complex conjugate pair of eigenvalues correspond to a distinct frequency, and we need to ensure that there are enough data to capture each of these frequencies accurately. To demonstrate this concept, a voltage waveform containing five distinct frequency components (fundamental frequency, two harmonics and two inter-harmonics) is considered. The fundamental frequency is at 60 Hz and contains integer frequencies of 120, 180 Hz and non-integer harmonics of 210, 312 Hz. The snapshot matrices Y_1 and Y_2 are created by stacking a minimum of ten shifted versions of the input waveform (as the input contains five distinct frequencies).

$$Y_1 = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_H \\ y_2 & y_3 & y_4 & \cdots & y_{H+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{10} & y_{11} & y_{12} & \cdots & y_{H+9} \end{bmatrix} \tag{29}$$

$$Y_2 = \begin{bmatrix} y_2 & y_3 & y_4 & \cdots & y_{H+1} \\ y_3 & y_4 & y_5 & \cdots & y_{H+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{11} & y_{12} & y_{13} & \cdots & y_{H+10} \end{bmatrix} \tag{30}$$

The proposed estimation procedure will return ten distinct eigenvalues and the corresponding frequency can be computed based on the algorithm described in Table 4. The input waveform and the corresponding extracted frequency components are shown in Fig. 8.

Motivated by the aforementioned facts, the number of overlapping segments (G) for Hankelization is suitably chosen to be 200, with the length of each segment fixed at $H = Q - G + 1$. As G is fixed at 200, the proposed approach will return 100 distinct frequency components. Literature shows that common harmonics in power systems can extend upto the 35^{th} order [49]. Therefore a high value for G is selected, preferably to accommodate the larger frequency variations that may occur in real-time distributed generation systems. However, suppose the proposed algorithm identifies any additional frequencies that are not present in the original input signal, the algorithm will detect them with a very low magnitude (typically in the range of 10^{-15}) and that can be safely ignored. Moreover, a lower value for G can be chosen

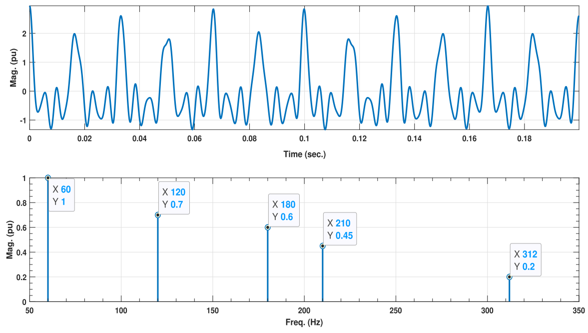

FIGURE 8. Illustration of the estimation procedure on a sample waveform.

if the number of frequency components is less and it will not affect the overall accuracy of the estimation process.

C. FLAT-TOP FIR FILTER PARAMETERS

The two important parameters of flat-top FIR filter are (1) filter order (M) and (2) window length (L). Based on the experiments conducted, filter order, $M = 5$ has attained the lowest main lobe error of all other orders. Thus, M is fixed to be 5 in this study. The window length for filter design is appropriately chosen to be 207 with the lowest error.

D. PERFORMANCE EVALUATION MEASURES

The accuracy in the estimation of frequency and amplitude components from the voltage waveform are evaluated based on error percentage [37]. It is a measure calculated as the ratio of the absolute difference between the estimated and actual values to the actual value.

$$Error\% = \frac{|Estimated - Actual|}{Actual} \times 100 \tag{31}$$

E. RESULTS AND COMPARISONS

This section describes the main results of the proposed hybrid methodology and comparisons with other benchmark methods. The experiments are performed under the following cases.

1) DETECTION OF MULTIPLE FREQUENCY OF VOLTAGE FLUCTUATIONS

The test waveform with fundamental frequency, $f_0 = 60$ Hz is defined as,

$$y(t) = [1 + 0.05\cos(2\pi 8t) + 0.1\cos(2\pi 13t) + 0.07\cos(2\pi 20t) + 0.1\cos(2\pi 29t)]\cos(2\pi f_0 t) \tag{32}$$

The test waveform is generated for one-second duration with a sampling frequency of 3840 Hz [2]. The evaluation results using the proposed methodology are shown in Fig. 9. The input voltage is shown in Fig. 9 (a). The extracted frequency components and the corresponding magnitude using the randomized low-rank DMD-based algorithm are shown in Fig. 9 (b). Fig. 9 (c) and Fig. 9 (d) represents the instantaneous tracking of fundamental frequency and magnitude

