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RESEARCH ARTICLE

Networked Tracking Control Based on Internal Model Principle and Adaptive Event-Triggering Mechanism

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ABSTRACT In this paper, the non-static error tracking control issue for the networked control system is considered. First, the unstable modes of the external disturbance signal and the output signal of the reference system are established to form a new series system with the controlled objects. Then, an internal model compensation controller is presented to achieve non-static tracking error. Furthermore, an adaptive event-triggering mechanism is introduced as a communication method to improve network resource utilization. By formulating the tracking control problem of the networked control system as the stabilization problem of a time-varying time-delay system, a sufficient condition such that the closed-loop system is asymptotically stable and satisfies the H_∞ output tracking performance is derived. Finally, the simulation examples illustrate the feasibility and effectiveness of the proposed approach.


INDEX TERMS The internal model principle, H_∞ output tracking control, networked control systems, adaptive event-triggering mechanisms, time-delay systems.

I. INTRODUCTION

In recent years, with the development of modern industry and military, tracking control has been widely applied in fields like industrial process control, robot control, marine information detection, and aerospace [1], [2], [3], [4], [5]. It should be mentioned that the practical control systems often suffer from external periodic disturbances, such as wind shear forces on aircraft [6], and wave forces on offshore platforms or ships [7]. These periodic disturbances can usually be described by some kinds of sinusoidal signals. For a tracking system, to reduce the tracking error, some strategies have been adopted in literature, such as internal model control (IMC) [8], [9], predictive control [10], [11], sliding mode

control [12], [13]. Among these methods, internal model control is an effective method to eliminate certain kinds of interferences like periodic sinusoidal disturbances. In addition, if the tracked signal is generated by a reference model, when the unstable modes of the external disturbance and the reference signal are implanted in the servo compensator, the tracking error can be asymptotically reduced to zero. Moreover, this method is insensitive to the parameter variations of the controlled system and compensator except for the internal model [14]. When the parameters of the controlled system and the compensator are perturbed, the controlled objects can still achieve non-static error tracking control as long as the closed-loop system is asymptotically stable, even if the range of the perturbation is quite large.

Another feature of current tracking control systems is that the components such as sensors, controllers, actuators,

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and controlled objects exchange data or information mostly through shared communication networks. Networked control systems have also become one of the main hot fields of control theory and engineering [15], [16]. Naturally, the tracking control problem of networked control systems has drawn much attention from scholars [17], [18]. Due to the introduction of networks, the tracking performance not only depends on the designed controller but also is affected by the network-induced delay and limited communication resources [19], [20]. This brings great challenges to the research of networked tracking control. In the traditional networked control systems, the time-triggering mechanism (TTM) is applied, where the controller is updated with a fixed period [21]. To release the restriction of this case, a well-designed event-triggering mechanism (ETM) is introduced into the networked tracking control [22], [23]. Different from the TTM, the ETM generally leads to a less conservatism usage of systems resources since the information of system states was sent to the controller unless the occurrence of an event [24], [25]. Recently, significant achievements have been made on the various ETMs. For instance, distributed event-triggered control for a multi-agent system was studied and the results were then extended to a self-triggered setup in [26]. In [27], the sampled-data-based event-triggered control and filtering for networked systems were investigated. For the issue of the networked output tracking control, a state-dependent ETM based on the T-S fuzzy model was reported in [28]. In [29], a fixed-time event-triggered control mechanism was proposed to investigate the time-varying formation tracking problem for multiagent systems. In [30], to perform the control in two different time scales, a new dual ETM was presented for a singularly perturbed system with structured state-space uncertainty. The periodic event-triggered control strategies were developed for linear systems in [31] and [32]. Further, a novel periodic event-triggered scheme and the predictor-based periodic event-triggered scheme were proposed for nonlinear systems in [33] and [34], respectively. The alternate periodic event-triggered control was investigated for the exponential synchronization of multilayer neural networks in [35]. The dynamic event triggering mechanism by the introduction of an internal dynamic variable was proposed in [36]. Moreover, the adaptive event-triggered controller design method was investigated in [37] for IT2 fuzzy networked control systems. Fuzzy event-triggered integral sliding mode control was addressed in [38] for TSFM-based nonlinear systems.

So far, in the aforementioned literature, the approach to tracking control is mostly to construct an augmented system with the states of the reference system and the plant, and the tracking error is used as the output. Thus, the tracking problem of the controlled systems is transformed into the stability problem of the augmented systems. However, when using this method to track a continuous signal produced by the reference system, even if the input signal of the reference

system and the external disturbance are both step signals, there will exist a steady-state tracking error for the controlled objects. It is difficult to eliminate the steady tracking error. Moreover, when the external disturbance is a periodically changing signal, the tracking performance of the controlled objects further deteriorates. It is no longer applicable to some systems that require precise docking. As alluded to above, the internal model control has non-static error tracking property and strong robustness, which is the motivation for introducing the internal model control into the networked tracking problem. At present, we have not found scholars who combine ETM with internal model control to study networked tracking control. More importantly, if the tracked signal is generated by a reference model, how to obtain the structural characteristic model of tracking signal $y_r(t)$ by the input signal $r(t)$ of the reference system is the difficulty of the proposed method. Another challenge of the considered approach is how to combine adaptive ETM and internal model compensation controller. And how to deal with adaptive parameter variables in LMIs is also an issue that needs to be considered.

Therefore, in this paper, an internal model controller with the adaptive ETM is designed for networked tracking control systems, which can suppress sinusoidal disturbance signals, achieve non-static error tracking control as well as save network bandwidth resources. And two numerical examples demonstrate the effectiveness and practicability of the proposed method.

The main contribution of the manuscript can be summarized as follows:

- New tracking method based on the internal model principle is proposed, in which the common unstable modes of the reference tracking system and the external disturbance is implanted into the servo compensator.
- The internal model compensation controller is designed for networked tracking control problems. Meanwhile, the adaptive ETM is introduced to improve bandwidth utilization.
- Considering the influence of network-induced delay, the networked control system is firstly modeled as a time-varying time-delay system that depends on sampling state error. Then, the networked tracking problem is further transformed into the stability problem of the closed-loop system.
- By constructing a Lyapunov-Krasovskii functional with the delay segmentation technique, we derive a sufficient condition such that the time-varying time-delay system is asymptotically stable and satisfies the H_∞ output tracking performance.

Notations: \mathbb{R}^m means the m -dimensional Euclidean space. $Sym(Y)$ represents $Y + Y^T$, and $*$ denotes the symmetric elements of a matrix. $n \times n$ identity matrix is expressed as I_n . A symmetric positive definite (semi-definite) matrix is defined by the representation $S > 0$ ($S \geq 0$).

