IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 6 February 2023, accepted 23 February 2023, date of publication 6 March 2023, date of current version 15 March 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3253512

RESEARCH ARTICLE

Analysis of Chaotic Maps for Global Optimization and a Hybrid Chaotic Pattern Search Algorithm for Optimizing the Reliability of a Bank

G. SANDHYA RANI[1], **SARADA JAYAN**[1], **AND BILAL ALATAS**[2]
[1]Department of Mathematics, Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Bengaluru 560035, India
[2]Department of Software Engineering, Firat University, 23119 Elaziğ, Turkey

Corresponding author: G. Sandhya Rani (sandhya.reddyj@gmail.com)

This work was supported by the Amrita Vishwa Vidyapeetham.

ABSTRACT Optimization is an imperative feature in almost all fields of Engineering, Economics, and Sciences. Due to the advent of high-end computers and the gradual increase in the complexity of optimization problems, algorithms for numerical optimization have been developed. Numerous existing numerical optimization algorithms suffer from premature convergence, poor local/global search abilities, and high computational complexity. A chaotic optimization algorithm and a chaotic map could help overcome most of these setbacks. This paper offers a detailed study and analysis of five chaotic maps used for global Optimization, namely Chebyshev, Cubic, ICMIC, Neuron, and Sine maps. This work also proposes a pioneering global optimization method, Hybrid Chaotic Pattern Search Algorithm (HCPSA), for finding the global minimum for multivariable unconstrained optimization problems. Numerical results over 12 benchmark functions and comparative results (comparison of accuracy and computational time) with some popular algorithms evidence the effectiveness of the proposed algorithm for higher dimensional non-linear functions. The efficient usage of chaotic maps has helped reduce the computational time to evaluate the optimum for higher dimensional non-linear functions. To showcase the use of HCPSA in a real-world problem, we have taken the problem of analyzing financial ratios for predicting bankruptcy. Banks predict bankruptcy from the start of their businesses to determine their financial stability. In this work, we initially perform Logistic Regression (LR) on the data obtained from the banks to get the reliability function with financial ratios as decision variables. After this, the function is maximized using HCPSA and a Chebyshev map. This methodology is beneficial for decision-makers within a bank to maximize the reliability of the financial ratios and, most essential, to protect the bank from disasters. Comparative results of reliability prediction using HCPSA and PSO and a non-parametric statistical test proves that the proposed algorithm is better in terms of accuracy.

INDEX TERMS Global optimization, multi-variable optimization, chaotic maps, pattern search, reliability, financial ratios, bankruptcy, logistic regression.

I. INTRODUCTION

Optimization techniques have received massive attention recently due to the advancement in computer technologies, especially the advent of high-speed processors and the accessibility to user-friendly software [1], [2], [3], [4], [5], [6], [7], [8], [9]. The classical optimization algorithms, in general, need to be more flexible to accommodate all optimization problems as they do not offer a general approach to solving problems with different types of variables (integer or real), objectives (linear/non-linear), and constraints (constrained or unconstrained) at the same time. Literature gives details of deterministic optimization algorithms and non-deterministic/stochastic optimization algorithms. This classification is based on using randomness in the initial values taken. Further, there are various stochastic computational

The associate editor coordinating the review of this manuscript and approving it for publication was Easter Selvan Suviseshamuthu.

search algorithms, including biology-based, chemical-based, physics-based, music based, swarm-based, etc., as given in [10], [11], [12], [13], [14], [15], and [16].

Most of these optimization algorithms could successfully obtain a local optimum solution. They were popular, too, in problems where the optimum was sought in an interval that mostly had a single optimum. However, many such algorithms botched for problems that specifically demanded a global solution. Chaotic Optimization Algorithms (COAs) that use chaotic numbers (numbers generated using chaotic maps) in place of random numbers can easily escape the local minima than the classic stochastic optimization algorithms [13], [17], [18], [19], [20], [21], [22], [23], [24], [25]. Pseudo-randomness, regularity, sensitivity, and ergodicity are the properties of chaotic numbers that help the COAs avoid the searching process getting trapped into local optima, reduce the computational complexity, and sustain the convergence criteria. These properties make COAs appropriate for global optimization problems.

Chaotic sequences that are used in COAs are generated using different chaotic maps. In terms of global search capabilities and optimization efficiency, these chaotic sequences from different maps have different probability distributions and search speeds. The chaotic behavior of a map can be studied in detail using its Lyapunov exponent, histogram, probability distribution, etc. This paper is initially devoted to a detailed analysis of five chaotic maps- the Chebyshev map, Cubic map, ICMIC map, Neuron map, and Sine map. Later these maps are used in a global optimization algorithm proposed in this work to search for an optimum for a set of higher dimensional benchmark functions.

The speed at which a chaos optimization algorithm takes the search to a neighborhood of the global optimum is very fast. However, once it reaches the neighborhood, it takes more computational effort to obtain the global optimum solution by examining several points [21]. This search leads to additional CPU time, thus increasing the computational cost. Considering this fact, we propose an optimization algorithm, a hybrid COA that has the benefit of the accuracy of the global solution and is computationally less expensive.

The proposed algorithm in this paper uses chaotic maps in one stage to reach the neighborhood of the global minimum and a unique pattern search in another stage to reach the global minimum.

As a practical application of the proposed algorithm, we considered the problem of maximizing a bank's reliability function, which is a function of the bank's financial ratios. The solution to this problem could help prevent the banks from getting bankrupt.

Banks and firms' bankruptcy is detrimental to any country's economy. Altman [40] made the first Bankruptcy predictions. Numerous methods have been developed to extend machine learning to soft computing to forecast bankruptcy in recent years. Recently, some exciting results have been published regarding bankruptcy forecasts. The bankruptcy

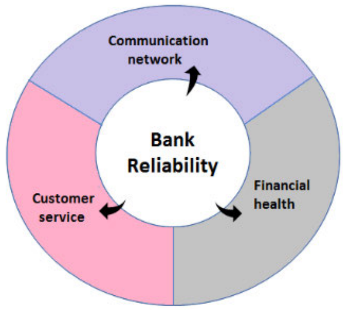


FIGURE 1. Doughnut chart for the reliability of a bank.

TABLE 1. Contributions and Research gaps in few related works.

Ref. No.	Contributions	Research gaps
[1-9, 29, 39]	These papers introduce a few algorithms for unconstrained non-linear optimization problems. Some of these algorithms work well for lower dimensional and convex problems.	Most of these algorithms fail when the objective is: non-convex, higher dimensional. A global solution is not always obtained as the algorithm gets trapped in a local solution.
[17-21, 34-38]	These papers explain some of the chaotic optimization algorithms.	A complete analysis of the maps concerning fixed points, Lyapunov exponents, etc., is missing.
[26]	Logistic Regression along with Particle Swarm Optimization is used to optimize the reliability of a bank.	The methodology fails to give a global solution for some of the banking variables (ratios) while maximizing the reliability since PSO sometimes gets trapped in a local solution.
[29]	Hooke Jeeves pattern search algorithm is described that helps in finding a solution for higher dimensional optimization problems.	The algorithm fails to obtain a global solution.
[38]	An algorithm by name CGSS is introduced in the paper. Computational results for benchmark functions are given along with a comparative study to demonstrate that the results using CGSS are better than many other heuristic algorithms.	This paper uses only two commonly used chaotic maps- Logistic and Chebyshev. CGSS fails to find a global optimum for the Rosen Brock function (F_5). Analysis of which map is better for which benchmark function is not present.

prediction on unbalanced datasets has been documented in [27]. A convolutional neural network was used on the imaged financial ratios data to forecast bankruptcy [28]. In order to define a bank's reliability, three connotations are made in [26]: (i) the reliability of its service levels, (ii) the reliability of its network, and (iii) the financial health of the bank, as demonstrated by financial ratios, such as solvency ratios, liquidity ratios, and profitability ratios (Figure1).

The first aspect is handled by a practical and holistic implementation of customer relationship management based on analytics. Second, the bank's communication network facilitates transactions in the shortest possible time by applying optimized algorithms. A bank's liquidity, capital adequacy, efficiency, solvency, profitability ratios, etc., are measured in the third facet. This paper concerns the third facet, where very little research has been conducted.

A detailed literature survey on few related works is captured in table1.

Section two gives preliminary information needed to understand the paper, which includes an understanding of the Lyapunov Exponent and its role in fixing the parameters of chaotic maps, and details regarding fixed points that help to use chaotic maps for global optimization problems effectively. This section also describes the Pattern Search Algorithm given by Hooke and Jeeves [29] and the Logistic regression methodology [30]. Section three provides a detailed analytical study of five chaotic maps, namely Chebyshev, Cubic, ICMIC, Neuron, and Sine maps. This section also explains how the selection of parameters for each of the five maps based on the LE and scatter diagram can be done. The list of fixed points for each map is also given here to avoid runtime errors while executing the algorithm. The proposed hybrid algorithm (HCPSA) is available in section four. Section five provides the results obtained using HCPSA on twelve benchmark functions [31] (eight higher dimensional - up to 1000 dimensions and four two-dimensional functions) using each of the five chaotic maps. Suitable maps for some benchmark functions and a discussion on reasons for the strength of the proposed algorithm are also given here.

As an application to HCPSA, we have considered the problem of finding financial ratios for banks to obtain maximum reliability using the data from three banks. Section six describes how Logistic regression (LR) is used to obtain the parameters of the reliability function, which is then maximized using HCPSA with the Chebyshev map. The details and results of this application are also discussed here, and finally, the paper is concluded with a mention of future scope in section seven.

The contributions of this paper are:

(1) A detailed analysis of five chaotic maps that shall help researchers to select which map to be used and how to use them to solve optimization problems.
(2) A general algorithm, HCPSA to solve non-linear unconstrained optimization problems.
(3) A technique to find the best parameters that maximize the reliability of a bank using HCPSA.

