IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 31 January 2023, accepted 12 March 2023, date of publication 17 March 2023, date of current version 29 March 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3258547

RESEARCH ARTICLE

Predefined-Time Consensus of Nonlinear Multi-Agent Input Delay/Dynamic Event-Triggered Under Switching Topology

XIAOLU MA[1], YIBO TAN[1], AND HONG MEI[2]

[1]School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China
[2]College of Aeronautics, Nanjing University of Aeronautics and Astronautics, Nanjing 210000, China

Corresponding author: Xiaolu Ma (maxiaolu@ahut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61472282, Grant 62172004, and Grant 62072002; in part by the Major Science and Technology Project of Anhui Province under Grant 202003a05020028; in part by the Anhui University Natural Science Research Key Project under Grant KJ2019A0065; in part by the Key Research and Development Project of Anhui Province under Grant 202004a0502001; and in part by the Open Research Project of Anhui Province Key Laboratory of Special Heavy Duty Robot under Grant TZJQR004-2020.

ABSTRACT The consensus control problem of multi-agent systems (MASs) is studied. Firstly, a predefined-time consensus control algorithm is proposed for nonlinear uncertain systems with the input delay and switching topology. Then, the problem of resource consumption is considered, a dynamic event-triggered predefined-time consensus control algorithm is presented by introducing internal dynamic variables, which can make the MASs achieve consensus in the preset time. The correctness of the algorithm is proved by algebraic graph theory and Lyapunov theory, and there is no Zeno behavior. Simulation comparison experiments verify the effectiveness and superiority of the proposed algorithm. Compared with the finite-time control algorithm, the convergence time of this algorithm is independent of the initial state. The upper bound of the system convergence time can be set by selecting a time parameter. Compared with the fixed-time control algorithm, the convergence time of this algorithm is independent of the controller parameters, only related to a single parameter, the setting is simple, and the estimated convergence time is less conservative. Compared with the static triggering mechanism, the dynamic triggering mechanism can avoid a large number of triggering.

INDEX TERMS Multi-agent systems (MASs), consensus control, the nonlinear uncertain system, input delay, switching topologies, dynamic event-triggered control, predefined-time consensus control.

I. INTRODUCTION

In recent years, the cooperative control problem of multi-agent systems (MASs) have received extensive attention [1], [2], [3], [4]. An important problem of MASs cooperative control is to design controllers so that all agents can reach consensus.

The asymptotic convergence result of cooperative control is given first, and the convergence time is infinite [5], [6], [7], [8]. However, for practical systems, the design of the control protocol should also consider the speed of convergence. Compared with asymptotic consensus control, finite-time consensus control has the characteristics of faster convergence speed, stronger robustness and better anti-interference ability [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20]. In [9], the H∞ consensus problem of first-order nonlinear MASs based on directed graph was discussed. In [10], [11], the consensus problem of high-order MASs under the influence of mismatch interference and uncertain nonlinearity was explored respectively.

However, the convergence time of finite-time consensus depends on the initial state of the system. If the initial state difference is large or unknown, the system control requirements cannot be met. Therefore, in order to solve the problem

The associate editor coordinating the review of this manuscript and approving it for publication was Shuai Liu.

that the convergence time of the system is affected by the initial state, researchers have carried out fixed-time control research [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34]. The first-order multi-agent system under fixed topology was considered in [21]. Reference [22] solved the consensus tracking problem of first-order nonlinear MASs. The average consensus problem under switching topology was developed in [23].

However, there are two main problems with fixed-time control results. Firstly, the convergence time bound determined by Lyapunov stability analysis is very conservative. Secondly, the convergence time bound of fixed time consistency is a complex function of system parameters, and the direct relationship between the adjustment parameters and the convergence time bound is not clear. Recently, researchers have proposed a predefined-time control [35], [36], [37], [38], [39], [40]. Unlike fixed-time control, the convergence time is only related to a single parameter, and the estimated convergence time is close to the actual convergence time. [35] solved the distributed optimization problem of MASs with equality constraints in predefined time under directed graphs. [36] proposed an adaptive fuzzy predefined time and precision tracking control scheme for strict-feedback nonlinear systems. In [37], the problem of predefined-time bipartite consensus control for uncertain nonlinear MASs under signed directed topology was developed. In [38], the problem of predefined-time consensus tracking for second-order MASs was addressed.

Due to the nonideality of data transmission, time delay often occurs in controlled system [5], [12], [13], [14], [24], [25], [26], [27], [32]. The existence of time delay will reduce the performance of the system. Therefore, it is very important to consider the delayed MASs. In [5], an event-triggered adaptive control scheme was proposed for a class of nonlinear uncertain systems with input delay and communication constraints. In order to ensure faster convergence speed, the finite-time consensus of MASs with time delay was achieved in [12], [13], and [14]. In [12], the finite-time observer-based leader-following consensus problem for a class of nonlinear MASs with non-uniform time-varying input delay was addressed. Reference [13] solved the consensus tracking problem for a class of nonlinear heterogeneous MASs with asymmetric state constraints and input delays. Reference [14] considered the event-based finite-time consensus problem for second-order MASs with input delay. Reference [24], [25], and [26] gave the corresponding fixed-time consistency results. Reference [24] studied the consensus problem of non-strict nonlinear uncertain MASs with state constraints and input delays. Reference [25] considered the proportional consensus problem of MASs with input delay under undirected graphs and directed graphs. Reference [26] solved the fixed-time leader-following consensus problem for second-order MASs with input delays.

In addition, due to the limited resources of embedded processors, the event-triggered approach was proposed to solve the consensus problem. The main results are as follows [6], [7], [8], [15], [16], [17], [18], [19], [20], [27], [28], [29], [30], [31], [32], [33], [34], [39], [40]. Reference [6] constructed an event-triggered adaptive distributed observer and proposed an event-triggered dynamic output feedback control law. In [7] and [8], the event-triggered tracking control problem for nonlinear second-order MASs with and without disturbances was considered, respectively. The problem of convergence speed was considered, [15], [16], [17], [18], [19], [20] proposed a finite-time event-triggered consensus algorithm. [15] addressed MASs with general linear dynamics and directed topology. Reference [16] studied the Zeno behavior in event-triggered MASs. Reference [17] solved the finite-time distributed event-triggered consensus control problem for MASs. Reference [18] discussed the finite-time consensus problem of second-order leader-following nonlinear MASs with event-triggered communication. Reference [19] achieved the finite-time leader-follower consensus for second-order MASs with uncertain disturbances. Reference [20] developed the fuzzy adaptive finite-time consensus control problem for high-order nonlinear MASs with unknown nonlinear dynamics. In [27], [28], [29], [30], [31], [32], [33], and [34], the problem of fixed-time event-triggered consensus control was studied. Reference [27] considered the event-triggered attitude consensus of MASs with fixed-time convergence guarantee. Reference [28] solved the event-triggered attitude consensus of MASs with fixed-time convergence guarantee. Reference [29] considered the fixed-time event-triggered consensus problem of uncertain nonlinear MASs. Reference [30] discussed the fixed-time event / self-triggered leader-follower consensus problem for MASs with nonlinear dynamics. In [31], a new dynamic event-triggered control scheme was proposed for the fixed-time consensus problem of MASs with nonlinear dynamics. In [32], the fixed-time average consensus problem for nonlinear MASs with input delay, external disturbances and switching topology was solved. Reference [33] studied the problem of team-triggered fixed-time consensus for a class of double-integrator agents with uncertain disturbances. Reference [34] considered the fixed-time event-triggered output consensus tracking problem of high-order MASs under directed interaction graphs. In [39] and [40], the consensus control problem triggered by predefined time events was studied. Reference [39] addressed the problem of resource allocation in cyber-physical systems. By applying a dynamic event-triggered mechanism and a continuous-time function, a new distributed scheduled-time algorithm and dynamic triggering conditions were designed. Reference [40] studied the design of event-triggered prescribed-time output feedback control for nonlinear interconnected systems with non-strict feedback control structure.

Motivated by the above literature, we consider two aspects. On the one hand, the input delay of MASs with nonlinear uncertainty is solved. On the other hand, the problem of resource loss is considered. The main contributions of this article are stated as follows.

1) Different from the fixed-time consensus results [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], the prescribed convergence time of the system is independent of the controller parameters, and the setting is simple and less conservative.

2) Compared with the static event-triggered consensus results [6], [7], [8], [15], [16], [17], [18], [19], [20], [27], [28], [29], [30], [32], [33], [34], [40], we introduce a dynamic event-triggered factor to reduce the number of controller triggers and resource consumption.

3) Different from the predefined-time consensus results [35], [36], [37], [38], we consider the case of input delay, event-triggered and switching topology.

