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RESEARCH ARTICLE

Sampled-Data $\mathcal{L}_2 - \mathcal{L}_\infty$ Filter-Based Fuzzy Control for Active Suspensions

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ABSTRACT In this paper, we present a new sampled-data $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output feedback fuzzy control design technique for active suspension systems subjected to hard constraints. The sampled-data problem of continuous-time suspension systems is solved using an input delay approach. In this manner, the conservatism that occurs when designing filters and controllers is effectively alleviated by using Wirtinger's inequality with the extended reciprocally convex combination bounding technique. We designed an $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output-feedback fuzzy controller by using the Lyapunov–Krasovskii theorem to ensure that the closed-loop system is robust against external disturbances and sampled noise. The results of simulations involving different types of road disturbances and noise demonstrate the effectiveness of the proposed approach.

INDEX TERMS Active suspension system, input delay approach, $\mathcal{L}_2 - \mathcal{L}_\infty$ filter, sampled-data system, Takagi–Sugeno fuzzy system.

I. INTRODUCTION

Factors involved in vehicle performance include acceleration, braking, steering, and ride comfort, and many studies have been conducted on improving them. Suspension systems, which play a role in isolating passengers from vibrations due to impact with the ground, are important not only in terms of ride comfort but also for improving handling performance [1]. In particular, with the advent of electric vehicles, the role of the suspension system in providing ride comfort has become more important as the effects of engine noise and vibration are reduced. Suspension systems can be classified into three categories depending on their structural configuration: passive, semi-active, and active. Active suspension systems (ASSs) provide superior performance compared to passive suspension systems through additional electronic equipment. Therefore, many researchers have attempted to design controllers to provide outstanding ride quality under the mechanical constraints of vehicle systems, such as H_∞ control [2], [3], [4], sliding mode control [5], adaptive control [6], [7], and output-feedback control [8], [9], [10], [11].

Modeling techniques such as fuzzy systems and neural networks have received increasing attention as descriptors for nonlinear systems. Especially, the approximation based on the Takagi-Sugeno (T-S) fuzzy model has become a popular topic in modern control theory [12], [13]. Since the T-S fuzzy method describes a nonlinear system as a set of linear subsystems, an engineer can easily design a controller or filter for nonlinear systems by directly applying the conventional linear system theory [14], [15], [16], [17], [18]. For example, linear quadratic regulator (LQR) controller design techniques for fuzzy systems were examined by [19] and [20], while T-S fuzzy Kalman filtering problems were investigated by [21] and [22]. Recently, the T-S fuzzy approach has been improved to guarantee the feasibility of systems under modeling uncertainties, unmeasurable premise variables, etc [23], [24].

Since the controller or state estimator is implemented through a digital computer, control engineers have to consider

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sampling issues for the data or signals. Accordingly, many researchers have studied sampled-data control techniques to handle a system where a continuous-time plant and a digital controller coexist [25], [26], [27]. Among several approaches for handling sampled-data systems, the input delay method, which models the sampling intervals of the control input as a time-varying delay, has recently become popular [28]. Since the sampled-data system can be converted into a problem providing a stability criterion of the continuous time-delay system, a solution can be easily obtained based on the well-established Lyapunov stability analysis and linear matrix inequality (LMI) approach [29], [30], [31]. Although this method can lead to conservatism in the resulting stability criteria, many researchers have come up with solutions for this problem through various mathematical methods [32]. Bounding techniques, such as free weighting matrices [33], Jensen's inequality [34], [35], and Wirtinger's inequality [36], have been used to reduce conservatism. Since certain matrix inequalities can lead to the reciprocally convex condition, the use of the reciprocally convex combination lemma [37], [38] or the extended reciprocally convex combination lemma [39] is recommended to solve these inequalities. Notably, as the number of decision components such as free weighting matrices, increases, and the design complexity inevitably increases, engineers must consider the trade-off between conservatism and design complexity.

Issues on state estimation are of great importance in signal processing and control theory. This is because, in practical applications, engineers do not have access to all of the state information of a dynamic system. Although the Kalman filter is widely used in this situation, its performance is degraded in models with parametric uncertainty or high nonlinearity and external disturbances [40]. Consequently, the H_∞ and $\mathcal{L}_2 - \mathcal{L}_\infty$ filters have attracted the attention of researchers from the viewpoint of addressing model uncertainties, unknown external disturbances, and noise [41], [42], [43]. Based on the H_∞ analysis, the filtering problem for vehicle sideslip angle estimation with sampled-data measurements was studied by [44]. The authors of [45] and [46] investigated the application of the $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering to time-delayed neural networks. Different from the H_∞, the $\mathcal{L}_2 - \mathcal{L}_\infty$ is a performance measure of the peak value of a signal against external disturbances. Based on many examples in the literature, the peak value of the error signal greatly affects the stability or output response characteristics of a system [47], [48]. Therefore, control or filter design issues utilizing the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance have gained ongoing research interest in this field [49], [50], [51]. However, to the best of our knowledge, the $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering problem has not yet been solved for ASSs with sampled measurements.

Based on the above literature review, we present a method for designing a controller and filter for a nonlinear sampled-data system based on a T-S fuzzy model and an input delay approach. Our main objective is to design a filter-based output-feedback controller that can minimize the peak value of the estimation error by introducing the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance. The conditions for the proposed controller and filter design by using the Lyapunov stability theory and LMI are presented. Moreover, we reduce the conservatism in the results that can occur due to the time-varying delay by introducing a new boundary technology called extended reciprocally convex combination. The main contributions of this paper are summarized as follows.

1) By introducing the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index, we explored for the first time the suspension filtering problem capable of attenuating the peak value of estimation error for external disturbances.
2) In order to solve the conservatism of the sampled-data system based on the input delay approach, the controller and filter for active suspension were designed using Wirtinger's inequality with the extended reciprocally convex combination bounding technique.
3) Through various simulation results, the efficiency and excellence of the proposed sampled-data $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output feedback fuzzy controller are verified. In addition, it was confirmed that the boundary technique used in this paper reduced conservatism and extended the input delay time compared to the existing approach.

The remainder of this paper is organized as follows. Section II contains the formulation of an ASS with sampled input as a T-S fuzzy model. Section III presents the LMIs for designing a PDC scheme controller and a sampled-data $\mathcal{L}_2 - \mathcal{L}_\infty$ filter. Section IV provides the results of a performance evaluation of the proposed approach in cases involving different types of road disturbances. Section V presents our concluding remarks.

Notation: The superscript T denotes the transposes of matrices and vectors. The expression $P > 0$ (or, $P \geq 0$) indicates that P is positive (or positive semi-) definite matrix. The asterisk $*$ adjacent to (i, j) in a matrix denotes the transpose of element (j, i) in the matrix. For simplicity, Diag$\{\cdot\}$ indicates a block-diagonal matrix and Sym$\{A\}$ denotes $A + A^T$.

II. PROBLEM FORMULATION

The overall framework of the proposed approach is shown in Figure 1. To facilitate the application to this framework in the real world, we implement the following steps. The vehicle mass is considered a time-varying variable because it may change in several situations, for instance, owing to payloads. The dynamic equation is established as follows:

$$\begin{aligned} m_s(t)\ddot{z}_s(t) + f_{c_s} + f_{k_s} &= u(t), \\ m_u(t)\ddot{z}_u(t) - f_{c_s} - f_{k_s} + f_{c_u} + f_{k_u} &= -u(t), \end{aligned} \tag{1}$$

where m_s and m_u denote the sprung and unsprung masses, respectively; f_{c_s} and f_{c_u} represent the damping forces, f_{k_s} and f_{k_u} indicate the spring forces of the suspension and the tire, they can be represented as follows:

$$\begin{aligned} f_{c_s} &= c_s(\dot{z}_s(t) - \dot{z}_u(t)), \; f_{k_s} = k_s(z_s(t) - z_u(t)), \\ f_{c_u} &= c_u(\dot{z}_u(t) - \dot{z}_r(t)), \; f_{k_u} = k_u(z_u(t) - z_r(t)), \end{aligned}$$

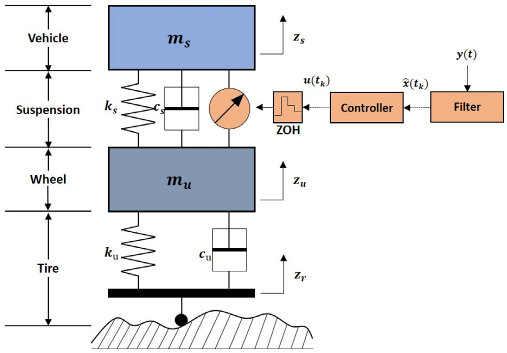


FIGURE 1. **Quarter-vehicle suspension model.**

where z_s and z_u denote the displacements of the sprung and unsprung masses, respectively; z_r denotes the displacement of the road surface; c_s and k_s imply the damping ratio and the spring constant of the suspension, respectively; c_u and k_u represent the damping ration and the compressibility of the tire, respectively; and $u(t)$ stands for the control input of the suspension.

The state variables considered in the state space analysis are defined as follows:

$$x(t) = [z_s(t) - z_u(t),\ z_s(t) - z_u(t),\ \dot{z}_s(t),\ \dot{z}_u(t)]^T.$$

The state variables include suspension deflection, tire deflection, vertical velocity of sprung mass, and vertical velocity of unsprung mass. By defining the external disturbance as $w(t) = \dot{z}_r(t)$, the system (1) can be rewritten as follows:

$$\dot{x}(t) = A(t)x(t) + B_u(t)u(t) + B_w(t)w(t), \tag{2}$$

where

$$A(t) = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s(t)} & 0 & -\frac{c_s}{m_s(t)} & \frac{c_s}{m_s(t)} \\ \frac{k_s}{m_u(t)} & -\frac{k_u}{m_u(t)} & \frac{c_s}{m_u(t)} & -\frac{c_s+c_u}{m_u(t)} \end{bmatrix},$$
$$B_u(t) = \begin{bmatrix} 0 & 0 & \frac{1}{m_s(t)} & -\frac{1}{m_u(t)} \end{bmatrix}^T,$$
$$B_w(t) = \begin{bmatrix} 0 & -1 & 0 & \frac{c_u}{m_u(t)} \end{bmatrix}^T.$$

In controller design, the three requirements pertaining to road holding stability, ride comfort performance, and mechanical constraint [52] must be considered.

(1-1) Vertical acceleration: Vehicle vibration can adversely influence the bodies of the vehicle occupants. Therefore, our primary objective is to design a controller that can minimize vertical acceleration of the vehicle body against uneven road disturbances:

$$\text{minimize } \ddot{z}_s(t). \tag{3}$$

(2-1) Constraint of suspension deflection: Considering the presence of mechanical structures, suspension deflection must not exceed a certain value:

$$|z_s(t) - z_u(t)| \le z_{\max}, \tag{4}$$

where $z_{\max}$ is the maximum permissible suspension deflection.

