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# **RESEARCH ARTICLE**

# A Novel Neural Network-Based Robust Adaptive Formation Control for Cooperative Transport of a Payload Using Two Underactuated Quadcopters

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**ABSTRACT** Designing a controller for the cooperative transport of a payload using quadcopter-type unmanned aerial vehicles (UAVs) is a very challenging task in control theory because these vehicles are underactuated mechanical systems. This paper presents a novel robust adaptive formation control design for the cooperative transport of a suspended payload by ropes using two underactuated quadcopters in the presence of external disturbances and parametric uncertainties. The structure of the proposed controller is divided into two subsystems: fully actuated and underactuated. An integral sliding mode adaptive control strategy is proposed for the fully actuated subsystem, and for the underactuated subsystem, an adaptive control strategy based on the combination of Backstepping and sliding mode is proposed. Then, the control parameters of the sliding surfaces of both control subsystems are adaptively tuned by a neural network. In addition, to improve the robustness of the proposed controller, a disturbance observer is incorporated to estimate and compensate for the lumped disturbances. The asymptotic stability of the cooperative transport system is verified with the Lyapunov theorem. Finally, numerical simulations are performed in MATLAB/Simulink environment, and the results show that the proposed controller successfully transports the payload safely and without oscillations. Moreover, the desired formation pattern is maintained throughout the flight task, even with external disturbances and parametric uncertainties.

**INDEX TERMS** Robust adaptive control, cooperative payload transportation, quadcopters, underactuated, UAVs.

# **I. INTRODUCTION**

#### A. CONTEXT AND MOTIVATIONS

<span id="page-0-1"></span><span id="page-0-0"></span>Quadcopters are very versatile unmanned aerial vehicles (UAV) due to their diverse capabilities for civilian and military applications [\[1\]. Co](#page-12-0)operative payload transport is one of the applications that has recently attracted the attention of researchers [\[2\] be](#page-12-1)cause it can be used for transporting medical packages to rural areas that are difficult to access [\[3\],](#page-12-2) transporting tanks with chemicals for crop spraying [\[4\], tra](#page-12-3)nsporting loads for construction processes [\[5\], etc](#page-12-4).

A quadcopter is an underactuated mechanical system with six degrees of freedom and four control inputs, i.e., the

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<span id="page-0-8"></span><span id="page-0-7"></span><span id="page-0-6"></span><span id="page-0-5"></span><span id="page-0-4"></span><span id="page-0-3"></span><span id="page-0-2"></span>control action cannot act independently in all degrees of freedom [\[6\], \[](#page-12-5)[7\]. N](#page-12-6)ow, when a quadcopter is attached to a suspended payload, the underactuated mechanical system is more difficult to control due to unknown nonlinear characteristics, dynamic coupling, and payload oscillations. In this context, research has been conducted on payload transport with a single quadcopter  $[8]$ ,  $[9]$ ,  $[10]$ ; these works mainly aim to stabilize the quadcopter and reduce payload oscillations. However, transporting a payload with a single quadcopter presents some limitations, such as payload weight, payload oscillations, and intolerance to failure [\[11\].](#page-12-10) The solution to this problem is using several quadcopters to carry a payload using a multi-agent system widely used in the literature for multi-quadcopter flight formation  $[12]$ , [\[13\],](#page-12-12) [\[14\]. U](#page-12-13)sing multi-quadcopters for transport improves

payload performance, i.e., heavier payloads can be transported, greater control of oscillations, and the ability to complete the transport task even if one quadcopter fails. However, designing a control strategy for the flight formation of multiple quadcopters carrying a payload is even more complicated. Such complications are due to the communication between the quadcopters and the suspended payload for flight formation, coupled nonlinear dynamics, and that quadcopters are underactuated mechanical systems, the latter being a crucial property for control design and has not been considered as such in many research works. Therefore, the control design for a quadcopter as an underactuated mechanical system requires a nonlinear control strategy suitable and reliable for the cooperative transport of a payload.

This paper investigates the control design for the cooperative transport of a payload using two underactuated quadcopters. Therefore, the main objective of the paper is to transport the payload safely and with minimal oscillation while maintaining the desired formation pattern and stability of the underactuated quadcopters with the payload suspended.

# B. LITERATURE REVIEW ON THE COOPERATIVE TRANSPORT OF A PAYLOAD USING QUADCOPTERS

There are several research works on cooperative payload transport using quadcopters, such as payload transport by gripping with a robotic arm [\[15\],](#page-12-14) [\[16\],](#page-12-15) [\[17\] a](#page-12-16)nd payload transport using cables or ropes [\[18\], \[](#page-12-17)[19\], \[](#page-12-18)[20\], \[](#page-12-19)[21\], \[](#page-12-20)[22\].](#page-12-21) Each of these methods has its advantages and disadvantages; in the use of gripping with a robotic arm, the payload does not oscillate, but it causes increased inertia, high cost, and high energy consumption. On the other hand, the use of cables or ropes is more popular due to its simplicity and low cost, but it causes an increase in the degrees of freedom in quadcopters due to payload oscillations which leads to a more complicated control design.

<span id="page-1-3"></span><span id="page-1-2"></span>In this context, several efforts have been made to address the control design for the cooperative transport of a suspended payload using ropes. For example, some linear control techniques such as Proportional, Integral and Derivative (PID) [\[23\], \[](#page-12-22)[24\], \[](#page-13-0)[25\], \[](#page-13-1)[26\] a](#page-13-2)nd Linear Quadratic Regulator (LQR) [\[27\], \[](#page-13-3)[28\] h](#page-13-4)ave been used. In research [\[23\] u](#page-12-22)sing two UAVs for cooperative transportation, an adaptive kinematic PID controller based on null space theory is proposed. In [\[24\],](#page-13-0) a trajectory planning method is proposed using three quadcopters for cooperative transport, and a dual cascade PID control is designed for trajectory tracking. In [\[25\], u](#page-13-1)sing four quadcopters for cooperative transport, a PID controller is proposed for quadcopter motion and PD for load swing control. In [\[26\], a](#page-13-2)n LQR-PID control strategy is designed for cooperative transport using four quadcopters and proposes a guidance algorithm using a virtual leader scheme based on payload position. In [\[27\], a](#page-13-3) leader-follower scheme is implemented using two quadcopters; the authors propose a hierarchical control; for the position, an LQR control

<span id="page-1-4"></span>is implemented, and for the attitude, a Quaternion-based controller is implemented. In [\[28\], t](#page-13-4)wo quadcopters are also used for cooperative transport, and an iterative LQR control is proposed. Some nonlinear control techniques have also been used to address the cooperative transport of a payload using quadcopters. Research works [\[29\], \[](#page-13-5)[30\], \[](#page-13-6)[31\] pr](#page-13-7)oposes a nonlinear control strategy based on feedback linearization for cooperative transport; however, such a control strategy is not useful for the control of underactuated mechanical systems because it cannot cancel the undesirable dynamics of underactuated quadcopters [\[6\]. In](#page-12-5) [\[32\], a](#page-13-8) predictive control strategy for transporting a payload employing four quadcopters is proposed. In [\[33\], B](#page-13-9)ackstepping control is proposed for cooperative transport under suspension failures, and a disturbance observer is designed to estimate the rope tension forces.

<span id="page-1-6"></span><span id="page-1-5"></span>In the previous research, different linear and nonlinear control strategies were designed to address for the cooperative transport of a payload. However, the authors did not consider the underactuated property of quadcopters for the control design, i.e., they designed one control action for each degree of freedom, which is not fulfilled in underactuated mechanical systems such as quadcopters that have four inputs and six outputs. In contrast, in our paper, we consider the underactuated property of quadcopters for control design, which is the main novelty of the paper, *i.e.*, for each underactuated quadcopter, only four control inputs are designed to perform the cooperative transport task.

# <span id="page-1-1"></span><span id="page-1-0"></span>C. STATE OF THE ART OF RELATED WORK

<span id="page-1-8"></span><span id="page-1-7"></span>In the literature, there are research works that did focus on the control design for a quadcopter as an underactuated mechanical system, i.e., four control inputs were designed for six degrees of freedom; these studies were based on sliding mode [\[34\],](#page-13-10) [\[35\],](#page-13-11) [\[36\],](#page-13-12) [\[37\],](#page-13-13) [\[38\] a](#page-13-14)nd Backstepping [\[39\], \[](#page-13-15)[40\] co](#page-13-16)ntrol techniques. However, only trajectory tracking control was addressed, but not formation control for a cooperative transport system, although it is worth noting that research works [\[37\], \[](#page-13-13)[38\] u](#page-13-14)sed neural networks to estimate unknown nonlinearities and external disturbances, improving the robustness of the controller. In this sense, using neural networks for control design provides an adaptive method to handle unknown external disturbances, which are good candidates for a cooperative payload transport application.

<span id="page-1-9"></span>To the best of our knowledge, a robust adaptive formation control for the cooperative transport of a payload using quadcopters as underactuated mechanical systems has not been designed in the literature. Although, recently in [\[41\], \[](#page-13-17)[42\],](#page-13-18) control design for payload transport with a single quadcopter as an underactuated mechanical system was addressed, where in [\[41\] pr](#page-13-17)oposes a sliding neuro-mode controller, and in [\[42\]](#page-13-18) proposes a finite-time sliding neuro-mode controller. However, they did not address formation control for cooperative transport; moreover, in their results, they only present the

temporal response of the quadcopter but not the temporal response of the suspended payload, thus the effectiveness of the proposed controller to suppress oscillations of a suspended payload, which is one of the main limitations of suspended payload transport with a single quadcopter, cannot be guaranteed. In contrast, our work does present the temporal response of the quadcopters and the suspended payload when cooperative transport is performed.

