IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 28 January 2023, accepted 27 March 2023, date of publication 3 April 2023, date of current version 6 April 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3263827

RESEARCH ARTICLE

Optimizing Operations Sequencing and Demulsifier Injection in Offshore Oil Production Platforms

EDUARDO RAUH MÜLLER[1], EDUARDO CAMPONOGARA[1], (Member, IEEE), LAIO ORIEL SEMAN[1,2], BRUNO FERREIRA VIEIRA[3], AND LUIS KIN MIYATAKE[3]
[1]Department of Automation and Systems Engineering, Federal University of Santa Catarina, Florianópolis 88040-900, Brazil
[2]Graduate Program in Applied Computer Science, University of Vale do Itajaí, Itajaí 21941-915, Brazil
[3]Petrobras Research Center (CENPES), Rio de Janeiro 20080-003, Brazil

Corresponding author: Laio Oriel Seman (laio@univali.br)

This work was supported in part by the Petróleo Brasileiro S.A. under Grant 2018/00217-3.

ABSTRACT Short-term production optimization of oil assets concerns routing decisions and equipment control settings that induce optimal steady-state operations. Typically, the decisions are made on the scale of hours to days with the aim of maximizing short-term economic and oil gains. Still an evolving field, this paper contributes by presenting a model and solution strategy for sequencing operations over time to drive the system towards an optimal state, while accounting for constraints that ensure smooth transitions until settling at the terminal steady-state. The constraints include the maximum number of wells that undergo a change in the control settings, maximum number and variation on the input signals for each well, and maximum number of changes in each time period, among others. Additionally, the proposed strategy can optimize the rate of injection of demulsifiers into wells considering the limits and impacts on fluid processing capacity. The paper reports simulated results for a real-world offshore production platform, demonstrating the effectiveness of the optimization strategy to cope with the failure and recovery of compression capacity, and to manage the injection of demulsifiers. The experiments show a gain of 11.5% in total oil production resulting from an optimal management of demulsifiers.

INDEX TERMS Production optimization, operations sequencing, demulsifier injection, nonlinear optimization, piecewise-linear approximation.

I. INTRODUCTION

Oil production optimization (OPO) concerns the design, planning, and operation of oil production assets to drive economic gains while accounting for operational and regulatory constraints, such as environmental impact. As such, OPO is a broad area attracting the interest of the service industry, operators and scientists alike for several decades. The growing complexity from isolated oil wells, through multiple wells sharing resources of a production platform, to multi-reservoir oil fields led the technical community to break OPO into layers of problems, known as the multilevel control hierarchy for oil production systems [1]. The top layer (Asset Management) addresses the long-term decision in a scale of years, including whether or not to drill a new well or install a new pipeline. The second layer (Reservoir Management) regards the decisions in the scale of months to years, typically relying on reservoir simulation models to propose a schedule of well controls that optimize long-term gains and oil recovery. The third layer (*Short-Term* Production Optimization) operates with decisions in the scale of hours to days, also known as daily production optimization, which concerns routing decisions and equipment control settings that define optimal steady state operations, while taking into account the system constraints. The fourth layer (Control and Automation) relies on the automation and instrumentation infrastructure

The associate editor coordinating the review of this manuscript and approving it for publication was Md. Abdur Razzaque.

to control the processes, performing trajectory tracking and rejecting disturbances that keep the system operating at the steady-state settings.

This present work addresses a practical problem in the short-term production optimization (STPO) layer, which regards the optimization of the sequence of operations that drive the production systems from the current operations conditions to optimal operating settings (steady state). Traditional problems and solution methodologies from the STPO layer are concerned with the computation of an optimal steady-state for a production system, such as an offshore production platform, considering a variety of configurations, constraints, and objectives. [2] framed the optimization of multiphase flow networks in graphs composed of wells, flows lines, risers and separators. They considered discrete decisions, such as routing and well action decisions, and nonlinear constraints involving pressure and temperature that led to a mixed-integer nonlinear programming (MINLP) formulation, which was solved with a spatial branch-and-bound algorithm. Several works in the literature address related STPO problems formalized as MINLPs, but instead rely on piecewise-linear approximation to render an MILPs that can be more robustly solved. Following piecewise-linear approximation, [3] reported results from an application of the to the Troll West Oil Field, in Norway, [4] proposed a formulation for computing the frequencies of electrical submersible pumps (ESP) to maximize oil production, and [5] presented a methodology to maximize oil production of large offshore oilfields, while coordinating the production of several platforms that share a subsea gas-pipeline network. Based on an extensive comparison between MINLP and MILP formulation for STPO, [6] proposed hybrid two-stage optimization strategy for maximizing the oil production subject system and environmental constraints. Reference [7] developed a generalized formulation for STPO to account complex production networks, well with dual completion that can operate with gas-lift injection and ESP, and flow assurance constraints to avoid hydrate formation.

Other works factored in uncertainty in modeling and measurements. Reference [8] reformulated an uncertain STPO problem in terms of an optimization problem following the column-wise and row-wise frameworks, with cardinality-constrained sets. These sets allow the operator to regulate the level of protection of a solution against model uncertainty, for instance in gas to oil ratio (GOR) and basic sediment and water (BSW). Reference [9] developed a stochastic model for STPO in offshore platforms that operate with satellite gas-lifted wells. Recourse actions were considered to meet system constraints for the uncertainty realizations of the stochastic variables. Reference [10] modeled the well-test data using sampling and regression techniques to account for uncertainties. The proposed optimization methodology produced stochastic solutions that outperformed their deterministic counterparts, yielding up to 4.5% gains in a field study considering the uncertainty in BSW.

To the best of our knowledge, few works have addressed the problem of sequencing operations over time to drive the production platform to a target steady-state, while considering major operations, such as well shut-in, well start-up, and changes in routing operations. The situation is not akin to the control problem entrusted to the control and automation layer, which concern the rejection of perturbations to keep the system operating at the steady-state, and trajectory tracking without structural system change. For instance, [11] presented an extremum-seeking controller to keep oil production of a gas-lifted well around the optimum point, which applies periodic perturbations into the process to obtain gradient information on the well-performance curve. Combining optimization and control, [12] reported results of multi-objective dynamic optimization applied to a gas-lift well to drive the well to an optimal steady state. Reference [13] developed a nonlinear model-based controller for a production platform operating gas-lifted oil wells, which performs dynamic linearization along the predicted trajectory in a manner akin, but simpler than sequential quadratic programming.

A key issue in operations sequencing is whether or not the dynamics should be factored into the problem, a topic that was addressed by [14] that stems from the practice of production optimization and reported case studies. Their findings support that most production optimization problems can be solved using steady-state models and static optimization methods, except in operation involving transients (cyclic behavior found in slug flow) and when reservoir dynamics is fast (shale gas formations). A chief hurdle to integrating dynamics into short-term production optimization include the computational difficulty of solving large nonlinear problems involving discrete decisions, the scales of time that can vary from seconds to hours, and the complexity of keeping models updated. Acknowledging these challenges, [15] proposed proxy models of well dynamics in the context of shut-in and start-up operations for well over a planning horizon, which are needed in the event of compressor failure and later recovery. Despite the use of proxy dynamic models, the identification, synthesis and maintenance of proxy well dynamic models was deemed costly for practical application by production engineers of the Brazilian Oil Company (Petrobras). If the proxy models are not sufficiently precise, the end results can deviate significantly from an optimal trajectory. Such difficulties motivated the research reported in this paper, which was pursued in close collaboration with Petrobras to deliver a simple model to serve as a support-decision tool for operations sequencing in STPO.

This present work offers the following contributions:

- A mixed-integer nonlinear programming (MINLP) formulation and mixed-integer linear programming (MILP) approximation for sequencing operations over a planning horizon, aiming to drive a production system from the current steady-state to an optimal steady-state (SS), while accounting for constraints that ensure a smooth transition from each SS to next until reaching the final SS.

- The modeling of demulsifier injection for improving the separation of water from crude oil emulsions, which can significantly impact the oil production in mature oil fields.
- Case studies demonstrating the potential of the proposed formulations for operations sequencing in the context of synthetic oil fields, which elicit the behaviors which are affected by parameters that impose a smooth transition from the current steady-state to the next along the planning horizon.

II. PROBLEM STATEMENT

The problem at hand is that of short-term planning of the operations in an offshore oil platform, in the scale from hours to days, according to the multilevel control hierarchy for oil production systems [1]. Recently, [7] advanced the technology for static optimization of offshore oil platforms by introducing features that are key in practice, such as the modeling of flow assurance constraints and dual completion (GLC and ESP), which yields optimal settings for the control variables to maximize oil production. However, that formulation does not consider the time dimension, which becomes relevant even for static models when operating conditions cannot be changed at once. Platform operators often prefer to act in a series of small steps to take the platform from the current operational condition to another to ensure stability and deal with unforeseen consequences of such changes.

In this work, we are concerned with finding an optimal operating point for an oil platform, while selecting a series of control inputs that drive the platform to reach such an operating point optimally, in a series of steps. Each of these steps represents a settling operation point for the system. We assume that the control automation layer [1] is responsible for taking the system from each settling point to the next, until reaching the optimal operating point at the end of the planning horizon. This means that the magnitude and number of changes in the control signals, within each time step, are sufficiently small for the system to reach a stable operation. This is not a fully automated system though. After each step is applied, operators are able to monitor the transients and assess the situation and decide whether or not to proceed with the next step. As such, the proposed methodology serves as a decision-support tool for the operators.

Motivated by practical applications, the proposed formulation additionally manages the injection of demulsifier agents into wells as a means to improve production. The decisions regarding the injection can become complex when a platform operates near capacity, such as water and liquid handling.

We consider the static model of a single oil platform, with multiple satellite wells that operate with lift-gas injection as depicted in Figure 1. This choice of scope is made to simplify the research and presentation of the model in a more succinct manner. It is possible to combine the methodology for operations sequence optimization with a more general scenario that considers multiple platforms, wells with manifolds, naturally flowing wells, flow assurance constraints, and other forms of artificial lifting such as the use of an electrical submersible pump (ESP) [7].

