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RESEARCH ARTICLE

On the Design Flow of the Fractional-Order Analog Filters Between FPAA Implementation and Circuit Realization

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ABSTRACT This work explicitly states the design flows of the fractional-order analog filters used by researchers throughout the literature. Two main flows are studied: the FPAA implementation and the circuit realization. Partial-fraction expansion representation is used to prepare the approximated fractional-order response for implementation on FPAA. The generalization of the second-order active RC analog filters based on opamp from the integer-order domain to the fractional-order domain is presented. The generalization is studied from both mathematical and circuit realization points of view. It is found that the great benefit of the fractional-order domain is that it adds more degrees of freedom to the filter design process. Simulation and experimental results match the expected theoretical analysis.

INDEX TERMS Fractional-order, analog filter, FPAA, circuit realization.

I. INTRODUCTION

Analog filters are one of the most important blocks in designing communications and electronics systems. Developing a filtering block allows the signal to be processed in many system stages. Active filters have many advantages over passive ones, especially when cascading many sections to acquire higher order filters [1]. In general, the order of the filter indicates how much one can accurately and smoothly achieve the required response. For discrete technologies, many inductor-less active RC filters have been developed in the literature [2], [3], [4], [5]. Most of these previous works aimed to achieve high-quality factors or reduce passive and active sensitivities. The design of the second-order active RC filters was of special concern to the authors as they could be cascaded to get higher-order filters without the need to design a complex higher-order filter as a whole [6]. An inductor would require a large chip area, developing inductor-less filters, i.e., those that incorporate only capacitors and resistors. became inevitable [1]. Integer-order calculus is a subset of a more general science called fractional-order calculus [7]. Generalizing any system from the integer-order domain to the fractional-order domain gives more flexibility and degrees of freedom in the system design process. This has been a hot research topic in the last decade, implying many applications in different fields like digital modelling on FPGA [8], chaotic systems realization [9] and bio-impedance modelling [10]. The application of the generalization concept of systems to the fractional-order domain is eligible the integrated circuits technologies, which led to many works in the literature. Extensive research on the applications of the fractional-order analog and chaotic systems for integrated circuits technologies was conducted in [11] and [12], respectively. Further recent applications for the fractional-order filters suitable for integrated circuits technologies were discussed in [13] and [14].

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Generalizing the analog filters to the fractional-order domain can be handled on the filter design theory and the filter circuit realization. Generalization of the filter design on the circuit realization can be made by substitution of the integer-order elements with the fractional-order elements [15], [16], [17], by optimization [18], [19], [20], [21] or by curve fitting techniques [22], [23]. A new technique in designing Butterworth filter depending on pole placement in the complex w-plane was presented in [24]. Another approximation technique for designing a fractional-order Butterworth filter that depends on the weighted sum of the integer-order Butterworth transfer function was discussed in [25]. In [26], tool-based approximation techniques were utilized to approximate the fractional-order filter transfer function with two independent orders with a curve fitting program and another expansion-based program (namely, Padé function toolbox). A fractional-order power-law shelving filter was implemented in [27] using Oustaloup approximation over Foster-I to realize the fraction-order capacitor. Many works studied the effect of designing specific filters in the fractional-order domain. The fractional-order notch filter was studied with clear realization in [28]. In [29], the design of both the fractional-order band-pass and notch filters was studied over different approximation techniques. An electronically adjustable fractional-order filter was introduced in [30].

Another design flow for systems such as the fractional-order filters includes the implementation of a Field-programmable Analog Array (i.e., FPAA) [31], [32]. In [33], a proposed fractional-order PID controller was implemented on FPAA using two different implementation techniques. Finally, in [34], a generalized form of a multi-output fractional-order filter with two independent orders was implemented on FPAA.

This work aims to study and compare the two design flows with a design case and introduce a catalog for the generalized active filters with the explicit transfer function for each filter family. The integer-order passive elements used in designing the filters were a subset of a more general term called the fractional-order elements (i.e., the fractance element) [35], [36]. The fractance element has the impedance of the form $Z(s) = ks^{\alpha}$ where it becomes a resistor for $\alpha = 0$, an inductor for $\alpha = 1$, a capacitor for $\alpha = -1$, and a frequency-dependent negative resistor (FDNR) for $\alpha = -2$ [37]. A fractional-order capacitor is achieved by choosing $-2 < \alpha < 0$ while a fractional-order inductor is achieved by choosing $0 < \alpha < 2$ [38].

While the fractance device is still commercially unavailable as a complete element and the usage of integer order analysis tools and methods is more reliable than that of the fractional order ones, the development of approximation techniques for the Laplacian operator is inevitable [39], [40] for the real-time realization of the CPE. In the Recent work, [41], the modelling and circuit realization of the fractional-order element was summarized, and the passive circuit realization of the CPE was reviewed in [42]. Analog approximation techniques could be categorized [43] as in Fig. 1. [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45].

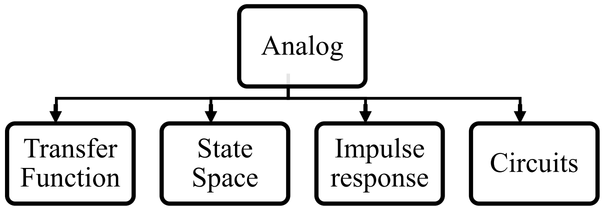


FIGURE 1. Analog approximation types [43], [44], [45].

This paper starts with some mathematical and theoretical basics for the fractional-order system in Section II and then goes through the design flow details, starting with the implementation of the approximated constant-phase element in Sections V-III. The design case of the filters on FPAA is then introduced in Section VI. Next, the catalog of the generalized filters is presented in Section VII and the paper finishes with the design case using the circuit approach in SectionVIII.

II. FRACTIONAL-ORDER MATHEMATICAL BACKGROUND

The conventional calculus that describes most engineering systems, including analog filters, is a subset of fractional-order calculus. In general, designing systems in the fractional-order domain gives more degrees of freedom for the design factors. Assuming a real-time continuous function $f(t)$, the Reimann-Liouville definition [46] for the continuous-time fractional-order derivative is as follows:

$$\frac{d^{\alpha}}{dt^{\alpha}} \equiv D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} f(\tau) d\tau, \quad (1)$$

where α is the fractional order and $0 < \alpha < 1$. As This definition is a continuous-time function, it mainly relates to the fractional-order analog systems applications. It is essential to understand the tendency of the fractance elements by applying the Laplace transform on Eqn. 1 by assuming a zero-time initial condition.

$$L\{{}_0d_t^{\alpha} f(t)\} = s^{\alpha} F(S), \quad (2)$$

which indicates that the fractance device (the fractional-order element) has an impedance that is proportional to s^{α}.

III. APPROXIMATION OF THE LAPLACIAN OPERATOR

Approximation of the Laplacian operator s^{α} has been an active research area in recent decades. The approximation techniques could be based on different criteria, like transfer-function based, circuit-based, state-space based or impulse-response based [45]. The most popular transfer function approximation techniques were continuous fraction expansion (CFE), Oustaloup, Matsuda and Carlson [43]. Another type of transfer function approximation was based on the weighted sum of high-pass filter sections. Considering the circuit-based techniques, the work of Valsa et al. [47]

TABLE 1. Comparison among different approximation techniques factors controllability.

Technique	(N)	(ω_c)	$(\omega_h - \omega_l)$	$(\Delta\Phi)$
Oustaloup	✓	x	✓	x
Matsuda	✓	x	✓	x
Carlson	x	✓	x	x
CFE	✓	✓	x	x
Valsa	✓	x	✓	✓

was the most significant approximation based on RC network expressions. Table 1 summarizes the configurable parameters for different approximation techniques.

A. CONTINUOUS FRACTION EXPANSION (CFE) APPROXIMATION

Also known as continued fractions, it is an approximation method used throughout history to approximate polynomials to rational functions (i.e., ratios of polynomials) [48]. The mathematical arguments and discussions on the continued fractions had been studied in the old textbooks as in [49]. In addition, the application of the CFE was associated with functions that converge much more rapidly than power series expansion [44]. This approximation method, alongside Carlson's method, is a mathematical approach to approximate the fractional-order Laplacian operator (s^α), which assumes it is a black box. This, in turn, reflects that both methods can only control the center frequency of the approximation, not the range of the frequencies. However, there is only one advantage of the CFE over Carlson: one can control the order of the resulting approximation fraction as listed in Table 1. The continued fractions have the following form:

$$a_0 + \frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \dots}}}, \tag{3}$$

which could be represented in Pringsheim notation as follows:

$$\left[a_0, \frac{b_k}{a_k}\right]_{k=1}^{+\infty}. \tag{4}$$

B. OUSTALOUP's (CRONE) APPROXIMATION

Oustaloup's approximation was first introduced as a part of the toolbox CRONE in [50]. The techniques distribute equal numbers of poles and zeros around the frequency range of interest, so the order of the approximation and frequency limits are controllable. The approximation has the following form [43]:

$$s^\alpha = C \prod_{m=1}^{N} \frac{1 + s/\omega_{z,m}}{1 + s/\omega_{p,m}}, \tag{5}$$

where

$$\omega_{z,m} = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(2m-1-\alpha)/2N}, \tag{6}$$

and

$$\omega_{p,m} = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(2m-1+\alpha)/2N}. \tag{7}$$

It is also worth mentioning that in the Oustaloup approximation, the order of the approximation beyond 8 (i.e., $N > 8$) does not significantly improve the performance [51].

