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RESEARCH ARTICLE

An Effective Method to Estimate the Aggregated Flexibility at Distribution Level

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ABSTRACT The exploitation of flexibility services provided by distributed energy resources (DERs) located in the distribution grid becomes increasingly important for the secure and reliable operation of power systems. These resources are typically small-scale and located close to end-users, however their aggregated flexibility can offset the imbalance caused by the stochastic variations of the Renewable Energy Sources (RES). This paper proposes a methodology to estimate with increased accuracy the available flexibility of DERs in an Active Distribution Grid (ADG) at the point of interconnection with the transmission system. An optimization-based approach is used to estimate the available maximum active and reactive power flexibility for specific search directions formed by constant active to reactive power ratios. In order to keep calculations manageable and estimate the available flexibility with high accuracy, the search directions are sampled from a distribution that is iteratively updated at each step. Three variations are proposed for the angle sampling, both in the angle domain and in the p-q domain. Each variation is tested on a modified 18-bus radial distribution system and its effectiveness and convergence is compared.

INDEX TERMS Active distribution grids, distributed energy resources, operational flexibility, optimal power flow (OPF), TSO-DSO coordination.

I. INTRODUCTION

The power supply in future energy systems is planned to be carbon-free, and, mostly based on Renewable Energy Sources (RES) such as solar, wind, biomass, geothermal, biogas, etc. A significant part of RES will be small-scale, Distributed Energy Resources (DERs) connected to the distribution networks, close to the loads. DERs comprising of distributed generators, flexible loads, distributed storage, electric vehicles etc., if properly controlled, can contribute to the stability and control of the bulk power system [1]. In particular, as the high penetration of RES at transmission level displaces central thermal units, such as gas units, that traditionally provide necessary flexibility, such flexibility services can be provided by DERs. In this context, Distributed System Operators (DSOs) assume a more active role in the overall system operation. DSOs operating Active Distribution Grids (ADGs) are expected to offer active and reactive power flexibility services, while accounting the uncertainty of local variable generation and load forecasts [2], [3].

The efficient exploitation of the flexibility provided by DERs requires a close coordination between DSOs and Transmission System Operators (TSOs) [4], [5], [6], and well-functioning flexibility markets to properly remunerate the participating DER owners [7], [8], [9]. Several European research and innovation projects have dealt with the practical applications of TSOs, DSOs, and end-customers coordination to efficiently utilize flexibility services and support power system operation [10], [11].

An interesting aspect of the flexibility offered by DERs lies in their reactive power capabilities and how they can be utilized to provide coordinated voltage ancillary services at the points of interconnection between the distribution and transmission network [12], [13]. It should be ensured that the

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activation of this flexibility does not violate the distribution network constraints or has undesirable effects in the distribution network operation.

The determination of the aggregate flexibility curve of an ADG is a challenging problem and has recently received significant academic interest. The proposed aggregation methodologies can be classified into two groups: (i) Monte-Carlo-based approaches [14], [15], [16] and (ii) optimization-based approaches [17], [18], [19], [20], [21], [22], [23], [24].

Monte-Carlo-based approaches rely on processing a large number of operating points randomly to estimate the feasible flexibility region [14]. The results of an AC power flow for each operating point are used to determine whether the operating point violates the distribution grid constraints. The identified feasible operating points approximate the flexibility area at the TSO-DSO interface in the form of a polygon. Reference [15] emphasizes the need to differentiate the terms feasibility and flexibility, while approximating the flexibility provided by fast and slow responding DERs. Reference [16] uses a Monte-Carlo approach, focusing on estimating the time-dependent flexibility from DERs.

The optimization-based approaches aim to directly identify the boundaries of the flexibility area around an operating point by maximizing the active-reactive power exchange at the TSO-DSO interconnection substation at different search directions. These approaches require fewer intermediate steps to estimate the flexibility area compared to Monte-Carlo approaches; however, the computational effort in each step is higher, since an optimization problem is solved at each step. Reference [17] identifies the boundary conditions of the flexibility area by minimizing the reactive power import from the TSO for a selected set of active power setpoints. Reference [18] presents a non-convex non-linear optimization-based approach that takes into account the cost of flexibility. Reference [19] investigates the impacts of the grid components, such as tap changing transformers, active/reactive generation, and demand on the estimated flexibility. In both [18] and [19], the flexibility area's extreme points are calculated first by minimizing or maximizing the active and reactive power exchange between the TSO and DSO. After this step, more precise calculations are made based only on the active power exchange in order to refine the boundaries of the flexibility area. Reference [20] proposes to explicitly formulate the time-variant active and reactive power capabilities of each type of flexibility-providing unit as constraint in a 2-step optimization problem. Reference [21] focuses on the uncertainty of demand and stochastic generation and proposes a scenario-based robust approach to determine the available flexibility. Reference [24] demonstrates how the distributed flexibility of thousands of DERs can be efficiently aggregated using an optimization-based approach and incorporated in the operational planning of a large real transmission system, such as the European network.

The optimization-based approaches present several advantages since they aim to directly identify the boundaries of the flexibility area around an operating point through multiple steps by maximizing the active-reactive power flexibility at a specified search direction. Nevertheless, the estimation of the flexibility curve relies on determining the angle/search directions beforehand without having any insight on the shape and the size of the flexibility curve to be estimated. The majority of the optimization-based approaches for estimating the flexibility curve use a fixed step-wise angle-based sampling to identify the boundaries of the flexibility curve at specific search directions. The performance of these methods is dependent on the angle step selection, thus a small number of steps fail to capture adequately the flexibility area. On the other hand a large number of steps requires long calculation times, while the accuracy of the flexibility curve is not guaranteed.

