IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 16 February 2023, accepted 5 March 2023, date of publication 14 March 2023, date of current version 20 March 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3257032

RESEARCH ARTICLE

Elastic Net Penalized Quantile Regression Model and Empirical Mode Decomposition for Improving the Accuracy of the Model Selection

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This research is supported by the Research Management Center (RMC), Universiti Malaysia Sabah, through the UMS/PPI-DPJ1 Journal Article Fund.

ABSTRACT In quantile regression models, numerous penalization methods have been developed to deal with ordinary least-squares method problems. Such methods are ridge penalized quantile regression, lasso penalized quantile regression, and elastic net penalized quantile regression which are used for variable selection and regularization and deals with the multicollinearity problem when it exists between the predictor variables. However, the variables of interest are often represented through time series processes, in which such time series data are often non-stationary and non-linear, which leads to poor accuracy of the resultant regression models and hence results with less reliability. The EMD-EnetQR method is proposed to address this issue, which consists of applying the empirical mode decomposition (EMD) algorithm to time series data and then using the resulting components in penalized quantile regression models. This study aims to apply the proposed EMD-QREnet method to determine the influence of the decomposition components of the original time series predictor variables on the response variable to build a model fit and improve prediction accuracy. Furthermore, this study addressed the multicollinearity between the decomposition components. Simulation studies and real dataset applications were conducted. The results show that the proposed EMD-QREnet method, in most cases, outperforms the other methods by improving prediction accuracy.

INDEX TERMS Elastic net penalty, quantile regression, empirical mode decomposition, model selection, multicollinearity.

I. INTRODUCTION

Linear regression models are widely used in statistical analyses and numerous fields, such as finance, economics, environmental science, and society. The traditional ordinary least squares (LS) regression approach was used to estimate regression models, representing the mean function of the response variable. However, the LS estimator is extremely sensitive to outliers or heavy-tailed distributions. In this case, the estimation efficiency is naturally reduced. Quantile regression (QR) Koenker and Bassett [16] has been used as an alternative to least squares in the presence of outliers. Over the past two decades, quantile regression (QR) has become a popular method for describing the distribution of a response variable given a collection of predictor variables. QR describes the effects of predictor variables on the complete conditional distributions of a response variable instead of only the average value and provides an overall assessment of the covariate effects at different quantile τ of the response Das [15], Tian and Song [27].

The associate editor coordinating the review of this manuscript and approving it for publication was Dost Muhammad Khan.

Empirical mode decomposition (EMD) was introduced by Huang et al. [13] for decomposing non-linear and non-stationary time series data into a finite set of decomposition components called Intrinsic Mode Functions (IMFs) and residual components via the sifting process. Unlike previous methods, such as wavelet decomposition Chui and Heil [6] and Fourier decomposition Titchmarsh [29], EMD does not assume any basis system on the dataset, such as stationary or linear. The EMD by the sifting method produces decomposition components with different wavelengths, amplitudes, and frequencies, indicating that they are functionally significant Huang [12]. These decomposition components can be used as new predictor variables to study their influence on the response variable. Al-Jawarneh and Ismail [3].

Variable selection methods have attracted considerable attention from researchers over the past few decades. Many penalization techniques have been proposed for simultaneous variable selection to produce sparse models, such as the ridge penalty Hoerl and Kennard [11], lasso penalty Tibshirani [28], and elastic-net penalty Zou and Hastie [38]. Moreover, penalized regularization methods have been conveniently used to improve the quantile regression method by enhancing prediction accuracy and improving variable selection. For example, Li and Zhu [17] studied a quantile regression with an L_1-norm penalty, Belloni and Chernozhukov [4] proposed a penalized quantile regression with the L_1-penalty in high-dimensional and Zheng et al. [37] combined quantile regression with a fully adaptive L_1-penalty. Then, Uniejewski and Weron [30] introduced a different approach that considers regularized quantile regression averaging (QRA), which utilizes lasso to select the relevant regressors automatically, Liu et al. [19] studied the generalized lasso penalty in a quantile regression with linear constraints on the parameters and Xu et al. [31] introduced both the sampling method and the lasso technique to QR to develop a sampling lasso quantile regression (SLQR) method. After that, Burgette et al. [5] presented two approaches based on lasso and elastic-net penalties to identify potentially essential predictors in quantile regression, and Ali et al. [1] proposed an optimal k-NN ensemble (Ok-NN-E) based on fitting stepwise regression for optimal model selection for regression. Recently, Hamraz et al. [10] used a robust fisher score approach to select discriminative genes or features, and Younas et al. [34] proposed an optimal causal trees (OCTE) method to select a subset of the best causal trees in terms of their strength.

Several penalization techniques have also been proposed to solve the problem of multicollinearity among predictor variables in quantile regression, such as Zaikarina et al. [35], who used lasso and ridge penalties in quantile regression to overcome the problem of multicollinearity. Subsequently, Sadig and Bager [24] used ridge and quantile regression with a parameter ridge to solve the multicollinearity problem. Similarly, Erişoğlu and Yaman [9] used ridge and quantile regression approaches to solve the multicollinearity problem. In contrast, Slawski [25] studied the structured elastic net regularized in conjugation with two significant loss functions: the checked loss of quantile regression and the hinge loss of support vector classification. Later Yan and Song [32] studied penalized quantile regression with an elastic net. Recently, Zhang et al. [36] proposed a framework for probability density forecasting of short-term wind speed based on quantile regression (QR) and kernel density estimation (KDE). They found that introducing the empirical mode decomposition (EMD) technique reduces raw wind speed series noise.