II. PROBLEM STATEMENT AND PRELIMINARIES

Let us consider the linear continuous-time plant as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t) \\ y(t) &= Cx(t) + Dw(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^q$, and $w(t) \in \mathbb{R}^q$ denote the state, control input, measured output, disturbance, and their corresponding dimensions respectively. A, B_1, B_2, C, D are the coefficient matrices with appropriate dimensions. The initial condition of the system (1) is $x(t_0) = x_0$. Assume that $\{A, B_1\}$ is completely controllable and $\{A, C\}$ is completely measurable.

The plant (1) is controlled to track the signal $y_r(t)$, which is generated by the reference system (2),

$$\begin{aligned} \dot{x}_r(t) &= Gx_r(t) + r(t) \\ y_r(t) &= Hx_r(t) \end{aligned} \tag{2}$$

where, $x_r(t) \in \mathbb{R}^m$ is the state of the reference system, $r(t) \in \mathbb{R}^m$ is the input of the reference system, $y_r(t) \in \mathbb{R}^p$ is the output signal of the reference system, G and H are appropriate dimensional constant matrices with G Hurwitz.

The controller is connected with the plant (1) via the internet shown in Fig.1. Our purpose is to design a class of controllers to make the closed-loop networked control system realize non-static error tracking control even if the external disturbance signal and the reference input signal both are periodic signals like sinusoids.

So, the ideas of internal model control and the adaptive ETM are introduced into networked control systems. The designed controller is composed of a stabilization compensator $u_1(t)$ and an internal model servo compensator $u_2(t)$. The stabilization compensator $u_1(t)$ is static state feedback, whose function is to make the controlled objects asymptotically stable. While the internal model servo compensator $u_2(t)$ provides the cancellation of unstable modes of the tracked reference system and the disturbance signal. The sum of the two compensators $u(t) = u_1(t) + u_2(t)$ is used as the quantity for the tracking control system to achieve that the error $e(t) = y(t) - y_r(t)$ tends asymptotically to zero.

III. DESIGN OF TRACKING CONTROLLER AND ADAPTIVE ETM

A. DESIGN OF INTERNAL MODEL CONTROLLER

In order to realize non-static error tracking control, the following equation should be satisfied: $\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} [y(t) - y_r(t)] = 0$.

First, for the q -dimensional disturbance signal, $w(t) = [w_1(t), w_2(t) \dots w_q(t)]^T$ take Laplace transform and obtain

$$\begin{aligned} \bar{W}(s) &= [\bar{W}_1(s), \bar{W}_2(s), \dots, \bar{W}_q(s)]^T \\ &= \begin{bmatrix} n_{w1}(s) & n_{w2}(s) & n_{wq}(s) \\ d_{w1}(s) & d_{w2}(s) & d_{wq}(s) \end{bmatrix}^T \end{aligned} \tag{3}$$

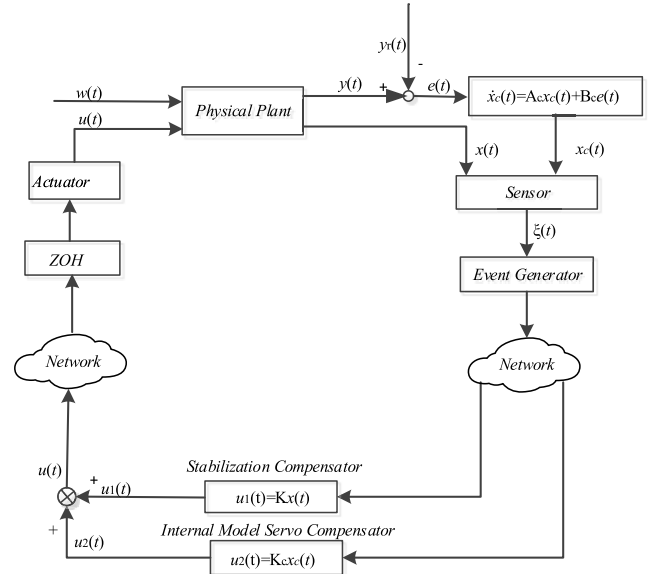


FIGURE 1. A diagram of networked tracking control system based on internal model principle and adaptive ETM.

From the structural characteristics of the denominator, it is easily found that $d_w(s) = \{d_{w1}(s), d_{w2}(s), \dots, d_{wq}(s)\}$ is the least common multiple, and n_w is the degree of the polynomial $d_w(s)$. Thus, the unstable model of the disturbance signal $w(t)$ can be derived as (4):

$$\begin{aligned} \dot{x}_w(t) &= A_w x_w(t) \\ w(t) &= C_w x_w(t) \end{aligned} \tag{4}$$

where, A_w is a $n_w \times n_w$ dimensional matrix that satisfies “minimum polynomial = $d_w(s)$ ”, C_w is a $q \times n_w$ dimensional matrix that satisfies the output variable is $w(t)$. Then, so does for the m -dimensional reference input signal $r(t) = [r_1(t), r_2(t), \dots, r_m(t)]^T$, and obtain the Laplace transform (5) and the least common multiple (6).

$$\begin{aligned} \bar{R}(s) &= [\bar{R}_1(s), \bar{R}_2(s), \dots, \bar{R}_m(s)]^T \\ &= \begin{bmatrix} n_{r1}(s) & n_{r2}(s) & n_{rm}(s) \\ d_{r1}(s) & d_{r2}(s) & d_{rm}(s) \end{bmatrix}^T \end{aligned} \tag{5}$$

$$d_r(s) = \{d_{r1}(s), d_{r2}(s), \dots, d_{rm}(s)\} \tag{6}$$

where n_r is the degree of the polynomial $d_r(s)$. Similarly, the structural characteristic model of the input signal $r(t)$ of the reference system can be derived as (7).

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) \\ r(t) &= C_p x_p(t) \end{aligned} \tag{7}$$

where, A_p is any $n_r \times n_r$ dimensional matrix that satisfies “minimum polynomial = $d_r(s)$ ”, C_p is any $q \times n_r$ dimensional matrix that satisfies the output variable is $r(t)$.

Combining the state-space model of the reference system (2), the structural characteristic model of the output $y_r(t)$ can

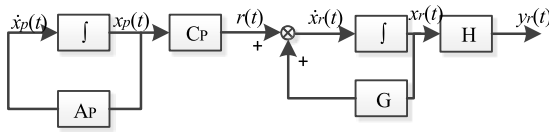


FIGURE 2. The structural characteristic model of $y_r(t)$.

be described by (8) as shown in Fig.2.