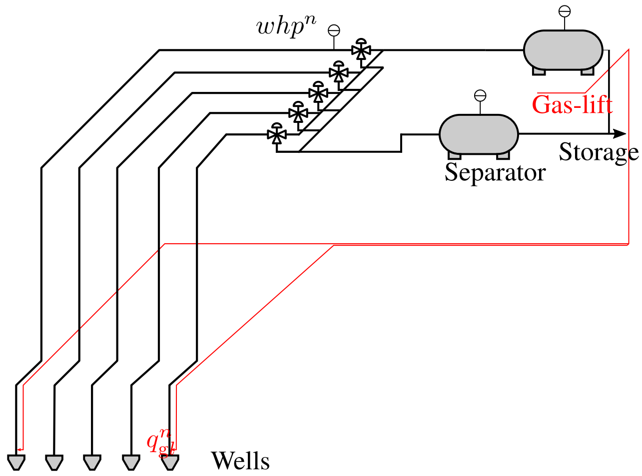


FIGURE 1. **Platform scheme. Wells (shown at the bottom) produce a mixture of oil, water and gas that is lifted through pipelines and can be routed to either separator. Individual well measurements for these flows are not available. whp^n is the wellhead pressure of well n. q^n_{gl} is the flow of lift-gas injected into well n.**

III. PROBLEM FORMULATION

Section III-A presents the model in a conceptual manner, in which, for brevity, well production curves are treated as non linear functions which are assumed to be known, and details regarding approximations are omitted. In section III-C we illustrate the procedure used to approximate the production curves with piecewise-linear functions built from data gathered in field tests and simulations.

A. CONCEPTUAL FORMULATION

Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of wells, in which each well n is characterized by a vector of control inputs $\theta^n[k] = (q^n_{\mathtt{gl}}[k], whp^n[k])$, and a vector of outputs $\gamma^n[k] = (q^n_{\mathtt{o}}[k], q^n_{\mathtt{g}}[k], q^n_{\mathtt{w}}[k])$, over time. The control inputs for each well n at time step k are the lift-gas injection flow $q^n_{\mathtt{gl}}[k]$ and wellhead pressure $whp^n[k]$, while the outputs are the flows of oil $q^n_{\mathtt{o}}[k]$, gas $q^n_{\mathtt{g}}[k]$, and water $q^n_{\mathtt{w}}[k]$. The output vector $\gamma^n[k]$ is a function of the inputs, and can be written as:

$$q^n_{\mathtt{o}}[k] = \widehat{q}^n_{\mathtt{o}}(q^n_{\mathtt{gl}}[k], whp^n[k]) \tag{1a}$$

$$q^n_{\mathtt{w}}[k] = \frac{BSW^n}{1 - BSW^n} q^n_{\mathtt{o}}[k] \tag{1b}$$

$$q^n_{\mathtt{g}}[k] = GOR^n q^n_{\mathtt{o}}[k] \tag{1c}$$

where GOR^n is the gas-oil ratio and $BSW^n \in [0, 1]$ is the *Basic Sediment and Water* that characterize the composition of the mixture of oil, gas, and water produced by well n. $\widehat{q}^n_{\mathtt{o}}(\cdot)$ is a function that maps the control inputs to oil production, and is generally unknown in explicit form but typically available in simulation models.

We assume that at time t_0 the platform is operating at a steady-state point defined by initial conditions

TABLE 1. List of sets.

Set	Description
General Sets:	
$\mathcal{N}$	Set of all wells in the platform
$\mathcal{N}_{\texttt{dss}}$	Set of wells with equipment for demulsifier injection
$\mathcal{N}^{\dagger}_{\texttt{dss}}$	subset of $\mathcal{N}_{\texttt{dss}}$ containing wells for which the injection flow of demulsifier is known.
$\mathcal{N}^{*}_{\texttt{dss}}$	subset of $\mathcal{N}_{\texttt{dss}}$ containing wells for which the injection flow of the demulsifier is estimated as a fraction of the liquid produced.
$\mathcal{K}$	Set of time periods in the control horizon.
$\mathcal{T}$	Set of time instants in the control horizon.
Sets for Piecewise-Linear Approximation:	
$\mathcal{R}^n$	Set of breakpoints (p, q) of well-head pressure and lift-gas injection for well n.
$\mathcal{P}^n_{\texttt{wh}}$	Set of breakpoints with well-head pressure for well n.
$\mathcal{Q}^n_{\texttt{gl}}$	Set of breakpoints with lift-gas injection for well n.

$\theta[0] = (\theta^n[0] : n \in \mathcal{N})$, which produces the outputs $\gamma[0] = (\gamma^n[0] : n \in \mathcal{N})$. Given a control horizon $\mathcal{K} = \{1, \ldots, K\}$, corresponding to time instants $\mathcal{T} = \{t_1, \ldots, t_K\}$, our goal is to find a sequence of control signals $\Theta = (\theta(1), \ldots, \theta(n))$ which leads to the maximum oil production in the platform, while conforming to physical and operational constraints, as well given bounds in the control variables and their rates of change along the time horizon.

Tables 1, 2 and 3 present respectively the sets, parameters and variables that characterize the formulation for the operations sequencing problem in oil platforms. The baseline model for the operations sequence optimization can be formulated as follows:

$$\max_{\Psi} f(\Psi) \tag{2a}$$

while, for all $n \in \mathcal{N}$, $k \in \mathcal{K}$, being subject to:

$$q^n_{\texttt{oil}}[k] = \begin{cases} \Big[\widehat{q}^n_{\texttt{oil}}\big(q^n_{\texttt{gl}}[k], whp^n[k]\big) \cdot y^n_{\texttt{std}}[k] \\ \quad + \widehat{q}^n_{\texttt{oil,dss}}\big(q^n_{\texttt{gl}}[k], whp^n[k]\big) \cdot y^n_{\texttt{dss}}[k]\Big] \cdot t^n[k] \\ \qquad \text{if } n \in \mathcal{N}_{\texttt{dss}} \\ \widehat{q}^n_{\texttt{oil}}[k]\big(q^n_{\texttt{gl}}[k], whp^n[k]\big) \cdot t^n[k] \\ \qquad \text{if } n \in \mathcal{N} \setminus \mathcal{N}_{\texttt{dss}} \end{cases} \tag{2b}$$

$$\begin{cases} q^n_{\texttt{liq}}[k] = BSW^n q^n_{\texttt{oil}}[k] \\ q^n_{\texttt{w}}[k] = \dfrac{BSW^n}{1 - BSW^n} q^n_{\texttt{o}}[k] \\ q^n_{\texttt{g}}[k] = GOR^n q^n_{\texttt{o}}[k] \end{cases} \tag{2c}$$

$$\begin{cases} whp^{n,\min}[k] \cdot t^n[k] \le whp^n[k] \le whp^{n,\max}[k] \cdot t^n[k] \\ q^{n,\min}_{\texttt{gl}}[k] \cdot t^n[k] \le q^n_{\texttt{gl}}[k] \le q^{n,\max}_{\texttt{gl}}[k] \cdot t^n[k] \end{cases} \tag{2d}$$

$$\begin{cases} t^n[k] = y^n_{\texttt{std}}[k] + y^n_{\texttt{dss}}[k] \\ y^n_{\texttt{std}}[k] + y^n_{\texttt{dss}}[k] \le 1 \end{cases} \tag{2e}$$

$$t^n[k], \quad y^n_{\texttt{std}}[k], \; y^n_{\texttt{dss}}[k] \in \{0, 1\} \tag{2f}$$

$$q^n_{\texttt{dss}}[k] = \begin{cases} dssr^n \cdot q^n_{\texttt{liq}}[k] \cdot y^n_{\texttt{dss}}[k], & \text{if } n \in \mathcal{N}^*_{\texttt{dss}} \\ q^{n,\dagger}_{\texttt{dss}} \cdot y^n_{\texttt{dss}}[k], & \text{if } n \in \mathcal{N}^{\dagger}_{\texttt{dss}} \end{cases} \tag{2g}$$

$$\begin{cases} \Delta q^n_{\texttt{gl}}[k] = q^n_{\texttt{gl}}[k] - q^n_{\texttt{gl}}(k-1), \\ \Delta q^n_{\texttt{gl}}[k] = \Delta q^{n,+}_{\texttt{gl}}[k] - \Delta q^{n,-}_{\texttt{gl}}[k], \\ \delta^{n,+}_{\texttt{gl}}[k] \Delta q^{\min}_{\texttt{gl}} \le \Delta q^{n,+}_{\texttt{gl}}[k] \le \delta^{n,+}_{\texttt{gl}}[k] \Delta q^{\max}_{\texttt{gl}}, \\ \delta^{n,-}_{\texttt{gl}}[k] \Delta q^{\min}_{\texttt{gl}} \le \Delta q^{n,-}_{\texttt{gl}}[k] \le \delta^{n,-}_{\texttt{gl}}[k] \Delta q^{\max}_{\texttt{gl}}, \\ \Delta q^{n,+}_{\texttt{gl}}[k], \Delta q^{n,-}_{\texttt{gl}}[k] \ge 0, \\ \delta^{n,+}_{\texttt{gl}}[k], \delta^{n,-}_{\texttt{gl}}[k] \in \{0, 1\}, \end{cases} \tag{2h}$$

$$\begin{cases} \Delta whp^n[k] = whp^n[k] - whp^n(k-1), \\ \Delta whp^n[k] = \Delta whp^{n,+}[k] - \Delta whp^{n,-}[k] \\ \delta^{n,+}_{\texttt{whp}}[k] \Delta whp^{\min} \le \Delta whp^{n,+}[k] \\ \qquad \le \delta^{n,+}_{\texttt{whp}}[k] \Delta whp^{\max}, \\ \delta^{n,-}_{\texttt{whp}}[k] \Delta whp^{\min} \le \Delta whp^{n,-}[k] \\ \qquad \le \delta^{n,-}_{\texttt{whp}}[k] \Delta whp^{\max}, \\ \Delta whp^{n,+}[k], \Delta whp^{n,-}[k] \ge 0, \\ \delta^{n,+}_{\texttt{whp}}[k], \delta^{n,-}_{\texttt{whp}}[k] \in \{0, 1\}, \end{cases} \tag{2i}$$

$$\begin{cases} \delta^n[k] \ge \delta^{n,-}_{\texttt{whp}}[k] + \delta^{n,+}_{\texttt{whp}}[k], \\ \delta^n[k] \ge \delta^{n,-}_{\texttt{gl}}[k] + \delta^{n,+}_{\texttt{gl}}[k], \\ \delta^n[k] \le \big(\delta^{n,-}_{\texttt{whp}}[k] + \delta^{n,+}_{\texttt{whp}}[k]\big) \\ \qquad + \big(\delta^{n,-}_{\texttt{gl}}[k] + \delta^{n,+}_{\texttt{gl}}[k]\big), \\ \delta^n[k] \in \{0, 1\} \end{cases} \tag{2j}$$

for all $n \in \mathcal{N}$, subject to:

$$\Big\{ \big(q^n_{\texttt{gl}}[0], whp^n[0]\big) = \theta^n[0] \tag{2k}$$

$$\begin{cases} \sum_{k \in \mathcal{K}} \delta^n[k] \le \delta^{\max}_{\texttt{well}} \delta^n, \\ \delta^n \in \{0, 1\} \end{cases} \tag{2l}$$

TABLE 2. List of parameters and functions.