C. MODIFIED OUSTALOUP APPROXIMATION

In [52], a modified version of oustaloup's algorithm was presented to give a better performance across the whole frequency range (ω_l, ω_h). The modified equation was presented as follows [53]:

$$s^\alpha = \left(\frac{d\omega_h}{b}\right)^\alpha \left(\frac{ds^2 + b\omega_h s}{d(1-\alpha)s^2 + b\omega_h s + d\alpha}\right) \dots \\ \dots \prod_{m=-N}^{N} \frac{1 + s/\omega_{z,m}}{1 + s/\omega_{p,m}}, \tag{8}$$

where $\omega_{z,m}$ and $\omega_{p,m}$ are given by Eqns. 6 and 7 respectively.

D. MATSUDA's APPROXIMATION

Matsuda approximation uses the continued fraction expansion of the single-input single-output fractional-order transfer functions [43]. The application of Matsuda approximation requires the distribution of an even number of frequency points between the frequency limits (i.e., $\omega_l : \omega_h$). This is because the algorithm gives $N/2$ number of poles and zeros [54]. Therefore, the algorithm could be written with Pringsheim notation as follows [43]:

$$G(s) = \left[d_0(\omega_0); \frac{s - \omega_{k1}}{d_k(\omega_k)+}\right]_{k=1}^{N}, \tag{9}$$

where

$$d_0(\omega) = |G(j\omega)|, \tag{10}$$

and

$$d_k(\omega) = \frac{\omega - \omega_{k-1}}{d_{k-1}(\omega) - d_{k-1}(\omega_{k-1})}. \tag{11}$$

E. VALSA's APPROXIMATION

In [47], Valsa introduced a circuit approximation technique for the fractional-order capacitor across a certain frequency range. The idea is similar to the transfer function approximations in that it realizes an RC network that distributes poles and zeros across the frequency range of interest. Two possible implementations for Valsa's networks are shown in Fig. 2. Valsa relied on the required phase variation $\Delta\phi$ across the frequency range to compute the values of the resistors and capacitors in the realized network. Given a required phase variation $\Delta\phi$, Valsa computes a and b parameters, as shown in Eqn. 12. The values of the first branch passives are related to the low-frequency limit ω_{min} by Eqn. 13. The initial branch value for the series network is shown in Eqn. 14 and for the

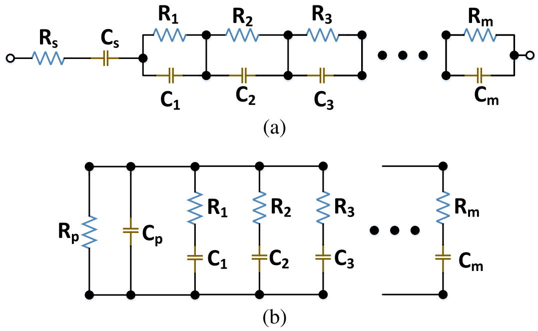


FIGURE 2. Valsa's approximation networks: (a) series *RC* network, (b) parallel *RC* network.

parallel network in Eqn. 15 where $1 < k \leq m$. The rest of the branches are calculated by Eqn. 16.

$$ab = \frac{0.24}{1 + \Delta\phi}, \tag{12a}$$

$$a = 10^{\alpha \log(ab)}, \tag{12b}$$

$$b = ab/a, \tag{12c}$$

$$R_1 C_1 = \frac{1}{\omega_{min}}. \tag{13}$$

$$R_s = \frac{R_1 a^m}{1 - a}, \tag{14a}$$

$$C_s = \frac{C_1(1 - b)}{b}, \tag{14b}$$

$$R_p = \frac{R_1(1 - a)}{a}, \tag{15a}$$

$$C_p = \frac{C_1 b^m}{1 - b}, \tag{15b}$$

$$R_k = R_1 a^{k-1}, \tag{16a}$$

$$C_k = C_1 b^{k-1}, \tag{16b}$$

IV. APPROXIMATION OF THE FRACTIONAL-ORDER RATIONAL CONTROLLERS

A. CHAREF's APPROXIMATION

Charef approximation was introduced in [55] as an approximation technique for the fractional-order controller of the form [44]:

$$H(s) = \frac{1}{(1 + \frac{s}{P_T})^{\alpha}} \approx \frac{\prod_{i=0}^{n-1}(1 + s/z_i)}{\prod_{i=0}^{n}(1 + s/p_i)}, \tag{17}$$

where poles and zeros are defined as:

$$p0 = p_T \sqrt{b}, \tag{18}$$

$$pi = p_0 (ab)^i, \tag{19}$$

$$zi = a p_0 (ab)^i, \tag{20}$$

where,

$$a = 10^{y/10(1-\alpha)}, \tag{21}$$

$$b = 10^{y/10\alpha}, \tag{22}$$

$$ab = 10^{y/10\alpha(1-\alpha)}, \tag{23}$$

It was also discussed in [55] that Charef's approximation could give a reasonable estimate for s^{α}.

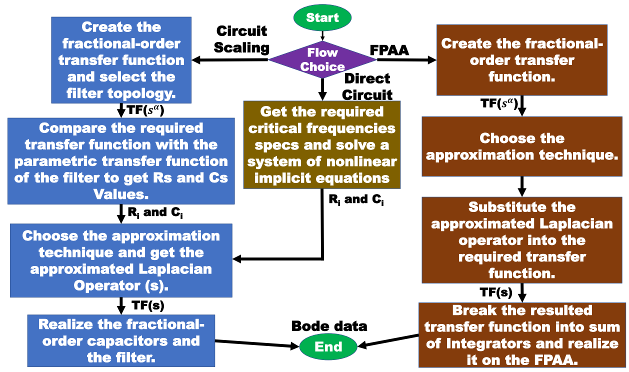


FIGURE 3. Fractional-order systems design flows.

B. PADÉ APPROXIMATION

Introduced in [56], the Padé method was meant to give an approximation for rational functions such as the lead/lag compensator presented in [57] or to be used as a reduction model as discussed in [58]. The approximation form is given by:

$$H(s) = R_{[m/n]} = \frac{(\frac{p_m}{q_n})s^m + (\frac{p_{m-1}}{q_n})s^{m-1} + \ldots + (\frac{p_0}{q_n})}{s^n + (\frac{q_{n-1}}{q_n})s^{n-1} + \ldots + (\frac{1}{q_n})} \tag{24}$$

where p_i and q_j can be calculated by solving a $m+n+1$ linear equations as shown in [57].

V. FRACTIONAL-ORDER ANALOG FILTERS DESIGN FLOWS

Two main flows can design the filters: circuit realization and FPAA implementation, as depicted in Fig. 3.

A. CASE STUDY FOR THE APPROXIMATION OF THE LAPLACIAN OPERATOR

The case to be studied implies approximating $s^{1/3}$ to a frequency range of 1 $10^4 Hz$. Four transfer function techniques have been chosen, namely, Oustaloup, Matsuda, Carlson and Continued Fraction Expansion (CFE), alongside one circuit approximation, Valsa's. Table 2 summarizes the MATLAB plot for the approximated functions. Phase variation $\Delta\Phi$ at frequency limits could be estimated as 8.67 % for Oustaloup, 9.3 % for Matsuda, 67.4 % for Carlson, 53.24 % for CFE.

VI. CASE STUDY FOR THE DESIGN OF A LOW-PASS FRACTIONAL-ORDER FILTER ON FPAA

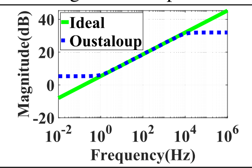
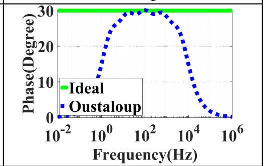
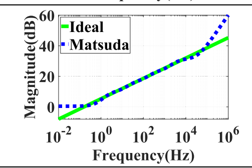
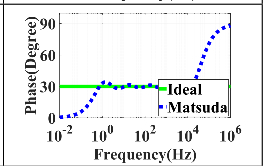
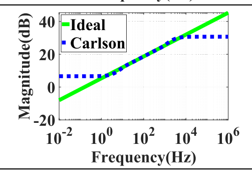
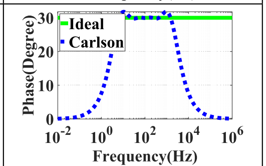
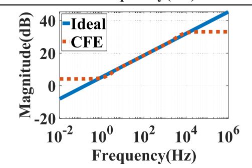
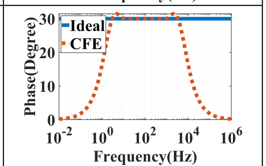
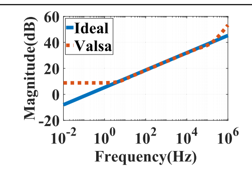
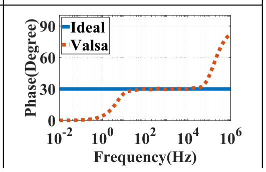
Considering the generalization of the normalized second-order Butterworth filter in the following equation:

$$T(s) = \frac{1}{s^2 + \sqrt{2}s + 1}, \tag{25}$$

to the fractional-order domain with one constant-phase element with order α. The fractional-order transfer function of the filter becomes:

$$T(s) = \frac{1}{s^{1+\alpha} + \sqrt{2}s^{\alpha} + 1}, \tag{26}$$

TABLE 2. Magnitude and phase response for different techniques.

