IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 9 March 2023, accepted 23 March 2023, date of publication 27 March 2023, date of current version 30 March 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3261963

RESEARCH ARTICLE

Cooperative Control of Multi-Time-Scale Agent Networks Under Digraphs

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This work was supported in part by the Guangxi Young and Middle-Aged Scientific Research Basic Ability Promotion Project under Grant 2021KY1560, and in part by the Interdisciplinary Scientific Research Foundation of Guangxi University under Grant 2022JCC023.

ABSTRACT This paper addresses cooperative control for a class of multi-agent networks. An interesting feature of such network is that each agent possesses the characteristic of multi-time-scale. By developing the relative states or the relative outputs based consensus protocols, some well-conditioned consensus criteria in terms of linear matrix inequalities are derived successively based on the ε-strict Lyapunov function. Compared with undirected graphs or strongly connected digraphs in existing works, a more general digraph that it contains at least a directed spanning tree is considered herein. Moreover, a fully distributed consensus protocol with time-varying coupling strength is proposed to avoid using global information. Finally, the obtained theoretical results are validated by two examples including standard and nonstandard agents.

INDEX TERMS Consensus, directed graph, fully distributed protocol, multi-time-scale feature, strict Lyapunov function.

I. INTRODUCTION

Cooperative control of multi-agent systems [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] has received much attention in past decades, and researchers from different disciplines have achieved abundant results including consensus, formation control, rendezvous, and so forth. Consensus, a fundamental problem in multi-agent systems, means that a group of agents reach a common value or state by its local interaction, and has become one of the attractive and challenging research directions in a variety of scientific communities, such as social networks [13], neural networks [14], unmanned swarm systems [15], smart girds [16], etc.

The agent's dynamics is an important factor in the design of consensus algorithms, which are primarily studied for single-integrator agents, please see [17], [18], and [19]. After that, consensus problems have been studied for second-order agents and high-order ones or their variations. Recently, general linear dynamical agents and nonlinear agents are also investigated. Despite these achievements, the dynamics in abovementioned works are assumed to be regular systems, in other words, all agents' dynamics evolve at a single time scale. In fact, many systems have multiple time scales features. For examples, in the longitudinal motion of a hypersonic vehicle, the 'phugoid mode' and the 'short period mode' coexist and reveal a two-time-scale property [20]. In the longitudinal dynamics of autonomous underwater vehicle, the speed of translational dynamics is much slower than that of orientation ones [21], which essentially reflects the two-time-scale feature. In the direct-driven single-link manipulator, the dynamics of armature current and the dynamics of angular of manipulator belong to the fast and slow dynamics, respectively. Unlike a single vehicle/manipulator, a team formation of multiple vehicles/manipulators can achieve more challenging and complex tasks, such as, battlefield surveillance, collaborative pick-and-place large payload, etc. Formally, these systems with multiple time scales features can be modeled as singularly perturbed systems [22], [23]. Compared with regular multi-agent networks, the main challenges for coordination of multi-time-scale agent networks include: (1) Conventional analysis methods for single-time-scale agent networks cannot

The associate editor coordinating the review of this manuscript and approving it for publication was Engang Tian.

be applied to multi-time-scale agent networks, otherwise it will lead to numerical ill-conditioned problem. How to design multi-time-scale collaborative analysis and design methods is challenging. (2) In the analysis and design of a single-time-scale agent network, the problem of estimating multi-time scale parameters is not involved. In this study, how to provide information about the upper bound estimation of multi-time scale parameters is also a challenging problem.

In the past decade, the collaborative behavior of multi-agent systems with multiple time scales has attracted much attention and various consensus algorithms have been developed. For examples, to achieve consensus of singularly perturbed multi-agent systems with global performance guarantees, [24] proposes the decentralized control strategy by using the state feedback controller. Recently, the results in [24] are extended to different situations. Reference [25] designs a distributed dynamic output-feedback consensus protocol to achieve asymptotically consensus with global performance guarantees, while [26] combines two Zeno-free event-triggered mechanisms and event-triggered controller to achieve guaranteed-cost consensus for two-time-scale agent networks. Reference [27] further proposes both the static and dynamic event-triggered mechanisms and distributed synchronization protocols for two-time-scale agent networks with switching topology. It is worth noticing that the singular perturbation decomposition-based technology utilized above is only applicable to the scene of the standard singular perturbation system. How to develop the corresponding analysis and design method applicable to both standard and non-standard singular perturbation system is one of the challenges of this paper. Recently, [28] constructs both the relative states and the relative outputs based adaptive consensus protocols for two-time-scale agent networks. However, it is pointed out that the communication topologies in the aforementioned literature are assumed to be undirected. In fact, the asymmetry of the Laplacian matrix induced by directed connected graphs makes the analysis difficult in this paper. On the other hand, the abovementioned works except [28] do not provide the upper bound of the singular perturbation parameter which is important for implementing the proposed consensus protocols. The limitation on the communication is relaxed in [29] and [30], where the topologies are directed and strongly connected. However, the achieved consensus criteria therein depend on the agent's dynamics and the size of networks, which maybe are unsolved as the size of networks is too larger. Although these contributions, some facts should be highlighted: 1) the communication topologies in above works are special cases of directed topologies with a spanning tree; 2) Apart from [25] and [28], the proposed consensus protocols given above rely heavily on the relative states of agent, which are usually impossible or too expensive in practice. Therefore, a natural question arises: Is it possible to develop an suitable consensus protocol for multi-time-scale agent networks with a general directed graphs by using the relative states or the relative output states of agents? Which remains unresolved so far.

Inspired by the above works, it is imperative to investigate the consensus of multi-time-scale agent network under a general digraph. The contributions of this paper are summarized as follows:

(1) A milder condition on communication topology is adopted herein. Specifically, unlike the undirected or strongly connected directed graphs employed in some existing works such as [24], [25], [26], [27], [28], [29], and [30], we consider the directed topology containing the spanning tree, such requirement of topology is relaxed effectively and is therefore more suitable for practical applications.

(2) The ε-strict Lyapunov function is constructed to analyze both relative states and relative outputs based consensus, some well-conditioned trackable consensus criteria are deduced and the upper bound of singular perturbation parameter is also estimated. Moreover, in contrast to [24], [26], and [27], the results obtained herein are applied to both the standard and nonstandard agents.

(3) Two new fully distributed consensus protocols with time-varying coupling strength are proposed. Compared with the adaptive consensus protocol in [28] for multi-time-scale agent network with undirected topology,, the proposed consensus protocols are independent on consensus states and can be implemented easily.

Notations: $\Re^n$ denotes the real n dimensional vectors. I_m is the identity matrix with dimension m, and vector $\mathbf{1}$ is a vector with its all elements equals to 1. For a matrix P, the expression $P > 0 (P < 0)$ means that P is symmetric positive (negative) definite. $A \otimes B$ stands for the Kronecker product of matrices A and B.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this section, model description and some preliminaries including definition and lemmas are given.

Consider a group of N identical general linear-time invariant agents with dynamics is in the form of

$$\begin{cases} \dot{x}_i(t) = A_{11}x_i(t) + A_{12}z_i(t) + B_1u_i(t), \\ \varepsilon\dot{z}_i(t) = A_{21}x_i(t) + A_{22}z_i(t) + B_2u_i(t), \\ y_i(t) = C_1x_i(t) + C_2z_i(t), i = 1, 2, \ldots, N. \end{cases} \quad (1)$$

where $x_i \in \Re^n$ and $z_i \in \Re^m$ are the slow and fast state vectors, respectively; $\varepsilon > 0$ is a small parameter that indicates the degree of the fast and slow dynamics separation; $u_i \in \Re^p$ and $y_i \in \Re^q$ are the control input vector and the measured output vector, respectively; $A_{ij}, B_i, C_i, i, j = 1, 2$ are the known constant matrices with appropriate dimensions. Moreover, the communication graph among all agents is represented by a directed graph $\mathcal{G}$.