When addressing the cooperative transport of a payload, the rope tension forces generated by the weight of the payload will significantly impact the dynamics and stability of the quadcopters because these tension forces cannot be measured directly. An efficient solution is to incorporate a disturbance observer into the controller to compensate for all types of disturbances, both external and internal. In [\[18\] an](#page-12-17)d [\[33\], th](#page-13-9)e authors consider the rope tensions as an external disturbance and estimate it with a disturbance observer. However, these works do not consider disturbances like wind gusts and parametric uncertainties.

# D. CONTRIBUTIONS

Motivated by the previous research works, this paper presents a novel control design for the cooperative transport of a payload using two quadcopters which are considered an underactuated mechanical system. The main contributions of the present paper are as follows:

- A novel robust adaptive formation controller is designed for the cooperative transport of a payload suspended by ropes using two underactuated quadcopters. For this purpose, the controller structure is divided into two subsystems: fully actuated and underactuated. An integral sliding mode adaptive control strategy is proposed for the fully actuated subsystem, and for the underactuated subsystem, an adaptive control strategy based on the combination of Backstepping and sliding mode is proposed. In addition, a neural network is used to adaptively tune the control parameters of the sliding surfaces in both control subsystems. To the best of our knowledge, this is the first time in the literature that control design for cooperative transport using quadcopters as underactuated mechanical systems is addressed.
- In this paper, external disturbances (unmeasurable rope tensions, wind gusts) and internal disturbances (parametric uncertainties) have been considered for the cooperative transport of a payload; all these disturbances constitute the total lumped disturbances and are estimated with a nonlinear disturbance observer.
- In comparison to  $[18]$  and  $[26]$ ,  $[33]$ , where three and four quadcopters are used to transport a payload with the same weight. The proposed control strategy in this paper uses only two underactuated quadcopters to transport the same payload, demonstrating the higher robustness of the proposed controller against disturbances.

<span id="page-2-1"></span>

**FIGURE 1.** The cooperative transport system of a payload using two quadcopters.

# E. MANUSCRIPT ORGANIZATION

In [II,](#page-2-0) the dynamic model is developed. In [III,](#page-3-0) the flight formation of the cooperative transport system is presented. In [IV,](#page-4-0) the four control inputs for each quadcopter are designed. The results of the numerical simulations are discussed in [V.](#page-8-0) Finally, in [VI,](#page-11-0) the conclusions of the paper are presented.

# <span id="page-2-0"></span>**II. DYNAMIC MODEL**

In this section, the dynamic model of the underactuated quadcopters and the suspended payload is developed. Fig. [1](#page-2-1) shows a representation of cooperative transport using two quadcopters, two ropes of length  $l_1$  and  $l_2$ , and a suspended payload. Throughout the paper, the subscript *i* corresponds to the *i*-th quadcopter  $(i = 1, 2, \ldots n)$ .

# A. QUADCOPTER DYNAMIC MODEL

The *i*-th quadcopter as a rigid body is characterized by a body-fixed frame  $B_i = \{x_B, y_B, z_B\}$  and an earth-fixed frame  $E_0 = \{x_E, y_E, z_E\}$  (see Fig. [1\)](#page-2-1). The vector  $\xi_i = [x_i, y_i, z_i]^T$ and the vector  $\eta_i = [\phi_i, \theta_i, \psi_i]^T$  describe the position and orientation of the *i*-th quadcopter in the *E* frame, where  $\phi$ ,  $\theta$ , and  $\psi$  represent the Euler angles roll, pitch and yaw respectively. The rotation matrix  $R_i: B_i \Rightarrow E$ , is the transformation matrix from the body-fixed frame to the earth-fixed frame and is defined by:

$$
R_{i} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}.
$$
 (1)

where  $c(\cdot) = \cos(\cdot), s(\cdot) = \sin(\cdot)$ . Using the Newton Euler mathematical approach, the translational and rotational dynamics of the *i*-th quadcopter are described by [\[20\]:](#page-12-19)

<span id="page-2-2"></span>
$$
\begin{cases}\nm_{qi}\ddot{\xi}_i = U_{1i}R_ie_z - m_{qi}ge_z - \mathbb{T}_i \times r_i + \delta_p(t), \\
J_i\ddot{\eta}_i = -\dot{\eta}_i \times J_i\dot{\eta}_i - \sum_{k=1}^4 J_{ri}(\dot{\eta}_i \times e_z)(-1)^{k+1}\Omega_k \\
&+ U_i^{\tau} + \delta_{\Theta}(t).\n\end{cases}
$$
\n(2)

Where  $\xi_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$ ,  $m_{qi} \in \mathbb{R}^+$ , and  $g \in \mathbb{R}^+$ represent the position, mass, and gravity acceleration of the *i*-th quadcopter.  $U_{1i} \in \mathbb{R}^+$  is the magnitude of the total thrust generated by the four rotors, and  $R_i \in \mathbb{R}^{3 \times 3}$  is the rotation matrix.  $\mathbb{T}_i = [\mathbb{T}_x, \mathbb{T}_y, \mathbb{T}_z]^T \in \mathbb{R}^3$  represents the vector of the rope tension force acting on each quadcopter. Likewise,  $\eta_i = [\phi_i, \theta_i, \psi_i]^T \in \mathbb{R}^3$ ,  $J_i \in \mathbb{R}^{3 \times 3}$  represents the orientation and moment of inertia, and similarly, the moment and velocity of inertia of the rotors of the *i*-th quadcopter are denoted respectively by  $J_{ri} \in \mathbb{R}^+$  and  $\Omega_k \in \mathbb{R}^+$ . The vector  $U_i^{\tau} = [g U_{2i}, g U_{3i}, \kappa U_{4i}]^T \in \mathbb{R}^3$  represents the input torques, where  $\zeta$  represents the distance from the rotors to the center of the quadcopter, i.e., arm length, and  $\kappa$  is the drag coefficient [\[43\]. T](#page-13-19)he matrix  $\dot{\eta}_i \times = S(\dot{\eta}_i) \in \mathbb{R}^{3 \times 3}$  is a skewsymmetric matrix, where  $\times$  denotes the vector cross product,  $e_z = [0, 0, 1]^T \in \mathbb{R}^3$  is a unit vector, and finally,  $\delta_p(t)$  and  $\delta_{\Theta}(t)$  are external disturbances of the translation and rotation dynamics, respectively.

# <span id="page-3-4"></span>B. PAYLOAD DYNAMIC MODEL

The payload is assumed to be on the ground initially. Therefore, the dynamics of payload is described by [\[20\]:](#page-12-19)

$$
\begin{cases}\nm_p \ddot{\xi}_p = -m_p g e_z + \sum_{i=1}^n \mathbb{T}_i \times r_i, \\
J_p \ddot{\eta}_p = -\dot{\eta}_p \times J_p \dot{\eta}_p - \sum_{k=1}^n \mathbb{T}_i \times r_i.\n\end{cases} \tag{3}
$$

where  $\xi_p \in \mathbb{R}^3$ ,  $\eta_p \in \mathbb{R}^3$ ,  $m_p \in \mathbb{R}^+$  and  $J_p \in \mathbb{R}^+$  represent the position, orientation, mass, and moment of inertia of the payload, respectively.  $\sum_{i=1}^{n} \mathbb{T}_i$  is the sum of the tension forces of the *n* ropes acting on the payload, and finally,  $r_i \in \mathbb{R}^3$  is a vector from the center of mass of the payload to the point of attachment of the ropes.

#### C. THE UDWADIA-KALABA EQUATION

The tension forces of the ropes connected to the quadcopters are constraint forces and can be obtained using the Udwadia-Kalaba equations [\[26\], \[](#page-13-2)[44\]. T](#page-13-20)his method provides explicit equations of the constraint forces, which are functions of the states of all the bodies involved and, therefore, are the most suitable for simulation purposes [\[45\].](#page-13-21)

Consider the *i*-th rope connecting the *i*-th quadcopter with the payload. The constraint of the *i*-th rope can be defined as:

<span id="page-3-5"></span><span id="page-3-1"></span>
$$
\varrho_i = \|\mathbf{f}_i\|^2 - l_i^2. \tag{4}
$$

where  $\mathbf{f}_i = \xi_i - \xi_p$ ,  $l_i$  is the nominal length of the *i*-th rope. Differentiating two times [\(4\)](#page-3-1), the cooperative transport system constraint can be formulated in its standard form as:

$$
A_i(p, v)\dot{v} = B_i(p, v). \tag{5}
$$

where:

$$
p = [\xi_1 \ \xi_2 \ \dots \ \xi_n \ \xi_p]^T,
$$
  

$$
v = [\dot{\xi}_1 \ \dot{\xi}_2 \ \dots \ \dot{\xi}_n \ \dot{\xi}_p]^T,
$$

Then, the constraint forces of all the ropes can be calculated by  $[26]$ :

<span id="page-3-2"></span>
$$
\mathbb{T} = M^{1/2} (AM^{-1/2})^{+} (B - 2\alpha \dot{\varrho} - \beta^2 \varrho - A \ddot{\chi}).
$$
 (7)

where,  $\mathbb{T} = [\mathbb{T}_1, \mathbb{T}_2, \dots, \mathbb{T}_n, \sum_{i=1}^n \mathbb{T}_i]^T$ ,  $\ddot{\chi}$  is the unconstrained acceleration of the cooperative transport system,  $(\cdot)^+$  denotes the Moore-Penrose pseudo inverse,  $\alpha$  and  $\beta$  are feedback gains, and finally  $M \in \mathbb{R}^{(n+1)\times(n+1)}$  is:

$$
M = \begin{bmatrix} m_1 I_{3 \times 3} & 0_{3 \times 3} & \dots & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & m_2 I_{3 \times 3} & \dots & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & \dots & m_n I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & \ddots & 0_{3 \times 3} & m_L I_{3 \times 3} \end{bmatrix}.
$$
 (8)

*Remark 1:* The tensions obtained in [\(7\)](#page-3-2) were calculated for simulation purposes. These tension forces cannot be measured directly; therefore, a nonlinear disturbance observer will estimate these forces in later sections.