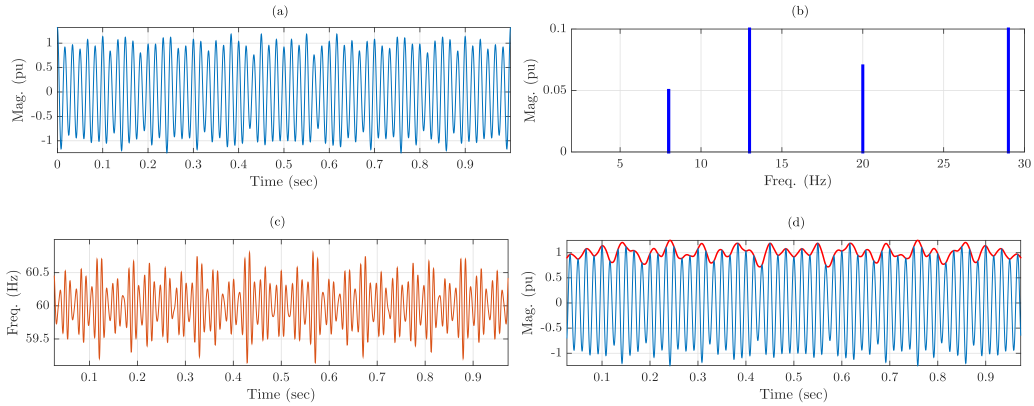

FIGURE 9. Illustration of the effectiveness of the proposed data-driven hybrid methodology. (a) input voltage signal, (b) detected frequency components and corresponding magnitude, (c) instantaneous fundamental frequency, (d) instantaneous magnitude envelope.

TABLE 5. Performance to detect non-integer frequency components with corresponding error%.

Method	f_1 (Hz)	Err.%	f_2 (Hz)	Err.%	f_3 (Hz)	Err.%	f_4 (Hz)	Err.%
FFT	8.00	3.60	14.00	2.90	20.00	2.90	30.00	0.67
Prony method - Order=130	8.32	2.40	13.61	0.10	20.15	0.10	29.80	0.00
Prony method - Order=80	8.92	7.40	13.27	2.43	20.17	0.35	29.80	0.00
Prony method - Order=20	11.01	32.00	13.04	4.00	19.94	0.80	29.78	0.07
Synchro-hybrid [2]	8.30	0.00	13.67	0.50	20.03	0.30	29.81	0.03
VMD [27]	-	-	14.23	4.63	20.38	1.39	29.30	1.68
Proposed hybrid methodology	**8.30**	**0.00**	**13.60**	**0.00**	**20.10**	**0.00**	**29.80**	**0.00**

envelope using flat-top FIR filter. It is obvious from Fig. 9 that the proposed hybrid methodology efficiently detected the four frequency components in the fluctuated voltage waveform. Further, the flat-top FIR filter accurately tracked the instantaneous variation of the fundamental component.

2) VOLTAGE FLUCTUATIONS WITH NON-INTEGER FREQUENCY COMPONENTS

To evaluate the performance efficiency of the proposed technique to detect non-integer frequency components, the signal defined in Eqn. 32 is generated with non-integer frequencies of 8.3, 13.6, 20.1, and 29.8 Hz [2]. The results of evaluation and comparison with FFT, Prony, Synchrosqueezing, and VMD methods are given in Table 5. The FFT method has low frequency resolution due to picket-fence and spectral leakage phenomena. The Prony method suffers from a narrow frequency range (0.01-30 Hz), thus not possible to detect low-frequencies. The Prony method of orders 20, 80, 130 are considered for comparison. The high estimation order of Prony is essential to detect the multiple frequency components in the input waveform. It is noticeable from Table 5 that the highest order, 130 gives better estimation results than the other two lower orders. However, computational complexity is directly proportional to the estimation order of the Prony method and which limits its suitability for real-time applications. Synchrosqueezing transform based hybrid method has a better detection rate. However, proposed methodology has superior performance. VMD method completely failed to detect the lower order frequency components (eg. 8.3 Hz) and thus not suitable for smart grid scenarios. The error% attained in each case confirms the superior performance of the proposed hybrid method.

