Received 20 March 2023, accepted 29 March 2023, date of publication 6 April 2023, date of current version 18 April 2023.
Digital Object Identifier 10.1109/ACCESS.2023.3265211

RESEARCH ARTICLE

Joint Optimization of Condition-Based Maintenance and Spare Ordering Strategy for a Multistate Competing Failure System

MENGYING HAN[1], ZHICHAO MA[2], SHUGANG MA[1], WEI LI[1], AND JIANHUA YANG[3]
[1]School of Business, Hebei University of Economics and Business, Shijiazhuang 050061, China
[2]School of Management Engineering and Business, Hebei University of Engineering, Handan 056038, China
[3]School of Economics and Management, University of Science and Technology at Beijing, Beijing 100083, China

Corresponding author: Zhichao Ma (1362189001@qq.com)

This work was supported in part by the Natural Science Foundation of Hebei Province under Grant G2022207003; in part by the Social Science Foundation of Hebei Province under Grant HB21GL025; in part by the Social Science Grand Research of the Hebei Education Department under Grant BJS2023029, Grant BJ2021070, and Grant SY2022002; and in part by the Social Science Research of Hebei University of Economics and Business under Grant 2023QN02.

ABSTRACT This paper develops a joint strategy of condition-based maintenance and spare ordering for a multistate system that is subject to competing failures due to external shocks and self-degradation. Failures of this system are hidden and can be divided into two types: a one-stage hard failure and a two-stage soft failure. The states of the system are identified by periodic inspections and replacement is executed preventively or correctively in response to the defective or failed state. When the operating time of the system is reached the predetermined threshold τ, the spare is ordered. Furthermore, a threshold level z is introduced to postpone the preventive replacement (PR) when the ordered spare is delivered before the defective state is first detected. Depending on the state of the ordered spare when replacement is required and the fault cause of the system when a corrective replacement (CR) is needed, all possible renewal events are analyzed to establish the joint optimization model. A modified artificial bee colony algorithm and discrete simulation algorithm are adopted to find the optimal solutions, and the correctness of the proposed model is verified by a numerical example. Moreover, the results from the numerical example indicate that the proposed strategy is superior to the comparative model, thus the applicability and effectiveness of the proposed model are illustrated.

INDEX TERMS Multistate competing failure, hidden failures, condition-based maintenance, spare ordering, delay-time model.

I. INTRODUCTION

With the increasing development of technology, complex systems, which are usually subject to multiple failure modes, have been extensively used in various industrial areas [1], [2], [3], [4], [5]. Generally, these complex systems are regarded as competing failure systems. Competing failures mean that the system fails as soon as one of the failures occurs. In the literature, the competing failure systems not only experienced soft failures but are also subjected to hard failures [6], [7], [8], [9], [10]. Soft failures refer to internal fatigue, wear, aging, and other reasons that cause a slow decline in the performance or function of the system. In other words, soft failures are degradation failures. Unlike soft failures, hard failures occur suddenly and those failures cause the system to fail instantaneously. Moreover, the performance of the system is often affected by external random shocks in the hard failures mode, and the shock that triggers the system to fail is called a fatal shock.

The maintenance problem for the competing failure systems subject to internal continuous degradation and external random shocks has been highly concern by many scholars [11], [12], [13]. In their papers, the Gamma process and the Inverse Gaussian process are frequently used to describe

The associate editor coordinating the review of this manuscript and approving it for publication was Xiao-Sheng Si.

the continuous degradation process, and the random shocks are often characterized by the Poisson process. With the rapid development of sensor and Internet of Things (IoT) technologies, although the states of a system can be monitored online continuously, the utilization rate of the continuous monitoring data is a huge challenge in some systems, since maintenance is required only when the system deteriorates to a certain extent [14]. Besides, the complexity of decision model analysis and calculation is increased due to the data obtained from continuous condition monitoring [15]. However, the maintenance decision modeling based on discrete states can effectively reduce the complexity of the model [16], and the system is commonly monitored periodically [17]. In fact, the degradation process of some competing failure systems, such as light emitting diodes [18], gyros [19], generators [20], and some bulk power electric systems [21], actually are not continuous but exhibit multi-stage features.

In practice, 40% of failure modes of complex systems are hidden, and 80% of these hidden failures can only be detected through monitoring or inspection technology [22]. Therefore, in recent years, the maintenance policies of multi-stage competing failure systems with hidden failures began to attract attention. For example, Yang et al. [23] divided the internal degradation process into normal and defective stages by using two Wiener processes, meanwhile assuming that external random shocks arrive at the system with a nonhomogeneous Poisson process. Zhang et al. [24], [25] utilized the delay-time concept [26] to model the degradation process of the two-stage competing failure system, and a homogeneous Poisson process is used to describe the random shocks process so as to seek the optimal periodic inspection strategy.

In the above studies on maintenance optimization of competing failure systems, spare parts are assumed to be always available when replacement is required. However, it is more reasonable to research the integration of maintenance schedules and spare parts management since these two factors influence and restrict each other. The existing studies on the joint optimization of maintenance and spare ordering mainly focus on single failure mode systems (degradation failure systems), which restricts the application of the existing joint optimization model [27], [28], [29]. As far as we know, only Zhao et al. [30] and Zhang et al. [31] investigated the joint optimization of maintenance and spare ordering for a competing failure system. Zhao et al. [30] focused on a single-unit system with two failure modes, and considered the influence of random shocks, which follow a homogeneous Poisson process, on the Wiener-based multi-stage degradation level and degradation rate. Moreover, according to the relationship between random shock amplitude and the dynamic threshold to decide whether to order a spare. Zhang et al. [31] utilized the Gamma process and homogeneous Poisson process to characterize the soft failure process and the hard failure process, respectively. On this basis, a semi-Markov decision process is formulated to derive the related indices for a series-parallel competing failure system. However, these results do not apply to competing failure systems with hidden failures.

In this paper, a joint optimization model of maintenance and spare ordering for a multistate competing failure system with hidden failures is established. The soft failure process of the system is described by the delay-time concept. Therefore, the system experiences a two-stage degradation process, i.e., from the new state to the initial point of a defective state and from that point to failed state. The two stages are modeled by different probability distributions. Besides, hard failures may lead the system to fail. In this mode, the failure process of the system only has one stage, i.e., from new state to failed state, due to the system being in the normal state until a fatal shock arrives, and the stage is characterized by a general probability distribution. Based on the abovementioned description, the states of the competing failure system consist of normal, defective, and failed states.

Furthermore, the periodic monitoring or inspection strategy is adopted in this paper to identify the system states, and the defective-based preventive and failed-based corrective replacement policies are implemented for the system. The spare ordering threshold is introduced and the random lead time is considered to judge the state of the spare when the system is required to be replaced. When a preventive replacement (PR) is required and the spare has already arrived, the commonly adopted strategy used in the literature is performing the replacement immediately [27], [28], [29]. However, we allow the PR to be postponed for an additional period of time z to improve the utilization of the system's useful life and reduce costs. Then, all possible renewal events are analyzed, and their occurrence probability is deduced. Eventually, based on renewal reward theory [32], a joint optimization model is constructed by taking the minimal expected cost rate as the objective, and the inspection interval, spare ordering threshold, and postponed PR threshold as the optimization variables. We use a modified artificial bee colony (ABC) algorithm to solve such multivariable decision problems since is superior to other algorithms [33], [34].

The main contributions of this paper can be summarized as follows: (1) a joint optimization strategy regarding both preventive maintenance and spare for a multistate competing failure system with hidden failures is developed; (2) a time threshold level is introduced to decide whether to place an order; (3) postponed replacement is scheduled when the ordered spare is delivered before the defective state is first detected; (4) the proposed strategy is compared with a special strategy in which the PR is carried out immediately when the defective state is first detected and the spare is in stock.

The remainder of this paper is organized as follows. Section II gives the description of the multistate competing failure system and the joint periodic inspection and spare ordering strategy, while the model formulation is presented in Section III. Section IV briefly describes the proposed ABC algorithm procedures. In Section V a comparative model is presented, and a numerical example is shown to demonstrate

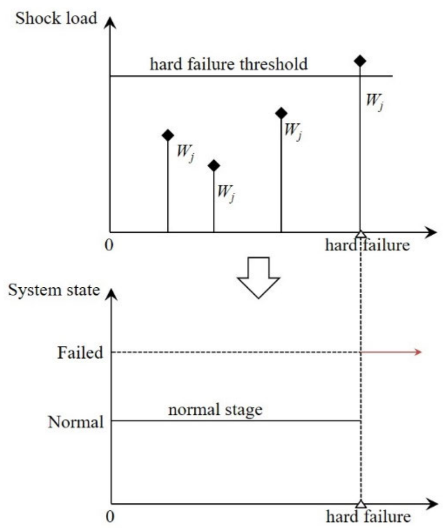

FIGURE 1. One-stage hard failure process.

