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RESEARCH ARTICLE

Dynamic Event-Triggered Communication-Based Adaptive Consensus Tracking of Uncertain Nonlinear Multiagent Systems in Pure-Feedback Form

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ABSTRACT This study is aimed at investigating a dynamic event-triggered communication-based adaptive distributed consensus control problem for a class of uncertain pure-feedback nonlinear multiagent systems with limited communication resources. The nonaffine nonlinear functions of multiagent systems are assumed to be unknown and heterogeneous, and intermittent inter-agent communication is considered to occur within a directed network. A novel adaptive distributed dynamic surface control strategy using neural networks is developed to manage non-differentiable virtual control laws associated with the intermittent communication between agents with uncertain nonaffine nonlinear parts. The key contribution of this research is the derivation of dynamic event-triggering conditions using distributed tracking errors to efficiently adjust inter-event times, while ensuring the consensus tracking performance and robustness against unexpected external disturbances. Compared with the existing static event-triggered communication approach, the proposed controller can alleviate the communication burden among agents. Using technical lemmas, it is shown that all signals of the considered system are semiglobally uniformly ultimately bounded, and Zeno behavior is strictly excluded. Comparative simulation results illustrate the effectiveness of the proposed dynamic event-triggered control approach.

INDEX TERMS Distributed consensus tracking, dynamic event-triggered inter-agent communication, dynamic surface control, neural networks, uncertain pure-feedback nonlinear systems.

I. INTRODUCTION

Distributed consensus control has attracted considerable attention from the control community due to its potential applications such as vehicle formation, traffic, and mobile networks [1], [2], [3], [4], [5]. Many researchers have focused on event-triggered communication-based consensus control approaches to increase the communication efficiency among connected agents In such approaches, event-triggered conditions using static thresholds [6], [7], [8], [9], [10] and dynamic thresholds [11], [12], [13] are incorporated to avoid continuous monitoring of the neighbors' state information. However, these methods [6], [7], [8], [9], [10], [11], [12], [13] are typically applicable only to linear systems. For more practical applications, various event-triggered consensus controllers using adaptive control technique [14] have been designed for uncertain nonlinear multiagent systems such as first-order [15], second-order [16], [17], and high-order nonlinear systems with matched nonlinearities [18], [19], [20], [21], [22], [23]. Tan et al. [22] addressed the event-triggered consensus tracking control problem for nonlinear multiagent systems with stochastic actuator faults. In [23], an observer-based event-triggered security control methodology was studied for interval type-2 fuzzy networked system with network attacks. Recursive design approaches for continuous communication-based adaptive consensus tracking

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have been reported for lower-triangular nonlinear systems with model uncertainties (see [24], [25], [26] and the references therein). Recently, several event-triggered consensus control problems have been investigated for multiagent models such as strict-feedback form [27] and nonstrict-feedback form [28]. Although these studies [27], [28] have reported event-triggered mechanisms for the control inputs of agents, the signals communicated among agents were required to be continuously monitored. To avoid the continuous monitoring of the neighbors' information, an event-triggered communication-based distributed adaptive consensus control methodology has recently been presented in [29]. However, this strategy involves the following limitations:

- (i) In [29], the event-triggered condition for inter-agent communication depends on a static threshold parameter. In several practical environments, it is inefficient to fix the triggering threshold. The threshold must be dynamically adjusted to compensate for the increase in the tracking errors owing to unexpected external disturbances and to avoid unnecessary communication among agents after consensus achievement. Furthermore, the transmission rates of certain wireless networks such as IEEE 802.11 WLAN are time-varying because of random wireless fading and variable interference. In particular, several data rates are allowed in the IEEE 802.11b standard [11]. The static threshold-based event-triggered approach is ineffective in such practical environments. Thus, the first motivation of this paper is to develop a dynamic event-triggered mechanism for inter-agent communication to realize distributed consensus tracking.
- (ii) The multiagent systems considered in [29] are in the affine nonlinear form, in which the known nonlinear functions are matched to the control input. In contrast, this study takes into account multiagent models including unknown *nonaffine* nonlinearities *unmatched* to the control input. Thus, more general nonlinear multiagent systems are treated in this study, compared to [29]. The second motivation of this paper is to address the unknown nonaffine and unmatched nonlinearities in the event-triggered inter-agent communication-based consensus tracking field.

To the best of our knowledge, none of the existing studies have focused on the dynamic event-triggered inter-agent communication-based adaptive distributed consensus control problem of uncertain pure-feedback nonlinear multiagent systems.

The aim of this paper is to establish a novel dynamic event-triggered inter-agent communication strategy for the distributed consensus tracking of uncertain nonaffine nonlinear multiagent systems under directed networks with limited bandwidths. First, the dynamic event-triggered conditions using the distributed consensus tracking are developed for inter-agent communication. Next, the recursive consensus control scheme for resolving the non-differentiability problem of the outputs of the event-triggered neighbors is presented to guarantee total closed-loop stability in the Lyapunov sense. Furthermore, we analyze the exclusion of Zeno behavior under the proposed dynamic thresholds. Finally, simulations are performed to demonstrate the performance of the proposed scheme in comparison with the existing strategy [29] using static thresholds.