This paper is structured as follows. Section II introduces preliminaries and problem formulation. Section III considers the predefined-time consensus of nonlinear uncertainty MASs with input delay. Section IV considers dynamic event-triggered predefined-time consensus. Section V is the simulation results and analysis. Section VI concludes this article.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. GRAPH THEORY

Consider a MASs with M agents whose communication topology is represented by an undirected graph $G = (V, \varepsilon, A)$, where $V = \{v_1, v_2, \cdots, v_m\}$ is the set of nodes, $\varepsilon \subseteq V \times V$ represents the set of edges, $A = [a_{ij}]_{m\times m}$ is an adjacency matrix. Each node v_i represents an agent i, and the existence of an edge set $(v_i, v_j) \in \varepsilon$ indicates that agents j and i can communicate with each other. For an adjacency matrix $A = [a_{ij}]_{m\times m}$ of an undirected graph G, when $(v_j, v_i) \in \varepsilon$, $a_{ij} = 1$, otherwise $a_{ij} = 0$. The Laplacian matrix L of an undirected graph G is defined as $L = D - A$, where $D = \text{diag}\{d_1, d_2, \cdots, d_m\}$ is a diagonal matrix, $d_j = \sum_{i=1, i\neq j}^{m} a_{ji}, j \in \{1, 2, \cdots, m\}$. An undirected graph G is said to be connected if there is at least one path between any two nodes.

B. DEFINITION AND LEMMAS

Consider the following system

$$\dot{x}(t) = f(x(t), t)x(0) = x_0 \tag{1}$$

where $x = [x_1, x_2, \cdots, x_m]^{\mathrm{T}} \in \mathbb{R}^m, f(x(t), t) : \mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}^m$ is a nonlinear function. Let the origin be an equilibrium point of system (1).

Definition 1 [41]: If the origin of system (1) is globally uniformly finite-time stable and the convergence time parameter T: $\mathbb{R}^m \to \mathbb{R}^+$ is globally bounded, then the origin is called the equilibrium point of global fixed-time convergence, i.e. there exists a finite constant $T_{\max} \in \mathbb{R}^+$ such that for all $t \geq T$ and $x_0 \in \mathbb{R}^m$ satisfying $T_s < T_{\max}, x(t) = 0$, then the origin of system (1) is pre-determined time convergent.

Lemma 1 [42]: If there exists a positive definite function $V(x) : \mathbb{R}^m \to \mathbb{R}, \forall x \in \mathbb{R}^m$, and $V(x) = 0 \Leftrightarrow x = 0$ such that the following holds

$$\dot{V}(x) \leq -\frac{\pi}{\alpha T}(V(x)^{1-\frac{\alpha}{2}} + V(x)^{1+\frac{\alpha}{2}}) \tag{2}$$

where constant $T > 0$, $0 < \alpha < 1$, then the origin of the system is stable for the predefined time T.

Lemma 2 [43]: A connected undirected graph G whose Laplacian matrix L is positive semidefinite with eigenvalues satisfying

$$0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_m(L)$$
$$\lambda_2(L) = \min_{\|x\|\neq 0, \sum_{i=1}^{m} x_i = 0} \frac{x^{\mathrm{T}}Lx}{\|x\|^2} \tag{3}$$

For $x = [x_1, x_2, \cdots, x_m]^{\mathrm{T}} \in \mathbb{R}^n$, there is

$$x^{\mathrm{T}}Lx = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}(x_i - x_j)^2 \tag{4}$$

Therefore, if $\mathbf{1}^{\mathrm{T}}x = 0$, that is $\sum_{i=1}^{m} x_i = 0$, then $\lambda_2(L)x^{\mathrm{T}}x \leq x^{\mathrm{T}}Lx \leq \lambda_m(L)x^{\mathrm{T}}x$.

Lemma 3 [44]: If $|y|$ denotes the absolute value of the real number y, then

$$\frac{d}{dy}|y|^{\alpha+1} = (\alpha + 1)sig(y)^{\alpha}$$
$$\frac{d}{dy}sig(y)^{\alpha+1} = (\alpha + 1)|y|^{\alpha}$$

where $sig(y)^{\alpha} = sign(y)|y|^{\alpha}$.

Lemma 4 [45]: For a real number $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m \in \mathbb{R}^+$, one has

$$m^{1-p}\left(\sum_{i=1}^{m}\varepsilon_i\right)^p \geq \sum_{i=1}^{m}\varepsilon_i^p \geq \left(\sum_{i=1}^{m}\varepsilon_i\right)^p, \quad 0 < 0 \leq 1$$
$$\sum_{i=1}^{m}\varepsilon_i^q \geq m^{1-q}\left(\sum_{i=1}^{m}\varepsilon_i\right)^q, \quad q > 1$$

C. PROBLEM DESCRIPTION

Consider nonlinear MASs with input delay disturbance

$$\begin{cases}\dot{x}_i(t) = u_i(t-\tau) + f(x_i(t), t) + d_i(x_i(t), t) \\ x(0) = x_0\end{cases} \tag{5}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$, $i = 1, 2, \cdots, m$ are state and the control input of agent i, τ is the known input delay, $f(x(t), t)$: $\mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}^m$ is nonlinear uncertainty, $d_i(x_i(t), t) \in \mathbb{R}^n$ can represent unknown disturbance and noise, etc..

Definition 2: For the input delay nonlinear MASs (5) with disturbance, there exists a predefined time constant T such that when $t \geq T$, the system has

$$\begin{cases}\lim_{t\to T}\left|x_i(t) - x_j(t)\right| = 0 \\ x_i(t) = x_j(t)\end{cases} \tag{6}$$

Then the system can achieve predefined time consistency.

Assumption 1: The nonlinear function $f(x(t),t)$ satisfies the following conditions

$$\left|f(x_i(t),t)-f(x_j(t),t)\right| \le \gamma_1+\gamma_2\left|x_i(t)-x_j(t)\right| \tag{7}$$

where $\gamma_1 \ge 0$, $\gamma_2 \ge 0$.

Assumption 2: There exists a known nonnegative constant D satisfying

$$|d_i(x_i(t),t)| \le D \tag{8}$$

III. PREDEFINED-TIME CONSENSUS UNDER INPUT DELAY

In this part, a distributed input delay predefined-time consensus control algorithm is proposed with $\gamma_2 = 0$. The MASs consensus under fixed topology and switching topology are considered.

Define

$$\xi_i(t)=\sum_{j=1}^{m} a_{ij}(\chi_i(t)-\chi_j(t)) \tag{9}$$

where $\chi_i(t)=x_i(t)+\int_{t-\tau}^{t} u_i(T)dT$, $i \in \{1,2,\cdots,m\}$.

A. PREDEFINED-TIME CONSENSUS UNDER FIXED TOPOLOGY

In order to achieve predefined-time consensus of MASs under input delay, a predefined-time control law is designed as

$$u_i(t)=-c_1 sig\left(\xi_i(t)\right)^{1+\alpha}-c_2 sig\left(\xi_i(t)\right)^{1-\alpha}-\beta sign\left(\xi_i(t)\right) \tag{10}$$

where

$$\begin{cases} c_1=\dfrac{\pi}{2\alpha T m^{-\frac{\alpha}{2}}(2\lambda_2(L))^{1+\frac{\alpha}{2}}} \\ c_2=\dfrac{\pi}{\alpha T(2\lambda_2(L))^{1-\frac{\alpha}{2}}} \end{cases}$$

$0<\alpha<1, \beta>0$ are the control gain constant, $\lambda_2(L)$ is the nonzero minimum eigenvalue of the Laplacian matrix L of MASs, and T is the preset convergence time.

Based on Newton-Leibniz formula, we have

$$\dot{\chi}_i(t)=u_i(t)+f(x_i(t),t)+d_i(x_i(t),t) \tag{11}$$

Theorem 1: When Assumptions 1, Assumptions 2 and the following conditions are satisfied.

$$\beta \ge \frac{1}{2}\gamma_1+D \tag{12}$$

Using controller (10) to satisfy input delay MASs achieves predefined-time consensus in fixed topology.