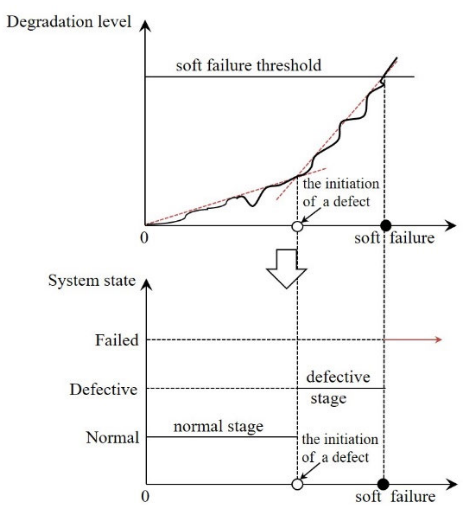

FIGURE 2. Two-stage soft failure process.

the proposed model and method. Section VI draws major conclusions and presents further research.

II. PROBLEM DESCRIPTION

A. THE SYSTEM STATEMENT

The system undergoes two failure modes: a one-stage hard failure and a two-stage soft failure, they compete with each other, and the earlier failure mode leads the system to fail. In the case of the first failure mode, the system will operate normally when the external shock is less than the hard failure threshold, while once the external shock exceeds the hard failures threshold the system will fail suddenly, as depicted in Figure 1, where the W_j represents the shock load. Therefore, in failure mode 1 the system consists of two states: normal and failed. In the case of the second failure mode, the system suffers from a two-stage degradation process, including the normal working stage and the defective stage, which is characterized by the delay-time concept. As such, in failure mode 2 the system has three possible states: normal, defective, and failed. In general, the degradation amount of the system increases in the defective stage at a higher rate than in the normal working stage, as shown in Figure 2.

In the hard failures mode, the duration of the system in the normal state is described by random variable X_1; in the soft failures mode, the durations of the system in the normal and defective states are characterized by two independent random variables X_2 and X_3, respectively. The probability density function and cumulative distribution function corresponding to the three parts are $f_i(\cdot)$ and $F_i(\cdot)$, $i = 1, 2, 3$, respectively. Moreover, the two failure modes are independent of each other, and we assume that the failures of the competing-risk system are hidden.

B. THE JOINT STRATEGY OF CONDITION-BASED MAINTENANCE AND SPARE ORDERING

In this paper, a periodic inspection scheme with the interval T is adopted to reveal the normal states, defects, and hidden failures of the system, and inspections are assumed to be perfect. If the system is detected in the defective state, a PR needs to be arranged. If the system is found in the failed state, a corrective replacement (CR) is required to be done. All replacement actions can bring the system to the ''as-good-as-new'' state. There is no doubt that an available spare is a key factor to perform replacement activity. The time when the system is in a new state is recorded as time 0, our study assumes that the spare is ordered at time $\tau (\tau \geq 0)$, and it will be delivered after a random lead time L, which follows a certain distribution, i.e., $L \sim \varepsilon(L)$.

It is clear that there exist three possible scenarios for the state of the spare when a PR or CR is needed. If the time of a replacement is required is smaller than the time of the spare ordering, namely, the spare is not ordered, an order is placed immediately, meanwhile keeping the system operating or retaining the failed state until the spare is delivered. If the spare has been ordered but not delivered, keep the state of the system as it is and wait for the spare until the spare become available. If the spare has already arrived, beyond all doubt, a CR should be conducted immediately to renew the system. However, considering that the holding cost per unit time is generally no larger than the penalty cost per unit time due to stockout, at the same time, in order to achieve better utilization of the system's useful life and avoid excessive maintenance, a PR cannot be executed until z time units later. In other words, a PR is postponed to z time units later instead of carried out immediately when a defect is identified and the spare is in stock. Consequently, when the replacement is required at the k^{th} inspection $T_k (T_k = kT; k = 1, 2, \ldots, \infty)$, the corresponding decisions should be made in accordance with the system state and the state of the spare, which are summarized in Table 1.

According to the above description, the proposed joint strategy of periodic inspection and spare ordering can be represented by a set (T, τ, z). Based on all possible renewal

TABLE 1. Joint decision making under different renewal scenarios.

Renewal scenarios	The system state	The spare states	Decision making
1	The system is identified to be in the defective state at T_k.	Not ordered	Order immediately: The system continues to operate and is replaced when the spare become available.
2		Ordered but not delivered	Wait for spare: The system continues to operate and is replaced when the spare become available.
3		Arrived	Replacement postponed to z time units later.
4	The system is identified to be in the failed state at T_k.	Not ordered	Order immediately: The system will be replaced when the spare become available.
5		Ordered but not delivered	Wait for spare: The system will be replaced when the spare become available.
6		Arrived	Replace the system immediately.

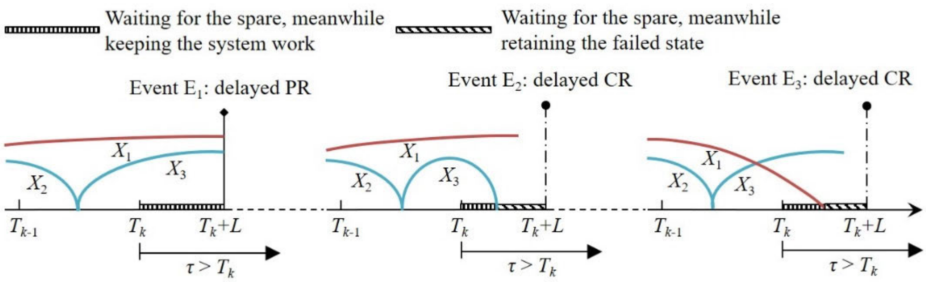

FIGURE 3. The defective state is first detected at T_k, before which no order is placed.

scenarios in Table 1, an optimization model that minimizes the expected cost rate is established by using the renewal reward theorem [32], and we intend to find the optimal strategy (T^*, τ^*, z^*). The expected cost rate can be expressed as

$$C(T, \tau, z) = \frac{EC(T, \tau, z)}{EL(T, \tau, z)}, \tag{1}$$

where $EC(T, \tau, z)$ and $EL(T, \tau, z)$ are the expected cost incurred during a renewal cycle and the expected length of a renewal cycle, respectively.

III. MODEL FORMULATION

This section aims to derive Eq. (1), thus the probabilities for all renewal scenarios should be firstly modeled, and accordingly, the expected renewal cycle cost and length of each scenario can be calculated.

A. SCENARIO 1

In this scenario, the spare has not been ordered when the system is required to replace preventively at an inspection $T_k (k = 1, 2, \ldots, \infty)$, i.e., the condition $\tau > T_k$ is met, and the replacement of the system has to be delayed to the time $T_k + L$. Since the system continues to operate after the spare is ordered at T_k, an inspection will be executed at the arrival time of the spare $T_k + L$, and there are three situations: (a) the system is still in the defective state, as shown in event E_1 in Figure 3; (b) the degradation process is in the failed state, as depicted in event E_2 in Figure 3; (c) the system in the failed state due to hard failures, see event E_3 in Figure 3. The occurrence probability of such three situations can be respectively formulated as

$$\begin{aligned} &P_1^{E_1}(defective) \\ &= P(X_1 > T_k + L, T_{k-1} < X_2 < T_k, \\ &\quad X_3 > T_k + L - X_2, L > 0) \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_{k-1}}^{T_k} [1 - F_1(T_k + L)][1 - F_3(T_k + L - X_2)] \\ &\quad \cdot f_2(x_2)\varepsilon(L) \mathrm{d}x_2 \mathrm{d}L \cdot I(\tau - T_k), \end{aligned} \tag{2}$$

where $I(m) = \begin{cases} 1, & m > 0 \\ 0, & m \le 0 \end{cases}$.

$$\begin{aligned} P_1^{E_2}(defective) &= P(X_1 > X_2 + X_3, T_{k-1} < X_2 < T_k, \\ &\quad T_k < X_2 + X_3 < T_k + L, L > 0) \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_{k-1}}^{T_k} \int_{T_k - x_2}^{T_k + L - x_2} [1 - F_1(x_2 + x_3)] \\ &\quad \cdot f_2(x_2) f_3(x_3) \varepsilon(L) \mathrm{d}x_3 \mathrm{d}x_2 \mathrm{d}L \cdot I(\tau - T_k), \end{aligned} \tag{3}$$

$$\begin{aligned} P_1^{E_3}(defective) &= P(T_k < X_1 < T_k + L, T_{k-1} < X_2 < T_k, \\ &\quad X_3 > X_1 - X_2, L > 0) \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_k}^{T_k + L} \int_{T_{k-1}}^{T_k} [1 - F_3(x_1 - x_2)] \\ &\quad \cdot f_1(x_1) f_2(x_2) \varepsilon(L) \mathrm{d}x_2 \mathrm{d}x_1 \mathrm{d}L\ cdot I(\tau - T_k), \end{aligned} \tag{4}$$

Accordingly, the expected cost and length of the renewal cycle $(0, T_k + L)$ caused by a defect detected at T_k can be

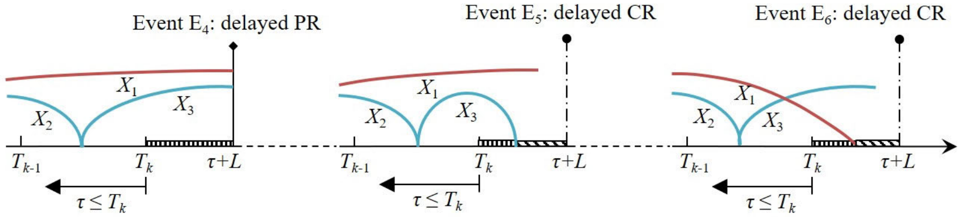

FIGURE 4. The defective state is first detected at T_k, before which an order is placed and after which the spare is arrived.