Let $\xi_i^T = \begin{bmatrix} x_i^T & z_i^T \end{bmatrix}$, $E(\varepsilon) = diag\{I_n, \varepsilon I_m\}$, and

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}.$$

Network (1) can be rewritten as

$$\begin{cases} E(\varepsilon)\dot{\xi}_i = A\xi_i + Bu_i, \\ y_i = C\xi_i, i = 1, 2, \ldots, N. \end{cases} \tag{2}$$

The communication links among N agents are described by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the sets of nodes, $\mathcal{E}$ is the sets of edges. An edge (v_i, v_j) belongs to the edge set $\mathcal{E}$, if v_j can access the information of v_i. If v_j and v_i can access the information from each other, the corresponding graph is undirected. A path between v_i and v_j is a sequence of edges. The graph $\mathcal{G}$ is said to be connected if there exists a path between v_i and v_j, $\forall i, j \in \{1, 2, \cdots, N\}$. The adjacency matrix $\mathcal{A} = [a_{ij}], i, j \in \{1, 2, \cdots, N\}$ is defined as $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ but 0 otherwise, when $i \neq j$, and $a_{ii} = 0$. The Laplacian matrix $\mathcal{L} = [l_{ij}], i, j \in \{1, 2, \cdots, N\}$ is defined as $l_{ii} =$ and $l_{ij} = -a_{ij}$ for $i = j$.

Throughout this paper, the following definition and lemmas are introduced.

Assumption: The directed graph $\mathcal{G}$ contains at least a rooted spanning tree.

Remark 1: Recently, the consensus problem has been studied for multi-time-scale agent networks with undirected topologies in [24], [25], [26], [27], and [28], the communication topology considered above can be seen as a special case of Assumption 1. The asymmetrical of Laplacian matrix associated with Assumption 1 makes significance difficult to achieve consensus.

Definition: For network (1) and a given positive scalar ε_*, it is said to achieve ε-uniformly consensus (ε-UC) if there exists a suitable protocol u_i such that $\lim_{t\to\infty} \|\xi_i(t)-\xi_j(t)\| = 0$, for all $i, j = 1, \ldots, N$ and for any $\varepsilon \in (0, \varepsilon_*]$.

Lemma 1: ([28], [31]): For some given matrices $M_i (i = 1, 2, 3)$ with $M_2 = M_2^T$ and positive constant ε_*, $E(\varepsilon)M(\varepsilon) = M^T(\varepsilon)E(\varepsilon) > 0$ holds for any $\varepsilon \in (0, \varepsilon_*]$, if $M_1 > 0$ and

$$\begin{bmatrix} M_1 & \varepsilon_* M_2^T \\ \varepsilon_* M_2 & \varepsilon_* M_3 \end{bmatrix} > 0, \tag{3}$$

where $M(\varepsilon) = \begin{bmatrix} M_1 & \varepsilon M_2^T \\ M_2 & M_3 \end{bmatrix}$.

Lemma 2 ([32]): For a matrix $\mathcal{L} \in \Re^{N\times N}$ with exactly one eigenvalue equal to zero and $N - 1$ eigenvalues with positive real parts. Then, for any positive definite Q and positive constant λ, there exists a positive definite matrix $P(\lambda) \in \Re^{N\times N}$ such that

$$P\mathcal{L} + \mathcal{L}^T P = Q - \lambda[P\vartheta_r\vartheta_l^T + \vartheta_l\vartheta_r^T P], \tag{4}$$

where ϑ_l and ϑ_r are, respectively, the left and right eigenvectors of the matrix $\mathcal{L}$ corresponding to the eigenvalue that is equal to zero.

III. MAIN RESULTS

In this section, we will propose both the relative states based protocol and the relative outputs based protocol, and deduce some sufficient consensus conditions under which ε-UC of MTSANs are ensured by constructing ε-dependent strict Lyapunov function.

A. THE RELATIVE STATES BASED CONSENSUS ANALYSIS

To achieve ε-UC of MTSANs (1) by using the relative states among the agents, the following protocol is given

$$u_i = cF(\varepsilon)\sum_{j=1}^{N} a_{ij}(\xi_i - \xi_j), \quad i = 1, 2, \ldots, N \tag{5}$$

where c is the coupling strength, a_{ij} is the (i, j)-th entry of the adjacency matrix associated with the communication network topology $\mathcal{G}$ and indicates the information flow between agents i and j with the definition that $a_{ij} > 0$ if the agent i receives the information of the agent j, otherwise $a_{ij} = 0$. Moreover, assume that there is no self-loops, $F(\varepsilon)$ is the feedback gain matrix will be designed later.

Theorem 1: Consider a MTSAN (1) with the topology satisfies Assumption 1. The ε-UC via protocol (5) is solvable if the coupling strength $c \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}$, where the matrices P and Q are defined in Lemma 2, and the matrix gain is chosen as $F(\varepsilon) = -B^T M^{-1}(\varepsilon)$, where $M(\varepsilon)$ is defined in Lemma 1 and satisfies

$$AM(0) + M^T(0)A^T - BB^T < 0, \tag{6}$$

$$AM(\varepsilon_*) + M^T(\varepsilon_*)A^T - BB^T < 0, \tag{7}$$

with a given positive scalar $\varepsilon_* > 0$.

Proof: It can be deduced from Lemma 1 and the conditions (6)–(7) that

$$E(\varepsilon)M(\varepsilon) = M^T(\varepsilon)E(\varepsilon) > 0, \quad \forall\varepsilon \in (0, \varepsilon_*], \tag{8}$$

$$AM(\varepsilon) + M^T(\varepsilon)A^T - BB^T < 0, \quad \forall\varepsilon \in (0, \varepsilon_*], \tag{9}$$

Let $\xi^* = \sum_{j=1}^{N} \vartheta_{l,j}\xi_j$ and $\bar{\xi}_i = \xi_i - \xi^*$, where ϑ_l is the left eigenvector of the matrix L corresponding to the eigenvalue that is equal to zero. It is easy to see that the consensus problem under the protocol (5) can be investigated by analyzing the asymptotical stability of $\bar{\xi}_i, i = 1, 2, \ldots, N$ since $\xi_1 = \xi_2 = \cdots = \xi_N$ is equivalent to $\bar{\xi}_i = 0$.

Denote $\bar{\xi}^T = [\bar{\xi}_1^T \ \bar{\xi}_2^T \ \ldots \ \bar{\xi}_N^T]$, combine (2) and (5), it can be achieved that

$$E(\varepsilon)\dot{\bar{\xi}}_i = A\bar{\xi}_i + cBF(\varepsilon)\sum_{j=1}^{N} a_{ij}(\bar{\xi}_i - \bar{\xi}_j), \quad i = 1, 2, \ldots, N. \tag{10}$$

or

$$(I_N \otimes E(\varepsilon))\dot{\bar{\xi}} = [I_N \otimes A + cL \otimes (BF(\varepsilon))]\bar{\xi}, \tag{11}$$

in a compact form.

Consider the following ε-dependent strict Lyapunov function candidate

$$V_1(\varepsilon, \bar{\xi}) = \bar{\xi}^T[P \otimes (E(\varepsilon)M^{-1}(\varepsilon))]\bar{\xi}, \tag{12}$$

where $M^{-1}(\varepsilon)$ exists for all $\varepsilon \in (0, \varepsilon_*]$ based on (8), and P is a positive definite matrix as shown in Lemma 2. Then, the time derivative of $V_1(\varepsilon, \bar{\xi})$ along the trajectory of (11) is given by:

$$\begin{aligned}\dot{V}_1(\varepsilon, \bar{\xi}) =& \ 2\dot{\bar{\xi}}^T[P \otimes (E(\varepsilon)M^{-1}(\varepsilon))]\bar{\xi} \\ =& \ \bar{\xi}^T[I_N \otimes A + cL \otimes (BF(\varepsilon))]^T[P \otimes M^{-1}(\varepsilon)]\bar{\xi} \\ &+ \bar{\xi}^T[P \otimes M^{-T}(\varepsilon)][I_N \otimes A + cL \otimes (BF(\varepsilon))]\bar{\xi} \\ =& \ \bar{\xi}^T[P \otimes (A^T M^{-1}(\varepsilon) + M^{-T}(\varepsilon)A) \\ &+ cL^T P \otimes (BF(\varepsilon))^T M^{-1}(\varepsilon) \\ &+ cPL \otimes (M^{-T}(\varepsilon)BF(\varepsilon))]\bar{\xi}, \\ =& \ \bar{\xi}^T[P \otimes (A^T M^{-1}(\varepsilon) + M^{-T}(\varepsilon)A) \\ &- cQ \otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi} \\ &+ c\lambda\bar{\xi}^T[(P\vartheta_r\vartheta_l^T + \vartheta_l\vartheta_r^T P) \\ &\otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi} \end{aligned} \tag{13}$$

by using $F(\varepsilon) = -B^T M^{-1}(\varepsilon)$ and Lemma 2.