In this paper a novel methodology is proposed that is able to estimate the size and the shape of the flexibility curve with high accuracy without resorting to an exhaustive evaluation of search directions. The search directions at each step are not fixed step selections but are chosen iteratively using a sampling distribution that is updated after each step based on the previous search directions. The methodology also employs a stopping criterion that ensures that the resulting flexibility curve meets certain shape characteristics with a specific level of accuracy.

Three different approaches for angle sampling and updating the sampling distribution are examined. The first approach proposes a *sampling from a discrete distribution in the angle domain* based on a Multiplicative Weight Update Method (MWUM). This approach assigns initial weights to a set of potential search directions and updates these weights multiplicatively and iteratively according to a feedback function. The second approach uses a Kernel Density Estimation (KDE)-based method to apply a *sampling from a continuous distribution in the angle domain*, by fitting a KDE model with a set of observations at potential angles and removing an observation at each step. The third approach proposes a *sampling from a continuous distribution in the p-q domain*, which uses the already estimated points of the flexibility curve to determine the next search direction.

The main contribution of the paper is that it proposes a methodology that manages to estimate the available flexibility curve with increased accuracy. This is achieved by determining effectively the search directions, using a distribution curve which is updated iteratively based on the results of the previous steps. This means that fewer searches are required for estimating the flexibility curve in optimization-based approaches. Moreover, the proposed methodology captures better the boundaries of the available flexibility, since convergence is achieved when a specific stopping criterion is met, while fixed-angle approaches use a predefined number of search directions.

The remainder of this paper is structured as follows: Section II presents the working principle of the optimization-based approach for estimating the available active-reactive power flexibility that can be provided from an ADG at the TSO-DSO interface. Section III presents

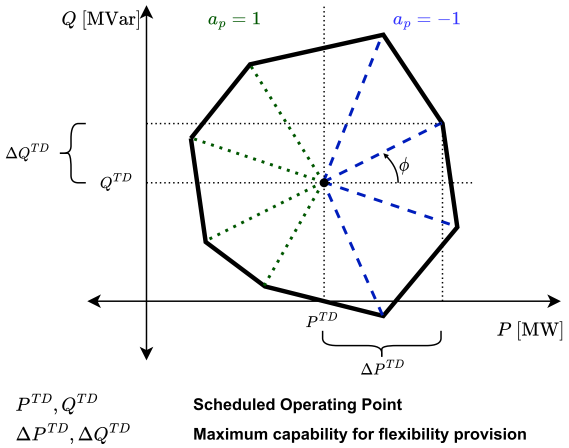

FIGURE 1. Example of estimating flexibility for an active distribution grid.

the proposed methodology for determining the search direction using an iteratively updated sampling distribution. In Section IV a case study is presented to illustrate the working principle of the proposed methodology for each different proposed sampling approach. In Section V the effectiveness and the convergence performance of the proposed methodology are compared with the fixed-angle sampling. Finally, conclusions are provided in Section VI.

II. OPTIMIZATION-BASED APPROACH TO ESTIMATE THE FLEXIBILITY CURVE

A. GENERAL ARCHITECTURE

Fig. 1 depicts an illustrative example of the optimization-based approach for the estimation of the available flexibility from aggregate DERs located in ADGs at the TSO-DSO interconnection point. P^{TD}, Q^{TD} denotes the scheduled active and reactive power exchange at the TSO-DSO interconnection point. The estimation of the scheduled power exchange is performed using the maximum forecasted active and reactive power exchange at this interconnection point and can be updated over time to increase its accuracy. The goal of the optimization-based approach is to estimate the flexibility capability ΔP^{TD}, ΔQ^{TD} of the distribution network considering the operational constraints of the network and the technical constraints of the DERs. The method proposed in the paper can be applied at various time intervals, ranging from day-ahead forecasts to very short-term forecasts, typically from 4 hours to 15 minutes ahead. The method becomes more accurate as more recent measurements are incorporated into the forecasts. This approach allows for a continuously updated and flexible management of the TSO-DSO interconnection point, helping to optimize the power exchange and improve the overall efficiency of the network. To the best of our knowledge, there is no obligation to exchange such information between TSOs and DSOs, however we believe that such information exchange would be beneficial for a closer collaboration between the two system operators in the future.

Active and reactive power provision are related by the power factor angle which defines the search direction of the method. The flexibility area is constructed following an iterative procedure for different search directions. The identified points are linearly interpolated and used to approximate the flexibility area as shown in Fig. 1. The great advantage of the optimization-based approaches is that the estimated flexibility area is a convex polygon. Therefore, the estimated flexibility area can be efficiently incorporated in commercial optimization tools.

B. MATHEMATICAL FORMULATION

The proposed method is based on the formulation of an optimization problem that estimates the maximum active-reactive power flexibility that an ADG can offer at a specified search direction.

The objective function is as follows:

$$\min \quad a_p \cdot \Delta P^{TD} + a_q \cdot \Delta Q^{TD} \tag{1}$$

where ΔP^{TD}, ΔQ^{TD} denotes the active and reactive power flexibility at the TSO-DSO boundary node and a_p, a_q are the objective function coefficients determined by power factor angle ϕ.

The optimization of the objective function is subject to the following constraints $\forall i \in B, \forall j \in G_i$:

$$2g(\Delta P^{TD}, \Delta Q^{TD}, \Delta P_{ij}, \Delta Q_{ij}, V_i, \theta_i) = 0 \quad \forall i \in B \tag{2a}$$

$$h(\Delta P^{TD}, \Delta Q^{TD}, \Delta P_{ij}, \Delta Q_{ij}, V_i, \theta_i) \leq 0 \quad \forall i \in B \tag{2b}$$

$$\Delta P^{TD} = \tan(\phi)\Delta Q^{TD} \tag{2c}$$

where ΔP_{ij} and ΔQ_{ij} denote the active and reactive power flexibility for each type of flexibility provider j connected at bus i, including distributed thermal and renewable generation, flexible loads, battery energy storage systems etc. V_i and θ_i denote the voltage magnitudes and angles of all the buses of the distribution system.