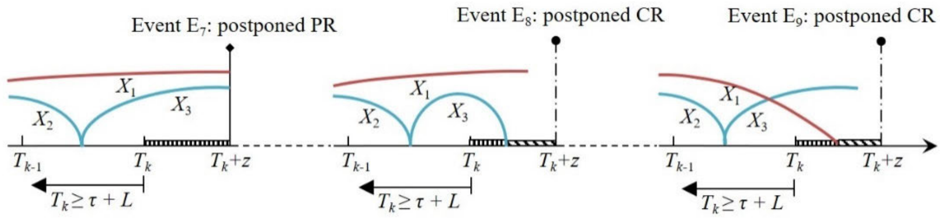

FIGURE 5. The defective state is first detected at T_k, before which the spare has been delivered.

respectively expressed as

$$\begin{aligned} &EC_1(T, \tau, z) \\ &= \sum_{k=1}^{\infty} \{[(k+1)C_i + C_r + C_p + C_w L] \cdot P_1^{E_1}(defective) \\ &\quad + [(k+1)C_i + C_r + C_f] \\ &\quad \cdot [P_1^{E_2}(defective) + P_1^{E_3}(defective)] \\ &\quad + [C_s(T_k + L - x_2 - x_3) + C_w(x_2 + x_3 - T_k)] \\ &\quad \cdot P_1^{E_2}(defective) \\ &\quad + [C_s(T_k + L - x_1) + C_w(x_1 - T_k)] \cdot P_1^{E_3}(defective)\}, \end{aligned} \tag{5}$$

where C_i, C_r, C_p, C_f, C_w, and C_s refer to the unit inspection cost, ordering costs of the spare (including the value of the spare), preventive replacement costs (including the installation fee and labor cost), corrective replacement costs (including the installation fee and labor cost), penalty cost per unit time for waiting for spare during system operation, and loss per unit time during system failure shutdown, respectively.

$$\begin{aligned} &EL_1(T, \tau, z) \\ &= \sum_{k=1}^{\infty} (T_k + L) \\ &\quad \cdot [P_1^{E_1}(defective) + P_1^{E_2}(defective) + P_1^{E_3}(defective)], \end{aligned} \tag{6}$$

B. SCENARIO 2

As illustrated in Figure 4, in the two-stage delay-time failure mode, the system enters its defective state in the inspection interval (T_{k-1}, T_k),where $k = 1, 2, \ldots, \infty$, and no fatal shock arrives at the system until T_k. Therefore, the system is revealed to be in a defective state at inspection T_k, since the spare has been ordered but not delivered, i.e., the condition $\tau \leq T_k < \tau + L$ is met, the replacement is delayed until the arrival time of the spare $\tau + L$. Similar to scenario 1, an inspection needs to be carried out at $\tau + L$ to determine the type of replacement. If a PR is performed at $\tau + L$, implies the system has not failed during the waiting period for spare, as described in event E_4 in Figure 4. However, if a CR is carried out at $\tau + L$, indicates the system has failed before the delivery of the spare, which may be caused by a soft or hard failure (see events E_5 and E_6 in Figure 4). Consequently, the occurrence probability of the three renewal events can be respectively given by

$$\begin{aligned} &P_2^{E_4}(defective) \\ &= P(X_1 > \tau + L, T_{k-1} < X_2 < T_k, \\ &\quad X_3 > \tau + L - X_2, L > T_k - \tau) \cdot I'(\tau - T_k) \\ &= \int_{T_k - \tau}^{\infty} \int_{T_{k-1}}^{T_k} [1 - F_1(\tau + L)][1 - F_3(\tau + L - x_2)] \\ &\quad \cdot f_2(x_2)\varepsilon(L)\mathrm{d}x_2\mathrm{d}L \cdot I'(\tau - T_k), \end{aligned} \tag{7}$$

where $I'(n) = \begin{cases} 1, & n \leq 0 \\ 0, & n > 0 \end{cases}$.

$$\begin{aligned} &P_2^{E_5}(defective) \\ &= P(X_1 > X_2 + X_3, T_{k-1} < X_2 < T_k, \\ &\quad T_k < X_2 + X_3 < \tau + L, L > T_k - \tau) \cdot I'(\tau - T_k) \\ &= \int_{T_k - \tau}^{\infty} \int_{T_{k-1}}^{T_k} \int_{T_k - x_2}^{\tau + L - x_2} [1 - F_1(x_2 + x_3)] \\ &\quad \cdot f_2(x_2) f_3(x_3)\varepsilon(L)\mathrm{d}x_3\mathrm{d}x_2\mathrm{d}L \cdot I'(\tau - T_k), \end{aligned} \tag{8}$$

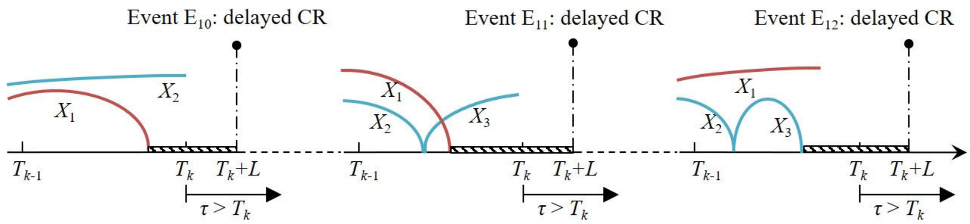

FIGURE 6. The failed state is first detected at T_k, before which no order is placed.

$$\begin{aligned}&P_2^{E_6}(defective)\\&=P(T_k<X_1<\tau+L,T_{k-1}<X_2<T_k,\\&\quad X_3>X_1-X_2,L>T_k-\tau)\cdot I'(\tau-T_k)\\&=\int_{T_k-\tau}^{\infty}\int_{T_k}^{\tau+L}\int_{T_{k-1}}^{T_k}[1-F_3(x_1-x_2)]\\&\quad\cdot f_1(x_1)f_2(x_2)\varepsilon(L)\mathrm{d}x_2\mathrm{d}x_1\mathrm{d}L\cdot I'(\tau-T_k),\end{aligned}\tag{9}$$

The expected cost and length of the renewal cycle $(0, \tau + L)$ caused by a defect detected at T_k can be respectively expressed as

$$\begin{aligned}&EC_2(T,\tau,z)\\&=\sum_{k=1}^{\infty}\{[(k+1)C_i+C_r+C_p+C_w(\tau+L-T_k)]\\&\quad\cdot P_2^{E_4}(defective)\\&\quad+[(k+1)C_i+C_r+C_f]\\&\quad\cdot[P_2^{E_5}(defective)+P_2^{E_6}(defective)]\\&\quad+[C_s(\tau+L-x_2-x_3)+C_w(x_2+x_3-T_k)]\\&\quad\cdot P_2^{E_5}(defective)\\&\quad+[C_s(\tau+L-x_1)+C_w(x_1-T_k)]\\&\quad\cdot P_2^{E_6}(defective),\}\end{aligned}\tag{10}$$

$$\begin{aligned}&EL_2(T,\tau,z)\\&=\sum_{k=1}^{\infty}(\tau+L)\\&\quad\cdot[P_2^{E_4}(defective)+P_2^{E_5}(defective)+P_2^{E_6}(defective)],\end{aligned}\tag{11}$$