Technique	Magnitude Response	Phase Response
Oustaloup		
Matsuda		
Carlson		
CFE		
Valsa		

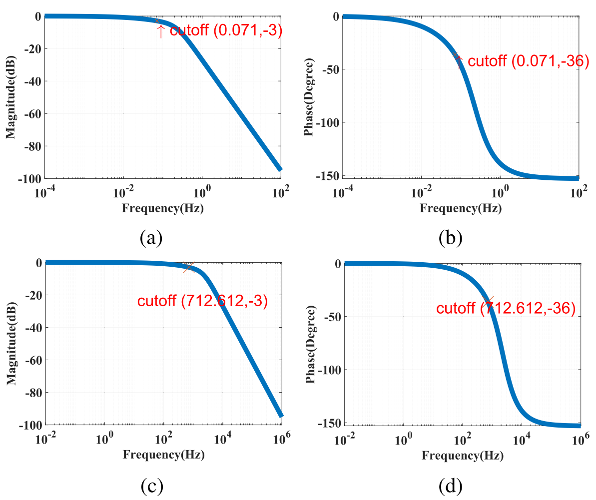


FIGURE 4. Butterworth ideal response: (a) normalized magnitude response, (b) normalized phase response, (c) scaled magnitude response, (d) scaled phase response.

which has the normalized response with cutoff frequency 0.071 Hz and phase -30^o as shown in Figs. 4a-4b. For more practical operation, a frequency scale factor of 10^4 is used to get the cutoff frequency of 714 Hz, which could be seen in Figs. 4c-4d.

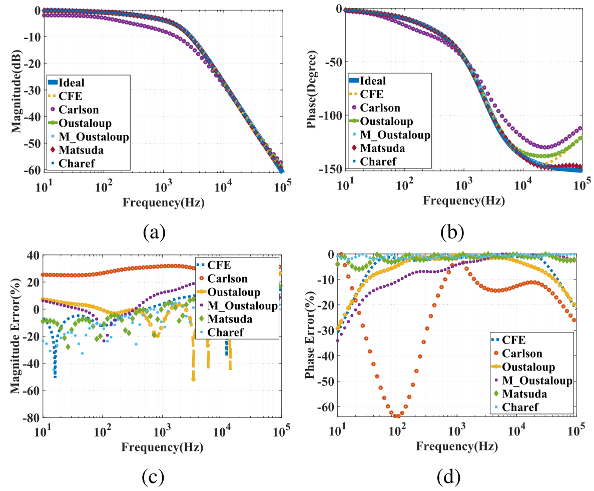


FIGURE 5. Butterworth approximated response: (a) magnitude responses, (b) phase responses, (c) magnitude response error, (d) phase response error.

Applying different approximation techniques to the fractional-order Laplacian operator $\alpha = 0.7$ and then substituting into the transfer function gives the following approximated response as in Figs. 5a-5b. The error of the magnitude and phase responses are shown in Figs. 5c-5d.

At this point, the approximated transfer functions can be broken into a sum of integrators, as shown in Fig. 7 for realization on FPAA. Using the residue function on MATLAB and getting the appropriate gain and time constants for integrators, Table 3 summarizes the realized values. First, AnadigmDesigner software was used to realize the filters on the FPAA AN231E04 kit, as shown in Fig. 6. Then the stimulus was injected using an NI ELVIS II kit to perform an AC sweep and get the bode plot for each filter. Finally, figure 8 shows the experimental results plotted using MATLAB.

VII. TRANSFORMATION OF SOME ACTIVE FILTERS TO THE FRACTIONAL-ORDER DOMAIN

A generalized form of the integer-order version of the filter should be derived to realize the fractional-order filter circuits, which is the first step in the design process. As the orders of the fractional-order capacitor could be of different values, it has become inevitable to determine the capacitor that contributes to the quality factor term in the integer-order version (i.e., the s term in the denominator). After that, the transfer function will be ready for comparison with the required transfer function to map the components' values. This will be followed by approximating and realizing the calculated fractional-order capacitor and then composing the circuit. This section presents the generalization of some of the well-known alongside some of the forgotten second-order filter circuits to the fractional-order domain.

A. BACH FAMILY

Figure 9 shows the Bach's low-pass filter generalized into fractional order domain with two fractional order capacitors

TABLE 3. Gain and time constants of realized integrators.

Integrator No.	CFE		Oustaloup		Mod Oustaloup		Matsuda		Charef	
	$k(mV/V)$	$\tau(\mu s)$	$k(mV/V)$	$\tau(\mu s)$	$k(mV/V)$	$\tau(\mu s)$	$k(mV/V)$	$\tau(\mu s)$	$k(mV/V)$	$\tau(\mu s)$
1	−70.9	24.8	526.2	100	−0.2	1.43	−0.5	1.16	−36.8	16.1
2	245.1	200	195.3	600	−3.7	5.89	−63.6	21.2	325	371
3	153.7	600	50	3500	−82.9	29.5	278.3	526.478	87	2620
4	86	2300	195.3	100	264.4	256.362	69	5000	18.7	21900
5	564.5	100	-	-	107.1	1030.763	707.7	100	4	194000
6	-	-	-	-	30.6	4449.225	-	-	0.9	1730000
7	-	-	-	-	35.7	1015800	-	-	0.2	15500000
8	-	-	-	-	648.5	100	-	-	601	100

TABLE 4. Fliege transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\left(\frac{R_1+R_2}{R_2R_4R_6C_\beta C_\alpha}\right)}{s^{\alpha+\beta}+\frac{s^\beta}{R_5C_\alpha}+\frac{R_1}{R_2R_4R_6C_\beta C_\alpha}}$
FO HPF	$\frac{s^{\alpha+\beta}\left(1+\frac{R_2}{R_1}\right)}{s^{\alpha+\beta}+\frac{s^\beta}{R_6C_\alpha}+\frac{R_2}{R_1R_3R_5C_\beta C_\alpha}}$
FO BPF	$\frac{\frac{s^\beta}{R_6C_\alpha}\left(1+\frac{R_2}{R_1}\right)}{s^{\alpha+\beta}+\frac{s^\beta}{R_6C_\alpha}+\frac{R_2}{R_1R_3R_5C_\beta C_\alpha}}$
FO Notch	$\frac{s^{\alpha+\beta}\left(1+\frac{R_2}{C_\alpha}(\frac{1}{R_2R_{6b}}-\frac{1}{R_1R_{6a}})\right)+\frac{R_2}{R_1R_3R_5C_\beta C_\alpha}}{s^{\alpha+\beta}+\frac{s^\beta}{R_6C_\alpha}+\frac{R_2}{R_1R_3R_5C_\beta C_\alpha}}$

C_1 and C_2 having α and β respectively. By direct analysis, the transfer function of the filter becomes:

$$H_{LP}(s) = \frac{\left(\frac{1}{R_1R_2C_\alpha C_\beta}\right)}{s^{\alpha+\beta}+\frac{s^\beta}{R_1C_\alpha}+\frac{1}{R_1R_2C_\alpha C_\beta}}. \tag{27}$$

B. RAUCH FILTERS FAMILY

The Rauch filter was presented In [59]. The fractional-order version of the filter is presented in Fig. 10 and the transfer function is shown in Eqn. 28.

$$H_{LP}(s) = \frac{\frac{-1}{R_1R_2C_\alpha C_\beta}}{s^{\alpha+\beta}+s^\beta\frac{1}{R_1C_\alpha}(1+\frac{R_1}{R_2}+\frac{R_1}{R_3})+\frac{1}{R_2R_3C_\alpha C_\beta}}. \tag{28}$$

C. FRACTIONAL-ORDER FLIEGE FAMILY

The Fliege filter was presented In [60]. The fractional-order version of the filter is presented in Fig. 11 and the transfer function is shown in Table 4.

D. DELIYANNIS FILTERS FAMILY

The Deliyannis filter was presented In [61]. The fractional-order version of the filter is presented in Fig. 12 and the transfer function is shown in Table 5.

E. MIKHAEL-BHATTACHARYYA (MB) FILTERS FAMILY

Mikhael and Bhattacharyya presented a filter family In 1975 in [62]. The fractional-order version of the filter is

TABLE 5. Fractional-order Delyannis transfer functions.

Filter Type	$H(s)$
Band-Pass (I)	$\frac{-\frac{s^\alpha}{R_1C_\beta}}{s^{\alpha+\beta}+\frac{s^\alpha}{R_2C_\beta}+\frac{s^\beta}{R_2C_\alpha}+\frac{1}{R_1R_2C_\alpha C_\beta}+\frac{1}{R_2R_3C_\alpha C_\beta}}$
Band-Pass (II)	$\frac{\frac{-s^\alpha}{R_1C_\alpha}(1+\frac{R_a}{R_b})}{s^{2\alpha}+s^\alpha(\frac{1}{R_2C_\alpha}-\frac{R_a}{R_1R_bC_\alpha})+s^\beta\frac{C_\beta}{R_2C_\alpha^2}+\frac{1}{R_1R_2C_\alpha^2}}$