# <span id="page-3-0"></span>**III. COOPERATIVE FLIGHT FORMATION**

In this paper, the cooperative transport system is considered as a flight formation of a multi-agent system since it can provide advantages such as uniform payload distribution, collision avoidance, and payload oscillation reduction. Therefore, this paper employs graph theory for the flight formation of the cooperative transport system.

# A. GRAPH THEORY

Graph theory is extensively used for a communication network in multi-agent systems [\[13\]. C](#page-12-12)onsider a group of *n* agents communicating with their neighbors described by the graph  $G = (V, E)$ , where *V* and *E* represent the nodes and edges, respectively. An edge  $(V_i, V_j)$  means that agent *i* can access the information of agent *j* and vice versa. The adjacency matrix of a graph is defined as *A*, whose elements are  $a_{ij} > 0$  if  $(V_i, V_j) \in E$  and  $a_{ij} = 0$  otherwise. The degree matrix is defined as  $D = diag(d_1, \ldots, d_n) \in R^{n \times n}$  with  $d_i = \sum_{j=1}^n a_{ij}$ . Then, the Laplacian matrix can be expressed as  $L = D - A$ . In multi-agent systems, one agent is called the leader, and the other agents are called followers; the diagonal matrix of the leader is defined as  $B = diag(b_1, \ldots, b_n)$ , where  $b_i > 0$  if agent *i* can access the information of the leader; otherwise  $b_i = 0$ .

<span id="page-3-6"></span><span id="page-3-3"></span>*Assumption 1:* The network is considered to be directed for flight formation of the cooperative transport system.

<span id="page-3-7"></span>*Lemma 1:* According to the assumption [1,](#page-3-3) all eigenvalues of the matrix  $L + B$  have positive real parts [\[46\].](#page-13-22)

For the flight formation of the cooperative transport system, a point is selected as the formation center, which is considered as the leader agent of the group. Under this premise and based on research works [\[12\],](#page-12-11) [\[26\], i](#page-13-2)n this paper the payload is considered as a virtual leader, which is represented

<span id="page-4-1"></span>

**FIGURE 2.** Communication topology of the desired formation pattern.

by the subscript *L*, and the quadcopters as followers, which are represented by the subscript *i*. Thereby, a desired formation pattern is established as a straight line shape in the *xy*plane, such that the quadcopters are in a symmetric position with respect to the payload position. The Fig. [2](#page-4-1) illustrates the graphical communication of the desired formation pattern for the cooperative transport system.

Next, the formation error and its derivative for the *i*-th quadcopter are defined as follows:

<span id="page-4-2"></span>
$$
e_i^{\xi} = \sum_{j=1}^n a_{ij} (\xi_j - \xi_i - (\Delta_j - \Delta_i)) + b_i (\xi_L^d - \xi_i + \Delta_i),
$$
  
\n
$$
e_i^V = \sum_{j=1}^n a_{ij} (V_j - V_i) + b_i (V_L^d - V_i).
$$
\n(9)

where  $\xi_i = [x_i, y_i, z_i]^T$  and  $V_i = [x_i, \dot{y}_i, \dot{z}_i]^T$  represent the position and velocity of the *i*-th quadcopter.  $\xi_L^d = [x_L^d, y_L^d, z_L^d]^T$  and  $V_L^d = [x_L^d, y_L^d, z_L^d]^T$  represent the desired position and velocity for the virtual leader.  $\Delta_i$  =  $[\Delta_{xi}, \Delta_{yi}, \Delta_{zi}]^T$  represents the expected relative formation distance between the *i*-th quadcopter and the virtual leader. Expressing [\(9\)](#page-4-2) in its general form for the cooperative transport system is given as follows:

$$
E_X = (L + B \otimes I_3)[X - \overline{X}_L - \Delta]^T,
$$
  
\n
$$
E_V = (L + B \otimes I_3)[V - \overline{V}_L]^T.
$$
 (10)

where  $\otimes$  denotes the Kronecker product,  $X = [\xi_1, \dots, \xi_n]^T$ and  $V = [\dot{\xi}_1, \dots, \dot{\xi}_n]^T$  represent the position and velocity vector of the agents, respectively, and finally,  $\overline{X}_L$ ,  $\overline{V}_L$ ,  $\Delta$  is defined as:

$$
\overline{X}_L = ([1, ..., 1]^T)_{n \times 1} \otimes [x_L^d, y_L^d, z_L^d]^T, \n\overline{V}_L = ([1, ..., 1]^T)_{n \times 1} \otimes [x_L^d, \dot{y}_L^d, \dot{z}_L^d]^T, \n\Delta = ([1, ..., 1]^T)_{n \times 1} \otimes [\Delta_i].
$$
\n(11)

# B. CONTROL OBJECTIVE

The control objective is:

- Transport the payload safely with minimal oscillation.
- Achieve the desired formation pattern and maintain it throughout the cooperative transport task.
- Guarantee the stability of the formation errors at the origin at a fixed time  $T_f$ , even in the presence of lumped

disturbances, i.e.

$$
\lim_{t \to T_f} e_i^{\xi} = 0,
$$
\n
$$
\lim_{t \to T_f} (\xi_i - \xi_L) = \Delta_i,
$$
\n
$$
\lim_{t \to T_f} (\xi_i - \xi_j) = (\Delta_i - \Delta_j).
$$
\n(12)

# <span id="page-4-0"></span>**IV. PROPOSED CONTROL DESIGN**

In this section, control inputs are designed for the cooperative transport of a payload using two quadcopters as underactuated mechanical systems, i.e., only four control inputs are designed for each quadcopter. The proposed controller structure is divided into two subsystems: fully actuated  $(U_{1i}, U_{4i})$ and underactuated  $(U_{2i}, U_{3i})$ . An integral sliding mode adaptive control strategy is proposed for the fully actuated subsystem, and for the underactuated subsystem, an adaptive control strategy based on the combination of Backstepping and sliding mode is proposed. The control parameters of the sliding surfaces of both subsystems are adaptively tuned with a neural network that is trained by the backpropagation algorithm to minimize the errors of the cooperative transport system. To improve the performance of the proposed controller, a disturbance observer is incorporated to estimate the disturbances, including rope tensions that dramatically affect the quadcopter dynamics.

# A. NONLINEAR DISTURBANCE OBSERVER

Due to the presence of unmeasurable states in the cooperative transport system, a nonlinear disturbance observer is incorporated into the controller to estimate and compensate for lumped disturbances. The equation [\(2\)](#page-2-2) can be written as the following nonlinear state space form:

<span id="page-4-3"></span>
$$
\dot{X}_T = X_{T+1}, \n\dot{X}_{T+1} = f(X) + g(X)U + d(t).
$$
\n(13)

where  $X_T = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$  is the vector of states,  $U = [U_{1i}, U_{2i}, U_{3i}, U_{4i}]^T$  are the control inputs, *f* and *g* are two known, real, nonlinear smooth functions of  $X_{\mathcal{T}}$ , and  $d(t)$  are the unknown lumped disturbances.

*Assumption 2:* The disturbances  $d(t)$  are limited by  $d(t) \leq$ *D*,  $t > 0$ , and *D* is a positive definite constant.

*Assumption 3:* The derivative of  $d(t)$  is bounded and satisfies  $\lim_{t\to\infty} d(t) = 0$ .

Estimation of the disturbances  $\hat{d}(t)$  are calculated using the nonlinear disturbance observer according to the following algorithm developed in [\[47\]:](#page-13-23)

<span id="page-4-6"></span><span id="page-4-4"></span>
$$
\dot{\Pi} = -\lambda \Pi - \lambda [\lambda \dot{\Gamma} + f(X) + g(X)U],
$$
  
\n
$$
\hat{d}(t) = \Pi + \lambda \dot{\Gamma},
$$
\n(14)

 $\Pi \in \mathbb{R}^6$  and  $\lambda = \lambda I_{6\times 6} \in \mathbb{R}^{6\times 6}$  denote the internal state vector and the observer gain matrix, respectively, and  $\Gamma =$ [ $x, y, z, \phi, \theta, \psi$ ]<sup>T</sup> are the states.

The estimation error of the disturbances and its time derivative are defined as follows:

<span id="page-4-5"></span>
$$
\widetilde{d}(t) = d(t) - \widehat{d}(t),
$$

$$
\vec{\hat{d}}(t) = \dot{\hat{d}}(t) - \hat{\vec{d}}(t). \tag{15}
$$

Substitute  $(13)$  and  $(14)$  into  $(15)$ , the following expression is obtained:

<span id="page-5-0"></span>
$$
\begin{aligned}\n\vec{d}(t) &= \dot{d}(t) - (\dot{\Pi} - \lambda \ddot{\Gamma}) \\
&= \dot{d}(t) - (-\lambda \Pi - \lambda [\lambda \dot{\Gamma} + f(X) + g(X)U] \\
&- \lambda (f(X) + g(X)U + d(t))) \\
&= \dot{d}(t) - (-\lambda \Pi - \lambda^2 \dot{\Gamma} + \lambda d(t)) \\
&= \dot{d}(t) - (-\lambda \hat{d}(t) + \lambda d(t)) \\
&= \dot{d}(t) - \lambda \tilde{d}(t).\n\end{aligned}
$$
\n(16)

*Lemma 2:* According to assumptions 2 and 3, the disturbances estimate  $\hat{d}(t)$  of the designed observer in [\(14\)](#page-4-4) can track the lumped disturbances if the observer gains are properly selected  $\lambda > 0$  [\[47\]. T](#page-13-23)he equation [\(16\)](#page-5-0) can be rewritten as:

$$
\vec{\tilde{d}}\,(t) + \lambda \tilde{d}(t) = 0,
$$

therefore, the estimation error of the disturbances  $\tilde{d}(t)$  is globally asymptotically stable.