C. SCENARIO 3

In scenario 3, a defect is first identified at $T_k(k = 1, 2, \ldots, \infty)$, and the PR is postponed to $T_k + z$ since the system is detected to be in the defective state after the delivery time of the spare, i.e., the condition $T_k \geq \tau + L$ is met. As can be seen from event E_7 in Figure 5, the degradation process is in a defective state at the $(k + 1)^{\text{th}}$ inspection, and a postponed PR is carried out at $T_k + z$. However, a soft or hard hidden failure may occur before $T_k + z$, thus, the CR has to be postponed to $T_k + z$, see events E_8 and E_9 in Figure 5. The corresponding renewal probability is obtained as

$$\begin{aligned}&P_3^{E_7}(defective)=P(X_1>T_k+z,T_{k-1}<X_2<T_k,\\&\quad X_3>T_k+z-X_2,0<L\leq T_k-\tau)\\&=\int_0^{T_k-\tau}\int_{T_{k-1}}^{T_k}[1-F_1(T_k+z)]\\&\quad\times[1-F_3(T_k+z-x_2)]\\&\quad\cdot f_2(x_2)\varepsilon(L)\mathrm{d}x_2\mathrm{d}L,\end{aligned}\tag{12}$$

$$\begin{aligned}P_3^{E_8}(defective)&=P(X_1>X_2+X_3,T_{k-1}<X_2<T_k,\\&\quad T_k<X_2+X_3<T_k+z,0<L\leq T_k-\tau)\\&=\int_0^{T_k-\tau}\int_{T_{k-1}}^{T_k}\int_{T_k-x_2}^{T_k+z-x_2}[1-F_1(x_2+x_3)]\\&\quad\cdot f_2(x_2)f_3(x_3)\varepsilon(L)\mathrm{d}x_3\mathrm{d}x_2\mathrm{d}L,\end{aligned}\tag{13}$$

$$\begin{aligned}P_3^{E_9}(defective)&=P(T_k<X_1<T_k+z,T_{k-1}<X_2<T_k,\\&\quad X_3>X_1-X_2,0<L\leq T_k-\tau)\\&=\int_0^{T_k-\tau}\int_{T_k}^{T_k+z}\int_{T_{k-1}}^{T_k}[1-F_3(x_1-x_2)]\\&\quad\cdot f_1(x_1)f_2(x_2)\varepsilon(L)\mathrm{d}x_2\mathrm{d}x_1\mathrm{d}L,\end{aligned}\tag{14}$$

The expected cost and length of the renewal cycle $(0, T_k + z)$ caused by a defect detected at T_k can be respectively expressed as

$$\begin{aligned}&EC_3(T,\tau,z)\\&=\sum_{k=1}^{\infty}\{[(k+1)C_i+C_r+C_p+C_h(T_k+z-\tau-L)]\\&\quad\cdot P_3^{E_7}(defective)+[(k+1)C_i+C_r+C_f+C_h(T_k\\&\quad+z-\tau-L)]\\&\quad\cdot[P_3^{E_8}(defective)+P_3^{E_9}(defective)]\\&\quad+C_s(T_k+z-x_2-x_3)]\\&\quad\cdot P_3^{E_8}(defective)+C_s(T_k+z-x_1)\cdot P_3^{E_9}(defective),\}\end{aligned}\tag{15}$$

where C_h denotes the holding cost per unit time.

$$\begin{aligned}&EL_3(T,\tau,z)\\&=\sum_{k=1}^{\infty}(T_k+z)\\&\quad\cdot[P_3^{E_7}(defective)+P_3^{E_8}(defective)+P_3^{E_9}(defective)],\end{aligned}\tag{16}$$

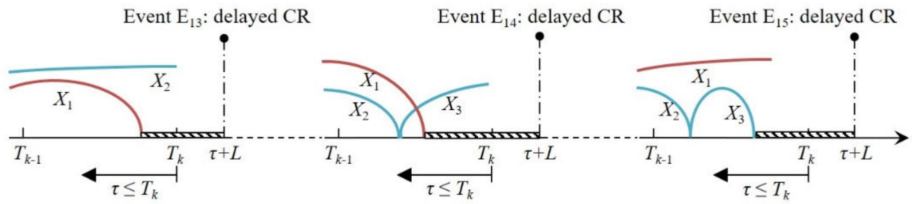

FIGURE 7. The failed state is first detected at T_k, before which an order is placed and after which the spare is arrived.

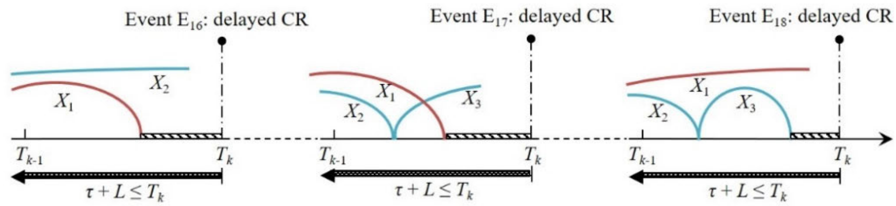

FIGURE 8. The failed state is first detected at T_k, before which the spare has been delivered.

D. SCENARIO 4

Analogous to scenario 1, the condition $\tau > T_k$ is met. When the competing failure system is detected to be in the failed state, the failure causes should be first concerned. If the system fails due to hard failures, there are two possible situations for the state of the system under the two-stage delay-time failure mode as follows: the first is in the normal state (see event E_{10} in Figure 6) and the second is in the defective state (see event E_{11} in Figure 6). If the system fails due to soft failures, the defective stage must be started and ends within the inspection interval (T_{k-1}, T_k), in which $k = 1, 2, \ldots, \infty$, as shown in event E_{12} in Figure 4. The corresponding occurrence probability for events E_{10}–E_{12} are given respectively as

$$\begin{aligned} P_4^{E_{10}}(failed) &= P(T_{k-1} < X_1 < T_k, X_2 > X_1, L > 0) \\ &\quad \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_{k-1}}^{T_k} [1 - F_2(x_1)] f_1(x_1) \varepsilon(L) \mathrm{d}x_1 \mathrm{d}L \\ &\quad \cdot I(\tau - T_k), \end{aligned} \tag{17}$$

$$\begin{aligned} P_4^{E_{11}}(failed) &= P(T_{k-1} < X_1 < T_k, T_{k-1} \\ &\quad < X_2 < X_1, X_3 > X_1 - X_2, L > 0) \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_{k-1}}^{T_k} \int_{T_{k-1}}^{x_1} [1 - F_3(x_1 - x_2)] \\ &\quad \times f_1(x_1) f_2(x_2) \varepsilon(L) \mathrm{d}x_2 \mathrm{d}x_1 \mathrm{d}L \cdot I(\tau - T_k), \end{aligned} \tag{18}$$

$$\begin{aligned} P_4^{E_{12}}(failed) &= P(X_1 > X_2 + X_3, T_{k-1} < X_2 < T_k, \\ &\quad T_{k-1} < X_2 + X_3 < T_k, L > 0) \cdot I(\tau - T_k) \\ &= \int_0^{\infty} \int_{T_{k-1}}^{T_k} \int_0^{T_k - x_2} [1 - F_1(x_2 + x_3)] \\ &\quad \times f_2(x_2) f_3(x_3) \varepsilon(L) \mathrm{d}x_3 \mathrm{d}x_2 \mathrm{d}L \cdot I(\tau - T_k), \end{aligned} \tag{19}$$

Accordingly, the expected cost and length of the renewal cycle $(0, T_k + L)$ caused by a failure detected at T_k can be respectively expressed as

$$\begin{aligned} EC_4(T, \tau, z) &= \sum_{k=1}^{\infty} \{[kC_i + C_r + C_f] \cdot [P_4^{E_{10}}(failed) \\ &\quad + P_4^{E_{11}}(failed) + P_4^{E_{12}}(failed)] \\ &\quad + C_s(T_k + L - x_1) \\ &\quad \cdot [P_4^{E_{10}}(failed) + P_4^{E_{11}}(failed)] \\ &\quad + C_s(T_k + L - x_2 - x_3) \cdot P_4^{E_{12}}(failed)\}, \end{aligned} \tag{20}$$

$$\begin{aligned} EL_4(T, \tau, z) &= \sum_{k=1}^{\infty} (T_k + L) \\ &\quad \cdot [P_4^{E_{10}}(failed) + P_4^{E_{11}}(failed) + P_4^{E_{12}}(failed)], \end{aligned} \tag{21}$$