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_r(t) \end{bmatrix} = \begin{bmatrix} A_p & 0 \\ C_p & G \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_r(t) \end{bmatrix}$$

$$y_r(t) = [0 \ H] \begin{bmatrix} x_p(t) \\ x_r(t) \end{bmatrix} \quad (8)$$

Since the signal model to be implanted in the controller is just the common unstable modes of the reference output signal and the disturbance signal, the smallest polynomials of the structural properties of $y_r(t)$ and $w(t)$ are obtained as $d_y(s)$ and $d_w(s)$, respectively. Then, take the factor product of $d_y(s)$ and $d_w(s)$ in the right half-plane of the complex plane as $\phi_y(s)$ and $\phi_w(s)$, and finally, take the least common multiple of the common unstable signal model $\phi(s) = \phi_y(s) \cdot \phi_w(s)$. Let $\phi(s) = s^l + a_{l-1}s^{l-1} + \dots + a_1s + a_0$, the coefficient matrices can be computed as follows.

$$A_c = \begin{bmatrix} \Gamma & & & \\ & \ddots & & \\ & & \Gamma & \\ & & & \Gamma \end{bmatrix}_{ql \times ql}, B_c = \begin{bmatrix} \beta & & & \\ & \ddots & & \\ & & \beta & \\ & & & \beta \end{bmatrix}_{ql \times q} \quad (9)$$

where

$$\Gamma = \begin{bmatrix} 0 & & & \\ \vdots & & & \\ 0 & & I_{l-1} & \\ -a_0 & -a_1 & \dots & -a_{l-1} \end{bmatrix}_{l \times l}, \beta = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{l \times 1}$$

Furthermore, the common unstable modes of the reference system output and the disturbance signal can be described as:

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c e(t) \\ y_c(t) &= x_c(t) \\ u_2(t) &= K_c x_c(t) \end{aligned} \quad (10)$$

where K_c is the gain matrix of the servo compensator, $x_c(t)$ is the state variable of the common unstable model, which includes stable variables $x_w(t)$, $x_p(t)$, $x_r(t)$ defined in (4) and (8). $u_2(t)$ is the control output of the internal mode controller.

Remark 1: The important advantages of the non-static tracking control system scheme based on the internal model principle are: it has strong insensitivity to the parameter changes of the controlled system and compensator except the internal model. When the parameters of the controlled system and the compensator are perturbed, even if the parameter perturbation range is quite large, as long as the closed-loop system remains asymptotically stable, the system must still have the property of non-static error tracking.

Remark 2: For the non-static error tracking control system based on the internal model principle, under the premise

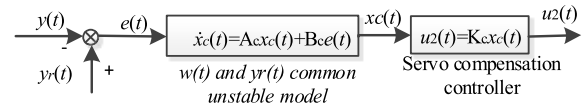


FIGURE 3. Internal model compensation controller.

that the system is asymptotically stable, the basic reason why the system can achieve asymptotic tracking and disturbance suppression is the internal model and the compensation effect generated by the internal model. The essence of internal model control is that the root of the common unstable algebraic equation $\phi(s)$ of reference tracking signal $y_r(t)$ and disturbance signal $w(t)$ can accurately cancel the unstable mode of $y_r(t)$ and $w(t)$, to achieve the goal of non-static error tracking.

Assuming that both the reference signal and the external disturbance signal are sinusoidal with frequencies ω_1 and ω_2 , respectively, the common unstable model is

$$\begin{aligned} \dot{x}_c(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 \omega_2^2 & 0 & -(\omega_1^2 + \omega_2^2) & 0 \end{bmatrix} x_c(t) \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [y(t) - y_r(t)] \\ y_c(t) &= x_c(t) \end{aligned} \quad (11)$$

The structure of the internal mode compensation controller is shown in Fig.3.

Taking the stabilization compensator as the state feedback of the controlled system, we have $u_1(t) = Kx(t)$. Then the state equation of the $(n+ql)$ -dimensional series system can be given:

$$\begin{aligned} \dot{\xi}(t) &= \tilde{A}\xi(t) + \tilde{B}_1 u(t) + \tilde{B}_2 w(t) + \tilde{E} y_r(t) \\ u(t) &= \tilde{K}\xi(t) \end{aligned} \quad (12)$$

where, $\xi(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}$, $\tilde{A} = \begin{bmatrix} A & 0 \\ B_c C & A_c \end{bmatrix}$, $\tilde{B}_1 = \begin{bmatrix} B_1 \\ B_c D \end{bmatrix}$, $\tilde{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$, $\tilde{E} = \begin{bmatrix} 0 \\ -B_c \end{bmatrix}$, $\tilde{K} = [K \ K_c]$.

For the series system (12), the necessary and sufficient condition for the existence of state feedback is that the system is fully controllable. If the non-static error tracking control based on the principle of the internal model is realized, the system (1) and the series system (12) are both controllable. The following lemma is necessary to prove that the series system (12) is controllable.

Lemma 1 [39]: For the series system (12), there exists a state feedback controller to make the tracking error approach to zero when $t \rightarrow \infty$, if the following conditions are satisfied.

- The input dimension of the controlled system is greater than or equal to the output dimension, i.e. $\dim(u) \geq \dim(y)$;

- For each root λ_i of the common unstable algebraic equation $\phi(s) = 0$ of the output of the reference system and the disturbance signal, the following equation holds:

$$\text{rank} \begin{bmatrix} \lambda_i I - A & B \\ -C & D \end{bmatrix} = n + q, i = 1, 2, \dots, l$$

B. DESIGN OF THE ADAPTIVE ETM

Consider a networked tracking control system as shown in Fig.1, assuming that sensors are time-triggered with a constant sampled period $h, h \in \mathbb{N}$, the sampled data is transmitted in a single packet, and neither packet losses nor disorder occurs in transmission.

The sampling sequence is described by the set $S_1 = \{0, h, 2h, \dots, lh, \dots\} (l \in \mathbb{N})$. The successfully transmitted sampled sequence at the sensors is described by the set $S_2 = \{0, t_1h, t_2h, \dots, t_kh, \dots\} (t_k \in \mathbb{N})$. To reduce the transmission of data packets, the time to start a transmission task t_kh is determined by adaptive event-triggering equipment. When the triggering condition is satisfied, the state sequence $\xi(t_kh)$ is transmitted to the controller through the network channel, then the control law is updated, and a new set point is sent to the zero-order holder (ZOH) via network again. ZOH keeps the current value for the actuator until the next transmitting datum come.

The adaptive ETM is implemented as the violation of the following inequality condition:

$$\rho^T(lh) W_1 \rho(lh) < \sigma(lh) \xi^T(lh) W_2 \xi(lh) \quad (13)$$

where $lh \in [t_kh, t_{k+1}h), \xi(lh)$ is the current sampling data, $\xi(t_kh)$ is the latest transmission data, $\rho(lh) = \xi(t_kh) - \xi(lh)$ is the current sampling state error, W_1, W_2 are quadratic positive weighting matrices on $\rho(lh)$ and $\xi(lh)$, respectively. The adaptive triggering threshold $\sigma(lh)$ is designed for given parameters $0 \leq \underline{\sigma} \leq \bar{\sigma}$ as $\sigma(lh) = \underline{\sigma} + (\bar{\sigma} - \underline{\sigma}) \cdot \exp(-\rho^T(lh) \rho(lh))$.