Proof: Construct the following Lyapunov function

$$V(t)=\frac{1}{2}\chi^{\mathrm{T}}(t)L\chi(t) \tag{13}$$

where $\chi(t)=[\chi_1(t),\chi_2(t),\ldots,\chi_m(t)]^{\mathrm{T}}$, the derivation of V is

$$\begin{aligned}\dot{V}(t)&=\frac{1}{2}\left(\chi^T(t)L\dot{\chi}(t)+\dot{\chi}^T(t)L\chi(t)\right)\\
&=\chi^T(t)L\dot{\chi}(t)\\
&=\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(\chi_i(t)-\chi_j(t)\dot{\chi}_i(t)\\
&=\sum_{i=1}^{m}\xi_i(t)\left(u_i(t)+f(x_i(t),t)+d_i(x_i(t),t)\right)\\
&=-c_1\sum_{i=1}^{m}|\xi_i|^{2+\alpha}(t)-c_2\sum_{i=1}^{m}|\xi_i(t)|^{2-\alpha}-\beta\sum_{i=1}^{m}|\xi_i(t)|\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(\chi_i(t)-\chi_j(t))f(x_i(t),t)\\
&\quad+\sum_{i=1}^{m}\xi_i(t)d_i(x_i(t),t)\end{aligned} \tag{14}$$

By Assumption 1, we have

$$\begin{aligned}&\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(\chi_i(t)-\chi_j(t))f(x_i(t),t)\\
&\le\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(\chi_i(t)-\chi_j(t))(f(x_i(t),t)-f(x_j(t),t))\\
&\le\frac{1}{2}\gamma_1\sum_{i=1}^{m}\xi_i(t)\end{aligned} \tag{15}$$

Substitute (15) into (14), we can get

$$\begin{aligned}\dot{V}(t)=&-c_1\sum_{i=1}^{m}\left(|\xi_i(t)|^2\right)^{1+\frac{\alpha}{2}}-c_2\sum_{i=1}^{m}\left(|\xi_i(t)|^2\right)^{1-\frac{\alpha}{2}}\\
&-\beta\sum_{i=1}^{m}|\xi_i(t)|+\frac{1}{2}\gamma_1\sum_{i=1}^{m}\xi_i(t)+\sum_{i=1}^{m}\xi_i(t)d_i(x_i(t),t)\end{aligned} \tag{16}$$

By Lemma 4, we have

$$\sum_{i=1}^{m}\left(|\xi_i(t)|^2\right)^{1+\frac{\alpha}{2}} \ge m^{-\frac{\alpha}{2}}\left(\sum_{i=1}^{m}\xi_i^2(t)\right)^{1+\frac{\alpha}{2}} \tag{17}$$

$$\sum_{i=1}^{m}\left(|\xi_i(t)|^2\right)^{1-\frac{\alpha}{2}} \ge \left(\sum_{i=1}^{m}\xi_i^2(t)\right)^{1-\frac{\alpha}{2}} \tag{18}$$

Applying Lemma 2, one has

$$2\lambda_2(L)V(t) \le \sum_{i=1}^{m}\left(\xi_i^2(t)\right) \le 2\lambda_m(L)V(t) \tag{19}$$

Then can be written as

$$\begin{aligned}\dot{V}(t)\le&-c_1m^{-\frac{\alpha}{2}}\left(\sum_{i=1}^{m}\xi_i^2(t)\right)^{1+\frac{\alpha}{2}}-c_2\left(\sum_{i=1}^{m}\xi_i^2(t)\right)^{1-\frac{\alpha}{2}}\\
&-\beta\sum_{i=1}^{m}|\xi_i(t)|+\left(\frac{1}{2}\gamma_1+\mathrm{D}\right)\sum_{i=1}^{m}\xi_i(t)\\
\le&-c_1m^{-\frac{\alpha}{2}}\left(2\lambda_2(L)V(t)\right)^{1+\frac{\alpha}{2}}-c_2\left(2\lambda_2(L)V(t)\right)^{1-\frac{\alpha}{2}}\end{aligned}$$

$$-\left(\beta-\frac{1}{2}\gamma_1-\mathrm{D}\right)\sum_{i=1}^{m}|\xi_i(t)|$$
$$\leq-\frac{\pi}{\alpha T}(V(t))^{1+\frac{\alpha}{2}}-\frac{\pi}{\alpha T}(V(t))^{1-\frac{\alpha}{2}} \quad (20)$$

It follows from Lemma 1 that $\lim_{t\to T}V(t)=0$. This means that when $t=T$, $u_i(t)$ is zero, and when $t\leq T+\tau_i$, $\int_{t-\tau}^{t}u_i(T)dT$ is zero. Therefore, when $t=T_s\leq T+\tau_i$, $\lim_{t\to T_s}\chi(t)=x(t)$, MASs achieves consistency within a predefined-time Ts.

B. PREDEFINED-TIME CONSENSUS UNDER SWITCHING TOPOLOGY

This section extends the fixed topology dynamic event-triggered protocol to the switching topology for analysis. Let the undirected graph set $G_s=\{G_1,G_2,\cdots,G_N\}$ represent the set of MASs communication topology graphs. The switching signal $s(t):[0,+\infty)\to\Omega$, $\Omega=\{1,2,\cdots,N\}$ is the graph G_s index set. The communication topology is defined as $G_{s(t_0)}$ when the sampling time is t_0, and the corresponding Laplacian matrix is $L(G_{s(t_0)})$.

In order to achieve predefined-time consensus of nonlinear MASs Eq. (5) under switching topology, a predefined-time control law is designed as

$$u_i(t)=-\tilde{c}_1 sig(\xi_i(t))^{1+\alpha}-\tilde{c}_2 sig(\xi_i(t))^{1-\alpha}-\beta sign(\xi_i(t)) \quad (21)$$

where

$$\begin{cases}\tilde{c}_1=\dfrac{\pi}{2\alpha Tm^{-\frac{\alpha}{2}}(2\lambda_2^{\min}(L))^{1+\frac{\alpha}{2}}}\\ \tilde{c}_2=\dfrac{\pi}{\alpha T(2\lambda_2^{\min}(L))^{1-\frac{\alpha}{2}}}\end{cases}$$
$$\lambda_2^{\min}(L)=\min\{\lambda_2(L(t_0)),\lambda_2(L(t_1)),\ldots\}.$$

Theorem 2: When Assumptions 1, Assumptions 2 and the following conditions are satisfied.

$$\beta\geq\frac{1}{2}\gamma_1+D \quad (22)$$

The controller (21) enables MASs to achieve predefined-time consensus under switching topologies.

Proof: Similar to the proof of Lemma 1, the derivative of V(t) is

$$\begin{aligned}&\dot{V}(t)\\ &\leq-c_1m^{-\frac{\alpha}{2}}(2\lambda_2(L)V(t))^{1+\frac{\alpha}{2}}-c_2(2\lambda_2(L)V(t))^{1-\frac{\alpha}{2}}\\ &\quad-\left(\beta-\frac{1}{2}\gamma_1-\mathrm{D}\right)\sum_{i=1}^{m}|y_i(t)|\\ &\leq-\tilde{c}_1m^{-\frac{\alpha}{2}}\left(2\lambda_2^{\min}(L)V(t)\right)^{1+\frac{\alpha}{2}}-\tilde{c}_2\left(2\lambda_2^{\min}(L)V(t)\right)^{1-\frac{\alpha}{2}}\\ &\quad-\left(\beta-\frac{1}{2}\gamma_1-\mathrm{D}\right)\sum_{i=1}^{m}|y_i(t)|\\ &\leq-\frac{\pi}{\alpha T}\left(V^{\frac{2-\alpha}{2}}(t)+V^{\frac{2+\alpha}{2}}(t)\right)\end{aligned} \quad (23)$$

Based on Lemma 1, under the switching signal $s(t):[0,+\infty)\to\Omega$, V(t) is stable at a predefined-time T, and MASs achieves consistency within a predefined-time Ts.

The proof is completed.

Remark 1: Different from the existing finite-time controls with input delay [12], [13], [14], the convergence time of the proposed predefined-time controls (10), (21) are independent of the initial state.

Remark 2: Compared with the existing fixed-time controls with input delay [24], [25], [26], [27], [32], the convergence time of the proposed predefined-time controls can be given as an exact controller parameter in advance.

Remark 3: Considering the input delay multi-agent predefined-time consensus control under switching topology, which provides a good reference value for the communication transformation of the system in practical applications.

IV. PREDEFINED-TIME CONSENSUS UNDER DYNAMIC EVENT TRIGGERING

In this section, a dynamic event-triggered predefined-time consensus control algorithm is designed without input delay ($\tau=0$).

A. PREDEFINED-TIME CONSENSUS UNDER FIXED TOPOLOGY

Define $y_i(t)=\sum_{j=1}^{m}a_{ij}(x_i(t)-x_j(t))$. Design the predefined-time control law as

$$u_i(t)=-k_1y_i^{1+\alpha}(t_k^i)-k_2y_i^{1-\alpha}(t_k^i)-k_3y_i(t_k^i)-\beta sign\left(y_i(t_k^i)\right) \quad (24)$$

where

$$\begin{cases}k_1=\dfrac{\pi}{2\alpha T(1-\varepsilon)m^{-\frac{\alpha}{2}}(\lambda_2(L))^{1+\frac{\alpha}{2}}}\\ k_2=\dfrac{\pi}{\alpha T(1-\varepsilon)(2\lambda_2(L))^{1-\frac{\alpha}{2}}}\end{cases}.$$

$k_3>0$, $\varepsilon\in(0,1)$, t_k^i is the latest trigger time, $k=0,1,2,\cdots$, $\omega>0$, $\beta>0$, $\kappa\geq0$ is a very small positive constant that can be set as needed.