Note the fact that

$$\begin{aligned}&\bar{\xi}^T[(P\vartheta_r\vartheta_l^T) \otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi} \\ &= \bar{\xi}^T[(P\vartheta_r\vartheta_l^T) \otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))] \\ &\quad \times [(I_N - \mathbf{1}_N\vartheta_l^T) \otimes I_{n+m}]\xi \\ &= \bar{\xi}^T[(P\vartheta_r\vartheta_l^T - P\vartheta_r\vartheta_l^T\mathbf{1}_N\vartheta_l^T) \otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\xi \\ &= 0, \end{aligned} \tag{14}$$

since $\vartheta_l^T\mathbf{1}_N = 1$. Then, it follows that

$$\begin{aligned}\dot{V}_1(\varepsilon, \bar{\xi}) =& \ \bar{\xi}^T[P \otimes (A^T M^{-1}(\varepsilon) + M^{-T}(\varepsilon)A) \\ &- cQ \otimes (M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi} \\ \leq& \ \bar{\xi}^T[P \otimes (A^T M^{-1}(\varepsilon) + M^{-T}(\varepsilon)A \\ &- c\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}(M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi}. \end{aligned} \tag{15}$$

Let $\tilde{\xi}_i = M^{-1}(\varepsilon)\bar{\xi}_i$ and $\tilde{\xi} = [\tilde{\xi}_1^T \ \tilde{\xi}_2^T \ \ldots \ \tilde{\xi}_N^T]$, the inequality (15) can be further rewritten as

$$\dot{V}_1(\varepsilon, \bar{\xi}) = \tilde{\xi}^T[P \otimes (AM(\varepsilon) + M^T(\varepsilon)A^T - BB^T)]\tilde{\xi} < 0, \tag{16}$$

by using the fact that $c \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}$ and the condition (9). In this case, $\dot{V}_1(\varepsilon, \bar{\xi}) < 0$ for any $\varepsilon \in (0, \varepsilon_*]$. That is, the ε-UC problem of MTSANs (1) is solved. This ends the proof.

Remark 2: Conditions (6) and (7) show that the existence condition of consensus protocol (5) is that $(E^{-1}(\varepsilon)A, E^{-1}(\varepsilon)B)$ is strongly controllable. On the other hand, we do not require that the matrix A_{22} should be nonsingular, which means that our method can be used for multi-time-scale agent networks with both the standard and nonstandard agents. Moreover, we have given the estimation of the upper bound of singular perturbation parameter ε_*.

Remark 3: Contrary to the undirected topologies considered in [24], [25], [26], [27], and [28], and the strongly connected digraphs considered in [29] and [30], the communication networks herein are assumed to be digraphs with spanning tree. The asymmetrical of Laplacian matrix makes significance difficult to achieve consensus. Moreover, in contrast to the consensus conditions in [29] and [30] which may be hard to verify when the size of networks is huge, the obtained consensus criteria can be easy tested by agent's dynamics.

B. THE RELATIVE OUTPUTS BASED CONSENSUS ANALYSIS

In general, it is difficult to access all states of agents in practice. In this subsection, by using only the agents' relative outputs, some well-conditioned sufficient consensus conditions are further presented for MTSANs (1).

In this respect, we design the following consensus protocol:

$$\begin{cases} E(\varepsilon)\dot{\hat{\xi}}_i = A\hat{\xi}_i + Bu_i + K(\varepsilon)(y_i - C\hat{\xi}_i), \\ u_i = c\hat{F}(\varepsilon)\sum_{j=1}^N a_{ij}(\hat{\xi}_i - \hat{\xi}_j), i = 1, 2, \ldots, N, \end{cases} \tag{17}$$

where $\hat{\xi}_i \in \Re^{n+m}$ is the estimation of ξ_i, c denotes the coupling strength, $\hat{F}(\varepsilon)$ and $K(\varepsilon)$ serve as, respectively, the consensus gain matrix and the observer gain matrix which will be designed later.

Combining (2) and (17), we have

$$\begin{cases} E(\varepsilon)\dot{\xi}_i = A\xi_i + cB\hat{F}(\varepsilon)\sum_{j=1}^N a_{ij}(\hat{\xi}_i - \hat{\xi}_j), \\ E(\varepsilon)\dot{e}_i = (A - K(\varepsilon)C)e_i, i = 1, 2, \ldots, N, \end{cases} \tag{18}$$

where $e_i = \xi_i - \hat{\xi}_i$ serves as the estimation error of agent i. Moreover, network (18) can be rewritten in a compact form

$$\begin{cases} (I_N \otimes E(\varepsilon))\dot{\xi} = [I_N \otimes A + cL \otimes (B\hat{F}(\varepsilon))]\xi \\ \qquad\qquad - cL \otimes (B\hat{F}(\varepsilon))e, \\ (I_N \otimes E(\varepsilon))\dot{e} = [I_N \otimes (A - K(\varepsilon)C)]e. \end{cases} \tag{19}$$

Let $\xi^* = \sum_{j=1}^N \vartheta_{l,j}\xi_j$, $\bar{\xi}_i = \xi_i - \xi^*$, and $\bar{\eta}_i^T = [\bar{\xi}_i^T \ e_i^T]^T$, it can be obtained that

$$\begin{aligned}&(I_N \otimes \mathbf{E}(\varepsilon))\dot{\bar{\eta}} \\ &\quad = \begin{bmatrix} I_N \otimes A + cL \otimes (B\hat{F}(\varepsilon)) & -cL \otimes (B\hat{F}(\varepsilon)) \\ 0 & I_N \otimes (A - K(\varepsilon)C) \end{bmatrix}\bar{\eta}. \end{aligned} \tag{20}$$

where $\mathbf{E}(\varepsilon) = diag\{E(\varepsilon), E(\varepsilon)\}$. Note that the ε UC problem for network (2) under the protocol (17) can be investigated by the asymptotical stability of $\bar{\eta}$.

Theorem 2: Consider a MTSAN (1) with the topology satisfies Assumption 1. The ε-UC via protocol (17) is solvable if the coupling strength c and the matrix gain is chosen as in Theorem 1, and the observer gain matrix is chosen as $K(\varepsilon) = \hat{M}^{-T}(\varepsilon)Y(\varepsilon)$, where $Y(\varepsilon) = Y_1 + \varepsilon Y_2$ and $\hat{M}(\varepsilon)$ satisfy the following LMIs

$$\hat{M}^T(0)A + A^T\hat{M}(0) - Y_1C - C^TY_1^T < 0, \tag{21}$$

$$\hat{M}_2^T(\varepsilon_*)A + A^T\hat{M}(\varepsilon_*) - Y(\varepsilon_*)C - C^TY^T(\varepsilon_*) < 0, \tag{22}$$

with a given positive scalar $\varepsilon_* > 0$.

Proof: It can be deduced from Lemma 1 and the conditions (21)–(22) that for any $\varepsilon \in (0, \varepsilon_*]$

$$\hat{M}^T(\varepsilon)(A - K(\varepsilon)C) + (A - K(\varepsilon)C)^T\hat{M}(\varepsilon) < 0 \tag{23}$$

with $\hat{M}(\varepsilon) = \begin{bmatrix} \hat{M}_1 & \varepsilon \hat{M}_2^T \\ \hat{M}_2 & \hat{M}_3 \end{bmatrix}$ and $Y(\varepsilon) = \hat{M}^T(\varepsilon)K(\varepsilon)$. Which implies that the second equation of (20) is asymptotically stable for any $\varepsilon \in (0, \varepsilon_*]$.