$g(.)$ denotes the power balance equations of the distribution system. $h(.)$ denotes all the inequality constraints of the optimization problem, including distribution line flow limits, voltage limits, active and reactive power flexibility limits of the various flexibility resources, that may differ depending on their technical characteristics.

The knowledge of the specific operational constraints of each type of DER from the DSO is crucial to ensure the effective integration and flexibility estimation of these resources. The DSO relies on forecasting to determine the maximum available flexibility of each DER. The operational constraints are taken into account as far as they are reflected in the time series input data used for forecasting. To effectively consider the DER limitations, probabilistic forecasting techniques and robust optimization should be applied for estimating more realistically the aggregated flexibility curve.

Constraint (2c), specifies the search direction of the methodology, since it bounds active and reactive power flexibility at the TSO-DSO boundary node with the power factor angle ϕ, as shown in Fig. 1. The search direction is determined using a_p and a_q as follows:

$$a_p = \begin{cases} -1, & 0^\circ \leq \phi < 90^\circ \\ 0, & \phi = 90^\circ \\ 1, & 90^\circ < \phi < 270^\circ \\ 0, & \phi = 270^\circ \\ 1, & 270^\circ < \phi < 360^\circ \end{cases} \tag{3}$$

$$a_q = \begin{cases} 0, & 0 \leq \phi < 90^\circ \\ -1, & \phi = 90^\circ \\ 0, & 90^\circ < \phi < 270^\circ \\ 1, & \phi = 270^\circ \\ 0, & 270^\circ < \phi < 360^\circ \end{cases} \tag{4}$$

Therefore the objective function (1) minimizes $-\Delta P^{TD}$ (or equivalently maximizes ΔP^{TD}) when $a_p = -1$, which corresponds to positive values of ΔP^{TD} in quadrants 1 and 4. Similarly, the objective function minimizes ΔP^{TD} when $a_p = +1$, which corresponds to negative values of ΔP^{TD} in quadrants 2 and 3. Therefore at each point the objective function aims to maximize the flexibility provision at the TSO-DSO interface substation. Similarly, for search directions $\phi = 90^\circ, 270^\circ$, the objective function aims to maximize the reactive power flexibility ΔQ^{TD} at the TSO-DSO interface and constraint (2c) is modified to set the active power flexibility ΔP^{TD} to zero as follows:

$$\Delta P^{TD} = 0 \tag{5}$$

C. FIXED STEP-WISE ANGLE

The methodology for estimating the flexibility curve requires to run the optimization problem for various angles/search directions. Following the estimated points of the flexibility curve can be linearly interpolated to approximate the whole flexibility curve. The fixed step-wise angle methodology that is currently proposed in the majority of related research works uses a predefined number of points K. The methodology, initially requires as input the sweeping angle $\Delta\phi$ which is calculated as:

$$\Delta\phi = \frac{360^\circ}{K} \tag{6}$$

The flexibility estimation procedure begins at $\phi = 0$, as illustrated in Fig. 2. Next, for each $\Delta\phi$ increment, the optimization methodology estimates a point of the flexibility curve. The procedure is terminated when the 360° circle is completed and the points are linearly interpolated to create the flexibility curve. The algorithm of the step-wise angle procedure for creating the flexibility map of an ADG is presented in Algorithm 1.

It is obvious that the performance of these methodologies depend on the angle interval selection and the accuracy of the estimation of the available flexibility depends on the number of points used. Since the shape and the size of the flexibility curve to be estimated is unknown, the proper selection of search direction is critical.

Algorithm 1 Estimating the Flexibility Area Using a Fixed Step-Wise Angle-Based Sampling

Require: Input $\Delta\phi$
$\phi \leftarrow 0$
while $\phi \leq 360^\circ$ **do**
Select a_p and a_q based on (3), (4)
Solve the optimization problem (1), (2b), (2a), (2c) for a_p, a_q, ϕ
$\phi \leftarrow \phi + \Delta\phi$
end while
Connect the points and create the polygon of the flexibility map

III. ITERATIVE UPDATE OF THE DISTRIBUTION

In this section a novel approach for determining the search directions is presented that is adaptive to the size and the shape of the flexibility curve. The proposed methodology differs from the fixed step-wise angle approaches since the search directions at each step are chosen effectively using a distribution curve that updates after each step based on the results of the previous search directions.

The algorithm of the iterative update of the distribution for estimating the flexibility map at the TSO-DSO interconnection point is presented in Algorithm 2. The methodology starts by running the optimization problem for a predefined number of search directions that will create the basic initial points of the flexibility curve. These points can typically be at $0^\circ, 90^\circ, 180^\circ, 270^\circ$. Alternatively, these points can estimate the maximum and minimum active and reactive power exchange between the transmission and the distribution system as in the setpoint-based sampling method.

A random angle selection in the space of [0, 360], determines the consecutive search directions of the methodology, sampling from an iteratively updated sampling distribution. The methodology is based on the observation that angles relatively close to the already chosen angles, have a lower probability to increase the area of the flexibility curve.

Within the context of this paper, three different approaches for updating the sampling distribution and sampling are proposed, as follows:

- Sampling from a discrete distribution in the angle domain.
- Sampling from a continuous distribution in the angle domain.
- Sampling from a continuous distribution in the p-q domain.