Substituting Eq. (24) into Eq. (5) yields, we get

$$\begin{aligned}\dot{x}_i(t)&=-k_1y_i^{1+\alpha}(t_k^i)-k_2y_i^{1-\alpha}(t_k^i)-k_3y_i(t_k^i)\\ &\quad-\beta sign\left(y_i(t_k^i)\right)+f(x_i(t),t)+d_i(x_i(t),t)\end{aligned} \quad (25)$$

The measurement error of agent i is defined as

$$\begin{aligned}&e_i(t)\\ &=k_1y_i^{1+\alpha}(t_k^i)+k_2y_i^{1-\alpha}(t_k^i)+k_3\left(y_i(t_k^i)\right)+\beta sign\left(y_i(t_k^i)\right)\\ &\quad-k_1y_i^{1+\alpha}(t)-k_2y_i^{1-\alpha}(t)-k_3(y_i(t))-\beta sign(y_i(t))\end{aligned} \quad (26)$$

The trigger function for constructing agent i is

$$\psi_i(t) = \theta\left(|e_i(t)| - \varepsilon k_1 \left|y_i^{1+\alpha}(t)\right| - \varepsilon k_2 \left|y_i^{1-\alpha}(t)\right| - \varepsilon k_3 |y_i(t)| - \varepsilon\beta\right) \tag{27}$$

where $\theta > 0$, $\varepsilon \in (0, 1)$ is the preset trigger parameter.

A new internal dynamic variable η_i is designed, which satisfies

$$\begin{aligned}\dot{\eta}_i(t) = \delta |y_i(t)| \Big(&\varepsilon k_1 \left|y_i^{1+\alpha}(t)\right| + \varepsilon k_2 \left|y_i^{1-\alpha}(t)\right| \\ &+\varepsilon k_3 |y_i(t)| + \varepsilon\beta - |e_i(t)|\Big) - k_4\eta_i^{1+\frac{\alpha}{2}}(t) \\ &- k_5\eta_i^{1-\frac{\alpha}{2}}(t) - k_6\eta_i^2(t)\end{aligned} \tag{28}$$

where

$$\begin{cases} k_4 = \dfrac{\pi}{2^{-\frac{\alpha}{2}} m^{-\frac{\alpha}{2}} \alpha T} \\ k_5 = \dfrac{\pi}{\alpha T} \end{cases}$$

$k_6 > 0$, $\eta_i(0) > 0$, $\delta \in (0, 1)$. Define the trigger condition as

$$t_{k+1}^i = \inf\left\{t > t_k^i | \psi_i(t) - \eta_i(t) \geq 0\right\} \tag{29}$$

This trigger condition guarantees that when $t \in (t_k^i, t_{k+1}^i)$, we have $\psi_i(t) \leq \eta_i(t)$, which means

$$|e_i(t)| \leq \varepsilon c_1 \left|y_i^{1+\alpha}(t)\right| + \varepsilon c_2 \left|y_i^{1-\alpha}(t)\right| + \varepsilon c_3 |y_i(t)| + \varepsilon\beta + \frac{\eta_i(t)}{\theta} \tag{30}$$

In addition, from Equation (29)(30), when $t > 0$, one has

$$\dot{\eta}_i(t) \geq -k_4\eta_i^{1+\frac{\alpha}{2}}(t) - k_5\eta_i^{1-\frac{\alpha}{2}}(t) - k_6\eta_i^2(t) - \frac{\delta}{\theta} |y_i(t)| \eta_i(t) \tag{31}$$

Applying the comparison principle, we have

$$\eta_i(t) \geq \eta_i(0)e^{\int_0^t \phi_i(t)ds} > 0 \tag{32}$$

where $\phi_i(t) = -k_4\eta_i^{\frac{\alpha}{2}}(t) - k_5\eta_i^{-\frac{\alpha}{2}}(t) - k_6\eta_i(t) - \frac{\delta}{\theta} |\xi_i(t)|$.

Theorem 3: When Assumption1, Assumption2, and the following conditions are met

$$(1-\varepsilon)k_3\lambda_2(L) \geq 2\gamma_2 \tag{33}$$

$$(2(1-\varepsilon)k_3k_6)^{\frac{1}{2}} = \frac{1-\delta}{\theta} \tag{34}$$

$$(1-\varepsilon)\beta \geq D + \frac{1}{2}\gamma_1 \tag{35}$$

Using controller (24), dynamic trigger function (27) and trigger condition (29), the predefined-time consensus of nonlinear uncertain MASs under fixed topology can be achieved.

Proof: Construct Lyapunov function

$$W(t) = V_1(t) + V_2(t) \tag{36}$$

where $V_1(t) = \frac{1}{2}x(t)^{\mathrm{T}}Lx(t) = \frac{1}{2}\left[\frac{1}{2}\sum_{i=1}^m\sum_{j=1}^m a_{ij}(x_i(t) - x_j(t))^2\right]$, $V_2(t) = \sum_{j=1}^m \eta_i(t)$, combined with Lemma 4, the derivation of $W(t)$ is

$$\begin{aligned}\dot{W}(t) &= x(t)^T L\dot{x}(t) + \sum_{i=1}^m \dot{\eta}_i(t) \\ &= \sum_{i=1}^m y_i(t)\left(u_i(t) + f(x_i(t), t) + d_i(x_i(t), t)\right) \\ &\quad - k_2(1-\varepsilon)\left(2\lambda_2(L)V_1(t)\right)^{1-\frac{\alpha}{2}} + \sum_{i=1}^m \dot{\eta}_i(t) \\ &\quad - k_4 m^{-\frac{\alpha}{2}}\left(V_2(t)\right)^{1+\frac{\alpha}{2}} - k_5\left(V_2(t)\right)^{1-\frac{\alpha}{2}} \\ &\leq -\frac{2^{\frac{\alpha}{2}}}{\alpha T}\pi\left(V_1(t)\right)^{1+\frac{\alpha}{2}} - \frac{\pi}{\alpha T}\left(V_1(t)\right)^{1-\frac{\alpha}{2}} \\ &\quad - \frac{2^{\frac{\alpha}{2}}}{\alpha T}\pi\left(V_2(t)\right)^{1+\frac{\alpha}{2}} - \frac{\pi}{\alpha T}\left(V_2(t)\right)^{1-\frac{\alpha}{2}} \\ &\leq -\frac{2^{\frac{\alpha}{2}}}{\alpha T}\pi\left((V_1(t))^{1+\frac{\alpha}{2}} + (V_2(t))^{1+\frac{\alpha}{2}}\right) \\ &\quad - \frac{\pi}{\alpha T}\left((V_1(t))^{1-\frac{\alpha}{2}} + \frac{\pi}{\alpha T}(V_2(t))^{1-\frac{\alpha}{2}}\right) \\ &\leq -\frac{\pi}{\alpha T}\left((W(t))^{1+\frac{\alpha}{2}} + (W(t))^{1-\frac{\alpha}{2}}\right)\end{aligned} \tag{37}$$

According to the formula (36)(37), combined with Lemma 2, we can get

$$|y_i(t)| \leq \|y(t)\| \leq \sqrt{2\lambda_m(L)V_1(t)} \leq \sqrt{2\lambda_m(L)V_1(0)} \tag{38}$$

Theorem 4: When Assumptions 1, Assumptions 2 and the following conditions are satisfied

$$(1-\varepsilon)k_3\lambda_2(L) \geq 2\gamma_2 \tag{39}$$

$$(2(1-\varepsilon)k_3k_6)^{\frac{1}{2}} = \frac{1-\delta}{\theta} \tag{40}$$

$$(1-\varepsilon)\beta \geq D + \frac{1}{2}\gamma_1 \tag{41}$$

Nonlinear uncertain MASs without Zeno behavior under triggering condition (29).