Different from the previous distributed consensus control results using event-triggered communication [11], [12], [13], [27], [28], [29], the main contributions are two-fold:

- (C1) Compared with [11], [12], [13], [27], [28], and [29], the range of applicable systems is extended to pure-feedback nonlinear multiagent models in the dynamic event-triggered communication-based control field. In addition, intermittent inter-agent communication makes virtual controllers non-differentiable, which makes it challenging to perform recursive backstepping. To overcome this difficulty, the dynamic surface control method is used to design the distributed consensus tracking scheme and a novel analysis strategy is established to ensure the dynamic stability of the non-differentiable filtering error.
- (C2) Contrary to the existing strategy [29] based on static thresholds, we derive a dynamic threshold using the distributed tracking error for event-based inter-agent communication. The proposed approach can reduce the number of unnecessary network transmissions among agents while achieving favorable consensus tracking performance, and ensure efficient inter-agent communication against unexpected external disturbances.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PROBLEM FORMULATION

Consider a scenario involving a time-varying leader and N followers described by the following heterogeneous pure-feedback nonlinear systems

$$\begin{aligned}\dot{x}_{i,p}(t) &= h_{i,p}(\bar{x}_{i,p}(t), x_{i,p+1}(t)) + d_{i,p}(t)\\ \dot{x}_{i,n_i}(t) &= h_{i,n_i}(\bar{x}_{i,n_i}(t), u_i(t)) + d_{i,n_i}(t)\\ y_i(t) &= x_{i,1}(t)\end{aligned} \tag{1}$$

where $i = 1, \ldots, N$, $p = 1, \ldots, n_i - 1$, $\bar{x}_{i,p} = [x_{i,1}, \ldots, x_{i,p}]^\top \in \mathbb{R}^p$ is the state vector of the ith follower, $y_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the control output and input of the ith follower, respectively, $h_{i,p}(\bar{x}_{i,p}, x_{i,p+1})$ and $h_{i,n_i}(\bar{x}_{i,n_i}, u_i)$ are unknown nonaffine C^1 nonlinear functions, and $d_{i,p}$, $p = 1, \ldots, n_i$ are unknown and bounded external disturbances.

The objective of this paper is to design distributed adaptive consensus control laws u_i based on the dynamic event-triggered inter-agent communication such that all outputs $y_i(t)$ of (1) follow the time-varying leader signal $y_r(t)$ with an adjustable bound.

The following assumptions and lemmas are used in this paper.

Assumption 1 [30]: Let $\xi_{i,p}(\bar{x}_{i,p}, x_{i,p+1}) = \partial h_{i,p}/\partial x_{i,p+1}$ with $p = 1, \ldots, n_i$ and $x_{i,n_i+1} = u_i$. The sign of $\xi_{i,p}(\bar{x}_{i,p}, x_{i,p+1})$ is known, and for constants $\xi_{i,p0} > 0$ and $\xi_{i,p1} > 0$, $0 < \xi_{i,p0} \leq |\xi_{i,p}| < \xi_{i,p1}$ is satisfied for all $\bar{x}_{i,p+1} \in \mathbb{R}^{p+1}$. Without loss of generality, $\xi_{i,p}$ is assumed to be strictly positive.

Assumption 2 [31]: For an unknown constant $\bar{d}_{i,p} > 0$, $|d_{i,p}| \leq \bar{d}_{i,p}$.

Assumption 3 [25]: The time-varying leader signal $y_r(t)$ and its time derivatives $\dot{y}_r$ and $\ddot{y}_r$ are continuous and bounded. In addition, only the followers with the leader as a neighbor can receive the leader information $y_r(t)$.

Lemma 1 [32]: For any constant $\omega > 0$ and $\Theta \in \mathbb{R}$, $0 \leq |\Theta| - \Theta \tanh(\Theta/\omega) \leq 0.2785\omega$.

Lemma 2 [33]: For any constant $\omega > 0$, $\Theta_1 \in \mathbb{R}$, and $\Theta_2 \in \mathbb{R}$, $|\Theta_1 \tanh(\Theta_1/\omega) - \Theta_2 \tanh(\Theta_2/\omega)| \leq 1.1997|\Theta_1 - \Theta_2|$.

B. GRAPH THEORY

Communication among N followers occurs with a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, which has a set of nodes $\mathcal{V} = \{1, \ldots, N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$. The adjacency weights a_{ij} are defined as follows: $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Self-loops are excluded as $a_{ii} = 0$. The set of neighbors for agent i is denoted as $\mathcal{N}_i$. An in-degree matrix is defined as $\mathcal{B} = \text{diag}[b_1, \ldots, b_N]$ with $b_i = \sum_{j=1}^{N} a_{ij}$ and the Laplacian matrix is written as $\mathcal{L} = \mathcal{B} - \mathcal{A}$. Another weighted graph $\bar{\mathcal{G}}$ is defined to describe the network interconnection between N followers and a leader robot labeled by '0'. The leader adjacency weights ν_i are defined as follows: $\nu_i > 0$ if y_r is directly accessible by agent i, and $\nu_i = 0$ otherwise. Then, $\bar{\mathcal{G}}$ represents a spanning tree with the leader as the root node. Thus, the matrix $Q = (\mathcal{L} + \Upsilon) + (\mathcal{L} + \Upsilon)^\top$ with $\Upsilon = \text{diag}[\nu_1, \ldots, \nu_N]$ is symmetric and positive [34].

