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RESEARCH ARTICLE

A Resource Allocation Algorithm for Collaborative Networks Using Inferred Information

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ABSTRACT Even with the advent of 5G wireless communications and the millimeter wave spectrum, there will always be crowded frequency bands where multiple uncoordinated networks will have to contend (or collaborate) to squeeze as much throughput as possible while avoiding interference. This work proposes a branch and bound algorithm for maximizing the overall sum rate over multiple interfering networks with a pre-fixed set of offered flows, as well as a heuristic algorithm that individual networks can follow to collaboratively share the available bandwidth with that same objective. The latter algorithm finds a greedy solution by independently optimizing the links and routes for each network, and then refines that solution by discarding inefficient and potentially harmful links. It does not require any direct communication between the networks, relying instead on location estimates which could be inferred from interference powers. Simulation results show that, when the networks have different traffic loads, the proposed algorithm outperforms the original greedy solution as well as those based on partitioning the resources among the networks for their exclusive use.

INDEX TERMS Collaborative networks, branch and bound, resource allocation, power and rate control, link scheduling, non-convex optimization, wireless networks.

I. INTRODUCTION

The number of wireless devices is increasing exponentially and expected to continue doing so in the foreseeable future. Furthermore, users and applications demand more data at faster speeds, lower latencies, and higher reliability. Some researchers and telecommunications providers have turned towards the mmWave band (above 6 GHz) to find the necessary bandwidth to accommodate such growing demand, but the hardware and propagation limitations associated with high frequencies make mmWave communications unsuitable for many applications. The congestion in the sub-6GHz band is not expected to be alleviated anytime soon and it is therefore imperative that we find ways for different networks to coexist sharing the same spectrum. Prior research has addressed this problem using either shared databases (e.g., TV whitespace [1] and CBRS [2]) or sensing-based techniques (e.g., DARPA XG [3] and 3GPP [4]), but these approaches substantially underperform information theoretic limits. Information theory tells us that nearly any performance measure of multiple networks operating in a shared environment is bounded by the fully centralized case, where all nodes follow the orders of a shared controller. Centralized performance is clearly unachievable in practice, but allowing some level of collaboration or information sharing could help us approach that bound.

The goal of this paper is to design an algorithm for autonomous resource allocation across multiple uncoordinated wireless networks operating in a shared interference-limited environment. Each network will have complete

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knowledge of the channels between its own nodes and the intended receiver for their transmissions, but it will not know the impact that those transmissions might have on the receivers of peer networks. It will also overhear the interference caused by peer networks on its receivers. Treating that interference as noise, each network could perform a marginal optimization of the frequency band and power for its own transmissions ignoring the effect on peer networks, but this would be highly sub-optimal. In order to maximize the sum of the rates over all the networks we will study the optimal (fully centralized) solutions for different random configurations and attempt to leverage their common traits in the design of a heuristic distributed algorithm.

We will use the information-theoretical capacity as a performance metric, ignoring latency, security, and other practical considerations. The concept of capacity in wireless networks was pioneered by Gupta and Kumar in [5]. Their paper focused on asymptotic bounds for the number of successful links that can be simultaneously scheduled under two interference models: the physical model, where transmissions are successful when the SINR is above a pre-fixed threshold, and the protocol model, where they are successful as long as there are no other transmitters within a certain distance of the receiver. Subsequent works adopted the same models and extended the results to consider probabilistic success in the transmissions [6] and heuristics with constant approximation guarantees to the previous bounds [7], [8], [9]. However, they still focused on cases with asymptotically large number of nodes and offered loads. One of the main differences between this paper and most of the existing literature is that it attempts to solve the problem for a pre-fixed set of flows with specific sources, sinks, and maximum data rates.

Furthermore, modern communication systems have a lot of flexibility in terms of rate adaptation: when the channel is strong the transmitter can increase the throughput by using higher order modulations and/or lower code rates, and when when the channel has high attenuation it can do the opposite. Therefore, we find that neither the physical nor the protocol models are suitable for practical systems. Instead, we model the data rate on a link to be dependent on the SINR. Another significant difference between this paper and most of the existing literature (e.g., [10], [11]) is that we will use the information-theoretical capacity as a performance metric, instead of the number of successful links.

In environments where background noise is significantly stronger than the potential interference between networks (noise-limited), the capacity maximization problem becomes trivial: every node should transmit with as much power as it can using as much bandwidth as it can. However, when the background noise is relatively small (interference-limited) the problem of designing a strategy to maximize the sum rate of a network is very hard, mostly because the optimal strategy is different depending on the noise power and the positions of the nodes (equivalently, the channel gains).

It is well known that the simultaneous power and bandwidth allocation problem is NP-hard [12]. Over the last decade there have been multiple attempts at finding efficient methods to solve it, mostly through decomposition [13]. Palomar et al. published a compilation of the first results in this direction [14] and analyzed the advantages and disadvantages for each of them. However, there has been significant progress in this area since that compilation was published. Iterative approaches such as [15], [16], and [17] have been shown to provide better results than methods purely based on decomposition. More recently, there have been multiple methods proposed based on fractional programming techniques [18], [19], [20], information-theoretical inequalities for the optimality of treating interference as noise [21], [22], or heuristic methods based on the distance between the links [23]. As with many other problems, there has also been a plethora of machine learning (ML) solutions proposed [11], [24], [25], [26], [27], [28], [29], [30]. However, ML methods often require computational- and energy-intensive training and they are hard to interpret; we advocate for a simpler and more transparent approach. In many applications, it is common for resources to be sliced (in time, in frequency, or both) and auctioned among multiple networks for their exclusive use. Some researchers have studied this scenario and attempted to optimize the bidding process [31]. Another common approach in modern software defined radio (SDR) -based networks is to map multiple virtual channels to a few shared physical ones [32]. Yang et. al. proposed using a dynamic program to optimize such mapping [33], allowing reuse of the physical channels when the collision probability is below a pre-fixed threshold.

However, all of these recent methods are aimed at solving the sum-rate maximization problem (or some variation of it) for a single network and frequency band. They seek the set of links that should be scheduled simultaneously to maximize a certain utility function, assuming that the nodes are somewhat coordinated and it is possible to sacrifice all transmissions from some nodes in favor of those from others. This paper, on the other hand, attempts to solve the problem for multiple uncoordinated networks and frequency bands. It will assume that the networks operate independently without a standardized physical layer to enable the decoding and relaying of each other's messages as proposed in [34] and [35]. It is therefore unclear how any of them would know to stop its transmissions for the benefit of others. We will propose a scheme that enables collaboration using only the information that the networks can infer from the power of overheard transmissions.

Some studies have attempted to facilitate collaboration by assuming that networks can share certain information, such as the GPS locations of their nodes and their planned transmissions, with their peers. One of these studies is the DARPA Spectrum Collaboration Challenge (SC2), where ensembles of intelligent SDR networks exhibited autonomous collaborative behaviors and outperformed traditional RF schemes with evenly divided or individually reserved spectrum bands. The challenge was a big success, but its outstanding performance relied heavily on collaboration messages between the networks through a side channel [36]. This paper, on the other hand, assumes that the networks lack a protocol for exchanging collaboration messages, and proposes a collaboration algorithm which only uses inferred information from other networks' transmissions. Specifically, since every node can overhear the transmissions of peer networks' nodes as interference, it will be assumed that nodes in the same network can compare their interference patterns and estimate the position and transmit power of peer nodes through triangulation. Recent research has shown that there are many scenarios where geographical location information is enough to optimize link scheduling [30].