(2-2) Road holding ability: Road holding is the ability of a vehicle to stay on the road surface. The following inequality ensures that the dynamic tire load is less than static tire load:

$$k_u(z_u(t) - z_r(t)) < (m_s(t) + m_u(t))g. \tag{5}$$

Therefore, the requirements can be divided into performance and constraint outputs as follows:

$$\begin{aligned} z_1(t) &= ddotz_s(t), \\ z_{2,1}(t) &= z_s(t) - z_u(t), \\ z_{2,2}(t) &= k_u(z_u(t) - z_r(t))/(m_s(t) + m_u(t))g. \end{aligned} \tag{6}$$

Then, the following system can be established to indicate the ASSs with uncertainty of the sprung and unsprung masses:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_u(t)u(t) + B_w(t)w(t), \\ z_1(t) &= C_1(t)x(t) + D_1(t)u(t), \\ z_2(t) &= C_2(t)x(t), \end{aligned} \tag{7}$$

where

$$C_1(t) = \begin{bmatrix} -\frac{k_s}{m_s(t)} & 0 & -\frac{c_s}{m_s(t)} & \frac{c_s}{m_s(t)} \end{bmatrix},\ D_1(t) = \frac{1}{m_s(t)},$$
$$C_2(t) = \begin{bmatrix} \frac{1}{z_{\max}} & 0 & 0\ 0 \\ 0 & \frac{k_u}{(m_s(t)+m_u(t))g} & 0\ 0 \end{bmatrix}.$$

We assume that $w(t) \in \mathcal{L}_2[0, \infty)$, and without loss of generality, $\|w\|_2^2 \le w_{\max} < \infty$.

Let $y(t)$ and $v(t)$ denote the measurement output vector and the sensor noise, respectively. Then, the measurement output equation can be expressed as follows:

$$y(t) = Cx(t) + Dv(t), \tag{8}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}.$$

where α_1 and α_2 imply the coefficient of sensor noises.

Remark) For a practical implementation of the proposed controller, estimating the state variable from the measured output is a very important issue. In case of the laboratory vehicles, all state variables of the suspension system is accessible by using sensors such as LVDT (Linear Variable Displacement Transducer) and LVT (Linear Velocity Transducer). However, it is impossible to equip industrial vehicles with all of these sensors due to problems such as cost [53]. In particular, the vertical velocity of sprung and unsprung masses is not available because LVT cannot be realized in real vehicles. Therefore, in this paper, we choose only suspension and tire deflection as measurement output from the view of the practical implementation of the controller.

As above mentioned, the sprung mass $m_s(t)$ and the unsprung mass $m_u(t)$ are considered as uncertainties which have the minimum and maximum values. Assume that the

TABLE 1. IF-THEN Rules.

Rule	ξ_s	ξ_u	Weight
1	Heavy	Heavy	$\mu_1(\xi(t)) = M_1(\xi_s(t)) \times N_1(\xi_u(t))$
2	Heavy	Light	$\mu_2(\xi(t)) = M_1(\xi_s(t)) \times N_2(\xi_u(t))$
3	Light	Heavy	$\mu_3(\xi(t)) = M_2(\xi_s(t)) \times N_1(\xi_u(t))$
4	Light	Light	$\mu_4(\xi(t)) = M_2(\xi_s(t)) \times N_2(\xi_u(t))$

sprung and unsprung masses can be measured through online and offline estimation methods [54], [55]. Let $m_{s,\min}$ and $m_{s,\max}$ denote the minimum and maximum weight of the sprung mass, respectively; and $m_{u,\min}$ and $m_{u,\max}$ indicate the minimum and maximum weight of the unsprung mass, respectively. By defining the premise variables as $\xi_s(t) = 1/m_s(t)$ and $\xi_u(t) = 1/m_u(t)$, the uncertainty terms can be expressed by using the sector nonlinear method as follows:

$$\begin{aligned} \xi_s(t) &= M_1(\xi_s(t))\acute{m}_s + M_2(\xi_s(t))\grave{m}_s, \\ \xi_u(t) &= N_1(\xi_u(t))\acute{m}_u + N_2(\xi_u(t))\grave{m}_u, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \acute{m}_s &:= \max \xi_s(t), \quad \grave{m}_s := \min \xi_s(t), \\ \acute{m}_u &:= \max \xi_u(t), \quad \grave{m}_u := \min \xi_u(t), \end{aligned}$$

and

$$\begin{aligned} M_1(\xi_s(t)) + M_2(\xi_s(t)) &= 1, \\ N_1(\xi_u(u)) + N_2(\xi_u(t)) &= 1. \end{aligned}$$

We can define the membership functions as follows:

$$\begin{aligned} &M_1(\xi_s(t)) = frac{\xi_s(t) - \grave{m}_s}{\acute{m}_s - \grave{m}_s}, M_2(\xi_s(t)) = \frac{\acute{m}_s - \xi_s(t)}{\acute{m}_s - \grave{m}_s}, \\ &N_1(\xi_u(t)) = frac{\xi_u(t) - \grave{m}_u}{\acute{m}_u - \grave{m}_u}, N_2(\xi_u(t))\frac{\acute{m}_u - \xi_u(t)}{\acute{m}_u - \grave{m}_u}. \end{aligned} \tag{10}$$

The membership functions $M_1(\xi_s(t))$ and $N_1(\xi_u(t))$ correspond to "Heavy", the membership functions $M_2(\xi_s(t))$ and $N_2(\xi_u(t))$ correspond to "Light." Therefore, based on the fuzzy rules presented in Table 1, the T-S fuzzy model for ASSs can be defined as follows:

$$\begin{aligned} \dot{x}(t) &= \bar{A}_i x(t) + \bar{B}_{u,i} u(t) + \bar{B}_{w,i} w(t), \\ z_1(t) &= \bar{C}_{1,i} x(t) + \bar{D}_{1,i} u(t), \\ z_2(t) &= BarC_{2,i} x(t), \\ y(t) &= Cx(t) + Dv(t), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \bar{A}_i &:= \sum_{i=1}^{4} \mu_i(\xi(t))A_i, \ \bar{B}_{u,i} := \sum_{i=1}^{4} \mu_i(\xi(t))B_{u,i}, \\ \bar{B}_{w,i} &:= \sum_{i=1}^{4} \mu_i(\xi(t))B_{w,i}, \ \bar{C}_{1,i} := \sum_{i=1}^{4} \mu_i(\xi(t))C_{1,i}, \\ \bar{D}_{1,i} &:= \sum_{i=1}^{4} \mu_i(\xi(t))D_{1,i}, \ \bar{C}_{2,i} := \sum_{i=1}^{4} \mu_i(\xi(t))C_{2,i}, \end{aligned}$$

and $\mu_i(\xi(t))$ represents the weighting functions that satisfy $\mu_i(\xi(t)) \geq 0$ and $\sum_{i=1}^{4} \mu_i(\xi(t)) = 1$.

The sampled-data fuzzy controller based on the PDC scheme can be expressed as follows:

$$u(t) = \bar{K}_j x(t_k), \tag{12}$$

where

$$\bar{K}_j := \sum_{j=1}^{4} \mu_j(\xi(t))K_j.$$

It is assumed that the membership functions between the system and PDC fuzzy controller can be real-time matching. By substituting (12) into (11), we can express the closed-loop system as follows:

$$\begin{aligned} \dot{x}(t) &= \bar{A}_i x(t) + \bar{B}_{u,i}\bar{K}_j x(t_k) + \bar{B}_{w,i} w(t), \\ z_1(t) &= BarC_{1,i} x(t) + \bar{D}_{1,i}\bar{K}_j x(t_k), \\ z_2(t) &= \bar{C}_{2,i} x(t), \\ y(t) &= Cx(t) + Dv(t). \end{aligned} \tag{13}$$

The objective of designing a fuzzy controller of the form given in (12) for the system expressed in (11) is to determine the fuzzy controller gain $\bar{K}_j$ $(j = 1, \ldots, 4)$, such that the following conditions are satisfied for all nonzero $w(t) \in L_2[0, \infty)$ under the zero initial condition: For the closed-loop system in (13) 1) the asymptotic stability; 2) $||z_1||_2 < \gamma_c ||w||_2$ with a given attenuation level $\gamma_c > 0$; 3) hard constraints $|u(t)| \leq u_{\max}, \quad |\{z_2(t)\}_r| \leq 1, \quad (r = 1, 2)$.

As mentioned previously, we assume that all state variables are not measurable owing to unavailable sensors like LVTs. By defining the filter state vector as $\hat{x}(t)$, the fuzzy filter equation with sampled measurements can be established as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \bar{A}_i \hat{x}(t) + \bar{B}_{u,i} u(t) + \bar{L}_i (y(t_k) - \hat{y}(t_k)), \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \tag{14}$$

where

$$\bar{L}_i := \sum_{i=1}^{4} \mu_i(\xi(t))L_i.$$

Then, the control input using estimated state can be expressed as follows:

$$u(t) = \bar{K}_j \hat{x}(t). \tag{15}$$

The filtering error state vector is defined as $e(t) = x(t) - \hat{x}(t)$. Subsequently, the fuzzy filtering error system can be established using (11) and (14).

$$\begin{aligned} \dot{e}(t) &= \bar{A}_i e(t) - \bar{L}_i Ce(t_k) + \bar{B}_{w,i} w(t) - \bar{L}_i Dv(t_k), \\ \tilde{z}(t) &= He(t). \end{aligned} \tag{16}$$

For the $\mathcal{L}_2 - \mathcal{L}_\infty$ filter design, the main aim is to select the filter gain $\bar{L}_i$ $(i = 1, \ldots, 4)$, such that the following requirements are satisfied for all $w, v \in L_2[0, \infty)$ under the zero condition:

1) The filtering error system in (16) is asymptotically stable.
2) Given an attenuation level $\gamma_f > 0$, the filtering error system in (16) guarantees the following performance:

$$\sup_{t\geq 0}\{\tilde{z}^T(t)\tilde{z}(t)\} < \gamma_f^2[\int_0^\infty w^T(t)w(t)dt + \sum_{k=0}^{\infty} h_k v^T(t_k)v(t_k)]. \quad (17)$$

By defining $h(t) = t - t_k$ for $t \in [t_k, t_{k+1})$, we can use the zero-order-hold (ZOH) sampling method as an input delay approach [28]. In this case, $0 \leq h(t) < t_{k+1} - t_k = h_k \leq h_M$, and $\dot{h}(t) = 1$, where h_M indicates the maximum allowable sampling time. By following this approach, the closed-loop system and error system can be redefined as follows:

$$\begin{aligned} \dot{x}(t) &= \bar{A}_i x(t) + \bar{B}_{u,i}\bar{K}_j x(t-h(t)) + \bar{B}_{w,i}w(t), \\ z_1(t) &= \bar{C}_{1,i}x(t) + \bar{D}_{1,i}\bar{K}_j x(t-h(t)), \\ z_2(t) &= \bar{C}_{2,i}x(t), \end{aligned} \quad (18)$$

and

$$\begin{aligned} \dot{e}(t) &= \bar{A}_i e(t) - \bar{L}_i Ce(t-h(t)) + \bar{B}_{w,i}w(t) \\ &\quad - \bar{L}_i Dv(t-h(t)), \\ \tilde{z}(t) &= He(t). \end{aligned} \quad (19)$$

Before proceeding, we introduce the following lemmas, which are indispensable to derive our main results.