# B. CONTROL DESIGN FOR THE FULLY ACTUATED **SUBSYSTEM**

The fully actuated control subsystem for the *i*-th quadcopter is formed by the inputs  $[U_{1i}, U_{4i}]^T$  and outputs  $[z_i, \psi_i]^T$ . Therefore, an integral sliding mode adaptive control strategy is proposed.

The fully actuated control subsystem errors are defined as:

<span id="page-5-1"></span>
$$
e_i^z = \sum_{j=1}^n a_{ij}(z_j - z_i - \Delta_{jiz}) + b_i(z_L^d - z_i + \Delta_{iz}),
$$
  
\n
$$
e_i^{\psi} = \psi_i^d - \psi_i.
$$
 (17)

The time derivative of  $(17)$  is given by:

$$
\dot{e}_{i}^{z} = \sum_{j=1}^{n} a_{ij}(\dot{z}_{j} - \dot{z}_{i}) + b_{i}(\dot{z}_{L}^{d} - \dot{z}_{i}),
$$
  
\n
$$
\dot{e}_{i}^{\psi} = \dot{\psi}_{i}^{d} - \dot{\psi}_{i}.
$$
\n(18)

The following sliding surfaces for the fully actuated control subsystem are introduced as:

<span id="page-5-2"></span>
$$
s_{z_i} = \dot{e}_i^z + \Xi_1 e_i^z + \Xi_2 \int_0^t e_i^z dt,
$$
  
\n
$$
s_{\psi_i} = \dot{e}_i^{\psi} + \Xi_3 e_i^{\psi} + \Xi_4 \int_0^t e_i^{\psi} dt.
$$
 (19)

where the parameters  $\Sigma_{\sigma}(\sigma = 1, \ldots, 4)$  are positive definite and will be tuned with a neural network later. The time derivative of  $(19)$  is given by:

<span id="page-5-3"></span>
$$
\dot{s}_{z_i} = \ddot{e}_i^z + \Xi_1 \dot{e}_i^z + \Xi_2 e_i^z, \n\dot{s}_{\psi_i} = \ddot{e}_i^{\psi} + \Xi_3 \dot{e}_i^{\psi} + \Xi_4 e_i^{\psi}.
$$
\n(20)

To design the control inputs, the following condition must be satisfied [\[34\]:](#page-13-10)

<span id="page-5-4"></span>
$$
\dot{s}_{z_i}=-\varepsilon_z \text{sign}(s_{z_i})-\eta_z s_{z_i},
$$

$$
\dot{s}_{\psi_i} = -\varepsilon_{\psi} \text{sign}(s_{\psi_i}) - \eta_{\psi} s_{\psi_i}.
$$
 (21)

According to [\(20\)](#page-5-3) and [\(21\)](#page-5-4), the control inputs  $U_{1i}$  and  $U_{4i}$  for the *i*-th quadcopter are designed as follows:

<span id="page-5-5"></span>
$$
\begin{cases}\nU_{1i} = \frac{\cos\phi_i \cos\theta_i}{m_{qi}} \bigg[ (b_i + \sum_{j=1}^n a_{ij})^{-1} (\Xi_1 \dot{e}_i^z + \Xi_2 e_i^z + b_i \ddot{z}_L^d \\
+ \sum_{j=1}^n a_{ij} \ddot{y}_j + \varepsilon_2 \text{sign}(s_{z_i}) + \eta_z s_{z_i}) + g + \widehat{T}_z - \widehat{\delta}_z(t) \bigg], \\
U_{4i} = \frac{J_{zi}}{\kappa} \bigg[ \Xi_3 \dot{e}_i^{\psi} + \Xi_4 e_i^{\psi} + \ddot{\psi}_i^d + \varepsilon_{\psi} \text{sign}(s_{\psi_i}) + \eta_{\psi} s_{\psi_i} \\
- \frac{(J_{xi} - J_{yi})}{J_{zi}} \dot{\phi}_i \dot{\theta}_i - \widehat{\delta}_{\psi} \bigg].\n\end{cases} (22)
$$

where  $\varepsilon_{\rho} > 0$  and  $\eta_{\rho} > 0$  ( $\rho = z, \psi$ ) are the switching parameters.

*Remark 2:* To solve the chattering problem, in the control laws designed in [\(22\)](#page-5-5), the *sign*(.) function is substituted by the *tanh*(.) function.

<span id="page-5-8"></span>*Theorem 1:* The control inputs designed in  $(22)$  with the nonlinear disturbance observer in  $(14)$  guarantee the asymptotic stability of the cooperative transport system such that the output states  $[z_i, \psi_i]^T$  converge to their desired values in a finite time. Furthermore the errors in [\(17\)](#page-5-1) converge to zero in a finite time.

*Proof 1:* To demonstrate the stability, the following candidate Lyapunov functions are defined as:

<span id="page-5-6"></span>
$$
\begin{bmatrix} V_i^z \\ V_i^{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} s_{z_i}^2 + \frac{1}{2} \tilde{d}_z^2 \\ \frac{1}{2} s_{\psi_i}^2 + \frac{1}{2} \tilde{d}_{\psi}^2 \end{bmatrix} .
$$
 (23)

The time derivative of  $(23)$  and substituting  $(16)$ , [\(19\)](#page-5-2), [\(20\)](#page-5-3) and [\(22\)](#page-5-5), the following expressions are obtained:

<span id="page-5-7"></span>
$$
\begin{bmatrix}\n\dot{V}_i^z \\
\dot{V}_i^{\psi}\n\end{bmatrix} = \begin{bmatrix}\n-\varepsilon_z |s_{z_i}| - \eta_z s_{z_i}^2 - \lambda \tilde{d}_z^2 \\
-\varepsilon_{\psi} |s_{\psi_i}| - \eta_{\psi} s_{\psi_i}^2 - \lambda \tilde{d}_{\psi}^2\n\end{bmatrix} \le 0.
$$
\n(24)

*Remark 3:* From [\(24\)](#page-5-7), it can be concluded that the fully actuated control subsystem guarantees the asymptotic stability of the cooperative transport system. Therefore, Theorem [1](#page-5-8) has been verified.

*Remark 4:* The sliding surfaces in [\(22\)](#page-5-5) have four design parameters  $\Xi_1$ ,  $\Xi_2$ ,  $\Xi_3$ ,  $\Xi_4$  and two switching parameters  $\varepsilon_z$ ,  $\varepsilon_{\psi}$ . These parameters are adaptively tuned with a neural network that is trained by the backpropagation algorithm to minimize the errors of the fully actuated control subsystem.

Next, the control parameters of the sliding surface  $s_{z_i}$  are designed in the following steps:

<span id="page-5-9"></span>1) First, the error function is defined as follows:

$$
E_z = \frac{1}{2} \left[ e_i^z \right]^2. \tag{25}
$$

2) By employing the steepest descent method [\[48\], t](#page-13-24)he following adaptive equations are obtained:

<span id="page-6-1"></span>
$$
\varepsilon_z = \varepsilon_{z,0} - \omega_z \frac{\partial E_z}{\partial \varepsilon_z},
$$
  
\n
$$
\Xi_1 = \Xi_{1,0} - \omega_z \frac{\partial E_z}{\partial \Xi_1},
$$
  
\n
$$
\Xi_2 = \Xi_{2,0} - \omega_z \frac{\partial E_z}{\partial \Xi_2}.
$$
 (26)

where  $\omega_z$  is the learning rate,  $\varepsilon_{z,0}$  and  $\Xi_{\sigma,0}(\sigma)$  = 1, 2) are the initial values of  $\varepsilon_z$  and  $\Xi_{\sigma}(\sigma = 1, 2)$ , respectively.

3) Using the chain rule for partial derivatives, the following equations are obtained:

<span id="page-6-0"></span>
$$
\frac{\partial E_z}{\partial \varepsilon_z} = \frac{\partial E_z}{\partial z} \frac{\partial z}{\partial U_1} \frac{\partial U_1}{\partial \varepsilon_z} = \frac{-m_q e_i^z}{\cos(\phi)\cos(\theta)} \tanh(s_z),
$$
  
\n
$$
\frac{\partial E_z}{\partial \Xi_1} = \frac{\partial E_z}{\partial z} \frac{\partial z}{\partial U_1} \frac{\partial U_1}{\partial s_z} \frac{\partial s_z}{\partial \Xi_1}
$$
  
\n
$$
= \frac{-m_q (e_i^z)^2}{\cos(\phi)\cos(\theta)} \left( \eta_z + \frac{4e^{-2s_z} \varepsilon_z}{(1 + e^{-2s_z})^2} \right),
$$
  
\n
$$
\frac{\partial E_z}{\partial \Xi_2} = \frac{\partial E_z}{\partial z} \frac{\partial z}{\partial U_1} \frac{\partial U_1}{\partial s_z} \frac{\partial s_z}{\partial \Xi_2}
$$
  
\n
$$
= \frac{-m_q e_i^z \int_0^t e_i^z dt}{\cos(\phi)\cos(\theta)} \left( \eta_z + \frac{4e^{-2s_z} \varepsilon_z}{(1 + e^{-2s_z})^2} \right). \quad (27)
$$