II. PRELIMINARIES

A. LYAPUNOV EXPONENT AND SCATTER DIAGRAM

Lyapunov Exponent (LE) [32], [33] is the average rate of exponential divergence or convergence of nearby trajectories. It is a quantitative measure of the predictability and sensitivity of a system to changes in the initial conditions. Thus, the LE helps in the study of the chaotic nature of a sequence produced by a map $x_{i+1} = f(x_i)$. A negative LE value indicates that adjacent points may finally move very close and merge into only one point. In such a case, the sequence will not have chaotic behavior. If LE is greater than zero, then even if the initial distance between two trajectories is minimal, the points may eventually separate, causing chaos in the sequence. As LE increases, the chaotic nature of the sequence increases.

The Lyapunov exponent, commonly denoted as λ is evaluated numerically using the formula:

$$\lambda = \frac{1}{n}\sum\nolimits_{i=1}^{n} ln\left|f'(x_i)\right| \tag{1}$$

where $x_i, i = 1, 2, \ldots, n$ is the sequence and $f'(x_i)$ is the first derivative of the map $f(x)$ at x_i. In the next section, the Lyapunov exponent λ is numerically evaluated and presented in a graph for each of the five maps discussed in this paper.

A scatter diagram shows the distribution of points obtained by a sequence. Scatter diagrams for the five chaotic maps with different parameter values are also shown in section three, which gives a clear indication that the distribution of points is better when the Lyapunov exponent is high. Based on the LE value and scatter diagram for each map, the best parameter value is decided for the parameters of each map that will make the chaotic map best suitable for the proposed COA.

B. FIXED POINT

A fixed point [32] is a point that gets repeated in the sequence. i.e., points where$x_i = f(x_i)$. If a fixed point appears in a sequence, the sequence will converge to that point, and chaos will vanish, thus making the map inappropriate for global optimization problems. Hence it is essential to ensure that the starting value for the optimization algorithm is not a fixed point, and also, if in between the sequence, a fixed point appears, the value needs to be changed to $x_i + (0.01)\,r$ where $r\epsilon(0, 1)$.

Geometrically the fixed point of a map $x_{i+1} = f(x_i)$ is the point where the line $y = x$ intersects the curve$y = f(x)$. Numerically, it can be evaluated by solving the equation $x = f(x)$. The fixed points and their geometrical representation for five maps are also specified in section III.

C. PATTERN SEARCH ALGORITHM

Pattern Search Algorithm is a direct search method that searches for a minimum of a non-linear unconstrained multivariable function. Among the direct search algorithms for unconstrained non-linear optimization problems, Hooke and Jeeve's pattern search method [29] stands out as a simple and effective optimization method, especially for higher-dimension functions.

1) ALGORITHM

Step 1: Choose a starting point $\boldsymbol{X_0} = [x_1, x_2, \ldots x_n]^T$, variable increments Δx_i (for i = 1,2, ...,n), coordinate directions $\boldsymbol{u_i}(for\, i = 1, 2, \ldots, n)$ and a termination parameter, ε. Set $k = 0$. Set$i = 1$, $\boldsymbol{Y_{k,i-1}} = \boldsymbol{X_k}$, compute $f_k = f(\boldsymbol{Y_{k,i-1}})$ and go to Step 2.

Step 2: (Exploratory move with base point$Y_{k,i-1}$):
Evaluate: $f = f\left(\boldsymbol{Y_{k,i-1}}\right), f^+ = f\left(\boldsymbol{Y_{k,i-1}} + (\Delta x_i)\boldsymbol{u_i}\right)$ and $f^- = f\left(\boldsymbol{Y_{k,i-1}} - (\Delta x_i)\boldsymbol{u_i}\right)$

$$Y_{k,i} = \left\{\begin{array}{ll} \boldsymbol{Y_{k,i-1}} + (\Delta x_i)\boldsymbol{u_i}, & \text{if} f^+ < \min\left(f, f^-\right) \\ \boldsymbol{Y_{k,i-1}} - (\Delta x_i)\boldsymbol{u_i}, & \text{if} f^- < \min\left(f^+, f\right) \\ \boldsymbol{Y_{k,i-1}}, & \text{if} f < \min\left(f^+, f^-\right) \end{array}\right\}$$

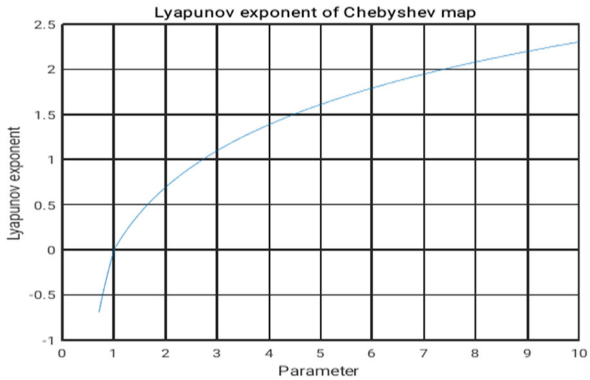


FIGURE 2. **Lyapunov exponent of the Chebyshev map.**

Set $i=i+1$, evaluate f, f^+ and f^- for the base point $Y_{k,i}$and repeat this procedure till we get$Y_{k,n}$.If $f(Y_{k,n}) < f(Y_{k,0})$ then $X_{k+1} = Y_{k,n}$ and go to step 4, Else go to step 3.

Step 3: Is $\|(\Delta x_1, \Delta x_2, \ldots, \Delta x_n)\| < \varepsilon$ If yes, terminate and mention X_{kC1} is the minimum; Else set $\Delta x_i = \frac{\Delta x_i}{2}$ for i =1,2,...,n and go to step 2.

Step 4: (Pattern move) Set k=k+1. Perform the pattern move: $Y_{k+1,0} = 2X_k - X_{k-1}$

Step 5: Perform another exploratory move using $Y_{k+1,0}$ as the base point (Step 2). Let the result be X_{k+1}.

Step 6: Is $f(X_{k+1}) < f(X_k)$)? If yes, go to Step 4; Else go to Step3.

D. LOGISTIC REGRESSION

A logistic regression model is the most standard method for estimating the probability of a binary reply based on the values of various features. Numerous fields, including the social sciences, economics, and health sciences, utilize it extensively. The Logistic regression model is given in eqn. (2).

$$y = p(x_1, x_2, \ldots x_N) = \frac{1}{1 + e^{-(b_0 + b_1 x_1 + \ldots + b_N x_N)}} \tag{2}$$

where $\mathbf{X}= (x_1, x_2, \ldots x_N) \in R^N$, $y \in \{0, 1\}$ and $b_0, b_1, \ldots b_N$ are the parameters of the logistic regression. These parameters can be obtained using maximum likelihood estimation and the Newton-Raphson method. Logistic regression [30] is the most straightforward, relatively accurate, and non-parametric discriminative classifier.

III. CHAOTIC MAPS

Chaotic maps are the ones that generate pseudo-random chaotic numbers [34], [35], [36], [37], [38], [39]. Though literature provides the definitions for various chaotic maps, a complete analysis of the maps concerning fixed points, Lyapunov exponents, etc., needs to be included. In this section, we will provide an analysis of five maps that helps in making them suitable for global optimization.

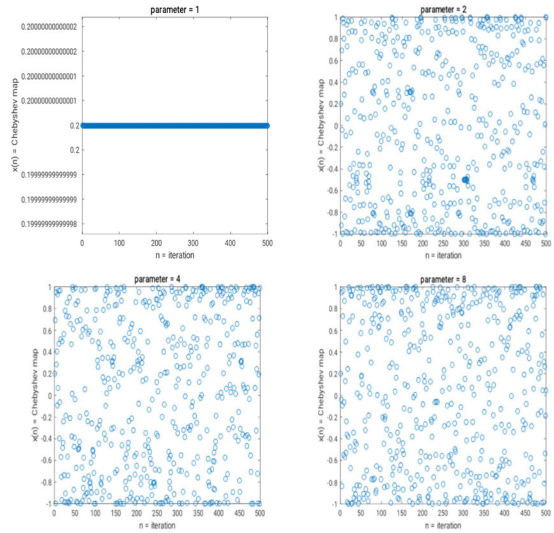


FIGURE 3. **Scatter diagrams of the Chebyshev map with different parameter values.**

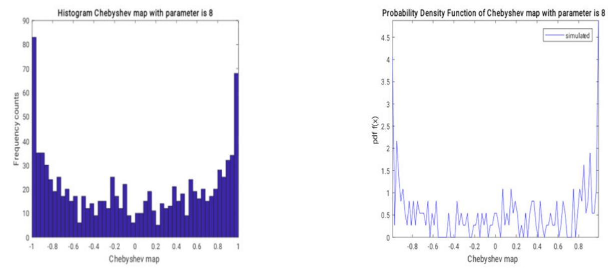


FIGURE 4. **Histogram and pdf of Chebyshev map for α= 8.**

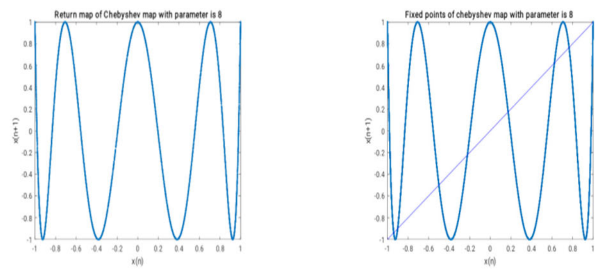


FIGURE 5. **Return map and fixed points of the Chebyshev map for α= 8.**

A. CHEBYSHEV MAP

Chebyshev map, defined in eqn.3, is a chaotic map that generates chaotic sequences in the interval $(-1, 1)$ for any parameter value α.

$$x_{n+1} = cos\left(\alpha cos^{-1}(x_n)\right) \tag{3}$$

The best value for α is chosen based on the Lyapunov Exponent graph.

The fixed points of the Chebyshev map are -0.9397, -0.9010, -0.5, -0.2225, 0.1736, 0.6235, 0.7660 *and* 1. The Chebyshev map's return map and fixed points are shown in Figure 5.