Lemma 1 [38]: For an arbitrary matrix $R > 0$, the scalars a and b with $0 < a < b$, and the vector $\alpha : [a, b] \to \mathbb{R}^n$, the following inequality holds:

$$(b-a)\int_a^b \dot{\alpha}^T(u)R\dot{\alpha}(u)du \geq \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^T \begin{bmatrix} R & 0 \\ * & 3R\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}, \quad (20)$$

where $\alpha_1 = \alpha(b) - \alpha(a)$ and $\alpha_2 = \alpha(a) + \alpha(b) - \frac{2}{b-a}\int_a^b \alpha(u)du$.

Lemma 2 [39]: For a real scalar $\beta \in (0, 1)$, the matrices $R_1 > 0$ and $R_2 > 0$, and the appropriate dimension matrices S_1 and S_2, the following matrix inequality holds:

$$\begin{bmatrix} \frac{1}{\beta}R_1 & 0 \\ 0 & \frac{1}{1-\beta}R_2 \end{bmatrix} \geq \begin{bmatrix} R_1 + (1-\beta)T_1 & (1-\beta)T_1 + \beta T_2 \\ * & R_2 + \beta T_2 \end{bmatrix}, \quad (21)$$

where $T_1 = R_1 - S_2 R_2^{-1} S_2^T$, and $T_2 = R_2 - S_1^T R_1^{-1} S_1$.

III. MAIN RESULTS

In this section, we present the new LMI conditions for a sampled-data $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output-feedback controller to ensure its asymptotic stability under hard constraints by using Wirtinger's inequality and the extended reciprocally convex combination lemma.

A. SAMPLED-DATA FUZZY CONTROLLER

Theorem 1: For the given constants $\rho > 0$ and $h_k > 0$, if there exist positive definite matrices P, Q_1, Q_2, R, and S_1; a symmetric matrix S_2; any appropriate dimension matrices T_1, T_2, U_1, U_2, and U_3; and a scalar $\gamma_c > 0$ for $i, j = (1, \ldots, 4)$, the following conditions hold:

$$\Theta_{1,ii} < 0, \quad (22)$$

$$\Theta_{2,ii} < 0, \quad (23)$$

$$\Theta_{1,ij} + \Theta_{1,ji} < 0, (i < j) \quad (24)$$

$$\Theta_{2,ij} + \Theta_{2,ji} < 0, (i < j) \quad (25)$$

$$\begin{bmatrix} -u_{\max}^2 P & \sqrt{\rho}\bar{K}_j^T \\ * & -I \end{bmatrix} < 0, \quad (26)$$

$$\begin{bmatrix} -P & \sqrt{\rho}\{\bar{C}_{2,i}\}_r^T \\ * & -I \end{bmatrix} < 0, \quad (27)$$

where

$$\Theta_{1,ij} = \begin{bmatrix} \Xi_{1,ij} & \Xi_{3,ij} & E_2^T T_1^T \\ * & -I & 0 \\ * & * & -2R_a \end{bmatrix},$$

$$\Theta_{2,ij} = \begin{bmatrix} \Xi_{2,ij} & \Xi_{3,ij} & E_1^T T_2 \\ * & -I & 0 \\ * & * & -R_a \end{bmatrix},$$

$$\begin{aligned}
\Xi_{1,ij} &= \Pi_1 - \Pi_2 - \Pi_3 + \mathrm{Sym}\{e_U^T e_{s,ij}\} - \gamma_c^2 e_7^T e_7, \\
\Xi_{2,ij} &= \Pi_1 - \Pi_4 - \Pi_5 + \mathrm{Sym}\{e_U^T e_{s,ij}\} - \gamma_c^2 e_7^T e_7, \\
\Xi_{3,ij} &= [\bar{C}_{1,i}e_1 + \bar{D}_{1,i}\bar{K}_j e_3]^T, \\
\Pi_1 &= e_1^T Q_1 e_1 + 2e_1^T P e_6 - e_3^T Q_1 e_3 + h_k^2 e_6^T R e_6, \\
\Pi_2 &= e_1^T S_1 e_1 - 2e_1^T S_1 e_2 + \mathrm{Sym}\{e_1^T S_2 e_2 + e_2^T S_2 e_2\} \\
&\quad + e_2^T(h_k Q_2 + S_1)e_2, \\
\Pi_3 &= 4E_1^T R_a E_1 + \mathrm{Sym}\{E_1^T T_1 E_2\} + E_2^T R_a E_2, \\
\Pi_4 &= e_1^T S_1 e_1 - 2e_1^T S_1 e_2 + 2h_k e_1^T S_1 e_6 - e_2^T(h_k Q_2 - S_1)e_2 \\
&\quad + 2h_k e_2^T S_1 e_6 + \mathrm{Sym}\{e_1^T S_2 e_2 - e_2^T S_2 e_2 - h_k e_2^T S_2^T e_6\} \\
&\quad - h_k^2 e_6^T R e_6, \\
\Pi_5 &= 2E_1^T R_a E_1 + \mathrm{Sym}\{E_1 T_2^T E_2\} + 2E_2^T R_a E_2, \\
E_l &= [e_l^T - e_{l+1}^T,\ e_l^T + e_{l+1}^T - 2e_{l+4}^T]^T, \quad (l = 1, 2) \\
e_{s,ij} &= \bar{A}_i e_1 + \bar{B}_{u,i}\bar{K}_j e_3 + \bar{B}_{w,i}e_7, \\
e_u &= U_1^T e_1 + U_2^T e_2 + U_3^T e_6, \\
e_k &= [0_{n\times(k-1)n},\ I_{n\times n},\ 0_{n\times(7-k)n}], \quad (k = 1, \ldots, 7) \\
R_a &= \mathrm{Diag}\{R, 3R\}.
\end{aligned}$$

Then, the closed-loop system (18) ensures asymptotic stability, and has an H_∞ performance under $w(t)$ for prescribed bound γ_c subject to the hard constraints, which satisfies $0 \leq h(t) = t - t_k \leq t_{k+1} - t_k = h_k$.

Proof) Consider the following LKF candidate:

$$V(t) = \sum_{i=1}^{6} V_i(t), \quad (28)$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t),\\
V_2(t) &= \int_{t-h_k}^{t} x^T(s)Q_1x(s)ds,\\
V_3(t) &= h_k\int_{-h_k}^{0}\int_{t+u}^{t}\dot{x}^T(s)R\dot{x}(s)dsdu,\\
V_4(t) &= h(t)(h_k-h(t))x^T(t-h(t))Q_2x(t-h(t)),\\
V_5(t) &= h_k(h_k-h(t))\int_{t-h(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds,\\
V_6(t) &= (h_k-h(t))(x(t)-x(t-h(t)))^T\\
&\quad\cdot\{S_1(x(t)-x(t-h(t)))+2S_2x(t-h(t))\}.
\end{aligned}$$

Let define

$$\eta(t) = [x^T(t), x^T(t-h(t)), x^T(t-h_k), \int_{t-h(t)}^{t}\frac{x^T(s)}{h(t)}ds, \int_{t-h_k}^{t-h(t)}\frac{x^T(s)}{h_k-h(t)}ds, \dot{x}(t), w^T(t)]^T.$$

In this case, we can calculate the time derivative of $V(t)$ as follows:

$$\dot{V}(t) = \eta^T(t)(\Pi_1+\bar{\Pi}_2^{h(t)})\eta(t) - J_1 - J_2, \tag{29}$$

where

$$\begin{aligned}
\bar{\Pi}_2^{h(t)} &= (h_k-2h(t))e_2^TQ_2e_2 + h_k(h_k-h(t))e_6^TR_2e_6\\
&\quad+2(h_k-h(t))\{e_1^TS_1e_6+e_2^T(S_2^T-S_1)e_6\}\\
&\quad-e_1^TS_1e_1+2e_1^T(S_1-S_2)e_2-e_2^T(S_1-2S_2)e_2,\\
J_1 &= h_k\int_{t-h_k}^{t}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta,\\
J_2 &= h_k\int_{t-h(t)}^{t}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta.
\end{aligned}$$

By applying Lemmas 1 and 2 to the integral terms of $\dot{V}(t)$, we obtain

$$\begin{aligned}
&h_k\int_{t-h_k}^{t}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta + h_k\int_{t-h(t)}^{t}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta\\
&\geq \eta^T(t)\{\frac{1}{\alpha}E_1^T(2R_a)E_1+\frac{1}{1-\alpha}E_2^TR_aE_2\}\eta(t)\\
&\geq \eta^T(t)\{\bar{\Pi}_3+(1-\alpha)\bar{\Pi}_4+\alpha\bar{\Pi}_5\}\eta(t),
\end{aligned}\tag{30}$$

where

$$\begin{aligned}
\bar{\Pi}_3 &= 2E_1^TR_aE_1+E_2^TR_aE_2,\\
\bar{\Pi}_4 &= \begin{bmatrix}E_1\\E_2\end{bmatrix}^T\begin{bmatrix}2R_a-T_2R_a^{-1}T_2^T & T_1\\ * & 0\end{bmatrix}\begin{bmatrix}E_1\\E_2\end{bmatrix},\\
\bar{\Pi}_5 &= \begin{bmatrix}E_1\\E_2\end{bmatrix}^T\begin{bmatrix}0 & T_2\\ * & R_a-T_1^T(2R_a)^{-1}T_1\end{bmatrix}\begin{bmatrix}E_1\\E_2\end{bmatrix},\\
\alpha &= \frac{h(t)}{h_k}.
\end{aligned}$$

For any appropriate dimension matrices U_i $(i = 1, 2, 3)$, the following equation can be derived from system (18).

$$\begin{aligned}
&2\{x^T(t)U_1+x^T(t-h(t))U_2+\dot{x}^T(t)U_3\}\\
&\quad\times\{\bar{A}_ix(t)+\bar{B}_{u,i}\bar{K}_jx(t-h(t))+\bar{B}_{w,i}w(t)-\dot{x}(t)\}\\
&= 2\eta^T(t)e_U^Te_{s,ij}\eta(t).
\end{aligned}\tag{31}$$

By applying (30)-(31) to (29) and adding $z_1^T(t)z_1(t)-\gamma_c^2w^T(t)w(t)$ on both sides, the following expression can be obtained:

$$\dot{V}(t)+z_1^T(t)z_1(t)-\gamma_c^2w^T(t)w(t)\leq\eta^T(t)\Pi_{ij}^{h(t)}\eta(t), \tag{32}$$

where

$$\begin{aligned}
\Pi_{ij}^{h(t)} &= \Pi_1+\bar{\Pi}_2^{h(t)}-\bar{\Pi}_3-(1-\alpha)\bar{\Pi}_4-\alpha\bar{\Pi}_5\\
&\quad+\text{Sym}\{e_U^Te_{s,ij}\}+\Xi_{3,ij}\Xi_{3,ij}^T-\gamma_c^2e_7^Te_7.
\end{aligned}$$

If $\Pi_{ij}^{h(t)} < 0$, we can easily establish the H_∞ performance under zero initial conditions for all nonzero $w(t)\in\mathcal{L}_2[0,\infty)$ as the integral from zero to ∞, that is, $\|z_1(t)\|_2 < \gamma_c\|w(t)\|_2$.