Proof: According to the measurement error formula (26), combined with the Lemma 3, the Dini derivative can be obtained.

$$\begin{aligned}&D^+|e_i(t)| \\ &\leq |\dot{e}_i(t)| \\ &= \left|\left(-k_1y_i^{1+\alpha}(t) - k_2y_i^{1-\alpha}(t) - k_3y_i(t) - \beta sign\left(y_i(t)\right)\right)'\right| \\ &\leq \left(k_1(1+\alpha)\left|y_i^{\alpha}(t)\right| + k_2(1-\alpha)\left|y_i^{-\alpha}(t)\right| + k_3\right)|\dot{y}_i(t)| \\ &\leq (\phi_1 + k_3)\left|\sum_{j=1}^m a_{ij}(\dot{x}_i(t) - \dot{x}_j(t))\right| \\ &\leq (\phi_1 + k_3)\left(\left|\sum_{j=1}^m a_{ij}(u_i(t) - u_j(t))\right|\right.\end{aligned}$$

$$+\left|\sum_{j=1}^{m} a_{ij}(f(x_i(t),t)-f(x_j(t),t))\right|$$
$$+\left|\sum_{j=1}^{m} a_{ij}(d_i(x_i(t),t)-d_j(x_j(t),t))\right|\Bigg)$$
$$\leq(\phi_1+k_3)\left(\left|\sum_{j=1}^{m} l_{ij}u_j(t_{k'}^j)\right|+l_{ii}(\gamma_1+2D)\right.$$
$$\left.+\gamma_2\sum_{j=1}^{m} a_{ij}\left|x_i(t)-x_j(t)\right|\right)$$
$$\leq(\phi_1+k_3)\left(\left|\sum_{j=1}^{m} l_{ij}u_j(t_{k'}^j)\right|+l_{ii}(\gamma_1+2D)\right.$$
$$\left.+\gamma_2 m^{\frac{1}{2}}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}\left(x_i(t)-x_j(t)\right)^2}\right)$$
$$\leq(\phi_1+k_3)\left(\phi_2(t_{k'}^j)+\phi_3\right) \tag{42}$$

where

$$\phi_1=k_1(1+\alpha)\left(2\lambda_m(L)V_1(0)\right)^{\frac{\alpha}{2}}+k_2(1-\alpha)\left(2\lambda_m(L)V_1(0)\right)^{-\frac{\alpha}{2}} \tag{43}$$
$$\phi_2(t_k^j)=\left|\sum_{j=1}^{m} l_{ij}\left(k_1\left(y_j(t_{k'}^i)\right)^{1+\alpha}+k_2\left(y_j(t_{k'}^i)\right)^{1-\alpha}+k_3y_j(t_{k'}^i)+\beta sign\left(y_j(t_{k'}^i)\right)\right)\right|+l_{ii}(\gamma_1+2D) \tag{44}$$
$$\phi_3=\gamma_2\sqrt{4mV_1(0)} \tag{45}$$

Due to $e_i(t_k^j)=0$, one has

$$|e_i(t)|\leq\int_{t_k^i}^{t}(\phi_1+k_3)\left(\phi_2(t_{k'}^j)+\phi_3\right)ds+\left|e_i(t_k^i)\right|$$
$$=\int_{t_k^i}^{t}(\phi_1+k_3)\left(\phi_2(t_{k'}^j)+\phi_3\right)ds \tag{46}$$

By the trigger condition (29) and the measurement error $|e_i(t)|$ upper bound (46), we have

$$\left|e_i(t_{k+1}^i)\right|=\varepsilon k_1\left|y_i^{1+\alpha}(t_{k+1}^i)\right|+\varepsilon k_2\left|y_i^{1-\alpha}(t_{k+1}^i)\right|+\varepsilon k_3\left|y_i(t_{k+1}^i)\right|+\varepsilon\beta+\frac{\eta_i(t_{k+1}^i)}{\theta}$$
$$\leq\int_{t_k^i}^{t_{k+1}^i}(\phi_1+k_3)\left(\bar{\phi}_2+\phi_3\right)ds$$
$$\leq(\phi_1+k_3)\left(\bar{\phi}_2+\phi_3\right)(t_{k+1}^i-t_k^i) \tag{47}$$

where

$$\bar{\phi}_2=\sum_{j=1}^{m}\left|l_{ij}\right|\left(k_1\left(2\lambda_m(L)V_1(0)\right)^{\frac{1+\alpha}{2}}+k_2\left(2\lambda_m(L)V_1(0)\right)^{\frac{1-\alpha}{2}}+k_3\left(2\lambda_m(L)V_1(0)\right)^{\frac{1}{2}}+\beta\right)+l_{ii}(\gamma_1+2D)$$

Trigger interval $t_{k+1}^i-t_k^i\geq\frac{|e_i(t_{k+1}^i)|}{(\phi_1+k_3)(\bar{\phi}_2+\phi_3)}\geq\frac{\varepsilon\beta}{(\phi_1+k_3)(\bar{\phi}_2+\phi_3)}>0$, which means no Zeno behavior in the system.

B. PREDEFINED-TIME CONSENSUS UNDER SWITCHING TOPOLOGY

In order to achieve predefined-time consensus of nonlinear MASs Eq. (5) under switching topology, a predefined-time control law is designed as

$$u_i(t)=-\tilde{k}_1y_i^{1+\alpha}(t_k^i)-\tilde{k}_2y_i^{1-\alpha}(t_k^i)-\tilde{k}_3y_i(t_k^i)-\beta sign\left(y_i(t_k^i)\right) \tag{48}$$

where

$$\begin{cases}\tilde{k}_1=\dfrac{\pi}{2\alpha T(1-\varepsilon)m^{-\frac{\alpha}{2}}(\lambda_2^{\min}(L))^{1+\frac{\alpha}{2}}}\\ \tilde{k}_2=\dfrac{\pi}{\alpha T(1-\varepsilon)(2\lambda_2^{\min}(L))^{1-\frac{\alpha}{2}}}\end{cases}$$

Theorem 5: When Assumption 1, Assumption 2, and the following conditions are met,

$$(1-\varepsilon)\tilde{k}_3\lambda_2^{\min}(L)\geq 2\gamma_1 \tag{49}$$
$$\left(2(1-\varepsilon)\tilde{k}_3k_6\right)^{\frac{1}{2}}=\frac{1-\delta}{\theta} \tag{50}$$
$$(1-\varepsilon)\beta\geq D+\frac{1}{2}\gamma_2 \tag{51}$$

Using controller (48), dynamic trigger function (27) and trigger condition (29), the predefined-time consensus of non-linear uncertain MASs under switching topology can be achieved.

Proof: Similar to Lemma 4, one has

$$\dot{W}(t)\leq-\tilde{k}_1(1-\varepsilon)m^{-\frac{\alpha}{2}}\left(2\lambda_2^{\min}(L)V_1(t)\right)^{1+\frac{\alpha}{2}}$$
$$-\left(\tilde{k}_3(1-\varepsilon)\lambda_2^{\min}(L)-2\gamma_1\right)V_1(t)$$
$$-\tilde{k}_2(1-\varepsilon)\left(2\lambda_2^{\min}(L)V_1(t)\right)^{1-\frac{\alpha}{2}}$$
$$-k_4m^{-\frac{\alpha}{2}}\left(V_2(t)\right)^{1+\frac{\alpha}{2}}-k_5\left(V_2(t)\right)^{1-\frac{\alpha}{2}}$$
$$\leq-\frac{2^{\frac{\alpha}{2}}}{\alpha T}\pi\left((V_1(t))^{1+\frac{\alpha}{2}}+(V_2(t))^{1+\frac{\alpha}{2}}\right)$$
$$-\frac{\pi}{\alpha T}\left((V_1(t))^{1-\frac{\alpha}{2}}+\frac{\pi}{\alpha T}(V_2(t))^{1-\frac{\alpha}{2}}\right)$$
$$\leq-\frac{\pi}{\alpha T}\left((W(t))^{1+\frac{\alpha}{2}}+(W(t))^{1-\frac{\alpha}{2}}\right) \tag{52}$$

Therefore, according to Lemma 1, MASs can achieve consensus within a predefined-time under switching topology.

Similarly, it can be proved that the system has no Zeno behavior.

The proof is completed.

Remark4: Different from the static event-triggered predefined-time consensus [40], the dynamic triggering mechanism can avoid a large number of triggering.

V. SIMULATION EXPERIMENT

Two sets of numerical examples are designed to verify the effectiveness and superiority of the proposed algorithm. The first set of numerical examples is the simulation results of Section II. The second set of numerical examples is the simulation results of Section III.

Without losing generality, the system is set to contain six agents, and the system dynamics model is

$$\begin{cases} \dot{x}_i(t) = u_i(t-\tau) + f(x_i(t), t) + d_i(x_i(t), t) \\ x(0) = x_0 \end{cases} \tag{53}$$

Example 1: Based on Section II, we set $\tau = 0.06s$, $f(x_i(t), t) = 0.2\cos(x_i(t))$, $d_i(x_i(t), t) = 0.3\sin(x_i(t))$. Therefore, we can set $D = 0.3$. Assuming that the initial state of each agent is $x(0) = [-5, 0, 4, 9, -3, 2]^{\mathrm{T}}$, and its communication topology is shown in Fig. 1, the Laplacian matrix is

$$\boldsymbol{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & -1 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

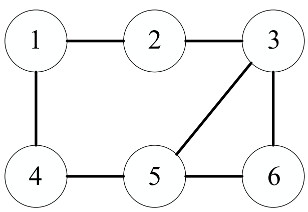


FIGURE 1. Communication topology.

Then the eigenvalue $\lambda_2(\boldsymbol{L}) = 1$ is obtained. Design parameter $\alpha = 0.5$, $\beta = 0.5$, $T = 2$, then under the condition of delayed input, the preset upper bound of the system convergence time is $T(x) \le T + \tau = 2.06$. Simulation results are shown in Fig. 2 and Fig. 3.