The rest of the paper is organized as follows. Section II establishes the system model to be considered throughout the rest of the paper. Sections III and IV present algorithms for optimizing the transmissions in a single frequency band. Section V proposes methods to extend the previous algorithms to multiple frequency bands. Finally, Section VI presents simulation results to illustrate the performance of the different algorithms and Section VII concludes the paper.

Notation: We use bold font to represent vectors (lower case) and matrices (upper case). The bold numbers $\mathbf{0}$ and $\mathbf{1}$ represent the all zeros and the all ones vectors, respectively, with dimensions that should be clear from the context. We use parentheses to construct column vectors from comma separated lists, i.e., $\mathbf{x} = (x_1, \ldots, x_n) = [\, x_1 \;\cdots\; x_n \,]^T$. The gradient of a scalar function respect to a vector is given by $\nabla_{\mathbf{x}} f = \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right)$. The comparison operators ($\le$, $\ge$, min, max) on vectors or matrices are understood as element-wise.

II. SYSTEM MODEL

The existing literature typically divides time into discrete time slots, and then optimizes the transmission schedule of the network within those slots. This is suitable for traditional radios where the transmitter and receiver signal processing chains are implemented in hardware, offering limited flexibility. A controller may switch the transmitter on or off and control the power in each time slot to achieve the desired schedule. However, with the recent advancement in SDRs, more dimensions can now be explored. An SDR transmitter can easily switch its frequency on demand, and cater to complicated signal processing requirements. We leverage this flexibility of SDRs and divide the available bandwidth into m identical bands or channels instead of dividing time. Theoretically, these are equivalent approaches, but dividing frequency instead of time offers some practical advantages. For instance, the nodes do not need to synchronize the time slots and multi-hop packets can be delivered to their destinations with less latency because the connecting hops can happen concurrently on different frequency channels, whereas in the time-division approach one hop may need to wait until the next cycle after the previous hop.

We consider N wireless networks deployed over a rectangular 2-dimensional region, with the i-th network consisting of n_i nodes distributed uniformly at random over the whole region. Each node is equipped with a single-antenna full-duplex transceiver, i.e., a node can be transmitting on one frequency band while it is receiving on another. Although some modern radios are capable of transmitting and receiving on the same frequency, this paper does not address that scenario. For simplicity, we will assume that nodes that transmit do so continuously in time, on whatever band they are assigned.

With $n = \sum_{i=1}^{N} n_i$ nodes and m frequency bands, there will be $L = nm$ potentially active links. These links are modeled as symmetric AWGN channels with path loss exponent $\alpha = 3$. Fast fading will be ignored, since our objective is maximizing the overall throughput over long periods. The signal received by node i at time t and channel f, assuming it is not transmitting on that channel, is given by

$$y_i^f(t) = \sum_{j=1}^{n} \sqrt{g_{ji}}\, x_j^f(t) + z_i^f(t), \tag{1}$$

where $x_j^f(t)$ is the signal transmitted by node j on channel f at time t (if any), g_{ji} the (power) path loss from node j to node i, and $z_i^f(t)$ the additive Gaussian noise at receiver i. For simplicity, we assume that the noise is white with the same variance σ^2 for all receiver nodes, that the path loss is identical for all the frequencies under consideration, and we neglect propagation delays. These assumptions could be easily removed, but it would complicate the equations unnecessarily.

The capacity of the ℓ-th ($\ell = 1, \ldots, L$) link is given by

$$c_\ell = \log(1 + \gamma_\ell), \tag{2}$$

where the unit of data rate is normalized by the bandwidth of each frequency channel and γ_ℓ denotes the SINR (signal-to-interference-and-noise ratio) of the link. Specifically, if the ℓ-th link goes from node ℓ_o to node ℓ_d,

$$\gamma_\ell = \frac{g_{\ell_o \ell_d} p_\ell}{\sum_{\nu \in \mathcal{T}_\ell \setminus \{\ell\}} g_{\nu_o \ell_d} p_\nu + \sigma^2}, \tag{3}$$

where $\mathcal{T}_\ell$ denotes the set of all links on the same frequency band as link ℓ, ν_o denotes the transmitter node for link ν, p_ν its transmit power, and σ^2 the noise level at ℓ_d. Without loss of generality, we assume unit maximum power, so $0 \le p_\ell \le 1$ for all ℓ.

Each network is being asked to deliver a certain number of flows with randomly chosen sources, sinks, and offered data rates. We use F to denote the total number of flows offered across all networks and $\mathbf{r} := (r_1, \ldots, r_F)$ to denote the corresponding vector of offered data rate for each flow. The data rate on each link is limited by its capacity in (2). We will only consider single-hop transmissions; flows cannot use relays to get from their transmitter to the receiver. This was done because introducing routing causes the problem to become exponentially more complex [13]. Each network can be centrally optimized with perfect knowledge of the path loss between its nodes as well as the source and destination for each flow, but no information can be exchanged between different networks.

However, we assume that each network can infer certain information about the others from their transmissions, namely the node locations and transmit powers. Since concurrent transmissions are generally scattered into different regions to avoid excessive interference, each network can estimate these through triangulation [37], [38]. Even if some nodes are mobile, we assume that the relative motion is slow and, with periodic re-estimations, the location and power estimates can be reasonably accurate for the period under consideration. We note that in some practical applications there could be pre-established collaboration protocols allowing networks to share the GPS locations and transmit powers of their nodes over a side channel, e.g., DARPA's Spectrum Collaboration Challenge [36]. This would render unnecessary the aforementioned information inference through triangulation.

The goal of this paper is to maximize the sum of the rates that can be delivered by all N networks. Unlike many prior works (e.g., [27], [39]), we do not include any fairness criteria in our objective function. Some networks or flows will be encouraged to deliver significantly more data than others if it increases the overall data rate. The data rate of a flow cannot be higher than its offered rate, but it can be lower, i.e., we allow partial delivery of the flows.

III. CENTRALIZED ALGORITHM

This section describes an efficient branch and bound method to find the optimal power allocation to maximize the total data rate in a single frequency band, subject to limits on the offered data rates for each flow. Many other papers have proposed branch and bound methods for the weighted sum-rate maximization problem [40], [41], but we did not find any which were suitable for limited offered rates.

The problem to be solved is

$$\begin{aligned} \text{maximize} \quad & \sum_{\ell=1}^{L} \min(r_\ell, \log(1+\gamma_\ell)) \\ \text{subject to} \quad & \gamma_\ell = \frac{g_{\ell_o \ell_d} p_\ell}{\sum_{\nu \in \mathcal{T}_\ell \setminus \{\ell\}} g_{\nu_o \ell_d} p_\nu + \sigma^2} \\ & \mathbf{0} \le \mathbf{p} \le \mathbf{1}. \end{aligned} \tag{4}$$

All branch and bound methods follow the same structure, summarized in Algorithm 1. They start from a feasible region guaranteed to include the desired solution, for example the vector of powers $\mathbf{p}$ with all components between 0 and 1. This region is then divided into two or more disjoint subregions, finding an upper and a lower bound for the objective value within each subregion. In a maximization problem such as the one we are dealing with, the lower bound can be the objective value of any feasible point within the subregion. The upper bound is slightly more complex and is normally obtained by solving a relaxed version of the original problem.