In several studies, such as medicine and economics, the relationships between natural processes are assessed through regression analyses using time series data. Such data are often non-stationary and non-linear, and a multicollinearity problem may exist. If these concerns are not considered, it can lead to poor accuracy in the resulting regression models and make the final result less accurate. To address these issues, the EMD-EnetQR method is proposed, which consists of applying the empirical mode decomposition (EMD) algorithm to time series data and then using the resulting components in penalized quantile regression models. The proposed EMD-EnetQR method addresses the problems associated with non-stationary and non-linear signals. The EMD algorithm decomposes the original non-stationary and non-linear signals of the data sets into a set of orthogonal IMF components and a residual component. Second, the decomposition components of EMD are used as orthogonal predictor variables in elastic net penalized quantile regression (EnetQR). This study aims to select the decomposition components of the original time-series predictors that exhibit the most substantial effects on the response variable to build a best-fitted model and address multicollinearity among the decomposition components to improve the performance of predictions further. The proposed method (EMD-EnetQR) is compared with traditional methods called Ridge regularized quantile regression QRR proposed by Zaikarina et al. [35], Lasso regularized quantile regression QRL Li and Zhu [17], Elastic net penalized quantile regression (QREnet) Yan and Song [32]. Mentioned also that Ridge regularized quantile regression based on EMD (EMD-QRR), and LASSO regularized quantile regression based on EMD (EMD-QRL) are also the proposed methods.

The remainder of this paper is organized as follows. Section II describes the EMD method, elastic net penalized quantile regression, the proposed EMD-EnetQR method, and prediction goodness measurements. Section III presents a simulation study and a real data analysis. Furthermore, report analysis and discussion are presented in Section III. Finally, Section IV concludes the study with a brief discussion.

II. METHODOLOGY

This section discusses the methods used in this study. First, the EMD method is used to decompose the original signal (predictor variables) using the sifting process technique. Secondly, an elastic net (Enet) penalized quantile regression was applied. Finally, the proposed EMD-EnetQR method is discussed in this section.

A. EMPIRICAL MODE DECOMPOSITION

The empirical mode decomposition (EMD) method was proposed by Huang et al. [13] and represented the first part of the Hilbert–Huang transform. The EMD technique decomposes the non-stationary and non-linear original signals into a finite set of decomposition components called Intrinsic Mode Functions (IMFs) and residual components via the sifting process. In EMD, the time domain of the signal is unchanged Huang [12]. Each IMF must fulfil the following two conditions: (1) the number of local extreme values and the number of zero-crossings in the entire data set must be the same or differ by one; and (2) at any time point, the mean value of the upper envelope defined by the local maximum and the lower envelope defined by the local minimum must be zero. The EMD process has the following simple expression Huang [12].

$$x\left(t\right)=\sum\nolimits_{k=1}^{K}C_k\left(t\right)+r\left(t\right) \tag{1}$$

where $x\left(t\right)$ indicates the original signal, $r(t)$ represents the residue of the original signal decomposition, and $C_k\left(t\right)$ represents the ith intrinsic mode function (IMF).

1) SIFTING PROCESS

The sifting process decomposes the original signal $x(t)$ into several $C_k\left(t\right)$ and $r(t)$. The detailed decomposition processes of the EMD are briefly summarized as follows:

Step 1: Insert the original signal $x\left(t\right)$ into the sifting process and the iteration index value was assumed to be $j=1$.

Step 2: Identify all local extrema (maxima and minima).

Step 3: Connect all the minima and maxima using cubic spline interpolation to form the lower $l\left(t\right)$ and upper $u\left(t\right)$ envelopes.

Step 4: Compute the local mean of the upper and lower envelope.

$$m_j\left(t\right)=\frac{u\left(t\right)+l\left(t\right)}{2} \tag{2}$$

Step 5: Subtract the mean $m_j\left(t\right)$ from signal $x\left(t\right)$ to obtain the first component IMF candidate.

$$h_j\left(t\right)=x\left(t\right)-m_j\left(t\right) \tag{3}$$

Step 6: Check the component $h_j\left(t\right)$ is an $IMF_i\left(t\right)$, according to $\text{IMF}_i\left(t\right)$ conditions. If the function $h_j\left(t\right)$ satisfies $IMF_i\left(t\right)$ conditions, it continues to Step 7. If not, return to step 2 and replace the value $h_j\left(t\right)$ with $x\left(t\right)$ and repeat Steps 2 to 4. Subsequently, the $IMF_i\left(t\right)$ result obtained in the previous step is saved. Then, the iteration index value is updated such that it equals $j=j+1$. Finally, using the $\text{IMF}_i\left(t\right)$ and the signal $x\left(t\right)$, the residue function $r_i\left(t\right)$ is obtained as follows:

$$r_i\left(t\right)=r_{i-1}\left(t\right)-IMF_i\left(t\right) \tag{4}$$

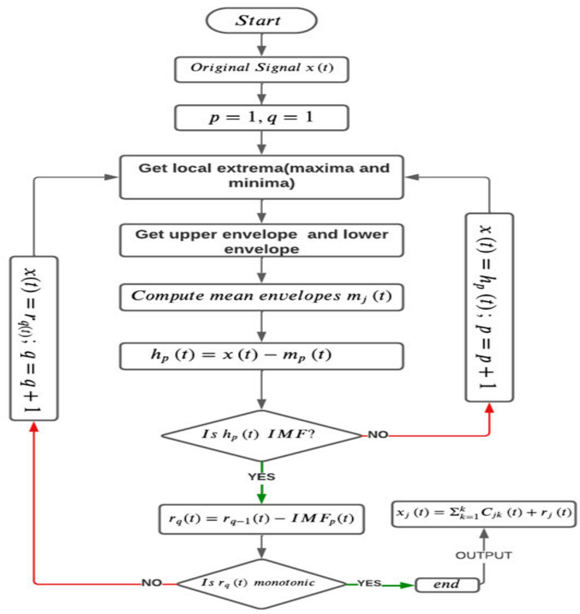

FIGURE 1. The EMD decomposition process.