Parameter	Description
Parameters and Functions of Well n:	
$\widehat{q}^n_{\mathrm{o}}(\cdot)$	Production curve that maps control inputs to oil production in well n when there is no injection of dss
$\widehat{q}^n_{\mathtt{oil,dss}}(\cdot)$	Production curve that maps control inputs to oil production in well n when dss is injected
BSW^n	Base Sediment and Water ratio. The ratio between water and total liquid produced
GOR^n	Gas-Oil Ratio. The ratio between gas and oil produced
$whp^{n,\min}[k]$	Lower bound for the wellhead pressure of well n at period k
$whp^{n,\max}[k]$	Upper bound for the wellhead pressure of well n at period k
$q^{n,\min}_{\mathtt{gl}}[k]$	Lower bound for the lift-gas injection flow of well n at period k
$q^{n,\max}_{\mathtt{gl}}[k]$	Upper bound for the lift-gas injection flow of well n at period k
$dssr^n$	The ratio between liquid produced and dss injected in well n, if dss is injected
$q^{n,\dagger}_{\mathtt{dss}}$	The flow of dss injected in well n, if dss is injected
$q^{\max}_{\mathtt{dss}}$	A large constant needed for the Big-M strategy applied to the dss injection
Platform Parameters:	
$q^{\max}_{\mathtt{liq}}[k]$	Liquid processing capacity of the platform for period k
$q^{\max}_{\mathtt{w}}[k]$	Water processing capacity of the platform for period k
$q^{\max}_{\mathtt{exp}}[k]$	Gas export capacity of the platform for period k
$q^{\min}_{\mathtt{flare}}[k]$	Minimum allowed flared gas of the platform for period k
$q^{\max}_{\mathtt{flare}}[k]$	Maximum allowed flared gas of the platform for period k
$q_{\mathtt{gtc}}[k]$	Gas compression capacity of the platform for period k
$q_{\mathtt{turbine}}$	Fuel (gas) needed to produce energy for the platform
Operations Sequence Parameters:	
$\delta^{\max}_{\mathtt{well}}$	Maximum number of operations allowed in a well
$\delta^{\max}_{\mathtt{period}}$	Maximum number of operations allowed in a time period
$\delta^{\max}_{\mathtt{all-wells}}$	Maximum number of allowed operations in total
$\delta^{\max}_{\mathtt{sys}}$	Maximum number of wells that can be subject to operations

and for all $k \in \mathcal{K}$, constrained by:

$$\begin{cases} \sum_{n\in\mathcal{N}} \left(q^n_{\mathtt{o}}[k] + q^n_{\mathtt{w}}[k]\right) \leq q^{\max}_{\mathtt{liq}}[k] \\ \sum_{n\in\mathcal{N}} q^n_{\mathtt{w}}[k] \leq q^{\max}_{\mathtt{w}}[k], \\ q^{\mathrm{total}}_{\mathtt{dss}}[k] = \sum_{n\in\mathcal{N}} q^n_{\mathtt{dss}}[k] \\ q_{\mathtt{exp}}[k] \leq q^{\max}_{\mathtt{exp}}[k] \\ q^{\min}_{\mathtt{flare}}[k] \leq q_{\mathtt{flare}}[k] \leq q^{\max}_{\mathtt{flare}}[k] \\ q_{\mathtt{gas-prod}}[k] = \sum_{n\in\mathcal{N}} q^n_{\mathtt{g}}[k], \\ q_{\mathtt{exp}}[k] = q_{\mathtt{gas-prod}}[k] - q_{\mathtt{flare}}[k] - q_{\mathtt{turbine}}, \\ q_{\mathtt{gas-lift}}[k] = \sum_{n\in\mathcal{N}} q^n_{\mathtt{gl}}[k], \\ q_{\mathtt{gas-prod}}[k] + q_{\mathtt{gas-lift}}[k] - q_{\mathtt{flare}}[k] \leq q_{\mathtt{gtc}}[k] \end{cases} \tag{2m}$$

$$\begin{cases} \sum_{n\in\mathcal{N}} \delta^n[k] \leq \delta^{\max}_{\mathtt{period}} \end{cases} \tag{2n}$$

$$\sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} \delta^n[k] \leq \delta^{\max}_{\mathtt{all-wells}}, \tag{2o}$$

$$\sum_{n\in\mathcal{N}} \delta^n \leq \delta^{\max}_{\mathtt{sys}} \tag{2p}$$

where:

- $dssr^n$ is an estimative for the ratio of injected demulsifier to the total liquid produced in a well. Usually, this is in the order of 100 parts per million (10^{-4}).
- $\mathcal{N}_{\mathtt{dss}} \subseteq \mathcal{N}$ is the subset of wells that are equipped for injection of a demulsifier which can improve the flow and thereby production. $\mathcal{N}_{\mathtt{dss}} = \mathcal{N}^*_{\mathtt{dss}} \cup \mathcal{N}^\dagger_{\mathtt{dss}}$ is partitioned in two disjunct sets, where $\mathcal{N}^*_{\mathtt{dss}}$ are the wells with a rate of demulsifier proportional to the liquid production, whereas $\mathcal{N}^\dagger_{\mathtt{dss}}$ has the well with a fixed rate regardless of the production.
- $t^n[k]$ is a binary variable that assumes the value 1 if well n is operating in time instant t_k, and is 0 otherwise.
- $y^n_{\mathtt{dss}}[k]$ is a binary variable that indicates if demulsifier is injected in well n at period k, and $y^n_{\mathtt{std}}[k]$ is binary variable that indicates the well is operating in standard mode.
- $\delta^{\max}_{\mathtt{well}}$ is the maximum number of allowed operations (changes in control inputs) that can be applied to a well along time horizon $\mathcal{K}$.
- $\delta^n[k]$ is a binary variable that signals if a change of control inputs (either wellhead pressure whp^n or lift gas injection flow $q^n_{\mathtt{gl}}$) is performed in well n at time instant t_k.
- δ^n is a binary variable that indicates if at least one adjustment in the control inputs is performed in well n, and 0 otherwise. Note that $\delta^n[k]$ can take the value 1, in whatever period k, only if $\delta^n = 1$.
- $\Delta whp^n[k]$ and $\Delta q^n_{\mathtt{gl}}[k]$ are, respectively, the variation in wellhead pressure and gas-lift injection rate implemented in the well n, during the period k, which occurs only if $\delta^n[k] = 1$.
- $\delta^{\max}_{\mathtt{period}}$ is the maximum number of allowed adjustments in the control signals, during any period k of the planning horizon.

TABLE 3. List of variables.

Variable	Description
Variables for Well n:	
$\theta^n[k]$	Input vector of well n. $\theta^n[k] = (q^n_{\mathtt{gl}}[k], whp^n[k])$
$q^n_{\mathtt{gl}}[k]$	lift-gas injection flow of well n in time k $[Nm^3/\mathtt{day}]$
$whp^n[k]$	wellhead pressure of well n in time k $[\mathtt{kgf/cm^2}]$
$\gamma^n[k]$	Output vector of well n. $\gamma^n[k] = (q^n_{\mathtt{o}}[k], q^n_{\mathtt{g}}[k], q^n_{\mathtt{w}}[k])$
$q^n_{\mathtt{o}}[k]$	oil flow of well n in time k $[m^3/\mathtt{day}]$
$q^n_{\mathtt{g}}[k]$	water flow of well n in time k $[m^3/\mathtt{day}]$
$q^n_{\mathtt{w}}[k]$	gas flow of well n in time k $[Nm^3/\mathtt{day}]$
$q^n_{\mathtt{liq}}[k]$	liquid flow from well n in time k $[m^3/\mathtt{day}]$
$q^n_{\mathtt{dss}}[k]$	flow of demulsifier from well n in time k $[m^3/\mathtt{day}]$
$t^n[k]$	Binary variable that indicates if well n is active at period k
$y^n_{\mathtt{dss}}[k]$	Binary variable that indicates if demulsifier is injected in well n at period k
$y^n_{\mathtt{std}}[k]$	Binary variable that indicates if well n is operating in standard mode at period k
$\delta^n[k]$	Binary variable that indicates if well n is subject to a change in control inputs between times $k-1$ and k
$\Delta q^n_{\mathtt{gl}}[k]$	Change in $q^n_{\mathtt{gl}}[k]$ between time periods $k-1$ and k
$\Delta q^{n,-}_{\mathtt{gl}}[k]$	Auxiliary variable for modeling the positive component of $\Delta q^n_{\mathtt{gl}}[k]$
$\Delta q^{n,+}_{\mathtt{gl}}[k]$	Auxiliary variable for modeling the negative component of $\Delta q^n_{\mathtt{gl}}[k]$
$\delta^{n,-}_{\mathtt{gl}}[k]$	Auxiliary binary variable that signals if there is an decrease of $q^n_{\mathtt{gl}}[k]$ between time periods $k-1$ and k
$\delta^{n,+}_{\mathtt{gl}}[k]$	Auxiliary binary variable that signals if there is an increase of $q^n_{\mathtt{gl}}[k]$ between time periods $k-1$ and k
$\Delta whp^n[k]$	Change in $whp^n[k]$ between time periods $k-1$ and k
$\Delta whp^{n,-}[k]$	Auxiliary variable for modeling the positive component of $\Delta whp^n[k]$
$\Delta whp^{n,*}[k]$	Auxiliary variable for modeling the negative component of $\Delta whp^n[k]$
$\delta^{n,-}_{\mathtt{whp}}[k]$	Auxiliary binary variable that signals if there is an decrease of $whp^n[k]$ between time periods $k-1$ and k
$\delta^{n,+}_{\mathtt{whp}}[k]$	Auxiliary binary variable that signals if there is an increase of $whp^n[k]$ between time periods $k-1$ and k
$\mu^{n,\mathtt{i}}_{p,q}[k]$	Auxiliary variable used in the convex combinations of the employed linearization, with $i \in \{\mathtt{std}, \mathtt{dss}\}$
$\kappa^{n,\mathtt{i}}_{p}[k]$	Auxiliary variable used in the convex combinations of the employed linearization considering the well-head pressure breakpoints p, with $i \in \{\mathtt{std}, \mathtt{dss}\}$
$\kappa^{n,\mathtt{i}}_{q}[k]$	Auxiliary variable used in the convex combinations of the employed linearization considering the gas lift breakpoints q, with $i \in \{\mathtt{std}, \mathtt{dss}\}$
Platform Variables:	
$q^{\text{total}}_{\mathtt{dss}}[k]$	Total flow of dss used in the platform in time k $[m^3/\mathtt{day}]$
$q_{\mathtt{exp}}[k]$	Gas export flow during time k $[Nm^3/\mathtt{day}]$
$q_{\mathtt{flare}}[k]$	Flow of gas being flared during time k $[Nm^3/\mathtt{day}]$
$q_{\mathtt{gas-prod}}[k]$	Total gas production rate during time k $[Nm^3/\mathtt{day}]$
$q_{\mathtt{gas-lift}}[k]$	Total flow of lift-gas during time k $[Nm^3/\mathtt{day}]$