The *Sampling from a discrete distribution in the angle domain* is based on a MWUM, while the *Sampling from a continuous distribution in the angle domain* approach is based on a KDE method. The main difference of these two approaches is that MWUM depends on a discrete probability distribution, while KDE depends on a continuous probability distribution. Additionally, we propose a *Sampling from a continuous distribution in the p-q domain*, which is a Gaussian-

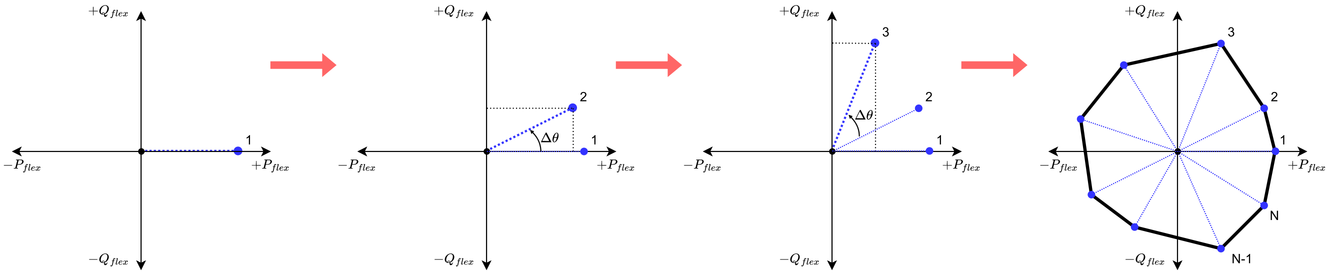

FIGURE 2. The step-wise procedure of creating the flexibility area at the TSO-DSO substation.

Algorithm 2 Estimating the Flexibility Area Using Sampling From an Iteratively Updated Distribution

Set convergence threshold ϵ
Initialize the methodology by solving the optimization problem for $\phi = 0°, 90°, 180°, 270°$
$S \leftarrow 0$
Initialize the sampling distribution
while $S \geq \epsilon$ **do**
    Sample an angle ϕ from the sampling distribution
    Select a_p and a_q based on (3), (4)
    Solve the optimization problem (1), (2b), (2a), (2c) for a_p, a_q, ϕ
    Update the sampling distribution
    Calculate the area of step i A_i
    Calculate the flexibility area increase rate of step i: $S \leftarrow \frac{A_i - A_{i-1}}{A_{i-1}}$
end while
Connect the points and create the polygon of the flexibility map

based methodology. This approach is more robust compared to the previous ones, since it accounts directly the already estimated active-reactive points of the flexibility curve.

For all methodologies, the termination criterion uses the surface difference of the estimated flexibility between two consecutive steps. In the following section the proposed methodologies are further presented and analyzed.

A. SAMPLING FROM A DISCRETE DISTRIBUTION IN THE ANGLE DOMAIN

The *Sampling from a discrete distribution in the angle domain* approach is based on a MWUM which is an algorithmic technique most commonly used for decision making and widely deployed in game theory and algorithm design [25]. In its simplest application, MWUM is used to predict a value based on a group of experts' advise, in which a decision making model iteratively decides which expert's advise follows. The method assigns initial weights to the experts and updates these weights multiplicatively and iteratively according to a feedback function, reducing it in case of poor performance and increasing it otherwise.

The proposed variation of MWUM, substitutes the experts with a finite number of potential angles ϕ in the space [0, 360]. Each angle's weight is considered as the probability of the given angle to be chosen. Therefore, the MWUM has as a constraint that the sum of all weights is equal to one. The weight's update method used is a variation of the weighted majority algorithm [25]. The steps of the update algorithm are the following:

- Initialize the weights of N_ϕ potential angles ϕ with a value equal to $w_i = \frac{1}{N_\phi}, \quad i = 1, \ldots, N_\phi$.
- At iteration t draw a random sample of angle ϕ_s from the discrete distribution created by the weights.
- Update the weight of the chosen angle ϕ_s using the following update function:

$$w_s^{t+1} = 0 \tag{7}$$

- Update the weights of the r nearest angles of angle ϕ_s using the following update functions:

$$w_{s+i}^{t+1} = w_{s+i}^{t}\left(1 - \frac{1}{i+1}\right), \quad i = 1, \ldots, r \tag{8}$$

$$w_{s-i}^{t+1} = w_{s-i}^{t}\left(1 - \frac{1}{i+1}\right), \quad i = 1, \ldots, r \tag{9}$$

- Update the rest weights that do not exist in the ensemble $[s-r, \ldots, s+r]$ using the following update function:

$$w_i^{t+1} = w_i^t + \frac{1}{N_\phi - (2r+1)} \sum_{j=s-r}^{s+r} w_{j+i}^{t+1} - w_{j+i}^{t} \tag{10}$$

B. SAMPLING FROM A CONTINUOUS DISTRIBUTION IN THE ANGLE DOMAIN

The *Sampling from a continuous distribution in the angle domain* approach extends the previous work in order to provide an iterative update method for a continuous sampling distribution. Therefore, a KDE-based methodology is proposed, which is a non-parametric method for estimating the probability density function of a population of finite observations. Assuming $(\phi_1, \ldots, \phi_{N_\phi})$ independent and identically distributed samples drawn from a univariate distribution with unknown density f_ϕ at any given point ϕ, KDE method approximates function f_ϕ using the following formula:

$$\hat{f}_h(\phi) = \frac{1}{N_\phi h} \sum_{i=1}^{N_\phi} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\phi-\phi_i}{h})^2} \tag{11}$$

where h is a non-negative smoothing parameter called bandwidth. Initially, in the proposed methodology, a finite number N_ϕ of potential angles ϕ are chosen in the space $[0, 360]$ and a KDE model is fitted in these observations. Due to its convenient mathematical properties, within the context of this paper the standard Gaussian kernel is used, as presented in (11). The update function method of the KDE model is described as follows:

- Draw a sample angle ϕ_s from the KDE
- Remove the observation closest to the drawn sample ϕ_s and re-fit the KDE model in the new set of observation
- The elimination and update process is repeated until the termination criterion is met and convergence of the flexibility curve's area is achieved.

The finite number of angles N_ϕ needs to be large enough, so that the KDE approximates a uniform distribution in the initial iterations. Following in every iteration, the removal of an observation from the potential angles set reduces the placed kernels in the next fitting process of the KDE model. The missing kernel creates a valley shape in the approximated distribution with the lowest point in the valley be the point that has been removed. The valley in the distribution models the notion that angles near already drawn angles do not increase dramatically the area of the estimated flexibility curve.