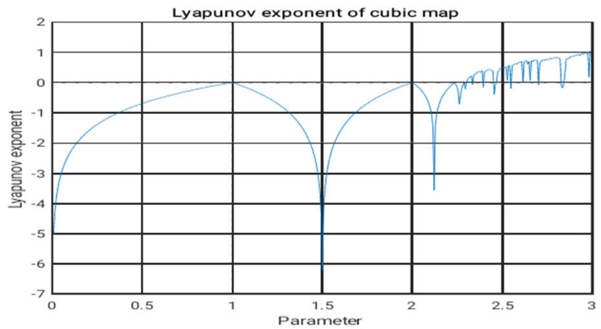


FIGURE 6. Lyapunov Exponent graph of the Cubic map.

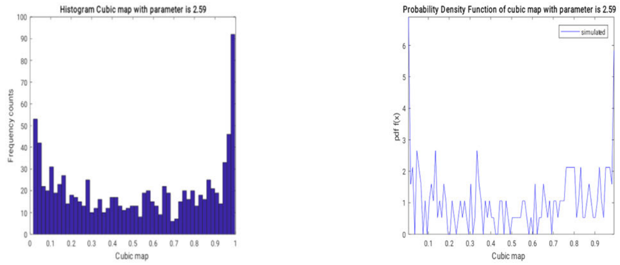


FIGURE 7. Histogram and pdf of the Cubic map with parameter β =2.59.

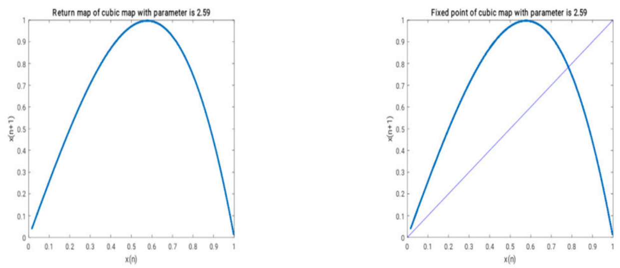


FIGURE 8. Return map and fixed point of the Cubic map with parameter is 2.59.

The Lyapunov Exponent λ for different parameter values is calculated numerically and is shown in Figure 2.

The scatter diagrams for various α values are shown in Figure 3. For α =1 eqn.3 converges to a line; hence, no chaos is shown in Figure 3. The simulation results show that $\alpha = 8$ can obtain the best chaotic sequence.

The distribution of the histogram and the probability density function of the chaotic sequences generated by the Chebyshev map in 500 iterations is depicted in Figure 4.

B. CUBIC MAP

A Cubic map is a chaotic map that generates sequences in the interval (0, 1). This map is defined as follows with the parameter β.

$$x_{n+1} = \beta x_n \left(1 - x_n^2\right), x_n \epsilon (0, 1) \tag{4}$$

The best value for β is chosen based on the Lyapunov Exponent graph which is given in Figure 6 for different values of β. The scatter diagrams are given in Figure 9. The graphs of scatter diagrams show that $\beta = 2.59$ can obtain best

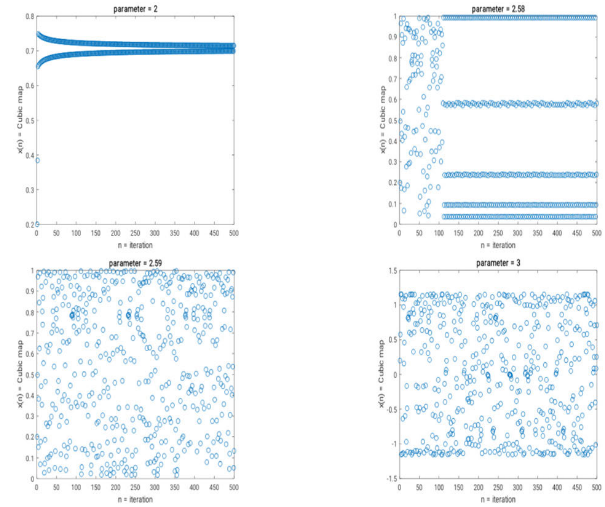


FIGURE 9. Scatter diagrams of the Cubic map for different parameters.

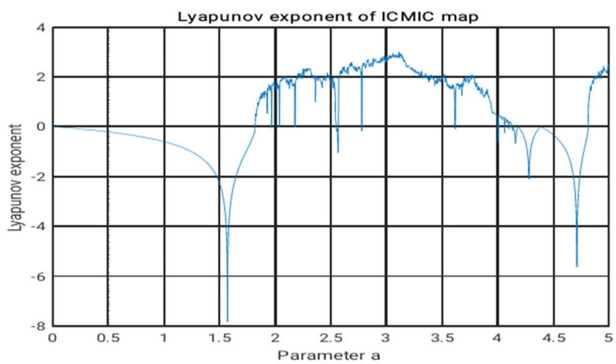


FIGURE 10. Lyapunov exponent of the ICMIC map.

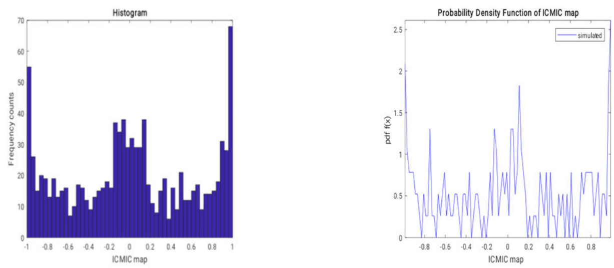


FIGURE 11. Histogram and pdf of the ICMIC map for *a*= 3.

chaotic sequence in the interval(0, 1). The distribution of the histogram and the probability density function(pdf) of the chaotic sequences generated by the cubic map in 500 iterations is shown in Figure 7.

The fixed point is obtained 0.7835 for the cubic map. The return map and the fixed point of the cubic map are visible in Figure 8.

C. ICMIC (ITERATIVE CHAOTIC MAP WITH INFINITE COLLAPSES) MAP

The one-dimensional ICMIC map is defined as:

$$x_{n+1} = sin\left(\frac{a}{x_n}\right), \quad a \in (0, \infty), \; x_n \in (-1, 1) \tag{5}$$

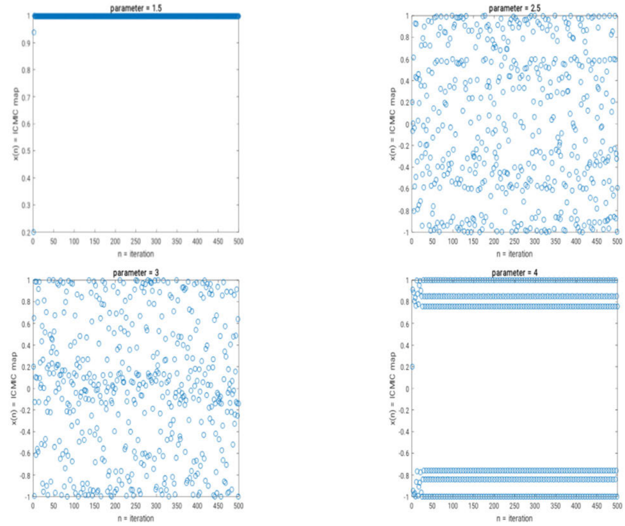


FIGURE 12. Scatter diagrams of the ICMIC map with different parameters.

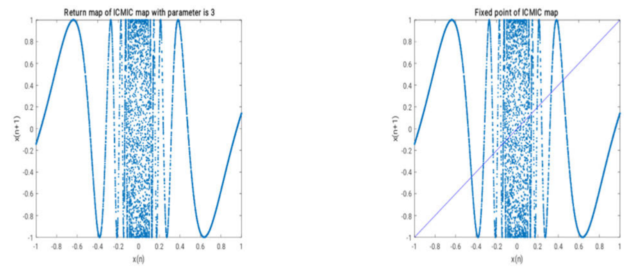


FIGURE 13. Return map and fixed point of the ICMIC map for *a*= 3.

The best value for the parameter a is chosen based on the Lyapunov exponent shown in Figure 10. It can be seen from the graph that, for some values of a Lyapunov exponent is negative hence the ICMIC map cannot generate a proper chaotic sequence at those points.

Figure 11 shows the distribution of chaotic numbers with a value as 1.5, 2.5, 3 and 4. For $a = 2.5$, 3 Lyapunov exponent is positive hence the ICMIC map can generate a chaotic sequence.

The fixed points of the ICMIC map are: ± 0.0952, ± 0.1065, ± 0.1188, ± 0.1373, ± 0.1578, ± 0.1934, ± 0.2343, ± 0.3301, ± 0.4448 and there are infinite number of fixed points in $(-0.1, 0.1)$. The histogram and the probability density function of the ICMIC map are shown in Figure 12 and the return map is shown in Figure 13.

D. NEURON MAP

The neuron map [39] is defined in eqn.6 as a chaotic map. With the attenuation factor α and proportionality factor β as the parameters.

$$x_{n+1} = \alpha - 2\, tanh\,(\beta)\, e^{-3x_n^2} \tag{6}$$

According to [37], the proportionality factor β should be taken as five. The Lyapunov exponent and scatter diagrams for the neuron map with $\beta = 5$, are given in Figures 14 and 15.

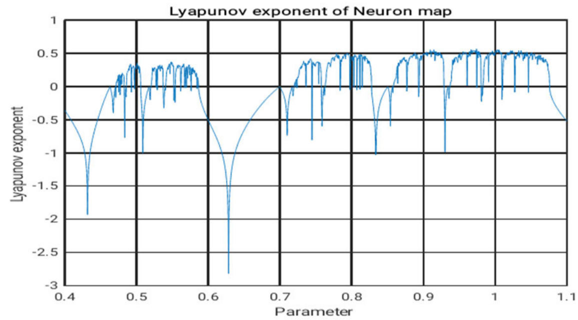


FIGURE 14. Lyapunov exponent of the Neuron map.