3) VOLTAGE FLUCTUATIONS WITH FUNDAMENTAL FREQUENCY DEVIATIONS

Many of the existing PQ analysis methods fail to give an accurate estimation during fundamental frequency deviations. The frequency deviations ($f = f_0 + \Delta f$) results in an asynchronous sampling situations and the estimations may not be accurate. The performance against fundamental frequency deviations is given in Table 6, for the waveform mentioned in Eqn. 33.

$$y(t) = [0.05\cos(2\pi 6t) + 0.02\cos(2\pi 10.13t) + 0.06\cos(2\pi 15.25t) + 0.09\cos(2\pi 24.5t)]\cos(2\pi f_0 t) \tag{33}$$

The waveform contains frequency components at 6, 10.13, 15.25, 24.5 Hz and the fundamental frequency is varied from 59.7 to 60.3 Hz. The results of the evaluation and comparison with synchro-hybrid method are tabulated in Table 6. It is obvious from the Table 6 that the proposed hybrid methodology precisely detected the frequency components.

4) VOLTAGE FLUCTUATIONS WITH HARMONICS AND INTER-HARMONICS

Detection of harmonic and inter-harmonic components from fluctuated voltage waveforms is a tedious task. Due to the presence of nonlinear loads, todays power grid is suffering from harmonics and inter-harmonics and thus, its detection

TABLE 6. Performance under fundamental frequency deviations.

Fund. freq.(Hz)	Proposed hybrid f_1	f_2	f_3	f_4	Synchro-hybrid [2] f_1	f_2	f_3	f_4
60.3	**6.00**	**10.13**	**15.25**	**24.5**	5.99	10.16	15.25	24.41
60.2	**6.00**	**10.13**	**15.25**	**24.5**	6.03	10.20	15.26	24.38
60.1	**6.00**	**10.13**	**15.25**	**24.5**	6.00	10.13	15.21	24.39
60.0	**6.00**	**10.13**	**15.25**	**24.5**	6.00	10.13	15.21	24.39
59.9	**6.00**	**10.13**	**15.25**	**24.5**	6.00	10.12	15.22	24.39
59.8	**6.00**	**10.13**	**15.25**	**24.5**	6.00	10.13	15.21	24.38
59.7	**6.00**	**10.13**	**15.25**	**24.5**	5.99	10.14	15.20	24.39

is essential. However, many of the conventional techniques fails to detect inter-harmonics as it is non-integer multiples of the fundamental frequency. The performance of the proposed hybrid methodology in detecting fluctuated voltage waveform with harmonic and inter-harmonic components is shown in Table 7. It is visible from Table 7 that the proposed methodology detected the fundamental and disturbance frequency and amplitude components accurately.

TABLE 7. Robustness of the proposed hybrid methodology to voltage fluctuations with harmonics and inter-harmonics.

Ref. freq. (Hz)	Est. freq. (Hz)	Ref. amp. (pu)	Est. amp. (pu)
4.000	4.000	0.004	0.004
10.00	10.00	0.002	0.002
28.00	28.00	0.006	0.006
120.0	120.0	0.800	0.800
75.00	75.00	0.200	0.200
Actual fund. freq = 60 Hz		Est. fund. freq = 60 Hz	

5) ROBUSTNESS TO NOISE AND HARMONICS CONTAMINATION

Robustness against noise and harmonics pollution is evaluated on a signal having a modulating magnitude of 1 pu with various modulating frequencies and inter-harmonics at 77.5 Hz. The results of the evaluation are shown in Table. 8. The estimated frequencies and corresponding error% are tabulated. The estimation with low error% highlights the accuracy of the proposed methodology. However the performance can be further improved by incorporating the noise-tolerant versions of DMD, such as Total DMD (TDMD) algorithm.

6) EFFECT OF PHASE ANGLES IN VOLTAGE FLUCTUATIONS

The performance advantage of the proposed method in flicker waveform with phase angles is evaluated. The voltage signal considered for evaluation is defined as,

$$y(t) = [1+0.008\cos(2\pi 6t+60^0)+0.003\cos(2\pi 15t+30^0) + 0.005\cos(2\pi 25t+36^0)]\cos(2\pi 60t+112.5^0) \tag{34}$$

The test waveform contains frequency components at 6, 15, 25 Hz with the fundamental frequency of 60 Hz. Fig. 10 (a) indicates the voltage waveform with and without phase angles. Fig. 10 (b) represents the eigenvalues visualized over the unit circle and Fig. 10 (c) represents the detected components frequency and corresponding magnitude. In this evaluation, it is observed that both test signals have obtained