4) Substituting  $(27)$  into  $(26)$ , the control parameters of the sliding surface  $s_{z_i}$  are given as:

<span id="page-6-2"></span>
$$
\varepsilon_{z} = \varepsilon_{z,0} + \omega_{z} \bigg[ \frac{m_{q} e_{i}^{z}}{\cos(\phi)\cos(\theta)} \tanh(s_{z}) \bigg],
$$
  
\n
$$
\Xi_{1} = \Xi_{1,0} + \omega_{z} \bigg[ \frac{m_{q} (e_{i}^{z})^{2}}{\cos(\phi)\cos(\theta)} \bigg( \eta_{z} + \frac{4e^{-2s_{z}} \varepsilon_{z}}{(1+e^{-2s_{z}})^{2}} \bigg) \bigg],
$$
  
\n
$$
\Xi_{2} = \Xi_{2,0} + \omega_{z} \bigg[ \frac{m_{q} e_{i}^{z} \int_{0}^{t} e_{i}^{z} dt}{\cos(\phi)\cos(\theta)} \bigg( \eta_{z} + \frac{4e^{-2s_{z}} \varepsilon_{z}}{(1+e^{-2s_{z}})^{2}} \bigg) \bigg].
$$
\n(28)

By following the same procedure as above (step [1](#page-5-9) to step [4\)](#page-6-2) the control parameters of the sliding surface  $s_{\psi}$  are obtained:

$$
\varepsilon_{\psi} = \varepsilon_{\psi,0} + \omega_{\psi} \left[ \frac{J_{z} e_{i}^{\psi}}{\kappa} \tanh(s_{\psi}) \right],
$$
  
\n
$$
\Xi_{3} = \Xi_{3,0} + \omega_{\psi} \left[ \frac{J_{z} (e_{i}^{\psi})^{2}}{\kappa} \left( \eta_{\psi} + \frac{4e^{-2s_{\psi}} \varepsilon_{\psi}}{(1 + e^{-2s_{\psi}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{4} = \Xi_{4,0} + \omega_{\psi} \left[ \frac{J_{z} e_{i}^{\psi} \int_{0}^{t} e_{i}^{\psi} dt}{\kappa} \left( \eta_{\psi} + \frac{4e^{-2s_{\psi}} \varepsilon_{\psi}}{(1 + e^{-2s_{\psi}})^{2}} \right) \right].
$$
\n(29)

# C. CONTROL DESIGN FOR THE UNDERACTUATED **SUBSYSTEM**

The underactuated control subsystem for the *i*-th quadcopter is formed by the inputs  $[U_{2i}, U_{3i}]^T$  and outputs

<span id="page-6-8"></span> $[x_i, y_i, \phi_i, \theta_i]^T$ . Clearly, it is impossible to design a control strategy for each output. Therefore, an adaptive control strategy based on the combination of Backstepping and sliding mode control techniques is proposed due to its good robustness characteristics to control underactuated mechanical systems.

The underactuated control subsystem errors are defined as:

<span id="page-6-3"></span>
$$
e_i^x = \sum_{j=1}^n a_{ij}(x_j - x_i - \Delta_{jix}) + b_i(x_L^d - x_i + \Delta_{ix}),
$$
  
\n
$$
e_i^y = \sum_{j=1}^n a_{ij}(y_j - y_i - \Delta_{jiy}) + b_i(y_L^d - y_i + \Delta_{iy}),
$$
  
\n
$$
e_i^\phi = \phi_i^d - \phi_i,
$$
  
\n
$$
e_i^\theta = \theta_i^d - \theta_i.
$$
\n(30)

The first time derivative of  $(30)$  is given as:

$$
\dot{e}_i^x = \sum_{j=1}^n a_{ij}(\dot{x}_j - \dot{x}_i) + b_i(\dot{x}_L^d - \dot{x}_i),
$$
  
\n
$$
\dot{e}_i^y = \sum_{j=1}^n a_{ij}(\dot{y}_j - \dot{y}_i) + b_i(\dot{y}_L^d - \dot{y}_i),
$$
  
\n
$$
\dot{e}_i^\phi = \dot{\phi}_i^d - \dot{\phi}_i,
$$
  
\n
$$
\dot{e}_i^\theta = \dot{\theta}_i^d - \dot{\theta}_i.
$$
\n(31)

Then, the first Lyapunov candidate function for  $e_i^{\phi}$  $\frac{\phi}{i}$  and  $e_i^{\theta}$  are chosen as:

<span id="page-6-4"></span>
$$
V_{1i}^{\phi} = \frac{1}{2} e_i^{\phi^2},
$$
  
\n
$$
V_{1i}^{\theta} = \frac{1}{2} e_i^{\theta^2}.
$$
 (32)

Deriving the first Lyapunov candidate functions [\(32\)](#page-6-4) and substituting the time derivative of the errors  $e_1^{\phi}$  $\frac{\phi}{1i}$  and  $e_{1i}^{\theta}$  gives the following expression:

<span id="page-6-7"></span>
$$
\dot{V}_{1i}^{\phi} = e_i^{\phi} \dot{e}_i^{\phi} = e_i^{\phi} (\dot{\phi}_i^d - \dot{\phi}_i),
$$
  
\n
$$
\dot{V}_{1i}^{\theta} = e_i^{\theta} \dot{e}_i^{\theta} = e_i^{\theta} (\dot{\theta}_i^d - \dot{\theta}_i).
$$
\n(33)

In order to stabilize  $e_1^{\phi}$  $\frac{\phi}{1i}$  and  $e_{1i}^{\theta}$ , the following virtual control inputs are introduced as follows: [\[49\]:](#page-13-25)

<span id="page-6-9"></span><span id="page-6-6"></span>
$$
\dot{\phi}_i^* = \dot{\phi}_i^d + k_{e_\phi} e_i^\phi + s_{\phi_i},
$$
  
\n
$$
\dot{\theta}_i^* = \dot{\theta}_i^d + k_{e_\theta} e_i^\theta + s_{\theta_i}.
$$
\n(34)

where  $k_{e_{\phi}}$  y  $k_{e_{\theta}}$  are positive definite constants. Inspired by [\[36\], \[](#page-13-12)[50\], t](#page-13-26)he following sliding surfaces are proposed for the underactuated control subsystem as follows:

<span id="page-6-10"></span><span id="page-6-5"></span>
$$
s_{\phi_i} = c_1 e_i^y + c_2 e_i^y + c_3 |e_i^y|^{\alpha_y} \text{sgn}(e_i^y) + e_i^{\phi} + c_4 e_i^{\phi} + c_5 |e_i^{\phi}|^{\alpha_{\phi}} \text{sgn}(e_i^{\phi}), s_{\theta_i} = c_6 e_i^x + c_7 e_i^x + c_8 |e_i^x|^{\alpha_x} \text{sgn}(e_i^x) + e_i^{\theta} + c_9 e_i^{\theta} + c_{10} |e_i^{\theta}|^{\alpha_{\theta}} \text{sgn}(e_i^{\theta}).
$$
 (35)

The coefficients of the sliding surfaces in  $(35)$  are obtained using the Hurwitz stability analysis  $[34]$ , where  $c_1$  =  $- \Xi_5 m_{qi}/(U_{1i} cos \psi_i), c_2 = -\Xi_6 m_{qi}/(U_{1i} cos \psi_i), c_3 = \Xi_7,$  $c_4 = \Xi_8$ ,  $c_5 = \Xi_9$ ,  $c_6 = \Xi_{10} m_{ai}/(U_{1i} cos \phi_i cos \psi_i)$ ,  $c_7 =$  $\Xi_{11}m_{qi}/(U_{1i}cos\phi cos\psi), c_8 = \Xi_{12}, c_9 = \Xi_{13}, c_{10} = \Xi_{14}.$ 

The parameters  $\Xi_{\sigma}(\sigma = 5, \ldots, 14)$  are positive definite and will be tuned with a neural network later,  $\alpha_{\rho}(\rho)$  $x, y, \phi, \theta$  are positive definite constants. The time derivative of the [\(35\)](#page-6-5) is as follows:

<span id="page-7-1"></span>
$$
\dot{s}_{\phi_i} = c_1 \ddot{e}_i^y + c_2 \dot{e}_i^y + c_3 \alpha_y \left| e_i^y \right|^{\alpha_y - 1} \dot{e}_i^y + \ddot{e}_i^{\phi} \n+ c_4 \dot{e}_i^{\phi} + c_5 \alpha_{\phi} \left| e_i^{\phi} \right|^{\alpha_{\phi} - 1} \dot{e}_i^{\phi}, \n\dot{s}_{\theta_i} = c_6 \ddot{e}_i^x + c_7 \dot{e}_i^x + c_8 \alpha_x \left| e_i^x \right|^{\alpha_x - 1} \dot{e}_i^x + \ddot{e}_i^{\theta} \n+ c_9 \dot{e}_i^{\theta} + c_{10} \alpha_{\theta} \left| e_i^{\theta} \right|^{\alpha_{\theta} - 1} \dot{e}_i^{\theta}.
$$
\n(36)

Next, the second Lyapunov candidate function are chosen as:

<span id="page-7-0"></span>
$$
V_{2i}^{\phi} = \frac{1}{2}e_i^{\phi^2} + \frac{1}{2}s_{\phi_i}^2,
$$
  
\n
$$
V_{2i}^{\theta} = \frac{1}{2}e_i^{\theta^2} + \frac{1}{2}s_{\theta_i}^2.
$$
\n(37)

The time derivative of the second Lyapunov candidate function  $(37)$ , and substituting  $(34)$  and  $(36)$  gives:

<span id="page-7-2"></span>
$$
\dot{V}_{2i}^{\phi} = -k_{e_{\phi}}e_{i}^{\phi^{2}} - s_{\phi_{i}}e_{i}^{\phi} + s_{\phi_{i}}(c_{1}\ddot{e}_{i}^{\nu} + c_{2}\dot{e}_{i}^{\nu} + \ddot{e}_{i}^{\phi} \n+ c_{3}\alpha_{y} |e_{i}^{y}|^{\alpha_{y}-1} \dot{e}_{i}^{y} + c_{4}\dot{e}_{i}^{\phi} + c_{5}\alpha_{\phi} |e_{i}^{\phi}|^{\alpha_{\phi}-1} \dot{e}_{i}^{\phi}),
$$
\n
$$
\dot{V}_{2i}^{\theta} = -k_{e_{\theta}}e_{i}^{\theta^{2}} - s_{\theta_{i}}e_{i}^{\theta} + s_{\theta_{i}}(c_{6}\ddot{e}_{i}^{x} + c_{7}\dot{e}_{i}^{x} + \ddot{e}_{i}^{\theta} \n+ c_{8}\alpha_{x} |e_{i}^{x}|^{\alpha_{x}-1} \dot{e}_{i}^{x} + c_{9}\dot{e}_{i}^{\theta} + c_{10}\alpha_{\theta} |e_{i}^{\theta}|^{\alpha_{\theta}-1} \dot{e}_{i}^{\theta}).
$$
\n(38)

According to [\(38\)](#page-7-2), the control inputs  $U_{2i}$  and  $U_{3i}$  for the *i*-th quadcopter are designed as follows:

<span id="page-7-3"></span>
$$
\begin{cases}\nU_{2i} = \frac{J_{xi}}{\varsigma} \left[ -e_i^{\phi} + k_{s_{\phi}} s_{\phi_i} + c_1 \left( b_i \ddot{y}_L^d - (v_{yi} - \widehat{T}_y) \right.\n\right. \\
\left. + \widehat{\delta}_y(t) \right) \left( \sum_{j=1}^n a_{ij} + b_i \right) + \sum_{j=1}^n a_{ij} \ddot{y}_j \right) + c_2 \dot{e}_i^y \\
+ c_3 \alpha_y \left| e_i^y \right|^{a_y - 1} \dot{e}_i^y + \ddot{\phi}_{1i}^d + c_4 \dot{e}_i^{\phi} + c_5 \alpha_{\phi} \left| e_i^{\phi} \right|^{a_{\phi} - 1} \\
\times \dot{e}_i^{\phi} + \varepsilon_{\phi} sign(s_{\phi_i}) + \eta_{\phi} s_{\phi_i} - \frac{J_{yi} - J_{zi}}{J_{xi}} \dot{\theta}_i \dot{\psi}_i \\
+ \frac{J_{ri}}{J_{xi}} \dot{\theta}_i \Omega_i - \widehat{\delta}_{\phi} \right], \\
U_{3i} = \frac{J_{yi}}{\varsigma} \left[ -e_i^{\theta} + k_{s_{\theta}} s_{\theta_i} + c_6 \left( b_i \ddot{x}_L^d - (v_{xi} - \widehat{T}_x) \right.\n\right. \\
\left. + \widehat{\delta}_x(t) \right) \left( \sum_{j=1}^n a_{ij} + b_i \right) + \sum_{j=1}^n a_{ij} \ddot{x}_j \right) + c_7 \dot{e}_i^x \\
+ c_8 \alpha_x \left| e_i^x \right|^{a_x - 1} \dot{e}_i^x + \ddot{\theta}_{1i}^d + c_9 \dot{e}_i^{\theta} + c_{10} \alpha_{\theta} \left| e_i^{\theta} \right|^{a_{\theta} - 1} \\
\times \dot{e}_i^{\theta} + \varepsilon_{\theta} sign(s_{\theta_i}) + \eta_{\theta} s_{\theta_i} - \frac{J_{zi} - J_{xi}}{J_{yi}} \dot{\phi}_i \dot{\psi}_i \\
+ \frac{J_{ri}}{J_{yi}} \dot{\phi}_i \Omega_i - \widehat{\delta}_{\theta} \right].\n\end{cases} \tag{39}
$$

where,  $v_{xi} = (c\phi_i s\theta_i c\psi_i + s\phi_i s\psi_i) U_{1i}$  and  $v_{yi}$  $(c\phi_i s\theta_i s\psi_i - s\phi_i c\psi_i) U_{1i}, k_{s\phi} > 0, k_{s\theta} > 0$  are positive definite constants, and the coefficients  $\varepsilon_{\rho} > 0$  and  $\eta_{\rho} > 0$  $(\rho = \phi, \theta)$  are the switching parameters.

*Remark 5:* To solve the chattering problem, in the control laws designed in [\(39\)](#page-7-3), the *sign*(.) function is substituted by the *tanh*(.) function.

<span id="page-7-6"></span>*Theorem 2:* The control inputs designed in [\(39\)](#page-7-3) with the nonlinear disturbance observer in  $(14)$  guarantee the asymptotic stability of the cooperative transport system such that the output states  $[x_i, y_i, \phi_i, \theta_i]^T$  converge to their desired values in a finite time. Furthermore the errors in  $(30)$  converge to zero in a finite time.

*Proof 2:* To demonstrate the stability, the following candidate Lyapunov functions are defined as:

<span id="page-7-4"></span>
$$
\begin{bmatrix} V_i^{\phi} \\ V_i^{\phi} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} e_i^{\phi^2} + \frac{1}{2} s_{\phi_i}^2 + \frac{1}{2} \tilde{d}_{\phi}^2 + \frac{1}{2} \tilde{d}_{\gamma}^2 \\ \frac{1}{2} e_i^{\phi^2} + \frac{1}{2} s_{\theta_i}^2 + \frac{1}{2} \tilde{d}_{\phi}^2 + \frac{1}{2} \tilde{d}_{\chi}^2 \end{bmatrix} . \tag{40}
$$

The time derivative of  $(40)$  and substituting  $(16)$ ,  $(33)$ ,  $(38)$ and [\(39\)](#page-7-3), the following expressions are obtained:

<span id="page-7-5"></span>
$$
\begin{bmatrix}\n\dot{V}_{i}^{\phi} \\
\dot{V}_{i}^{\theta}\n\end{bmatrix} = \begin{bmatrix}\n-k_{e_{\phi}}e_{i}^{\phi^{2}} - k_{s_{\phi}}s_{\phi_{i}}^{2} - \varepsilon_{\phi}|s_{\phi}| - \lambda_{\phi}\tilde{d}_{\phi}^{2} - \lambda_{y}\tilde{d}_{y}^{2} \\
-k_{e_{\theta}}e_{i}^{\theta^{2}} - k_{s_{\theta}}s_{\theta_{i}}^{2} - \varepsilon_{\theta}|s_{\theta}| - \lambda_{\theta}\tilde{d}_{\theta}^{2} - \lambda_{x}\tilde{d}_{x}^{2}\n\end{bmatrix}
$$
\n
$$
\leq 0.
$$
\n(41)

*Remark 6:* From [\(41\)](#page-7-5), it can be concluded that the underactuated control subsystem guarantees the asymptotic stability of the cooperative transport system. Therefore, Theorem [2](#page-7-6) has been verified.

*Remark 7:* The sliding surfaces in [\(39\)](#page-7-3) have ten design parameters  $\Xi_5$ ,  $\Xi_6$ ,  $\Xi_7$ ,  $\Xi_8$ ,  $\Xi_9$ ,  $\Xi_{10}$ ,  $\Xi_{11}$ ,  $\Xi_{12}$ ,  $\Xi_{13}$ ,  $\Xi_{14}$ and two switching parameters  $\varepsilon_{\phi}$ ,  $\varepsilon_{\theta}$ . These parameters are adaptively tuned with a neural network that is trained by the backpropagation algorithm to minimize the errors of the underactuated control subsystem.

Next, the control parameters of the sliding surface  $s_{\phi_i}$  are designed in the following steps:

<span id="page-7-8"></span>1) First, the error function is defined as follows:

$$
E_{y} = \frac{1}{2} [e_i^y]^2.
$$
 (42)

2) By employing the steepest descent method [\[48\], t](#page-13-24)he following adaptive equations are obtained:

<span id="page-7-7"></span>
$$
\begin{cases}\n\varepsilon_{\phi} = \varepsilon_{\phi,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \varepsilon_{\phi}},\\ \n\Xi_{5} = \Xi_{5,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \Xi_{1}},\\ \n\Xi_{6} = \Xi_{6,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \Xi_{2}},\\ \n\Xi_{7} = \Xi_{7,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \Xi_{3}},\\ \n\Xi_{8} = \Xi_{8,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \Xi_{4}},\\ \n\Xi_{9} = \Xi_{9,0} - \omega_{\phi} \frac{\partial E_{y}}{\partial \Xi_{5}}.\n\end{cases}
$$
\n(43)

where  $\omega_{\phi}$  is the learning rate,  $\varepsilon_{\phi,0}$  and  $\Xi_{\sigma,0}(\sigma$  = 5, ..., 9) are the initial values of  $\varepsilon_{\phi}$  and  $\Xi_{\sigma}(\sigma =$  $5, \ldots, 9$ , respectively.