Moreover, by applying the convex combination bounding technique, the inequality $f(\alpha) := (1-\alpha)\bar{\Pi}_4+\alpha\bar{\Pi}_5 > 0$ holds if $f(0) > 0$ and $f(1) > 0$. Therefore, $\Pi_{ij}^{h(t)} < 0$ if $\Pi_{ij}^{h(t)=0} < 0$ and $\Pi_{ij}^{h(t)=h_k} < 0$, and the conditions (22)-(25) can be derived using the Schur complement.

As described in an existing study [9], according to the LKF, $x^T(t)Px(t) < \rho$, where $\rho = V(0)+\gamma_c^2w_{\max}$. Therefore,

$$\begin{aligned}
\max_{t>0}|\{z_2(t)\}_r|^2 &= \max_{t>0}||x^T(t)\{\bar{C}_{2,i}\}_r^T\{\bar{C}_{2,i}\}_rx(t)||_2\\
&< \rho\cdot\lambda_{\max}(P^{-\frac{1}{2}}\{\bar{C}_{2,i}\}_r^T\{\bar{C}_{2,i}\}_rP^{-\frac{1}{2}}),
\end{aligned}\tag{33}$$

$$\begin{aligned}
\max_{t>0}|(u(t_k))|^2 &= \max_{t>0}||x^T(t-h(t))\bar{K}_j^T\bar{K}_jx(t-h(t))||_2\\
&< \rho\cdot\lambda_{\max}(P^{-\frac{1}{2}}\bar{K}_j^T\bar{K}_jP^{-\frac{1}{2}}),
\end{aligned}\tag{34}$$

where $\lambda_{\max}(\cdot)$ indicates the maximal eigenvalue. As a result, the input and output constraints are guaranteed if

$$\rho P^{-\frac{1}{2}}\bar{K}_j^T\bar{K}_jP^{-\frac{1}{2}} < u_{\max}^2I, \tag{35}$$

$$\rho P^{-\frac{1}{2}}\{\bar{C}_{2,i}\}_r^T\{\bar{C}_{2,i}\}_rP^{-\frac{1}{2}} < I, \tag{36}$$

where $i = 1,\ldots,4$, $j = 1,\ldots,4$, and $r = 1,2$. According to the Schur complement, the conditions in (26) and (27) are equivalent to (35) and (36), respectively.

If the controller gain $\bar{K}_j$ is unknown, the conditions in Theorem 1 are not LMIs because of the terms $\Xi_{1,ij}$ and $\Xi_{2,ij}$. Therefore, we cannot find the solutions easily by using any of the existing LMI tools. For this reason, we transform the conditions in (22)-(25) appropriately to calculate the controller gains $\bar{K}_j$ $(j = 1,\ldots,4)$. By using the following theorem, we present the LMI conditions required to obtain the desired controller gains.

Theorem 2: Given the constants $\rho > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, and $h_k > 0$, if there exist positive definite matrices $\hat{P}$, $\hat{Q}_1$, $\hat{Q}_2$, $\hat{R}$, and $\hat{S}_1$; a symmetric matrix $\hat{S}_2$; any appropriate dimension matrices $\bar{G}_j$, $\hat{T}_1$, $\hat{T}_2$, and U; and a scalar $\gamma_c > 0$ for $i, j = (1, \ldots, 4)$,

$$\hat{\Theta}_{1,ii} < 0, \tag{37}$$

$$\hat{\Theta}_{2,ii} < 0, \tag{38}$$

$$\hat{\Theta}_{1,ij} + \hat{\Theta}_{1,ji} < 0, (i < j) \tag{39}$$

$$\hat{\Theta}_{2,ij} + \hat{\Theta}_{2,ji} < 0, (i < j) \tag{40}$$

$$\begin{bmatrix} -u_{\max}^2 \hat{P} & \sqrt{\rho}\bar{G}_j^T \\ * & -I \end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix} -\hat{P} & \sqrt{\rho}U\{\bar{C}_{2,i}\}_r^T \\ * & -I \end{bmatrix} < 0, \tag{42}$$

where

$$\hat{\Theta}_{1,ij} = \begin{bmatrix} \hat{\Xi}_{1,ij} & \hat{\Xi}_{3,ij} & E_2^T \hat{T}_1^T \\ * & -I & 0 \\ * & * & -2\hat{R}_a \end{bmatrix},$$

$$\hat{\Theta}_{2,ij} = \begin{bmatrix} \hat{\Xi}_{2,ij} & \hat{\Xi}_{3,ij} & E_1^T \hat{T}_2 \\ * & -I & 0 \\ * & * & -\hat{R}_a \end{bmatrix},$$

$$\begin{aligned}
\hat{\Xi}_{1,ij} &= \hat{\Pi}_1 + \hat{\Pi}_2 - \hat{\Pi}_3 + \mathrm{Sym}\{e_{\hat{u}}^T e_{\hat{s},ij}\} - \gamma_c^2 e_7^T e_7, \\
\hat{\Xi}_{2,ij} &= \hat{\Pi}_1 + \hat{\Pi}_4 - \hat{\Pi}_5 + \mathrm{Sym}\{e_{\hat{u}}^T e_{\hat{s},ij}\} - \gamma_c^2 e_7^T e_7, \\
\hat{\Xi}_{3,ij} &= [\bar{C}_{1,i} e_1 + \bar{D}_{1,i} \bar{G}_j e_3]^T, \\
\hat{\Pi}_1 &= e_1^T \hat{Q}_1 e_1 + 2e_1^T \hat{P} e_6 - e_3^T \hat{Q}_1 e_3 + h_k^2 e_6^T \hat{R} e_6, \\
\hat{\Pi}_2 &= e_1^T \hat{S}_1 e_1 - 2e_1^T \hat{S}_1 e_2 + \mathrm{Sym}\{e_1^T \hat{S}_2 e_2 + e_2^T \hat{S}_2 e_2\} \\
&\quad + e_2^T (h_k \hat{Q}_2 + \hat{S}_1) e_2, \\
\hat{\Pi}_3 &= 4E_1^T \hat{R}_a E_1 + \mathrm{Sym}\{E_1^T \hat{T}_1 E_2\} + E_2^T \hat{R}_a E_2, \\
\hat{\Pi}_4 &= e_1^T \hat{S}_1 e_1 - 2e_1^T \hat{S}_1 e_2 + 2h_k e_1^T \hat{S}_1 e_6 - e_2^T (h_k \hat{Q}_2 - \hat{S}_1) e_2 \\
&\quad + 2h_k e_2^T e_6 + \mathrm{Sym}\{e_1^T \hat{S}_2 e_2 - e_2^T \hat{S}_2 e_2 - h_k e_2^T \hat{S}_2^T e_6\} \\
&\quad - h_k^2 e_6^T \hat{R} e_6, \\
\hat{\Pi}_5 &= 2E_1^T \hat{R}_a E_1 + \mathrm{Sym}\{E_1 \hat{T}_2^T E_2\} + 2E_2^T \hat{R}_a E_2, \\
e_{\hat{s},ij} &= \bar{A}_i U^T e_1 + \bar{B}_{u,i} \bar{G}_j e_3 + \bar{B}_{w,i} e_7, \\
e_{\hat{u}} &= e_1 + \epsilon_1 e_2 + \epsilon_2 e_6, \\
\hat{R}_a &= \mathrm{Diag}\{\hat{R}, 3\hat{R}\}.
\end{aligned}$$

Then, a T-S fuzzy controller exists that can ensure the asymptotic stability, and has an H_∞ performance under $w(t)$ for prescribed bound γ_c, which satisfies $0 \leq h(t) = t - t_k \leq t_{k+1} - t_k = h_k$. The fuzzy controller gain can be calculated as $\bar{K}_j = \bar{G}_j U^{-T}$.

Proof) We define $U = U_1^{-1}$, $U_2 = \epsilon_1 U_1$, $U_3 = \epsilon_2 U_1$, $\Omega_1 = \mathrm{Diag}\{U, U, U, U, U, U, I, I, U, U\}$, $\Omega_2 = \mathrm{Diag}\{U, I\}$, and $J = \mathrm{Diag}\{U^{-1}, U^{-1}\}$. And, redefine $P = U^{-1}\hat{P}U^{-T}$, $Q_1 = U^{-1}\hat{Q}_1 U^{-T}$, $Q_2 = U^{-1}\hat{Q}_2 U^{-T}$, $R = U^{-1}\hat{R}U^{-T}$, $S_1 = U^{-1}\hat{S}_1 U^{-T}$, $S_2 = U^{-1}\hat{S}_2 U^{-T}$, $T_1 = J\hat{T}_1 J^T$, $T_2 = J\hat{T}_2 J^T$, and $\bar{G}_j = \bar{K}_j U^T$ in Theorem 1. By pre- and post-multiplying (22)-(25) with Ω_1 and Ω_1^T, we can obtain (37)-(40), respectively. Also, (41)-(42) can be derived by pre- and post-multiplying with Ω_2 and Ω_2^T to (26) and (27). Then, the PDC controller gain can be obtained as $\bar{K}_j = \bar{G}_j U^{-T}$ from solving LMI conditions (37)-(42).This completes the proof of Theorem 2.