C. SAMPLING FROM A CONTINUOUS DISTRIBUTION IN THE P-Q DOMAIN

This methodology is a more robust approach compared to the previous ones, since it uses the already estimated points (p, q) of the flexibility curve to determine the following search direction. Similarly to the previous ones the methodology begins by estimating the initial (p, q) points at the following search direction $0°,\ 90°,\ 180°,\ 270°$.

At each step the proposed methodology aims to determine points that are far away from the already estimated flexibility curve points of the previous steps. Therefore, we create a 2d likely-hood surface plane using the observed (p, q) of the already estimated flexibility curve, as follows:

$$\hat{f_H}(\boldsymbol{x}) = \frac{1}{n_{obs}} \sum_{i=1}^{n_{obs}} \boldsymbol{K_H}(\boldsymbol{x} - \boldsymbol{x_i}) \tag{12}$$

$$\boldsymbol{K_H}(\boldsymbol{x}) = (2\pi)^{-1}|\boldsymbol{H}|^{-\frac{1}{2}} e^{-\frac{1}{2}\boldsymbol{x}^T \boldsymbol{H}^{-1}\boldsymbol{x}} \tag{13}$$

where $\boldsymbol{x_i} = [p_i, q_i]$ denotes the already observed (p,q) points of the flexibility curve, $\boldsymbol{x} = [p, q]$ denotes the independent variables of the (p,q) point that determines the next search direction, $\boldsymbol{K_H}$ denotes the bi-variate normal kernel and $\boldsymbol{H}$ denotes the bandwidth or smoothing matrix acting as the covariance matrix.

Based on the assumption that search directions that proximate to the already selected pairs will have low or no added value, we determine the next search direction using the $\hat{p}, \hat{q}$ that maximize the likely-hood to increase the flexibility curve. Therefore, the minimization of function $\hat{f_H}(\boldsymbol{x})$ in the $p - q$ domain is used to locate the $\hat{p}, \hat{q}$ pairs with the lowest likelihood; thus the points with the most added value towards the flexibility curve surface. The angle of the $\hat{p}, \hat{q}$ determines the next search direction, which will be used in the optimization problem to determine the next point of the estimated flexibility curve.

To further extend the accuracy of the methodology, we may also introduce a limit to the selection of $\hat{p}, \hat{q}$ points, introducing a set of linear constraints that bound the selection of $\hat{p}, \hat{q}$ points with the latest polygon of the estimated flexibility curve that was produced in the previous step. The combination of these two functions in each iteration determines the next search direction as the point on the polygon which minimizes the objective function (12).

As a result, the selection of the next point $\hat{p}, \hat{q}$ is made based on the following process:

$$\hat{\boldsymbol{x}} = [\hat{p}, \hat{q}] = \min \quad \hat{f_H}(\boldsymbol{x}) \qquad s.t{:}\ \hat{\boldsymbol{x}} \in f_{polygon} \tag{14}$$

TABLE 1. Distribution network line ratings and voltage limits.

Line Rating [MVA]	Distribution lines				
27	1-2	2-3	3-4	4-5	5-6
14	6-7	3-12	12 - 13	13-15	15-17
9	7-8 15-16	8-9 17-18	9-10	4-11	13-14
Voltage limit [p.u.]	**Distribution buses**				
[0.9, 1.1]	all buses				

IV. SIMULATION

The proposed methodology in this paper is implemented in MATLAB using YALMIP [26] as the modeling layer and IPOPT as the solver. The performance of the different sampling approaches is analyzed on a laptop with a processor Intel i7 CPU @ 2.6 GHz and 16.00 GB of RAM.

We illustrate the performance of the proposed methodologies for estimating the available active-reactive power flexibility that can be provided at the TSO-DSO interface, using a radial 18-bus distribution system, based on [27]. The line thermal ratings and the voltage limits are adjusted as shown in Table 1. The demand of the distribution system in buses 4, 5, 20, 25 is considered flexible and offers both upward and downward flexibility up to 20% of its scheduled value. Additionally, 29 MW of PV are distributed, with a capability to offer up to 20% of the installed capacity for downward flexibility. To better reflect the performance of each proposed method, the following results refer to a single time-step. Nevertheless, the analysis can be extended to estimate the time-dependent available flexibility, incorporating inter-temporal constraints into the optimization problem. Following the performance of each proposed methodology is presented and commented.

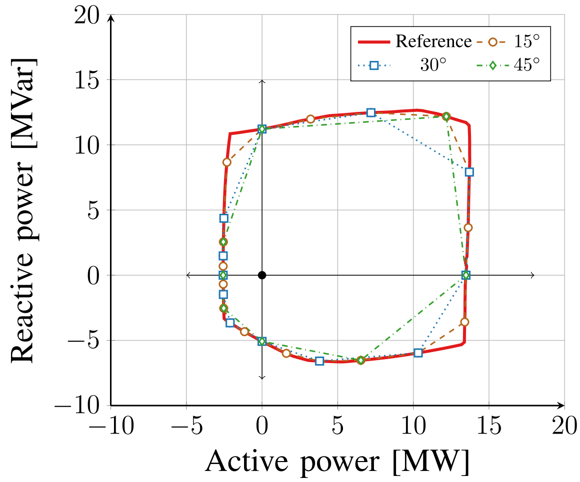

FIGURE 3. Fixed angle-based sampling for different angle intervals.