Step 7: Check whether the residue function $r_i\left(t\right)$ acquired from step 6 is a monotonic function or satisfies the stopping criterion of the standard deviation (SD) for two consecutive successive sifting of the results, where a typical value for SD can be set between 0.2 and 0.3, as shown in the following formula:

$$SD_j=\sum\nolimits_{t=0}^{T}\frac{h_{j-1}\left(t\right)-h_j\left(t\right)^2}{h_{j-1}^2\left(t\right)} \tag{5}$$

If not, replace $r_i\left(t\right)$ with $x\left(t\right)$ and then repeat the operations from step 2, setting $j=j+1$. If yes, save the residue and all the IMFs obtained, and stop the sifting process. Figure 1 describes a flowchart that summarizes all the sifting process steps.

2) INTRINSIC MODE FUNCTION (IMF)

IMFs represent a simple oscillatory mode as an alternative to a straightforward harmonic function. An IMF is defined as any function with the same number of extrema and zero crossings whose envelopes are symmetric concerning zero. According to the EMD algorithm presented in the previous section, the IMF produced by the sifting process must satisfy the following two conditions: Huang et al. [13], Huang [12]

- In the entire data set, the number of local extreme values (local maxima and local minima) and the number of zero crossings must be equal or differ at most by one.
- At any point, the local mean value between the upper and lower envelopes was zero. The first condition is necessary for oscillation data. This indicates that each IMF has only one local maximum or local minimum between two consecutive zero crossings. The second condition assumes that the IMF is stationary, making its analysis easier. However, an IMF can exhibit

amplitude modulation as well as changing frequency Al-Jawarneh et al. [2], Lu [20], Raghuram et al. [23].

B. PENALIZED QUANTILE REGRESSION

Consider the following typical linear regression model:

$$y = X\beta + \varepsilon \tag{6}$$

where

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \boldsymbol{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & \cdots & x_{np} \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}, \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

y is an $(n \times 1)$ vector of observations on the response variable, X is an $(n \times p)$ matrix of observations on the predictor variables, β is a $(p \times 1)$ vector of unknown regression coefficients, ε is an $(n \times 1)$ vector of random errors that are supposed to be normally distributed with $E(\varepsilon) = 0$ and $E\left(\varepsilon\varepsilon^T\right) = \sigma^2 \mathrm{I}_n$.

The τth conditional quantile function can then be estimated by solving the following optimization problem:

$$\hat{\beta}_\tau = \min_\beta \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^T \beta\right) \tag{7}$$

where x_i^T is i^{th} row of X and $\rho_\tau(u) = \tau(u - I(u< 0))$ for $u \in \mathbb{R}$ is the check loss function with $I(.)$ being the indicator function and quantile level $\tau \in (0, 1)$. Under the regularization framework and to improve quantile regression Koenker [14] suggested penalized version, where we consider the following penalized optimization problem:

$$\hat{\beta}_\tau = \min_\beta \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^T \beta\right) + \lambda P(\beta) \tag{8}$$

where $\lambda > 0$ is the penalization parameter and $P(\beta)$ is the penalty function.

C. ELASTIC NET PENALIZED QUANTILE REGRESSION

Zou and Hastie [38] introduced the elastic net penalty technique, which was a convex combination of the L_1-norm penalty (lasso) by Tibshirani [28] and the L_2-norm penalty (ridge) by Hoerl and Kennard [11]. The L_1-norm part of the elastic net penalty reduces the number of predictor variables by shrinking some regression coefficients to zero. The L_2-norm part of the elastic net penalty deal with the high correlation between the predictor variables Al-Jawarneh [2], Liu and Li [18]. The formula for the elastic net penalty is as follows:

$$P_\lambda(\beta) = \lambda_1 \sum_{j=1}^{p} \left|\beta_j\right| + \lambda_2 \sum_{j=1}^{p} \beta_j^2 \tag{9}$$

where $P_\lambda(\beta)$ is the Elastic Net (Enet) convex penalty function; $\|\beta\|_2^2 = \sum_{j=1}^{p} \beta_j^2$ is the L_2-norm of vector β and $\|\beta\|_1 = \sum_{j=1}^{p} \left|\beta_j\right|$ is the L_1-norm of vector β. Moreover, λ_1 and λ_2 are tuning parameters that control the amount of shrinkage for regression parameters and non-negative parameters $(\lambda_1, \lambda_2 \geq 0)$, which are automatically selected by cross-validation (CV) Al-Jawarneh et al. [2], Masselot et al. [21], Zou and Hastie [38].

D. PROPOSED EMD-EnetQR METHOD

In this section, the elastic net penalized quantile regression model based on Empirical Mode Decomposition (EMD-EnetQR) is presented to select decomposition components that have the most significant effect on the response variable and deal with the multicollinearity between the decomposition components. The proposed method can be summarized as follows:

Step 1: The original signals $x_j(t)$ are decomposed by EMD into several components named $C_{jk}(t)$ and residual component $r_j(t)$. These decomposed components can be expressed by Equation (11) Qin et al. [22]

$$x_j(t) = \sum_{k=1}^{K} C_{jk}(t) + r_j(t) \tag{10}$$

Step 2: All the decomposition components and residuals obtained in Step 1 are used as predictor variables to predict the behaviour of the response variable $y(t)$ as in Equation (11) Al-Jawarneh et al. [2], Masselot et al. [21].

$$y(t) = \sum_{j=1}^{p} \left[\sum_{k=1}^{K} C_{jk}\beta_{jk} + r_{jk}(t)\beta_{jk}\right] + \varepsilon(t) \tag{11}$$

Step 3: Using the correlation coefficient and the variance inflation factor (VIF) test to check whether there is a multicollinearity problem among the decomposition components.