B. SAMPLED-DATA $\mathcal{L}_2 - \mathcal{L}_\infty$ FILTER

Theorem 3: For given constant $h_k > 0$, if there exist the positive definite matrices $\mathcal{P}$, $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{R}$, and $\mathcal{S}_1$; appropriate dimension matrices $\mathcal{S}_2 = \mathcal{S}_2^T$, $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{X}_1$, $\mathcal{X}_2$, and $\mathcal{X}_3$; and a scalar $\gamma_f > 0$ for $i = (1, \ldots, 4)$, the following conditions hold:

$$\begin{bmatrix} \Psi_{1,i} + \Psi_2 - \Psi_3 & \hat{E}_2^T \mathcal{T}_1^T \\ * & -2\mathcal{R}_a \end{bmatrix} < 0, \tag{43}$$

$$\begin{bmatrix} \Psi_{1,i} + \Psi_4 - \Psi_5 & \hat{E}_1^T \mathcal{T}_2 \\ * & -\mathcal{R}_a \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} \mathcal{P} & \mathcal{H}^T \\ * & \gamma_f^2 I \end{bmatrix} > 0, \tag{45}$$

where

$$\begin{aligned}
\Psi_{1,i} &= \hat{e}_1^T \mathcal{Q}_1 \hat{e}_1 - \hat{e}_3^T \mathcal{Q}_1 \hat{e}_3 + \mathrm{Sym}\{\hat{e}_1^T \mathcal{P} \hat{e}_6 + \hat{e}_{\mathcal{X}}^T \hat{e}_{s,i}\} \\
&\quad + h_k^2 \hat{e}_6^T \mathcal{R} \hat{e}_6 - \hat{e}_7^T \hat{e}_7 - \hat{e}_8^T \hat{e}_8, \\
\Psi_2 &= \mathrm{Sym}\{\hat{e}_0^T \mathcal{S}_2 \hat{e}_2\} - h_k \hat{e}_2^T \mathcal{Q}_2 \hat{e}_2 - \hat{e}_0^T \mathcal{S}_1 \hat{e}_0, \\
\Psi_3 &= \mathrm{Sym}\{\hat{E}_2^T \mathcal{T}_2 \hat{E}_1\} + 2\hat{E}_1^T \mathcal{R}_a \hat{E}_1 + 2\hat{E}_2^T \mathcal{R}_a \hat{E}_2, \\
\Psi_4 &= \mathrm{Sym}\{h_k \hat{e}_0^T \mathcal{S}_1 \hat{e}_6 + h_k \hat{e}_2^T \mathcal{S}_2 \hat{e}_6 + \hat{e}_0^T \mathcal{S}_2 \hat{e}_2\} \\
&\quad + h_k \hat{e}_2^T \mathcal{Q}_2 \hat{e}_2 + h_k^2 \hat{e}_6^T \mathcal{R} \hat{e}_6 - \hat{e}_0^T \mathcal{S}_1 \hat{e}_0, \\
\Psi_5 &= \mathrm{Sym}\{\hat{E}_1^T \mathcal{T}_1 \hat{E}_2\} + 4\hat{E}_1^T \mathcal{R}_a \hat{E}_1 + \hat{E}_2^T \mathcal{R}_a \hat{E}_2, \\
\hat{E}_l &= [\hat{e}_l^T - \hat{e}_{l+1}^T, \hat{e}_l^T + \hat{e}_{l+1}^T - 2\hat{e}_{l+3}^T]^T, \quad (l = 1, 2) \\
\hat{e}_{s,i} &= \bar{A}_i \hat{e}_1 - \bar{L}_i C \hat{e}_2 + \bar{B}_{w,i} \hat{e}_7 - \bar{L}_i D \hat{e}_8 - \hat{e}_6, \\
\hat{e}_{\mathcal{X}} &= \mathcal{X}_1^T \hat{e}_1 + \mathcal{X}_2^T \hat{e}_2 + \mathcal{X}_3^T \hat{e}_6, \\
\hat{e}_0 &= \hat{e}_1 - \hat{e}_2, \\
\hat{e}_k &= [0_{(k-1)\times n}, I_n, 0_{(8-k)\times n}], \quad (k = 1, \ldots, 8) \\
\mathcal{R}_a &= \mathrm{Diag}\{\mathcal{R}, 3\mathcal{R}\}.
\end{aligned}$$

Then, the filtering error system (19) is asymptotically stable, and has an $\mathcal{L}_2 - \mathcal{L}_\infty$ performance (17) under $w(t)$, and $v(t)$ for a given attenuation level γ_f, which satisfies $0 \leq h(t) = t - t_k \leq t_{k+1} - t_k = h_k$.

Proof) Consider the following LKF candidate;

$$V(t) = \sum_{i=1}^{6} V_i(t), \tag{46}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t) \mathcal{P} e(t), \\
V_2(t) &= \int_{t-h_k}^{t} e^T(s) \mathcal{Q}_1 e(s) ds, \\
V_3(t) &= h_k \int_{-h_k}^{0} \int_{t+s}^{t} \dot{e}^T(u) \mathcal{R} \dot{e}(u) du ds, \\
V_4(t) &= h(t)(h_k - h(t)) e^T(t - h(t)) \mathcal{Q}_2 e(t - h(t)), \\
V_5(t) &= h_k(h_k - h(t)) \int_{t-h(t)}^{t} \dot{e}^T(s) \mathcal{R} \dot{e}(s) ds, \\
V_6(t) &= (h_k - h(t))(e(t) - e(t - h(t)))^T \\
&\quad \cdot \{\mathcal{S}_1(e(t) - e(t - h(t))) + 2\mathcal{S}_2 e(t - h(t))\}.
\end{aligned}$$

Let define

$$\zeta(t) = [e^T(t), e^T(t-h(t)), e^T(t-h_k), \int_{t-h(t)}^{t} \frac{e^T(s)}{h(t)}ds, \int_{t-h_k}^{t-h(t)} \frac{e^T(s)}{h_k-h(t)}ds, \dot{e}^T(t), w^T(t), v^T(t_k)]^T.$$

Then, we can calculate the time derivative of $V(t)$ as follows:

$$\dot{V}(t) = \zeta^T(t)(\bar{\Psi}_1 + \bar{\Psi}_2^{h(t)})\zeta(t) - \bar{J}_1 - \bar{J}_2, \quad (47)$$

where

$$\begin{aligned}
\bar{\Psi}_1 &= 2\hat{e}_1^T\mathcal{P}\hat{e}_6 + \hat{e}_1^T\mathcal{Q}_1\hat{e}_1 - \hat{e}_3^T\mathcal{Q}_1\hat{e}_3 + h_k^2\hat{e}_6^T\mathcal{R}\hat{e}_6,\\
\bar{\Psi}_2^{h(t)} &= -\hat{e}_0^T\mathcal{S}_1\hat{e}_0 + 2\hat{e}_0^T\mathcal{S}_2\hat{e}_2 + (h_k-2h(t))\hat{e}_2^T\mathcal{Q}_2\hat{e}_2\\
&\quad +2(h_k-h(t))(\hat{e}_0^T\mathcal{S}_1\hat{e}_6 + \hat{e}_2^T\mathcal{S}_2\hat{e}_6)\\
&\quad +h_k(h_k-h(t))\hat{e}_6^T\mathcal{R}\hat{e}_6,\\
\bar{J}_1 &= h_k\int_{t-h_k}^{t} \dot{e}^T(\theta)\mathcal{R}\dot{e}(\theta)d\theta,\\
\bar{J}_2 &= h_k\int_{t-h(t)}^{t} \dot{e}^T(\theta)\mathcal{R}e(\theta)d\theta.
\end{aligned}$$

By applying Lemma 1, we obtain the following expression:

$$\begin{aligned}
&h_k\int_{t-h_k}^{t} \dot{e}^T(\theta)\mathcal{R}\dot{e}(\theta)d\theta\\
&\quad\geq \zeta^T(t)\{\frac{1}{\alpha}\hat{E}_1^T\mathcal{R}_a\hat{E}_1 + \frac{1}{1-\alpha}\hat{E}_2^T\mathcal{R}_a\hat{E}_2\}\zeta(t),\\
&h_k\int_{t-h(t)}^{t} \dot{e}^T(\theta)\mathcal{R}\dot{e}(\theta)d\theta\\
&\quad\geq \frac{1}{\alpha}\zeta^T(t)\hat{E}_1^T\mathcal{R}_a\hat{E}_1\zeta(t).
\end{aligned} \quad (48)$$

According to Lemma 1, the reciprocally convex terms can be calculated as follows:

$$\begin{aligned}
&\frac{1}{\alpha}\hat{E}_1^T(2\mathcal{R}_a)\hat{E}_1 + \frac{1}{1-\alpha}\hat{E}_2^T\mathcal{R}_a\hat{E}_2\\
&\quad\geq \bar{\Psi}_3 + (1-\alpha)\bar{\Psi}_4 + \alpha\bar{\Psi}_5,
\end{aligned} \quad (49)$$

where

$$\begin{aligned}
\bar{\Psi}_3 &= 2\hat{E}_1^T\mathcal{R}_a\hat{E}_1 + \hat{E}_2^T\mathcal{R}_a\hat{E}_2,\\
\bar{\Psi}_4 &= \begin{bmatrix}\hat{E}_1\\ \hat{E}_2\end{bmatrix}^T \begin{bmatrix}2\mathcal{R}_a - \mathcal{T}_2\mathcal{R}_a^{-1}\mathcal{T}_2^T & \mathcal{T}_1\\ * & 0\end{bmatrix} \begin{bmatrix}\hat{E}_1\\ \hat{E}_2\end{bmatrix},\\
\bar{\Psi}_5 &= \begin{bmatrix}\hat{E}_1\\ \hat{E}_2\end{bmatrix}^T \begin{bmatrix}0 & \mathcal{T}_2\\ * & \mathcal{R}_a - \mathcal{T}_1^T(2\mathcal{R}_a)^{-1}\mathcal{T}_1\end{bmatrix} \begin{bmatrix}\hat{E}_1\\ \hat{E}_2\end{bmatrix}.
\end{aligned}$$

For any appropriate dimension matrices $\mathcal{X}_i$ $(i = 1, 2, 3)$, the following equation can be derived from system (16).

$$\begin{aligned}
&2\{e^T(t)\mathcal{X}_1 + e^T(t-h(t))\mathcal{X}_2 + \dot{e}^T(t)\mathcal{X}_3\}\\
&\quad\times\{\bar{A}_ie(t) - \bar{L}_iCe(t-h(t)) + \bar{B}_{w,i}w(t) - \bar{L}_iDv(t_k) - \dot{e}(t)\}\\
&= 2\zeta^T(t)\hat{e}_{\mathcal{X}}^T\hat{e}_{s,i}\zeta(t).
\end{aligned} \quad (50)$$

By adding $-w^T(t)w(t) - v^T(t_k)v(t_k)$ to both sides of the inequality derived from (47)-(50), we can obtain the following expression:

$$\dot{V}(t) - w^T(t)w(t) - v^T(t_k)v(t_k) \leq \zeta^T(t)\bar{\Psi}^{h(t)}\zeta(t), \quad (51)$$

where

$$\bar{\Psi}_i^{h(t)} = \Psi_{1,i} + \bar{\Psi}_2^{h(t)} - \bar{\Psi}_3 - \bar{\Psi}_4 - \bar{\Psi}_5.$$

If $\bar{\Psi}_i^{h(t)} < 0$, then

$$\dot{V}(t) < w^T(t)w(t) + v^T(t_k)v(t_k). \quad (52)$$

By integrating (52) from 0 to t under the zero initial condition, we obtain

$$0 \leq V(t) - V(0) < \int_0^t w^T(s)w(s)ds + \sum_{k=0}^{n} h_kv^T(t_k)v(t_k). \quad (53)$$

Moreover, the LMI condition (45) indicates that

$$\mathcal{P} - \frac{1}{\gamma_f^2}\mathcal{H}^T\mathcal{H} > 0. \quad (54)$$

By using this equation, we can derive the $\mathcal{L}_2-\mathcal{L}_\infty$ performance as follows:

$$\begin{aligned}
\tilde{z}^T(t)\tilde{z}(t) &= e^T(t)\mathcal{H}^T\mathcal{H}e(t)\\
&< \gamma_f^2e^T(t)\mathcal{P}e(t)\\
&\leq \gamma_f^2V(t)\\
&< \gamma_f^2[\int_0^t w^T(s)w(s)ds + \sum_{k=0}^{n} h_kv^T(t_k)v(t_k)]\\
&\leq \gamma_f^2[\int_0^\infty w^T(t)w(t)dt + \sum_{k=0}^{\infty} h_kv^T(t_k)v(t_k)].
\end{aligned} \quad (55)$$

Therefore, if the inequalities (52) and (55) hold, the state estimation error system (19) can guarantee asymptotic stability of the system and has an $\mathcal{L}_2-\mathcal{L}_\infty$ performance against external disturbances and sampled noise.