TABLE 6. MB transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{(\frac{R_6(R_2+R_7)}{R_2(R_5+R_6)})(\frac{R_2(R_5+R_6)}{C_1C_2R_5R_6R_7R_{10}})}{s^2+s\frac{R_2R_9}{C_1R_4R_7R_8}+\frac{R_2(R_5+R_6)}{C_1C_2R_5R_6R_7R_{10}}}$
FO HPF	$\frac{s^2\frac{R_2(R_4+R_9)}{R_4(R_1+R_2)}}{s^2+s\frac{R_1R_2R_9}{C_1R_4R_7R_8(R_1+R_2)}+\frac{R_1R_2}{C_1C_2R_6R_7R_{10}(R_1+R_2)}}$
FO BPF	$\frac{s\frac{R_9(R_2+R_7)}{C_1R_3R_7R_8}}{s^2+s\frac{R_2R_9(R_3+R_4)}{C_1R_3R_4R_7R_8}+\frac{R_2}{C_1C_2R_6R_7R_{10}}}$
FO Notch	$\frac{(\frac{R_2(R_4+R_9)}{R_4(R_1+R_2)})(s^2+\frac{R_1}{R_5R_7R_{10}C_1C_2})}{s^2+s\frac{R_1R_2R_9}{R_4R_7R_8(R_1+R_2)C_1}+\frac{R_1R_2(R_3+R_6)}{R_3R_6R_7R_{10}(R_1+R_2)C_1C_2}}$

TABLE 7. PMG transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\frac{R_2^2R_6^2(R_2+R_3)}{R_2R_3R_4R_5R_9C_1^2C_2^2}}{s^2+s\frac{R_2R_6(R_3+R_8)}{R_1R_3R_4R_8C_1}+\frac{R_6(R_7+R_9)}{R_4R_5R_7R_9C_1C_2}}$
FO HPF	$\frac{s^2\frac{R_2^2R_6(R_2+R_3)C_3}{R_2R_3C_2(C_1+C_3)}}{s^2+s\frac{R_2R_6}{R_1R_3R_4(C_1+C_3)}+\frac{R_6(R_7+R_9)}{R_4R_5R_7R_9C_2(C_1+C_3)}}$
FO BPF	$\frac{s\frac{R_2^2R_6^2}{R_1R_4R_8C_1^2C_2}}{s^2+s\frac{R_2R_6(R_3+R_8)}{R_1R_3R_4R_8C_1}+\frac{R_6}{R_4R_5R_7C_1C_2}}$
FO Notch	$\frac{(\frac{R_2R_6(R_2+R_3)C_3}{R_3C_2(C_1+C_3)^2})(s^2+\frac{R_6}{R_4R_5R_9C_2C_3})}{s^2+s\frac{R_2R_6}{R_1R_3R_4(C_1+C_3)}+\frac{R_6(R_7+R_9)}{R_4R_5R_7R_9C_2(C_1+C_3)}}$

presented in Fig. 13, and the transfer function is shown in Table 6.

F. PADUKONE-MULAWKA-GHAUSI (PMG) FILTERS FAMILY

This filter family was presented in [63]. The fractional-order version of the filter is presented in Fig. 14 and the transfer function is shown in Table 7.

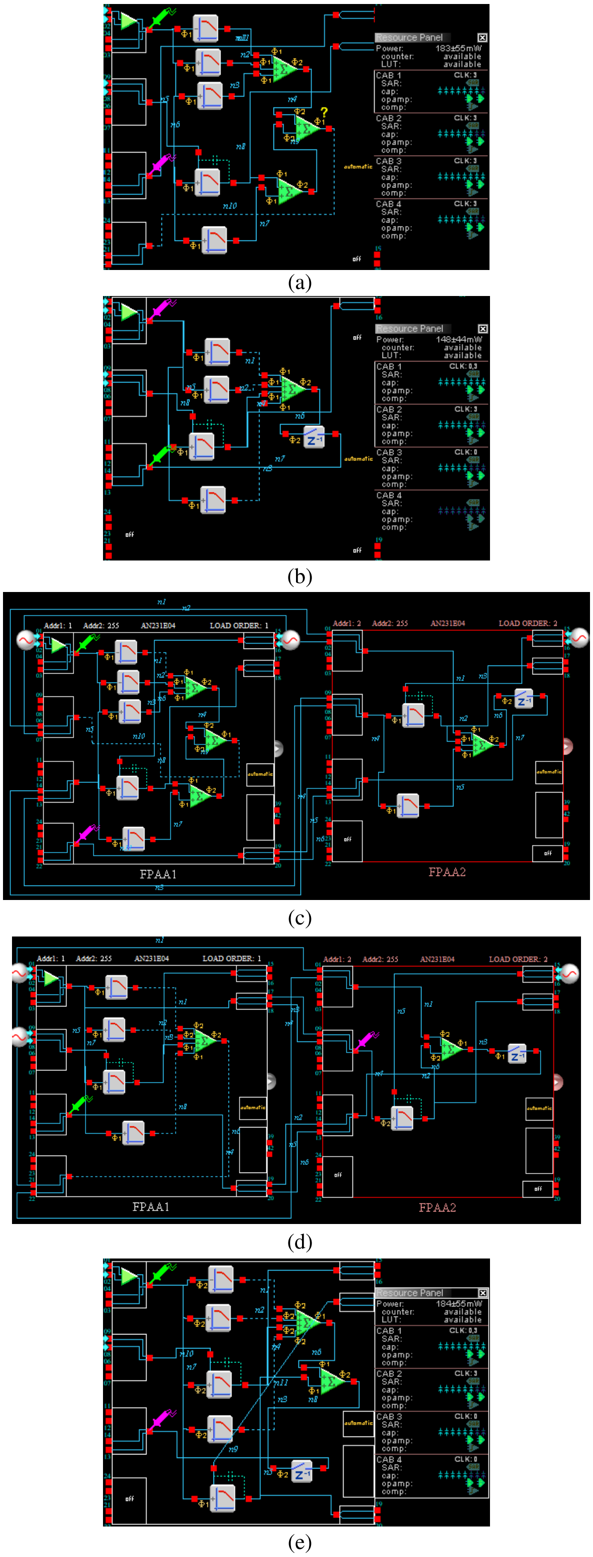


FIGURE 6. FPAA realization of the FO filter using approximation: (a) CFE, (b) Oustaloup, (c) Modified Oustaloup, (d) Matsuda, (e) Charef.

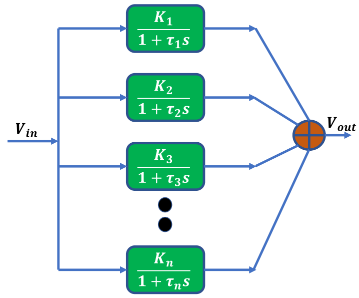


FIGURE 7. Partial fraction expansion representation of the approximated transfer function.

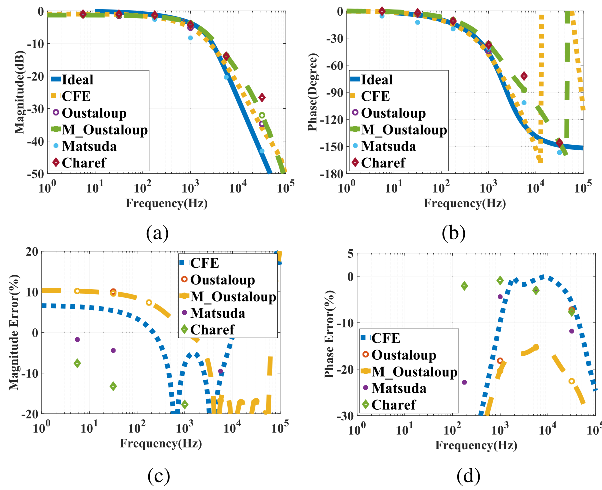


FIGURE 8. Experimental Results: (a) magnitude responses, (b) phase responses, (c) magnitude response error, (d) phase response error.

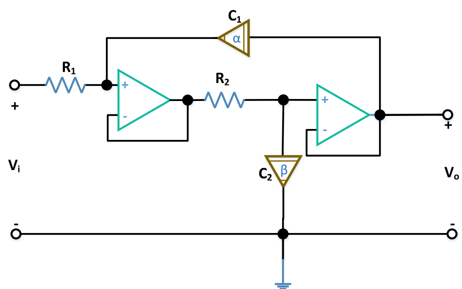


FIGURE 9. Bach's FOLPF circuit schematics.

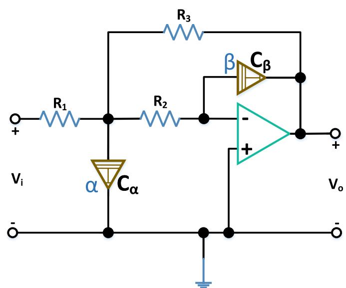


FIGURE 10. Rauch's FOLPF circuit schematics.

G. BERKA-HERPY FAMILY

The BH filter family is a universal building block family that was first proposed in 1981 in [64].

H. FRACTIONAL-ORDER Soliman72 FILTER

Presented in [65], the fractional-order version of this filter is presented in Fig. 16 and the transfer function is shown in Table 9.

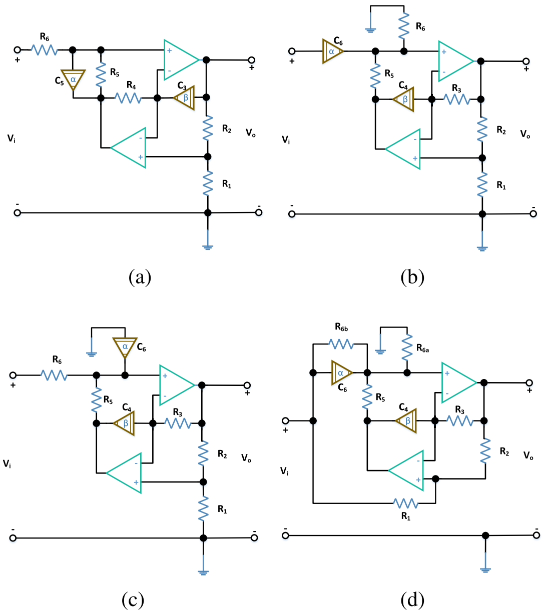


FIGURE 11. Fractional-order Fliege Filters Family: (a) FOLPF, (b) FOHPF, (c) FOBPF, (d) FONF.