- $\delta^{\max}_{\mathtt{all-wells}}$ is the maximum number allowed for adjustments in the control signals, considering all wells and time intervals along the planning horizon.
- $\delta^{\max}_{\mathtt{sys}}$ is the maximum number of wells that can be subject to adjustments in the control signals.
- $\Delta whp^{\max}$ ($\Delta whp^{\min}$) is the maximum (minimum) value of variation in wellhead pressure to be implemented in a well n, if an adjustment performed during the period k, which is indicated by $\delta^n[k] = 1$. $\Delta q^{\max}_{\mathtt{gl}}$ ($\Delta q^{\min}_{\mathtt{gl}}$) are analogous for the lift-gas injection flow.
- $whp^{n,\min}[k]$ and $whp^{n,\max}[k]$ are the lower and upper bounds for the wellhead pressure of well n at period k. $q^{n,\min}_{\mathtt{gl}}[k]$ and $q^{n,\min}_{\mathtt{gl}}[k]$ are analogous for the lift-gas injection flow.
- $q_{\mathtt{turbine}}$ is the gas consumed as fuel to power the platform.
- $q_{\mathtt{gas-prod}}[k]$ is the total gas produced by all wells at period k.
- $q_{\mathtt{flare}}[k]$ is the gas flared (burned) in the platform at period k.
- $q_{\mathtt{exp}}[k]$ is the (produced) gas exported by the platform at period k., after discounting gas flared or consumed.
- $q_{\mathtt{gas-lift}}[k]$ is the total flow of gas lift to all wells at period k.
- Considering the possibility of variation in the compression and processing capacities of the platform, either for unexpected or planned reasons, the model above assumes that several platform parameters can vary in the horizon of schedule $k = 1, \ldots, K$: Liquid processing capacity $q^{\max}_{\mathtt{liq}}[k]$; water processing capacity $q^{\max}_{\mathtt{w}}[k]$; gas export capacity $q^{\max}_{\mathtt{exp}}[k]$; minimum and maximum bounds for gas flaring $q^{\min}_{\mathtt{flare}}[k]$ and $q^{\max}_{\mathtt{flare}}[k]$; and gas compression capacity $q_{\mathtt{gtc}}[k]$.
- Ψ is a vector collecting all of the decision variables.

To promote understanding of the formulation above, we now give some brief explanation of the semantics for the constraints. Eq. (2b) defines the oil production stream $q^n_{\mathtt{o}}$ of well n depending on the decision regarding the injection of demulsifier, besides the control variables for well-head pressure and lift-gas injection rate. The streams of total liquid, water, and gas are obtained from the oil stream based on the process (GOR and BSW) parameters according to Eq. (2c). The bounds for well-head pressure and lift-gas injection rate are imposed on producing well by Eq. (2d). Eq. (2g) defines the rate of demulsifier injected in a well, depending on decision variables $y^n_{\mathtt{dss}}$ and whether the injection is pre-defined or varies proportionally to the liquid production. The magnitude of the allowed change in lift-gas injection rate $q^n_{\mathtt{gl}}[k]$ between consecutive time steps is bounded by Eq. 2h. Similarly, Eq. 2i bound the allowed change in $whp^n[k]$. Together with Eq.2j, these equations guarantee that $\delta^n[k]$ flags when an operation is performed in well n in time step k. Eq. (2k) implements the initial conditions, while Eq. (2l) constrains the number of operations a well can be subject to. Eq. (2m) models constraints related to the platform processing capacities. Finally, (2n), (2o) and (2p) model constraints related to the operations sequencing: the number of operations allowed in a time step, the total number of operations, and the maximum number of operations allowed for each well, respectively.

The formulation given in Eq. (2) is a Mixed-Integer Nonlinear Programming (MINLP) problem which, to be optimized, would need an explicit expression for the production functions $\widehat{q}^n_{\mathtt{oil}}$ and $\widehat{q}^n_{\mathtt{oil,dss}}$. Owing to the nonlinear nature of such functions, combined with discrete decisions to be made over time, renders the MINLP a challenging problem to be solved. Even finding feasible solutions may be not trivial when dealing with MINLP optimization. The alternative that has proven effective in practice consists in using a proxy model as discussed below.

B. OBJECTIVES IN OPERATIONS SEQUENCING

According to the experience of systems operators, the principal goal in optimizing operations sequencing is to drive the platform from the current state to a terminal state $\theta(K)$, within a horizon of K steps, that maximizes the steady-state production. It so happens that multiple control-input trajectories can take the platform form the initial condition to the optimal terminal conditions, giving rise to the opportunity to optimize other criteria along the trajectory and reduce the degrees of freedom. A strategy that proves very effective in practice when there is a clear order of priorities among different objectives is lexicographic optimization [16], in part because it spares the user from tuning multiple objectives, but also because the strategy relates to physical and economic figures.

The overall problem is solved by optimizing 9 different problems (stages) in sequence, and fixing the solution of each stage as constraints for the subsequent stages. First, this strategy considers the maximization of the following objectives on the *terminal state*:

1) Oil production at the terminal state: $f_1(\Psi) = \sum_{n\in\mathcal{N}} q^n_{\mathtt{o}}[K]$
2) Demulsifier injection: $f_2(\Psi) = -\sum_{n\in\mathcal{N}} q^n_{\mathtt{dss}}[K]$.
3) Flared gas: $f_3(\Psi) = -q^n_{\mathtt{flare}}[K]$
4) Lift-gas use: $f_4(\Psi) = -\sum_{n\in\mathcal{N}} q^n_{\mathtt{gl}}[K]$

After the first four optimization stages, the terminal state is well defined. The next step in the process is to find the minimum number of operations needed to achieve this state, which is modelled with the objective:

5)
$$f_5(\Psi) = -\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}\Big(\delta^{n,-}_{\mathtt{gl}}[k] + \delta^{n,+}_{\mathtt{gl}}[k] + \delta^{n,-}_{\mathtt{whp}}[k] + \delta^{n,+}_{\mathtt{whp}}[k]\Big)$$

Finally, the last four stages are analogous to the first four, but considering the entire time horizon instead of only the terminal state:

6) Oil production: $f_1(\Psi) = \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} q^n_{\mathtt{o}}[k]$
7) Demulsifier injection: $f_2(\Psi) = -\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} q^n_{\mathtt{dss}}[k]$.
8) Flared gas: $f_3(\Psi) = -\sum_{k\in\mathcal{K}} q^n_{\mathtt{flare}}[k]$.
9) Lift-gas use: $f_4(\Psi) = -\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} q^n_{\mathtt{gl}}[k]$.

Formally, the problem to be solved in stage $s \in \mathcal{S} = \{1, 2, \ldots, 9\}$ is defined as follows:

$$J_s = \max\ f_s(\Psi) \tag{3a}$$
$$\text{s.t. } f_j(\Psi) \geq J_j - \epsilon, \quad j = 1, \ldots, s-1 \tag{3b}$$
$$\mathtt{Eq.}\ (2) \tag{3c}$$

which amounts to maximizing the objective f_s, while constraints are introduced to limit degradation in the objective of the previous stages by a small margin, given by $\epsilon > 0$, which is needed to prevent numerical issues. Notice that the objectives of the stages $s \in \mathcal{S} \setminus \{1, 6\}$ are effectively minimized, but were defined as maximization with negative signs on the functions to simplify the notation and yield a single problem formulation (3).

The cascading optimization approach is well suited for resolving the multiple objectives in operations sequencing since there is a clear ranking of the importance of the objectives in the form of lexicographic preferences. According to [16], "*In lexicographic optimization a finite number of objective functions is considered which are to be optimized on a feasible set in a lexicographic order, i.e. low priority objectives are optimized as far as they do not interfere with the optimization of higher priority objectives*."