Algorithm 1 Branch and bound
Goal: Maximize obj($\mathbf{p}$) With $\mathbf{p} \in R_0$, With Tolerance ϵ

```
α := −∞;
list := {(R_0, ∞)};
while list ≠ ∅ do
    Pop (R, U) with largest U_R from list;
    if U ≥ α + ε then
        Split R into R^(1) and R^(2);
        for i ∈ {1, 2} do
            U^(i) := upper bound for obj(p), ∀p ∈ R^(i);
            L^(i) := any obj(p) with p ∈ R^(i);
            α := max(α, L^(i));
            if U^(i) ≥ α + ε then
                Push (R^(i), U^(i)) into list;
return α
```

If the upper bound of a subregion is smaller than the best (largest) of the lower bounds found so far, that subregion is discarded. Otherwise, it is stored in a list of candidate regions to be searched. The regions in the list are then processed in the same way as the initial one: subdivide, bound, and discard subregions if possible. As the subregions become smaller, the gap between the upper and lower bounds narrows, and more regions get discarded. Eventually, the remaining area to be searched (regions in the list) is small enough for us to claim that we have found the optimum within a certain tolerance.

In order to make the algorithm more efficient, we took advantage of a few special features of our problem:

1) *Initialization:* There exists an optimal solution where at least one node is transmitting at maximum power. This is due to the fact that the objective value in problem (4) does not decrease if all the powers are scaled by the same constant. Hence, our initial list of candidate regions is

$$\text{list} = \{(R_1, \infty), \ldots, (R_L, \infty)\}, \tag{5}$$

with $R_i = \{\mathbf{0} \le \mathbf{p} \le \mathbf{1} | p_i = 1\}$ and trivial infinite upper bounds.

2) *Splitting:* After a few iterations, each element (R, U) in the list consists of a power interval for each transmitter $R = \{\mathbf{l} \le \mathbf{p} \le \mathbf{u}\}$ and an upper bound U for the objective value obj($\mathbf{p}$) when $\mathbf{p} \in R$. The splitting is done by dividing the interval for one transmitter into two halves. The choice of transmitter can mean the difference between discarding both subregions or adding them to the list, so we tried all L possible splittings and picked the one yielding the lowest upper bounds $U^{(i)}$.

3) *Bounding:* The function

$$C(x, \mathbf{y}, z) = \log\left(1 + \frac{x}{\mathbf{a}^T \mathbf{y} + bz + c}\right) \tag{6}$$

is monotonically increasing in x and convex monotonically decreasing in $\mathbf{y}$ and z, as long as all coefficients and variables are positive. Hence, when $(x, \mathbf{y}, z)$ are lower and upper bounded by $(l_x, \mathbf{l}_y, l_z)$ and $(u_x, \mathbf{u}_y, u_z)$, respectively, the following linear bound holds:

$$C(x, \mathbf{y}, z) \leq C(u_x, \mathbf{l}_y, z) \tag{7}$$

$$\leq \rho(l_z) + (z - l_z)\frac{\rho(u_z) - \rho(l_z)}{u_z - l_z}, \tag{8}$$

where $\rho(\cdot) = C(u_x, \mathbf{l}_y, \cdot)$ and (8) follows from Jensen's inequality.

Finally, we are ready to explain the most complicated step in our branch and bound algorithm: the computation of the upper bounds U for the subregions. The goal is to find an upper bound for the achievable data rate

$$\text{obj}(\mathbf{p}) = \sum_{\ell=1}^{L} \min\left(r_\ell, \log\left(1 + \frac{g_{\ell_o \ell_d} p_\ell}{\sum_{\nu \neq \ell} g_{\nu_o \ell_d} p_\nu + \sigma^2}\right)\right) \tag{9}$$

in a subregion $\mathbf{l} \leq \mathbf{p} \leq \mathbf{u}$. We will compute one bound U_k based on each link power p_k, $k = 1, \ldots, L$ and then keep the tightest (lowest) one. The key idea is to assume minimum interference on link k and upper bound the data rate on the other links with either a linear function of p_k or with their offered rates r_i. The latter subset of links, which use r_i as a bound, is denoted by V. Algorithm 2 summarizes the main steps in the computation of the bound; the rest of this section provides a more elaborate description.

For each component $k = 1, \ldots, L$, first observe that for any subset of links $V \in \{1, \ldots, L\}$ that does not include k

$$\text{obj}(\mathbf{p}) \leq \sum_{\ell \in V} r_\ell + \sum_{\ell \notin \{V,k\}} \min\left(r_\ell, C_\ell(p_\ell, \mathbf{p}_{\setminus\{\ell,k\}}, p_k)\right) + \min\left(r_k, \log\left(1 + \lambda_k p_k\right)\right) \tag{10}$$

with

$$\lambda_k = \frac{g_{k_o k_d}}{\sum_{\nu \neq k} g_{\nu_o \ell_d} l_\nu + \sigma^2} \tag{11}$$

and

$$C_\ell(p_\ell, \mathbf{p}_{\setminus\{\ell,k\}}, p_k) = \log\left(1 + \frac{g_{\ell_o \ell_d} p_\ell}{\sum_{\nu \notin \{\ell,k\}} g_{\nu_o \ell_d} p_\nu + g_{k_o \ell_d} p_k + \sigma^2}\right), \tag{12}$$

where $\mathbf{p}_{\setminus\{\ell,k\}}$ denotes the vector of powers without components p_ℓ and p_k. Then we consider two cases, comparing the offered rate r_k with the link capacity $\log(1 + \lambda_k l_k)$.

If $r_k \leq \log(1 + \lambda_k l_k)$, the last term in (10) can be replaced with r_k. We then choose $V = \emptyset$, $p_\ell = u_\ell$, and $\mathbf{p}_{\setminus\{\ell\}} = \mathbf{l}_{\setminus\{\ell\}}$ (maximum signal power and minimum interference) to obtain the bound

$$U_k := r_k + \sum_{\ell \neq k} \min\left(r_\ell, C_\ell(u_\ell, \mathbf{l}_{\setminus\{\ell,k\}}, l_k)\right). \tag{13}$$

If $r_k \geq \log(1 + \lambda_k l_k)$, let $V = \{\ell | r_\ell \leq C(u_\ell, \mathbf{l}_{\setminus\{\ell,k\}}, u_k)\}$ be the set of indices for which we adopt the offered rate r_i as the upper bound and observe that (12) has the same form as (6). Hence, we can use the linear bound in (8) to rewrite (10) in the form

$$\text{obj}(\mathbf{p}) \leq a_k p_k + b_k + \log(1 + \lambda_k p_k), \tag{14}$$

where

$$a_k = \sum_{\ell \notin \{V,k\}} \frac{\rho_{\ell,k}(u_k) - \rho_{\ell,k}(l_k)}{u_k - l_k} \tag{15}$$

$$b_k = \sum_{\ell \in V} r_\ell - a_k l_k + \sum_{\ell \notin \{V,k\}} \rho_{\ell,k}(l_k) \tag{16}$$

$$\rho_{\ell,k}(p_k) = C_\ell(u_\ell, \mathbf{l}_{\setminus\{\ell,k\}}, p_k). \tag{17}$$

In order to find a global bound to $\text{obj}(\mathbf{p})$ that holds for all $\mathbf{l} \leq \mathbf{p} \leq \mathbf{u}$, we need to find the p_k which maximizes (14). Let

$$\delta(p_k) = a_k + \frac{\lambda_k}{1 + \lambda_k p_k} \tag{18}$$

be the derivative of the right hand side of (14) with respect to p_k. Since λ_k is always positive, $\delta(p_k)$ is monotonically decreasing. We derive a different bound depending on which of the following cases happens:

- When $\delta(u_k) \geq 0$, the right hand side of (14) is non-decreasing for $l_k \leq p_k \leq u_k$. We plug $p_k = u_k$ into (14) to obtain the bound

$$U_k := \sum_{\ell \in V} r_\ell + \sum_{\ell \notin \{V,k\}} \rho_{\ell,k}(u_k) + \log\left(1 + \lambda_k u_k\right). \tag{19}$$

- When $\delta(l_k) \leq 0$, the right hand side of (14) is non-increasing for $l_k \leq p_k \leq u_k$. If $r_i \leq \rho_{i,k}(l_k)$ for any $i \notin V$, we add i to V, recompute (14) and (18), and reconsider these three cases. Otherwise, we plug $p_k = l_k$ into (14) to obtain the bound

$$U_k := \sum_{\ell \in V} r_\ell + \sum_{\ell \notin \{V,k\}} \rho_{\ell,k}(l_k) + \log\left(1 + \lambda_k l_k\right). \tag{20}$$

- When $\delta(u_k) \leq 0 \leq \delta(l_k)$, the maximum is attained inside the interval $p_k \in (l_k, u_k)$. We solve for $\delta(p_k) = 0$ and obtain $p_k^\star = -\frac{\lambda_k + a_k}{\lambda_k a_k}$. If $r_i \leq \rho_{i,k}(p_k^\star)$ for any $i \notin V$, we add i to V, recompute (14) and (18), and reconsider these three cases. Otherwise, we plug $p_k^\star$ into (14) to obtain the bound

$$U_k := b_k - \frac{a_k + \lambda_k}{\lambda_k} + \log\left(-\frac{\lambda_k}{a_k}\right). \tag{21}$$

In summary, the upper bound for the achievable data rate in a subregion $\mathbf{l} \leq \mathbf{p} \leq \mathbf{u}$ is found as $U = \min_{1 \leq k \leq L} U_k$ and the upper bound U_k associated with the k-th link is given by one of (13), (19), (20) or (21).

Fig. 2 shows an example of the optimal transmissions found for an ensemble of 4 networks with 8 nodes each and $\alpha = 3$ attenuation exponent. Observing many such examples showed that the transmissions tend to be rather sparse, with most of the nodes not transmitting at all. Furthermore, the nodes that do transmit often do so with maximum power.

Algorithm 2 Upper Bound for Region R
Goal: Find $\mathrm{U} \geq \mathrm{obj}(\mathbf{p})$ for $\mathbf{l} \leq \mathbf{p} \leq \mathbf{u}$

```
Function Main:
    for k = 1, 2, ..., L do
        if r_k ≤ log(1 + λ_k l_k) then
            U_k := (13);
        else
            V := {i | r_i ≤ ρ_{ℓ,k}(u_k)};
            U_k := find_bound(V, k);
    return U := min(U_k);
Function find_bound(V, k):
    δ(p) defined as in (18);
    if δ(u_k) ≥ 0 then
        return U_k from (19);
    else if δ(l_k) ≤ 0 then
        W := {i ∉ V | r_i ≤ ρ_{i,k}(l_k)};
        if W = ∅ then
            return U_k from (20);
    else
        W := {i ∉ V | r_i ≤ ρ_{i,k}(p*_k)};
        if W = ∅ then
            return U_k from (21);
    return U_k := find_bound(V ∪ W, k);
```

IV. DISTRIBUTED ALGORITHMS

With multiple networks sharing the same time-frequency resources, we consider two algorithms that the networks may use: greedy and collaborative. In the first algorithm, every network behaves greedily trying to maximize its own data rate irrespective of the interference it causes onto other networks. In the second algorithm, the networks attempt to achieve a better data rate as a whole than the purely greedy approach.

Note that if the networks were able to share perfect information (sources and sinks, offered flow rates, etc.) with each other, then they would be able to find the optimal (centralized) strategy using the centralized algorithm in Section III. However, such computation can be costly because of the super-exponential growth in search space when the centralized controller needs to consider all networks' nodes. Additionally, practical scenarios generally do not allow the different networks to share every piece of information due to limited inter-network communications capability, privacy concerns, etc.

Still, it is generally possible for every network to keep track of the expected amount of noise plus interference from other networks' transmissions at each of its own nodes. Our goal is to find a collaborative algorithm that takes advantage of the limited information that each network can infer about its peers and steers the networks to a transmission configuration that resembles what would be found by a centralized controller.

Algorithm 3 Greedy Algorithm (gradient ascent)

```
p := 0.5;
μ := 0.01;
I := received signal+interference powers;
∇_p(1^T θ*) := (25);
while ∇_p(1^T θ*) ≥ ε do
    p := (min(1, p^(i) + μ∇_p(1^T θ*)))_+;
    I := update signal+interference powers;
    ∇_p(1^T θ*) := (25);
```

A. GREEDY ALGORITHM

In the greedy algorithm, a network does not care about the performance of its peers; it optimizes its transmission powers treating the interference as white noise [22]. The σ^2 term in (3) is therefore replaced with the corresponding noise plus interference estimates.

Let $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{n_i})$ denote the data rates on the n_i links in network i. Then naturally our objective is to maximize their sum $\mathbf{1}^T\boldsymbol{\theta}$ with $0 \leq \theta_\ell \leq \min(r_\ell, c_\ell)$ and c_ℓ given by (2). Given a vector $\mathbf{p}$ of transmit powers for each link, the optimization problem can be formulated as

$$\begin{aligned} \text{maximize} \quad & \mathbf{1}^T\boldsymbol{\theta} \\ \text{subject to} \quad & \boldsymbol{\theta} \leq \mathbf{c}(\mathbf{p}) \\ & \boldsymbol{\theta} \leq \mathbf{r} \\ & \mathbf{0} \leq \mathbf{p} \leq \mathbf{1}, \end{aligned} \tag{22}$$

with optimal solution $\boldsymbol{\theta}^\star = \max(\mathbf{r}, \mathbf{c}(\mathbf{p}))$. Unfortunately, the link capacities $\mathbf{c}(\mathbf{p})$ are not a concave function of the transmit powers, so optimizing $\mathbf{p}$ is not a convex problem.

It is possible use the centralized algorithm from Section III to solve this problem, but numerical simulations have shown that when multiple networks successively perform such sudden updates to their transmit powers, the ensemble often converges to bad solutions or fails to converge altogether. Instead, we use gradient ascent to perform somewhat smoother updates: initialize $\mathbf{p}^{(0)}$ with a feasible set of values (e.g., $1/2$ on all links) and then iteratively update $\mathbf{p}$ with a step size of μ along the gradient of the objective. In order to ensure that the updated $\mathbf{p}$ is feasible, its components are cropped to be between 0 and 1, i.e.,

$$\mathbf{p}^{(i+1)} = \left(\min\left(\mathbf{1}, \mathbf{p}^{(i)} + \mu\nabla_{\mathbf{p}}(\mathbf{1}^T\boldsymbol{\theta}^\star)\right)\right)_+, \tag{23}$$

where $(\cdot)_+$ is a function that replaces all negative elements with zeros. After each update, the interference from peer networks is re-assessed to incorporate its changes in the computation of subsequent gradient directions. Numerical simulations have shown that when all the networks follow this procedure, their transmit powers usually converge to a locally optimal solution for the ensemble problem.