Hence, the next event-triggering time instant $t_{k+1}h$ is determined as follows:

$$t_{k+1}h = t_kh + \min_{l \geq t_k, l \in \mathbb{N}} \left\{ lh | \rho^T(lh) W_1 \rho(lh) \geq \sigma(lh) \xi^T(lh) W_2 \xi(lh) \right\} \quad (14)$$

Remark 3: The proposed adaptive event-triggered condition only is judged at the sampling instant, the lower bound of the internal execution time is the sampling period $h, (h > 0)$, hence avoiding the occurrence of Zeno behavior.

Remark 4: According to (13), the sampling data is not sent to the controller unless satisfying the trigger condition. This may cost a bit of computing time of (13), but it can considerably relieve the transmission pressure and save the bandwidth use of the network. In networked control systems, the bandwidth resources are usually limited, and communication consumes more energy than information processing does. Therefore, it is significant to improve bandwidth utilization by the adaptive ETM.

Due to the influence of the network-induced delay, the state sequence $\xi(t_kh)$ is sent at the time t_kh by the adaptive ETM and arrives at the actuator at the moment $t_kh + \tau_{t_k}$. Afterward, the control input will remain a constant through ZOH until the next state sequence $\xi(t_{k+1}h)$ arrives. where $\tau_{t_k} = \tau_{t_k}^{sc} + \tau_{t_k}^{ca} \in [\tau_{\min}, \tau_{\max}], \tau_{\min}, \tau_{\max} \in \mathbb{R}, (\tau_{t_k}$ includes the transport delays both from the sensor to the controller $\tau_{t_k}^{sc}$ and from the controller to the actuator $\tau_{t_k}^{ca}$). Hence, the state feedback control law based on the internal model principle is chosen as follows:

$$\begin{aligned} u(t) &= u_1(t) + u_2(t) \\ &= Kx(t_kh) + K_c x_c(t_kh) \\ &= \tilde{K} \xi(t_kh) \end{aligned} \quad (15)$$

where, $t \in I_a = [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}), \tilde{K} = [K \ K_c], \xi(t) = \begin{bmatrix} x(t_k) \\ x_c(t_k) \end{bmatrix}, K_c$ is the gain matrix of the servo-compensated controller and K is the gain matrix of the stabilization controller.

As in the literature [40], the time interval I_a is partitioned as $I_a = \bigcup_{l=t_k}^{t_{k+1}-1} I_l, l \in \mathbb{N}$ with $I_l \triangleq [lh + \tau_l, lh + h + \tau_{l+1}), t_k \in \mathbb{N}$ and $\tau_l \leq h + \tau_{l+1}$, which ensures that the sequence $\{lh + \tau_l\}$ is strictly increasing. For $\forall t \in I_l$, we denote the time delay as $\eta_t = t - lh$, where, η_t is bounded by $\underline{\eta} = \underline{\tau} \leq \tau_l \leq \eta_t < 1 + \tau_{l+1} \leq 1 + \bar{\tau} = \bar{\eta}$, and $\underline{\eta}, \bar{\eta}$ represent the minimum and maximum allowable communication delay, respectively. Then, according to (15), the feedback control law is given by:

$$u(t) = \tilde{K} (\rho(t - \eta_t) + \xi(t - \eta_t)), t \in I_l \quad (16)$$

Considering the impact of delay on the system, the adaptive event-triggering condition can be written as: $\rho^T(t - \eta_t) W_1 \rho(t - \eta_t) < \sigma(t) \xi^T(t - \eta_t) W_2 \xi(t - \eta_t), t \in I_l$, with $\rho(t - \eta_t) = \xi(t_kh) - \xi(t - \eta_t)$.

Combining (12) and (16), one can derive an augmented closed-loop time-delay system based on the internal model control and adaptive ETM:

$$\begin{aligned} \dot{\xi}(t) &= \tilde{A} \xi(t) + \tilde{B}_1 \tilde{K} \xi(t - \eta_t) \\ &\quad + \tilde{B}_1 \tilde{K} \rho(t - \eta_t) + \tilde{B}_2 w(t) + \tilde{E} y_r(t) \\ e(t) &= -\tilde{C} \xi(t) + y_r(t) \end{aligned} \quad (17)$$

where the system matrices are defined in (12).

According to the closed-loop system (17), the networked tracking control system (1) that based on the internal model principle, and the adaptive ETM is modeled as a time-varying time-delay system that depends on the sampling state error. Therefore, the time-delay-dependent method is suitable for the analysis and synthesis of the system.

IV. MAIN RESULTS

The following two Lemmas will be essential for the proof.

Lemma 2 [41]: For any vectors a , and positive definite matrix Z , the following holds:

$$2a^T b \leq a^T Z^{-1} a + b^T Z b$$

Lemma 3: For any constant matrix $M \in R^{n \times n}$, $M = M^T > 0$, scalar $r \leq 0$, vector function $\zeta : [-r, 0] \rightarrow R^n$, the following holds:

$$-r \int_{-r}^0 \dot{\zeta}^T(t + \alpha) M \dot{\zeta}(t + \alpha) d\alpha \leq (\zeta^T(t) \ \zeta^T(t-r)) \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \begin{pmatrix} \zeta(t) \\ \zeta(t-r) \end{pmatrix}$$

Proof: Note that Jensen's inequality

$$r \int_0^r w^T(\beta) M w(\beta) d\beta \geq \left(\int_0^r w(\beta) d\beta \right)^T M \left(\int_0^r w(\beta) d\beta \right)$$

Then, according to Jensen's inequality, we have

$$\begin{aligned} & -r \int_{-r}^0 \dot{\zeta}^T(t + \alpha) M \dot{\zeta}(t + \alpha) d\alpha \\ &= -r \int_0^r \dot{\zeta}^T(t + \alpha - r) M \dot{\zeta}(t + \alpha - r) d\alpha \\ &\leq -\left(\int_0^r \dot{\zeta}(t + \alpha - r) d\alpha \right)^T M \left(\int_0^r \dot{\zeta}(t + \alpha - r) d\alpha \right) \\ &= -(\zeta(t + \alpha - r) \Big|_0^r)^T M (\zeta(t + \alpha - r) \Big|_0^r) \\ &= -[\zeta(t) - \zeta(t-r)]^T M [\zeta(t) - \zeta(t-r)] \\ &= (\zeta^T(t) \ \zeta^T(t-r)) \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \begin{pmatrix} \zeta(t) \\ \zeta(t-r) \end{pmatrix} \end{aligned}$$

This completes the proof of the Lemma 3.