E. SCENARIO 5

The system is revealed to be in a failed state by an inspection $T_k (k = 1, 2, \ldots, \infty)$ after the spare is ordered, but before the arrival of the spare, therefore, the CR is delayed until $\tau + L$, as illustrated in Figure 7. Event E_{13} indicates that the fatal shock arrives at the system before the degradation process enters the defective state. Event E_{14} means that the fatal shock arrives at the system when the degradation process is in a defective state, and Event E_{15} implies that a soft failure occurred before the fatal shock arrives at the system. Thus, we obtain the occurrence probability of the three cases respectively as follows

$$\begin{aligned} & P_5^{E_{13}}(failed) \\ &= P(T_{k-1} < X_1 < T_k, X_2 > X_1, L > T_k - \tau) \cdot I'(\tau - T_k) \\ &= \int_{T_k - \tau}^{\infty} \int_{T_{k-1}}^{T_k} [1 - F_2(x_1)] f_1(x_1) \varepsilon(L) \mathrm{d}x_1 \mathrm{d}L \\ &\quad \cdot I'(\tau - T_k), \end{aligned} \tag{22}$$

$$
\begin{aligned}
&P_5^{E_{14}}(failed) \\
&= P(T_{k-1} < X_1 < T_k, T_{k-1} < X_2 < X_1, \\
&\quad X_3 > X_1 - X_2, L > T_k - \tau) \cdot I'(\tau - T_k) \\
&= \int_{T_k-\tau}^{\infty} \int_{T_{k-1}}^{T_k} \int_{T_{k-1}}^{x_1} [1 - F_3(x_1 - x_2)] \\
&\quad \times f_1(x_1) f_2(x_2) \varepsilon(L) \mathrm{d}x_2 \mathrm{d}x_1 \mathrm{d}L \cdot I'(\tau - T_k),
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
&P_5^{E_{15}}(failed) \\
&= P(X_1 > X_2 + X_3, T_{k-1} < X_2 < T_k, \\
&\quad T_{k-1} < X_2 + X_3 < T_k, L > T_k - \tau) \cdot I'(\tau - T_k) \\
&= \int_{T_k-\tau}^{\infty} \int_{T_{k-1}}^{T_k} \int_{0}^{T_k-x_2} [1 - F_1(x_2 + x_3)] f_2(x_2) f_3(x_3) \\
&\quad \cdot \varepsilon(L) \mathrm{d}x_3 \mathrm{d}x_2 \mathrm{d}L \cdot I'(\tau - T_k),
\end{aligned} \tag{24}
$$

The expected cost and length of the renewal cycle $(0, \tau + L)$ caused by a failure detected at T_k can be respectively expressed as

$$
\begin{aligned}
&EC_5(T, \tau, z) \\
&= \sum_{k=1}^{\infty} \{[kC_i + C_r + C_f] \cdot [P_5^{E_{13}}(failed) \\
&\quad + P_5^{E_{14}}(failed) + P_5^{E_{15}}(failed)] + C_s(\tau + L - x_1) \\
&\quad \cdot [P_5^{E_{13}}(failed) + P_5^{E_{14}}(failed)] \\
&\quad + C_s(\tau + L - x_2 - x_3) \cdot P_5^{E_{15}}(failed)\},
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&EL_5(T, \tau, z) \\
&= \sum_{k=1}^{\infty} (\tau + L) \\
&\quad \cdot [P_5^{E_{13}}(failed) + P_5^{E_{14}}(failed) + P_5^{E_{15}}(failed)],
\end{aligned} \tag{26}
$$

F. SCENARIO 6

As can be seen from Figure 8, a CR can be performed immediately since the spare is available when the failed state is found at $T_k (k = 1, 2, \ldots, \infty)$, and the condition $T_k \geq \tau + L (k = 1, 2, \ldots, \infty)$ is met, Similarly, depending on the states of the system in two failure modes at the k^{th} inspection, three renewal cases should be considered, as shown in events E_{16}–E_{18} in Figure 8, and the occurrence probability for each event is obtained as

$$
\begin{aligned}
&P_6^{E_{16}}(failed) \\
&= P(T_{k-1} < X_1 < T_k, X_2 > X_1, 0 < L \leq T_k - \tau) \\
&= \int_{0}^{T_k-\tau} \int_{T_{k-1}}^{T_k} [1 - F_2(x_1)] f_1(x_1) \varepsilon(L) \mathrm{d}x_1 \mathrm{d}L,
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
&P_6^{E_{17}}(failed) \\
&= P(T_{k-1} < X_1 < T_k, T_{k-1} < X_2 < X_1, X_3 > X_1 \\
&\quad - X_2, 0 < L \leq T_k - \tau) \\
&= \int_{0}^{T_k-\tau} \int_{T_{k-1}}^{T_k} \int_{T_{k-1}}^{x_1} [1 - F_3(x_1 - x_2)] \\
&\quad \times f_1(x_1) f_2(x_2) \varepsilon(L) \mathrm{d}x_2 \mathrm{d}x_1 \mathrm{d}L,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
&P_6^{E_{18}}(failed) \\
&= P(X_1 > X_2 + X_3, T_{k-1} < X_2 < T_k, \\
&\quad T_{k-1} < X_2 + X_3 < T_k, 0 < L \leq T_k - \tau) \\
&= \int_{0}^{T_k-\tau} \int_{T_{k-1}}^{T_k} \int_{0}^{T_k-x_2} [1 - F_1(x_2 + x_3)] f_2(x_2) f_3(x_3) \\
&\quad \cdot \varepsilon(L) \mathrm{d}x_3 \mathrm{d}x_2 \mathrm{d}L,
\end{aligned} \tag{29}
$$

The expected cost and length of the renewal cycle $(0, T_k)$ caused by a failure detected at T_k can be respectively expressed as

$$
\begin{aligned}
EC_6(T, \tau, z) = \sum_{k=1}^{\infty} &\{[kC_i + C_r + C_f + C_h(T_k - \tau - L)] \\
&\cdot [P_6^{E_{16}}(failed) \\
&+ P_6^{E_{17}}(failed) + P_6^{E_{18}}(failed)] + C_s(T_k - x_1) \\
&\cdot [P_6^{E_{16}}(failed) + P_6^{E_{17}}(failed)] \\
&+ C_s(T_k - x_2 - x_3) \cdot P_6^{E_{18}}(failed)\},
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
EL_6(T, \tau, z) = \sum_{k=1}^{\infty} &[T_k] \cdot [P_6^{E_{16}}(failed) + P_6^{E_{17}}(failed) \\
&+ P_6^{E_{18}}(failed)],
\end{aligned} \tag{31}
$$

G. EXPECTED COST RATE

Based on the above analysis, the expected renewal cycle cost $EC(T, \tau, z)$ is the sum of Eqs. (5), (10), (15), (20), (25), and (30), and the expected renewal cycle length $EL(T, \tau, z)$ is the sum of Eqs. (6), (11), (16), (21), (26), and (31). Therefore, $EC(T, \tau, z)$ and $EL(T, \tau, z)$ can be calculated respectively as

$$
EC(T, \tau, z) = \sum_{w=1}^{6} EC_w(T, \tau, z) \tag{32}
$$

$$
EL(T, \tau, z) = \sum_{w=1}^{6} EL_w(T, \tau, z) \tag{33}
$$

Substituting Eqs. (32)–(33) into Eq. (1), the objective function of the proposed joint condition-based maintenance and spare ordering strategy can be obtained, and we seek for the optimal strategy (T^*, τ^*, z^*) by minimizing the expected cost rate $C(T, \tau, z)$.

IV. OPTIMIZATION METHODOLOGY

We use a modified artificial bee colony (ABC) algorithm to optimize Eq. (1) which has multiple decision variables. The bees in the algorithm have three different roles: employed, onlooker, and scout bees. Assume that the number of employed and onlooker bees is NP, and the percentages of the two types of bees are 50%. Since the bee has the worst food source is seen as a scout bee, the number of scout bees is assumed as 1, where the food source represents the solution of Eq. (1). The quality of the food source represents the value of Eq. (1), and the maximum number of iterations is denoted as $n_{\max}$.

The ABC algorithm consists of four phases: initialization, employed bee, onlooker bee and scout bee. In the initialization phase, we generate $NP/2$ initial food source in the search space of D dimensions, where each source is denoted as v_h, $h = 1, 2, \ldots, NP/2$, and D is the number of decision variables. The location of the h^{th} bee is denoted as $v_h = (v(h, 1), v(h, 2), \ldots, v(h, D))$, and the corresponding quality of the food source is represented as $fitness(v_h)$. In the employed bee phase, each bee renews the location of the food source according to the update mode, and the h^{th} bee's location is denoted as $v'_h = (v'(h, 1), v'(h, 2), \ldots, v'(h, D))$. Then, we compare the corresponding expected cost rate $fitness(v_h)$ and $fitness(v'_h)$, if $fitness(v'_h) < fitness(v_h)$, indicating that the quality of the food source is improved, therefore, the bad quality of the food source $fitness(v_h)$ is replaced by $fitness(v'_h)$, the old food source v_h is replaced by v'_h, and the non-improvement number $npro_h$ is reset to 0; otherwise, $npro_h = npro_h + 1$. In order to avoid a premature convergence of solutions, the onlooker bee phase is executed. In this phase, onlooker bees are selected by using the roulette wheel selection method, and the new location of an onlooker bee is denoted as $b_h = (b(h, 1), b(h, 2), \ldots, b(h, D))$, if $fitness(b_h)$ is smaller than $fitness(v_h)$, the v_h is replaced by b_h, and $npro_h = 0$; otherwise, $npro_h = npro_h + 1$. The main function of the scout bee phase is to avoid falling into local optimum. The scout bee can be determined by maximizing the $npro_h$, if the value is larger than the threshold $n_{\lim}$, this solution is replaced by a new solution.

The pseudo code of the ABC algorithm is as follows.