The gradient in (23) can be found using the chain rule as follows

$$\frac{\partial(\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}^{\star})}{\partial \mathrm{p}_{\ell}} = \boldsymbol{\lambda}^{\mathrm{T}} \cdot \frac{\partial \mathbf{c}(\mathbf{p})}{\partial \mathrm{p}_{\ell}}, \tag{24}$$

where $\boldsymbol{\lambda} := \nabla_{\mathbf{c}}(\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}^{\star}) = (\lambda_1, \ldots, \lambda_{\mathrm{n_i}})$ is the gradient of the optimal objective value with respect to the channel capacity vector $\mathbf{c}$ and can be obtained as the dual variables corresponding to the first constraint in problem (22), *i.e.* , $\lambda_\ell = 1$ when $\mathrm{c}_\ell \geq \mathrm{r}_\ell$ and $\lambda_\ell = 0$ otherwise [42]. Applying (2) and (3), (24) can be expressed as

$$\frac{\partial(\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}^{\star})}{\partial \mathrm{p}_{\ell}} = \frac{\lambda_\ell \mathrm{g}_{\ell_\mathrm{o}\ell_\mathrm{d}}}{\mathrm{I}(\ell_\mathrm{d})} - \sum_{\nu \in \mathcal{T}_\ell \setminus \{\ell\}} \left(\frac{\lambda_\nu \mathrm{g}_{\ell_\mathrm{o}\nu_\mathrm{d}} \mathrm{g}_{\nu_\mathrm{o}\nu_\mathrm{d}} \mathrm{p}_\nu}{\mathrm{I}(\nu_\mathrm{d})[\mathrm{I}(\nu_\mathrm{d}) - \mathrm{g}_{\nu_\mathrm{o}\nu_\mathrm{d}} \mathrm{p}_\nu]} \right), \tag{25}$$

where $\mathrm{I}(\ell_\mathrm{d}) := \sum_{\nu \in \mathcal{T}_\ell} \mathrm{g}_{\nu_\mathrm{o}\ell_\mathrm{d}} \mathrm{p}_\nu + \sigma^2$ represents the total power (signal, interference, and noise) being received by node ℓ_d. The other variables are defined in the same way as in (3).

The greedy algorithm is able to provide a feasible vector of transmit powers, but they are clearly sub-optimal. An optimal configuration should only allow a link to transmit when it yields a throughput greater than the total decrease in throughput that its interference is causing on other networks. The greedy algorithm neglects this consideration and results in a transmission plan that is often too crowded. Every network tries to "shout" as loudly as possible to get their data through, causing severe interference to its peers and having a counterproductive effect. Although it may locally appear that increasing the power of a link is beneficial to the overall capacity, turning it off completely can offer a larger gain in peer networks' capacity that offsets the loss. Fig. 2 in Section VI shows a typical sample of the transmission links found by the different algorithms. This and other empirical evidence suggest that dropping links according to the rate-to-power ratio typically performs better than gradient ascent.

B. COLLABORATIVE ALGORITHM

Algorithm 4 Collaborative Algorithm

```
p ← Greedily optimize link powers within network;
links ← active links sorted by rate-to-power ratio;
for ℓ ∈ links do
    gain := ΔC_S + λΔE[C_P] by (28) and (29);
    if gain > 0 then
        p_ℓ := 0;
        Î_i(j) := interference estimates by (27);
        p := reoptimize remaining active link powers;
    else
        return p;
```

In light of the above considerations, we wish to design a distributed protocol that the networks can follow to decide whether a link should be dropped and how much power to allocate otherwise, with the goal of maximizing the sum rate across the whole ensemble. There will be no direct communication between different networks, but it is assumed that all of them can estimate the position of peer nodes and their transmit powers based on the received interference. Our algorithm starts from the greedy solution described in subsection IV-A and then refines it by discarding links carrying low data rates but whose interference could be having a significant impact on peer networks.

Let $\mathcal{S}$ be the set of nodes in a network and $\mathcal{P}$ those in all peer networks. The greedy algorithm in subsection IV-A uses (24) and (25) to find the gradient of the network capacity $\mathrm{C}_\mathcal{S} = \mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}$ with respect to the transmit powers. One natural extension to this scheme is to include the expected capacity of peer networks $\mathrm{E}[\mathrm{C}_\mathcal{P}]$ into the objective function of problem (22). Since a network can estimate the locations and transmit powers of peer nodes through triangulation, it can also estimate their total received radio power. For a node i, denote its estimated location and transmit power as $\hat{\mathrm{x}}_\mathrm{i}$ and $\hat{\mathrm{p}}_\mathrm{i}$, respectively, and let $\hat{\mathrm{g}}_{\mathrm{ij}} := \|\hat{\mathrm{x}}_\mathrm{i} - \hat{\mathrm{x}}_\mathrm{j}\|^{-\alpha}$ be the estimated channel gain between nodes i and j. Although every network is aware of its peers' transmissions, it does not know their intended receivers, nor which nodes belong to which network. Thus the algorithm assumes that every peer node is equally likely to be the receiver of any peers' transmission. This way, it can express the total expected capacity of peer networks as follows:

$$\mathrm{E}[\mathrm{C}_\mathcal{P}] = \frac{1}{|\mathcal{P}| - 1} \sum_{\mathrm{i} \in \mathcal{P}} \sum_{\mathrm{j} \in \mathcal{P} \setminus \{\mathrm{i}\}} \log\left(1 + \frac{\hat{\mathrm{g}}_{\mathrm{ij}} \hat{\mathrm{p}}_\mathrm{i}}{\hat{\mathrm{I}}_\mathrm{i}(\mathrm{j})}\right), \tag{26}$$

where $\hat{\mathrm{I}}_\mathrm{i}(\mathrm{j})$ is the estimated noise-plus-interference at node j when receiving the signal from node i. It can be computed as

$$\hat{\mathrm{I}}_\mathrm{i}(\mathrm{j}) = \sigma^2 + \sum_{\mathrm{k} \in \mathcal{S} \cup \mathcal{P}} \hat{\mathrm{g}}_{\mathrm{kj}} \hat{\mathrm{p}}_\mathrm{k} - \hat{\mathrm{g}}_{\mathrm{ij}} \hat{\mathrm{p}}_\mathrm{i}. \tag{27}$$

Adding this to the objective function in problem (22) results in $\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta} + \lambda \mathrm{E}[\mathrm{C}_\mathcal{P}]$, where λ is a parameter that determines how "considerate" each network is in weighing its own data rate with respect to that of peer networks. Unfortunately, this function is not concave and gradient ascent is not guaranteed to converge to a global maximum. Instead, we study the effect of dropping each active link by making its transmit power equal to 0.