C. MILP APPROXIMATION

The operations sequencing problem (2) is a MINLP of considerable complexity given the nonlinear nature of the well production functions $\widehat{q}^n_{\mathtt{oil}}(\cdot)$ and $\widehat{q}^n_{\mathtt{oil,dss}}(\cdot)$. A practical alternative consists in approximating the MINLP with a MILP by piecewise-linearizing the production functions. This approximation is achieved by performing sensitivity analysis with well simulation models, aiming to gather breakpoints consisting of input pairs with lift-gas injection rates and well-head pressure and the corresponding oil production output. With the sets of breakpoints, the resulting piecewise-linear approximation honoring the feasible domain can be expressed in MILP using one of the formalisms from the literature [17]. Among these formalisms, we choose the SOS2 constraints [18] for being effective and arguably the easiest to implement in algebraic modeling languages [19].

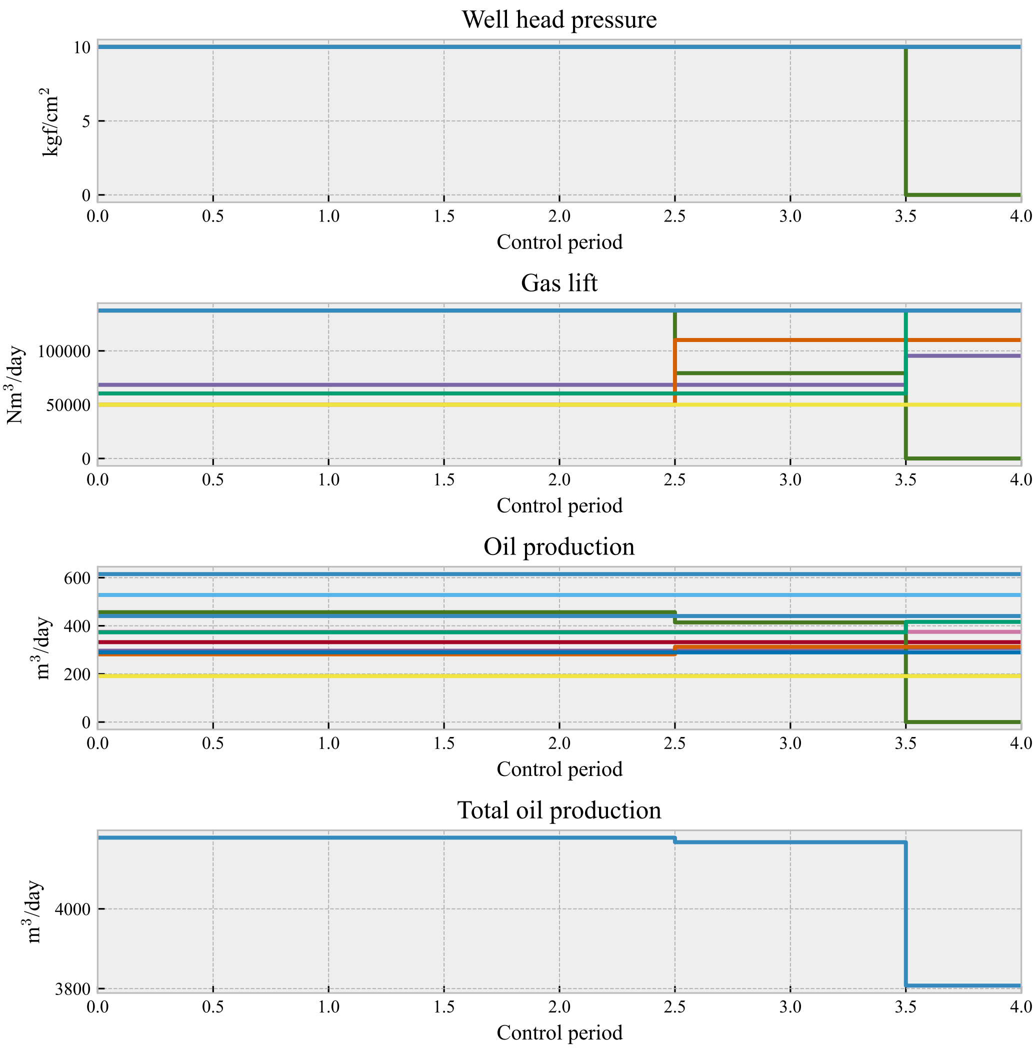


FIGURE 2. Shutting down a well: (a) well head pressure for each well in the production system, each well represented by a color; (b) gas-lift injection for each well in the production system, each well represented by a color; (c) oil production for each well in the production system, each well represented by a color; (d) total oil production of the system considering all wells.

SOS2 constraints are natively supported by several commercial solvers and have some properties that allow them to be handled more efficiently by a branch-and-bound procedure. The method involves approximating the function with linear segments over small intervals of the domain, and ensuring that the resulting approximation respects the order constraints.

Using SOS2 constraints, the piecewise-linear model for the production function of well n, at time k, is presented below:

$$q_{\circ}^{n}[k] = \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{std}}[k]\cdot\widehat{q}_{\circ}^{n}(p,q) + \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{dss}}[k]\cdot\widehat{q}_{\circ,\mathtt{dss}}^{n}(p,q) \tag{4a}$$

$$whp^{n}[k] = \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{std}}[k]\cdot p + \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{dss}}[k]\cdot p \tag{4b}$$

$$q_{\mathtt{gl}}^{n}[k] = \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{std}}[k]\cdot q + \sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{dss}}[k]\cdot q \tag{4c}$$

$$\sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{std}}[k] = y_{\mathtt{std}}^{n}[k] \tag{4d}$$

$$\sum_{(p,q)\in\mathcal{R}^n} \mu_{p,q}^{n,\mathtt{dss}}[k] = y_{\mathtt{dss}}^{n}[k] \tag{4e}$$

$$\kappa_{p}^{n,\mathtt{std}}[k] = \sum_{q\in\mathcal{Q}_{\mathtt{gl}}^{n}} \mu_{p,q}^{n,\mathtt{std}}[k], \quad \forall p\in\mathcal{P}_{\mathtt{wh}}^{n} \tag{4f}$$

$$\kappa_{q}^{n,\mathtt{std}}[k] = \sum_{p\in\mathcal{P}_{\mathtt{wh}}^{n}} \mu_{p,q}^{n,\mathtt{std}}[k], \quad \forall q\in\mathcal{Q}_{\mathtt{gl}}^{n} \tag{4g}$$

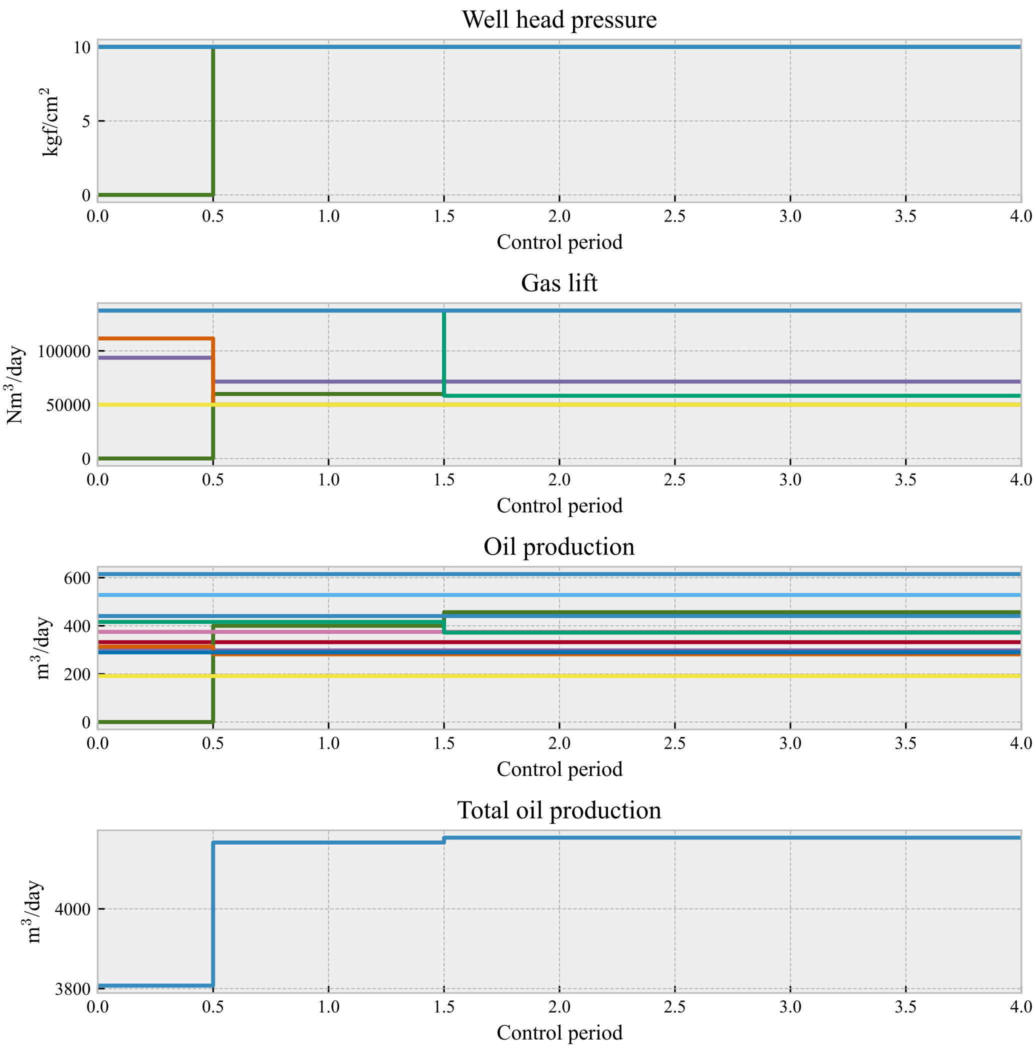


FIGURE 3. **Opening a well: (a) well head pressure for each well in the production system, each well represented by a color; (b) gas-lift injection for each well in the production system, each well represented by a color; (c) oil production for each well in the production system, each well represented by a color; (d) total oil production of the system considering all wells.**

$$\kappa_p^{n,\mathtt{dss}}[k] = \sum_{q \in \mathcal{Q}_{\mathtt{gl}}^n} \mu_{p,q}^{n,\mathtt{dss}}[k], \quad \forall p \in \mathcal{P}_{\mathtt{wh}}^n \tag{4h}$$

$$\kappa_q^{n,\mathtt{dss}}[k] = \sum_{p \in \mathcal{P}_{\mathtt{wh}}^n} \mu_{p,q}^{n,\mathtt{dss}}[k], \quad \forall q \in \mathcal{Q}_{\mathtt{gl}}^n \tag{4i}$$

$$(\kappa_p^{n,\mathtt{std}}[k])_{p \in \mathcal{P}_{\mathtt{wh}}^n} \text{ and } (\kappa_q^{n,\mathtt{std}}[k])_{q \in \mathcal{Q}_{\mathtt{gl}}^n} \in \mathtt{SOS2} \tag{4j}$$

$$(\kappa_p^{n,\mathtt{dss}}[k])_{p \in \mathcal{P}_{\mathtt{wh}}^n} \text{ and } (\kappa_q^{n,\mathtt{dss}}[k])_{q \in \mathcal{Q}_{\mathtt{gl}}^n} \in \mathtt{SOS2} \tag{4k}$$

where $(p, q) \in \mathcal{R}^n$ are breaking points extracted from a phenomenological simulator (*i.e.*, MARLIM [20]) for the expected working conditions. In other words, $p \in \mathcal{P}_{\mathtt{wh}}^n$ are the well head pressure points and $q \in \mathcal{Q}_{\mathtt{gl}}^n$ are the proposed gas lift operating points for a given well n. Notice that for each pair of breakpoints (p, q), two oil flows are simulated, one considering the use of demulsifiers and one not considering it.