Assuming that the coefficient matrices \tilde{A} , \tilde{B}_1 , \tilde{B}_2 , \tilde{E} and the controller gain matrix \tilde{K} are known, we study the conditions for the augmented closed-loop system to achieve non-static error tracking control. Theorem 1 shows that if some matrices can satisfy certain LMIs, then the system can achieve non-static error tracking control.

Theorem 1: Consider closed-loop system (17), given scalars $\bar{\eta} \geq \delta \geq \eta \geq 0, \gamma > 0, \sigma_l \geq 0, l = 1, 2$ and matrix \tilde{K} , if there exist symmetric positive definite matrices $P, Q_1, Q_2, M_i (i = 1, 2, 3), W_1, W_2$, such that

$$\begin{bmatrix} \Pi_{11l} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0 \quad (18)$$

where

$$\begin{aligned} \Pi_{12} &= [\eta \Phi_1^T M_1^T (\delta - \eta) \Phi_1^T M_2^T (\bar{\eta} - \delta) \Phi_1^T M_3^T \Phi_2^T \Phi_3^T] \\ \Pi_{22} &= \text{diag} \{ -M_1 - M_2 - M_3 - I - W_1 \} \\ \Phi_1 &= [\tilde{A} \ 0 \ 0 \ \tilde{B}_1 \tilde{K} \ 0 \ \tilde{B}_1 \tilde{K} \ \tilde{B}_2 \ \tilde{E}] \\ \Phi_2 &= [-\tilde{C} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ I] \\ \Phi_3 &= [\tilde{K}^T \tilde{B}_1^T P \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \Gamma_1 &= \tilde{A}^T P + P \tilde{A} + Q_1 + Q_2 - M_1 \end{aligned}$$

$$\Pi_{11l} = \begin{bmatrix} \Gamma_1 & M_1 & 0 & P \tilde{B}_1 \tilde{K} \\ * & -Q_1 - M_1 - M_2 & M_2 & 0 \\ * & * & -M_2 - M_3 & 0 \\ * & * & * & \sigma_l W_2 \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ 0 & 0 & P \tilde{B}_2 & P \tilde{E} \\ 0 & 0 & 0 & 0 \\ M_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -M_3 - Q_2 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & 0 \end{bmatrix}$$

then the closed-loop system (17) is asymptotically stable and has the H_∞ performance γ .

Proof: Firstly, based on the idea of time-delay segmentation, the Lyapunov-Krasovski functional candidates are chosen as follows:

$$V(\xi, t) = V_1(\xi, t) + V_2(\xi, t) + V_3(\xi, t), t \in I_m \quad (19)$$

with

$$\begin{aligned} V_1(\xi, t) &= \xi^T(t) P \xi(t) \\ V_2(\xi, t) &= \int_{t-\eta}^t \xi^T(\alpha) Q_1 \xi(\alpha) d\alpha + \int_{t-\bar{\eta}}^t \xi^T(\alpha) Q_2 \xi(\alpha) d\alpha \\ V_3(\xi, t) &= \eta \int_{-\eta}^0 \int_{t+\beta}^t \dot{\xi}^T(\alpha) M_1 \dot{\xi}(\alpha) d\alpha d\beta \\ &\quad + (\delta - \eta) \int_{-\delta}^{-\eta} \int_{t+\beta}^t \dot{\xi}^T(\alpha) M_2 \dot{\xi}(\alpha) d\alpha d\beta \\ &\quad + (\bar{\eta} - \delta) \int_{-\bar{\eta}}^{-\delta} \int_{t+\beta}^t \dot{\xi}^T(\alpha) M_3 \dot{\xi}(\alpha) d\alpha d\beta \end{aligned}$$

The time derivatives of $V(\xi, t)$ can be calculated as

$$\begin{aligned} \dot{V}_1(\xi, t) &= 2\dot{\xi}^T(t) P \xi(t) \\ &= 2 \left(\tilde{A} \xi(t) + \tilde{B}_1 \tilde{K} \xi(t - \eta_t) + \tilde{B}_1 \tilde{K} \rho(t - \eta_t) \right. \\ &\quad \left. + \tilde{B}_2 w(t) + \tilde{E} y_r(t) \right)^T P \xi(t) \\ &= \xi^T(t) \left(\tilde{A}^T P + P \tilde{A} \right) \xi(t) \\ &\quad + 2\dot{\xi}^T(t - \eta_t) \tilde{K}^T \tilde{B}_1^T P \xi(t) \\ &\quad + 2\rho^T(t - \eta_t) \tilde{K}^T \tilde{B}_1^T P \xi(t) \\ &\quad + 2w^T(t) \tilde{B}_2^T P \xi(t) + 2y_r^T(t) \tilde{E}^T P \xi(t) \end{aligned}$$

Combining Lemma 2 and adaptive ETM (13), we have:

$$\begin{aligned} & 2\rho^T(t - \eta_t) \tilde{K}^T \tilde{B}_1^T P \xi(t) \\ & \leq \rho^T(t - \eta_t) W_1 \rho(t - \eta_t) \\ & \quad + \xi^T(t) P \tilde{B}_1 \tilde{K} W_1^{-1} \tilde{K}^T \tilde{B}_1^T P \xi(t) \\ & \leq \sigma(t) \xi^T(t - \eta_t) W_2 \xi(t - \eta_t) \\ & \quad + \xi^T(t) P \tilde{B}_1 \tilde{K} W_1^{-1} \tilde{K}^T \tilde{B}_1^T P \xi(t) \end{aligned}$$

$d_w(s)$ and $d_y(s)$, then take the least common multiple of the common unstable signal model $\phi(s) = \phi_w(s) \cdot \phi_y(s)$.

S4: Calculate the roots of $\phi(s) = 0$. Check if the following equation holds for each root λ_i

$$\text{rank} \begin{bmatrix} \lambda_i I - A & B \\ -C & D \end{bmatrix} = n + q, i = 1, 2, \dots, l$$

If the equation holds, continue to the next step; otherwise, go to S8.

S5: Determine block coefficient matrices

$$\Gamma = \begin{bmatrix} 0 & & & \\ \vdots & & I_{l-1} & \\ 0 & & & \\ -a_0 & -a_1 & \dots & -a_{l-1} \end{bmatrix}_{l \times l}, \beta = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{l \times 1}$$

Determine the coefficient matrices of the common unstable model

$$A_c = \begin{bmatrix} \Gamma & & \\ & \ddots & \\ & & \Gamma \end{bmatrix}_{ql \times ql}, B_c = \begin{bmatrix} \beta & & \\ & \ddots & \\ & & \beta \end{bmatrix}_{ql \times q}$$

S6: Construct the state equation of the $(n + ql)$ -dimensional series system

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ B_c C & A_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_c D \end{bmatrix} u(t) \\ &+ \begin{bmatrix} B_2 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ -B_c \end{bmatrix} y_r(t) \\ u(t) &= [K \ K_c] \xi(t) \end{aligned}$$

S7: Solve the gain of controller by stability condition Theorem 2.