On the other hand, it is easy to conclude from Theorem 1 that the subsystem $\bar{\xi}$ is also asymptotically stable for any $\varepsilon \in (0, \varepsilon_*]$. Then, we can draw the conclusion from Lemma 4.7 in [23] that system (20) is asymptotically stable for any $\varepsilon \in (0, \varepsilon_*]$. Which means that the ε-UC problem of MTSANs (1) is solved. This ends the proof.

Remark 4: Compared with the consensus strategies proposed in [24], [26], [27], [29], and [30] require full knowledge of the relative states of agents, which are usually impossible or too expensive in practice, the consensus protocol (17) depends on only the relative output states of agents, which can be implemented more easily.

Remark 5: The use of time-varying coupling gains is related with consensus protocol. The author of [32] and [33] proposed for the first time to use time-varying coupling weights to avoid using global information. Compared with [32], and [33], the contributions of this paper lie in the following two aspects: (1) we extend the results of [32] and [33] to the situation that each agent is described singularly perturbed systems, and it is reported that the analysis and synthesis for singularly perturbed systems is more complicated. Specifically, we construct the ε-strict Lyapunov function to analyze multi-time-scale networks; (2) only full-state-based consensus protocol is given in [32] and [33], we not only studies the full-state-based consensus issue, but also performs the output-state-based consensus protocol design.

C. FULLY DISTRIBUTED CONSENSUS ANALYSIS

Note that in Theorems 1 and 2, the coupling strength c should selected as $c \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}$, which means that we have to calculate the maximum eigenvalue of P and the minimum eigenvalue of Q. Therefore, from Lemma 2, the proposed consensus protocols are not fully distributed since they rely on knowledge of the Laplacian L. To address such issue, in this subsection, motivated by [33], the coupling strength c in above is redefined as a "slowly" strictly increasing time-varying function. Therefore, the consensus protocols (5) and (17) can respectively be modified as

$$u_i = c(t)F(\varepsilon)\sum_{j=1}^{N} a_{ij}(\xi_i - \xi_j), i = 1, 2, \ldots, N \tag{24}$$

and

$$\begin{cases} E(\varepsilon)\dot{\hat{\xi}}_i = A\hat{\xi}_i + Bu_i + K(\varepsilon)(y_i - C\hat{\xi}_i), \\ u_i = c(t)\hat{F}(\varepsilon)\sum_{j=1}^{N} a_{ij}(\hat{\xi}_i - \hat{\xi}_j), i = 1, 2, \ldots, N, \end{cases} \tag{25}$$

Theorem 3: Consider a MTSAN (1) with the topology satisfies Assumption 1. The ε-UC via protocol (17) is solvable if the coupling strength c is selected as a 'slowly' strictly increasing time-varying function and the matrix gain is chosen as in Theorem 1.

Proof: Consider again ε-dependent strict Lyapunov function candidate in (12). Proceeding as in the proof of Theorem 1, it can be obtained that

$$\begin{aligned} \dot{V}_1(\varepsilon, \bar{\xi}) \leq\ & \bar{\xi}^T[P \otimes (A^T M^{-1}(\varepsilon) + M^{-T}(\varepsilon)A \\ & -c(t)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}(M^{-T}(\varepsilon)BB^T M^{-1}(\varepsilon))]\bar{\xi} \\ =\ & \tilde{\xi}^T[P \otimes (M^T(\varepsilon)A^T + AM^1(\varepsilon) \\ & -c(t)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}BB^T]\tilde{\xi}. \end{aligned} \tag{26}$$

Denote $S_c(\varepsilon, t) = M^T(\varepsilon)A^T - c(t)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}BB^T + AM^1(\varepsilon)$ and $S_c(\varepsilon) = AM(\varepsilon) + M^T(\varepsilon)A^T - BB^T$, respectively, then it follows that

$$\dot{V}_1(\varepsilon, \bar{\xi}) \leq \tilde{\xi}^T[P \otimes (S_c(\varepsilon, t) - S_c(\varepsilon)) + S_c(\varepsilon)]\tilde{\xi}. \tag{27}$$

Since $c(t)$ is strictly increasing, it can be deduced that $T := \min\{t \geq 0 : c(t) \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}\} \in [0, \infty)$ exists. Therefore, $\|S_c(\varepsilon, t) - S_c(\varepsilon)\| = (1 - c(t)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)})\|BB^T\| \leq \|BB^T\| := \beta$ for any $t \in [0, T)$ and $S_c(\varepsilon, t) - S_c(\varepsilon) \leq 0$ for $t \geq T$. It further follows that for $t \in [0, T)$

$$\begin{aligned} \dot{V}_1(\varepsilon, \bar{\xi}) &\leq \|\tilde{\xi}^T[P \otimes (S_c(\varepsilon, t) - S_c(\varepsilon))]\tilde{\xi}\| \\ &\leq \beta\|P\|\|\tilde{\xi}\|^2 = \beta\|P\|\|M^{-1}(\varepsilon)\|^2\|\bar{\xi}\|^2 \\ &\leq \frac{\beta\|P\|\|M^{-1}(\varepsilon)\|^2}{\lambda_{\min}(P \otimes (E(\varepsilon)M^{-1}(\varepsilon)))}V_1(\varepsilon, \bar{\xi}) \\ &:= \bar{\beta}V_1(\varepsilon, \bar{\xi}), \end{aligned} \tag{28}$$

and for $t \in [0, T)$

$$\begin{aligned} \dot{V}_1(\varepsilon, \bar{\xi}) &\leq \tilde{\xi}^T[P \otimes S_c(\varepsilon)]\tilde{\xi} \\ &\leq \lambda_{\min(S_c(\varepsilon))}\|M^{-T}(\varepsilon)PM^{-1}(\varepsilon)\|\|\bar{\xi}\|^2 \\ &\leq \frac{\lambda_{\min(S_c(\varepsilon))}\|M^{-T}(\varepsilon)PM^{-1}(\varepsilon)\|}{\lambda_{\max}(P \otimes (E(\varepsilon)M^{-1}(\varepsilon)))}V_1(\varepsilon, \bar{\xi}) \\ &:= \bar{\lambda}_{\min(S_c(\varepsilon))}V_1(\varepsilon, \bar{\xi}). \end{aligned} \tag{29}$$

It can be obtained by integrating both sides of above inequalities that

$$\begin{aligned} V_1(\varepsilon, \bar{\xi}) &\leq e^{\bar{\beta}t}V_1(\varepsilon, \bar{\xi}_0) \\ &\leq e^{\bar{\beta}t}e^{-\bar{\lambda}_{\min(S_c(\varepsilon))}t}V_1(\varepsilon, \bar{\xi}_0)e^{\bar{\lambda}_{\min(S_c(\varepsilon))}t} \\ &\leq e^{(\bar{\beta} - \bar{\lambda}_{\min(S_c(\varepsilon))})T}V_1(\varepsilon, \bar{\xi}_0)e^{\bar{\lambda}_{\min(S_c(\varepsilon))}t}, \\ & \quad t \in [0, T], \end{aligned} \tag{30}$$

and

$$\begin{aligned} \dot{V}_1(\varepsilon, \bar{\xi}) &\leq e^{\bar{\lambda}_{\min(S_c(\varepsilon))}(t-T)}V_1(\varepsilon, \bar{\xi}_T) \\ &\leq e^{(\bar{\beta} - \bar{\lambda}_{\min(S_c(\varepsilon))})T}V_1(\varepsilon, \bar{\xi}_0)e^{\bar{\lambda}_{\min(S_c(\varepsilon))}t}, \forall t \geq T. \end{aligned} \tag{31}$$

Which means that the ε-UC problem of MTSANs (1) with the time-varying coupling strength is solved. This ends the proof.

Similarly, the following result can be deduced immediately based on Theorems 2 and 3.