Step 4: The Elastic Net penalized quantile regression is used between the response variable $y(t)$ and all decomposed components obtained from the predictor variable $x(t)$ via EMD to select the subset of components that exhibited the most impact. The EnetQR method was then used in the following formula: Al-Jawarneh and Ismail [3], Sadig and Bager [24]

$$\begin{aligned} &\hat{\beta}_{EnetQR} \\ &\quad = \underset{\beta}{argmin} \frac{1}{2n} \sum_{i=1}^{n} \rho_\tau (y_i(t) \\ &\qquad - \sum_{j=1}^{p} \left(\sum_{k=1}^{K} C_{jk}(t)\beta_{jk-r_j(t)\beta_{jk+1}}\right)\Big)^2 + \lambda P(\beta) \\ &\lambda P(\beta) \\ &\quad = \lambda \left(\alpha \sum_{k=1}^{p} \left[\sum_{k=1}^{K} \beta_{jk}^2\right]\right. \\ &\qquad \left. + \frac{(1-\alpha)}{2} \sum_{k=1}^{p} \left[\sum_{k=1}^{K+1} \beta_{jk}^2\right]\right) \end{aligned} \tag{12}$$

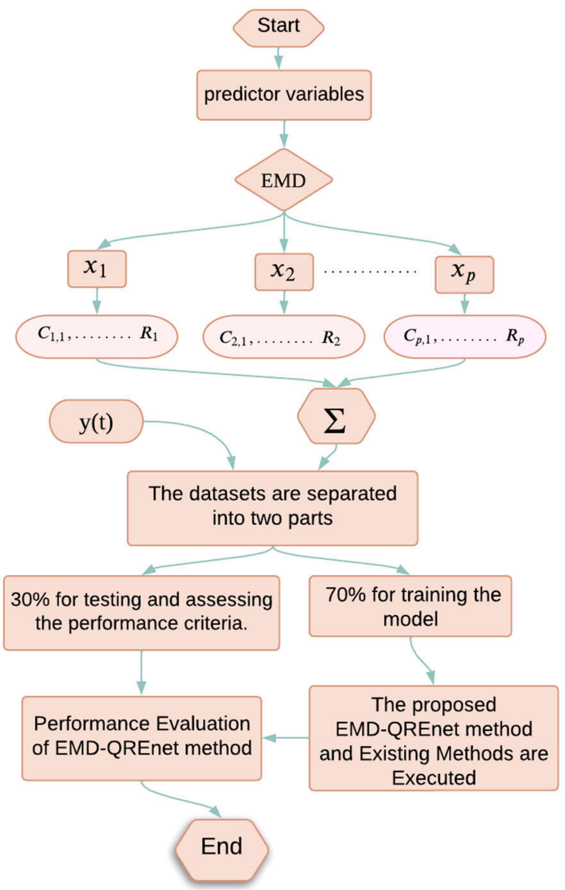

FIGURE 2. Schematic representation of the proposed EMD-QREnet method.

Step 5: Finally, the performance of the proposed EMD-EnetQR models was compared with that of the existing methods, namely, EMD-RQR, EMD-LQR, RQR, LQR, and EnetQR. The main steps in applying the proposed EMD-QREnet method are summarized in Figure 2.

E. PREDICTION GOODNESS MEASUREMENTS

The performance of the proposed predictive method was evaluated and compared using five test criteria. The test criteria include the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), Mean Absolute Percentage Error (MAPE) and Residual Sum of Squares (RSS), which are used to evaluate the performance of the predictive models and are computed as follow:

- Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum\nolimits_{i=1}^{n} \left|y_i - \hat{y}_i\right| \tag{13}$$

- The root mean square error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum\nolimits_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2} \tag{14}$$

- mean absolute scaled error (MASE)

$$MASE = \frac{1}{n}\sum\nolimits_{i=1}^{n}\left(\frac{\left|y_i - \hat{y}_i\right|}{\frac{1}{n-1}\sum_{i=2}^{n}\left|y_i - y_{i-1}\right|}\right) \tag{15}$$

- Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100\%}{n}\sum\nolimits_{i=1}^{n}\frac{y_i - \hat{y}_i}{y_i} \tag{16}$$

- The residual sum of squares (RSS)

$$RSS = \sum\nolimits_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{17}$$

F. MULTICOLLINEARITY

Multicollinearity occurs when strong correlations exist between two or more predictor variables Yang and Wen [33]. The variance inflation factor test and Interpredictor Correlations Matrix test methods were used to check for the presence of multicollinearity among the predictor variables.

1) INTERPREDICTOR CORRELATION MATRIX

The interpredictor correlation matrix will be used to examine bivariate correlations among the decomposition components to check the correlation among the decomposition components. If the correlation coefficient between each two decomposition components is large, it indicates a multicollinearity problem among the decomposition components. The correlation coefficient is calculated as follows:

$$\rho_{ij} = \frac{cov(C_i, C_j)}{\sqrt{var\left(C_i\right) \times var\left(C_j\right)}} \tag{18}$$

where $-1 \le \rho_{ij} \le 1$ is the inter-predictor correlation between two decomposition components $\left(C_i, C_j\right)$, $cov\left(C_i, C_j\right)$ is the covariance between two the decomposition components $\left(C_i, C_j\right)$, $var\left(C_i\right)$ is the variance of C_i and $var\left(C_j\right)$ is the variance of C_j Thompson et al. [26].