The remaining nonlinear constraint (2g) can be linearized by applying the Big-M strategy, for well $n \in \mathcal{N}_{\mathtt{dss}}^*$ and time $k \in \mathcal{K}$, as follows:

$$dssr^n \cdot q_{\mathtt{liq}}^n[k] - q_{\mathtt{dss}}^{\max}\left(1 - y_{\mathtt{dss}}^n[k]\right) \leq q_{\mathtt{dss}}^n[k] \leq dssr^n \cdot q_{\mathtt{liq}}^n[k] + q_{\mathtt{dss}}^{\max}\left(1 - y_{\mathtt{dss}}^n[k]\right) \tag{5a}$$

$$q_{\mathtt{dss}}^n[k] \leq q_{\mathtt{dss}}^{\max} \cdot y_{\mathtt{dss}}^n[k] \tag{5b}$$

where $q_{\mathtt{dss}}^{\max}$ is a sufficiently large constant. Likewise, the big-M strategy is applied to a well $n \in \mathcal{N}_{\mathtt{dss}}^{\dagger}$, at time $k \in \mathcal{K}$, as follows:

$$q_{\mathtt{dss}}^{n,\dagger} - q_{\mathtt{dss}}^{\max}\left(1 - y_{\mathtt{dss}}^n[k]\right) \leq q_{\mathtt{dss}}^n[k] \leq q_{\mathtt{dss}}^{n,\dagger} + q_{\mathtt{dss}}^{\max}\left(1 - y_{\mathtt{dss}}^n[k]\right) \tag{6a}$$

$$q_{\mathtt{dss}}^n[k] \leq q_{\mathtt{dss}}^{\max} \cdot y_{\mathtt{dss}}^n[k] \tag{6b}$$

By replacing the definition of $q_{\mathrm{o}}^n[k]$ in Eq. (2b) with the piecewise-linear model introduced in Eq. (4), and substituting

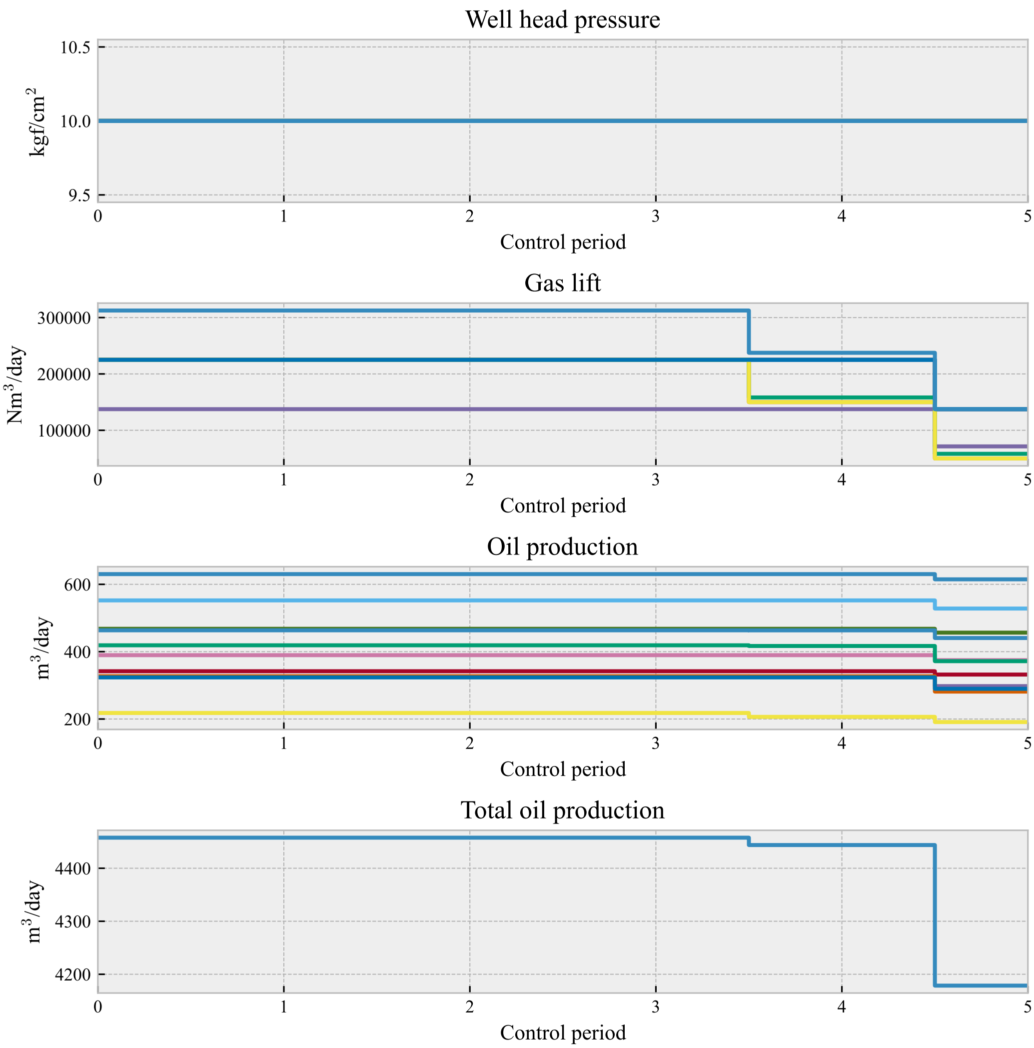


FIGURE 4. **Reducing the compression capacity: (a) well head pressure for each well in the production system, each well represented by a color; (b) gas-lift injection for each well in the production system, each well represented by a color; (c) oil production for each well in the production system, each well represented by a color; (d) total oil production of the system considering all wells.**

Eq. (2g) with Eqs. (5) and (6), the MINLP formulation for the operations sequencing problem is approximated as a MILP.

IV. CASE STUDIES

This section presents simulated results from optimizing operations sequencing and demulsifier injection in an offshore production platform. The studies aim to show how the methodology can assist operators in situations of shortage and recovery of processing capacity, such as preventive maintenance of compressors, and shutting down and restarting of wells. The experiments further illustrate the impact of optimal management of demulsifier injection.

A. PRODUCTION SYSTEM

The case studies regard an offshore production platform operated by Petrobras in the Campos Basin. The platform handles the production of 11 wells that operate with continuous lift-gas injection, each one connected directly to the platform with a dedicated riser, in a configuration known as satellite wells, as depicted in Figure 1. The multiphase flows from the wells are gathered and separated at the top side into streams of oil, gas, and water. The oil is transferred to onshore terminals by shuttle tankers, the water is processed before reinjection into the reservoir and discharge, and the gas is compressed to supply lift-gas and turbine that generate electric power, while the remaining gas is exported in a subsea gas pipeline to the onshore terminal. When production streams cannot be processed due, for instance, a failure in one of the compressors, the gas is burned in the flare as it cannot be reinjected in the wells nor exported. The compression system comprises two parallel turbo-generators that can yield a gas processing capacity of 3.2 MNm^3/day.

The MARLIM simulator was used to obtain the production curves $\widehat{q}_{\texttt{oil}}(\cdot)$ and $\widehat{q}_{\texttt{oil,dss}}(\cdot)$ for all experiments performed.