Consider an active link $\ell \in \mathcal{S}$ with power p_ℓ. If it is turned off, we lose the data rate currently carried by that link $\boldsymbol{\theta}_\ell$, but the expected capacity for the remaining links will increase due to the reduced interference. The gain in the link's own network can be expressed as

$$\Delta \mathrm{C}_\mathcal{S} = \sum_{\nu \in \mathcal{S} \setminus \{\ell\}} \log\left(1 + \frac{\hat{\mathrm{g}}_{\nu_\mathrm{o}\nu_\mathrm{d}} \mathrm{p}_\nu}{\hat{\mathrm{I}}_{\nu_\mathrm{o}}(\nu_\mathrm{d}) - \hat{\mathrm{g}}_{\ell_\mathrm{o}\nu_\mathrm{d}} \mathrm{z}}\right)\Bigg|_{\mathrm{z}=0}^{\mathrm{z}=\mathrm{p}_\ell}. \tag{28}$$

For peer networks, since the network does not know the routing information, it assumes that every peer node is equally likely to be the receiver of every transmission. Then, it can find the total expected gain in peer networks as follows:

$$\Delta \mathrm{E}[\mathrm{C}_{\mathcal{P}}] = \frac{1}{|\mathcal{P}|-1} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P} \setminus \{i\}} \log\left(1 + \frac{\hat{g}_{ij}\hat{p}_i}{\hat{I}_i(j) - \hat{g}_{\ell_o j} z}\right)\Bigg|_{z=0}^{z=p_\ell}. \quad (29)$$

If the network finds that the data rate on link ℓ is sufficiently smaller than the overall expected gain in capacity, it should drop the link, and vice versa. In other words, the condition that link ℓ should be dropped is

$$\boldsymbol{\theta}_\ell < \Delta \mathrm{C}_{\mathcal{S}} + \lambda \Delta \mathrm{E}[\mathrm{C}_{\mathcal{P}}]. \quad (30)$$

The mean value theorem states that $f(a) - f(0) = a \cdot f'(\xi)$ for some $0 < \xi < a$, as long as $f(x)$ is a continuous and differentiable real function. Applying this theorem to (28) and (29), we find that the condition in (30) is equivalent to

$$\frac{\boldsymbol{\theta}_\ell}{p_\ell} < \sum_{\nu \in \mathcal{T}_\ell \setminus \{\ell\}} \frac{\hat{g}_{\ell_o \nu_d}\hat{g}_{\nu_o \nu_d} p_\nu}{K_{\nu_o \nu_d}(K_{\nu_o \nu_d} + \hat{g}_{\nu_o \nu_d} p_\nu)} + \frac{1}{|\mathcal{P}|-1} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P} \setminus \{i\}} \frac{\hat{g}_{\ell_o j}\hat{g}_{ij}\hat{p}_i}{K_{ij}(K_{ij} + \hat{g}_{ij}\hat{p}_i)}, \quad (31)$$

for some $0 < \xi < p_\ell$, where $K_{ij} := \hat{I}_i(j) - \hat{g}_{\ell_o j}\xi$.

This suggests that the dropped links should be those with low rate-to-power ratio (often referred to as energy efficiency [43], [44]). This makes intuitive sense: if a node transmits with high power, thereby causing strong interference, but only provides a small throughput, it should stop transmitting. The exact rate-to-power threshold below which the links should be dropped is unknown, since the mean value theorem guarantees the existence of ξ but does not specify how it can be found. The rate-to-power threshold that determines whether a link is dropped cannot be a fixed constant. Networks with different parameters have different optimal thresholds. For instance, if the network is densely packed with nodes competing to transmit, it is desirable to limit the number of simultaneous transmissions to reduce interference, so more links should be dropped; however, if the nodes are scattered across a very large region, the background noise dominates the interference and it is better to allow more concurrent transmissions, and thus a lower threshold should be used.

We propose that each network sorts its links in increasing order of rate-to-power ratios and drops them in that order as long as doing so increases the expected ensemble capacity according to (30). It is important to note that the estimated interference $\hat{I}_i(j)$ for each pair of nodes (i, j) should be updated every time a link is dropped, accounting for any links that peer networks might have dropped. After dropping a link, each network should re-optimize the transmit power of its remaining active links, using either the greedy or the centralized (branch and bound) algorithms. Since dropped links are never reactivated, the algorithm is guaranteed to converge. This procedure is summarized in Algorithm 4.

Real wireless scenarios are generally dynamic, with moving nodes, load variations, and changing channel conditions. Therefore, the resource allocation configuration should be reassessed periodically, or whenever a network detects significant changes in the environment.

V. EXTENSION TO MULTIPLE BANDS

A very simple method for minimizing inter-network interference and contention is to split the time and/or bandwidth into multiple resource blocks and assign each of them to one of the networks for its exclusive use. This is very easy to implement and hence has become the basis for many standards, but such partition of resources is suboptimal because it precludes parallel transmissions reusing the same time and frequency. So, the question naturally arises of whether the reduced interference makes up for the lack of parallel transmissions by different networks. Furthermore, one may wonder whether splitting the resources into multiple blocks might be beneficial even without exclusive assignments, i.e. having all networks share multiple smaller resource blocks instead of a single bigger one. This would enable the activation of different sets of links in different resource blocks (henceforth assumed frequency bands) instead of using a fixed configuration.

Prior sections proposed protocols for maximizing the amount of data that could be simultaneously transferred over a single frequency band. This section will extend those protocols to scenarios with multiple bands, but first it will prove that both sharing the bandwidth and splitting the available resources into multiple blocks are indeed beneficial in some cases, but not always. Subsequent simulation results in subsection VI-B will show that the gains for both are comparable, i.e. sharing a single band yields similar results to splitting it into multiple (non-shared) channels. The best results, however, are obtained when we use multiple shared channels.

As previously mentioned, the information-theoretical capacity of a link is concave with respect to the signal power and convex with respect to the interference. Hence, the sum rate of multiple interfering links is neither convex nor concave. This makes its maximization a non-convex problem. As with many non-convex problems, the highest objective value is not achieved with a fixed solution but rather by time-sharing (or frequency-sharing in our case) multiple configurations. In layman's terms, this means that rather than having a fixed set of active links, we can sometimes achieve a higher overall throughput by cycling through multiple different configurations. The following example proves this claim.

Example 1: Consider the highly simplified scenario shown in Fig. 1 of four nodes located in the corners of a $D \times D$ square, with quadratic signal attenuation and additive white Gaussian noise (AWGN). If the two nodes on the left want to transmit to the two nodes on the right, the question arises: would it be better to have both nodes transmit simultaneously or to give each of them exclusive use of half of the channel? The answer depends on the noise power, the distance between the nodes, and how they are paired. If the links go along the sides of the square, the overall capacity for each strategy is

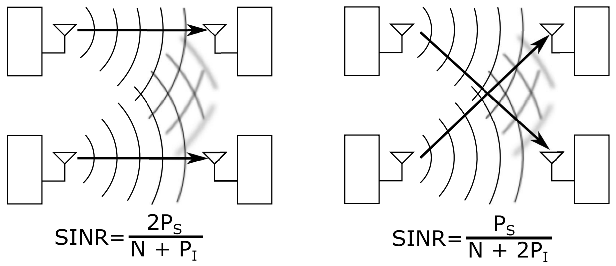

FIGURE 1. Signal to interference and noise ratio for two pairs of nodes in the corners of a square and $\alpha = 2$.

respectively given by

$$C_{sim} = 2\log\left(1 + \frac{2P}{N_0 + P}\right) \tag{32}$$

$$C_{half} = \log\left(1 + \frac{2P}{N_0}\right), \tag{33}$$

where $N_0/2$ is the noise power and $P = P_T/D^2$ denotes the received signal power, given by the transmit power divided by the distance squared. When $N_0 = P$ we have that $C_{sim} > C_{half}$, but the inequality inverts when N_0 becomes small enough. Similarly, if the nodes are attempting to transmit along the diagonals of the square, the overall capacities for the two strategies are given by

$$C_{sim} = 2\log\left(1 + \frac{P}{N_0 + 2P}\right) \tag{34}$$

$$C_{half} = \log\left(1 + \frac{P}{N_0}\right). \tag{35}$$

When $N_0 = P$ we have the opposite conclusion as in the previous case: $C_{sim} < C_{half}$, but the inequality once again inverts if the noise power becomes large enough.