S8: Stop calculate.

V. NUMERICAL EXAMPLES

In this section, two examples are given for the networked tracking control problem where both the external disturbances and the reference inputs are sinusoidal signals. It will confirm that the proposed method can save network bandwidth resources and enable the networked control system to achieve non-static error tracking control.

Example 1: Consider a continuous-time linear time-invariant controlled system (1) and a reference tracking model (2) with the following coefficient matrices.

$$\begin{aligned} A &= \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ C &= [1 \ 1], D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, G = -10, H = -1. \end{aligned}$$

Here, $n = 2, p = 1$ and $q = 1$. First of all, we know that the controlled system (1) is fully controllable from $\text{rank}[B_1 \ AB_1] = 2$, and the dimensionality relationship $\dim(u) \geq \dim(y)$ between input and output is satisfied by $\dim(u) = \dim(y) = 1$. From the above analysis, the common

unstable model of the external disturbance and the reference tracking system (i.e., the internal model) obtained is:

$$\begin{aligned} \dot{x}_c(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 \omega_2^2 & 0 & -(\omega_1^2 + \omega_2^2) & 0 \end{bmatrix} x_c(t) \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [y(t) - y_r(t)] \\ y_c(t) &= x_c(t) \end{aligned}$$

From $\text{rank}[B_c \ A_c B_c \ A_c^2 B_c \ A_c^3 B_c] = 4$, we know that the internal model is controllable. Assuming that the external disturbance is given as $w(t) = 5\sin 0.1t$, the external input of the reference system is $r(t) = \sin 0.5t$, based on which the coefficient matrices of the series system is obtained as:

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -0.0025 & 0 & -0.26 & 0 \end{bmatrix}, \\ \tilde{B}_1 &= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tilde{B}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \end{aligned}$$

In addition, the sampling period $h = 10\text{ms}$, the network induction delay $\eta = 10\text{ms}, \bar{\eta} = 30\text{ms}, \delta = 20\text{ms}$, the adaptive event triggering parameters $\underline{\sigma} = 0.1, \bar{\sigma} = 0.5$, and the state of the controlled objects is initialized by $x_0 = [-0.1 \ 0.1]^T$, and the initial state of the reference model is $x_{r0} = 0.5$. By solving the LMI (21) in Theorem 2, the gain matrices of the internal mode controller and the state feedback controller are obtained as: $K_c = [-100.7269 \ -279.8896 \ -414.1911 \ -197.8802]$, $K = [243.7811 \ 105.3280]$. The power matrices of the adaptive ETM are:

$$W_1 = \begin{bmatrix} 302.2090 & -282.4243 & 5.3640 \\ -282.4243 & 279.8309 & -4.8350 \\ 5.3640 & -4.8350 & 294.12 \\ -0.2660 & -0.3105 & -0.6407 \\ 0.4195 & -0.4308 & -79.3431 \\ 0.3100 & 1.6804 & -0.6501 \\ -0.2660 & 0.4195 & 0.3100 \\ -0.3105 & -0.4308 & 1.6804 \\ -0.6407 & -79.3431 & -0.6501 \\ 78.8644 & -0.0326 & -24.7977 \\ -0.0326 & 24.5654 & 0.2813 \\ -24.7977 & 0.2813 & 10.93621 \end{bmatrix}$$

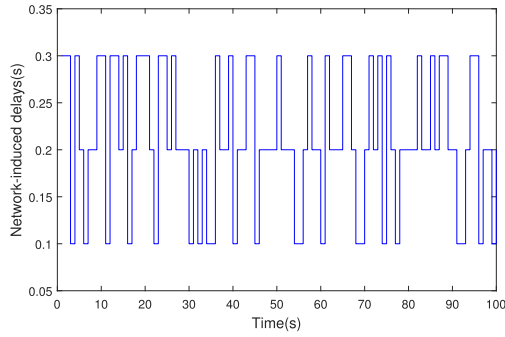


FIGURE 4. Network-induced delay.

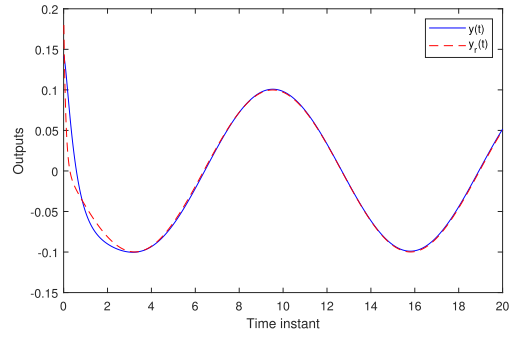


FIGURE 6. The tracking curves of the system under the action of the internal model compensator.

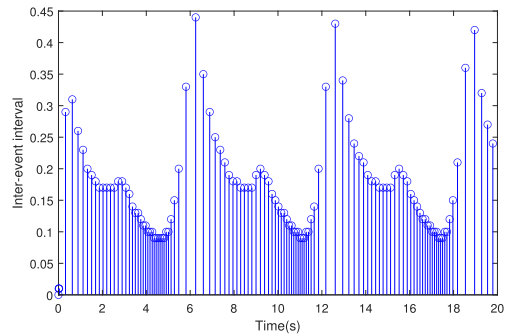


FIGURE 5. Inter-event interval of adaptive ETM.

$$W_2 = \begin{bmatrix} 321.5418 & -294.3152 & 6.9218 \\ -294.3152 & 285.7625 & -6.1652 \\ 6.9218 & -6.1652 & 374.2215 \\ -0.1408 & -0.2982 & -1.1138 \\ 0.2484 & -0.2600 & -100.2892 \\ 0.7196 & 1.5215 & -0.3078 \\ -0.1408 & 0.2484 & 0.7196 \\ -0.2982 & -0.2600 & 1.5215 \\ -1.1138 & -100.2892 & -0.3078 \\ 100.0238 & 0.0603 & -31.0592 \\ 0.0603 & 30.5980 & 0.4994 \\ -31.0592 & 0.4994 & 13.2425 \end{bmatrix}$$

The network-induced delay is shown in Fig.4. Fig.5 shows the inter-event interval of adaptive ETM. From Fig.5, we can see that when the adaptive event-triggering parameter is $\underline{\sigma} = 0.1, \bar{\sigma} = 0.5$, the update rate of the control signal under the adaptive ETM is 5.9 %, which illustrates a reduction in the transmission of data packets. Fig.6 depicts the tracking curve of the system under the action of the internal mode compensation controller when the input of the reference system and the external disturbance are sinusoidal signals; From Fig.6 we can see that, due to the addition of the internal model controller, the common unstable modes of the external disturbance and the reference system is canceled, and the networked control system realizes the tracking without steady error.