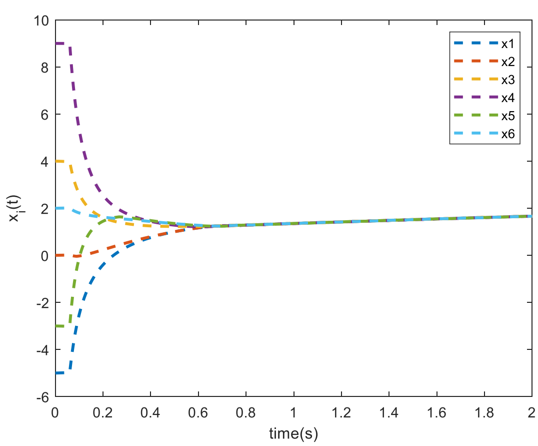


FIGURE 2. State evolution.

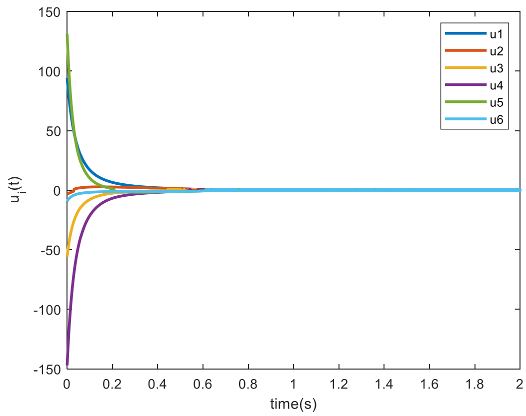


FIGURE 3. Control input evolution.

Fig. 2 and Fig. 3 show the state evolution and control input evolution of the six agents. As can be seen from the figure, under the action of the predefined-time controller (10), the system can achieve system state consistency within about 0.7s, satisfying $T(x) \le T + \tau = 2.06$.

In order to verify the switching topology, each agent can also achieve consistency within a predefined-time under the action of controller (21). Set the communication topology as shown in Fig. 4.

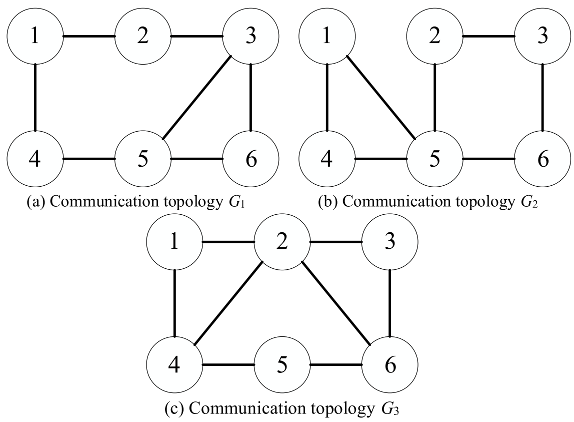


FIGURE 4. Switching topologies.

When $t < 0.3s$ is set, the topology graph G_1 is adopted in Fig.4 (a). When $0.3s \le t < 1.2s$ is set, the topology graph G_2 is adopted. When $t > 1.2s$ is set, the topology graph G_3 is adopted. Thus, $\lambda_2^{\min}(L(G_\Omega)) = 0.7639$ is obtained, and the controller parameters $\alpha = 0.5, \beta = 0.5, T = 2$ are designed. At this time, the upper bound of the preset system convergence time is $T(x) \le T + \tau = 2.06$. The simulation results are shown in Fig.5 and Fig.6.

As can be seen from Fig. 5 and Fig. 6, under the condition of switching topology, the system can achieve system state consistency within about 0.6s, satisfying $T(x) \le T + \tau = 2.06$.

To verify the superiority of the proposed predefined-time controller with input delay, we will compare it with the fixed time controller proposed in [26].

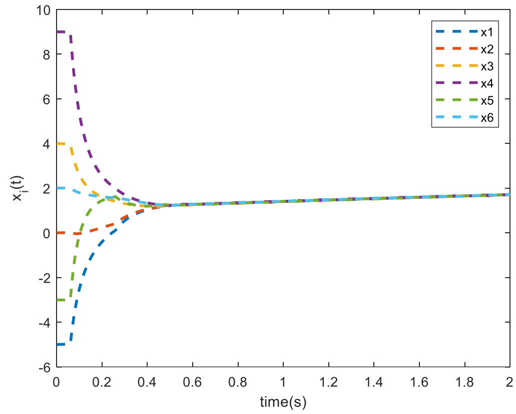


FIGURE 5. State evolution.

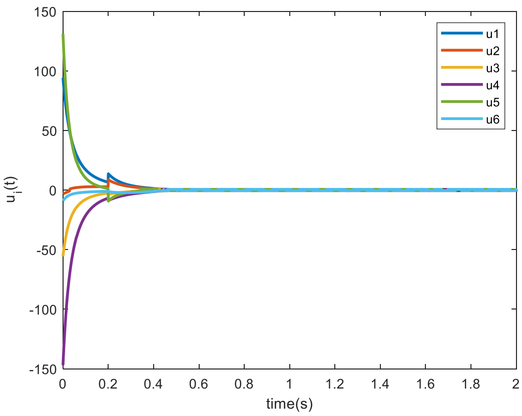


FIGURE 6. Control input evolution.

Because this paper is different from the system discussed in [26], for fairness, we obtain a fixed-time controller suitable for the system discussed in this paper according to the controller design strategy in [26]. For convenience, the algorithm in this paper and the algorithm in literature [26] are shown in formula (54) and formula (55).

$$u_i(t) = -\frac{\pi}{2\alpha T m^{-\frac{\alpha}{2}} (2\lambda_2(L))^{1+\frac{\alpha}{2}}} sig\left(\xi_i(t)\right)^{1+\alpha} - \frac{\pi}{\alpha T (2\lambda_2(L))^{1-\frac{\alpha}{2}}} sig\left(\xi_i(t)\right)^{1-\alpha} - \beta sign\left(\xi_i(t)\right) \quad (54)$$

$$u_i(t) = -\mathrm{w} sig\left(\xi_i(t)\right)^{\frac{\mu}{\varpi}} - v sig\left(\xi_i(t)\right)^{\frac{p}{q}} - \beta sign\left(\xi_i(t)\right) \quad (55)$$

where the controller (55) parameters $w = 0.5, v = 10, \mu = 7, \varpi = 5, p = 3, q = 5, \beta = 0.5$ are selected. In this paper, the controller parameters $\alpha = 0.4, \beta = 0.5$ are selected. It is worth noting that the time parameter T of the controller (54) is selected. In order to obtain fairness, the parameters are determined by the time function of [26]. Substituting the fixed-time controller (55) into the proof of Theorem 1 in this paper, it is obtained that

$$\dot{V}(t) \le -wm^{\frac{1}{2}-\frac{\mu}{2\varpi}} \left(2\lambda_2(L)\right)^{\frac{1}{2}+\frac{\mu}{2\varpi}} \left(V(t)\right)^{\frac{1}{2}+\frac{\mu}{2\varpi}} - v\left(2\lambda_2(L)\right)^{\frac{1}{2}+\frac{p}{2q}} \left(V(t)\right)^{\frac{1}{2}+\frac{p}{2q}} \quad (56)$$

It can be obtained from the preset convergence time function of the system in [26] that

$$T \le T_{\max} := \frac{1}{wm^{\frac{1}{2}-\frac{\mu}{2\varpi}} \left(2\lambda_2(L)\right)^{\frac{1}{2}+\frac{\mu}{2\varpi}} \left(\left(\frac{1}{2}+\frac{\mu}{2\varpi}\right)-1\right)} + \frac{1}{v\left(2\lambda_2(L)\right)^{\frac{1}{2}+\frac{p}{2q}} \left(1-\left(\frac{1}{2}+\frac{p}{2q}\right)\right)} \quad (57)$$

After calculation, T = 6.2677. Consensus-keeping Error Metric (CKM) in MASs is defined as

$$CKM = \sqrt{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left(\chi_i(t) - \chi_j(t)\right)^2} \quad (58)$$

Fig. 7 shows the CKM comparison simulation results of the system under two control laws.

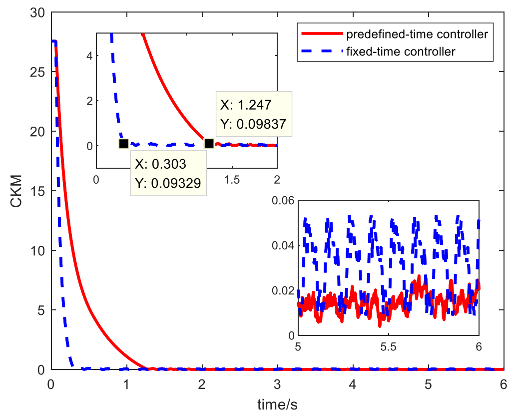


FIGURE 7. The CKM comparison.