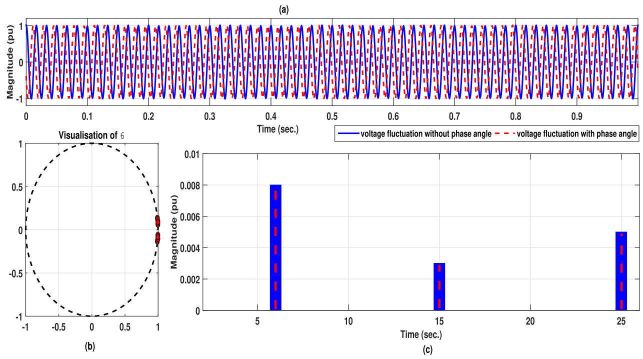

FIGURE 10. **Illustration of the performance of the proposed data-driven hybrid methodology. (a) input voltage signal with and without phase angle, (b) computed eigenvalues, (c) detected frequency components in both test signals with and without phase angles.**

the same detection results. Thus, it is concluded that the proposed data-driven hybrid method performs irrespective of the presence of phase angle in the modulated voltage signal.

Further, the fundamental dynamic phasor of the voltage waveform is extracted using the flat-top FIR filter tuned to the identified fundamental frequency. Fig. 11 represents the results of the fundamental dynamic phasor using the proposed methodology. It is visible from Fig. 11 that the proposed method efficiently tracked the magnitude envelope, fundamental frequency and phase of the fluctuated voltage signal.

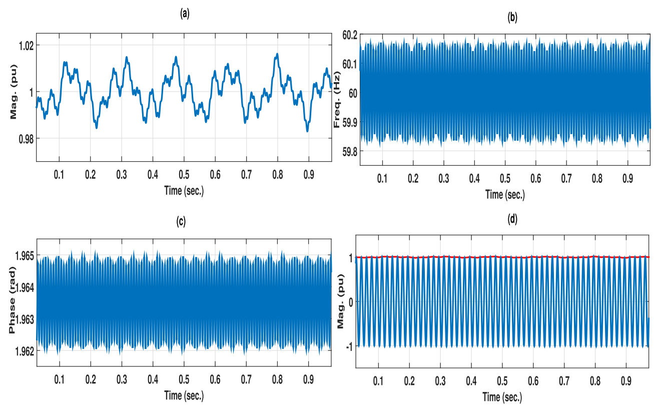

FIGURE 11. **Illustration of the fundamental dynamic phasor estimation using the proposed hybrid methodology. (a) magnitude envelope, (b) fundamental frequency, (c) fundamental phase, (d) voltage waveform with magnitude envelope.**

7) REAL-TIME VALIDATION

To evaluate the performance of the proposed hybrid voltage fluctuation monitoring system in the real-time scenarios, the voltage fluctuations data collected from the IEEE task force group is utilized. It contains arc furnace 138 kV flicker disturbances. The proposed hybrid methodology is used for the fundamental frequency estimation and the results are given in Table 9. The proposed hybrid methodology is capable of learning the dynamic fluctuations happening over the time and thus the proposed data-driven model is efficient for real-time voltage fluctuation monitoring.

TABLE 8. Performance obtained for various SNRs.

Mod. freq. (Hz)	SNR = 40 dB				SNR = 50 dB				SNR = 60 dB			
	Proposed		FFT		Proposed		FFT		Proposed		FFT	
	Freq. (Hz)	Err.%	Freq. (Hz)	Err.%	Freq. (Hz)	Err.%	Freq. (Hz)	Err.%	Freq. (Hz)	Err.%	Freq. (Hz)	Err.%
12.00	**12.01**	**0.08**	12.00	0.00	**12.00**	**0.00**	12.00	0.00	**12.00**	**0.00**	12.00	0.00
12.50	**12.63**	**1.04**	12.00	4.00	**12.50**	**0.00**	12.00	4.00	**12.49**	**0.08**	12.00	4.00
8.30	**8.40**	**1.20**	8.00	3.60	**8.30**	**0.00**	8.00	3.60	**8.29**	**0.12**	8.00	3.60
77.50	**77.40**	**0.10**	77.00	0.64	**77.49**	**0.01**	77.00	0.64	**77.50**	**0.00**	77.00	0.64

TABLE 9. Real-time validation of the proposed methodology.

Methods	Proposed	Adaptive Notch	DFT-Phasor	VMD
Est.freq (Hz)	**59.99**	60.05	59.39	59.98
Error %	**0.02**	0.08	1.02	0.03

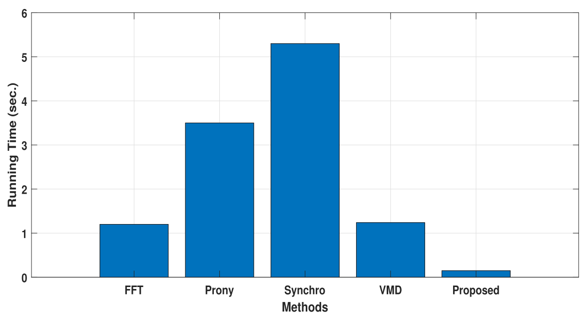

FIGURE 12. Illustration of the running time of different methods for detecting of non-integer frequency components.