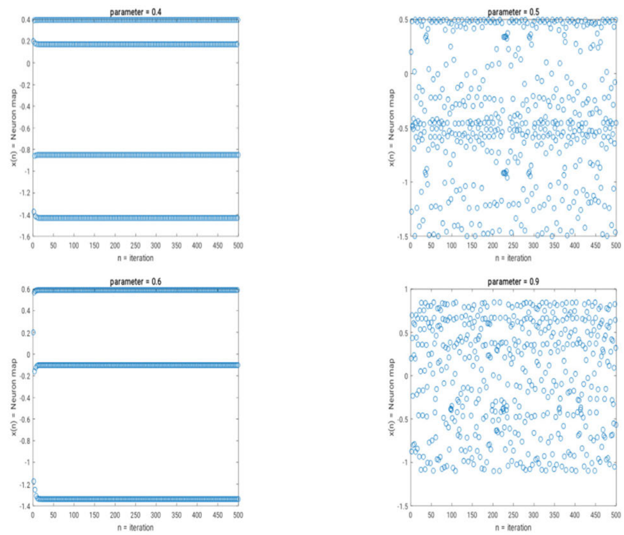


FIGURE 15. Scatter diagram for the Neuron map with α =0.4,0.5,0.6,0.7,0.8 and 0.9.

The best values for parameter α is chosen based on them. For$\alpha = 0.5$, λ is positive and the neuron map produces a chaotic sequence in $(-1.5, 0.5)$ (Figure 15), thus reducing the sequence to:

$$x_{n+1} = 0.5 - 2\, tanh\,(5)\, e^{-3x_n^2} \tag{7}$$

For $\alpha = 0.8$, λ is positive, the neuron map produces a chaotic sequence in $(-1.2, 0.8)$ thus making the sequence as

$$x_{n+1} = 0.8 - 2\, tanh\,(5)\, e^{-3x_n^2} \tag{8}$$

For $\alpha = 0.9$, λ is positive, the neuron map produces a chaotic sequence in $(-1.1, 0.9)$ (Figure 15) and the sequence will then be

$$x_{n+1} = 0.9 - 2\, tanh\,(5)\, e^{-3x_n^2} \tag{9}$$

For 500 iterations, Figure 15 shows scatter diagrams of neuron maps for $\alpha = 0.4, 0.5, 0.6$ *and* 0.9.

For $\alpha = 0.4, 0.6$ the Lyapunov exponent λ is negative and hence no chaos can be seen.

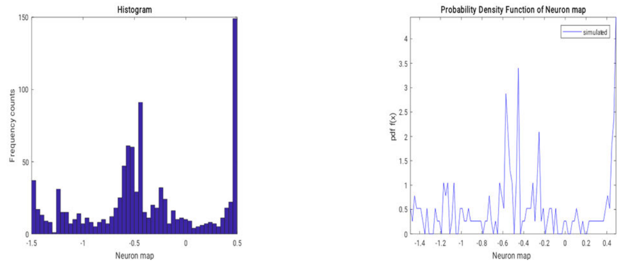


FIGURE 16. Neuron map at α= 0.5with histogram and pdf.

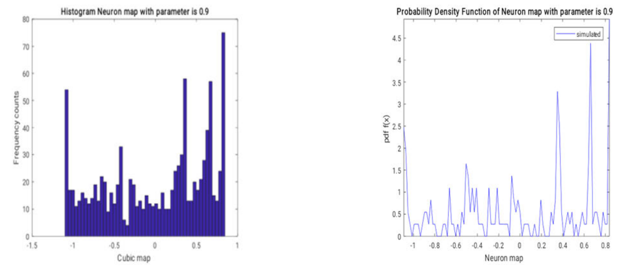


FIGURE 17. Neuron map at α= 0.9with histogram and pdf.

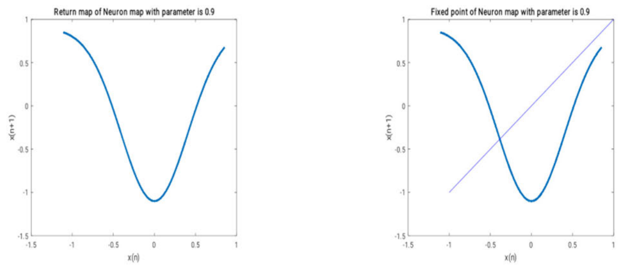


FIGURE 18. Return map and fixed point of the Neuron map for α = 0.9.

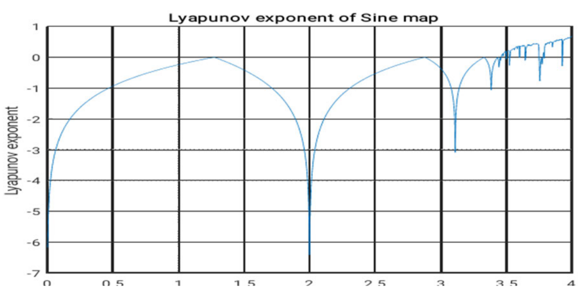


FIGURE 19. Lyapunov exponent of the sine map.

Figure 16 shows the histogram and pdfs of the neuron map for $\alpha = 0.5$ and $\beta = 5$. In Figure 17, the histogram, and pdfs of the neuron maps at $\alpha = 0.9$ and $\beta = 5$ are shown.

Return map of neuron map in eqn. (9) (for α =0.9) is given in Figure 18. The fixed point of the Neuron map in eqn. (9) is - 0.3842 (Figure 18).

E. SINE MAP

The sine map is a unimodal chaotic map defined by eqn. (10) with parameter α.

$$x_{n+1} = \frac{\alpha}{4} sin(\pi x_n), \quad x_n \in (0, 1) \tag{10}$$

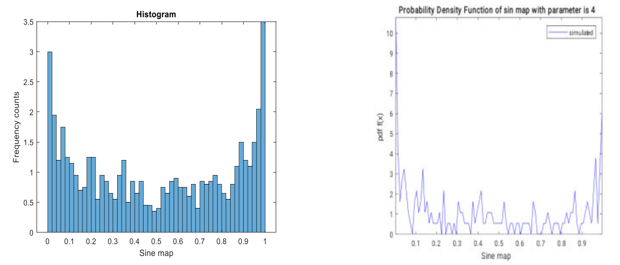


FIGURE 20. Histogram and pdf of the sine map with α= 4.

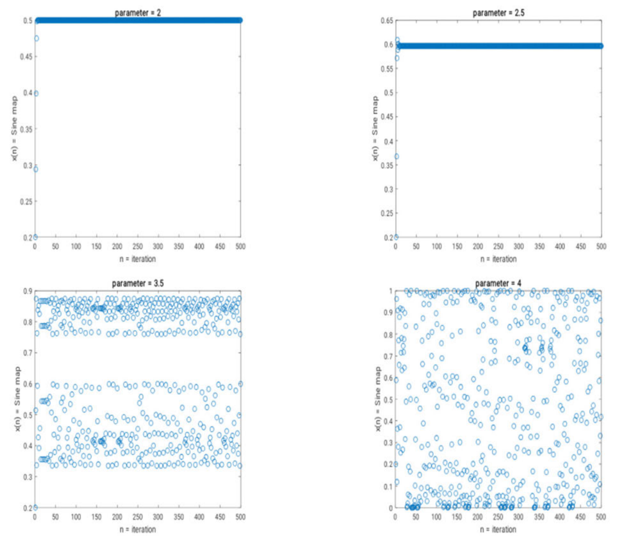


FIGURE 21. Scatter diagrams of the sine map with different parameters.

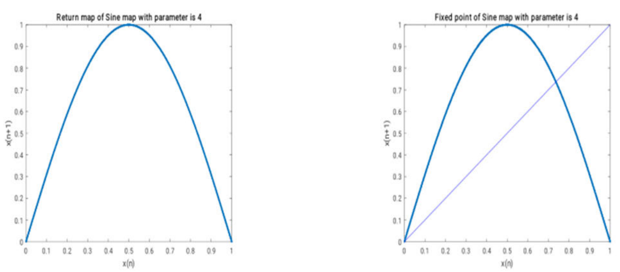


FIGURE 22. Return map and fixed point of the sine map.

The Lyapunov exponent (λ) graph for the same is shown in Figure 19. The histogram and pdfs for $\alpha = 4$ is given in Figure 20. For $\alpha = 4$, λ is positive and the sine map produces a chaotic sequence in (0, 1) (Figure 2).

Fixed point of the sine map with $\alpha = 4$is numerically evaluated and found to be 0.7365(Figure 22).

IV. HYBRID CHAOTIC PATTERN SEARCH ALGORITHM (HCPSA)

Chaotic search always helps to get to the neighbourhood of the global solution in a few iterations, and pattern search is a well-known technique to obtain the optimum for higher dimensional problems. Here we propose the Hybrid Chaotic Pattern Search (HCPSA) algorithm that ensures the global solution for higher dimensional optimization problems at a minimal computational cost. The algorithm has two stages and is as follows:

Stage I:

Step1: Input the objective function f, the lower bound L and upper bound U. Choose the number of iterations for stage I as N. Set $k = 1$, initialize the dimension and choose $fmin = +\infty$.

Step2: Choose a random number as the initial point for the chaotic variable c^k. This initial value is same for all dimension to reduce the functional evaluations and computational time. The initial value should be selected from the defined interval for each chaotic map and should not be any of the fixed points of the respective chaotic maps.

Step3: Map the chaotic variable c^k to the variable $X_i^{(k)} i = 1, 2, \ldots, n$ by using one of the following equations.

$$X_i^{(k)} = \left(\frac{1.5U + 0.5L}{2}\right) + \left(\frac{U - L}{2}\right) c^k, i = 1, 2, \ldots, n \quad (11)$$

This equation maps variables in (-1.5, 0.5) to the optimization variables in (L, U). Transformation (13) is used for the Neuron map.

Similarly, the transformation in eqn. (12) is suitable for the Logistic and the Cubic map.

$$X_i^{(k)} = L + (U - L)\, c^k, i = 1, 2, \ldots, n \quad (12)$$

For the Chebyshev map and ICMIC map, we use transformation in eqn.(13).

$$X_i^{(k)} = \left(\frac{U + L}{2}\right) + \left(\frac{U - L}{2}\right) c^k i = 1, 2, \ldots, n \quad (13)$$

Step4: Compute the function value $f\left(\mathrm{X}^{(\boldsymbol{k})}\right) = f\left(X_1^{(k)}, X_2^{(k)}, \ldots, X_n^{(k)}\right)$.