2) VARIANCE INFLATION FACTOR (VIF)

The variance inflation factor (VIF) test is used to detect multicollinearity between the predictor variables. The VIF is computed using the following formula Thompson et al. [26]:

$$VIF_j = \frac{1}{1 - R_j^2}, \tag{19}$$

where VIF_j is the variance inflation factor for the j^{th} predictor R_j^2 is the multiple correlation coefficient. When VIF_j of 10 or above indicates that high multicollinearity exists among the predictor variables, whereas if VIF_j is less than 10, it indicates that no multicollinearity exists Davino et al. [7].

TABLE 1. ADF test for original variables.

Variables	ADF	p.value
$x_1(t)$	1.0543	0.09921
$x_2(t)$	0.87758	0.99

III. NUMERICAL STUDIES

In this section, we implement a simulation study to evaluate the finite sample performance of EMD-EnetQR, illustrate it with an empirical analysis of real dataset application and compare it with its competitors such as EMD-RQR, EMD-LQR, RQR, LQR and EnetQR.

A. SIMULATION STUDY

In this section, we describe simulations conducted using the sine function to investigate the performance of the proposed method. The datasets are created for non-stationary and non-linear predictor variables and the response variable. The datasets for the predictor variables and response variable were simulated from signals selected from Al-Jawarneh and Ismail [3] and Qin et al. [22]. We carried out the analysis using R software, and the simulation experiments were replicated 5000 times with a sample size of $n = 300$ and the time domain was $(0 \leq t \leq 9)$.

The tuning parameters were selected based on 10-fold cross-validation using the statistical computing environment R 4.1.3 and its freely accessible packages. The simulation study considered three different quantile levels: $\tau = 0.25$, $\tau = 0.5$ and $\tau = 0.75$. We are using the three quartiles because they represent the three locations of the data: the lower tail, the median, and the upper tail. The datasets were split into 70% for training the model and 30% for testing and assessing performance criteria. The formula for the function test of the response variable and the predictor variables are presented as follows:

$$y(t) = 0.5t + sin(\pi t) + sin(2\pi t) + cos(6\pi t)$$
$$x_1(t) = 0.8t + sin(0.3\pi t) + sin(2\pi t) + sin(7\pi t) + sin(9\pi t)$$
$$x_2(t) = 0.4t + sin(0.2\pi t) + sin(6\pi t) + sin(5\pi t) + sin(12\pi t)$$

To test for stationarity of the original signals time series, we performed the Augmented Dickey-Fuller (ADF) Dickey and Fuller [8] unit root test. The results reported in Table 1 above showed the p-value is greater than 0.05 for all predictor variables, which implies that the time series are non-stationary.

Figure 3 shows the results of the EMD decomposition of the original predictor variables. It can be seen that the predictor $x_1(t)$ decomposes into seven IMFs components and one residual component, while the predictor $x_1(t)$ decomposes into eight IMFs components and one residual component.

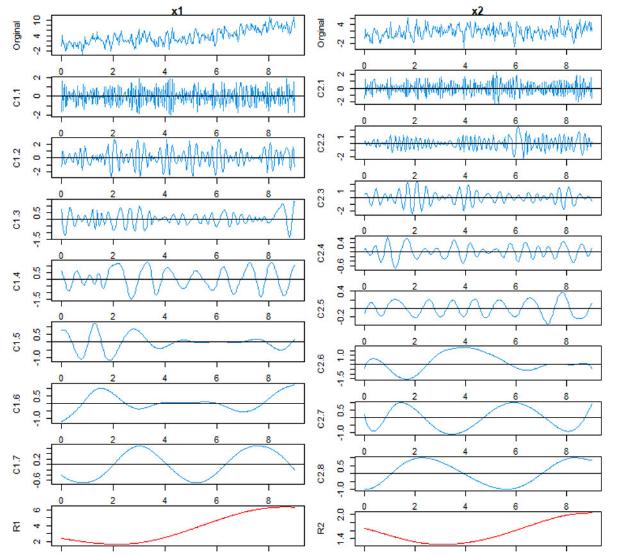

FIGURE 3. The decomposition process of the original signal of $x_1(t)$ and $x_2(t)$ via EMD.

TABLE 2. Descriptive statistics simulation data.

Variables	Mean	SD	Skewness	Kurtosis
$y(t)$	2.3205	1.7281	0.12746	2.4849
$x_1(t)$	3.8439	2.6363	0.41219	2.6359
$x_2(t)$	1.8689	1.6618	0.03803	2.8944

TABLE 3. Comparison of different methods for simulation study.

τ	Methods	λ_{min}	λ_{1se}	SSR	VS
0.25	EMD-QRR	0.09882	0.18655	48.324	All
	EMD-QRL	0.00988	0.01704	45.922	14
	EMD-QREnet	0.01412	0.03395	**45.011**	14
	QRR	0.10750	0.25084	102.39	-
	QRL	0.01075	0.04736	94.772	-
	QREnet	0.02688	0.08748	95.219	-
0.50	EMD-QRR	0.13675	0.28269	35.624	All
	EMD-QRL	0.02505	0.05178	**27.921**	13
	EMD-QREnet	0.03428	0.06671	27.971	13
	QRR	0.14270	0.35373	57.668	-
	QRL	0.03432	0.10201	50.101	-
	QREnet	0.03567	0.17208	49.481	-
0.75	EMD-QRR	0.12579	0.37389	59.316	All
	EMD-QRL	0.01258	0.07499	58.983	13
	EMD-QREnet	0.06289	0.23105	**57.967**	13
	QRR	0.14810	0.40201	92.635	-
	QRL	0.01128	0.10271	90.755	-
	QREnet	0.05338	0.12085	90.739	-

Table 2 presents the descriptive statistics of the response variable $\boldsymbol{y(t)}$ and the original predictor variables $x_1(t)$ and $x_2(t)$. The results showed that the mean of $\boldsymbol{Y(t)}$, $x_1(t)$, and $x_2(t)$ are 2.3205, 3.8439, and 1.8689, respectively. Moreover, the skewnesses have a positive sign, which indicates that the distribution of variables is skewed to the right.