8) COMPUTATIONAL COMPLEXITY AND REAL-TIME IMPLEMENTATION

Computational complexity is a major concern when considering the real-time implementation of any approach to power system-related applications. To achieve real-time monitoring of voltage fluctuations in distributed generation systems, the methodology should be computationally efficient, faster and less complex. As the proposed methodology is based on the randomized low-rank approximation, the computational complexity is relatively lower. The running time taken by each method for fluctuation monitoring in presence of non-integer harmonics is shown in Fig. 12 and the proposed methodology achieved the lowest running time. Thus, the low computational complexity and high speed of the proposed method is suitable for real-time voltage fluctuations monitoring. Usage of parallel computing and GPU implementations will further reduce the computational cost in real-time [43], [44].The proposed methodology can be further extended by using non-linear variants of DMD, such as Kernel DMD, which can effectively capture the non-linear nature of the underlying system.

V. CONCLUSION AND FUTURE SCOPE

Voltage fluctuations monitoring and exact estimation of frequency components and associated magnitude variations are crucial in power quality monitoring. In this paper, a novel method for voltage fluctuations monitoring in grid-connected distributed generation systems are proposed. The proposed data-driven hybrid approach composed of Hankelization, randomized, low-rank dynamic mode decomposition approximation and flat-top FIR filter for the extraction of frequencies, associated magnitudes and fundamental dynamic phasor in fluctuated voltage waveforms. The key features of the proposed hybrid methodology are:

- The methodology is data-adaptive and thus extracts meaningful relevant information from available data.
- The data-driven hybrid methodology is adapted to monitor the dynamic fluctuations happening over the grid and are suitable for real-time voltage fluctuation monitoring.
- The methodology maps the data into a lower-dimensional space using the randomized low-rank matrix approximation and hence it is computationally efficient and is free from tedious learning stages.

To confirm the performance advantage of the proposed methodology, various test scenarios are considered, including multiple frequency modulation, frequency deviations, inter-frequencies, noises and harmonics. Further validation of the proposed model is done using real-time field measurements. The improved voltage fluctuation monitoring and fast computation suggest that the proposed methodology can be used as an effective technique for voltage fluctuation monitoring and PQ analysis. It also has applications in the distribution grid to enhance situational awareness. Given its superior performance, the proposed methodology can have significant implications for various fields that rely on accurate frequency analysis such as, communication systems. The implementation of the proposed methodology using other computationally efficient, non-linear versions of DMD, such as Kernel DMD, in a parallel computing platform, further speeds up performance in real-time monitoring applications. This is considered as a potential future scope of this work.

REFERENCES

[1] M. I. Marei, T. K. Abdel-Galil, E. F. El-Saadany, and M. M. A. Salama, "Hilbert transform based control algorithm of the DG interface for voltage flicker mitigation," *IEEE Trans. Power Del.*, vol. 20, no. 2, pp. 1129–1133, Apr. 2005.

[2] G. W. Chang, Y.-Y. Chen, and Y.-L. Lin, "A synchrosqueezing transform-based hybrid method for voltage fluctuations assessment," *IEEE Trans. Power Del.*, vol. 33, no. 5, pp. 2541–2550, Oct. 2018.

[3] D. Kumar, F. Zare, and A. Ghosh, "DC microgrid technology: System architectures, AC grid interfaces, grounding schemes, power quality, communication networks, applications, and standardizations aspects," *IEEE Access*, vol. 5, pp. 12230–12256, 2017.

[4] M. Farajollahi, A. Shahsavari, E. M. Stewart, and H. Mohsenian-Rad, "Locating the source of events in power distribution systems using micro-PMU data," *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 6343–6354, Nov. 2018.

[5] A. von Meier, E. Stewart, A. McEachern, M. Andersen, and L. Mehrmanesh, "Precision micro-synchrophasors for distribution systems: A summary of applications," *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 2926–2936, Nov. 2017.

[6] N. Mohan and K. P. Soman, "Power system frequency and amplitude estimation using variational mode decomposition and Chebfun approximation system," in *Proc. 24th Nat. Conf. Commun. (NCC)*, Feb. 2018, pp. 1–6.