If $f\left(\boldsymbol{X}^{(\boldsymbol{k})}\right) < fmin$ then $fmin = f(\boldsymbol{X}^{(\boldsymbol{k})})$ and the optimal solution is $\boldsymbol{Xmin} = \boldsymbol{X}^{(\boldsymbol{k})}$

Step 5: Generate the next chaotic variable c^{k+1} by applying the chaotic map.

Step 6: If $k \leq N$ then $k = k + 1$, go to step 3, else go to The output of stage I is ***Xmin***, which is in the neighbourhood of global optima

Stage II:

Step7: Choose ***Xmin*** as the initial point of stage II.

Step8: Use Pattern Search Algorithm to find the global optimum.

V. RESULTS AND DISCUSSIONS

The MATLAB R2017b and Python3 simulation of the algorithms with which we got all the results presented in this work was carried out in an Intel i5 6GB machine.

HCPSA is tested on twelve benchmark functions [31], eight of them (F1 to F8) up to 1000 dimensions and four standard two-dimensional functions (F9 to F12) using all the five chaotic maps discussed in section III.

TABLE 2. Details of the benchmark functions.

	Search Space	Nature	Global Minimum point	Global Minimum function value
F_1	$[-500, 500]^n$	U	$(1, 2\ldots, n)$	0
F_2	$[-5.12, 5.12]^n$	M	(0, 0…,0)	0
F_3	$[-600, 600]^n$	M	(0, 0…,0)	0
F_4	$[-1, 1]^n$	U	(0, 0…,0)	-1
F_5	$[-2.04, 2.04]^n$	M	(1, 1…,1)	0
F_6	$[-100, 100]^n$	M	(0, 0…,0)	0
F_7	$[-1, 1]^n$	M	(0, 0…,0)	0
F_8	$[-500, 500]^n$	M	$\pm(\sqrt{1}, \sqrt{2}, \ldots\ldots, \sqrt{n})$	0
F_9	$[-2\pi, 2\pi]$	M	(4.70104, 3.15294) and (-1.58214, -3.13024)	-106.764537
F_{10}	[-500, 500]	M	(3,2), (-2.8051, 3.2831), (-3.7793, -3.2831) and (3.5844, -1.8481)	0
F_{11}	[-10,10]	U	(1,1)	0
F_{12}	[-10,10]	M	$(\pm\frac{\pi}{2}, 0)$	−10.872300

$$F_1 = \text{ShiftedSphere} = \sum_{i=1}^{n} (x_i - i)^2$$

$$F_2 = \text{Rastrigin} = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$$

$$F_3 = Griewank = \left(\frac{1}{4000}\sum_{i=1}^{n} x_i^2\right) - \left(\prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right)\right) + 1$$

$$F_4 = \text{Exponential} = -\,exp\left(-0.5\sum_{i=1}^{n} x_i^2\right)$$

$$F_5 = \text{Rosenbrock} = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$$

$$F_6 = \text{Salamon} = 1 - \cos\left(2\pi\sqrt{\sum_{i=1}^{n} x_i^2}\right) + 0.1\sqrt{\sum_{i=1}^{n} x_i^2}$$

$$F_7 = \text{Csendes} = \sum_{i=1}^{n} x_i^6 (2 + \sin\left(\frac{1}{x_i}\right))$$

$$F_8 = \text{Qing} = \sum_{i=0}^{n} (x_i^2 - i)^2$$

$$F_9 = \text{Bird} = \sin(x_1)\, e^{(1-\cos(x_2))^2} + \cos(x_2)\, e^{(1-\sin(x_1))^2} + (x_1 - x_2)^2$$

$$F_{10} = \text{Himmelblau} = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

$$F_{11} = \text{Cube} = 100(x_2 - x_1^3)^2 + (1 - x_1)^2$$

$$F_{12} = \text{TesttubeHolder} = -4(\sin(x1)\cos(x2)\, e^{\cos\left(\frac{x_1^2 + x_2^2}{200}\right)})$$

The bounds for these standard functions [31], the modality of these functions-Unimodal (U) or Multimodal(M), the global minimum point, and the global minimum value of these functions are given in Table 2. Figure 23 presents a

TABLE 3. Best, Worst and Standard deviation of the minimum function value obtained for the functions using the Chebyshev, the Cubic and the ICMIC (B: Best; W: Worst; SD=Standard deviation).

	Chebyshev			Cubic			ICMIC		
	B	W	SD	B	W	SD	B	W	SD
F_1	0	0	0	0	0	0	0	0	0
F_2	0	0	0	0	0	0	0	0	0
F_3	0	0	0	0	0	0	0	0	0
F_4	-1	-1	1.41E-15	-1	-1	8.00E-16	-1	-1	7.10E-16
F_5	0	6.74E-28	4.77E-28	1.54E-29	5.64E-28	3.88E-28	3.08E-30	2.77E-27	1.96E-27
F_6	0	0	0	0	0	0	0	0	0
F_7	0	0	0	0	0	0	0	0	0
F_8	1.20E-28	1.20E-28	0	1.20E-28	1.20E-28	0	1.20E-28	1.20E-28	0

TABLE 4. Best, Worst and SD of the minimum function value obtained for the functions using the Neuron and the Sine maps (B : Best; W : Worst; SD : Standard deviation).

	Neuron map			Sine map		
	B	W	SD	B	W	SD
F_1	0	0	0	0	0	0
F_2	0	0	0	0	0	0
F_3	0	0	0	0	0	0
F_4	-1	-1	1.39E-15	-1	-1	1.41E-15
F_5	1.27E-29	2.33E-27	1.63E-27	0	4.56E-28	3.22E-28
F_6	0	0	0	0	0	0
F_7	0	0	0	0	0	0
F_8	1.20E-28	1.20E-28	0	1.20E-28	1.20E-28	0

TABLE 5. Minimum point and minimum value obtained in stage 1 and stage 2 of the proposed algorithm (HCPSA) using neuron map.

	Stage1		Stage2	
	Minimum Point	Minimum value	Global Minimum Point	Global Minimum value
F_1	(1.2388, 1.2388)	0.6363	(1,2)	0
F_2	(0.0043, 0.0043)	0.0077	[-9.54e-10, -9.54e-10]	0
F_3	(0.0918, 0.0918)	0.0063	[9.56e-09, 9.56e-09]	0
F_4	(0.0011,0.0011)	-0.9999	[3.25e-09, 7.72e-04]	-1
F_5	(0.9960, 0.9960)	0.0015	[1.000,1.000]	4.93e-32
F_6	(-0.0549, -0.0549)	0.1245	[0,0]	0
F_7	(-7.8046e-04, -7.8046e-04)	1.1128e-18	[-6.52e-55, -6.52e-55]	0
F_8	(-0.4951, -0.4951)	3.6495	[-1,-1.41]	1.97e-31
F_9	(-2.3552, -2.3552)	-26.07	[-1.58, -3.13]	-106.7645
F_{10}	(2.6195, 2.6195)	8.4640	(3, 2)	0
F_{11}	(0.8180, 0.8180)	7.3564	[1.00,1.00]	1.7749e-30
F_{12}	(0.7789, 0.7789)	-5.4360	[1.57,6.23e-09]	-10.87230

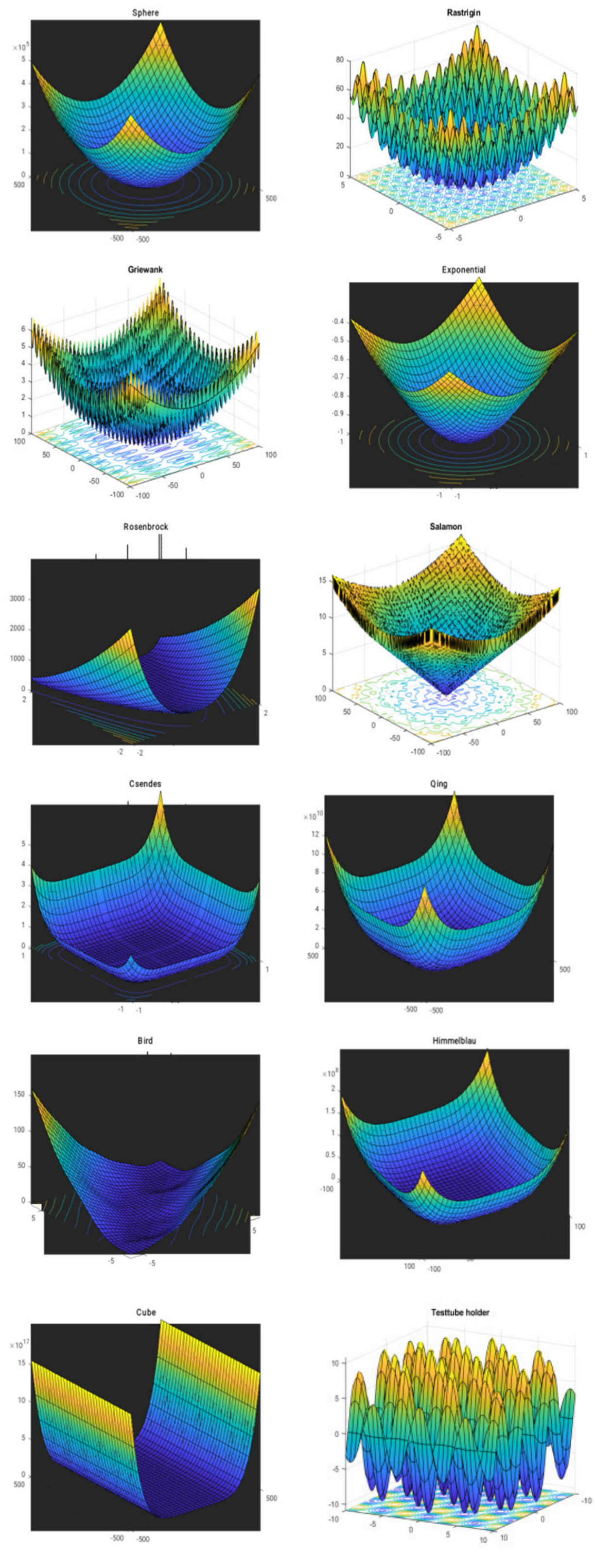


FIGURE 23. Plots of two-dimensional benchmark functions.

two-dimensional plot of each of these 12 test functions to illustrate the modality and nature of the minimum points.