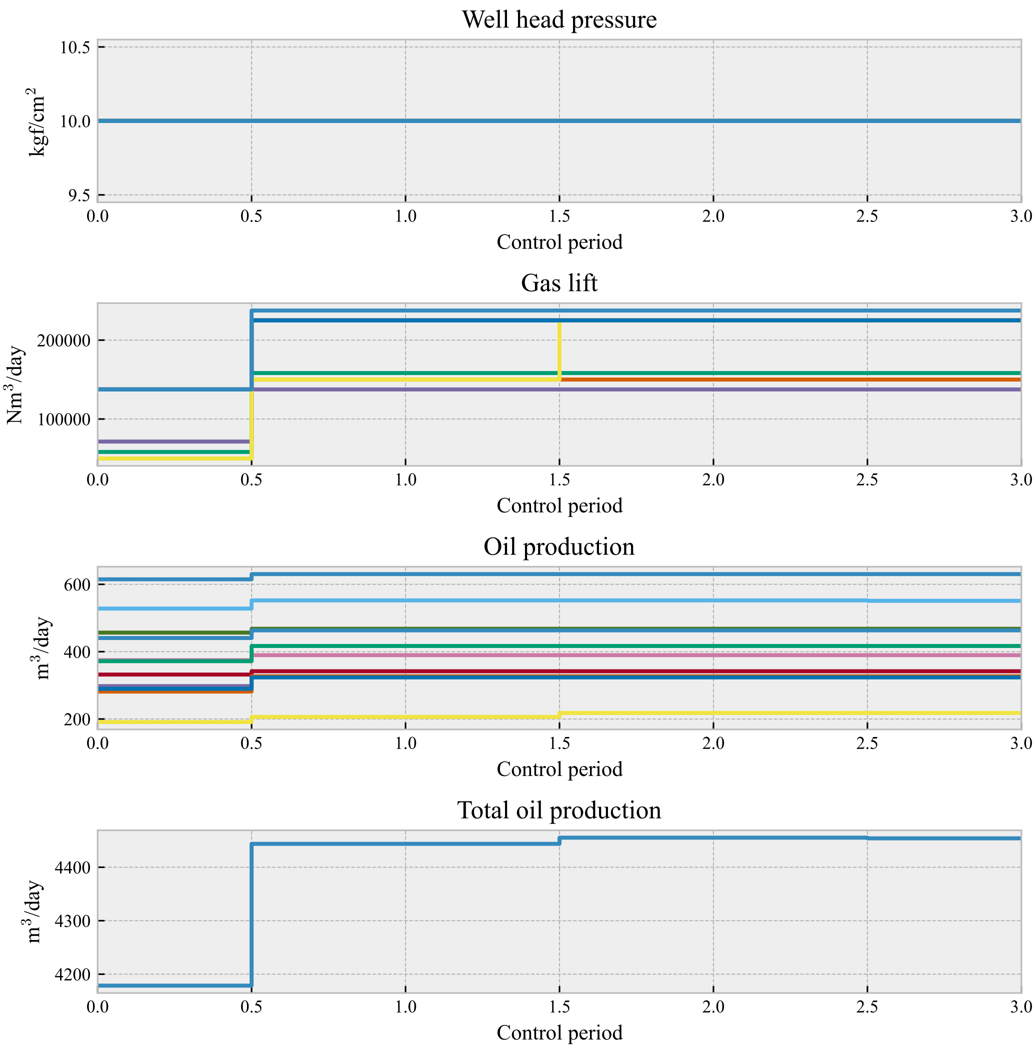


FIGURE 5. Rising the compression capacity: (a) well head pressure for each well in the production system, each well represented by a color; (b) gas-lift injection for each well in the production system, each well represented by a color; (c) oil production for each well in the production system, each well represented by a color; (d) total oil production of the system considering all wells.

The input data for such simulations was supplied by Petrobras for the eleven wells that are operated by the platform. Besides production simulation, the MARLIM simulator is used for sensitivity and flow assurance assessment of naturally flowing wells and wells equipped with continuous gas-lift or electrical submersible pumps [20]. Once the production system configuration is defined, the simulator computes one of the following three variables of interest given the other two: equilibrium flow, downstream pressure, or upstream pressure.

All experiments were performed on a workstation equipped with two 10-core Intel® Xeon® CPU E5-2630 v4 @ 2.20 GHz and 64 GB of RAM, running Ubuntu 20.04.1. The MILP approximation of the operations sequencing optimization problem (2) was obtained by piecewise-linear approximation of the production functions $\widehat{q}_{\texttt{oil}}(\cdot)$ and $\widehat{q}_{\texttt{oil,dss}}(\cdot)$, using SOS2 constraints according with the developments in Eq. (4), and the linearization of the DSS injection function (2g) according with Eqs. (5)-(6). The resulting MILP problem was coded in Python, using Pyomo, and solved with the Gurobi solver version 9.5.1.

B. IMPACT OF OPERATIONS SEQUENCING

In order to illustrate the behavior of the impact of operations sequencing, four scenarios are presented. First, we consider shutting down a well, which could then be aligned to a test separator, as presented in Figure 2. For Figures 2(a), 2(b), and 2(c), each line of a distinct color corresponds to a different well. Except for the well being shut down, indicated in color green in Figure 2(a), the well-head pressure is unaffected for the wells that remain in production. Notice that in this scenario, in order to maximize the oil production even during the shutdown event, the idle compression capacity resulting from the closure is used to increase the injection of gas lift from other wells, as illustrated in

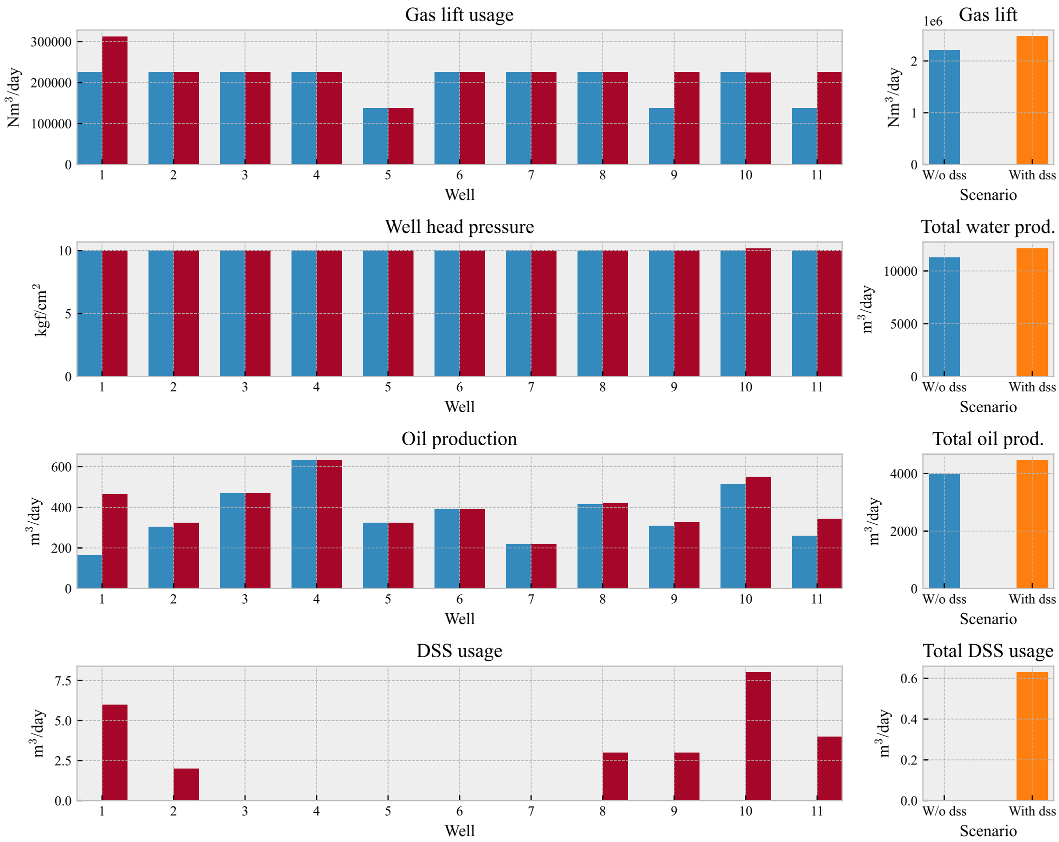


FIGURE 6. Bar chart comparison of the illustrative scenario without (blue) and with DSS injection (red).

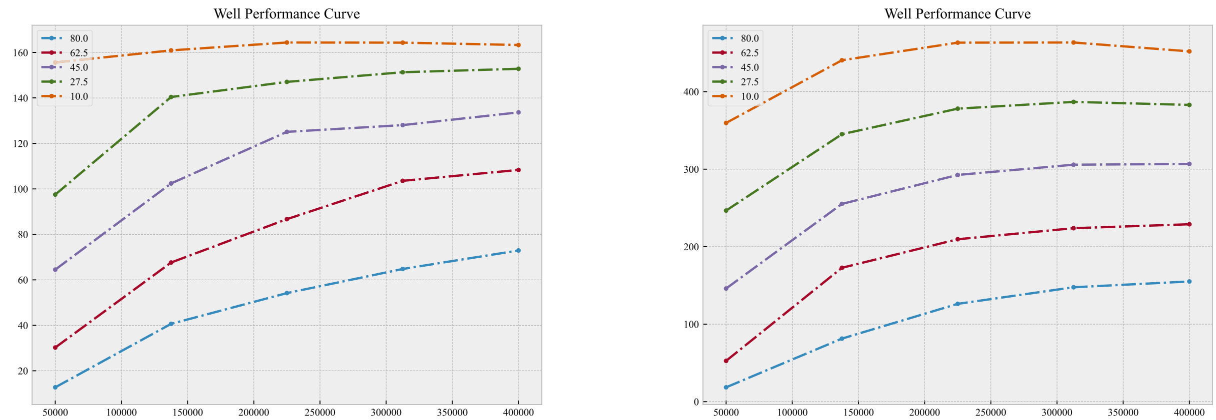
Figures 2(b) and 2(c), in an attempt to maintain high oil production. The resulting behavior reveals that the well is shut down at the last period of the planning horizon, in order to keep the production as high as possible, which is enforced by the cascading optimization strategy that maximizes oil production first.

On our second experiment, we consider adding/reconnecting a new well to the system that is already working at maximum compression capacity. It is possible to observe in Figure 3 that, considering the maximum oil production at the end of the sequencing horizon, the optimization problem decides to reduce the gas-lift injection of the least productive actives wells, in order to leverage the oil production from the newly added well. This behavior can be noticed in Figures 3(b) and 3(c). Overall, the behaviors emerging from this scenario are the opposite of the first one, *i.e.*, the actions are taken in order to bring the new well in production as soon as possible, aiming at the maximum oil production over the horizon as shown in 3(d).

For our third and fourth illustrative scenarios, we consider reducing and raising the compression capacity, as shown in Figure 4 and Figure 5, respectively. For the case of compression capacity shortfall, the lift-gas injection rate is reduced for all wells incurring a loss of production, whereas the rate is increased accordingly for the case of compression capacity recovery. As expected, the system acts in order to prevent oil loss, always maintaining the maximum oil production during the transition. Generally, the observed temporal behavior of the wells is similar to the previous scenarios: actions that increase production are implemented as soon as possible, whereas actions that reduce production are deferred until the last possible moment.

C. IMPACT OF DSS

In order to illustrate the gains induced by the DSS injection, we present in Figure 6 simulated results from a real platform consisting of 11 wells. In the first optimization, the oil production is maximized without considering the DSS injection, *i.e.*, only seeking the best operation conditions (well head pressure and gas lift injection). In the second optimization, besides seeking the best operation conditions, the optimization is also free to decide between the injection or not of the demulsifier. It is possible to observe that the latter can obtain considerable gains. Notice that in the second optimization, in addition to gains induced by the DSS, it was possible to reduce the usage of gas lift in some wells while achieving a high oil production. The experiments show a gain of 11.5% in total oil production resulting from an optimal management of demulsifiers.