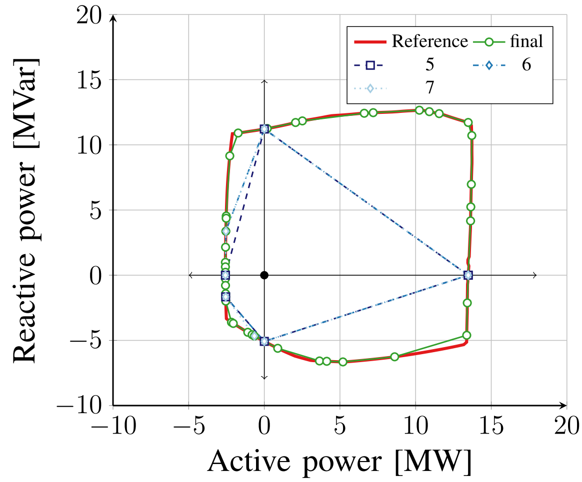

FIGURE 4. Sampling from a discrete distribution in the angle domain.

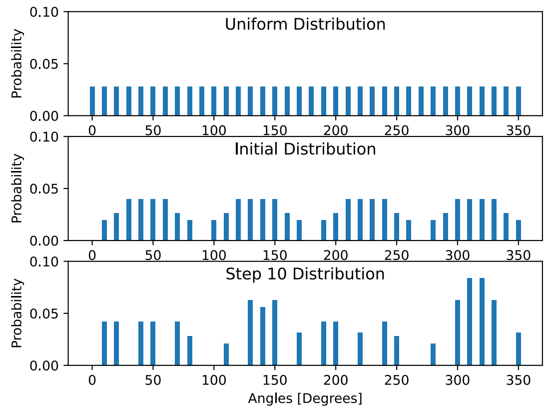

FIGURE 5. Iterative update of the discrete distribution in the angle domain.

A. FIXED ANGLE-BASED SAMPLING

Fig. 3 presents the results of the estimated flexibility for different angle interval ranging from $\Delta\phi = 15°$ up to $\Delta\phi = 45°$. The red curve presents the reference flexibility curve which is estimated using a very small angle interval ($\Delta\phi = 1°$) and is considered as the flexibility curve to be estimated. The dotted curves show the estimated flexibility curve for the selected angle interval. Using an angle interval of 45°, the basic shape of the flexibility map is obtained comprising 9 points. The estimated flexibility curve is shrunk compared to the actual available curve especially at the edges of the curve. More information for the shape of the flexibility map is obtained using a 30° angle interval since the procedure requires 12 intermediate steps for the estimation of the flexibility curve. Despite the calculation of flexibility at more points, the resulting curve misses an important part of the flexibility curve which was identified when using a 45° angle interval. Using a 15° angle interval, we manage to capture the flexibility map with 24 points and, as expected with higher accuracy.

It becomes clear that with lower rotating angle interval, we have higher accuracy in forming the flexibility curve. Nevertheless, the higher accuracy requires additional computational effort since the procedures for estimating the boundaries of the flexibility curve is performed more times. From this first example it becomes evident how crucial and difficult is to select the proper angle interval in order to have a perfect compromise between capturing as much of the available flexibility curve and reducing the computational effort by converging to the solution in fewer steps.

B. SAMPLING FROM A DISCRETE DISTRIBUTION IN THE ANGLE DOMAIN

Fig. 4 presents the estimated flexibility curve and the initial intermediate steps using a sampling from a discrete distribution in the angle domain. The finite number of potential angles is chosen to be 360 with an angle interval of 1°. In Fig. 4 the number of the nearest angles r that their weight is reduced is selected to be 1. Therefore, at each step, we draw a random sample of angle ϕ_s from the discrete distribution, the weight of angle ϕ_s is set to 0 and the weight of the nearest angles ϕ_{s+1} and ϕ_{s-1} is reduced while the weight of the remaining angles is respectively increased.

The methodology starts by estimating the available flexibility for a predefined number of search directions, creating the basic shape of the flexibility curve. Following, at each step a new angle is selected from the updated distribution and the available flexibility is estimated at the specific search direction. Fig. 5 presents the discrete distribution at the first steps of the methodology, showing how the distribution is updated at each step.

The proper selection of parameter r, which defines the neighboring angles whose weights are updated, is crucial for the convergence of the methodology, since it regulates the locality applied to the already selected angles. Under the assumption that proximate angles will add less value on the estimated flexibility curve, parameter r must be selected properly in order to model this assumption, but not

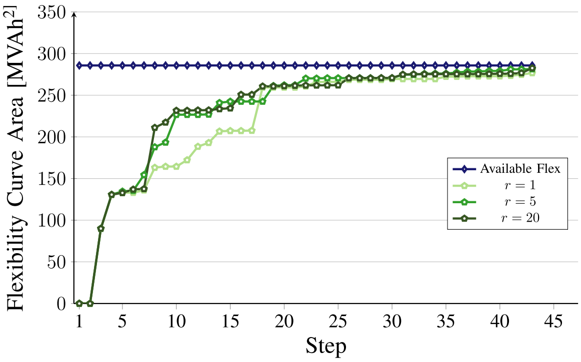

FIGURE 6. Convergence comparison sampling from a discrete distribution in the angle domain for different *r*.

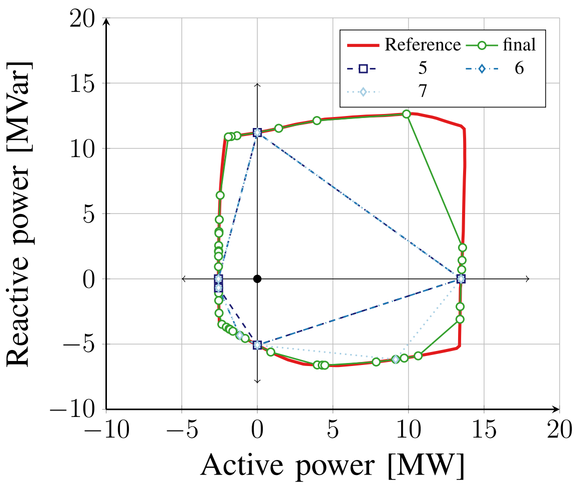

FIGURE 7. Sampling from a continuous distribution in the angle domain.