The code for the proposed algorithm with 1000 iterations was run 50 times to find the global minimum of eight 30-dimensional benchmark functions (F1 to F8). The best, worst, and standard deviation of the minimum function value thus obtained by the five chaotic maps are revealed in tables 3 and 4. The average time to execute the code was less than two seconds for each function.

TABLE 6. **Comparison of proposed algorithm with six other algorithms for finding minimum of five 30-dimensional functions.**

Functions	Algorithms	Best	Worst	Mean	Std.
F_1	Harmony Search	1.8E+00	1.2E+01	5.41E+00	2.79E+00
	Improved harmony search	1.9	2.0E+01	6.68E+00	3.42E+00
	Global-best harmony search	3.8E+08	8.41E-02	1.23E-02	1.97E-02
	Self-adaptive global-best harmony search	3.06E-10	3.64E-09	1.62E-09	8.69E-10
	Novel global harmony search	5.15E-16	5.83E-13	3.69E-14	1.05E-13
	Chaotic Golden section search Algorithm	0	0	0	0
	HCPSA(Proposed)	**0**	**0**	**0**	**0**
F_2	Harmony Search	1.1E+00	1.1E+01	4.6E+00	2.1E+00
	Improved harmony search	2.3E+00	1.2E+01	7.0E+00	2.2E+00
	Global-best harmony search	7.63E-07	1.76E-01	2.79E-02	4.81E-02
	Self-adaptive global-best harmony search	9.11E-08	9.95E-01	1.68E-01	3.76E-01
	Novel global harmony search	2.30E-14	9.9E+01	1.3E+01	3.4E+01
	Chaotic Golden section search Algorithm	0	0	0	0
	HCPSA(Proposed)	**0**	**0**	**0**	**0**
F_3	Harmony Search	1.0E+00	1.1E+00	1.0E+00	2.78E-02
	Improved harmony search	1.0E+00	1.1E+00	1.0E+00	2.21E-02
	Global-best harmony search	1.82E-08	2.99E-01	3.50E-02	7.30E-02
	Self-adaptive global-best harmony search	4.37E-04	1.62E-01	6.54E-02	4.00E-02
	Novel global harmony search	0	2.61E-01	0	5.85E-02
	Chaotic Golden section search Algorithm	0	0	0	0
	HCPSA(Proposed)	**0**	**0**	**0**	**0**
F_5	Harmony Search	5.2E+02	1.2E+04	3.1E+03	2.9E+03
	Improved harmony search	9.9E+02	1.3E+04	4.7E+03	2.9E+03
	Global-best harmony search	1.06E-01	7.5E+03	3.4E+02	1.3E+03
	Self-adaptive global-best harmony search	1.1E-01	8.7E+03	5.8E+02	1.8E+03
	Novel global harmony search	7.3E-04	7.9E+03	7.2E+02	1.7E+03
	Chaotic Golden section search Algorithm	2.8E+01	2.8E+01	2.8E+01	3.64E-15
	HCPSA(Proposed)	**2.2E-30**	**1.2E-28**	**6.21E-29**	**8.45E-29**

TABLE 7. **Test results on eight benchmark functions of 100, 500 and 1000 dimensions (M: Minimum value, E: Execution time (s), N: No. of function calls.**

	D=100			D=500			D=1000		
	M	E	N	M	E	N	M	E	N
F_1	0	3.56	127919	0	29.7	695435	0	96.409	1506265
F_2	0	3.25	114185	0	24.2	558575	0	86.347	1123582
F_3	0	2.56	115697	1.88 e-15	41.5	564185	6e-16	151.40	1117629
F_4	-1	2.30	120672	-1	23.3	562575	-1	82.937	1107568
F_5	3.40 e-28	3.18	262810	7.56 e-28	77.7	1928188	1.e-28	629.92	8737268
F_6	0	2.52	122622	0	25.4	586609	0	92.440	1216623
F_7	0	3.25	220556	0	199.6	1101760	0	765.24	2177741
F_8	4.32 e-27	2.443	125959	4.86 e-25	27.4	639392	3e-24	97.600	1330676

In order to showcase the effectiveness of the hybrid search with the chaotic search in the first stage, the minimum obtained in both stages is separately tabulated while searching for the global minimum of all 12 benchmark functions, using the proposed algorithm in 1000 iterations. Table 5 presents these results. The computational strength of the algorithm is established with the test results on eight benchmark functions of 100, 500, and 1000 dimensions provided in table 7. The minimum function value obtained, the execution time, as well as the number of function calls while running the algorithm with a neuron map are given in this table.

Chaotic golden section search (CGSS) is presented in the paper [38] in which the comparative results of CGSS with

TABLE 8. **Comparison of accuracy and the computational time while finding minimum of 2D Rosen Brock function (F_5).**

	Computational time (seconds)	Global minimum value
Genetic Algorithm	45.027	0.00012
Differential Evolution	35.38	2.7e-05
HCPSA	0.8	2.6e-29

TABLE 9. **Best parameters of five maps.**

Map	Best parameters
Chebyshev map	$\alpha = 8$
Cubic map	$\beta = 2.59$
ICMIC map	$\alpha = 3$
Neuron map	$\beta = 5, \alpha = 0.5 \; or \; 0.9$
Sine map	$\alpha = 4$

TABLE 10. **Result for the Rosen brock function for different number of iterations.**

For 30 D	For 500 iterations	For 1000 iterations	For 2000 iterations
F_5	8.45E-15	8.45E-21	8.45E-30

Harmony search, Improved harmony search, Global-best harmony search, Self-adaptive global-best harmony search, Novel global harmony search with CGSS are presented to demonstrate the superiority of CGSS over other algorithms.

In Table 6, we provide the comparative results of our proposed algorithm with the algorithms in [38] and the CGSS algorithm. Most algorithms fail to get the global minimum of the Rosen brock function among the twelve benchmark functions. The computational time and the global minimum point obtained for using the most popular algorithms- Genetic algorithm (with population size 10000) and Differential evolution (with population size 10000) are compared with HCPSA in Table 8. The parameters of different chaotic maps are given in Table 9.

To showcase how the results of HCPSA vary with respect to the number of iterations, we are showing in table 10 the optimum value obtained for the Rosen brock function in 500,1000 and 2000 iterations of HCPSA with the Neuron map.

A. RESULT ANALYSIS

Tables 3 and 4 show that the proposed algorithm is one of the best to find the global minimum for all types of functions using any of the mentioned five chaotic sequences. Among the different chaotic maps, we had worked on, we found that these five maps provided sequences that work exceptionally well for global optimization problems. Among these five maps, the Cubic map and ICMIC were found to be best for the exponential function, F4. Sine map and neuron map would be suggested for functions with trigonometric terms.

The proposed algorithm uses the same chaotic sequence for all dimensions to reduce the function evaluations and, thus, the computational time. Due to this reason, the stage

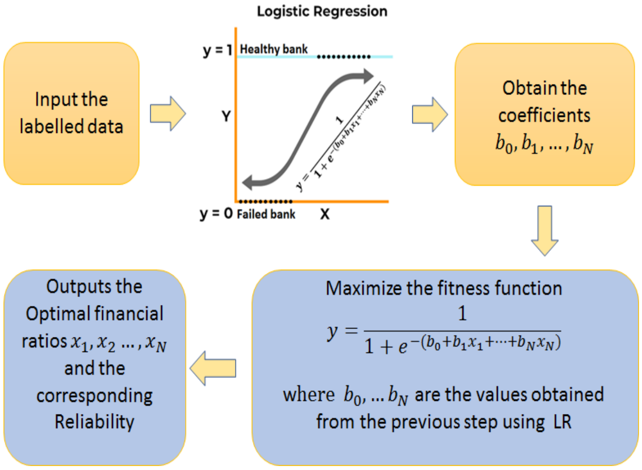


FIGURE 24. **Basic framework of the proposed methodology.**

one search significantly impacts the search (Table 5), unlike other chaotic/stochastic searches. Stage two search refines the search and gives the exact optimum point.

Due to the usage of pattern search in stage 2 of HCPSA, it is the most appropriate algorithm for higher dimensional functions. Unlike the other heuristic algorithms, HCPSA can preserve the accuracy of even 1000-dimensional functions (Table 5).

From Table 6, it can be noted that our proposed algorithm (HCPSA) works better than all six algorithms. Particularly for the Rosen Brock function (F5), even CGSS fails to find a global optimum, but HCPSA could find it in lesser computations. The results of two-dimensional F5, given in Table 8, prove that the computational time for HCPSA is much lesser than the most widely used GA and DE. For higher dimensional F5, GA and DE could not even reach the neighborhood of the global solution.

VI. RELIABILITY OF A BANK

A dramatic increase in of data reported on enterprises' bankruptcy has occurred nationally and internationally in recent years. Many large companies, as well as entry-level firms, have been filing for bankruptcy.

Financial services do not meet the definition of reliability in its strict sense. Thus, to bridge this gap, the financial ratios are used in this paper to measure a bank's reliability.

Here in this section, we analyse the financial ratios for predicting bankruptcy and thus determine a bank's financial stability.

The financial ratios of a bank are under control when a bank has high reliability. Applying Logistic Regression (LR) to the previous data from the bank, the reliability function is obtained, which is then maximized using the proposed HCPSA. The steps for this are as follows:

Step 1: Estimate the parameters of the reliability function based on a logistic regression model over the entire data set and save the results (predictive analytics).

Step 2: The reliability function obtained in stage one with financial ratios as decision variables is maximized using HCPSA (explained in section IV). In each continuous decision variable, the minimum and maximum values for each financial ratio in the data are used as lower and upper bounds, respectively.

The outcome of the proposed algorithm is the optimum financial ratios that will help the bank to remain healthy by having maximum reliability.

The basic framework of the proposed methodology is given in Figure 24.