[7] E. Hossain, M. R. Tür, S. Padmanaban, S. Ay, and I. Khan, "Analysis and mitigation of power quality issues in distributed generation systems using custom power devices," *IEEE Access*, vol. 6, pp. 16816–16833, 2018.

[8] K. Srinivasan, "Digital measurement of voltage flicker," *IEEE Trans. Power Del.*, vol. 6, no. 4, pp. 1593–1598, Oct. 1991.

[9] C.-J. Wu and Y.-J. Chen, "A novel algorithm for precise voltage flicker calculation by using instantaneous voltage vector," *IEEE Trans. Power Del.*, vol. 21, no. 3, pp. 1541–1548, Jul. 2006.

[10] T. Zheng and E. B. Makram, "Wavelet representation of voltage flicker," *Electric Power Syst. Res.*, vol. 48, no. 2, pp. 133–140, Dec. 1998.

[11] S.-J. Huang and C.-W. Lu, "Enhancement of digital equivalent voltage flicker measurement via continuous wavelet transform," *IEEE Trans. Power Del.*, vol. 19, no. 2, pp. 663–670, Apr. 2004.

[12] H. M. Al-Hamadi, "Fuzzy logic voltage flicker estimation using Kalman filter," *Int. J. Electr. Power Energy Syst.*, vol. 36, no. 1, pp. 60–67, 2012.

[13] N. Kose, O. Salor, and K. Leblebicioglu, "Kalman filtering based approach for light flicker evaluation of power systems," *IET Gener., Transmiss. Distrib.*, vol. 5, no. 1, pp. 57–69, 2011.

[14] I. Sadinezhad and V. G. Agelidis, "Frequency adaptive least-squares-Kalman technique for real-time voltage envelope and flicker estimation," *IEEE Trans. Ind. Electron.*, vol. 59, no. 8, pp. 3330–3341, Aug. 2012.

[15] A. Elnady and M. M. A. Salama, "Unified approach for mitigating voltage sag and voltage flicker using the DSTATCOM," *IEEE Trans. Power Del.*, vol. 20, no. 2, pp. 992–1000, Apr. 2005.

[16] T. K. Abdel-Galil, E. F. El-Saadany, and M. M. A. Salama, "Online tracking of voltage flicker utilizing energy operator and Hilbert transform," *IEEE Trans. Power Del.*, vol. 19, no. 2, pp. 861–867, Apr. 2004.

[17] E. A. Feilat, "Detection of voltage envelope using Prony analysis-Hilbert transform method," *IEEE Trans. Power Del.*, vol. 21, no. 4, pp. 2091–2093, Oct. 2006.

[18] M. A. Eldery, E. F. El-Saadany, and M. M. A. Salama, "A simple energy operator computational method for voltage flicker assessment," *IEEE Trans. Power Del.*, vol. 21, no. 3, pp. 1743–1750, Jul. 2006.

[19] F. Li, Y. Gao, Y. Cao, and R. Iravani, "Improved Teager energy operator and improved chirp-Z transform for parameter estimation of voltage flicker," *IEEE Trans. Power Del.*, vol. 31, no. 1, pp. 245–253, Feb. 2016.

[20] M. S. Reza, M. M. Hossain, and M. Ciobotaru, "Teager energy operator for fast estimation of three-phase grid frequency," *IEEE Trans. Instrum. Meas.*, vol. 70, pp. 1–10, 2021.

[21] W. M. Al-Hasawi and K. M. El-Naggar, "A genetic based algorithm for voltage flicker measurement," in *Proc. 11th IEEE Medit. Electrotechnical Conf.*, May 2002, pp. 600–604.

[22] D. Sun, Q. Gao, Y. Lu, and M. Zhu, "A sparse representation-based algorithm for the voltage fluctuation detection of a power system," *Digit. Signal Process.*, vol. 48, pp. 259–268, Jan. 2016.

[23] A. K. Al-Othman and K. M. El-Nagger, "Voltage flicker measurement using particle swarm optimization technique for power quality assessment," in *Proc. IEEE Medit. Electrotechnical Conf.*, 2006, pp. 1077–1082.

[24] N. Eghtedarpour, E. Farjah, and A. Khayatian, "Effective voltage flicker calculation based on multiresolution S-transform," *IEEE Trans. Power Del.*, vol. 27, no. 2, pp. 521–530, Apr. 2012.

[25] W. Yao, Q. Tang, Z. Teng, Y. Gao, and H. Wen, "Fast S-transform for time-varying voltage flicker analysis," *IEEE Trans. Instrum. Meas.*, vol. 63, no. 1, pp. 72–79, Jan. 2014.