TABLE 4. Simulation results: Quantile: Mean performance criteria.

τ	Methods	EMD-QRR	EMD-QRL	EMD-QREnet	QRR	QRL	QREnet
0.25	MAE	0.54243	0.51395	**0.51164**	0.82492	0.78561	0.78763
	RMSE	0.67519	0.65819	**0.65164**	0.98280	0.94556	0.94778
	MASE	1.15196	1.09147	**1.08657**	1.81918	1.73249	1.73694
	MAPE (%)	1.83896	**1.71213**	1.72484	2.82555	2.85122	2.82859
0.50	MAE	0.50056	**0.43389**	0.43526	0.59058	0.54897	0.54488
	RMSE	0.57972	**0.51323**	0.51369	0.73759	0.68749	0.68323
	MASE	1.20336	**1.04308**	1.04637	1.34257	1.24798	1.23869
	MAPE (%)	1.20282	1.19183	**1.18173**	1.29603	1.58371	1.58856
0.75	MAE	0.614039	0.60977	**0.60941**	0.73102	0.73479	0.72969
	RMSE	0.7480522	0.74595	**0.73949**	0.93483	0.92530	0.92522
	MASE	1.406701	1.39691	**1.39610**	1.64272	1.65121	1.63974
	MAPE (%)	1.673794	1.96231	**1.86966**	1.91658	1.94015	1.84042

TABLE 5. Descriptive statistics for daily exchange rates.

Variables	Mean	SD	Skewness	Kurtosis
China	6.742283	0.23492	-0.23458	1.91359
Japan	109.4827	2.91254	-0.05950	2.72243
Sri Lanka	173.1182	16.94317	-0.05484	1.61133

Table 3 shows the results of evaluating the performance of the methods that depended on the lambda values, which were selected by averaging the cross-validation (CV) error using a 10-fold CV. It was evident that EMD-EnetQR has the best performance because it has the smallest SSR when quantile levels are 0.25 and 0.75. However, when the quantile level is 0.50, EMD-LQR has the smallest value. From Table 3, for variable selection, EMD-EnetQR and EMD-LQR perform well by shrinking the regression coefficients of partially redundant variables to zero and selecting only 14 important variables. These variables had the greatest effect on the response variable to build the regression model when the quantile level was 0.25, whereas 13 important variables are selected at 0.50 and 0.75, by the EMD-EnetQR and EMD-LQR methods.

Table 4 demonstrates the mean of the performance criteria in terms of the MAE, RMSE, MASE and MAPE used in this study for all regression methods. Table 4 shows that the proposed EMD-EnetQR method outperforms the other four methods because it has the smallest values in MAE, RMSE, MASE, and MAPE for quantile levels of 0.25, 0.75, and 0.50; the smallest values are investigated by the EMD-LQR method.

B. REAL DATA ANALYSIS

In this section, to illustrate the application of the proposed method, we consider the three countries' daily close exchange rates from 03/10/2016 to 29/10/2021 against the US dollar (USD). All datasets were collected from the Wall Street Journal Database (https://www.wsj.com/). The dataset contains two of the original predictor variables for daily exchange rates: Sri Lanka and Japan, whereas the response variable is the daily exchange rate of China. These datasets were split into 70% for the training model and 30% for testing and assessing their performance criteria.

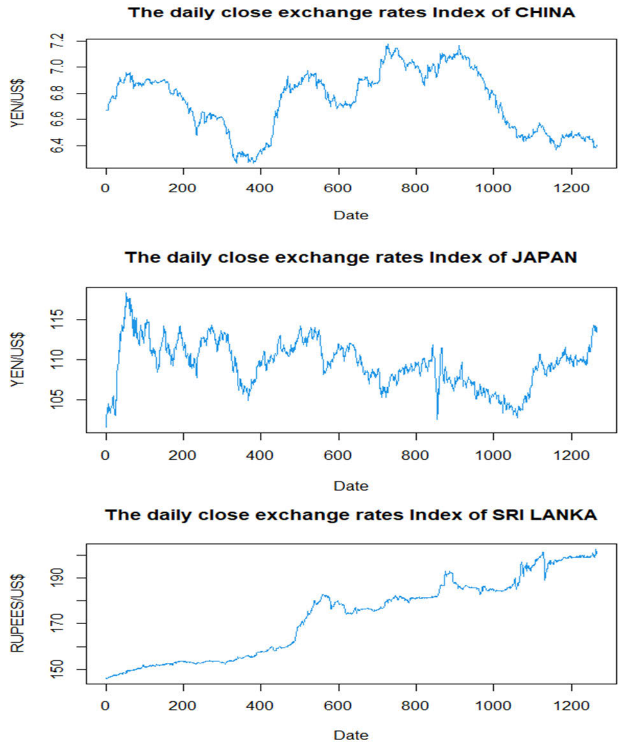

FIGURE 4. The daily close exchange rates index are plotted over time.

Figure 4 illustrates a graphical depiction of the original daily close exchange rates for Japan and Sri Lanka as predictor variables and China as the response variable. The predictor variables and the response variable neither show any constant value over time nor fluctuate around the zero lines. which indicates that the signals are non-stationary and non-linear. Figure 5 shows the decomposition results of the original predictors in Sri Lanka and Japan via EMD. The Japan signal was decomposed into eight IMFs and one residue

TABLE 6. Comparison of different methods for simulation study.