Therefore, we can conclude that there are noise values and node positions for which it is better to have both links simultaneously active and others for which it is better to split the resources. Even in a simplified scenario such as this one, finding the optimal strategy requires carefully analyzing the received signal, interference, and noise powers. In a more realistic situation with a larger number of nodes in arbitrary locations the problem becomes exponentially more complex.

We now show how the protocols from prior sections can be extended to a scenario with multiple sub-bands. The branch and bound algorithm described in Section III can be seamlessly applied to such scenario by defining the initial search regions to include the transmit power of each node on each band. However, the time required for the algorithm to converge increases exponentially with the dimension of the search regions, so its execution becomes impractical for all but a very small number of sub-bands. Instead, we propose a greedy application of the algorithm where the bands are optimized sequentially.

The same idea is used to extend the greedy and collaborative algorithms: Algorithms 3 and 4 are repeatedly executed to decide the transmit powers of every node on each frequency sub-band, updating their offered loads after each decision is made. It is important to take into account that the link capacities must be scaled by the width of each sub-band. When the offered loads are relatively small compared with the total capacity of the network, this results in a disproportionate amount of information being transmitted in the first few sub-bands, leaving the latter ones unused because all the offered load has been delivered. However, since our performance metric is the total sum-rate over all networks, we are not concerned with this load unbalance. On the other hand, when the offered loads are much larger than the link capacities, this greedy application of the algorithms results in an identical configuration of active links for every sub-band. Again, this is logical, since the system is ignoring the offered load limits and attempting to maximize the the transmission rate on every sub-band.

VI. NUMERICAL RESULTS

This section simulates multiple networks with nodes uniformly distributed over a 100 m × 100 m square region and background noise variance of $\sigma^2 = 10^{-8}$. Each network will be asked to carry a few flows with randomly chosen sources and sinks. Subsection VI-A focuses on a simplified scenario with a single frequency band and subsection VI-B then presents simulation results for networks with multiple bands and requirements. These results support the validity of our algorithms and illustrate their performance in a complex scenario.

Before presenting our results, we briefly discuss the Kesselheim algorithm given in [10] which will be used as a framework for comparison. The goal of the algorithm is to acquire a subset of feasible links under the physical model, where transmissions are successfully received if and only if the SINR at the receiver is greater than a given value β. It starts with an empty set of active links M and then, it traverses the links in increasing order of length activating those that satisfy $a_M(\ell) \leq \tau$, with

$$a_M(\ell) = \sum_{\nu \in M} \frac{\hat{g}_{\nu_0 \ell_d} + \hat{g}_{\ell_o \nu_d}}{\hat{g}_{\nu_o \nu_d}} \tag{36}$$

$$\tau = \frac{1}{2 \cdot 3^\alpha \cdot (4\beta + 2)}. \tag{37}$$

After the active links have been selected, their transmission powers are recursively set as follows. We traverse the active links in decreasing order of length, assigning power 1 to the longest link, and scaling the remaining ones according to

$$p(\ell) = 4\beta \sum_{\substack{\nu \in M \\ \hat{g}_{\nu_0 \nu_d} < \hat{g}_{\ell_0 \ell_d}}} p(\nu) \frac{\hat{g}_{\nu_0 \ell_d}}{\hat{g}_{\ell_0 \ell_d}} \tag{38}$$

This guarantees that all active links can successfully transmit simultaneously with SINR $\geq \beta$.

A. SINGLE-BAND

Before we attempt to study a complex scenario with multiple frequency bands, we want to analyze the capacity of

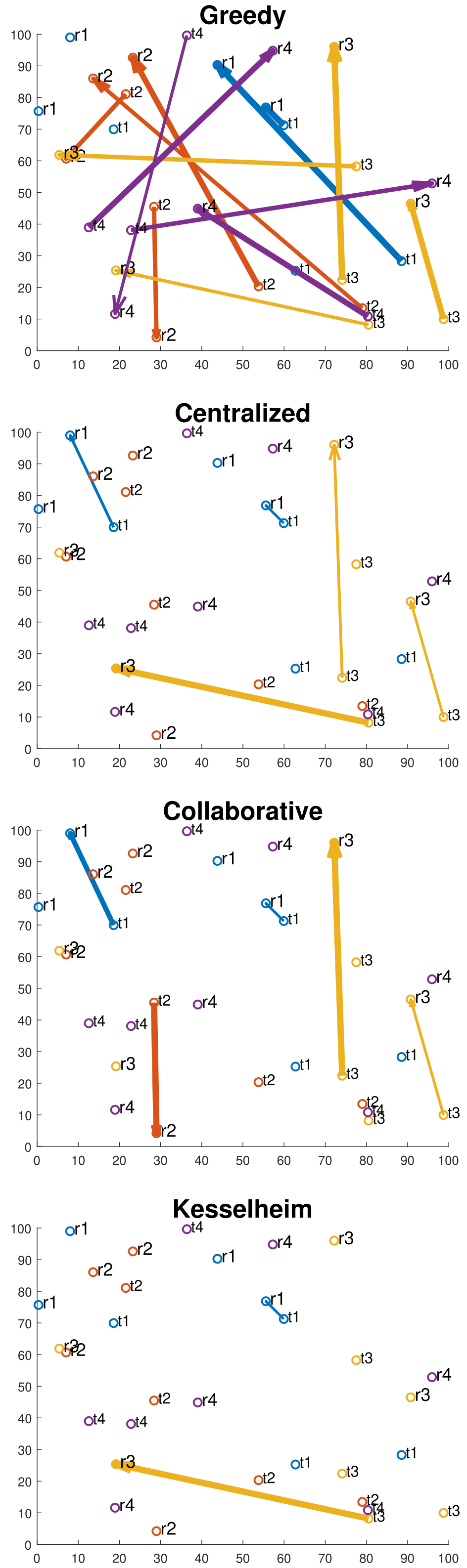

FIGURE 2. Sample links and powers with different algorithms.

our algorithm in a simplified case with only one band. The problem is reduced to finding the set of links that should be simultaneously activated and their transmit powers so as to maximize the total throughput of the ensemble. Specifically, we implemented our simulations in a scenario with $N = 4$ networks with $n_i = 8$ nodes each, $F = 16$ flows (4 per network), path-loss $\alpha = 3$, and $\beta = 1.2$. N and n_i have been chosen arbitrarily to evaluate the behaviour of the networks with an acceptable computational time. We study the performance of our algorithm with limited offered rates. Therefore for each network configuration (i.e., source and sink positions) we attempt to maximize the overall throughput when the flow rates $\boldsymbol{\theta}$ are constrained to be below a vector of values $\mathbf{r}$. The components of this last vector ($\mathbf{r}$) are uniformly distributed between 0 and a pre-fixed maximum data rate per flow. Our collaborative algorithm was simulated by cycling through the networks and dropping one link at a time when necessary. When updating the interference estimates by (27) each network took into account the links dropped by its peers. The reported results correspond to the final configuration once all networks have converged and are no longer dropping any links.

Fig. 2 provides some insight into the links and transmission powers suggested by the four algorithms. It illustrates a sample of the different configurations, where the colors represent the different networks and the thickness of an arrow indicates the power of the link. Each node has associated a letter, r for receivers and t for transmitters, as well as a number that represents the network to which it belongs. A visual comparison of these chosen links and powers shows an important difference between them: the centralized (optimal), collaborative, and Kesselheim solution have a lot fewer links than the greedy solution.