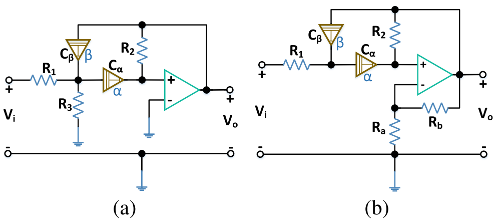


FIGURE 12. Fractional-order Delyannis Filters: (a) FOBPF I, (b) FOBPF II.

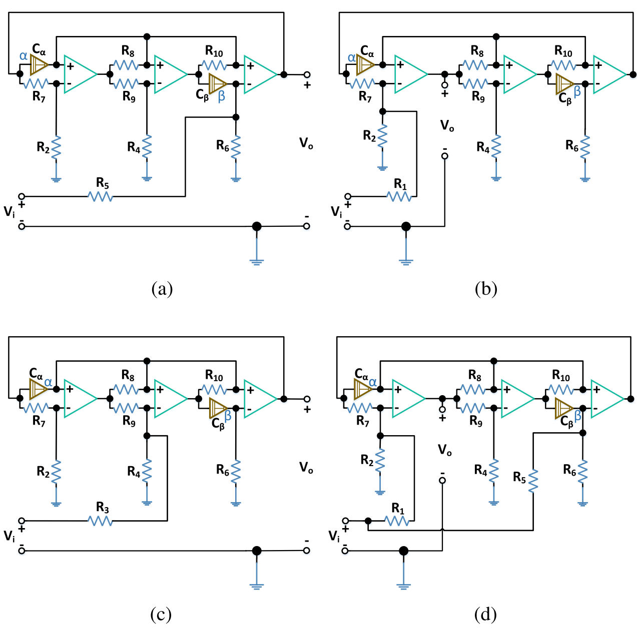


FIGURE 13. Fractional-order MB Filters Family: (a) FOLPF, (b) FOHPF, (c) FOBPF, (d) FONF.

I. FRACTIONAL-ORDER Soliman73 FILTER

Presented in [66], the fractional-order version of this filter is presented in Fig. 17 and the transfer function is shown in Table 10.

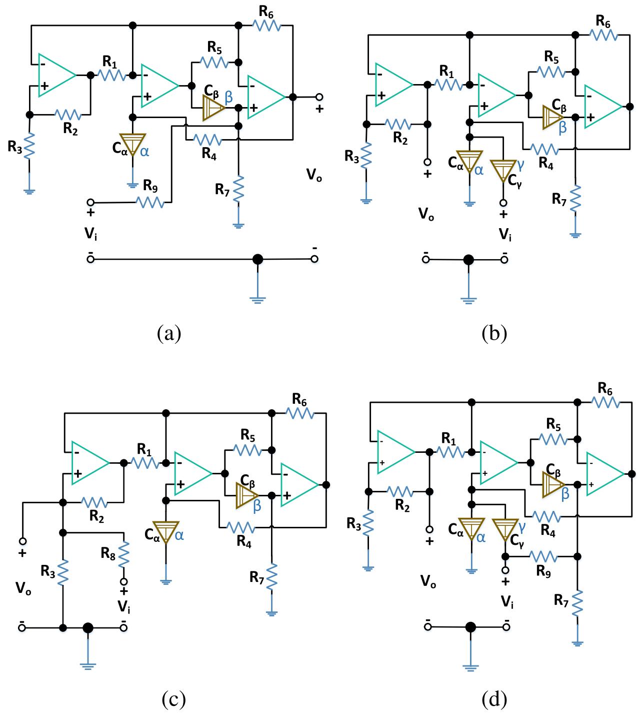


FIGURE 14. Fractional-order PMG Filters Family: (a) FOLPF, (b) FOHPF, (c) FOBPF, (d) FONF.

TABLE 8. BH transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\frac{R_6}{C_1C_2R_1R_2R_5}}{s^2+s\frac{1}{C_1R_3}+\frac{R_6}{C_1C_2R_1R_2R_5}}$
FO HPF	$\frac{s^2}{s^2+s\frac{1}{C_1R_3}+\frac{R_6}{C_1C_2R_1R_2R_5}}$
FO BPF	$\frac{-s\frac{R_8(R_5+R_6)}{R_1R_5(R_7+R_8)C_1}}{s^2+s\frac{1}{C_1R_3}+\frac{R_6}{C_1C_2R_1R_2R_5}}$

TABLE 9. Fractional-order Soliman72 filter transfer functions.

Filter Type	$H(s)$
FO Notch	$\frac{s^{2\alpha}+(\frac{1}{CR})^2}{s^{2\alpha}+s^{\alpha}(\frac{4}{KRC})+(\frac{1}{CR})^2}$

TABLE 10. Fractional-order Soliman73 filter transfer functions.

Filter Type	$H(s)$
All-Pass ($a=\frac{1}{1+1/2Q^2}$)	$a\frac{s^{2\alpha}-s^{\alpha}(\frac{R_b(1-a)-2aR}{aR_bRC})+(\frac{\sqrt{R}}{\sqrt{R_b}RC})^2}{s^{2\alpha}+s^{\alpha}(\frac{2}{R_bC})+(\frac{\sqrt{R}}{\sqrt{R_b}RC})^2}$
All-pass	$\frac{s^{2\alpha}-s^{\alpha}(\frac{2R_2}{RR_1C})+(\frac{1}{RC})^2}{s^{2\alpha}+s^{\alpha}(\frac{2(3R_a-R_b)}{RR_aC})+(\frac{1}{RC})^2}$

J. FRACTIONAL-ORDER Soliman74 FILTER

Presented in [67], the fractional-order version of this filter is presented in Fig. 18 and the transfer function is shown in Table 11.

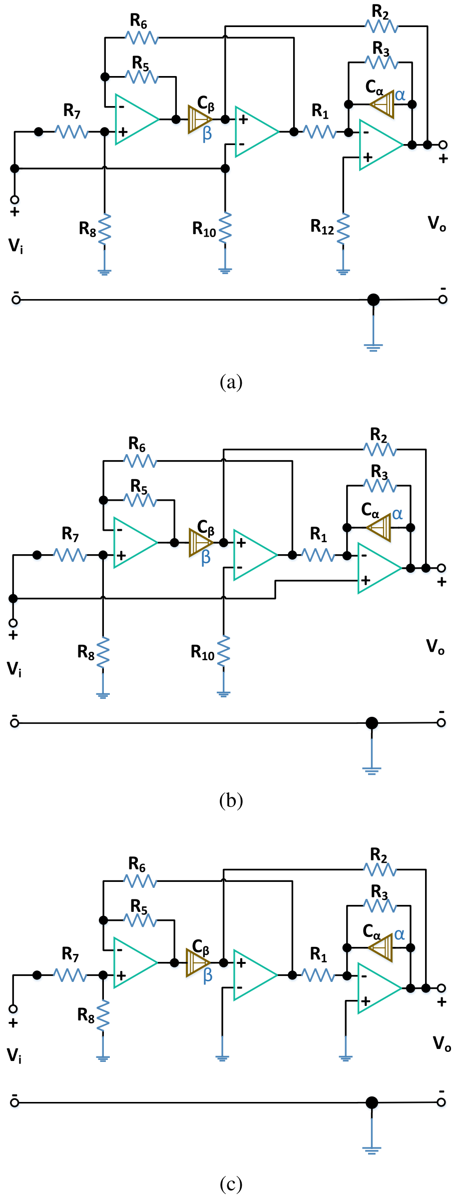


FIGURE 15. Fractional-order BH Filters Family: (a) FOLPF, (b) FOHPF, (c) FOBPF, (d) FONF.

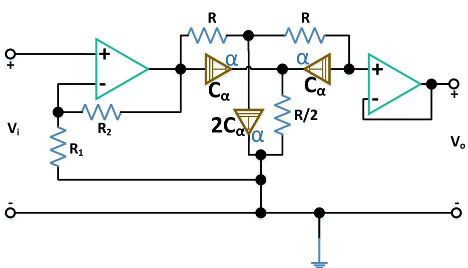


FIGURE 16. Fractional-order Soliman72 Filter Schematics.

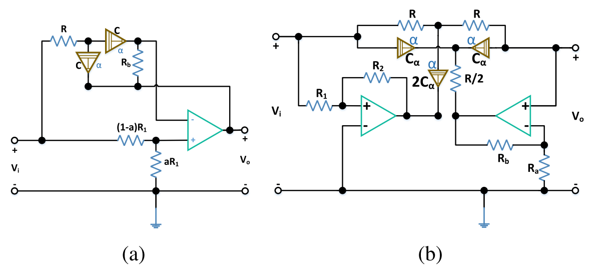


FIGURE 17. Fractional-order Soliman73 Filters Family: (a) APF I, (b) APF II.

TABLE 11. Fractional-order Soliman74 filter transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\frac{(R_a+R_b)(R_c+R_d)}{R_aR_cR_1R_2C_1C_2}}{s^2+s(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1})+\frac{R_aR_c+R_b(R_c+R_d)}{R_aR_cR_1R_2C_1C_2}}$

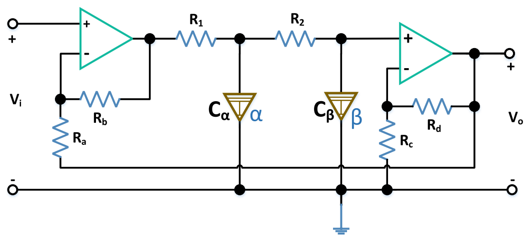


FIGURE 18. Fractional-order Soliman74 filter schematics.