(a) Without DSS injection.

(b) With DSS injection.

FIGURE 7. **Well production curve.**

To illustrate how a well can be affected by the injection of demulsifier, Figure 7a and Figure 7b present, respectively, the performance gain for a satellite well with and without DSS injection. Notice that a considerable offset can be observed in the oil production.

D. DISCUSSION

The case studies presented in this section involved optimizing the sequencing of operations and demulsifier injection in an offshore production platform operated by Petrobras in the Campos Basin. The platform handles the production of 11 satellite wells and has a gas processing capacity of 3.2 MNm3/day. The experiments were conducted using the MARLIM simulator and a mixed-integer linear programming (MILP) approximation of the optimization problem, which was solved using the Gurobi solver. The experiments demonstrated the ability of the methodology to assist operators in situations such as preventive maintenance of compressors and shutting down and restarting wells, as well as the impact of optimal management of demulsifier injection. The results showed that the methodology was able to significantly improve the oil production and reduce the volume of gas flared in the flare stack.

V. CONCLUSION

This work formalized the problem of optimizing the sequence of operations in production platforms to account for situations like connecting or disconnecting wells from the production pool, handling changes in processing capacity arising from the reduction in compression capacity during preventive and corrective maintenance, and recovery of processing capacity. The formulation extends existing models by introducing a discrete time dimension, allowing for constraints on the maximum number of changes and variation on control settings that can be handled during each time step, ensuring smooth transitions. The proposed formulation yields a sequence of operations (*e.g.*, changes in well-head pressure and lift-gas injection) over a prediction horizon that drives the production platform from the current steady-state to an optimal state reflecting the prevailing conditions.

In addition to the sequencing of operations, the proposed formulation can optimize production by optimally handling the injection of demulsifier in the wells of interest, given by piecewise-linear models of both well performance curves (*i.e.*, with and without demulsifier). Experiments revealed an increase of 11.5% in total oil production due to optimal demulsifier management.

In order to illustrate the behavior of the formulation, a case study considering a production system with 11 production wells was simulated and subject to different conditions of processing capacity and well connections. It was observed that the selected optimal sequence of operation points generally consisted in either: (i) implementing changes that increased production as soon as possible; or (ii) decreasing production to comply with constraints at the last possible time step, in order to maximize the oil production, a situation clearly enforced by the order of the cascading optimization strategy.

For future works, the proposed formulation for operations sequencing can be extended to account for other features, including modeling of subsea manifolds, multiple separation units, other types of artificial lifting (e.g., electrical submersible pumping), and flow assurance constraints, among others.

REFERENCES

[1] B. A. Foss and J. P. Jensen, "Performance analysis for closed-loop reservoir management," *SPE J.*, vol. 16, no. 1, pp. 183–190, Mar. 2011.
[2] B. Grimstad, B. Foss, R. Heddle, and M. Woodman, "Global optimization of multiphase flow networks using spline surrogate models," *Comput. Chem. Eng.*, vol. 84, pp. 237–254, Jan. 2016.
[3] V. Gunnerud and B. Foss, "Oil production optimization—A piecewise linear model, solved with two decomposition strategies," *Comput. Chem. Eng.*, vol. 34, no. 11, pp. 1803–1812, Nov. 2010.
[4] A. Hoffmann and M. Stanko, "Short-term model-based production optimization of a surface production network with electric submersible pumps using piecewise-linear functions," *J. Petroleum Sci. Eng.*, vol. 158, pp. 570–584, Sep. 2017.

[5] E. Camponogara, A. F. Teixeira, E. O. Hulse, T. L. Silva, S. Sunjerga, and L. K. Miyatake, ''Integrated methodology for production optimization from multiple offshore reservoirs in the Santos Basin,'' *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 2, pp. 669–680, Apr. 2017.

[6] R. R. Carpio, T. C. dAvila, D. P. Taira, L. D. Ribeiro, B. F. Viera, A. F. Teixeira, M. M. Campos, and A. R. Secchi, ''Short-term oil production global optimization with operational constraints: A comparative study of nonlinear and piecewise linear formulations,'' *J. Petroleum Sci. Eng.*, vol. 198, Mar. 2021, Art. no. 108141.

[7] E. R. Müller, E. Camponogara, L. O. Seman, E. O. Hülse, B. F. Vieira, L. K. Miyatake, and A. F. Teixeira, ''Short-term steady-state production optimization of offshore oil platforms: Wells with dual completion (gas-lift and ESP) and flow assurance,'' *TOP*, vol. 30, no. 1, pp. 152–180, Apr. 2022.

[8] E. O. Hülse and E. Camponogara, ''Robust formulations for production optimization of satellite oil wells,'' *Eng. Optim.*, vol. 49, no. 5, pp. 846–863, May 2017.

[9] C. Gamboa, T. Silva, D. Valladão, B. K. Pagnoncelli, T. Homem-de-Mello, B. Vieira, and A. Teixeira, ''A stochastic optimization model for short-term production of offshore oil platforms with satellite wells using gas lift,'' *TOP*, vol. 28, no. 3, pp. 549–574, Feb. 2020.

[10] G. Chaves, D. Monteiro, M. C. Duque, V. F. Filho, J. Baioco, and B. F. Vieira, ''Short-term production optimization under water-cut uncertainty,'' *SPE J.*, vol. 26, no. 5, pp. 3054–3074, Oct. 2021.

[11] A. J. Peixoto, D. Pereira-Dias, A. F. S. Xaud, and A. R. Secchi, ''Modelling and extremum seeking control of gas lifted oil wells,'' *IFAC-PapersOnLine*, vol. 48, no. 6, pp. 21–26, 2015.

[12] L. D. S. Santos, K. M. F. de Souza, M. R. Bandeira, V. R. R. Ahón, F. C. Peixoto, and D. M. Prata, ''Dynamic optimization of a continuous gas lift process using a mesh refining sequential method,'' *J. Petroleum Sci. Eng.*, vol. 165, pp. 161–170, Jun. 2018.

[13] F. C. Diehl, C. S. Almeida, T. K. Anzai, G. Gerevini, S. S. Neto, O. F. Von Meien, M. C. M. M. Campos, M. Farenzena, and J. O. Trierweiler, ''Oil production increase in unstable gas lift systems through nonlinear model predictive control,'' *J. Process Control*, vol. 69, pp. 58–69, Sep. 2018.

[14] B. Foss, B. R. Knudsen, and B. Grimstad, ''Petroleum production optimization—A static or dynamic problem?'' *Comput. Chem. Eng.*, vol. 114, pp. 245–253, Jun. 2018.

[15] E. O. Hülse, T. L. Silva, E. Camponogara, V. R. Rosa, B. F. Vieira, and A. F. Teixeira, ''Introducing approximate well dynamics into production optimization for operations scheduling,'' *Comput. Chem. Eng.*, vol. 136, May 2020, Art. no. 106773.

[16] H. Isermann, ''Linear lexicographic optimization,'' *Oper.-Res.-Spektrum*, vol. 4, no. 4, pp. 223–228, Dec. 1982.

[17] J. P. Vielma, S. Ahmed, and G. Nemhauser, ''Mixed-integer models for nonseparable piecewise-linear optimization: Unifying framework and extensions,'' *Oper. Res.*, vol. 58, no. 2, pp. 303–315, Apr. 2010.

[18] E. M. L. Beale, ''Branch and bound methods for numerical optimization of non-convex functions,'' in *Proc. Comput. Statist. (COMPSTAT)*, vol. 80, 1980, pp. 11–20.

[19] A. B. Keha, I. R. de Farias, and G. L. Nemhauser, ''Models for representing piecewise linear cost functions,'' *Oper. Res. Lett.*, vol. 32, no. 1, pp. 44–48, 2004.

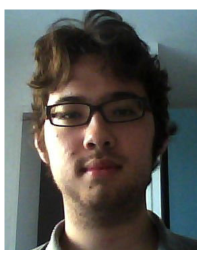
[20] L. O. Seman, L. K. Miyatake, E. Camponogara, C. M. Giuliani, and B. F. Vieira, ''Derivative-free parameter tuning for a well multiphase flow simulator,'' *J. Petroleum Sci. Eng.*, vol. 192, Sep. 2020, Art. no. 107288.

EDUARDO RAUH MÜLLER received the B.Eng. degree in control and automation engineering and the M.E. and Ph.D. degrees in systems engineering from the Federal University of Santa Catarina, Brazil, in 2011, 2013, and 2018, respectively. From 2013 to 2015, he was a Consultant of state and municipal agencies in urban and freeway traffic and public transportation systems. Since 2018, he has been a Postdoctoral Researcher with the Department of Automation and Systems, Federal University of Santa Catarina. His current research interests include control and optimization applied to traffic and transportation systems, and to oil production.

EDUARDO CAMPONOGARA (Member, IEEE) received the Ph.D. degree in electrical and computer engineering from Carnegie Mellon University, USA, in 2000. After being a Postdoctoral Fellow with the Institute for Complex Engineered Systems, USA, he joined as a Faculty Member with the Department of Automation and Systems Engineering, Federal University of Santa Catarina, Brazil, in 2002. His research interests include systems optimization, distributed decision making, and traffic control engineering.

LAIO ORIEL SEMAN received the Ph.D. degree in electrical engineering from the Federal University of Santa Catarina, in 2017. He is currently a University Professor with the University of Vale do Itajaí, Brazil. His research interests include strategies for static and dynamic optimization, along with applications in traffic control, cyber-physical systems, and oil and gas production systems.

BRUNO FERREIRA VIEIRA received the Ph.D. degree in production engineering from the Federal University of Rio de Janeiro, in 2014. He is currently a Researcher with CENPES, Petrobras. His research interests include the optimization of oil and gas systems, machine learning, and reconciliation.



LUIS KIN MIYATAKE received the master's degree in automation and control systems from COPPE/UFRJ. He is a Petroleum Engineer with Petrobras.