A. DESCRIPTION OF THE DATA SET

Different banks have different financial ratios. Data from banks in Spanish, the United Kingdom, and Turkey were studied for the study. It was found that Spanish banks [26] suffered the most severe crisis between 1977 and 1985, which resulted in a loss of twelve billion dollars. This data set holds 66 banks and their nine financial ratios. Among these 66 banks, 37 went bankrupt, and 29 stayed strong. The nine financial ratios taken in Spanish bank's data are

(i) Current Assets/Total Assets
(ii) Current Assets-Cash/Total Assets
(iii) Current Assets/Loans
(iv) Reserves/Loans
(v) Net Income/Total Assets
(vi) Net Income/Total Equity Capital
(vii) Net Income/Loans
(viii) Cost of Sales/Sales
(ix) Cash Flow/Loans.

The UK banks' data set [26], [42] consists of 60 samples, 30 in good health and 30 in financial trouble. The 12 financial ratios in the UK banks data set are

(i) Sales
(ii) Profit before tax/capital employed (%)
(iii) Funds flow/Total liabilities
(iv) (Current liabilities + longterm debit)/total assets
(y) Current liabilities/total assets
(vi) Current assets/current liabilities
(vii) Current assets-stock/Current liabilities
(viii) Current assets-current liabilities/total assets
(ix) LAG (Number of days between account year end and the date of annual report
(x) Age
(xi) changed auditor or not in previous three years
(xii) Has company auditor or not

The Turkish Banks [26], [41], [42] Association posts 49 financial ratios on its website, the ratios and the data relating to the ratios were attained from the location http://www.tbb.org.tr/english/bulten/yillik/2000/ratios.xls. After applying a univariate analysis of variance (ANOVA) test to those 49 ratios, 12 ratios were calculated and are given in [41]. The data for these 12 ratios were used for our experiment. According to thisdata set, 22 banks are healthy, and 18 went bankrupt. The 12 financial ratios in the Turkish banks data set are

(i) Interest Expenses/Average Profitable Assets
(ii) Interest Expenses/Average Non-Profitable Assets

TABLE 11. The financial ratios for the highest reliability obtained.

Data set	N	Stage1-Logistic Regression	Stage2-Chaotic Chebyshev Pattern search algorithm		
		Parameters of Logistic Regression	Optimal financial ratios	Reliability by HCPSA	Reliability by PSO
Spanish	9	$b_0 = 1.91382716$, $b_1 = 0.08344141$, $b_2 = -0.10437725$, $b_3 = 0.01294489$, $b_4 = 0.16643533$, $b_5 = 0.03124887$, $b_6 = 0.70735078$, $b_7 = 0.03303706$, $b_8 = -0.93331710$, $b_9 = 0.12311964$	X= [0.7671, 0.0883009521484375, 0.807, 0.299, 0.0226, 0.8109, 0.0278, 0.34780146484375, 0.183]	90.9%	89.5%
UK	12	$b_0 = -0.1419174800$, $b_1 = -0.0000100560$, $b_2 = -0.0275541492$, $b_3 = -0.0954533044$, $b_4 = -0.2471175740$, $b_5 = 0.1576719500$, $b_6 = -0.7797706210$, $b_7 = -0.6952260380$, $b_8 = -0.2455640520$, $b_9 = 0.0127544142$, $b_{10} = -0.0306457180$, $b_{11} = 0.2615038010$, $b_{12} = -0.9216310370$	$X = [2857, -37.3497, -0.328300, 3.53360, 1.48650, 0.497400, 0.284700, -0.747000, 421.000, 2,1,0]$	99.9%	92%
Turkish	12	$b_0 = 0.04950322$, $b_1 = -0.04283653$, $b_2 = 0.70069470$, $b_3 = -0.01569109$, $b_4 = -0.00764001$, $b_5 = 0.09983753$, $b_6 = 0.28806351$, $b_7 = -0.43970000$, $b_8 = 0.25428222$, $b_9 = -0.25383226$, $b_{10} = -0.32204017$, $b_{11} = 0.50599222$, $b_{12} = -0.57339864$	$X = [2.37964244, 16.65571138, 47.2343231, 122.79766497, -124.32272136, 25.65666827, -215.14912132, 14.61005686, 105.41581001, 29.94747963, 57.67638068, 6.67199326]$	100%	87%

(iii) (Share Holders' Equity+ Total Income)/ (Deposits + Non-Deposit Funds)
(iv) Interest Income/Interest Expenses
(v) *(Share Holders' Equity + Total Income)/Total Assets*
(vi) (Share Holders' Equity + Total Income)/ (Total Assets + Contingencies & Commitments)
(vii) Networking Capital/Total Assets
(viii) (Salary- and Employees' Benefits + Reserve for Retirement)/No. Of Personnel,
(ix) Liquid Assets/ (Deposits + Non-Deposit Funds)
(x) Interest Expenses/Total Expenses
(xi) LiquidAssets/Total Assets
(xii) Standard Capital Ratio.

The data from each of the banks were used to model a reliability function using logistic regression, which was then optimized using HCPSA with a Chebyshev map. The regression coefficients thus obtained and the optimum values for the financial ratios for each bank are tabulated in Table 11.

B. RESULT ANALYSIS

Logistic regression is a simple and reasonably accurate nonparametric classifier without user-defined parameters or hyperparameters. That is why our work considers logistic regression for the approximation of reliability. The comparative results of Table 11 give us another evidence for affirming the superiority of HCPSA over PSO.

HCPSA and PSO were run 100 times to obtain the reliability value (in percentage). The financial ratios for the highest reliability obtained are tabulated in Table 11.

Statistical analysis with hundred outputs (reliability values) of HCPSA and PSO was carried out to statistically ascertain that the reliability obtained using HCPSA is better than PSO. The test that was conducted was Mann-Whitney U Test, with

H_0: Accuracy of PSO $\geq$ Accuracy of HCPSA

H_1: Accuracy of PSO $<$ Accuracy of HCPSA

The statistical values obtained for the Spanish, UK, and Turkish banks are -6.069, -4.034, and -3.217. Since the statistic value in each case lies in the critical region, the null hypothesis is rejected, and the alternate hypothesis is accepted, thus giving us evidence for our claim that HCPSA gives better reliability than PSO.

If the banks keep the optimal financial ratios obtained using HCPSA as targets, they will be able to achieve high reliability and stay healthy.

VII. CONCLUSION AND FUTURE RESEARCH

The paper discusses five chaotic maps - Chebyshev, Cubic, ICMIC, Neuron, and Sine. The Lyapunov exponent value, a measure of the predictability, sensitivity, and, thus, the system's chaos, is evaluated numerically for all these five maps. The best parameter for the maps is then found based on the Lyapunov exponent. These parameter values can make the chaotic map suitable for chaotic optimization algorithms. The scatter diagram, histogram, probability density function, and return map for the chosen parameter value of each of the five maps are also illustrated in the paper. The fixed points for each map are found and specified in the paper, which could ascertain the global solution while using the maps in chaotic optimization algorithms.

A hybrid chaos pattern search algorithm is proposed that can be used to find the global minimum of a function using different chaotic maps. Chaotic variables are transformed as optimization variables in the algorithm to search for the global minimum in the desired interval. Due to the usage of the same chaotic sequence for all dimensions, the functional evaluations and computational time is drastically reduced, which makes the proposed algorithm computationally intense. To test the numerical performance of the proposed algorithm with different chaotic maps, twelve benchmark functions, some of which are convex and some non-convex, are tested in this paper. The simulation results show that the proposed algorithm is highly competent with good speed too. Also, the comparative results depict that its performance is better than many other popular algorithms.

The proposed algorithm is a general one. We can get different algorithms by selecting different chaotic maps in stage one. Using the Cubic map and the ICMIC map in stage one of HCPSA gave good results for functions with exponential terms. Similarly, the Sine map and the neuron map in stage one gave optimum results for functions with trigonometric terms. So, a suitable chaotic map needs to be chosen based on the optimization function to get the best results.

A sensitivity analysis is also done with respect to the number of iterations used in the algorithm.

Later in work, an advanced study of bankruptcy research is done by combining both predictive and prescriptive analytics. We employ logistic regression and the proposed HCPSA algorithm for maximizing bank reliability concerning the given ratios to define and estimate the bank's reliability with a given set of financial ratios. The proposed methodology is demonstrated to be effective on three well-known bank datasets. The study recommends the ideal healthy bank scenario, where maximum reliability can be assured if the prescribed financial ratios are targeted and achieved. Maximum reliability sometimes involves financial ratios reaching the upper or lower bounds, making the recommendation mathematically meaningful. Comparative results of reliability prediction using HCPSA and PSO are tabulated in the work along with a non-parametric statistical test to establish that the proposed algorithm is better in terms of accuracy.

A. LIMITATIONS AND FUTURE SCOPE

Though the proposed chaotic algorithm works well for higher dimensional functions, every chaotic map may not be suitable for every function that has to be optimized. Using the numerical results that we obtained, we are broadly giving a conclusion. However, more detailed research is needed to analyze which map to be used for which function.

Chaotic optimization algorithms for constrained optimization problems are studied very little. Also, currently, we are using one-dimensional chaotic maps only for solving two-dimensional optimization problems. Using two or higher-dimensional chaotic maps for solving higher-dimensional problems needs to be ventured.

The usage of chaotic optimization for facet two definitions of the reliability of banks is also a future scope.

ACKNOWLEDGMENT

The authors would like to thank the management of Amrita Vishwa Vidyapeetham for their continuous support and encouragement for research.

REFERENCES

[1] A. V. Zuylen and D. P. Williamson, "Deterministic algorithms for rank aggregation and other ranking and clustering problems," in *Proc. Int. Workshop Approximation Online Algorithms*. Berlin, Germany: Springer, Oct. 2007, pp. 260–273.

[2] S. A. Khan and S. Rehman, "Iterative non-deterministic algorithms in on-shore wind farm design: A brief survey," *Renew. Sustain. Energy Rev.*, vol. 19, pp. 370–384, Mar. 2013.

[3] G. S. Rani, S. Jayan, and K. V. Nagaraja, "An extension of golden section algorithm for n-variable functions with MATLAB code," *IOP Conf. Ser., Mater. Sci. Eng.*, vol. 577, no. 1, Nov. 2019, Art. no. 012175.