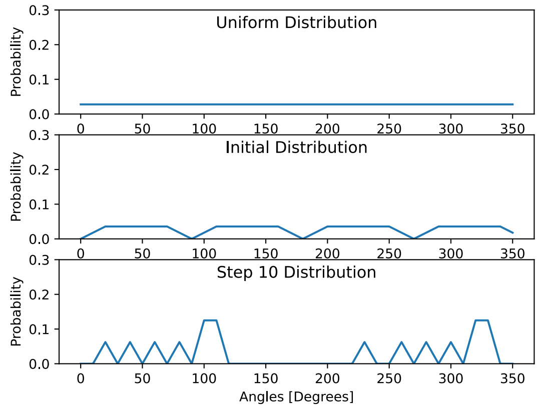

FIGURE 8. Iterative update of the continuous distribution in the angle domain.

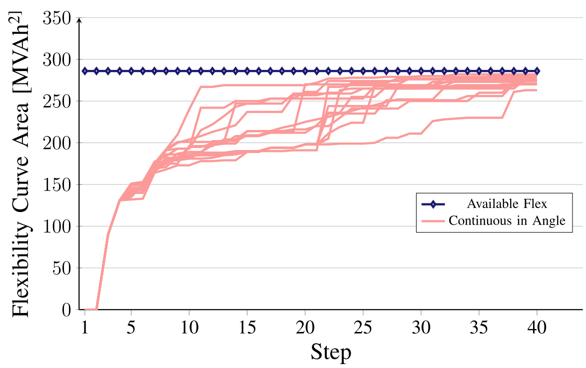

FIGURE 9. Convergence comparison sampling from a continuous distribution in the angle domain.

exclude values in the angle domain which might be important in the flexibility curve estimation. To demonstrate the impact of selection of r, we present in Fig. 6 the surface of the estimated flexibility curve in the above-mentioned example for different values of r.

C. SAMPLING FROM A CONTINUOUS DISTRIBUTION IN THE ANGLE DOMAIN

Although the selection of a large number of search directions increases the accuracy of the estimation of the flexibility curve, the discretization of the selection does not guarantee that all optimal points are covered. Therefore, in order to further increase the performance, the sampling of the search directions from a continuous distribution is used. For this, a methodology that fits a standard Gaussian kernel at each potential angle is proposed. Similarly to the previous methodology, the finite number of potential angles is chosen to be 360 with an angle interval of 1°.

Fig. 7 presents the estimated flexibility curve and the initial intermediate steps using an angle sampling from a continuous distribution in the angle domain for the same example. At each step, we draw a random sample of angle ϕ_s from the continuous distribution, and we estimate the available flexibility at the specific search direction. Next, the closest Gaussian kernel is removed and the sampling distribution is updated, as shown in Fig. 8.

The great advantage of this approach is that the sampling is not limited to a finite number of discrete angles but can have any value in the space [0,360]. The convergence of the methodology relies on the proper selection of the finite number of the observations that the Gaussian kernel will be fitted and the number of the nearest observations that are removed at each step.

Fig. 9 compares the surface of the estimated flexibility curve in the above-mentioned example for different executions of the same KDE-based methodology. The results indicate that the proposed methodology has better convergence, however the convergence is highly dependent to the sampling. The proper selection of the number of observations that will be selected to fit a Gaussian Kernel and the number of Kernels that are removed at each step is expected to impact the convergence performance of the proposed methodology.

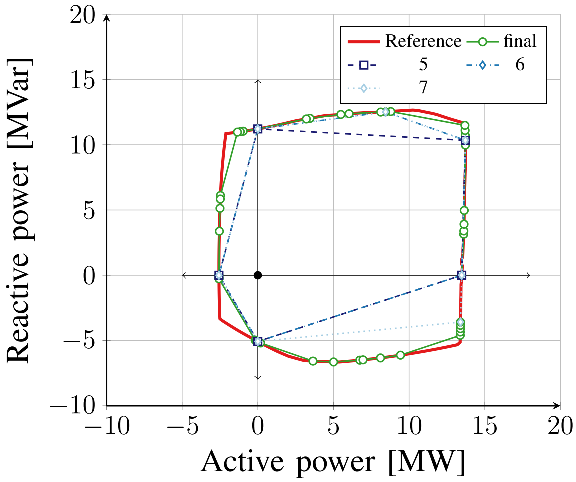

FIGURE 10. Sampling from a continuous distribution in the p-q domain.

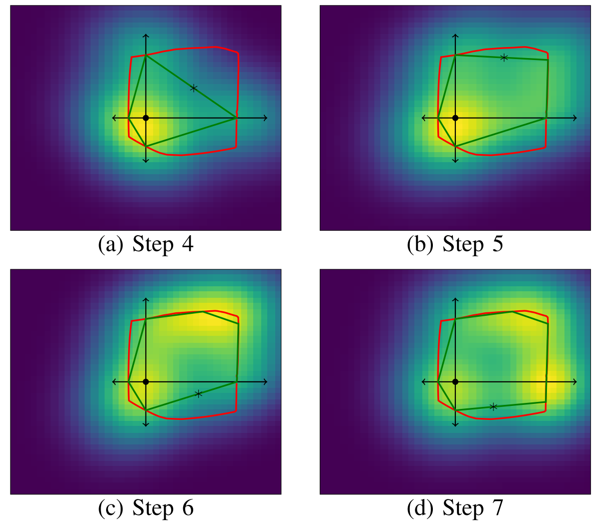

FIGURE 11. Iterative update of the p-q domain at steps 4-7.