Using the convex combination technique, the inequality $\bar{\Psi}_i^{h(t)} < 0$ can be considered equivalent to $\bar{\Psi}_i^{h(t)=0} < 0$ and $\bar{\Psi}_i^{h(t)=h_k} < 0$. Then, we can obtain Theorem 1 by applying the Schur complement. This completes the proof.

Next, we transform the conditions in Theorem 3 into LMIs when the filter gain is not given. In the following theorem, the LMI conditions and its proof are presented to obtain the filter gains L_i $(i = 1, \ldots, 4)$,

Theorem 4: For the constants μ_1, μ_2, and $h_k > 0$, if there exist the matrices $\mathcal{P} > 0$, $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{R} > 0$, $\mathcal{S}_1 > 0$, $\mathcal{S}_2 = \mathcal{S}_2^T$, $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{X}$, and $\bar{M}_i$ with appropriate dimensions and a scalar $\gamma_f > 0$, such that

$$\begin{bmatrix}\hat{\Psi}_{1,i} + \Psi_2 - \Psi_3 & \hat{E}_2^T\mathcal{T}_1^T\\ * & -2\mathcal{R}_a\end{bmatrix} < 0, \quad (56)$$

$$\begin{bmatrix}\hat{\Psi}_{1,i} + \Psi_4 - \Psi_5 & \hat{E}_1^T\mathcal{T}_2\\ * & -\mathcal{R}_a\end{bmatrix} < 0, \quad (57)$$

$$\begin{bmatrix}\mathcal{P} & \mathcal{H}^T\\ * & \gamma_f^2I\end{bmatrix} > 0, \quad (58)$$

TABLE 2. Vehicle parameters.

Parameter	k_s	k_u	c_s	c_u
Unit	N/m	N/m	$N \cdot s/m$	$N \cdot s/m$
Value	42720	101115	1095	14.6

TABLE 3. Attenuation levels corresponding to different sampling times.

h_k	$1(ms)$	$5(ms)$	$10(ms)$	$15(ms)$	$20(ms)$
γ_c	12.7459	14.2902	15.3211	17.5828	18.4011
γ_f	3.6747	3.8551	4.2034	4.6230	4.9119

TABLE 4. Maximum allowable sampling time as determined using various methods.

Methods	[35]	[37]	[38]	Theorem 2	Theorem 4
Controller	0.226s	0.266s	0.290s	**0.307s**	-
Filter	0.152s	0.164s	0.188s	-	**0.202s**

where

$$
\begin{aligned}
\hat{\Psi}_1 &= \hat{e}_1^T \mathcal{Q}_1 \hat{e}_1 - \hat{e}_3^T \mathcal{Q}_1 \hat{e}_3 + \mathrm{Sym}\{\hat{e}_1^T \mathcal{P} \hat{e}_6 + \hat{e}_{\hat{\mathcal{X}}}^T \hat{e}_{\hat{s},i}\},\\
\Psi_2 &= \mathrm{Sym}\{\hat{e}_0^T \mathcal{S}_2 \hat{e}_2\} - h_k \hat{e}_2^T \mathcal{Q}_2 \hat{e}_2 - \hat{e}_0^T \mathcal{S}_1 \hat{e}_0,\\
\Psi_3 &= \mathrm{Sym}\{\hat{E}_2^T \mathcal{T}_2 \hat{E}_1\} + 2\hat{E}_1^T \mathcal{R}_a \hat{E}_1 + 2\hat{E}_2^T \mathcal{R}_a \hat{E}_2,\\
\Psi_4 &= \mathrm{Sym}\{h_k \hat{e}_0^T \mathcal{S}_1 \hat{e}_6 + h_k \hat{e}_2^T \mathcal{S}_2 \hat{e}_6 + \hat{e}_0^T \mathcal{S}_2 \hat{e}_2\}\\
&\quad + h_k \hat{e}_2^T \mathcal{Q}_2 \hat{e}_2 + h_k^2 \hat{e}_6^T \mathcal{R} \hat{e}_6 - \hat{e}_0^T \mathcal{S}_1 \hat{e}_0,\\
\Psi_5 &= \mathrm{Sym}\{\hat{E}_1^T \mathcal{T}_1 \hat{E}_2\} + 4\hat{E}_1^T \mathcal{R}_a \hat{E}_1 + \hat{E}_2^T \mathcal{R}_a \hat{E}_2\\
&\quad + h_k^2 \hat{e}_6^T \mathcal{R} \hat{e}_6 - \hat{e}_7^T \hat{e}_7 - \hat{e}_8^T \hat{e}_8,\\
\hat{e}_{\hat{s},i} &= \mathcal{X}\bar{A}_i \hat{e}_1 - \bar{M}_i C \hat{e}_2 + \mathcal{X}\bar{B}_{w,i} \hat{e}_7 - \bar{M}_i D \hat{e}_8 - \mathcal{X}\hat{e}_6,\\
\hat{e}_{\hat{\mathcal{X}}} &= \hat{e}_1 + \mu_1 \hat{e}_2 + \mu_2 \hat{e}_6.
\end{aligned}
$$

Then, an $\mathcal{L}_2 - \mathcal{L}_\infty$ fuzzy filter exists that can guarantee the asymptotic stability and has an $\mathcal{L}_2 - \mathcal{L}_\infty$ performance under $w(t)$, and $v(t)$ for an given attenuation level γ_f, which satisfies $0 \le h(t) = t - t_k \le t_{k+1} - t_k = h_k$. The $\mathcal{L}_2 - \mathcal{L}_\infty$ fuzzy filter gain can be obtained as $\bar{L}_i = X^{-1}\bar{M}_i$.

Proof: By defining $\mathcal{X} = \mathcal{X}_1$, $\mathcal{X}_2 = \mu_1 \mathcal{X}$, $\mathcal{X}_3 = \mu_2 \mathcal{X}$, and $\bar{M}_i = \mathcal{X}\bar{L}_i$, we can obtain (56) and (57) from (43) and (44) in Theorem 3. Then, the $\mathcal{L}_2 - \mathcal{L}_\infty$ fuzzy filter gain can be calculated as $\bar{L}_i = X^{-1}\bar{M}_i$ from solving LMI conditions (56)-(58). This completes the proof of Theorem 4.

IV. SIMULATION RESULTS

This section presents the results of simulations involving different types of road disturbances to demonstrate the effectiveness of the proposed method. The parameters of the suspension system are summarized in Table 2. The mass of the vehicle is assumed to be $m_s \in [940\text{kg}, \ 1006\text{kg}]$ and $m_u \in [106\text{kg}, \ 122\text{kg}]$. The sampling period h_k is assumed to be 10 [ms]. The output and input constraint are set to $z_{\max} = 0.035$ [m] and $u_{\max} = 1500$ [N] respectively. In this study, we choose $\alpha_1 = \alpha_2 = 0.546$, $\rho = 1$, $\epsilon_1 = \epsilon_2 = 0.45$, and $\mu_1 = \mu_2 = 0.45$. It is assumed that the sensor noise $v(t)$ is Gaussian noise with a zero mean.

First, we consider the sampled-data H_∞ fuzzy controller design for ASSs in (11). By solving the LMIs (37)-(42) in Theorem 2, we can obtain the controller gain matrices as follows:

$$
\begin{aligned}
K_1 &= 10^3 \times [6.7966, -27.1725, -5.5776, 0.0286],\\
K_2 &= 10^3 \times [3.8405, -21.2493, -6.1195, 0.1257],\\
K_3 &= 10^3 \times [5.7589, -27.8985, -5.7612, 0.0628],\\
K_4 &= 10^3 \times [2.7556, -21.8822, -6.3126, 0.1621].
\end{aligned}
$$

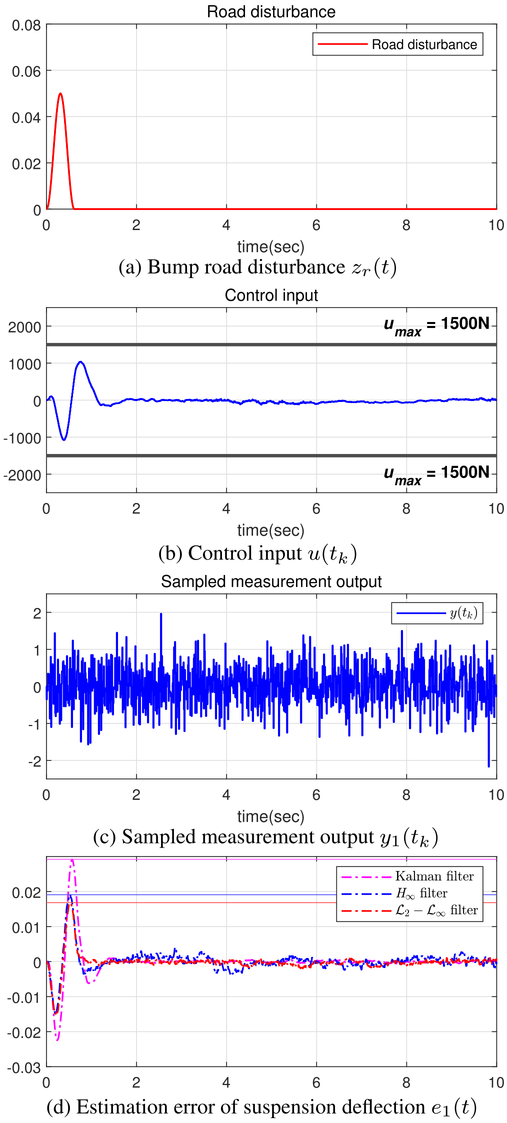


FIGURE 2. Bump road case.