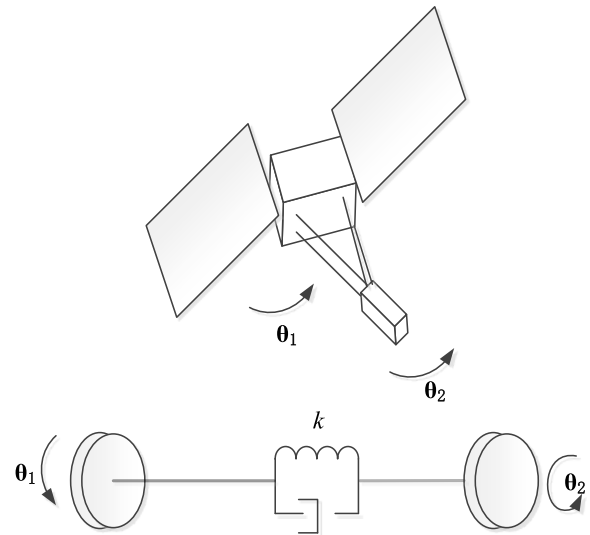


FIGURE 7. Schematic of the satellite.

Example 2: Take the satellite control problem from [42], [43], [44], and [45] as an example, and Fig.7 shows a schematic diagram of the satellite.

The satellite system consists of two rigid bodies connected by a flexible linkage. This linkage is modeled as a spring with a torque constant k and a viscous damping constant f . Denoting the yaw angles of the two bodies by θ_1 and θ_2 , the control torque by $u(t)$, the moments of inertia of the two bodies by J_1 and J_2 , and the torque disturbance by $w(t)$, then the dynamic equations are given by (22):

$$\begin{aligned} J_1 \ddot{\theta}_1(t) + f(\dot{\theta}_1(t) - \dot{\theta}_2(t)) + k(\theta_1(t) - \theta_2(t)) &= u(t) \\ J_2 \ddot{\theta}_2(t) + f(\dot{\theta}_2(t) - \dot{\theta}_1(t)) + k(\theta_2(t) - \theta_1(t)) &= w(t) \end{aligned} \tag{22}$$

If we choose the state variable $x(t) = [\theta_1 \ \theta_2 \ \dot{\theta}_1 \ \dot{\theta}_2]^T$ and the output $y(t) = \theta_2$, then the state space model of the satellite system can be described as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix}$$

$$\begin{aligned} & \times \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) \\ & + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(t) \\ y(t) &= [0 \ 1 \ 0 \ 0] \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix} \end{aligned}$$

where $J_1 = J_2 = 1, k = 0.09, f = 0.04$.

Suppose the reference model is given by:

$$\begin{aligned} x_r(t) &= -x_r(t) + r(t) \\ y_r(t) &= 0.5x_r(t) \end{aligned}$$

It is assumed that the network-induced delay $\eta = 5$ ms, $\bar{\eta} = 10$ ms, $\delta = 7$ ms, adaptive event-triggering parameter $\sigma = 0.1, \bar{\sigma} = 0.5$. The state of the satellite system is initialized by $x_0 = [-0.5 \ 1.3 \ 0.3 \ -0.3]^T$, and the initial state of the reference model is $x_{r0} = 0.5$.

Our goal is to design a control law (16) based on the internal-mode principle and the adaptive ETM to guarantee the output of the satellite system to track the output of the reference model well while reducing the communication frequency. In addition, the update rate of the control signal under the adaptive ETM is evaluated by $f_k = (n_s/n_k) \times 100\%$, where n_s and n_k represent the number of packets sent and sampled, respectively.

We assume that both the disturbance signal and the external input signal of the reference system are sinusoidal signals:

$$w(t) = 0.5 \sin 0.1t, r(t) = \sin 0.5t \tag{23}$$

By solving the LMIs (21) in Theorem 2, we obtained the gain matrices of the internal mode controller and the state feedback controller:

$$\begin{aligned} K_c &= 1.0e + 004 * [-0.0002 \ 0.0393 \ -0.0054 \ 0.2224], \\ K &= 1.0e + 004 * [-0.0036 \ -1.5199 \ -0.0017 \ -0.3699]. \end{aligned}$$

The weight matrices of the adaptive ETM are:

$$W_1 = \begin{bmatrix} 0.2888 & 0.0805 & 0.1181 & -0.2365 \\ 0.0805 & 0.4162 & -0.4403 & 0.5463 \\ 0.1181 & -0.4403 & 0.8595 & 0.6235 \\ 0.0351 & 0.1733 & -0.2256 & 0.2859 \\ 0.6354 & 0.2378 & 0.5637 & -0.8569 \\ 0.8616 & 0.1754 & 0.8745 & 0.6952 \\ 0.2135 & -0.5412 & 0.8654 & 0.2145 \\ -0.754 & 10.1245 & 0.8123 & -0.3251 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0.3874 & 0.0351 & 0.2365 & 0.1452 \\ 0.1784 & 0.1733 & 0.2584 & -0.0125 \\ 0.2479 & -0.2256 & 0.2541 & 0.2147 \\ 0.6354 & 0.1523 & -0.8512 & -0.3512 \\ 0.4125 & 0.4353 & 0.8745 & 0.5423 \\ -0.5367 & 0.9201 & 0.6241 & 0.7324 \\ 0.8741 & -0.1745 & -0.3541 & 0.2174 \\ -0.1745 & 0.7152 & -0.6854 & -0.3526 \\ 0.6354 & 0.2378 & 0.5637 & 0.8569 \\ 0.1181 & -0.4403 & 0.8595 & 0.6235 \\ 0.5281 & 3.3589 & 0.0128 & 0.8547 \\ 0.9359 & 0.3532 & 0.8564 & 0.2647 \\ 0.2888 & 0.0805 & 0.1181 & 0.2365 \\ 0.8616 & 0.1754 & 0.8745 & 0.6952 \\ -0.2816 & 0.1926 & 0.3325 & 0.1735 \\ 0.3287 & 0.5439 & 0.9513 & 0.5583 \\ 0.4125 & 0.4353 & -0.2584 & 0.3354 \\ 0.2479 & 0.2256 & 0.8521 & 0.2147 \\ 0.2653 & 0.3365 & -0.8554 & 0.6542 \\ 3.2756 & 0.3265 & 0.2543 & 0.8465 \\ 0.3874 & 0.0351 & 0.1563 & -0.1349 \\ 0.5367 & 0.9201 & 0.3254 & 0.2465 \\ 0.1146 & 0.8741 & 0.2314 & 0.8167 \\ -0.5546 & 0.6322 & -0.2541 & 0.5943 \end{bmatrix}$$