TABLE 12. Fractional-order Soliman76 filter transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\frac{-as}{RC}}{s^{2\alpha}+s^{\alpha}(\frac{b+1-a}{RC})+\frac{1}{R_2C_2}}$

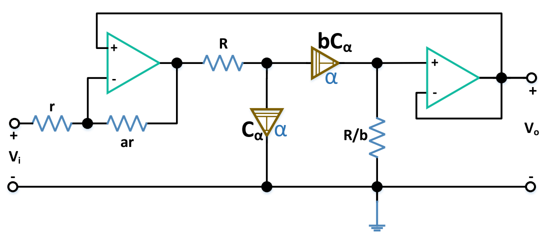


FIGURE 19. Fractional-order Soliman76 filter schematics.

TABLE 13. Fractional-order Soliman79 filter transfer functions.

Filter Type	$H(s)$
FO LPF	$\frac{\frac{-sa\beta}{R_1C_2}}{s^2+s(\frac{1}{R_1C_1}+\frac{1}{R_1C_2}-\frac{a}{R_2C_2})+\frac{1}{R_1R_2C_1C_2}}$

K. FRACTIONAL-ORDER Soliman76 FILTER

Presented in [68], the fractional-order version of this filter is presented in Fig. 19 and the transfer function is shown in Table 12.

L. FRACTIONAL-ORDER Soliman79 FILTER

Presented in [69], the fractional-order version of this filter is presented in Fig. 20 and the transfer function is shown in Table 13.

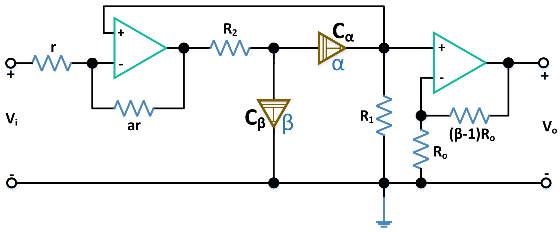


FIGURE 20. Fractional-order Soliman79 filter schematics.

TABLE 14. Fractional-order Soderstrand filter transfer functions.

Filter Type	$H(s)$
FO BPF	$\frac{(\frac{K_1K_2}{1-K_1K_2})\frac{s^\beta}{R_1C_\alpha}}{s^{\alpha+\beta}+\frac{1}{(1-K_1K_2)}(\frac{s^\alpha}{R_2C_\beta}+\frac{s^\beta}{R_2C_\alpha})+(\frac{1}{R_1R_2C_\alpha C_\beta(1-K_1K_2)})}$

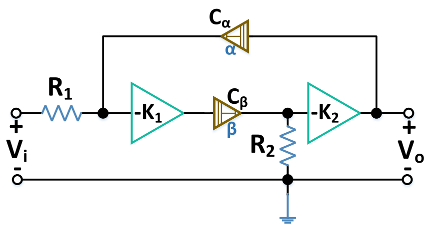


FIGURE 21. Fractional-order Soderstrand filter schematics.

TABLE 15. Fractional-order AM transfer functions.

Filter Type	$H(s)$
Low-Pass	$\frac{-\frac{R_3}{C_1C_2R_1R_2R_5}}{s^2+s\frac{1}{C_2R_6}+\frac{R_3}{C_1C_2R_2R_4R_5}}$
High-Pass	$\frac{-\frac{C_3}{C_2}s^2}{s^2+s\frac{1}{C_2R_6}+\frac{R_3}{C_1C_2R_2R_4R_5}}$
Band-Pass	$\frac{-\frac{1}{r_4C_2}}{s^2+s\frac{1}{C_2R_6}+\frac{R_3}{C_1C_2R_2R_4R_5}}$
Notch	$\frac{-\frac{C_3}{C_2}s^2+\frac{R_2}{r_1R_1R_3C_1C_3}}{s^2+s\frac{1}{C_2R_6}+\frac{R_3}{C_1C_2R_2R_4R_5}}$

TABLE 16. Fractional-order HS transfer functions.

Filter Type	$H(s)$
Low-Pass	$\frac{\frac{1}{R_1R_2C}}{s^{2\alpha}+s^\alpha\frac{2}{R_QC}+\frac{(1+2R/R_Q)}{R^2C^2}}$
High-Pass	$\frac{s^{2\alpha}}{s^{2\alpha}+s^\alpha\frac{2}{R_QC}+\frac{(1+2R/R_Q)}{R^2C^2}}$
Band-Pass	$\frac{\frac{s^\alpha}{R_1C}}{s^{2\alpha}+s^\alpha\frac{2}{R_QC}+\frac{(1+2R/R_Q)}{R^2C^2}}$

M. FRACTIONAL-ORDER SODERSTRAND FAMILY

Presented in [70], the fractional-order version of this filter is presented in Fig. 21 and the transfer function is shown in Table 14.

N. FRACTIONAL-ORDER AKERBERG-MOSSBERG FAMILY

Presented in [71], the fractional-order version of this filter is presented in Fig. 22 and the transfer function is shown in Table 15.

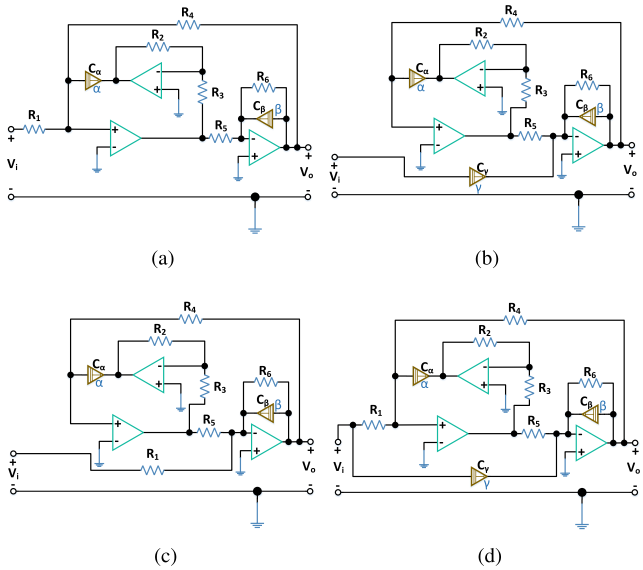


FIGURE 22. Fractional-order AM Filters Family: (a) FOLPF, (b) FOHPF, (c) FOBPF, (d) FONF.

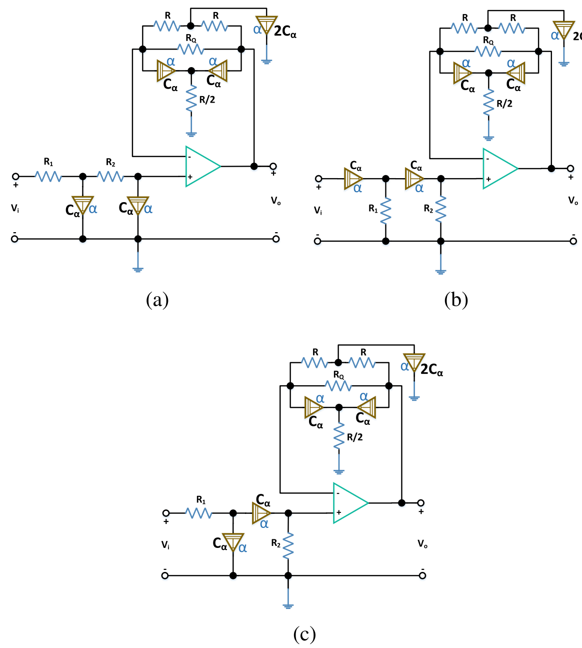


FIGURE 23. Fractional-order HS Filters Family: (a) FOLPF, (b) FOLPF, (c) FOHPF, (d) FOBPF.

O. FRACTIONAL-ORDER HAMILTON-SEDRA 1972 (HS I)

Presented in [72], the fractional-order version of this filter is presented in Fig. 23 and the transfer function is shown in Table 16.

P. FRACTIONAL-ORDER BHATTACHARYYA-MIKHAEL-ANTONIOU (BMA)

Presented in [73], the fractional-order version of this filter is presented in Fig. 24 and the transfer functions with conditions on the ports are shown in Table 17.

TABLE 17. Fractional-order BMA family.