D. SAMPLING FROM A CONTINUOUS DISTRIBUTION IN THE P-Q DOMAIN

The methodologies using sampling in the angle domain are based on the assumption that angles relatively close to the already chosen angles, have a lower probability to increase the area of the flexibility curve. The methodology using sampling from a continuous distribution in the p-q domain is proposed to further increase the convergence using directly the active and reactive power information of the already estimated points of the flexibility curve.

The estimated flexibility curve and the initial intermediate steps of this methodology are presented and compared to the reference curve in Fig. 10. The increased performance is demonstrated from the very first iterations. Following the formation of the basic flexibility curve based on the estimation of the initial (p,q) points at the search directions $0°, 90°, 180°, 270°$, the methodology creates a 2d likelyhood surface plane as shown in Fig. 11. The next search direction is chosen from $\hat{p}, \hat{q}$, which have the lowest likelihood, over the already estimated flexibility curve in order to obtain the most added value towards the coverage of the flexibility curve surface.

To further extend the accuracy of the methodology, we may also introduce a limit to the selection of $\hat{p}, \hat{q}$ points, introducing a set of linear constraints that bound the selection of $\hat{p}, \hat{q}$ points with the latest polygon of the estimated flexibility curve that was produced in the previous step. The combination of these two functions in each iteration determines the next search direction as the point on the polygon which minimizes the objective function (12).

This point is marked with an asterisk in Fig. 11 and the search direction of the next iteration is determined as $\phi = \arctan \hat{q}/\hat{p}$. The methodology manages to capture the crucial points of the flexibility curve in the first quadrant in steps 5 and 6. Step 7 manages to capture the other important edge of the flexibility curve in the fourth quadrant.

V. COMPARISON OF METHODOLOGIES

The effectiveness of the proposed methodologies is compared with the fixed-angle sampling for the radial 18-bus distribution system and the results are presented in Fig. 12. Each proposed methodology is compared with the fixed angle-based sampling with angle interval $10°, 15°, 30°$.

It is important to note that the computational time is determined by several factors, such as the choice of solver, the size and characteristics of the network and resources and the computational capabilities of the machine used to run the optimization. In our analysis, the optimization solution takes approximately 3.5 seconds per iteration to determine one point on the flexibility curve given a specific search direction for the radial 18 bus distribution system. Moreover, the computational time to determine the next search direction does not add any complexity or delay, since all proposed methods rely on single sampling procedures. Therefore, the computational performance of each method is determined by the iterations required to converge to the estimated flexibility curve. Table 2 presents the computational time of the proposed methods for estimating the flexibility curve with a specific accuracy. For example, to estimate the 90% of the flexibility curve of the presented test case, the method with a fixed angle-based sampling requires 109 seconds with angle interval 10°, and 77 seconds with angle interval 15°. The fixed angle with angle interval 30° estimates only the 85% of the flexibility curve. The proposed methods present increased computational efficiency since for the estimation of the 90% of the flexibility curve, the discrete sampling method in the angle-domain requires 56 seconds, the continuous sampling method in the angle-domain requires 38.5 seconds and the continuous sampling method in the p-q domain requires 31.5 seconds.

The results indicate that the discrete sampling method in the angle-domain reaches higher convergence compared to the fixed angle-based sampling. The continuous sampling

TABLE 2. Comparison of the computational performance of the proposed methods.

Estimation of the flexibility curve [%]	Computational time [sec] 80%	90%	95%
Fixed angle 10	94.5	109	119
Fixed angle 15	66.5	77	80.5
Fixed angle 30	38.5	-	-
Discrete in angle	28	56	73.5
Continuous in angle	35	38.5	77
Continuous in p-q	24.5	31.5	77

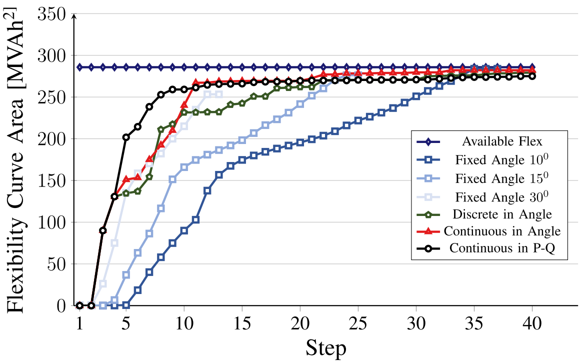

FIGURE 12. Convergence comparison of the proposed methodologies.

method in the angle-domain presents similar performance to the discrete sampling, despite the more accurate continuous sampling that is performed. This is reasonable considering that Gaussian Kernels are fitted in a large number of potential angles. Nevertheless, considering that we do not have prior knowledge for the proper selection of angle interval, the main advantage of these methodologies is that they will estimate the flexibility curve until the stopping criterion is reached. Therefore at the end of the operation they will manage to capture higher part of the available flexibility compared to the fixed angle-based sampling.

The continuous sampling in the p-q domain method appears to be the most advantageous methodology since it implicitly considers the available shape information from the previous estimated points. As shown in Fig. 12, the methodology manages to capture over the 90% of the surface of the flexibility curve in less than 10 steps. From this point and on the increase rate of the flexibility area at each step of the methodology is reduced and several steps are required to capture the whole flexibility potential of the active distribution grid.

VI. CONCLUSION

Distributed energy resources located in the distribution grid are sources of valuable flexibility for the whole power system. The exploitation of the provision of flexibility from these resources requires closer interaction and coordination between the TSOs and DSOs. This paper proposes a methodology to estimate the available flexibility of an active distribution grid at the point of interconnection with the transmission system with increased accuracy. An optimization-based approach is proposed to optimize the search directions to find the flexibility curve.

The proposed methodology presents several advances compared to the existing ones since the search directions are sampled from a distribution that is iteratively updated at each step. Three variations of angle sampling are examined, both in the angle domain and in the p-q domain. The proposed methodologies are a step forward towards a more robust and computational efficient estimation of the aggregated flexibility curve of active distribution grids at the TSO-DSO interconnection point.

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