The network-induced delay is shown in Fig.8. Fig.9 shows the inter-event interval of adaptive ETM. From Fig.9, we can see that when the adaptive event-triggering parameter is $\sigma = 0.1, \bar{\sigma} = 0.5$, the update rate of the control signal under the adaptive ETM is 6.39 %, which illustrates a reduction in the transmission of data packets. Fig.10 depicts the changing trend of adaptive parameter $\sigma(t)$. We can see that the threshold value of the adaptive ETM is constantly changing according to the system parameters. When the system parameters change drastically, the threshold value also changes drastically. Furthermore, when the control input of the system tends to be stable, that is tracking error approaches zero at about 7s, and the threshold value also tends to be constant. Fig.11 compares the results between tracking curves under the action of a general controller in [42], [44], and [45], and tracking curves under the action of the internal model compensation controller at the same time. From Fig.11 we can see that, compared with the general controller, the tracking error of the system under the proposed controller goes to zero asymptotically. This indicates that the satellite tracking system based on the internal model principle not only makes the tracking error of the yaw angle zero and improves the tracking control accuracy, but also enhances the anti-interference capability and has certain robustness, which is conducive to further improving the accuracy and stability of the satellite tracking system.

In addition, Table 1 compares the transmission rate of the existing literature with that of the adaptive ETM in this paper. From Table 1, it can be seen that the data transmission rate is 16.6 % in [44] and 11 % in [45] for the external disturbance and the reference input (23) respectively; while

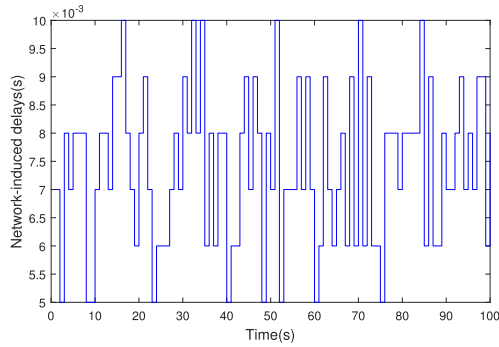


FIGURE 8. Network-induced delay.

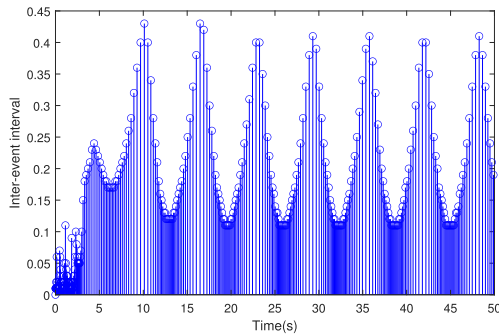


FIGURE 9. Inter-event interval of adaptive ETM.

TABLE 1. The transmission rate of adaptive ETM.

adaptive ETM	Theorem 2	[45]	[44]
f_k	6.39 %	11 %	16.6 %

TABLE 2. The computational complexity of proposed method.

The number of LMIs	The dimension of LMIs	The number of variables
2	$11(n + ql + 2)$	$8(n + ql)^2 + (n + ql)$

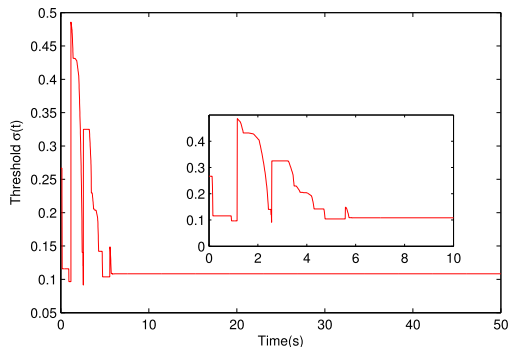


FIGURE 10. The adaptive change trend of parameter $\sigma(t)$.

in this paper, the data transmission rate is 6.39 %. This also indicates that the controller based on the adaptive ETM and internal model principle designed in this paper can better save network bandwidth resources on the premise of ensuring the tracking performance of the system. Furthermore, the number

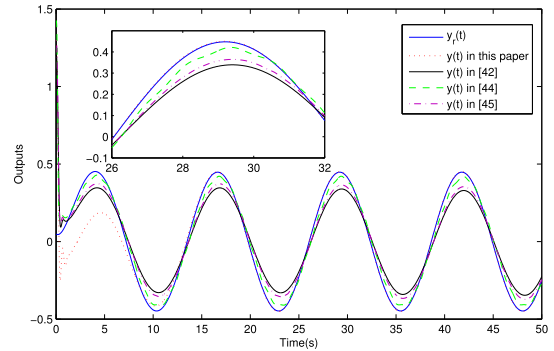


FIGURE 11. The tracking curves of the system under different controllers.

TABLE 3. Minimum value of γ for different delay interval.

The method	$\eta(t) \in [5\text{ms}, 30\text{ms}]$	$\eta(t) \in [5\text{ms}, 10\text{ms}]$
[42]	0.1267	/
[45]	/	0.07
[46]	0.0721	0.0520
Theorem 2	0.0645	0.0501

and dimension of LMIs and the number of variables that are applied to scale the computational burden for solving feasible solution of \tilde{K} are listed in Table 2.

To illustrate the feasibility of the proposed LMI problems, the minimum guaranteed H_∞ output tracking performance γ for different delay intervals $\eta(t)$ is listed in Table 3. From Table 3, it can be seen the obtained results in this paper is considerably less conservative than the previous results. In addition, the number of free variables in [44] is $6n^2 + n$, and that of variables in this paper is $8(n+ql)^2 + (n+ql)$. Hence, the range of solutions of the proposed method in this paper is larger.

VI. CONCLUSION

The problem of non-static error tracking control for a networked control system is investigated in a situation where both external disturbance and reference input are sinusoidal signals. Considering the influence of network-induced delay and limited bandwidth resources, and at the same time, to offset the common unstable modes of external disturbance signal and the output signal of the reference system, a compensation controller based on the internal model principle and an adaptive ETM is proposed in this paper. Firstly, the problem of non-static error tracking control for the networked control system is transformed into the stability problem of the closed-loop time-varying time-delay system. Then, by constructing a Lyapunov-Krasovskii functional with the delay segmentation technique, we derive a sufficient condition such that the closed-loop system is asymptotically stable and satisfies the H_∞ output tracking performance. Finally, the simulation results verify the feasibility and effectiveness of the proposed method.

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