Algorithm 1 Artificial Bee Colony (ABC) Algorithm

Input parameters: $f_i(\cdot)(i = 1, 2, 3)$, C_i, C_r, C_p, C_f, C_w, C_s, C_h, NP, $n_{\max}$, $n_{\lim}$
Initialization phase
For $h = 1$ **to** $NP/2$
  $v(h, j) = l_j + r(u_j - l_j)$; // generate the initial solutions v_h
   where r is uniformly distributed within range [0, 1]; l_j and u_j are the lower and upper bound of the j^{th} dimension search space; $j = 1, 2, \ldots, D$.
   $npro_h = 0$;
End
$N = 1$;
While $N \le n_{\max}$
Employed bee phase
For $h = 1$ **to** $NP/2$
  $v'(h, m) = v(h, m) + r'(v(h, m) - v(g, m))$;
   where r' is uniformly distributed within range [–1, 1]; $m \in \{1, 2, \ldots, D\}$; $g \in \{1, 2, \ldots, NP/2\}$, and $g \neq h$.
 If $m = j$
  $v'(h, j) = v'(h, m)$;
 Else
  $v'(h, j) = v(h, j)$;
 End//generate new solutions v'_h
 If $fitness(v'_h) < fitness(v_h)$
  $v_h = v'_h$; $fitness(v_h) = fitness(v'_h)$; $npro_h = 0$;
 Else
 $npro_h = npro_h + 1$;
 End
End
Onlooker bee phase
For $h = 1$ **to** $NP/2$
  $pro_h = (0.9 \times fitness(v_h)/\sum_{h=1}^{NP/2} fitness(v_h)) + 0.1$; // calculate the probability of the roulette wheel selection
  If $r < pro_h$
   $b(h, m) = v(h, m) + r'(v(h, m) - v(g, m))$;
  If $m = j$
   $b(h, j) = b(h, m)$;
  Else
   $b(h, j) = v(h, j)$;
  End//generate new solutions b_h
  If $fitness(b_h) < fitness(v_h)$
   $v_h = b_h$; $fitness(v_h) = fitness(b_h)$; $npro_h = 0$;
  Else
   $npro_h = npro_h + 1$;
  End
  End
End

V. OPTIMIZATION METHODOLOGY

A. THE SPECIAL CASE OF THE JOINT OPTIMIZATION MODEL

To show the effectiveness of the proposed model presented in Section III (model 1), we introduce one further model (model 2). Model 2 assumes that the PR is performed immediately when the defective state is first detected and the spare is in stock, i.e., $z = 0$ in Eqs. (12), (15), and (16) of model 1. Meanwhile, $P_3^{E_8}(defective) = 0$ in Eqs. (13), (15), and (16), and $P_3^{E_9}(defective) = 0$ in Eqs. (14), (15), and (16). Moreover, in Eq. (15) $(k + 1)C_i$ should be changed to kC_i. Thus, the objective function of model 2 can be obtained, and the inspection interval T and the spare ordering point τ are decision variables.

B. INITIAL MODELING PARAMETERS

To solve the two models based on the algorithm devised in Section IV, we assume that X_2 follows Weibull distribution, X_1 and X_3 follow two different exponential distributions according to Peng's study [20], and L follows normal distribution. The parameters of these distributions are shown in Table 2. The cost parameters are provided in Table 3, and the values of the control parameters in the ABC algorithm are presented in Table 4.

C. RESULTS ANALYSIS AND COMPARISON

The optimization calculation based on the ABC algorithm is carried out on a computer with Intel(R) Core(TM) i7-8550U CPU, and 8GB RAM. We make 10 independent runs to

Algorithm 1 *(Continue.)* Artificial Bee Colony (ABC) Algorithm

Scout bee phase
If $npro_e > n_{\lim}$
where $npro_e = \max(npro_h)$; $h = 1, 2, \ldots, NP/2$;
$e \in \{1, 2, \ldots, NP/2\}$; $v(e, j) = l_j + r(u_j - l_j)$; // generate new solution for the e^{th} bee
End
$N = N + 1$;
End
Find the minimum $fitness(v_h)^*$ and the corresponding solution v_h^*.
Output: the optimal solution is v_h^* and $fitness(v_h)^*$

TABLE 2. Distributions for X_1, X_2, X_3 and L.

Distributions	Parameters
$f_1(x_1) = \lambda_1 e^{-\lambda_1 x_1}$	$\lambda_1 = 0.015$
$f_2(x_2) = \alpha\beta(\alpha x_2)^{\beta-1} e^{-(\alpha x_2)^\beta}$	$\alpha = 0.018, \beta = 1.81$
$f_3(x_3) = \lambda_2 e^{-\lambda_2 x_3}$	$\lambda_2 = 0.037$
$\varepsilon(L) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(L-\mu)^2}{2\sigma}\right)}$	$\mu = 10, \sigma = 3$

TABLE 3. Parameters for the costs.

C_i	C_r	C_p	C_f	C_w	C_s	C_h
100	2000	200	500	50	150	10

eliminate stochastic discrepancy, the maximum iterative number is set to 100, and the average calculation time is about 150 seconds. Figure 9 shows the process of the ABC algorithm in obtaining the expected cost rate over the iterations, it is noted that the expected cost rate decreases along with the iterations, which indicates that the quality of the food source improves as the running time of the ABC algorithm increases. The optimal solution of model 1 is $(T^*, \tau^*, z^*) = (17, 6, 12)$ with $C(T^*, \tau^*, z^*) = 88.7378$, in this case, inspection should be carried out every 17 days, the spare is ordered at the 6$^{\text{th}}$ day, and when a defect is first detected, the spare has been delivered, PR is performed 12 days later. The optimal solution of model 2 is $(T^*, \tau^*) = (18, 8)$ with $C(T^*, \tau^*) = 90.5705$, in such a case, inspection is executed every 18 days and the spare is ordered at the 8$^{\text{th}}$ day.

In order to verify the correctness of the proposed model, we design a simulation algorithm based on discrete events, which is described in Figure 11 in Appendix. We simulate 100000 times, and the simulation results of the two models are as follows. For model 1, $T = 17$, $\tau = 6$, $z = 12$ are the optimal values of the decision variables, corresponding to the minimum expected cost rate 88.7538, which is close to

TABLE 4. Parameters for the ABC algorithm.

NP	$n_{\max}$	$n_{\lim}$	(l_1, l_2, l_3)	(u_1, u_2, u_3)
10	100	5	(5,0,0)	(30,30,30)

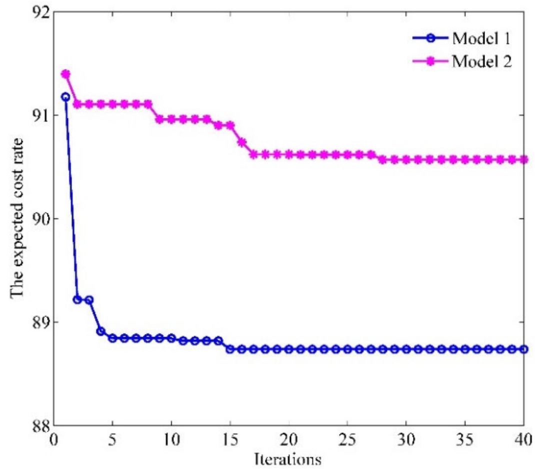

FIGURE 9. Expected cost rate of each iteration for models 1 and 2.

88.7378. For model 2, $T = 18$, $\tau = 8$ are the optimal values of the decision variables, corresponding to the minimum expected cost rate 90.5562, which is close to 90.5705. The tiny differences in the optimal expected cost rate between the ABC and the simulation algorithm can attribute to the randomness of the simulations.

Clearly, the proposed joint strategy provides the minimum expected cost rate compared with the special case. This implies that "allowing PR to be postponed for an additional time period when the ordered spare is delivered before the defective state is first identified" plays an important role in the joint optimization of the periodic inspection and spare ordering strategy. Furthermore, how long to postpone it motivates us to explore the influence of different postponement intervals z on the expected cost rate. Thus, we treat the expected cost rate $C(T, \tau, z)$ as a function of z given the optimal $T^* = 17$ and $\tau^* = 6$, as shown in Figure 10. Two distinctive features are observed: $C(17, 6, 0) < C(17, 6, 1)$; and the expected cost rate first decreases and then increases with z when $z > 0$. The former indicates that $\frac{EC(17,6,0)}{EL(17,6,0)} < \frac{\Delta EC(17,6,1)}{\Delta EL(17,6,1)}$, where $\Delta EC(17, 6, 1) = EC(17, 6, 1) - EC(17, 6, 0)$, $\Delta EL(17, 6, 1) = EL(17, 6, 1) - EL(17, 6, 0)$. The latter reveals that extending the system's useful life makes cost-effective when z is less than 12; However, the probability of a defect accumulating into a failure increases when z is larger than 12 that makes cost-prohibitive; Moreover, $z = 12$ is the trade-off point between the better utilization of the system's useful life and the higher costs due to the risk of failure.