τ	Methods	λ_{min}	λ_{1se}	SSR	VS
0.25	EMD-QRR	0.45019	1.42162	633.61	All
	EMD-QRL	0.04929	0.09887	626.23	13
	EMD-QREnet	0.02037	0.12899	615.14	12
	QRR	0.05217	0.38439	639.14	-
	QRL	0.00522	0.04609	636.86	-
	QREnet	0.05217	0.38439	639.14	-
0.50	EMD-QRR	0.05302	1.06045	345.23	All
	EMD-QRL	0.0143	0.03160	334.11	9
	EMD-QREnet	0.01589	0.05332	321.23	6
	QRR	0.56231	0.56231	386.33	-
	QRL	0.05623	0.05623	390.06	-
	QREnet	0.56231	0.56231	386.33	-
0.75	EMD-QRR	0.07635	1.52692	587.20	All
	EMD-QRL	0.01202	0.15269	576.75	13
	EMD-QREnet	0.01432	0.21813	577.48	13
	QRR	1.08442	1.11774	627.94	-
	QRL	0.11177	0.11177	590.29	-
	QREnet	1.08442	1.11774	627.94	-

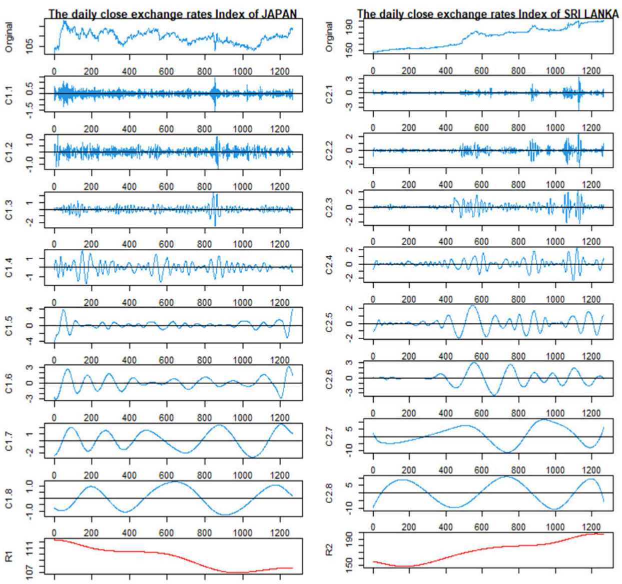

FIGURE 5. EMD decomposition results of Sri Lanka and Japan signals.

component. The Sri Lanka signal was decomposed into eight IMFs and one residual component.

Table 5 presents the descriptive statistics of the response variable is the daily exchange rate of China and the original predictor variables for the daily exchange rates of Sri Lanka and Japan. The results revealed that the mean daily exchange rate of China, Sri Lanka, and Japan are 6.742283, 109.4827, and 173.1182, respectively. Furthermore, the skewnesses have a negative sign, which indicates that the distribution of variables is skewed to the left.

Figure 6 shows the plots of the 10-fold CV of the EMD-EnetQR method to select the optimal k at quantile (0.25, 0.50, 0.75). The y-axis represents the mean square error (MSE), whereas the x-axis represents the log The upper horizontal

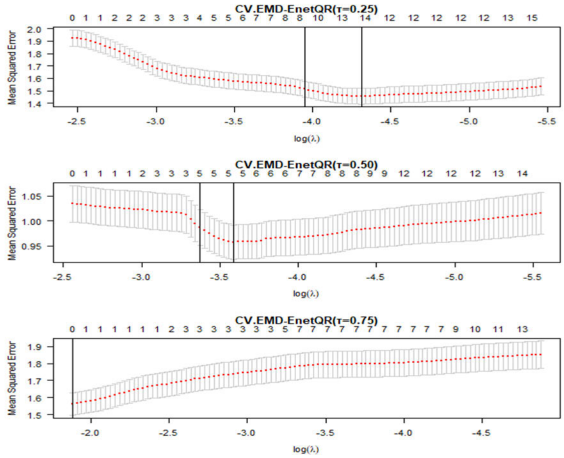

FIGURE 6. 10-fold cross-validation estimation of the MSE as the Log (k) for the proposed method at τ = (0.25, 0.50, 0.75).

line represents the number of non-zero coefficients chosen at the log() value. The location of the point chosen at minimum MSE (minM) is indicated by the first vertical dotted line from the left, while the location of the point chosen at minimum MSE using the one-standard-error (1se) criterion is indicated by the second vertical line. The CV plot shows that increasing leads to a reduction in the number of non-zero coefficients entering the final model.

Table 6 presents the optimal values of the tuning parameter λ in the models obtained via 10-fold cross-validation. In addition, as shown in Table 6, the EMD-EnetQR method selects the number of variables that are not equal to zero closer to the real number than other methods, while the EMD-RQR methods are invalid for variable selection. Overall, the EMD-EnetQR method outperforms the other models in terms of SSR error, which has the smallest residual sum of squares (RSS) of the other methods used.

Table 7 displays the prediction accuracy performance criteria using RMSE, MAE, MASE, and MAPE to compare the proposed method with the existing methods. The results show that the proposed EMD-EnetQR method has the smallest value in terms of RMSE, MAE, and MASE at the quantile level (0.25, 0.50). At the same time, the EMD-LQR method has the smallest value in terms of MASE, MAE, and MAPE for 0.75 quartiles. Nevertheless, our proposed method outperformed its competitors in terms of prediction accuracy and robustness.

The introduction of EMD significantly improved the prediction accuracy of the EnetQR model. This can be seen in Table 7, where without EMD, the lowest MAE, RMSE, MASE, and MAPE of the daily close exchange rates were all obtained by the EnetQR method is 0.8430, 1.0083, 9.1642, and 1.211876% respectively. With EMD, the lowest MAE, RMSE, MASE and MAPE of the daily close exchange rates obtained by the EMD-EnetQR method are 0.7217, 0.9194, 7.9967 and 1.0326%, respectively. The prediction accuracy increased after EMD was considered.