Fig. 3 shows the overall data rate as a function of the maximum possible offered data rate per link. The results were averaged over five network instances. The total throughput increases with the maximum offered data rate per flow for all algorithms, but the data rates provided by our collaborative algorithm are almost as good as the centralized (optimal) ones when the maximum offered rates are below 2. This makes sense, since at low data rates resources are abundant and all the flows can be easily delivered. When the offered rates increase, the gap between our algorithm and the optimal widens. However, our solutions are always better than the ones achieved by the greedy or Kesselheim approaches. The greedy algorithm is severely limited by the high interference that it causes. Consequently, when the maximum offered rate increases beyond 1, its performance starts weakening until it saturates. The Kesselheim algorithm is based on the distances between the nodes, it is oblivious to the offered rates. The number of active links and their transmit powers only depends on how the nodes are placed. It is worth noting that Kesselheim is a centralized method, the algorithm is applied as if all nodes were part of a single network, whereas our collaborative solution evaluates each network independently without information about the performance of others. Even with this lack of information, the average data rate with our algorithm is closer to the optimal. The collaborative algorithm is heuristic, therefore for some network configurations it may

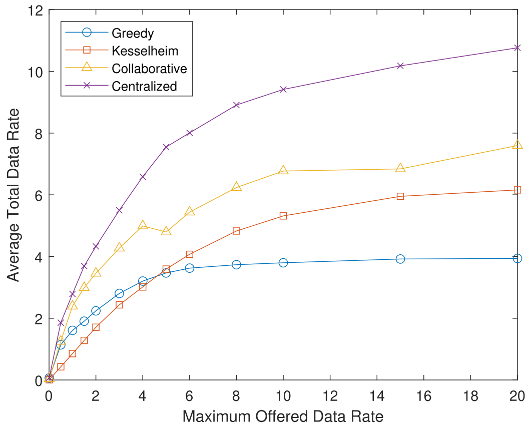

FIGURE 3. Overall data rate over a single band as a function of the maximum possible offered rate per link. The results have been averaged over five network instances.

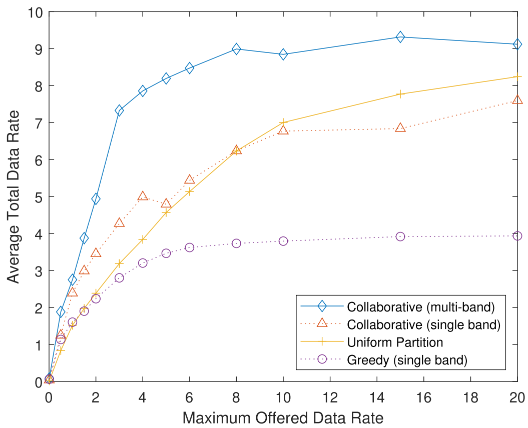

FIGURE 4. Overall data rate over four sub-bands as a function of the maximum possible offered data rate per link. The results have been averaged over five network instances.

delete or keep too many links, leading to situations where the total data rate diminishes despite the maximum offered data rate increases. This explains the dip in the curve from Fig. 3 at maximum offered data rate equals to 5.

All the results so far have assumed a constant path-loss exponent $\alpha = 3$ and that the trade-off parameter in (30) was set to $\lambda = 1$. We experimented with different path loss exponents, between 2 and 4, but the results were similar. This is due to the fact that our system is interference-limited, with relatively small SINR values. We also experimented with different values for λ, between 0.1 and 10, but the average data rate was lower.

B. MULTI-BAND

This section illustrates the performance of our algorithms when multiple frequency bands are considered. It will be assumed that the same bandwidth available in the previous subsection has been divided into multiple identical sub-bands and each network can choose to restrict each link to only some of those sub-bands. Without loss of generality, it will be

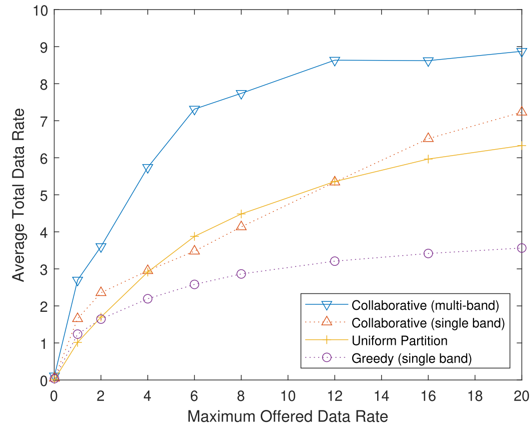

FIGURE 5. Overall data rate as a function of the maximum offered rate per link for 4 networks with heterogeneous offered loads. The results have been averaged over 10 ensemble instances.

further assumed that all sub-bands are occupied continuously in time.

The overall data rates in this section should increase with respect to the ones in the previous subsection, since the networks have an additional degree of freedom that they can choose whether or not to exploit. If the same active links transmit on all sub-bands, the solution would be identical to that in the single band case. Fig. 4 shows the average data rate for the same 5 network instances as in Fig. 3 when the overall bandwidth has been divided into four sub-bands. The curve labeled as "Uniform Partition" corresponds to assigning one sub-band to each network for its exclusive use, as described in subsection V. This is the typical solution proposed by many standards. The other two curves correspond to running our collaborative algorithm with and without multiple sub-bands. It can be seen that the results with a uniform partition are similar to those of the collaborative algorithm with a single band. The rate obtained by the collaborative algorithm with multiple sub-bands is indeed greater than with a single one, but the gain decreases quickly as the load increases.

The difference is larger when we study scenarios with unbalanced loads. Fig. 5 shows the sum rate averaged over 10 instances of 4 networks with 8 nodes each, uniformly distributed over the same 100 m $\times$ 100 m square area as in the previous simulations. However, in this case their offered data rates were scaled by a multiplicative factor of $\frac{1}{8}$, $\frac{1}{2}$, $\frac{5}{4}$, and 2, respectively. The overall offered load is very similar to that in Fig. 4, but the distribution over the networks is different. It was assumed that there were $m = 4$ available frequency channels for transmission, all of them with equal bandwidth, unit transmit power, and background noise $\sigma^2 = 10^{-8}$. It can be seen that the uniform partition curve is still similar to that for the collaborative algorithm with a single band, but both of them yield significantly lower average rates than the collaborative multiband algorithm. This is explained by the fact that a uniform distribution of resources is clearly suboptimal when the networks have different loads. The collaborative algorithm, on the other hand, is able to compensate for the unbalance and provides higher average rates. The greedy algorithm is still the worst performing, since the networks greedily attempt to transmit in all sub-bands resulting in high interference levels across the whole bandwidth.

VII. CONCLUSION

This paper studied the problem of collaborative resource allocation across multiple uncoordinated wireless networks sharing the same time-frequency resources. It first derived an efficient branch and bound algorithm to find the optimal resource allocation that maximizes the sum rate over the whole ensemble. Based on the experimental insights obtained by running that algorithm on multiple ensemble instances, the paper then proposed a heuristic collaborative algorithm that each network can autonomously follow to reduce interference and increase the overall throughput. The collaborative algorithm does not require any side information exchanged between the networks, only the location of peer networks' nodes, which can be inferred from interference powers through triangulation. Numerical simulations show that the collaborative algorithm significantly outperforms the non-collaborative greedy approach and the uniform splitting of resources.

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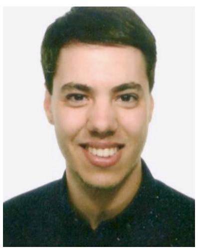

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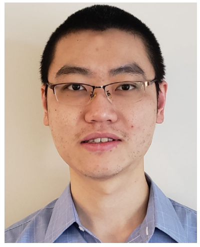

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