Theorem 4: Consider a MTSAN (1) with the topology satisfies Assumption 1. The ε-UC via protocol (17) is solvable if the coupling strength c is selected as a "slowly"

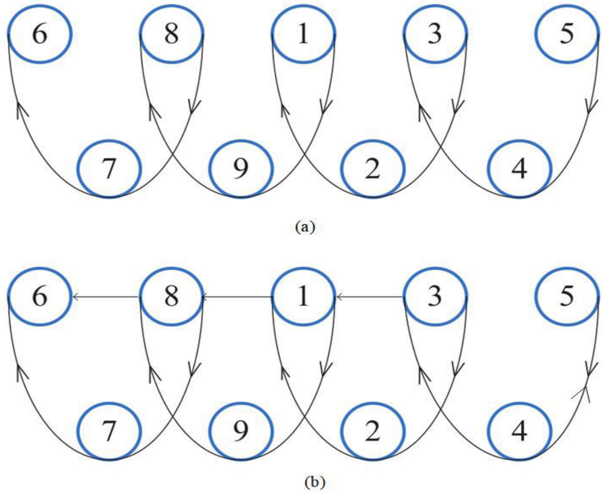


FIGURE 1. The communication network topology among 9 rolls.

strictly increasing time-varying function, the consensus matrix gain and the observer gain matrix are chosen as in Theorems 1 and 2, respectively.

Remark 6: Theorems 3 and 4 show that the ε-UC problem of MTSANs (1) can be solved by introducing the time-varying coupling strength $c(t)$, differenting from the adaptive consensus protocol in [28] for multi-time-scale agent network with undirected topology, the coupling strength is independent on consensus states and is therefore fully distributed.

Remark 7: A simpler and useful form of the coupling strength is that $c(t) := \ln(\kappa + t)$ with $\kappa > 1$. Moreover, it has be shown in [33] that under such choice, the corresponding control input for each agent remains bounded.

Remark 8: The existing of ε will lead to the ill-conditioning and stiffness problem and how to determine the upper bound ε_* is a fundamental problem and has attracted much attention. In this paper, we construct ε-dependent strict Lyapunov function to analyze the consensus of multi-time-scale networks, and provide some well-conditioned trackable consensus criteria. Moreover, to obtain an upper bound of ε, we can use one dimensional search algorithm or other methods, such as differential evolution algorithm, which has been employed in some existing works, see [31].

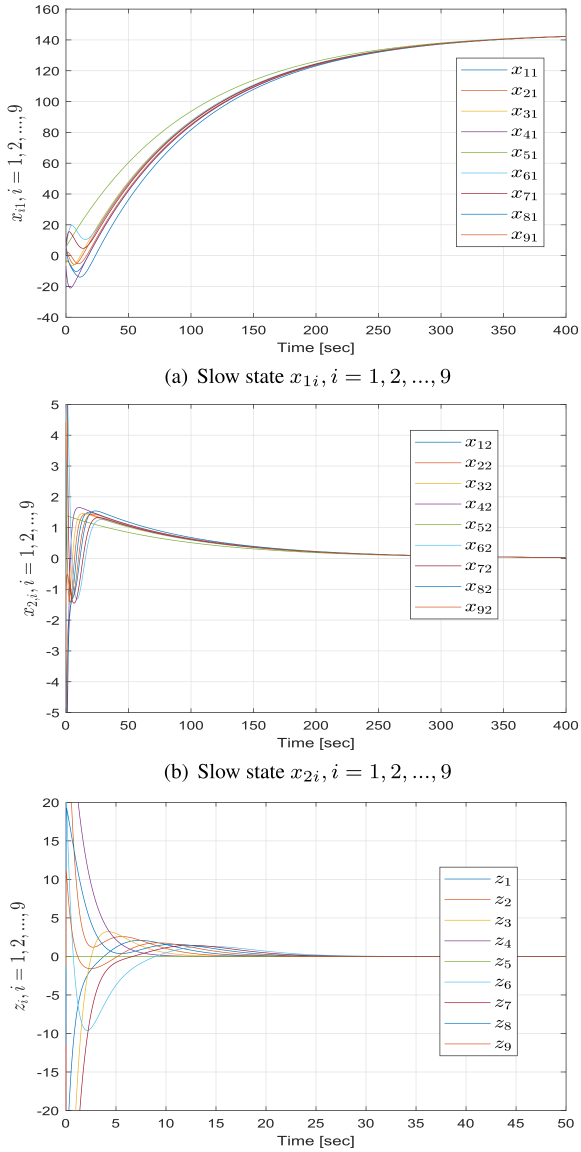
IV. SIMULATION RESULTS

In this section, we give two numerical examples to illustrate the effectiveness of the theoretical results obtained in this paper.

Example 1: We first consider a drying section of a paper converting machine [34] with 9 rolls, whose angles have to reach the same level to avoid tearing up the paper. Here, the network topology among 9 rolls is demonstrated by FIGURE 1, and the each roll system is described by (2) with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.01 & 0.2 \\ 0 & 0 & -1.25 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

First, we investigate the effectiveness of Theorems 1 and 2.



(a) Slow state $x_{1i}, i = 1, 2, ..., 9$

(b) Slow state $x_{2i}, i = 1, 2, ..., 9$

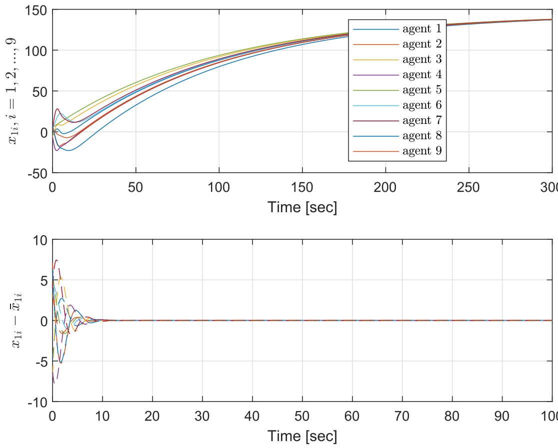
(c) Fast states $z_i, i = 1, 2, ..., 9$

FIGURE 2. State trajectories of 9 agents under protocol (17).

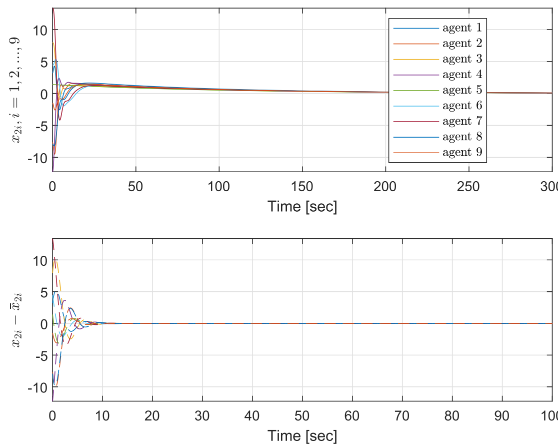
For FIGURE 1 (a), we choose the upper bound of singular perturbation parameter $\varepsilon_* = 0.1, \lambda = 0.2, Q = I_9$, it can be deduced that the coupling strength $c \geq 9.1709$. Then, by solving all LMIs in Theorems 1, we find that Theorem 1 is feasible with

$$F(0.1) = \begin{bmatrix} -0.0289 & -2.1257 & -0.0451 \end{bmatrix},$$
$$M(0.1) = \begin{bmatrix} 257.8917 & -3.4996 & -0.0311 \\ -3.4996 & 0.1087 & -0.2882 \\ -0.3106 & -2.8822 & 18.0452 \end{bmatrix}.$$

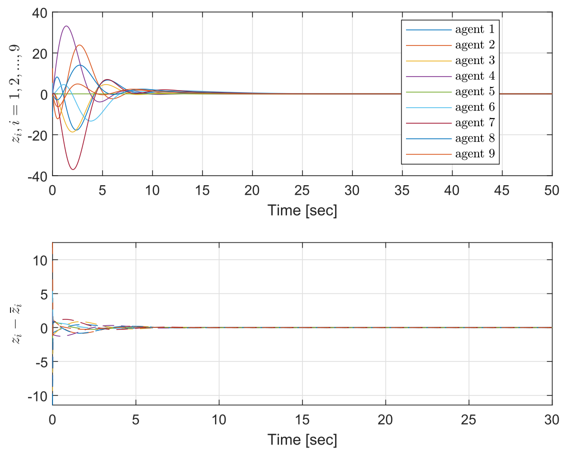
Which means that all rolls under the consensus protocol (5) achieve consensus for any $\varepsilon \in (0, 0.1]$. To verify Theorem 2, we further solve the conditions (21) and (22) in Theorem 2,