Similarly, based on the LMIs (56)-(58) in Theorem 4, the filter gain matrices can be calculated as follows:

$$
L_1 = \begin{bmatrix} 61.3754 & -16.5774 & -217.5601 & -440.6738 \\ -61.3191 & 16.5824 & 217.1726 & 438.1614 \end{bmatrix}^T,
$$

$$
L_2 = \begin{bmatrix} 61.6219 & -16.5977 & -218.2178 & -526.7450 \\ -61.5654 & 16.5986 & 217.8032 & 527.8501 \end{bmatrix}^T,
$$

$$
L_3 = \begin{bmatrix} 61.4134 & -16.6063 & -213.5678 & -439.5115 \\ -61.3571 & 16.6110 & 213.6146 & 437.1237 \end{bmatrix}^T,
$$

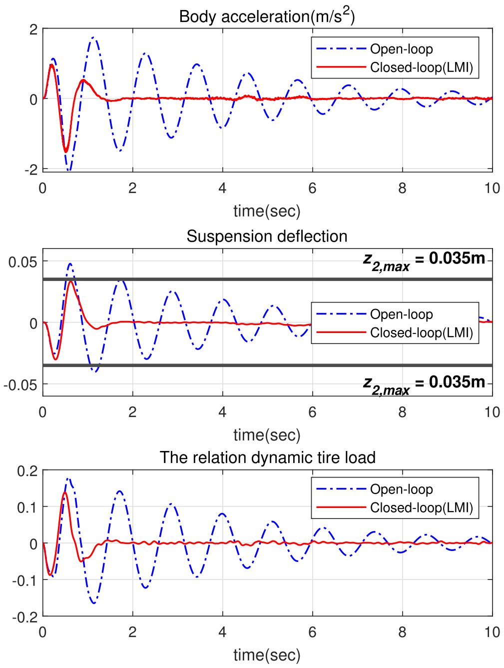


FIGURE 3. Controlled output for bump road disturbance.

$$L_4 = \begin{bmatrix} 61.3151 & -16.5277 & -214.3661 & -524.7152 \\ -61.2684 & 16.5335 & 214.4570 & 525.8312 \end{bmatrix}^T.$$

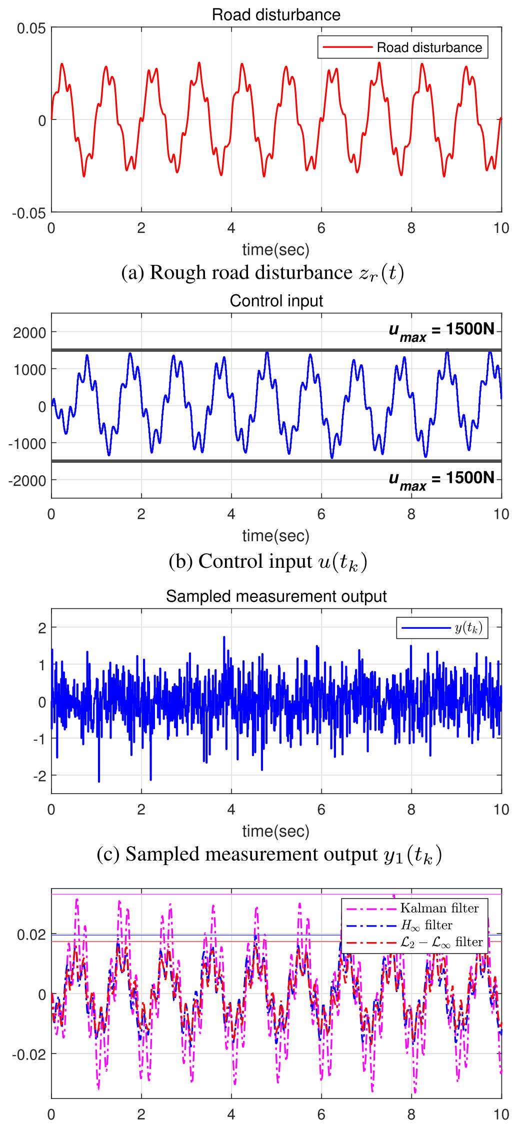
As mentioned previously, we solved the sampled-data problem by using the input delay approach. In order to reduce the conservatism that occurs in this situation, Wirtinger's inequality with the extended reciprocally convex combination bounding technique was used. Table 3 presents the minimum attenuation levels of the controller and filter corresponding to different sampling periods. In Table 4, we calculate the maximum allowable sampling times for some conventional bounding techniques. As shown in this table, Wirtinger's inequality with the extended reciprocally convex combination bounding technique guarantees more allowable sampling time than others in the case of the controller and filter, respectively.

A. BUMP ROAD

First, we simulate the performance of the proposed approach in a case involving a bump disturbance. A single bump is assumed to be present on a smooth road, and it is expressed as follows:

$$z_r(t) = \begin{cases} \frac{A}{2}(1-\cos(\frac{2\pi V}{L}t)), & 0 \le t \le \frac{L}{V} \\ 0, & otherwise \end{cases} \quad (59)$$

where A and L are the amplitude and length of the bump disturbance, respectively. V is the velocity of the ego-vehicle. It is assumed that the vehicle velocity is constant at $V = 35$ [km/h]. The road parameters are set as $A = 50$ [mm] and $L = 6$ [m]. A bump road disturbance signal is depicted in Figure 2(a). We verified the estimation performance of the proposed filter by comparing it with the H_∞ filter and the Kalman filter. Figure 2(c) shows the measurement output of suspension deflection sampled by ZOH method. Using this output signal, the proposed filter in (14) generates state estimation signals. Figure 2(d) illustrates estimation error signals of the suspension deflection according to different filters. The solid line represents the error peak value for the filter estimation corresponding to each color. As shown in this figure, the error peak value of the proposed filter estimation is much smaller than that of others under external disturbance. Therefore, the proposed filter is excellent in minimizing the peak value of the state estimation error that affects the controller performance in the presence of disturbance. Based on this state estimation result, the input signal generated by the proposed controller in (15) is shown in Figure 2(b). It is confirmed that the generated control



(a) Rough road disturbance $z_r(t)$

(b) Control input $u(t_k)$

(c) Sampled measurement output $y_1(t_k)$

(d) Estimation error of suspension deflection $e_1(t)$

FIGURE 4. Rough road case.

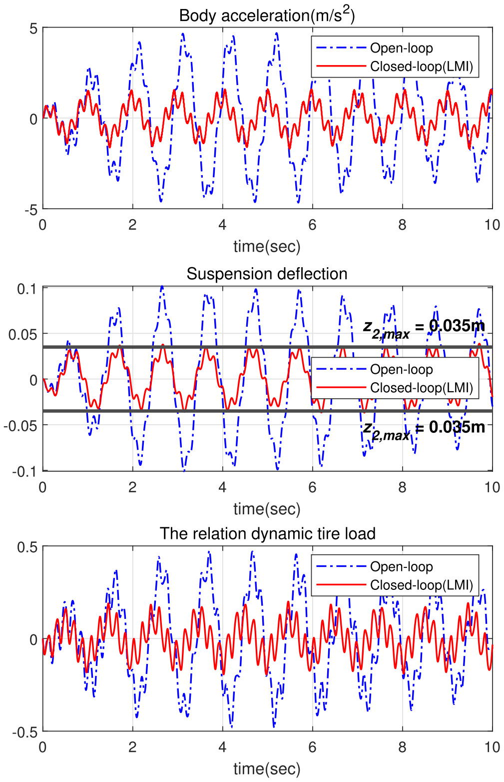


FIGURE 5. Controlled output for rough road disturbance.

input is within the prescribed constraint. Figure 3 represents the body acceleration, suspension deflection, and relational dynamic tire load response for open- and closed-loop systems, respectively. The black solid line indicates the physical constraint for suspension deflection. As shown in this figure, the closed-loop system with the proposed controller shows better responses than the open-loop system concerning these signals. Also, we can verify that all the states are within a range that satisfies the constraints of the suspension system.

B. ROUGH ROAD

Consider a rough road consisting of waves of various frequencies and amplitudes, as follows:

$$z_r(t) = 0.0254\sin(2\pi t) + 0.005\sin(10.5\pi t) + 0.001\sin(21.5\pi t). \quad (60)$$

A rough road disturbance depicted in Figure 4(a) is similar to the vehicle body resonance frequency (1 Hz) under a high-frequency disturbance condition. The sampled measurement for a rough road disturbance is shown in Figure 4(c). The proposed filter in (14) estimates an unmeasured state information by using this signal. Similar to Figure 2(d), the error signals for different filters are illustrated in Figure 4(d). The solid line indicates the peak value of error signals. This figure shows that the error peak value of the $\mathcal{L}_2 - \mathcal{L}_\infty$ filter is smaller than other filters. Therefore, it can be proven that the proposed filter is more suitable for minimizing the peak value of the estimation error under the external disturbance. Then, the control input generated by the state estimation is illustrated in Figure 4(b). This figure implies that the control input does not exceed the prescribed input constraint. Figure 5 presents the controlled outputs $z_1(t)$ and $z_2(t)$ for a rough road disturbance. As in the bump case, it can be seen that the closed-loop system with the controller designed from Theorem 2 not only performs better than the open-loop system in terms of all key performances of the suspension system but also satisfies the design constraints. Therefore, the applicability of the proposed controller has been effectively proven.

V. CONCLUSION

We proposed a novel sampled-data $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output-feedback fuzzy control design method for uncertain ASSs under hard constraints. The input delay approach was used to solve the sampled-data problem for continuous-time ASSs. The conservatism encountered in the design of sampled-data controllers and filters was effectively reduced by using Wirtinger's inequality with the extended reciprocally convex combination bounding technique. Moreover, hard limitations such as mechanical constraints and control inputs were considered when designing the controller. Based on the Lyapunov–Krasovskii theorem, an $\mathcal{L}_2 - \mathcal{L}_\infty$ filter-based output-feedback fuzzy controller was designed using the LMI technique to guarantee the robustness of the closed-loop system against external disturbances and sampled noise. The simulation results obtained using different types of road disturbances and measurement noise demonstrated the effectiveness of the proposed approach.