[26] Y. Önal, Ö. N. Gerek, and D. G. Ece, "Empirical mode decomposition application for short-term flicker severity," *TURKISH J. Electr. Eng. Comput. Sci.*, vol. 24, no. 2, pp. 499–509, 2016.

[27] P. D. Achlerkar, S. R. Samantaray, and M. S. Manikandan, "Variational mode decomposition and decision tree based detection and classification of power quality disturbances in grid-connected distributed generation system," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3122–3132, Jul. 2018.

[28] A. M. Alkandari and S. A. Soliman, "Measurement of a power system nominal voltage, frequency and voltage flicker parameters," *Int. J. Electr. Power Energy Syst.*, vol. 31, nos. 7–8, pp. 295–301, Sep. 2009.

[29] A. Dejamkhooy, A. Dastfan, and A. Ahmadyfard, "Modeling and forecasting nonstationary voltage fluctuation based on grey system theory," *IEEE Trans. Power Del.*, vol. 32, no. 3, pp. 1212–1219, Jun. 2017.

[30] M. I. Marei, E. F. El-Saadany, and M. M. A. Salama, "Envelope tracking techniques for flicker mitigation and voltage regulation," *IEEE Trans. Power Del.*, vol. 19, no. 4, pp. 1854–1861, Oct. 2004.

[31] D. L. Geiger and S. M. Halpin, "Assessing voltage fluctuations and lamp flicker using RMS voltages," *IEEE Trans. Power Del.*, vol. 32, no. 6, pp. 2481–2488, Dec. 2017.

[32] M. Mazadi, S. H. Hosseinian, W. Rosehart, and D. T. Westwick, "Instantaneous voltage estimation for assessment and monitoring of flicker indices in power systems," *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1841–1846, Jul. 2007.

[33] P. J. Schmid, "Dynamic mode decomposition of numerical and experimental data," *J. Fluid Mech.*, vol. 656, pp. 5–28, Jan. 2010.

[34] J. N. Kutz, S. L. Brunton, B. W. Brunton, and J. L. Proctor, *Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems*. Philadelphia, PA, USA: SIAM, 2016.

[35] B. W. Brunton, L. A. Johnson, J. G. Ojemann, and J. N. Kutz, "Extracting spatial–temporal coherent patterns in large-scale neural recordings using dynamic mode decomposition," *J. Neurosci. Methods*, vol. 258, pp. 1–15, Jan. 2016.

[36] N. Mohan, K. Soman, and S. S. Kumar, "A data-driven strategy for short-term electric load forecasting using dynamic mode decomposition model," *Appl. Energy*, vol. 232, pp. 229–244, Dec. 2018.

[37] E. Barocio, B. C. Pal, N. F. Thornhill, and A. R. Messina, "A dynamic mode decomposition framework for global power system oscillation analysis," *IEEE Trans. Power Syst.*, vol. 30, no. 6, pp. 2902–2912, Nov. 2015.

[38] S. Mohapatra and T. J. Overbye, "Fast modal identification, monitoring, and visualization for large-scale power systems using dynamic mode decomposition," in *Proc. Power Syst. Comput. Conf. (PSCC)*, Jun. 2016, pp. 1–7.

[39] N. Mohan and K. P. Soman, "A data-driven technique for harmonics monitoring in emerging power grids using noise-aware dynamic mode decomposition," *Meas. Sci. Technol.*, vol. 31, no. 1, Jan. 2020, Art. no. 015016.

[40] K. Soman and R. Ramanathan, *Digital Signal and Image Processing—The Sparse Way*. India: ISA, 2012.

[41] S. S. Kumar, K. Manjusha, and K. Soman, "Novel SVD based character recognition approach for Malayalam language script," in *Recent Advances in Intelligent Informatics*. Berlin, Germany: Springer, 2014, pp. 435–442.

[42] N. Halko, P. G. Martinsson, and J. A. Tropp, "Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions," *SIAM Rev.*, vol. 53, no. 2, pp. 217–288, 2011.

[43] N. B. Erichson and C. Donovan, "Randomized low-rank dynamic mode decomposition for motion detection," *Comput. Vis. Image Understand.*, vol. 146, pp. 40–50, May 2016.

[44] N. B. Erichson, S. Voronin, S. L. Brunton, and J. N. Kutz, "Randomized matrix decompositions using R," 2016, *arXiv:1608.02148*.

[45] K. Duda, T. P. Zieliński, and S. Barczentewicz, "Perfectly flat-top and equiripple flat-top cosine windows," *IEEE Trans. Instrum. Meas.*, vol. 65, no. 7, pp. 1558–1567, Jul. 2016.