TABLE 5. Sensitivity analysis of cost parameters.

Cost parameters		Optimal results of model 1					Optimal results of model 2			
		T^*	τ^*	z^*	$C'(T^*,\tau^*,z^*)$	$\Delta C(/\%)$	T^*	τ^*	$C'(T^*,\tau^*)$	$\Delta C'(/\%)$
C_i	80	15	4	12	87.4461	-1.456	17	7	89.4137	-1.277
	120	18	7	13	89.9341	1.348	19	9	91.6506	1.193
C_r	1600	15	5	10	79.6918	-10.194	16	6	80.8681	-10.713
	2400	19	8	15	97.4644	9.834	22	12	99.9061	10.308
C_p	160	17	6	12	88.5679	-0.191	18	8	90.2532	-0.35
	240	17	6	13	88.9065	0.19	18	8	90.8878	0.35
C_f	400	17	6	12	86.9434	-2.022	18	8	88.9754	-1.761
	600	17	6	12	90.5322	2.022	18	8	92.1656	1.761
C_w	40	17	6	12	88.7333	-0.005	18	8	90.5589	-0.013
	60	17	6	12	88.7422	0.005	18	8	90.582	0.008
C_s	120	23	12	16	83.7088	-5.667	23	35	85.0101	-6.14
	180	14	3	10	92.6479	4.406	14	4	94.4581	4.292
C_h	8	16	5	12	87.4222	-1.483	17	7	89.4238	-1.266
	12	17	7	12	90.0191	1.444	19	9	91.6898	1.236

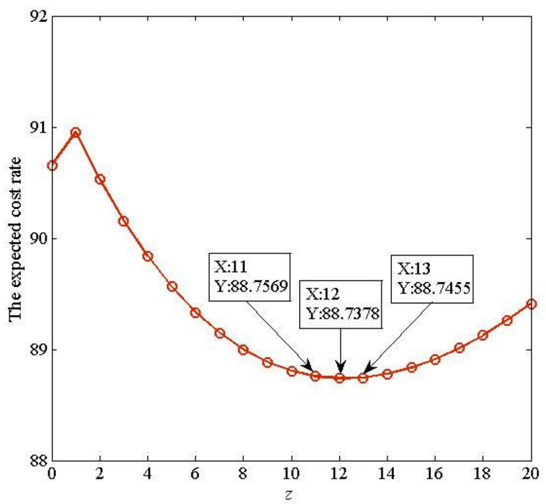

FIGURE 10. Expected cost rate in model 1 as a function of *z* for the optimal $T* = 17$ and $\tau* = 6$.

D. SENSITIVITY ANALYSIS

The influence of the cost parameters on the optimal solutions is investigated by performing a sensitivity analysis, and the results can be observed in Table 5. In the analysis, the percentage of each parameter's decrease or increase is -20%, $+20\%$, respectively, while other parameters are unvaried. Correspondingly, the obtained minimal expected cost rate for models 1 and 2 are denoted by $C'(T^*, \tau^*, z^*)$ and $C'(T^*, \tau^*)$, respectively. The ΔC and $\Delta C'$ are calculated to be $\Delta C = \frac{C'(T^*,\tau^*,z^*)-C(T^*,\tau^*,z^*)}{C(T^*,\tau^*,z^*)} \times 100\%(C(T^*, \tau^*, z^*) = 88.7378)$ and $\Delta C' = \frac{C'(T^*,\tau^*)-C(T^*,\tau^*)}{C(T^*,\tau^*)} \times 100\%(C(T^*, \tau^*) = 90.5705)$, respectively, which are used to compare the sensitivities of cost parameters' variation.

It can be noted from Table 5 that the minimal expected cost rate $C'(T^*, \tau^*, z^*)$ or $C'(T^*, \tau^*)$ increases with the increase of any cost parameter, and decreases as the cost parameter reduces. It indicates that the cost parameters affect the minimal expected cost rate. For model 1, the optimal inspection interval T^*, spare ordering threshold τ^*, and postponed PR threshold z^* have increasing trends with the increases of C_i or C_r. This finding indicates that in order to minimize the expected cost rate, less frequent inspections, later spare ordering, and a larger additional period of time for postponed replacement are recommended when the inspection cost or the value of the spare is high. Moreover, we find that when C_h increases from 8 to 12 with step size 2, τ^* increases strictly monotonically, and T^* has an increasing trend but increasing not monotonically, however, z^* keeps the value unvaried. It illustrated that if the holding cost changes, managers should pay more attention to the time of the spare ordering. It is also noted that the optimal decisions are the same when C_f changes from 400 to 600 or C_w changes from 40 to 60. It shows that the small change of C_f or C_w will only affect the total expected cost rate, but not affect the optimal decisions. Analogously, the change of C_p from 160 to 240 will not affect the optimal inspection interval and the spare ordering threshold, but lead to the optimal postponed PR threshold having a not monotonically increasing trend.

It can be concluded that when C_p, C_f, or C_w takes values as shown in Table 5, the minimal expected cost rate is mainly affected by C_p, C_f, or C_w, but the inspection cost almost does not play a role. We also find that the optimal decisions T^*, τ^*, and z^* have decreasing trends as C_s increases. It indicates that if C_s is high, managers are prone to carry out more frequent inspections, early spare ordering, and shorten the time limit for a postponement so as to reduce the expected cost rate.

Form the column given the results of $\Delta C(/\%)$, it can be seen that the minimum expected cost rate is the most sensitive to the ordering costs of the spare (including the value of the spare)C_r, next to the loss per unit time during system failure shutdown C_s. Therefore, when the external environment is unstable and the value of the spare changes, managers should be concerned with decisions for the inspection interval T, spare ordering threshold τ, and postponed PR threshold z. Additionally, managers should seek effective methods and take measures to cut down the loss per unit time during system failure shut down so that a lower expected cost rate can be obtained. For model 2, the sensitivities of the optimal inspection interval and the optimal spare ordering threshold on cost parameters are similar to model 1.

Furthermore, an interesting finding is that the sum of the optimal spare ordering threshold and the expected lead time of the order is almost equal to the optimal inspection interval. This finding illustrates that when the first inspection is performed, improving the probability of the spare arriving at time T^* can significantly reduce the expected cost rate. It is worth noting from Table 5 that the minimal expected cost rate of model 2, $C'(T^*, \tau^*)$, is always larger than the minimal expected cost rate of model 1, $C'(T^*, \tau^*, z^*)$, with the cost parameters increase or decrease. It illustrates the proposed joint strategy in our paper is more cost-saving.

VI. CONCLUSION

In this research, we developed a joint periodic inspection and spare ordering strategy for a multistate competing failure system subject to hidden failures under a novel assumption: we allow PR to be postponed when a defect is first detected and the spare is in stock at an inspection. This is in contrast to all previous works in the literature that assume PR is carried out immediately when a defect is first identified and the spare has arrived. The proposed joint strategy has three main benefits. First is that spare parts resources and maintenance schedules are planned simultaneously can help to reduce costs. Second is that it offers an opportunity for adequate planning of other maintenance resources, such as manpower and equipment, so as to reduce the probability of poor installation and improve the responsiveness of necessary resources. Third is that better utilization of the system's useful life meanwhile reduces the probability of ineffective early replacement. For the system, two failure modes, i.e., a one-stage hard failure and a two-stage delay-time failure, are considered. Periodic inspections are executed to identify defects and hidden failures. When a defective or failed state is detected at an inspection, if the spare is not in stock, PR or CR has to be delayed until the spare has arrived; otherwise, PR allows to be postponed for an additional period of time, but CR is carried out immediately. The optimization model of the joint strategy is formulated by minimizing the expected cost rate to find the optimal inspection interval, the spare ordering threshold, and the postponed PR threshold. A modified artificial bee colony algorithm is proposed to seek optimal solutions, and a simulation algorithm based on discrete events is also presented to illustrate the correctness of the optimization model. More critically, the traditional joint strategy in which the instantaneous PR is performed when a defect is first detected and the spare is in stock is also modeled, and the proposed optimization model is illustrated through a numerical example. The results from the numerical example indicate that allowing the PR to be postponed when a defect is identified and the spare is in stock at an inspection is more cost-saving compared to the traditional joint policy.

Also, the sensitivity analysis concluded that: (a) managers should pay more attention to the optimal solutions when the ordering costs of the spare (including the value of the spare) change since it is the most sensitive one; (b) Effective measures should be taken to cut down the loss per unit time during system failure shutdown so that the minimum expected cost rate can be reduced significantly. (c) It is also important to reduce the inspection cost, preventive replacement cost, corrective replacement cost, penalty cost, and holding cost so as to reduce the expected cost rate. The proposed joint strategy can provide a reference for the maintenance and spare ordering schedule of a two-stage competing failure system with hidden failures.