3) Using the chain rule for partial derivatives, the following equations are obtained:

<span id="page-8-1"></span>
$$
\frac{\partial E_y}{\partial \varepsilon_{\phi}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial \varepsilon_{\phi}} = \frac{-J_x e_i^y}{\varsigma} \tanh(s_{\phi}),
$$
\n
$$
\frac{\partial E_y}{\partial \varepsilon_{5}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial s_{\phi}} \frac{\partial s_{\phi}}{\partial \varepsilon_{5}}
$$
\n
$$
= \frac{J_x e_i^y e_i^y m_q}{\varsigma(\cos \psi U_1)} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^2} \right),
$$
\n
$$
\frac{\partial E_y}{\partial \varepsilon_{6}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial s_{\phi}} \frac{\partial s_{\phi}}{\partial \varepsilon_{6}}
$$
\n
$$
= \frac{J_x (e_i^y)^2 m_q}{\varsigma(\cos \psi U_1)} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^2} \right),
$$
\n
$$
\frac{\partial E_y}{\partial \varepsilon_{7}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial s_{\phi}} \frac{\partial s_{\phi}}{\partial \varepsilon_{7}}
$$
\n
$$
= \frac{-J_x e_i^y |e_i^y|^{\alpha_y} \operatorname{sgn}(e_i^y)}{\varsigma}
$$
\n
$$
\frac{\partial E_y}{\partial \varepsilon_{8}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial s_{\phi}} \frac{\partial s_{\phi}}{\partial \varepsilon_{8}}
$$
\n
$$
= \frac{-J_x e_i^y e_i^{\phi}}{\varsigma} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^2} \right),
$$
\n
$$
\frac{\partial E_y}{\partial \varepsilon_{9}} = \frac{\partial E_y}{\partial y} \frac{\partial y}{\partial U_2} \frac{\partial U_2}{\partial s_{\phi
$$

<span id="page-8-2"></span>4) Substituting equation [\(44\)](#page-8-1) into [\(43\)](#page-7-7), the control parameters of the sliding surface  $s_{\phi}$  are given as:

$$
\varepsilon_{\phi} = \varepsilon_{\phi,0} + \omega_{\phi} \left[ \frac{J_{x} e_{i}^{y}}{\varsigma} \tanh(s_{\phi}) \right],
$$
  
\n
$$
\Xi_{5} = \Xi_{5,0} - \omega_{\phi} \left[ \frac{J_{x} e_{i}^{y} e_{i}^{y} m_{q}}{\varsigma(\cos \psi U_{1})} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{6} = \Xi_{6,0} - \omega_{\phi} \left[ \frac{J_{x} (e_{i}^{y})^{2} m_{q}}{\varsigma(\cos \psi U_{1})} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{7} = \Xi_{7,0} + \omega_{\phi} \left[ \frac{J_{x} e_{i}^{y} |e_{i}^{y}|^{\alpha_{y}} \operatorname{sgn}(e_{i}^{y})}{\varsigma} \right] \times \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{8} = \Xi_{8,0} + \omega_{\phi} \left[ \frac{J_{x} e_{i}^{y} e_{i}^{\phi}}{\varsigma} \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{9} = \Xi_{9,0} + \omega_{\phi} \left[ \frac{J_{x} e_{i}^{y} |e_{i}^{\phi}|^{\alpha_{\phi}} \operatorname{sgn}(e_{i}^{\phi})}{\varsigma} \right] \times \left( \eta_{\phi} + \frac{4e^{-2s_{\phi}} \varepsilon_{\phi}}{(1 + e^{-2s_{\phi}})^{2}} \right) \right].
$$
(45)

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By following the same procedure as above (step [1](#page-7-8) to step [4\)](#page-8-2) the control parameters of the sliding surface  $s_{\theta}$  are obtained:

$$
\varepsilon_{\theta} = \varepsilon_{\theta,0} + \omega_{\theta} \left[ \frac{J_{y} e_{i}^{x}}{\varsigma} \tanh(s_{\theta}) \right],
$$
  
\n
$$
\Xi_{10} = \Xi_{10,0} + \omega_{\theta} \left[ \frac{J_{y} e_{i}^{x} \dot{e}_{i}^{x} m_{q}}{\varsigma(\cos\phi\cos\psi U_{1})} \left( \eta_{\theta} + \frac{4e^{-2s_{\theta}} \varepsilon_{\theta}}{(1 + e^{-2s_{\theta}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{11} = \Xi_{11,0} + \omega_{\theta} \left[ \frac{J_{y} (e_{i}^{x})^{2} m_{q}}{\varsigma(\cos\phi\cos\psi U_{1})} \left( \eta_{\theta} + \frac{4e^{-2s_{\theta}} \varepsilon_{\theta}}{(1 + e^{-2s_{\theta}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{12} = \Xi_{12,0} + \omega_{\theta} \left[ \frac{J_{y} e_{i}^{x} |e_{i}^{x}|^{\alpha_{x}} \operatorname{sgn}(e_{i}^{x})}{\varsigma} \right] \times \left( \eta_{\theta} + \frac{4e^{-2s_{\theta}} \varepsilon_{\theta}}{(1 + e^{-2s_{\theta}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{13} = \Xi_{13,0} + \omega_{\theta} \left[ \frac{J_{y} e_{i}^{x} e_{i}^{\theta}}{\varsigma} \left( \eta_{\theta} + \frac{4e^{-2s_{\theta}} \varepsilon_{\theta}}{(1 + e^{-2s_{\theta}})^{2}} \right) \right],
$$
  
\n
$$
\Xi_{14} = \Xi_{14,0} + \omega_{\theta} \left[ \frac{J_{y} e_{i}^{x} |e_{i}^{\theta}|^{\alpha_{\theta}} \operatorname{sgn}(e_{i}^{\theta})}{\varsigma} \right] \times \left( \eta_{\theta} + \frac{4e^{-2s_{\theta}} \varepsilon_{\theta}}{(1 + e^{-2s_{\theta}})^{2}} \right).
$$
(46)

#### <span id="page-8-0"></span>**V. RESULTS AND DISCUSSIONS**

In this section, the results of numerical simulations are presented to validate the efficiency and performance of the proposed controller for the cooperative transport of a payload using two underactuated quadcopters. The physical parameters of the cooperative transport system and the parameters for the control design are presented in Appendix [A,](#page-11-1) Tables [1](#page-11-2)[-2.](#page-12-23) The initial position of the quadcopters is set as  $\xi_1^0$  =  $(1, 0.5, 0)$ m,  $\xi_2^0 = (-1, -0.5, 0)$ m, and the initial position of the payload is set as  $\xi_p^0 = (0, 0, 0)$ m.

The desired formation pattern is set as a straight line shape so that the quadcopters are in a symmetric position with respect to the payload position, which is considered the virtual leader of the cooperative transport system. The desired formation distance for each quadcopter is chosen as  $\Delta_1 = [1, 0, \sqrt{3}]^T$  y  $\Delta_2 = [-1, 0, \sqrt{3}]^T$ . Two cases with different scenarios for the reference trajectory of the virtual leader were analyzed and are presented below:

# A. CASE 1: WITHOUT DISTURBANCES

In this case, disturbances such as wind gusts and parametric uncertainties are not considered. The desired trajectory for the virtual leader is chosen as  $\xi_L^d$  =  $[2.5 \cos(0.2t), 2.5 \sin(0.2t), 0.267 + 0.1t]^T$  m. Assuming that the desired orientation for each quadcopter is equal to  $\eta^d$  =  $[\phi^d, \theta^d, \psi^d]^T = [0, 0, 0]^T$ rad.

The simulation results are presented in Figs. [3](#page-9-0)[-8.](#page-10-0) In Fig. [3,](#page-9-0) a three-dimensional view of the cooperative transport of a payload following a helicoidal trajectory is illustrated, where it can be seen that the two quadcopters transport the payload safely and without oscillations.

The Fig. [4](#page-9-1) display the temporal response of the position of the cooperative transport system, as can be observed that the quadcopters and the payload achieve the desired formation

<span id="page-9-0"></span>

**FIGURE 3.** Cooperative transport of a payload with two UAVs for Case 1.

<span id="page-9-1"></span>

**FIGURE 4.** Temporal response of position of the cooperative transport system.

pattern in a short time and follow the desired trajectory of the virtual leader; moreover, it is observed that the desired formation pattern is successfully maintained while transporting the payload. The time response of the orientation of the two quadcopters is presented in Fig. [5,](#page-9-2) where it is shown that the Euler angles converge to zero in a short time; it is worth mentioning that the Euler angles are maintained at zero because it is the value that was assigned to the desired orientation of the quadcopters.

The formation errors of both quadcopters are displayed in Fig. [6,](#page-9-3) from which it can be seen that errors  $e_i^x$  and  $e_i^y$ *i* converge to the origin at time  $t = 5$  s, while error  $e_i^z$  converges to the origin in a shorter time  $t = 3$  s. This is explained by the underactuated property of the quadcopters since errors  $e_i^x$  and  $e_i^y$  $\mathbf{v}_i^y$  belong to the underactuated control subsystem, which has only two control actions for four outputs. In comparison, error  $e_i^z$  belongs to the fully actuated control subsystem, which has

<span id="page-9-2"></span>

**FIGURE 5.** Temporal response of quadcopters orientation.

<span id="page-9-3"></span>



two control actions for two outputs. Despite this, the control strategy proposed in this paper achieves that the formation errors converge to the origin in a short time and stabilize at the origin, even with the rope tension forces generated by the payload weight.

The estimation of the tension forces of the two ropes acting on each quadcopter is presented in Fig. [7,](#page-10-1) where the high initial values are due to the initial movements of the two quadcopters, before they achieve the desired formation pattern. The four control inputs  $(U_1, U_2, U_3, U_4)$  designed in  $(22)$  and  $(39)$ , are presented in Fig. [8.](#page-10-0) It can be seen that the total thrust  $U_1$  is finally around 22.091 N, which is equal to the gravity force of the quadcopter attached to the payload. The rest control inputs  $(U_2, U_3, U_4)$  are maintained at  $(0, 0, 0)$  N.m respectively.