Filter Type	Conditions	$H(s)$
FOLPF I (Port 4)	• $Y_2 = s^\alpha C_\alpha$ • $Y_3 = s^\beta C_\beta + G_3$ • $Y_6 = Y_7 = 0$	$\frac{\frac{1}{R_1 R_5 C_\alpha C_\beta}(1+\frac{R_8}{R_4})}{s^{\alpha+\beta}+s^\alpha \frac{1}{R_3 C_\beta}+\frac{1}{R_1 R_4 R_5 R_8 C_\alpha C_\beta}}$
FOLPF II (Port 3)	• $Y_1 = s^\alpha C_\alpha$ • $Y_4 = s^\beta C_\beta + G_4$ • $Y_8 = s^\gamma C_\gamma + G_8$ • $Y_5 = 0$	$\frac{\frac{1}{R_3 R_7 C_\alpha C_\beta}(1+\frac{R_6}{R_2})}{s^{\alpha+\beta}+s^\alpha \frac{1}{R_4 C_\beta}+s^\gamma \frac{C_\gamma}{R_2 R_3 R_6 C_\alpha C_\beta}+\frac{R_6(R_7+R_8)}{R_2 R_3 R_7 R_8 C_\alpha C_\beta}}$
FOHPF I (Port 3)	• $Y_3 = s^\alpha C_\alpha$ • $Y_7 = s^\alpha C_\alpha$ • $Y_8 = s^\gamma C_\gamma + G_8$ • $Y_5 = 0$	$\frac{s^{\alpha+\beta}(1+\frac{R_2}{R_6})}{s^{\alpha+\beta}+s^{\alpha+\gamma}\frac{C_\gamma}{C_\beta}+s^\alpha \frac{1}{R_8 C_\beta}+\frac{R_2}{R_1 R_4 R_6 C_\alpha C_\beta}}$
FOHPF II (Port 3)	• $Y_3 = s^\alpha C_\alpha$ • $Y_7 = \frac{s^\beta C_\beta}{1+s^\beta C_\beta R_7}$ • $Y_5 = Y_8 = 0$	$\frac{s^{\alpha+\beta}(1+\frac{R_2}{R_6})}{s^{\alpha+\beta}+s^\beta \frac{R_2 R_7}{R_1 R_4 R_6 C_\alpha}+\frac{R_2}{R_1 R_4 R_6 C_\alpha C_\beta}}$
FOBPF I (Port 4)	• $Y_2 = s^\alpha C_\alpha$ • $Y_3 = s^\beta C_\beta$ • $Y_5 = s^\gamma C_\gamma$ • $Y_7 = 0$	$\frac{s^\gamma \frac{C_\gamma}{R_1 C_\alpha C_\beta}(1+\frac{R_8}{R_4})}{s^{\alpha+\beta}+s^\gamma \frac{R_8 C_\gamma}{R_1 R_4 C_\alpha C_\beta}+\frac{R_8}{R_1 R_4 R_6 C_\alpha C_\beta}}$
FOBPF II (Port 3)	• $Y_1 = s^\alpha C_\alpha$ • $Y_7 = s^\gamma C_\gamma$ • $Y_4 = s^\beta C_\beta + G_4$ • $Y_5 = 0$	$\frac{s^\gamma \frac{R_6 C_\gamma}{R_2 R_3 C_\alpha C_\beta}(1+\frac{R_2}{R_6})}{s^{\alpha+\beta}+s^\alpha(\frac{1}{R_4 C_\beta})+s^\gamma(\frac{R_6 C_\gamma}{R_2 R_3 C_\alpha C_\beta})+\frac{R_6}{R_2 R_3 R_8 C_\alpha C_\beta}}$
FOBPF III (Port 3)	• $Y_3 = s^\alpha C_\alpha$ • $Y_8 = s^\beta C_\beta + G_8$ • $Y_5 = 0$	$\frac{s^\alpha \frac{1}{R_7 C_\beta}(1+\frac{R_2}{R_6})}{s^{\alpha+\beta}+s^\alpha \frac{R_7+R_8}{R_7 R_8 C_\beta}+\frac{R_2}{R_1 R_4 R_6 C_\alpha C_\beta}}$
FONF I (Port 4)	• $Y_1 = s^\alpha C_\alpha$ • $Y_4 = s^\beta C_\beta$ • $Y_8 = s^\gamma C_\gamma$ • $Y_6 = 0$	$\frac{s^{\alpha+\beta}+s^{\alpha+\gamma}\frac{C_\gamma}{C_\beta}+\frac{R_5}{R_2 R_3 R_7 C_\alpha C_\beta}}{s^{\alpha+\beta}+s^\gamma \frac{R_5 C_\gamma}{R_2 R_3 C_\alpha C_\beta}+\frac{R_5}{R_2 R_3 R_7 C_\alpha C_\beta}}$
FONF II (Port 4)	• $Y_3 = s^\alpha C_\alpha$ • $Y_7 = s^\beta C_\beta$ • $Y_6 = 0$	$\frac{s^{\alpha+\beta}+\frac{R_2}{R_1 R_4 R_5 C_\alpha C_\beta}(1+\frac{R_4}{R_8})}{s^{\alpha+\beta}+s^\alpha \frac{1}{R_8 C_\beta}+\frac{R_2}{R_1 R_4 R_5 C_\alpha C_\beta}}$
FONF III (Port 2)	• $Y_3 = s^\alpha C_\alpha$ • $Y_7 = s^\beta C_\beta$ • $Y_8 = s^\gamma C_\gamma + G_8$	$\frac{s^{\alpha+\beta}+\frac{R_2}{R_1 R_4 R_5 C_\alpha C_\beta}}{s^{\alpha+\beta}+s^{\alpha+\gamma}\frac{C_\gamma}{C_\beta}+s^\alpha \frac{C_\gamma}{C_\beta}+\frac{R_2(R_5+R_6)}{R_1 R_4 R_5 R_6 C_\alpha C_\beta}}$

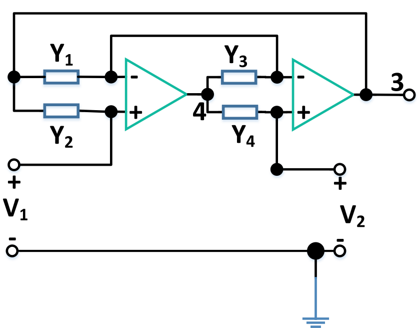


FIGURE 24. BMA general building block.

VIII. CASE STUDY FOR THE DESIGN OF A LOW-PASS FRACTIONAL-ORDER FILTER BY CIRCUIT SCALING IMPLEMENTATION

Considering the implementation of the transfer function in Eqn. 26 with the fractional-order Bach's filter in Fig. 9. Comparing the required transfer function with that of the filter in Eqn. 27 and using frequency scaling of 10^4 and magnitude scaling of 10^3, the values of the required passive elements are summarized in Table 18 with $\alpha = 1$ and $\beta = 0.7$. The fractional order capacitor was implemented using four approximation techniques: CFE, Carlson, Matsuda and Oustaloup. The capacitor was then realized using the FosterI RC network, as shown in Fig. 25a. Values for resistances and capacitances of the Foster network are summarized in Table 19. Using TL084C Chip as active elements and using the fractional-order capacitor on NISC PCB, Fig. 25b shows the setup for the filter experiment on the NI ELVIS II kit. Figure 26 shows the phase realization errors for the fractional-order capacitors in the four cases where ideal, approximation and realizable cases are compared. The realizable case is plotted using the values Table 19 to account for the round-off error in computing Foster I, which gives an

TABLE 18. Passive elements values for the circuit scaling design case.

Element	Value
$R_1 = R_2$	$1\ K\Omega$
C_α	$70\ nF$
C_β	$2.24\ \mu F/s^{1-\beta}$

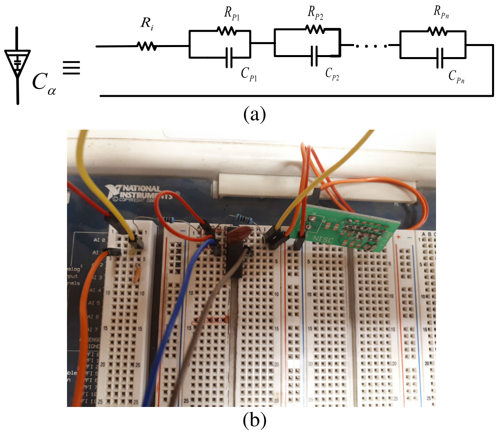


FIGURE 25. Fractional-order filter experiment: (a) Foster I schematics, (b) Setup.

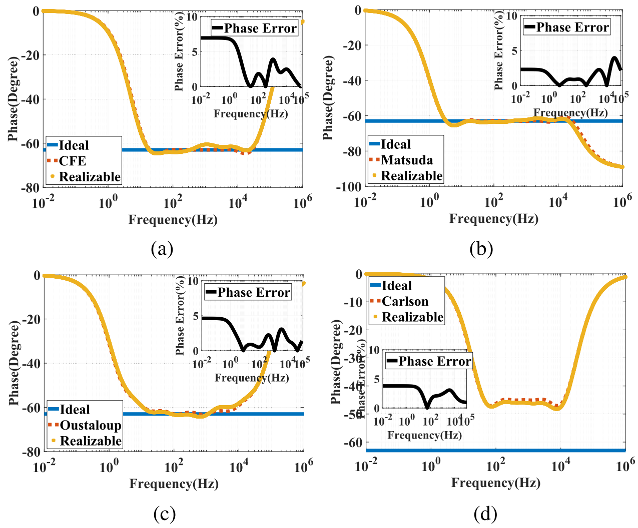


FIGURE 26. Phase response and phase error for the fractional-order capacitor realization by: (a) CFE, (b) Matsuda, (c) Oustaloup, (d) Carlson.

intuition for the best realizable CPE. Data was taken from the NI ELVIS bode analyzer and plotted on MATLAB in Figs. 27-28.