It can be seen from Fig. 7 that under the input delay, the predefined-time controller designed in this paper has lower conservatism and lower jitter than the controller of the fixed-time control method under the CKM index, which means that the predefined-time controller has better convergence effect.

Example 2: Based on Section III, $f(x_i(t), t) = 0.3x_i(t) - 0.2\sin(x_i(t)), d_i(x_i(t), t) = 0.3\cos(x_i(t))$ are set. Therefore, we have $\gamma_1 = \gamma_2 = D = 0.3$. Assume that the initial state of each agent is $x(0) = [-5, 0, 4, 9, -3, 2]^{\mathrm{T}}$. Consider the fixed topology Fig.1, then $\lambda_2(\boldsymbol{L}) = 1$. In addition, the controller (24) parameters $\alpha = 0.5, \beta = 0.9, \varepsilon = 0.5, \theta = 0.5, \delta = 0.5, k_3 = 6/5, k_6 = 5/6, T = 2, \eta_i(0) = 20$ are set. The simulation results are shown in Fig. 8- Fig. 10.

Fig. 8, Fig. 9 and Fig. 10 show the state evolution, trigger time and dynamic variable evolution of the six agents. As can be seen from the figure, under the action of a predefined-time controller (24), a trigger function (27), a dynamic variable (28), and a trigger condition (29), the system can achieve system state consistency within approximately 0.6s.

In order to verify the switching topology, each agent can also achieve consistency within a predefined-time under the action of controller (48). Switching topology Fig. 4, switching time identical to Example 1.

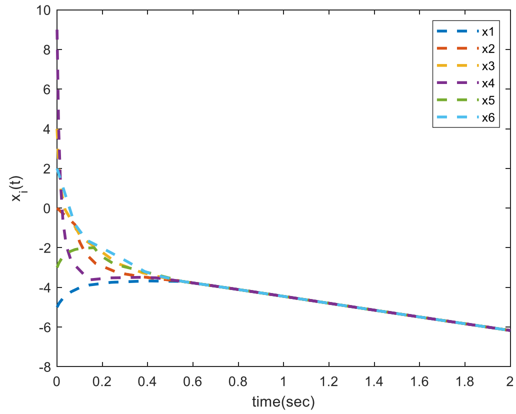


FIGURE 8. State evolution.

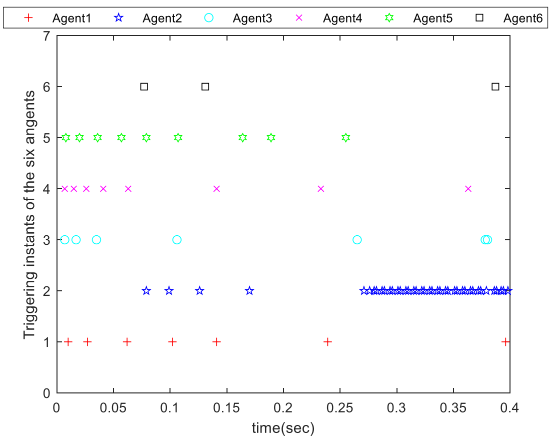


FIGURE 9. Triggering instants.

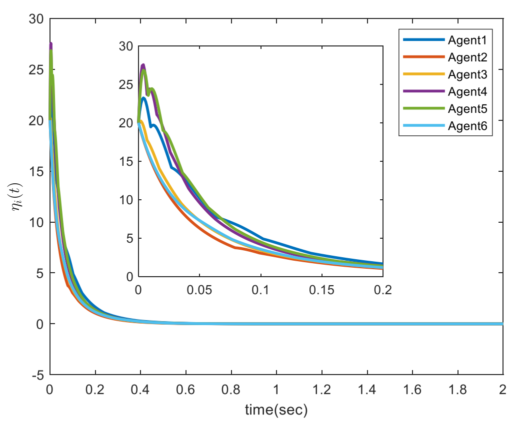


FIGURE 10. Dynamic variable evolution.

Parameters $\lambda_2^{\min}(L(G_\Omega)) = 0.7639, \alpha = 0.5, \beta = 0.9, \varepsilon = 0.5, \theta = 0.5, \delta = 0.5, k_3 = 3/2, k_6 = 2/3, T = 2, \eta_i(0) = 20$. The simulation results are shown in Fig. 11-Fig. 13.

From Fig. 11, Fig. 12 and Fig. 13, it can be seen that in the case of switching topology, using the predefined-time controller (48), the system can achieve system state consistency within about 0.6s. Similarly, fewer triggers in the system convergence phase.

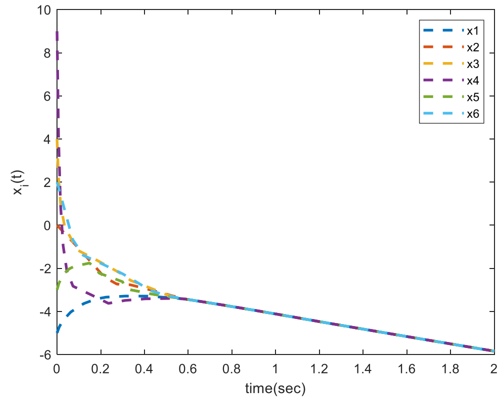


FIGURE 11. State evolution.

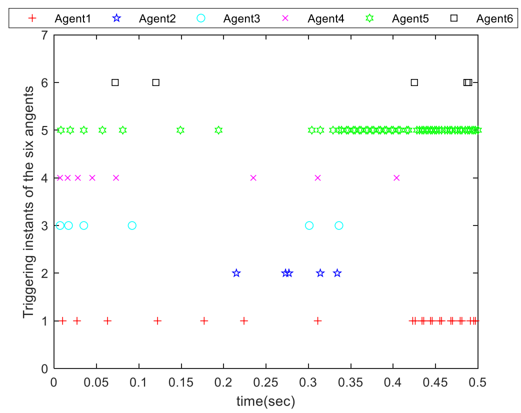


FIGURE 12. Triggering instants.

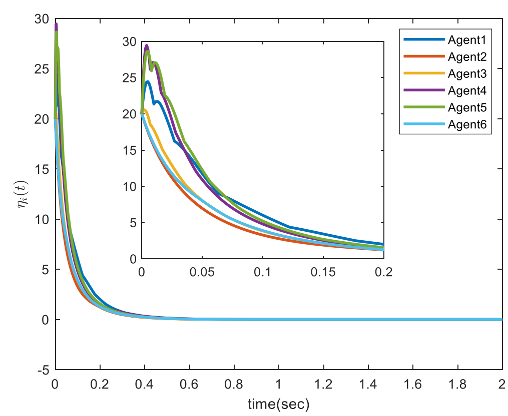


FIGURE 13. Dynamic variable evolution.

In order to reflect the superiority of the dynamic event-triggered predefined-time control algorithm proposed in this paper, the following will be compared with the dynamic event-triggered fixed-time control algorithm in [31] on the basis of Example 2. For convenience, the dynamic event-triggered fixed-time controller in the algorithm of this paper and [31] is shown in Equations (46) and (47).

$$u_i(t) = -\frac{\pi}{2\alpha T(1-\varepsilon)m^{-\frac{\alpha}{2}}(\lambda_2(L))^{1+\frac{\alpha}{2}}} y_i^{1+\alpha}(t_k^i)$$

$$- \frac{\pi}{\alpha T(1-\varepsilon)(2\lambda_2(L))^{1-\frac{\alpha}{2}}} y_i^{1-\alpha}(t_k^i) - k_3 y_i(t_k^i) - \beta sign\left(y_i(t_k^i)\right) \quad (59)$$

$$u_i(t) = -c_1 y_i^p(t_k^i) - c_2 y_i(t_k^i) - c_3 \tanh(\kappa y_i(t_k^i)) \quad (60)$$

where the controller (60) parameters are consistent with the original. $c_1 = 3, c_2 = 16c_3 = 6, p = 7/5, \kappa = 100$.

In addition, the trigger function and internal dynamic variable of agent i in [31] satisfy

$$\psi_i(t) = \theta\left(|e_i(t)| - \varepsilon c_1 \left|y_i^p(t)\right| - \varepsilon c_2 |y_i(t)| - \varepsilon c_3\right) \quad (61)$$

$$\dot{\eta}_i(t) = \delta |y_i(t)| \left(\varepsilon c_1 \left|y_i^p(t)\right| + \varepsilon c_2 |y_i(t)| + \varepsilon c_3 - |e_i(t)|\right) - c_4 \eta_i^{\frac{p+1}{2}}(t) - c_5 \eta_i^{\frac{1}{2}}(t) - c_6 \eta_i^2(t) \quad (62)$$

where $\theta = 0.1, \delta = 0.5, \varepsilon = 0.5, c_4 = 0.9, c_5 = 1, c_6 = 25/16, \eta_i(0) = 10$.