# B. CASE 2: WITH DISTURBANCES

In this case, lumped disturbances are considered as; **(i)** Rope tension forces  $\mathbb{T} = [\mathbb{T}_x, \mathbb{T}_y, \mathbb{T}_z]^T$ . (ii) External wind gusts that are injected into the translational and rotational dynamics of the underactuated quadcopters, and are

<span id="page-10-1"></span>

**FIGURE 7.** Estimation of rope tensions.

<span id="page-10-0"></span>

**FIGURE 8.** Control inputs for each quadcopter.

defined respectively as  $\delta_p = [0.5 + 0.15\sin(0.5\pi t), 0.5 +$  $0.15\sin(0.5\pi t), 0.2\sin(0.5\pi t)\right]$ <sup>T</sup>N,  $\delta_{\Theta} = [0.1\cos(0.01 \ t),$  $0.1\cos(0.05\pi t)$ ,  $0.1\cos(0.2\pi t)$ <sup>T</sup> N.m. (iii) Parametric uncertainties of  $+30\%$  in the mass  $m_{qi}$  and moments of inertia  $J_i =$  $diag(J_x, J_y, J_z)$ . The reference trajectory for the virtual leader is chosen as  $\xi_L^d = [2.5 \cos(0.2t), 2.5 \sin(0.4t), 3.267]^T m$ . Assuming that the desired orientation for each quadcopter is the same as in Case 1.

The simulation results are presented in Figs. [9-](#page-10-2)[14.](#page-11-3) In Fig. [9](#page-10-2) illustrates a three-dimensional view of the cooperative transport of a payload following an  $\infty$ -shaped trajectory in the presence of lumped disturbances; clearly, it can be seen that safe transport of the payload without oscillations is achieved.

The Fig. [10](#page-10-3) displays the temporal response of the position of the cooperative transport system; as in the previous case, it can be seen that the quadcopters and the payload achieve the desired formation pattern and maintain stability following the desired trajectory of the virtual leader. Fig. [11](#page-11-4) displays the temporal response of the orientation of the two quadcopters; in the same way, the Euler angles converge to their desired values in a short time. However, orientation  $\phi_1$ ,  $\phi_2$  does not converge completely to zero but remains in a reasonable range.

<span id="page-10-2"></span>

**FIGURE 9.** Cooperative transport of a payload with two UAVs for Case 2.

<span id="page-10-3"></span>

**FIGURE 10.** Temporal response of position of the cooperative transport system with disturbances.

From the above Figs. [10-](#page-10-3)[11,](#page-11-4) it can be observed that the lumped disturbances injected into the quadcopter dynamics do not affect the stability of the cooperative transport system of a payload. To support this statement, the formation errors of both quadcopters are presented in Fig. [12,](#page-11-5) where it can be observed that the formation errors are still maintained close to the origin, with small overshoots in errors  $e_i^x$  and  $e_i^y$  $\hat{i}$ ; the fact of the presence of lumped disturbances in the three axes explains this. Moreover, the convergence time of errors  $e_i^x$  and  $e_j^y$  $e_i^y$  is longer than error  $e_i^z$ ; as mentioned in the previous case, this is due to the underactuated property of the quadcopters. Despite this, the stability and the desired formation pattern of the cooperative transport system are still guaranteed, demonstrating the robustness of the control strategy proposed in this paper against lumped disturbances.

The estimation of the disturbances lumped along the three axes are presented in Fig. [13.](#page-11-6) The four control inputs

<span id="page-11-4"></span>

**FIGURE 11.** Temporal response of quadcopters orientation with disturbances.

<span id="page-11-5"></span>

**FIGURE 12.** Temporal response to formation errors.

 $(U_1, U_2, U_3, U_4)$  designed in  $(22)$  and  $(39)$  are presented in Fig. [14.](#page-11-3) In this case the total thrust is  $U_1 = 25.054$ N, as can be seen the magnitude is higher than in Case 1  $(U_1 = 22.091 \text{ N})$ , this is explained by the fact that in this case a parametric uncertainty of  $+30\%$  on the mass  $m_{qi}$  and moments of inertia  $J_i = diag(J_x, J_y, J_z)$  was considered, which leads the controller to make a greater effort to achieve the stability of the cooperative transport system. The rest control inputs  $(U_2, U_3, U_4)$  are maintained at  $(0, 0, 0)$  N.m respectively.

#### <span id="page-11-0"></span>**VI. CONCLUSION**

This paper presented a novel control design for the cooperative transport of a payload suspended by ropes using two quadcopters which are considered underactuated mechanical systems subject to lumped disturbances. The control structure was divided into two subsystems: fully actuated and underactuated. An integral sliding mode adaptive control strategy was proposed for the fully actuated subsystem, and an adaptive control strategy based on the combination of Backstepping and sliding mode was proposed for the underactuated control subsystem. The control parameters of the sliding surfaces

<span id="page-11-6"></span>

**FIGURE 13.** Estimation of external disturbances.

<span id="page-11-3"></span>

**FIGURE 14.** Control inputs for each quadcopter.

were adaptively tuned with a neural network by the backpropagation algorithm. In addition, a nonlinear disturbance observer was incorporated to estimate and compensate for lumped disturbances (string tension forces, wind gusts, parametric uncertainties). Simulation results demonstrate that the controller proposed in this paper successfully transported the payload safely and without oscillations using two underactuated quadcopters. Moreover, the desired formation pattern was successfully maintained throughout the flight task, thus guaranteeing the stability of the cooperative transport system, even in the presence of disturbances such as rope tensions, wind gusts, and parametric uncertainties.

# <span id="page-11-1"></span>**APPENDIX A PARAMETER VALUES USED FOR THE SIMULATIONS**

<span id="page-11-2"></span>



# <span id="page-12-23"></span>**TABLE 2.** Control parameters.



# **APPENDIX B ALGORITHMS**



- 1: Define quadcopters and payload parameters:  $m_{qi}, m_p, J_x, J_y, J_z, J_{ri}, \varsigma, \kappa$
- 2: Define the parameters of the Udwadia-Kalaba equation: *li*, α, β
- 3: Define controller parameters:

 $\Xi_{\sigma,0}, \varepsilon_{\phi,0}, \varepsilon_{\theta,0}, \varepsilon_{\psi,0}, \varepsilon_{z,0}, \omega_{\phi,0}, \omega_{\theta,0}, \omega_{\psi,0}, \omega_{z,0},$  $\eta_{\phi,\theta},\eta_{z,\psi},k_{s_\phi},k_{s_\theta},\alpha_{\phi,\theta}$ 

- 4: Define formation distance:  $\Delta_x$ ,  $\Delta_y$ ,  $\Delta_z$
- 5: **while** Repeat **do**

# 6: **Fully actuated control subsystem**

- 7: Acquires current status:  $z_i$ ,  $\psi_i$
- 8: Error computation:  $e_i^z$ ,  $e_i^{\psi}$ *i*
- 9: Derivation of errors:  $\dot{e}_i^z$ ,  $\dot{e}_i^{\psi}$
- *i* 10: Define sliding surfaces:  $s_{z_i}$ ,  $s_{\psi_i}$
- 11: Derivation of sliding surfaces:  $s_{z_i}$ ,  $s_{\psi_i}$
- 12: Calculation of control inputs:  $U_{1i}$ ,  $U_{4i}$
- 13: **Neural network**
- 14: Calculate adaptive parameters of  $s_{z_i}: \varepsilon_z, \Xi_1, \Xi_2$
- 15: Calculate adaptive parameters of  $s_{\psi_i}: \varepsilon_{\psi}, \Xi_3, \Xi_4$
- 16: **Underactuated control subsystem**
- 17: Acquires current status:  $x_i$ ,  $y_i$ ,  $\phi_i$ ,  $\theta_i$
- 18: Error computation:  $e_{1i}^x$ ,  $e_1^y$  $\frac{y}{1i}$ ,  $e_1^{\phi}$  $\frac{\phi}{1i}$ ,  $e_{1i}^{\theta}$
- 19: Derivation of errors:  $\ddot{e}_{1i}^x$ ,  $\ddot{e}_1^y$  $\ddot{i}_i, \ddot{e}_1^{\phi}$  $_{1i}^{\phi},\dot{e}_{1i}^{\theta}$
- 20: Define sliding surfaces:  $s_{\phi_i}$ ,  $s_{\theta_i}$
- 21: Derivation of sliding surfaces:  $\dot{s}_{\phi_i}$ ,  $\dot{s}_{\theta_i}$
- 22: Calculation of control inputs:  $U_{2i}$ ,  $U_{3i}$
- 23: **Neural network**
- 24: Calculate adaptive parameters of  $s_{\phi_i}$ :  $\varepsilon_{\phi}$ ,  $\Xi_5$ ,  $\Xi_6$ ,  $\Xi_7$ ,  $\Xi_8$ ,  $\Xi_9$
- 25: Calculate adaptive parameters of  $s_{\theta_i}$ :  $\varepsilon_{\theta}$ ,  $\Xi_{10}$ ,  $\Xi_{11}$ ,  $\Xi_{12}$ ,  $\Xi_{13}$ ,  $\Xi_{14}$
- 26: **Nonlinear disturbance observer**
- 27: Acquire control inputs:  $U_{1i}$ ,  $U_{2i}$ ,  $U_{3i}$ ,  $U_{4i}$
- 28: Acquire current status:  $x_i$ ,  $y_i$ ,  $z_i$ ,  $\phi_i$ ,  $\theta_i$ ,  $\psi_i$
- 29: Observer output:  $\hat{d}_x$ ,  $\hat{d}_y$ ,  $\hat{d}_z$
- 30: **end while**

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