IX. CASE STUDY FOR THE DESIGN OF A LOW-PASS FRACTIONAL-ORDER FILTER BY DIRECT CIRCUIT IMPLEMENTATION

In the direct implementation of a fractional-order order filter, three critical frequencies are to be evaluated and mapped to the passive elements to achieve the required performance. The critical frequencies are the maxima frequency, as shown in Eqn. 29, the half-power frequency shown in 30, and the right-phase frequency shown in Eqn. 31 [74].

$$\frac{d}{d\omega}|H(j\omega)|_{\omega_m} = 0. \tag{29}$$

$$|H(j\omega)|_{w_m} = \frac{1}{\sqrt{2}}|H(j\omega_{passband}|. \tag{30}$$

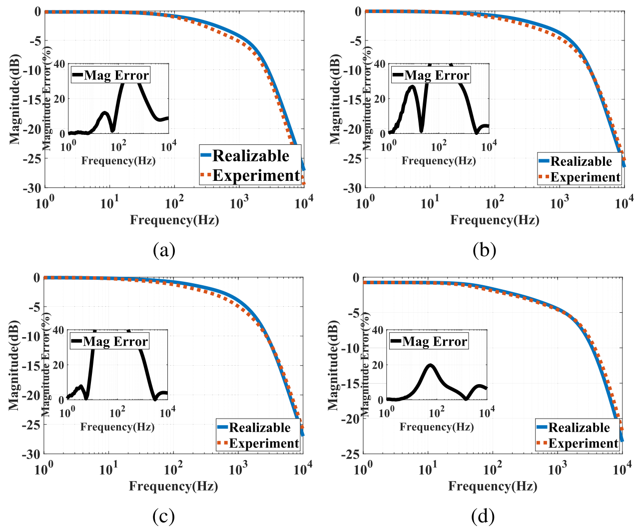


FIGURE 27. Experimental magnitude response for the filter with the fractional-order capacitor realized by: (a) CFE, (b) Matsuda, (c) Oustaloup, (d) Carlson.

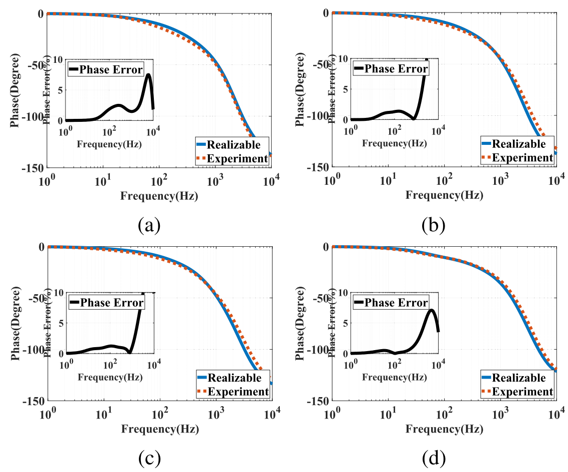


FIGURE 28. Experimental phase response for the filter with the fractional-order capacitor realized by: (a) CFE, (b) Matsuda, (c) Oustaloup, (d) Carlson.

$$\angle H(j\omega)_{\omega_{rp}} = \pm\frac{\pi}{2}. \tag{31}$$

The first design step is to factorize the filter's TF. For example, Bach's FOLPF TF in 27 gives the factorized transfer function in Eqn. 32.

$$H(s) = \frac{b}{s^{\alpha+\beta} + as^{\beta} + b}, \tag{32}$$

where a and b are the factors to be computed from solving the critical frequencies equations. The second step is to apply the three frequency equations to Eqn. 32, which gives the three implicit nonlinear equations Eqn. 33.

$$AA' + BB' = 0, \tag{33a}$$

$$A^2 + B^2 = b^2, \tag{33b}$$

$$A = 0, \tag{33c}$$

where A, A', B, and B' are summarized in Table 20.

TABLE 19. Foster I Rs and Cs values for realization of the fractional-order capacitor.

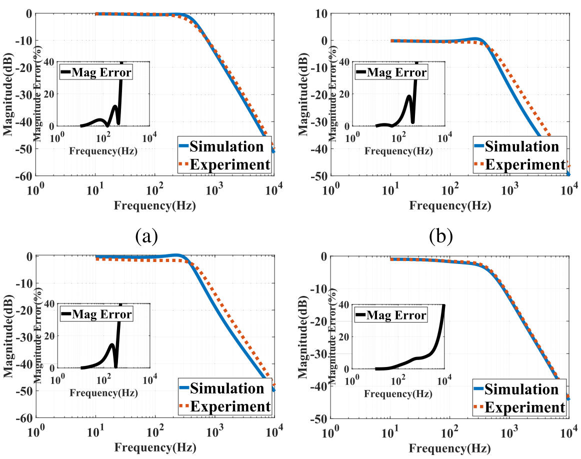
Index	CFE		Oustaloup		Matsuda		Carlson	
	$R(K\Omega)$	$C(nF)$	$R(K\Omega)$	$C(nF)$	$R(K\Omega)$	$C(nF)$	$R(K\Omega)$	$C(nF)$
i	0.027	-	39	-	-	-	130	-
1	0.01	150	0.051	150	0.082	68	0.3	100
2	0.160	470	0.22	220	0.22	220	0.47	330
3	0.270	560	0.91	470	0.56	470	1.1	599
4	0.620	1000	3.3	780	1.8	740	9.1	740
5	2.2	1000	15	1220	7.5	1000	-	-
6	51	560	100	1150	180	800	-	-

TABLE 20. Factors of the implicit critical equations.

A	$\omega^{\alpha+\beta}cos(\frac{\alpha+\beta}{2}\pi)+a\omega^{\beta}cos(\frac{\beta\pi}{2})+b$
B	$\omega^{\alpha+\beta}sin(\frac{\alpha+\beta}{2}\pi)+a\omega^{\beta}sin(\frac{\beta\pi}{2})$
A'	$(\alpha+\beta)\omega^{\alpha+\beta-1}cos(\frac{\alpha+\beta}{2}\pi)+a\beta\omega^{\beta-1}cos(\frac{\beta\pi}{2})$
B'	$(\alpha+\beta)\omega^{\alpha+\beta-1}sin(\frac{\alpha+\beta}{2}\pi)+a\beta\omega^{\beta-1}sin(\frac{\beta\pi}{2})$

TABLE 21. Passive elements values for the direct circuit design case.

Element	Value
R_1	$6\ K\Omega$
R_2	$1.31\ K\Omega$
C_α	$100\ nF$
C_β	$2.24\ \mu F/s^{1-\beta}$

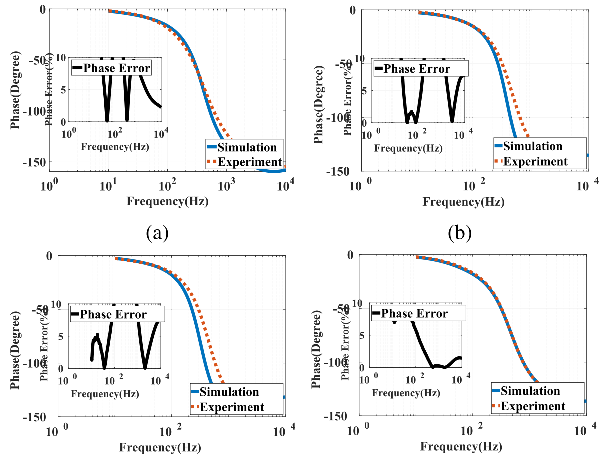


(a) (b) (c) (d)

FIGURE 29. Simulation and experimental magnitude response for the filter with the fractional-order capacitor realized by: (a) CFE, (b) Matsuda, (c) Oustaloup, (d) Carlson.

Considering the same realized CPE in Table 19, the set of the critical equations in 33 could be solved for a half power frequency of $f_h = 450Hz$ with $\alpha = 1$, $\beta = 0.7$, and $C_\beta = 2.24\mu F/s^{1-\beta}$. The calculated values for the rest of the passive elements are summarized in Table 21.

Figures 29-30 show the magnitude and phase responses of the simulation results performed on OrCAD and experimental results using ELVIS II kit and the realized CPEs in Table 21.



(a) (b) (c) (d)

FIGURE 30. Simulation and experimental phase response for the filter with the fractional-order capacitor realized by: (a) CFE, (b) Matsuda, (c) Oustaloup, (d) Carlson.

X. CONCLUSION

This work summarized the main possible design flows of the fractional-order filters. These flows can be generalized to the design of any fractional-order analog system if used with the correct corresponding target system's design equations. The work presents two design methods starting, for both methods, with creating the targeted fractional-order transfer function of the desired system.

The first design method is the circuit implementation which consists of selecting the filter topology. The next step is comparing the topology's transfer function with the desired system to acquire values for the filter's resistors and capacitors. This is followed by realizing the fractional-order capacitors (CPEs) using an appropriate approximation technique and realization network. The final step is to compose the circuit and run a simulation and experiment for verification.

The second method is the design of FPAA. This method consists of choosing an appropriate approximation technique for the Laplacian operators in the targeted transfer function. The next step is substituting the Laplacian operator in the transfer function with the approximated Laplacian operator, which results in an integer-order transfer function. The final step is transforming the approximated integer-order transfer function into a state-space representation such as PFE.

The performance of the fractional-order filter is highly dependent on the approximation process of the CPE. An appropriate reduction technique could reduce the approximation order of the Laplacian operator. The fractional-order filters' design flow using FPAA differs from the circuit's. Both flows were presented with a design case. Giving the entire transfer function of the fractional-order filters active topologies is essential to highlight the fractional S domain contributor. So, a short survey was presented on some generalized analog filter topologies. This work could be considered a catalog for fractional-order filters and analog systems designers. Future work will consider mapping more design specs, like quality and shaping factors, to the critical frequencies.

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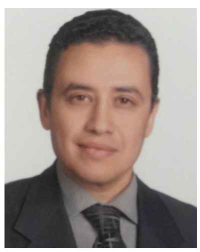

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