The parameters $\varepsilon = 0.5, \theta = 0.1, \delta = 0.5, \eta_i(0) = 10$ of trigger function (28) and internal dynamic variable (29) in this paper are consistent with that in [31]. It is worth noting that the time parameter T of the controller is selected. In order to obtain fairness, the parameters are determined by the time function of [31], that is

$$T \leq T_{\max} = \frac{2}{\hat{\beta}\phi} + \frac{2^{(p+1)/2}}{\hat{\alpha}\phi} \quad (63)$$

where $\hat{\alpha} = \min\left\{c_1(1-\varepsilon)m^{(1-p)/2}(2\lambda_2(L))^{(1+p)/2}, c_4 m^{(1-p)/2}\right\}$, $\hat{\beta} = \min\left\{c_3(1-\varepsilon)(2\lambda_2(L))^{1/2}, c_5\right\}, \phi \in (0, 1)$.

After calculation, T = 5.5220 can be selected. Fig. 14 and Fig. 15 are the comparative simulation results of the system under the action of two control laws.

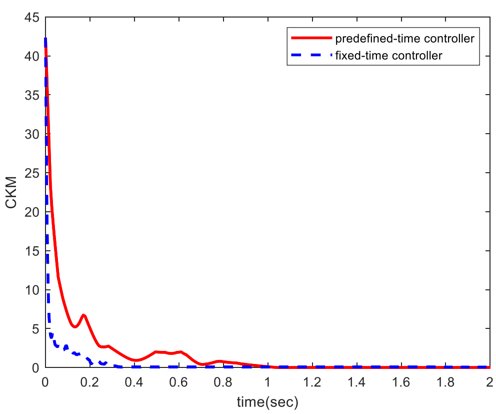
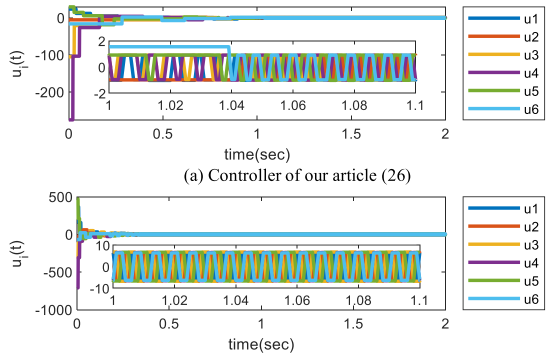


FIGURE 14. The CKM comparison.

From Fig. 14 and Fig. 15, it can be seen that the dynamic trigger predefined-time controller designed in this paper has lower conservatism under the CKM index than the controller of the dynamic trigger fixed-time control method, and the control input jitter amplitude is small, which has better protection for the hardware in practical applications.



(a) Controller of our article (26)

(b) Controller of [31]

FIGURE 15. Control input evolution.

In order to verify the advantages of the dynamic triggering mechanism in this paper, the following will be compared with the static triggering method in [29], [45], and [46]. For the sake of fairness, the controller is the controller of this paper, and only the trigger method is changed. The trigger parameters in [45] and [46] are the same as those in the original text. For convenience, the static triggering method in [29], [45], and [46] are as follows.

In [45], the event-triggered condition of agent i is defined as

$$|e_i(t)| \leq \sigma_i |y_i(t)| \quad (64)$$

where $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 0.006$.

In [46], the event-triggered condition of agent i is defined as

$$|e_i(t)| \leq \frac{1}{2}\tau |y_i(t)| \quad (65)$$

where $\tau = 1.2$.

In [29], the design of the trigger function is similar to the method of this paper. The difference is that there is no internal dynamic variable. According to the method, the event trigger condition of agent i can be defined as

$$|\psi_i(t)| \leq |e_i(t)| - \varepsilon c_1 \left|y_i^{1+\alpha}(t)\right| - \varepsilon c_2 \left|y_i^{1-\alpha}(t)\right| - \varepsilon c_3 |y_i(t)| - \varepsilon\beta \quad (66)$$

where the parameters of Equation (66) are the same as those of Equation (29).

Fig. 16, Fig. 17, and Table 1 show the comparative simulation results of the system under static and dynamic triggering mechanisms.

From Fig.16, Fig. 17 and Table 1, it can be seen that the dynamic triggering mechanism adopted by the controller in this paper has less triggering than the static triggering mechanism, which greatly reduces the resource consumption.

In summary, for MASs with nonlinear uncertainty, the predefined-time consensus control algorithm under input delay and dynamic event-triggered predefined-time consensus control algorithm proposed in this paper are effective and superior. The simulation results show that under the action of

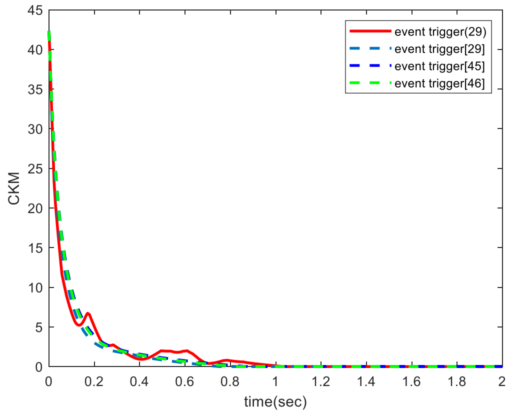


FIGURE 16. The CKM comparison.

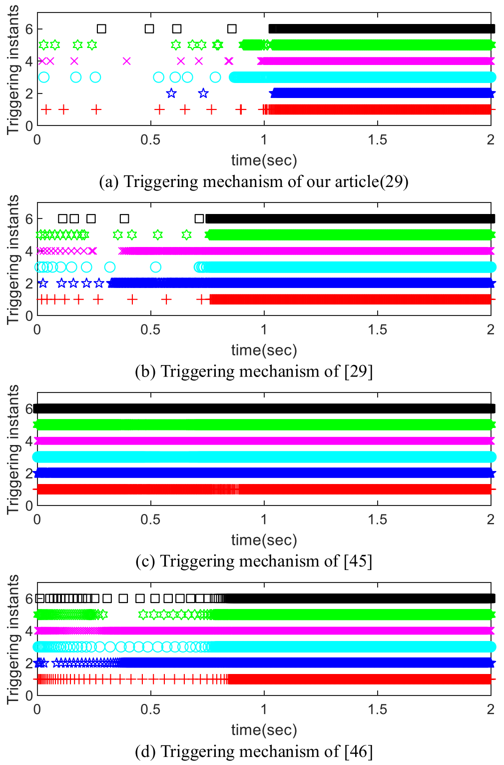


FIGURE 17. Triggering instants.

TABLE 1. Number of events triggered.

Mechanism \ Agents	T1	T2	T3	T4	T5	T6
Our article	341	322	381	346	362	325
[29]	423	553	432	552	428	419
[45]	933	998	955	1000	929	999
[46]	619	836	646	890	662	605
None	2000	2000	2000	2000	2000	2000

the controller, the system can be stable in the preset time, the convergence performance is exceptional, and the convergence speed is independent of the initial state of the system, the resource consumption is low, and the topological conditions are relaxed.

VI. CONCLUSION

This paper aims to study the MASs consensus control problem with nonlinear uncertainties. Considering the input delay and switching topology of MASs, predefined-time consensus controllers (10), (21) are proposed. Considering the problem of resource loss, dynamic event-triggered predefined-time consensus controllers (24), (48) are proposed. The correctness of the algorithm is proved by Lyapunov stability theory and algebraic graph theory. By proving that the minimum trigger time interval is greater than a positive number, it is concluded that the system does not have Zeno behavior. The superiority of the provided controller in convergence time and resource saving is verified by simulation experiments. In this paper, first-order nonlinear MASs are considered, and further research can be extended to second-order or high-order MASs.

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XIAOLU MA received the M.S. degree from the Hefei University of Technology, in 2005, and the Ph.D. degree from the Nanjing University of Aeronautics and Astronautics, in 2012. His current research interests include embedded systems, the Internet of Vehicles, motion planning of robotic systems, multi-robot formation control, cooperative control, multi-agent systems, and predefined-time control.

YIBO TAN received the B.S. degree from the Henan Institute of Technology, Xinxiang, China, in 2020. He is currently pursuing the M.E. degree with the Anhui University of Technology. His current research interests include multi-robot formation control, cooperative control, multi-agent systems, and predefined-time control.

HONG MEI received the M.S. degree from the Anhui University of Technology, in 2020. He is currently pursuing the Ph.D. degree with the Nanjing University of Aeronautics and Astronautics. His current research interests include multi-robot formation control, cooperative control, multi-agent systems, and predefined-time control.