(a) Slow state x_{1i} and its estimation error



(b) Slow state x_{2i} and its estimation error



(c) Fast states z_i and its estimation error

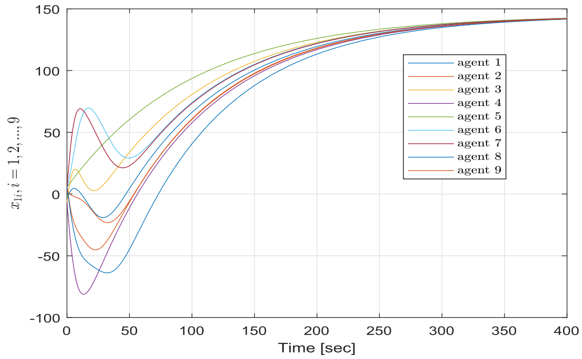
FIGURE 3. State trajectories of 9 agents under protocol (5).

and find that Theorem 2 is also feasible with

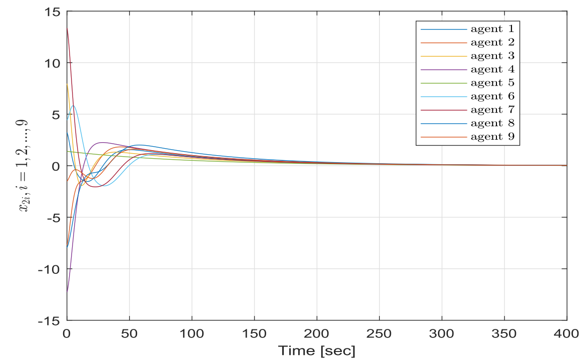
$$K(0.1) = \begin{bmatrix} 0.9347 & 1.3326 & -0.2025 \end{bmatrix}^T .$$

Which means that the ε-UC of MTSNs (1) under the consensus protocol (17) is achieved.

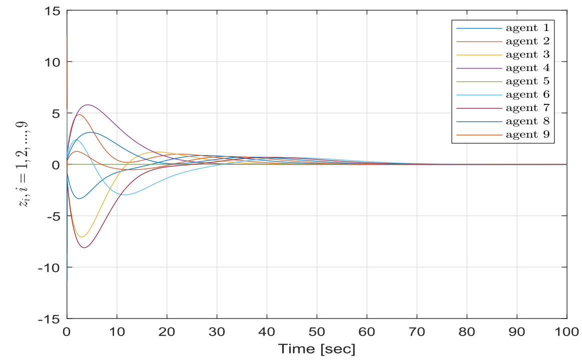
In the numerical simulation, the initial states of each agent are chosen randomly within $[-15, 15]$, the coupling strength and singular perturbation parameter are selected as $c = 9.2$ and $\varepsilon = 0.01$, respectively, the simulation results are depicted in FIGURE 2 and FIGURE 3. It can be observed that the consensus among all rolls are achieved, which agrees with our result in Theorems 1 and 2.



(a) Slow state $x_{1i}, i = 1, 2, ..., 9$



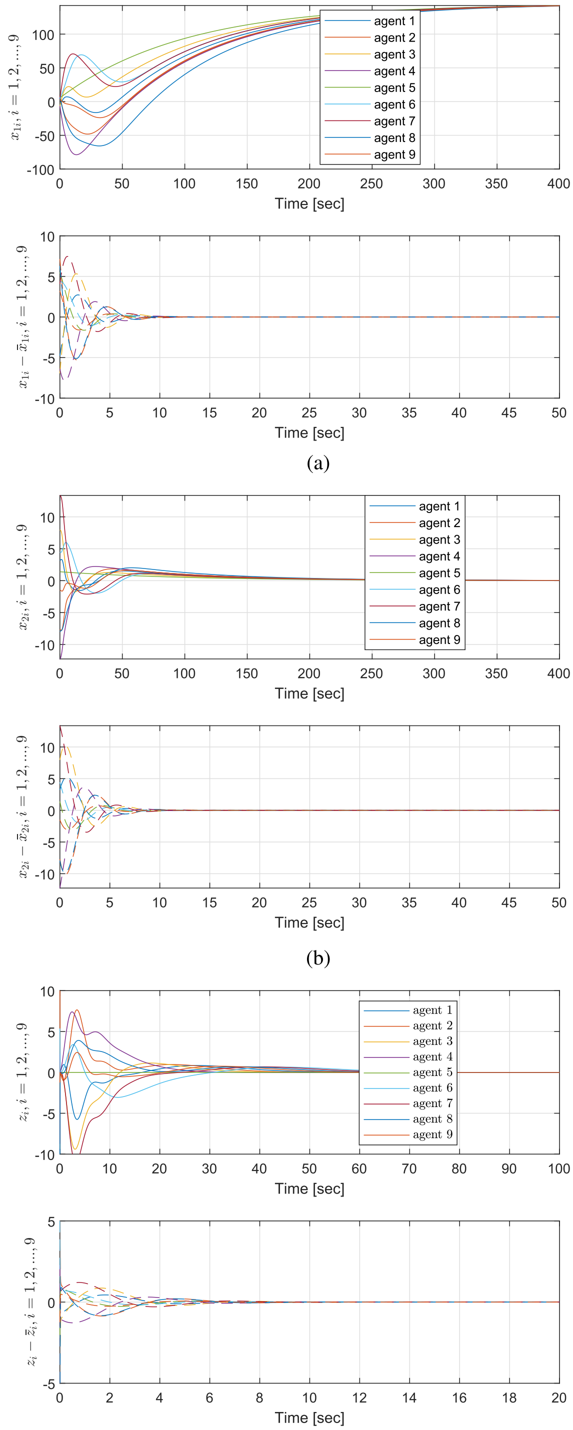
(b) Slow state $x_{2i}, i = 1, 2, ..., 9$



(c) Fast state $z_i, i = 1, 2, ..., 9$

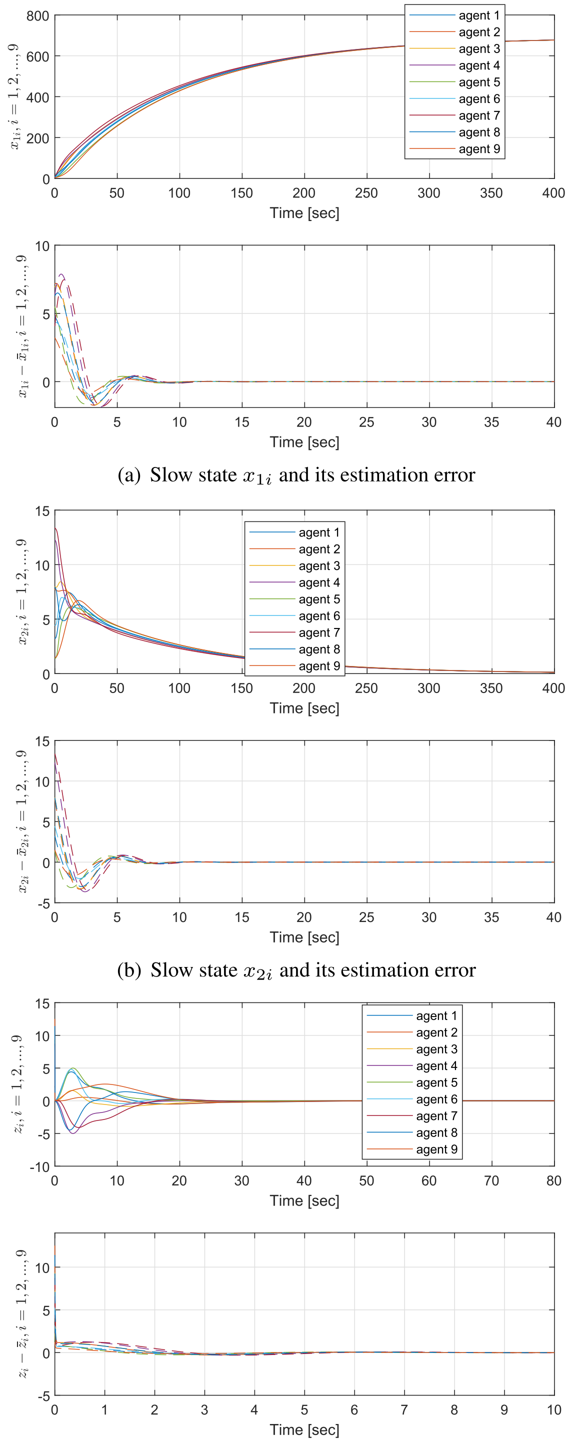
FIGURE 4. State trajectories of 9 agents under protocol (24).