There are some extensions to the current work. Firstly, extending the model to imperfect maintenance, and meanwhile considering the dependence between hard failure process and degradation process since they are more in line with practical applications; Secondly, it would be beneficial to extend to multiunit complex competing failure system so that relax the assumption of one spare unit is ordered and stored; Thirdly, it is difficult but worth to decide the optimal PR threshold, CR threshold, and spare ordering threshold by using real-time condition monitoring data, so that the data-driven joint optimization of maintenance and spare ordering policy can be developed.

APPENDIX

The flow chart of the simulation algorithm for model 1 in Figure 11 is as follows.

First, initialize values for all variables of the model by using the parameters in Table 2 and Table 3, and initialize the decision variables T, τ, z. The simulation runs until the number of iterations q exceeds the threshold value $N_{\max}$. Let EC and EL denote the total cost and length incurred by the simulation process, respectively. Next, we generate the random durations X_1, X_2, X_3 and L, therefore, the arrival time of the fatal shock x_1, the end points of two degradation processes x_2, y, and the lead time of the spare ordering L are determined. Meanwhile, we let t and k refer to the time and

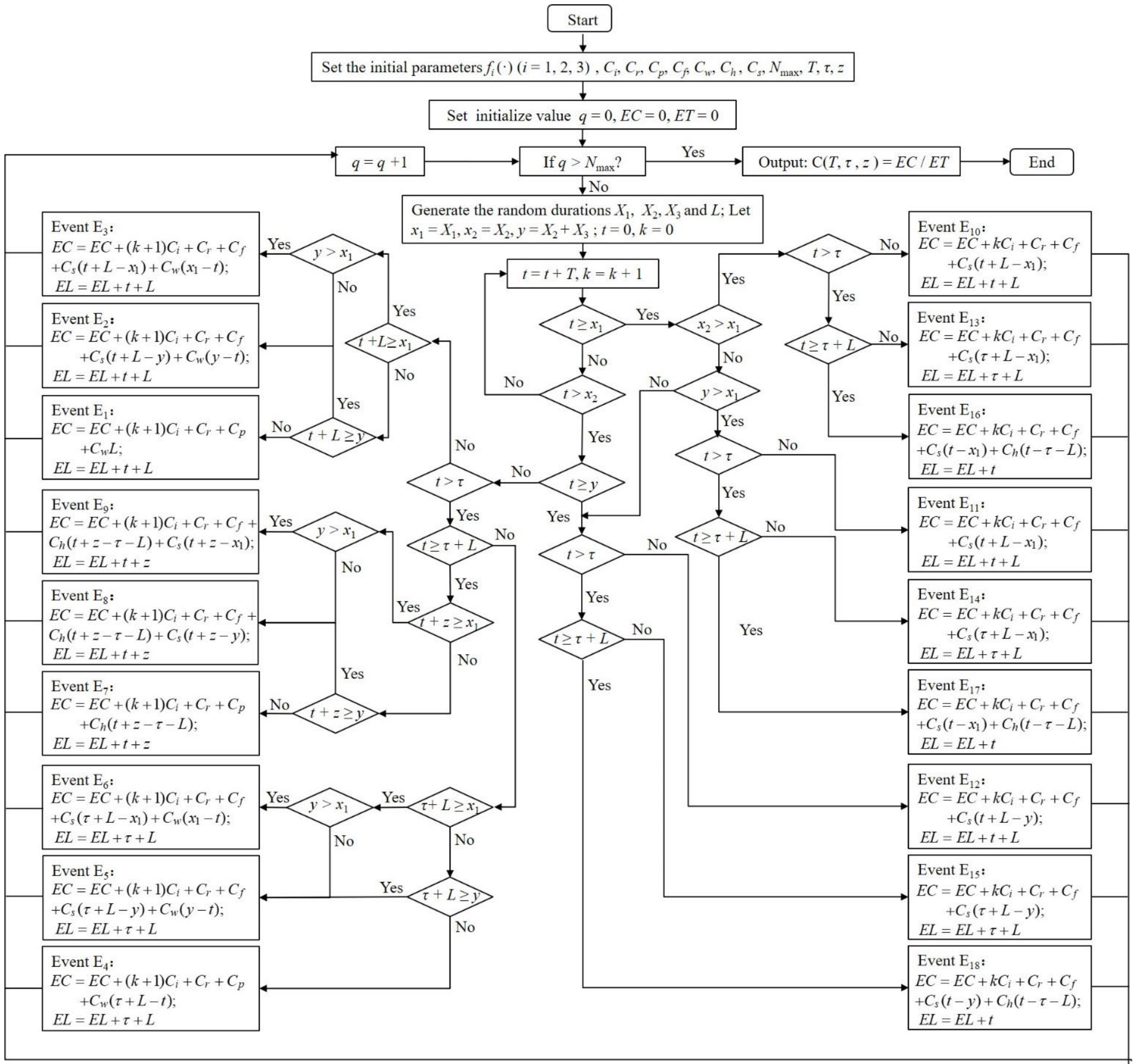

FIGURE 11. The simulation procedure.

the number of inspections in a renewal cycle, respectively. Then as the simulation goes: (a) a hard failure may occur before an inspection at t since $t \geq x_1$, if $x_2 > x_1$, implying that the degradation process is in the normal state when a fatal shock arrives at the system; otherwise, if $y > x_1$, indicating that the degradation process is in the defective state when the CR caused by a hard failure is required at t; (b) however, if the conditions $t \geq x_1$ and $y \leq x_1$ are satisfied, meaning that a soft failure occurs before the fatal shock arrives at the system so that the CR due to soft failure needs to be carried out at t; (c) if the normal state is detected in the case of $t < x_1$, then the state of the system in the soft failures mode needs to further identify by condition $t > x_2$. If it is rejected, the system is detected at the next inspection time; otherwise, two possible scenarios are considered as follows: the first is the degradation process is in the defective state in the case of $t < y$ so that a PR is required at t; the second is the degradation process is in the failed state in the case of $t \geq y$ and a CR is required at t. (d) when the replacement is needed at t, the state of the spare is judged by the case of $t > \tau$ or $t \geq \tau + L$. According to the decisions shown in Table 1, if $t \leq \tau$, an order is placed at t and a delayed PR/CR is carried out at the arrival time of the spare $t+L$; otherwise, the PR/CR is delayed to $\tau + L$ in the case of $t < \tau + L$, or, the PR is postponed to $t + z$ and the CR carried out immediately at t in the case of $t \geq \tau + L$. (e) since the system keeps working when the PR is required at t, let PRT represents the time of the replacement is performed, the state of the system at PRT should be further determined. Similar to the above discussion, there are also three situations: the first is the system still be in the defective state in the cases of PRT $< x_1$ and PRT $< y$ so that a PR is carried out at PRT, as stated in events E_1, E_4, and E_7; the second is a hard failure result in the system fail in the cases of PRT $\geq x_1$ and $y > x_1$, and a CR is executed at PRT, as described in events E_3, E_6, and E_9; the last is a soft failure result in the system fail in the cases of PRT $\geq x_1$ and $y \leq x_1$, or PRT $< x_1$ and PRT $\geq y$, therefore, the CR is performed at PRT, as stated in events E_2, E_5, and E_8. Finally, we obtain the expected cost rate $C(T, \tau, z)$ under the different combinations of T, τ, and z, which is calculated as the total cost EC divided by the length EL in the simulation process.

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MENGYING HAN was born in Shijiazhuang, Hebei, China, in 1990. She received the M.S. and Ph.D. degrees in management from the University of Science and Technology Beijing, China, in 2016 and 2021, respectively. She is currently a Lecturer in management science with the Hebei University of Economics and Business. Her research interests include reliability for complex systems, management decision analysis, and supply chain management.

ZHICHAO MA was born in Zhoukou, Henan, China. He received the M.S. and Ph.D. degrees in management from the University of Science and Technology Beijing, China, in 2012 and 2016, respectively. He is currently a Lecturer and a Master's Tutor in logistics engineering and management with the Hebei University of Engineering. His research interests include operations and supply chain management.

SHUGANG MA was born in Xingtai, Hebei, China. He received the B.E. and M.E. degrees in computer science from Yanshan University and the Ph.D. degree in management from the University of Science and Technology Beijing, China. He is currently an Associate Professor and a Master's Advisor in management science with the Hebei University of Economics and Business. His research interests include digital manufacture and supply chain management.

WEI LI is currently an Associate Professor with the School of Business, Hebei University of Economics and Business. His research interests include electronic commerce, digital economics, and business intelligence.

JIANHUA YANG was born in Weifang, Shandong, China. He received the B.E., M.E., and Ph.D. degrees in management from the Nanjing University of Science and Technology, China, in 1990 and 2002, respectively.

He is currently a Professor and a Ph.D. Advisor in management science and engineering with the University of Science and Technology Beijing. His research interests include operations and supply chain management, big data application and management decision analysis, security engineering and management, quality management, and service science.