TABLE 7. Simulation results: Quantile: Mean performance criteria.

τ	Methods	EMD-QRR	EMD-QRL	EMD-QREnet	QRR	QRL	QREnet
0.25	MAE	0.54243	0.51395	**0.51164**	0.82492	0.78561	0.78763
	RMSE	0.67519	0.65819	**0.65164**	0.98280	0.94556	0.94778
	MASE	1.15196	1.09147	**1.08657**	1.81918	1.73249	1.73694
	MAPE (%)	1.83896	**1.71213**	1.72484	2.82555	2.85122	2.82859
	MAE	0.50056	**0.43389**	0.43526	0.59058	0.54897	0.54488
	RMSE	0.57972	**0.51323**	0.51369	0.73759	0.68749	0.68323
0.50	MASE	1.20336	**1.04308**	1.04637	1.34257	1.24798	1.23869
	MAPE (%)	1.20282	1.19183	**1.18173**	1.29603	1.58371	1.58856
	MAE	0.614039	0.60977	**0.60941**	0.73102	0.73479	0.72969
	RMSE	0.7480522	0.74595	**0.73949**	0.93483	0.92530	0.92522
0.75	MASE	1.406701	1.39691	**1.39610**	1.64272	1.65121	1.63974
	MAPE (%)	1.673794	1.96231	**1.86966**	1.91658	1.94015	1.84042

TABLE 8. Correlation Matrix between the decomposition components.

Variance Inflation Factor (VIF)																		
	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	R_1	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}	C_{28}	R_2
	1	1.02	1.02	1.02	1.07	1.04	1.94	4.2	**17.69**	1.03	1.04	1.04	1.07	1.24	1.18	**15.65**	**18.79**	**15.82**
Interpredictor Correlation Matrix																		
	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	R_1	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}	C_{28}	R_2
C_{11}	1	0.02	-0.01	-0.03	0.0	-0.02	0.01	-0.01	0.01	0.03	-0.01	-0.01	-0.01	-0.01	-0.01	0	-0.01	-0.01
C_{12}		1	0.09	-0.02	-0.01	0.01	0.03	-0.01	-0.03	0.02	0.09	-0.03	-0.03	-0.01	-0.01	0.02	-0.01	0.03
C_{13}			1	0.03	-0.01	0.01	-0.01	0.01	0.01	-0.01	0.09	0.01	-0.03	-0.03	-0.01	-0.01	0.03	-0.01
C_{14}				1	0.02	0.02	-0.02	-0.01	0.01	-0.01	-0.01	0.02	0.13	-0.02	0.0	-0.02	0.01	-0.03
C_{15}					1	0.01	0.07	-0.01	-0.08	-0.01	0.02	0.01	0.03	0.15	-0.01	0.01	-0.01	0.01
C_{16}						1	0.12	0.02	-0.08	-0.01	0.01	0.01	-0.01	-0.03	0.04	0.04	0.03	0.08
C_{17}							1	-0.12	-0.03	0.0	0.0	0.0	-0.01	0.0	-0.01	0.09	0.08	0.03
C_{18}								1	0.05	0.02	-0.01	-0.01	0.01	0.09	0.0	**-0.51**	**0.59**	0.19
R_1									1	0.01	-0.01	-0.01	0.0	-0.01	0.01	-0.26	0.09	**-0.89**
C_{21}										1	-0.1	-0.12	-0.06	-0.03	-0.02	-0.01	0.01	0.0
C_{22}											1	0.1	-0.02	-0.02	-0.01	-0.01	0.01	0.0
C_{23}												1	0.09	-0.04	-0.03	0.02	-0.03	0.01
C_{24}													1	0.17	0.08	-0.01	0.02	-0.01
C_{25}														1	0.35	0.05	-0.01	0.03
C_{26}															1	0.1	-0.06	-0.02
C_{27}																1	**-0.89**	0.09
C_{28}																	1	-0.02
R_2																		1

Table 8 presents the VIF test and correlation matrix to check for multicollinearity among all decomposition (IMFs and residual) components. From Table 8, the correlation matrix of the predictor variables shows a significant correlation between decomposition components. The results indicate the presence of high correlations among some decomposition components, such as (C_{18}/C_{27}, C_{18}/C_{28}, R_1/R_2, C_{27}/C_{27}). In addition, The VIF test in Table 8 reveals the presence of multicollinearity among all the decomposition components, where the VIF values of some of the decomposition components are more extensive than 10 (VIF>10), which indicates that multicollinearity exists among the decomposition components.

IV. CONCLUSION

In this study, we employed the elastic net penalized quantile regression based on the EMD method to determine the influence of the decomposition components of the original time series predictor variables on the response variable and robust parameter estimation to enhance the prediction accuracy of the model selection. The EMD method decomposes non-stationary and non-linear time-series data into a finite set of decomposition components and a residual component. EnetQR was used to study the effect of the decomposition components on the response variable and tackle the multicollinearity problem among the decomposition components to ensure the accuracy and reliability of the fitting model.

To illustrate its strength, we discuss the techniques of EMD-EnetQR modelling and conduct simulations and real applications for daily close exchange rates. The simulation and numerical results show that the EMD-EnetQR method outperforms EMD-RQR, EMD-LQR, RQR, LQR, and EnetQR methods, where the proposed EMD-EnetQR method provided high prediction accuracy and produced a consistent model by selecting the decomposition components that exhibit the strongest effect on the response variable.

ACKNOWLEDGMENT

The authors wish to acknowledge computing facilities support from Universiti Sains Malaysia, Sebha University, Jerash University, and the Faculty of Computing and Informatics, Universiti Malaysia Sabah.

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