Note that to select some appropriate couple strength c for carrying out the proposed consensus protocols, we have to solve the equation in Lemma 2, in this situation, the global information of the communication topology is needed. To fix this issue, the coupling strength $c(t)$ is selected as $c(t) := \ln(\kappa + t)$, with $\kappa = 1.2$, while both the consensus and observer matrices are designed as above. For the sake of fair comparison, the same initial states are used, the simulation results are portrayed in FIGURE 4 and FIGURE 5, which depict the time evolution of the slow and fast states as well as their estimation errors under the proposed consensus protocols (24) and (25),



(a) (b) (c)

FIGURE 5. **State trajectories of 9 agents under protocol (25).**



(a) Slow state x_{1i} and its estimation error

(b) Slow state x_{2i} and its estimation error

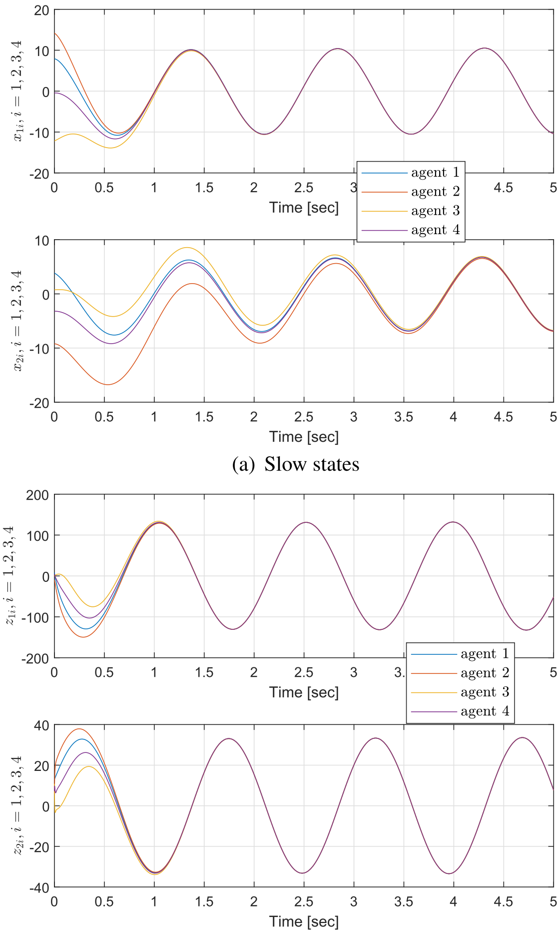
(c) Fast states z_i and its estimation error

FIGURE 6. **State trajectories of 9 agents under protocol (25).**

respectively. Obviously, the entire states of the considering networks finally achieve the desired consensus.

Moreover, we conduct simulation on a digraph with two roots. The topology is shown as FIGURE 1(b), where roots are agents 4 and 5. To be brief, only the simulation results associated with consensus protocol (25) are given here. In the numerical simulation, the initial states, the coupling strength and singular perturbation parameter are selected the same values, the simulation results are depicted in FIGURE 6.

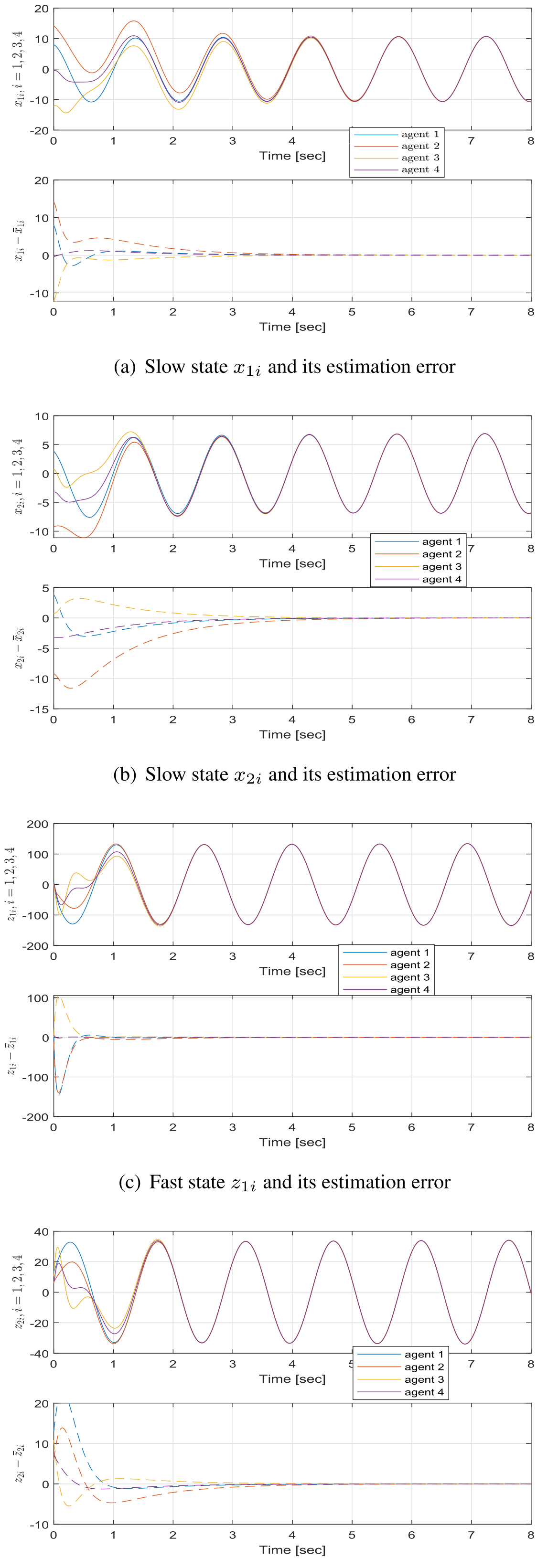
Example 2: This example is given to show that our proposed method is suitable for multi-time-scale agent network with nonstandard nodes. Consider network (1) consisting of



(a) Slow states

(b) Fast states

FIGURE 7. **State trajectories of 9 agents under protocol (5).**



(a) Slow state x_{1i} and its estimation error

(b) Slow state x_{2i} and its estimation error

(c) Fast state z_{1i} and its estimation error

(d) Fast state z_{2i} and its estimation error

FIGURE 8. **State trajectories of 9 agents under protocol (17).**

$N = 4$ agents, where the agent system is described by

$$A = \begin{bmatrix} -1 & 0 & 0.375 & 0.1 \\ 0 & -1 & 0.1 & -0.5 \\ 0 & 0 & -0.25 & -1 \\ 0.2 & -0.1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The Laplacian matrix corresponding to the network topology among 4 agents is given by

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Obviously, each agent in this case is described by nonstandard singularly perturbed system since $A_{22} = \begin{bmatrix} -0.25 & -1 \\ 0 & 0 \end{bmatrix}$ is singular matrix. Besides, the considered communication topology among these agents is directed and also contains a spanning tree. By selecting $\lambda = 0.46$ and $Q = I_4$, we have $c \geq 1.7425$. Moreover, it is easy to verify that Theorems 1 and 2 are feasible with the upper bound of singular perturbation parameter $\varepsilon = 0.05$ and the following protocol gains:

$$F(0.05) = \begin{bmatrix} -2.0431 & -0.0787 & 0.3145 & 2.7744 \end{bmatrix},$$
$$K(0.05) = \begin{bmatrix} 1.3994 & 0.2538 & 3.7460 & 0.0733 \\ 1.4666 & -0.1449 & 1.4300 & -0.1773 \end{bmatrix}^T.$$

Which means that network (1) can achieve ε-UC for any $\varepsilon \in (0, 0.05]$ under the proposed protocols (5) and (17).

Simulation results are then displayed in FIGURE 7 and FIGURE 8, where the initial states of networks are chosen randomly within [0, 10], singular perturbation parameter $\varepsilon = 0.01$, and the coupling strength $c = 1.75$.