REFERENCES

[1] D. Hrovat, "Survey of advanced suspension developments and related optimal control applications," *Automatica*, vol. 33, no. 10, pp. 1781–1817, 1997.
[2] H. Chen and K.-H. Guo, "Constrained H_∞ control of active suspensions: An LMI approach," *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 3, pp. 412–421, May 2005.
[3] W. Sun, H. Gao, and O. Kaynak, "Finite frequency H_∞ control for vehicle active suspension systems," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 2, pp. 416–422, Mar. 2011.
[4] H. Li, H. Liu, S. Hand, and C. Hilton, "Design of robust H_∞ controller for a half-vehicle active suspension system with input delay," *Int. J. Syst. Sci.*, vol. 44, no. 4, pp. 625–640, 2013.
[5] Y. M. Sam, J. H. S. Osman, and M. R. A. Ghani, "A class of proportional-integral sliding mode control with application to active suspension system," *Syst. Control Lett.*, vol. 51, nos. 3–4, pp. 217–223, Mar. 2004.
[6] Y. Huang, J. Na, X. Wu, X. Liu, and Y. Guo, "Adaptive control of nonlinear uncertain active suspension systems with prescribed performance," *ISA Trans.*, vol. 54, pp. 145–155, Jan. 2015.
[7] S. Liu, H. Zhou, X. Luo, and J. Xiao, "Adaptive sliding fault tolerant control for nonlinear uncertain active suspension systems," *J. Franklin Inst.*, vol. 353, no. 1, pp. 180–199, 2015.
[8] K. Matsumoto, K. Matsumoto, M. Yamashita, Y. Suzuki, K. Fujimori, and H. Kimura, "Robust H_∞-output feedback control of decoupled automobile active suspension systems," *IEEE Trans. Autom. Control*, vol. 44, no. 2, pp. 392–396, Feb. 1999.
[9] H. Li, X. Jing, and H. R. Karimi, "Output-feedback-based H_∞ control for vehicle suspension systems with control delay," *IEEE Trans. Ind. Electron.*, vol. 61, no. 1, pp. 436–446, Jan. 2014.
[10] H. D. Choi, C. K. Ahn, M. T. Lim, and M. K. Song, "Dynamic output-feedback H_∞ control for active half-vehicle suspension systems with time-varying input delay," *Int. J. Control, Automat. Syst.*, vol. 14, no. 1, pp. 59–68, 2016.

[11] W. Li, Z. Xie, J. Zhao, P. K. Wong, and P. Li, "Fuzzy finite-frequency output feedback control for nonlinear active suspension systems with time delay and output constraints," *Mech. Syst. Signal Process.*, vol. 132, pp. 315–334, Oct. 2019.

[12] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985.

[13] L.-X. Wang, "Design and analysis of fuzzy identifiers of nonlinear dynamic systems," *IEEE Trans. Autom. Control*, vol. 40, no. 1, pp. 11–23, Jan. 1995.

[14] K. Tanaka, "Stability and stabilizability of fuzzy-neural-linear control systems," *IEEE Trans. Fuzzy Syst.*, vol. 3, no. 4, pp. 438–447, Nov. 1995.

[15] K. Tanaka, T. Hori, and H. O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 582–589, Aug. 2003.

[16] J. Wang, J. Xia, H. Shen, M. Xing, and J. H. Park, "H_∞ synchronization for fuzzy Markov jump chaotic systems with piecewise-constant transition probabilities subject to PDT switching rule," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 10, pp. 3082–3092, Oct. 2021.

[17] K. Jeong and S. B. Choi, "Takagi–Sugeno fuzzy observer-based magnetorheological damper fault diagnosis using a support vector machine," *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 4, pp. 1723–1735, Jul. 2022.

[18] J. Wang, C. Yang, J. Xia, Z.-G. Wu, and H. Shen, "Observer-based sliding mode control for networked fuzzy singularly perturbed systems under weighted try-once-discard protocol," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 6, pp. 1889–1899, Jun. 2022.

[19] H. S. Ko and J. Jatskevich, "Power quality control of wind-hybrid power generation system using fuzzy-LQR controller," *IEEE Trans. Energy Convers.*, vol. 22, no. 2, pp. 516–527, Jun. 2007.

[20] C. W. Tao, J. S. Taur, and Y. C. Chen, "Design of a parallel distributed fuzzy LQR controller for the twin rotor multi-input multi-output system," *Fuzzy Sets Syst.*, vol. 161, no. 15, pp. 2081–2103, Aug. 2010.

[21] D. Simon, "Kalman filtering for fuzzy discrete time dynamic systems," *Appl. Soft Comput.*, vol. 3, no. 3, pp. 191–207, Nov. 2003.

[22] J. D. J. Rubio, E. Lughofer, J. A. Meda-Campaña, L. A. Páramo, J. F. Novoa, and J. Pacheco, "Neural network updating via argument Kalman filter for modeling of Takagi–Sugeno fuzzy models," *J. Intell. Fuzzy Syst.*, vol. 35, no. 2, pp. 2585–2596, Aug. 2018.

[23] X. Liu, J. Xia, J. Wang, and H. Shen, "Interval type-2 fuzzy passive filtering for nonlinear singularly perturbed PDT-switched systems and its application," *J. Syst. Sci. Complex.*, vol. 34, no. 6, pp. 2195–2218, Dec. 2021.

[24] Y. Pan, Q. Li, H. Liang, and H.-K. Lam, "A novel mixed control approach for fuzzy systems via membership functions online learning policy," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 9, pp. 3812–3822, Sep. 2022.

[25] T. Chen and B. A. Francis, "H_∞-optimal sampled-data control: Computation and designs," *Automatica*, vol. 32, no. 2, pp. 223–228, 1996.

[26] W. Chang, J. B. Park, Y. H. Joo, and G. Chen, "Design of sampled-data fuzzy-model-based control systems by using intelligent digital redesign," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 49, no. 4, pp. 509–517, Apr. 2002.

[27] H. Gao, S. Xue, S. Yin, J. Qiu, and C. Wang, "Output feedback control of multirate sampled-data systems with frequency specifications," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 5, pp. 1599–1608, Sep. 2017.

[28] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: An input delay approach," *Automatica*, vol. 40, no. 8, pp. 1441–1446, 2004.

[29] H. Gao, W. Sun, and P. Shi, "Robust sampled-data H_∞ control for vehicle active suspension systems," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 1, pp. 238–245, Jan. 2010.

[30] X.-L. Zhu, B. Chen, D. Yue, and Y. Wang, "An improved input delay approach to stabilization of fuzzy systems under variable sampling," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 330–341, Apr. 2012.

[31] J. Qiu, W. Ji, I. J. Rudas, and H. Gao, "Asynchronous sampled-data filtering design for fuzzy-affine-model-based stochastic nonlinear systems," *IEEE Trans. Cybern.*, vol. 51, no. 8, pp. 3964–3974, Aug. 2021.

[32] L. An and G.-H. Yang, "Byzantine-resilient distributed state estimation: A min-switching approach," *Automatica*, vol. 129, Jul. 2021, Art. no. 109664.

[33] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.

[34] O. M. Kwon and J. H. Park, "On improved delay-dependent robust control for uncertain time-delay systems," *IEEE Trans. Autom. Control*, vol. 49, no. 11, pp. 1991–1995, Nov. 2004.

[35] X.-L. Zhu and G.-H. Yang, "Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay," *IET Control Theory Appl.*, vol. 2, no. 6, pp. 524–534, Jun. 2008.

[36] A. Seuret and F. Gouaisbaut, "On the use of the Wirtinger inequalities for time-delay systems," *IFAC Proc. Volumes*, vol. 45, no. 14, pp. 260–265, 2012.

[37] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[38] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.

[39] C.-K. Zhang, Y. He, J. Liang, M. Wu, and Q.-G. Wang, "An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay," *Automatica*, vol. 85, pp. 481–485, Nov. 2017.

[40] M.-G. Park, Y.-H. Kim, K.-H. Cha, and M.-K. Kim, "H_∞ filtering for dynamic compensation of self-powered neutron detectors—A linear matrix inequality based method," *Ann. Nucl. Energy*, vol. 18, no. 26, pp. 1669–1682, 1999.

[41] H. Zhang, A. S. Mehr, and Y. Shi, "Improved robust energy-to-peak filtering for uncertain linear systems," *Signal Process.*, vol. 90, no. 9, pp. 2667–2675, Sep. 2010.

[42] H. D. Choi, C. K. Ahn, H. R. Karimi, and M. T. Lim, "Filtering of discrete-time switched neural networks ensuring exponential dissipative and $l_2 - -l_\infty$ performances," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3195–3207, Oct. 2017.

[43] X. Lin, H. E. Perez, J. B. Siegel, and A. G. Stefanopoulou, "Robust estimation of battery system temperature distribution under sparse sensing and uncertainty," *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 3, pp. 753–765, May 2020.

[44] C. Zhang, Q. Chen, and J. Qiu, "Robust H_∞ filtering for vehicle sideslip angle estimation with sampled-data measurements," *Trans. Inst. Meas. Control*, vol. 39, no. 7, pp. 1059–1070, 2017.

[45] C. K. Ahn and M. K. Song, "L_2-L_∞ filtering for time-delayed switched Hopfield neural networks," *Int. J. Innov. Comput., Inf. Control*, vol. 7, no. 4, pp. 1831–1844, 2011.

[46] H. D. Choi, C. K. Ahn, P. Shi, M. T. Lim, and M. K. Song, "L_2-L_∞ filtering for Takagi–Sugeno fuzzy neural networks based on Wirtinger-type inequalities," *Neurocomputing*, vol. 153, pp. 117–125, Apr. 2015.

[47] C. Lu, X.-M. Zhang, M. Wu, Q.-L. Han, and Y. He, "Energy-to-peak state estimation for static neural networks with interval time-varying delays," *IEEE Trans. Cybern.*, vol. 48, no. 10, pp. 2823–2835, Oct. 2018.

[48] Z. Xu, Z.-G. Wu, H. Su, P. Shi, and H. Que, "Energy-to-peak filtering of semi-Markov jump systems with mismatched modes," *IEEE Trans. Autom. Control*, vol. 65, no. 10, pp. 4356–4361, Oct. 2020.

[49] K. M. Grigoriadis and J. T. Watson, "Reduced-order H_∞ and L_2-L_∞ filtering via linear matrix inequalities," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 4, pp. 1326–1338, Oct. 1997.

[50] J. Xia, S. Xu, and B. Song, "Delay-dependent $L_2 - -L_8$ filter design for stochastic time-delay systems," *Syst. Control Lett.*, vol. 56, nos. 9–10, pp. 579–587, 2007.

[51] H. Zhang, X. Huang, J. Wang, and H. RezaKarimi, "Robust energy-to-peak sideslip angle estimation with applications to ground vehicles," *Mechatronics*, vol. 30, pp. 338–347, Sep. 2015.

[52] H. Li, H. Liu, H. Gao, and P. Shi, "Reliable fuzzy control for active suspension systems with actuator delay and fault," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 342–357, Apr. 2012.

[53] A. Chamseddine and H. Noura, "Control and sensor fault tolerance of vehicle active suspension," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 416–433, May 2008.

[54] S. K. Shimp III, "Vehicle sprung mass parameter estimation using an adaptive polynomial-chaos method," Ph.D. dissertation, Dept. Mech. Eng., Virginia Tech., Blacksburg, VA, USA, 2008.

[55] H. K. Fathy, D. Kang, and J. L. Stein, "Online vehicle mass estimation using recursive least squares and supervisory data extraction," in *Proc. Amer. Control Conf.*, Jun. 2008, pp. 1842–1848.

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