[46] K. Duda and T. P. Zieliński, "FIR filters compliant with the IEEE standard for M class PMU," *Metrol. Meas. Syst.*, vol. 23, no. 4, pp. 623–636, Dec. 2016.

[47] I. S. Reljin, B. D. Reljin, and V. D. Papic, "Extremely flat-top windows for harmonic analysis," *IEEE Trans. Instrum. Meas.*, vol. 56, no. 3, pp. 1025–1041, Jun. 2007.

[48] J. H. Tu, "Dynamic mode decomposition: Theory and applications," Ph.D. thesis, Dept. Mech. Aerosp. Eng., Princeton Univ., Princeton, NJ, USA, 2013.

[49] P. Top, M. R. Bell, E. Coyle, and O. Wasynczuk, "Observing the power grid: Working toward a more intelligent, efficient, and reliable smart grid with increasing user visibility," *IEEE Signal Process. Mag.*, vol. 29, no. 5, pp. 24–32, Sep. 2012.

NEETHU MOHAN received the Ph.D. degree in data-driven techniques for power system analysis. She is currently an Assistant Professor with the Centre for Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India. She has authored several national and international publications. Her research interests include power quality monitoring, data-driven analysis, artificial intelligence, and bio-medical data analytics.

S. SACHIN KUMAR received the Ph.D. degree in natural language processing. He is currently an Assistant Professor with the Centre for Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India. He has several years of experience in working on data science related problems. His research interests include artificial intelligence, bio-sequence, biomedical data, cybersecurity, data-driven approaches and applications for text, forecasting, machine vision systems, and multimodal data analysis. He is a reviewer of several journals, including IEEE Access, IEEE Transactions on Cybernetics, *Applied Mathematical Modeling* (Elsevier), and *Applied Energy*.

K. P. SOMAN is currently the Head and a Professor with the Center for Computational Engineering and Networking (CEN) and the Dean of the Amrita School of Artificial Intelligence, Amrita Vishwa Vidyapeetham, Coimbatore, India. He has more than 25 years of research and teaching experience in artificial intelligence and data science related subjects with the Amrita School of Engineering, Coimbatore. He has around 450 publications to his credit in reputed journals, such as IEEE Transactions, IEEE Access, *Applied Energy*, and conference proceedings. He published four books, namely, *Insight Into Wavelets*, *Insight Into Data Mining*, *Support Vector Machines and Other Kernel Methods*, and *Signal and Image Processing-The Sparse Way*. He is the most cited author in Amrita Vishwa Vidyapeetham in the areas of artificial intelligence and data science. He was listed among the Top-10 Computer Science Faculty by the DST, Government of India, from 2009 to 2013, the Career 360 and MHRD, from 2017 to 2018, and also in the list of the most prolific authors in the world, prepared by Springer Nature.

V. G. SUJADEVI is currently the joint Director of the Centre for Internet Studies and Artificial Intelligence, Amrita Vishwa Vidyapeetham, Kollam Campus. Her research interests include deep learning, natural language processing, artificial intelligence, and machine learning. In addition to these areas, she has also been researching on health data and signal analytics.

PRABAHARAN POORNACHANDRAN is currently a Professor and the Director of the Centre for Internet Studies and Artificial Intelligence, Amrita Vishwa Vidyapeetham, Kollam Campus. He has more than two decades of experience in computer science and security areas. His research interests include big data, the IoT, machine learning, malware, critical infrastructure security, digital healthcare, and complex binary analysis.

U. RAJENDRA ACHARYA received the Ph.D., D.Eng., and D.Sc. degrees. He is currently a Professor with the School of Mathematics, Physics and Computing, University of Southern Queensland, Springfield, Australia. He is also a Distinguished Professor with Kumamoto University, Japan, and an Associate Faculty Member of the Singapore University of Social Sciences, Singapore. He has published more than 550 papers in refereed international SCI-IF journals (500), international conference proceedings (42), and books (17) with more than 65,000 citations in Google Scholar (with an H-index of 128). In addition, he has worked on various funded projects with grants worth more than six million SGD. According to the Essential Science Indicators of Thomson, he has been ranked in the top 1% of the Highly Cited Researchers for the last seven consecutive years in computer science from 2016 to 2022. His research interests include biomedical signal processing, biomedical imaging, artificial intelligence, visualization, and biophysics for better healthcare design, delivery, and therapy. He is on the editorial board of many journals and has served as a guest editor for many journals.