Remark 9: In [24], [26], and [27], each agent is assumed to be an standard singularly perturbed system, in other words, the matrix A_{22} should be nonsingular, then singular perturbation decomposition method is employed to decouple the fast and slow states of every agent. However, in this example the considered model is nonstandard one, which implies that the methods in [24], [26], and [27] are unapplicable. On the other hand, the upper bound of singular perturbation parameter has not been given in [24], [26], and [27], in this paper, we give the estimation of the upper bound of ε. Therefore, our results have superiority compared with these existing results.

V. CONCLUSION

The consensus problem of multi-time-scale agent networks over general digraph has been investigated by introducing the so-called ε-strict Lyapunov function in this paper. Both the relative states and the relative output states based consensus protocols have been designed. Meanwhile, the corresponding consensus conditions have been obtained, and the upper bound of singular perturbation parameter for ensuring consensus has been estimated. Numerical simulations with standard and nonstandard agents have been validated the effectiveness of theoretical results.

REFERENCES

[1] J. Qin, Q. Ma, Y. Shi, and L. Wang, "Recent advances in consensus of multi-agent systems: A brief survey," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4972–4983, Jun. 2017.

[2] L. Ding, Q.-L. Han, X. Ge, and X.-M. Zhang, "An overview of recent advances in event-triggered consensus of multiagent systems," *IEEE Trans. Cybern.*, vol. 48, no. 4, pp. 1110–1123, Apr. 2018.

[3] D. Zhang, G. Feng, Y. Shi, and D. Srinivasan, "Physical safety and cyber security analysis of multi-agent systems: A survey of recent advances," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 2, pp. 319–333, Feb. 2021.

[4] S. Hattab and W. L. Chaari, "A generic model for representing openness in multi-agent systems," *Knowl. Eng. Rev.*, vol. 36, pp. 1–24, Jan. 2021.

[5] Q. Zhang, Y. Yang, X. Xie, C. Xu, and H. Yang, "Dynamic event-triggered consensus control for multi-agent systems using adaptive dynamic programming," *IEEE Access*, vol. 10, pp. 110285–110293, 2022.

[6] K. Chen, J. Wang, Z. Zhao, G. Lai, and Y. Lyu, "Output consensus of heterogeneous multiagent systems: A distributed observer-based approach," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 1, pp. 370–376, Jan. 2022.

[7] K. Chen, J. Wang, Y. Zhang, and Z. Liu, "Leader-following consensus for a class of nonlinear strick-feedback multiagent systems with state time-delays," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 50, no. 7, pp. 2351–2361, Jul. 2020.

[8] Y. Sun, B. Li, D. Zhang, and Y. Hu, "Guaranteed-cost consensus control for high-dimensional multi-agent systems with input delays and parameter uncertainties," *IEEE Access*, vol. 10, pp. 122112–122124, 2022.

[9] X. Pu, L. Zhang, and X. Sun, "Couple-group consensus of heterogeneous multi-agents systems with Markov switching and cooperative-competitive interaction," *IEEE Access*, vol. 10, pp. 118718–118735, 2022.

[10] J. Liu, T. Yin, D. Yue, H. R. Karimi, and J. Cao, "Event-based secure leader-following consensus control for multiagent systems with multiple cyber attacks," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 162–173, Jan. 2021.

[11] Z.-H. Pang, W.-C. Luo, G.-P. Liu, and Q.-L. Han, "Observer-based incremental predictive control of networked multi-agent systems with random delays and packet dropouts," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 68, no. 1, pp. 426–430, Jan. 2021.

[12] Z.-H. Pang, C.-B. Zheng, C. Li, G.-P. Liu, and Q.-L. Han, "Cloud-based time-varying formation predictive control of multi-agent systems with random communication constraints and quantized signals," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 3, pp. 1282–1286, Mar. 2022.

[13] Y. Zou and Z. Meng, "Targeted bipartite consensus of opinion dynamics in social networks with credibility intervals," *IEEE Trans. Cybern.*, vol. 52, no. 1, pp. 372–383, Jan. 2022.

[14] S. K. Joshi, "Synchronization of coupled Hindmarsh-rose neuronal dynamics: Analysis and experiments," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 3, pp. 1737–1741, Mar. 2022.

[15] X. Dong, B. Yu, Z. Shi, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles: Theories and applications," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 1, pp. 340–348, Jan. 2015.

[16] X.-K. Liu, C. Wen, Q. Xu, and Y.-W. Wang, "Resilient control and analysis for DC microgrid system under DoS and impulsive FDI attacks," *IEEE Trans. Smart Grid*, vol. 12, no. 5, pp. 3742–3754, Sep. 2021.

[17] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Mar. 2003.

[18] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[19] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[20] W. Ren, B. Jiang, and H. Yang, "Fault-tolerant control of singularly perturbed systems with applications to hypersonic vehicles," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 6, pp. 3003–3015, Dec. 2019.

[21] M. Lei, "Nonlinear diving stability and control for an AUV via singular perturbation," *Ocean Eng.*, vol. 197, Feb. 2020, Art. no. 106824.

[22] P. Kokotovic, H. K. Khalil, and J. O'reilly, *Singular Perturbation Methods in Control: Analysis and Design*. Philadelphia, PA, USA: SIAM, 1999.

[23] H. K. Khalil, *Nonlinear Systems*. Englewood Cliffs, NJ, USA: Prentice-Hall, 2002.

[24] J. B. Rejeb, I.-C. Morarescu, and J. Daafouz, "Control design with guaranteed cost for synchronization in networks of linear singularly perturbed systems," *Automatica*, vol. 91, pp. 89–97, May 2018.

[25] E. S. Tognetti, T. R. Calliero, I.-C. Morarescu, and J. Daafouz, "Synchronization via output feedback for multi-agent singularly perturbed systems with guaranteed cost," *Automatica*, vol. 128, Jun. 2021, Art. no. 109549.

[26] Y. Lei, Y.-W. Wang, I.-C. Morarescu, and J.-W. Xiao, "Guaranteed cost for an event-triggered consensus strategy for interconnected two time-scales systems with structured uncertainty," *IEEE Trans. Cybern.*, vol. 52, no. 6, pp. 4370–4380, Jun. 2022.

[27] Y. Lei, Y.-W. Wang, X.-K. Liu, and Z.-W. Liu, "Distributed event-triggered synchronization of interconnected linear two-time-scale systems with switching topology," *IEEE Trans. Cybern.*, vol. 52, no. 12, pp. 13714–13726, Dec. 2022.

[28] L.-M. Wei, W. Yang, and Y.-W. Wang, "Adaptive consensus of two-time-scale multi-agent systems," *Int. J. Control*, vol. 94, no. 4, pp. 943–951, Apr. 2021.

[29] W. Yang, Y.-W. Wang, J.-W. Xiao, and W.-H. Chen, "Modulus consensus in a network of singularly perturbed systems with collaborative and antagonistic interactions," *Int. J. Control*, vol. 90, no. 12, pp. 2667–2676, Dec. 2017.

[30] W. Yang, Y.-W. Wang, J.-W. Xiao, and Z.-W. Liu, "Coordination of networked delayed singularly perturbed systems with antagonistic interactions and switching topologies," *Nonlinear Dyn.*, vol. 89, no. 1, pp. 741–754, Jul. 2017.

[31] C. Yang and Q. Zhang, ''Multiobjective control for T–S fuzzy singularly perturbed systems,'' *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 104–115, Feb. 2009.
[32] E. Panteley, A. Loria, and S. Sukumar, ''Strict Lyapunov functions for consensus under directed connected graphs,'' in *Proc. Eur. Control Conf. (ECC)*, May 2020, pp. 935–940.
[33] M. Dutta, E. Panteley, A. Loria, and S. Sukumar, ''Strict Lyapunov functions for dynamic consensus in linear systems interconnected over directed graphs,'' *IEEE Control Syst. Lett.*, vol. 6, pp. 2323–2328, 2022.
[34] A. Mosebach and J. Lunze, ''Optimal synchronization of circulant networked multi-agent systems,'' in *Proc. Eur. Control Conf. (ECC)*, Jul. 2013, pp. 3815–3820.

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