[4] K. Kavya, M. G. Dechamma, and B. S. Kumar, "Extraction of retinal blood vessel using artificial bee-colony optimization," *J. Theor. Appl. Inf. Technol.*, vol. 88, no. 3, pp. 535–540, 2016.

[5] M. L. Pai, K. S. Varsha, and R. Arya, "Application of artificial neural networks and genetic algorithm for the prediction of forest fire danger in Kerala," in *Proc. Int. Conf. Intell. Syst. Design Appl.* Cham, Switzerland: Springer, Dec. 2018, pp. 935–942.

[6] P. Joshy and P. Supriya, "Implementation of robotic path planning using ant colony optimization algorithm," in *Proc. Int. Conf. Inventive Comput. Technol. (ICICT)*, Aug. 2016, pp. 1–6.

[7] A. S. Kumar, G. Manikutty, R. R. Bhavani, and M. S. Couceiro, "Search and rescue operations using robotic Darwinian particle swarm optimization," in *Proc. Int. Conf. Adv. Comput., Commun. Informat. (ICACCI)*, Sep. 2017, pp. 1839–1843.

[8] K. Ganesan, K. Barathi, P. Chandrasekar, and D. Balaji, "Selective harmonic elimination of cascaded multilevel inverter using BAT algorithm," *Proc. Technol.*, vol. 21, pp. 651–657, Jan. 2015.

[9] T. Ananthapadmanabha, "Genetic algorithm based network reconfiguration in distribution systems with multiple DGs for time varying loads," *Proc. Technol.*, vol. 21, pp. 460–467, Jan. 2015.

[10] S. Akyol and B. Alatas, "Plant intelligence based metaheuristic optimization algorithms," *Artif. Intell. Rev.*, vol. 47, no. 4, pp. 417–462, Apr. 2017.

[11] B. Alatas and H. Bingol, "Comparative assessment of light-based intelligent search and optimization algorithms," *Light Eng.*, vol. 28, no. 6, pp. 51–59, Dec. 2020.

[12] B. Alatas and H. Bingol, "A physics based novel approach for travelling tournament problem: Optics inspired optimization," *Inf. Technol. Control*, vol. 48, pp. 373–388, Sep. 2019.

[13] H. Bingol and B. Alatas, "Chaos based optics inspired optimization algorithms as global solution search approach," *Chaos, Solitons Fractals*, vol. 141, Dec. 2020, Art. no. 110434.

[14] Y. Xue, B. Xue, and M. Zhang, "Self-adaptive particle swarm optimization for large-scale feature selection in classification," *ACM Trans. Knowl. Discovery Data*, vol. 13, no. 5, pp. 1–27, Oct. 2019.

[15] Y. Xue, Y. Wang, J. Liang, and A. Slowik, "A self-adaptive mutation neural architecture search algorithm based on blocks," *IEEE Comput. Intell. Mag.*, vol. 16, no. 3, pp. 67–78, Aug. 2021.

[16] Y. Xue, X. Cai, and F. Neri, "A multi-objective evolutionary algorithm with interval based initialization and self-adaptive crossover operator for large-scale feature selection in classification," *Appl. Soft Comput.*, vol. 127, Sep. 2022, Art. no. 109420.

[17] E. V. Altay and B. Alatas, "Bird swarm algorithms with chaotic mapping," *Artif. Intell. Rev.*, vol. 53, no. 2, pp. 1373–1414, 2019.

[18] H. Bingol and B. Alatas, "Chaotic league championship algorithms," *Arabian J. Sci. Eng.*, vol. 41, no. 12, pp. 5123–5147, Dec. 2016.

[19] B. Alatas, "Chaotic bee colony algorithms for global numerical optimization," *Expert Syst. Appl.*, vol. 37, no. 8, pp. 5682–5687, Feb. 2010.

[20] B. Alatas, "Chaotic harmony search algorithms," *Appl. Math. Comput.*, vol. 216, no. 9, pp. 2687–2699, Jul. 2010.

[21] D. Yang, G. Li, and G. Cheng, "On the efficiency of chaos optimization algorithms for global optimization," *Chaos, Solitons Fractals*, vol. 34, no. 4, pp. 1366–1375, Nov. 2007.

[22] A. A. Heidari, R. A. Abbaspour, and A. R. Jordehi, "An efficient chaotic water cycle algorithm for optimization tasks," *Neural Comput. Appl.*, vol. 28, no. 1, pp. 57–85, 2017.

[23] A. R. Jordehi, "A chaotic-based big bang–big crunch algorithm for solving global optimisation problems," *Neural Comput. Appl.*, vol. 25, no. 6, pp. 1329–1335, Nov. 2014.

[24] R. Jordehi, "Chaotic bat swarm optimisation (CBSO)," *Appl. Soft Comput.*, vol. 26, no. 1, pp. 523–530, Jan. 2015.

[25] A. R. Jordehi, "A chaotic artificial immune system optimisation algorithm for solving global continuous optimisation problems," *Neural Comput. Appl.*, vol. 26, no. 4, pp. 827–833, May 2015.

[26] V. Ravi and V. Madhav, "Optimizing the reliability of a bank with logistic regression and particle swarm optimization," in *Data Management, Analytics and Innovation*. Singapore: Springer, 2021, pp. 91–107.

[27] D. Veganzones and E. Séverin, "An investigation of bankruptcy prediction in imbalanced datasets," *Decis. Support Syst.*, vol. 112, pp. 111–124, Aug. 2018.

[28] T. Hosaka, "Bankruptcy prediction using imaged financial ratios and convolutional neural networks," *Expert Syst. Appl.*, vol. 117, pp. 287–299, Mar. 2019.

[29] R. Hooke and T. A. Jeeves, "Direct search solution of numerical and statistical problems," *J. ACM*, vol. 8, no. 2, pp. 212–229, 1961.

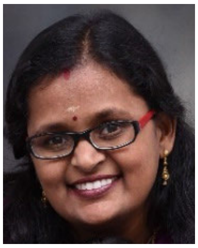
[30] D. G. Kleinbaum, K. Dietz, M. Gail, M. Klein, and M. Klein, *Logistic Regression*. New York, NY, USA: Springer-Verlag, 2002.

[31] M. Jamil and X.-S. Yang, "A literature survey of benchmark functions for global optimization problems," 2013, *arXiv:1308.4008*.

[32] J. M. T. Thompson, H. B. Stewart, and R. Turner, "Nonlinear dynamics and chaos," *Comput. Phys.*, vol. 4, no. 5, pp. 562–563, 1990.

[33] H. Kantz, "A robust method to estimate the maximal Lyapunov exponent of a time series," *Phys. Lett. A*, vol. 185, no. 1, pp. 77–87, Jan. 1994.
[34] J. Zhang, Y. Yang, and Q. Zhang, "The particle swarm optimization algorithm based on dynamic chaotic perturbations and its application to K-means," in *Proc. Int. Conf. Comput. Intell. Secur.*, vol. 1, Dec. 2009, pp. 282–286.
[35] S. Arora and P. Anand, "Chaotic grasshopper optimization algorithm for global optimization," *Neural Comput. Appl.*, vol. 31, no. 8, pp. 4385–4405, Aug. 2019.
[36] D. Yang, Z. Liu, and J. Zhou, "Chaos optimization algorithms based on chaotic maps with different probability distribution and search speed for global optimization," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 19, no. 4, pp. 1229–1246, 2014.
[37] J. Feng, J. Zhang, X. Zhu, and W. Lian, "A novel chaos optimization algorithm," *Multimedia Tools Appl.*, vol. 76, no. 16, pp. 17405–17436, Aug. 2017.
[38] J. A. Koupaei, S. M. M. Hosseini, and F. M. M. Ghaini, "A new optimization algorithm based on chaotic maps and golden section search method," *Eng. Appl. Artif. Intell.*, vol. 50, pp. 201–214, Apr. 2016.
[39] G. S. Rani and S. Jayan, "A global optimization algorithm for unconstrained non-linear optimization problems using chaotic neuron map and golden number," *Webology*, vol. 18, no. 5, pp. 1118–1136, Dec. 2021.
[40] E. I. Altman, "Financial ratios, discriminant analysis and the prediction of corporate bankruptcy," *J. Finance*, vol. 23, no. 4, pp. 589–609, Sep. 1968.
[41] S. Canbas, A. Cabuk, and S. B. Kilic, "Prediction of commercial bank failure via multivariate statistical analysis of financial structures: The Turkish case," *Eur. J. Oper. Res.*, vol. 166, no. 2, pp. 528–546, Oct. 2005.
[42] V. Arora and V. Ravi, "Data mining using advanced ant colony optimization algorithm and application to bankruptcy prediction," *Int. J. Inf. Syst. Social Change*, vol. 4, no. 3, pp. 33–56, 2013.

G. SANDHYA RANI received the M.Sc. degree in mathematics from Acharya Nagarjuna University and the M.Phil. degree in mathematics from Madurai Kamaraj University. She is currently pursuing the Ph.D. degree with Amrita Vishwa Vidyapeetham, Bengaluru Campus, India. She is a Data Scientist with Genpact, Bengaluru. Her research interests include global optimization, multivariable optimization, chaotic maps, machine learning, and deep learning.



SARADA JAYAN received the master's degree in mathematics from the Indian Institute of Technology Madras, Chennai, and the Ph.D. degree in mathematics from Amrita Vishwa Vidyapeetham, Bengaluru Campus. She is currently an Associate Professor with the Department of Mathematics, Amrita Vishwa Vidyapeetham, Bengaluru Campus. Her research interests include numerical analysis, optimization, linear algebra, and its applications to AI and finite element methods.

BILAL ALATAS received the B.S., M.S., and Ph.D. degrees from Firat University, Elaziğ, Turkey. He is currently the Head of the Department of Software Engineering, Firat University, where he works as a Professor of software engineering. He is also the Founder Head of the Department of Computer Engineering, Munzur University, and the Department of Software Engineering, Firat University. He has published more than 100 papers in many well-known international journals and proceedings of the refereed conferences. His research interests include artificial intelligence, data mining, social network analysis, meta-heuristic optimization, and machine learning. He has been an editor of 12 journals, four of which are indexed in SCI, and a reviewer of 60 SCI-indexed journals.