C. FUNCTION APPROXIMATION

Using the universal function approximation property, radial basis function neural networks (RBFNNs) [35], [36] are used to estimate unknown continuous functions $M_{i,p}(\zeta_{i,p}) : \Omega_{\zeta_{i,p}} \mapsto \mathbb{R}$ derived from the control design steps as follows:

$$M_{i,p}(\zeta_{i,p}) = \theta_{i,p}^\top W_{i,p}(\zeta_{i,p}) + \delta_{i,p}(\zeta_{i,p}) \tag{2}$$

where $i \in \mathcal{V}$, $p = 1, \ldots, n_i$, $\zeta_{i,p} \in \Omega_{\zeta_{i,p}}$ is the input of the RBFNN, $\Omega_{\zeta_{i,p}}$ is a compact set, $\theta_{i,p} \in \mathbb{R}^{l_{i,p}}$ is the ideal constant weighting vector with the node number $l_{i,p}$, $\delta_{i,p}(\zeta_{i,p})$ represents the reconstruction errors bounded as $|\delta_{i,p}(\zeta_{i,p})| \leq \bar{\delta}_{i,p}$ with the constant $\bar{\delta}_{i,p}$, and $W_{i,p}(\zeta_{i,p}) \in \mathbb{R}^{l_{i,p}}$ is the Gaussian function vector. An ideal weighting vector $\theta_{i,p}$ is bounded as $|\theta_{i,p}| \leq \bar{\theta}_{i,p}$ with a constant $\bar{\theta}_{i,p}$ [31], [37].

III. DISTRIBUTED CONSENSUS TRACKING BASED ON DYNAMIC EVENT-BASED INTER-AGENT COMMUNICATION

A. DESIGN OF THE DYNAMIC EVENT-TRIGGERED MECHANISM

The dynamic event-triggering condition for neighboring agent j is presented as

$$t_{k+1}^j = \inf\{t > t_k^j \mid |y_j(t) - \bar{y}_j(t)| \geq \varpi_j(t)\} \tag{3}$$

$$\dot{\varpi}_j(t) = -\mu_j \varpi_j(t) - \rho_j \varpi_j^2(t) + m_j \exp\left(-\frac{|\sum_{l=1}^{N} a_{jl}(y_j - \bar{y}_l) + \nu_j(y_j - y_r)|}{n_j}\right) \tag{4}$$

where $j \in \mathcal{V}$, $\varpi_j(0) > 0$, $\exp(\cdot)$ is the exponential function, μ_j, ρ_j, m_j, and n_j are positive design constants, $k \in \mathbb{Z}^+$, $t_1^j = 0$, and $\bar{y}_j(t) = y_j(t_k^j)$ for $t_k^j < t < t_{k+1}^j$ denotes the output information of neighboring agent j updated at t_k^j, which is transmitted to agent i with $a_{ij} = 1$. When the triggering condition (3) is satisfied, the transmitted output information is updated at the next event time t_{k+1}^j. The quadratic term $-\rho_j \varpi_j^2(t)$ in (4) is introduced to eliminate the effect of the distributed measurement error term $\sum_{j=1}^{N} a_{ij}|y_j - \bar{y}_j|$ derived in the stability analysis of the closed-loop system, which is described later.

Lemma 3: If $\varpi_j(0) > 0$, then $\varpi_j(t) > 0$ for $t \in [0, \infty)$, where $j \in \mathcal{V}$.

Proof: Because of $m_j \exp(-|\sum_{l=1}^{N} a_{jl}(y_j - \bar{y}_l) + \nu_j(y_j - y_r)|/n_j) \geq 0$, (4) satisfies

$$\dot{\varpi}_j(t) \geq -\rho_j \varpi_j^2(t) - \mu_j \varpi_j(t). \tag{5}$$

According to the comparison lemma [38], $\varpi_j(t) \geq \mu_j \varpi_j(0)/[e^{\mu_j t}(\rho_j \varpi_j(0) + \mu_j) - \rho_j \varpi_j(0)]$. Moreover, because of $\varpi_j(0) > 0$, $\varpi_j(t) > 0$ is ensured for $t \in [0, \infty)$. ■

Remark 1: Unlike the event-triggered consensus tracking approach based on the fixed constant threshold [29], the proposed triggering condition (3) for event-triggered inter-agent communication is based on the dynamic threshold (4) using the distributed consensus tracking error. As the consensus tracking error increases, the dynamic threshold decreases, and thus the tracking error can be rapidly recovered by increasing the number of events. In contrast, the threshold increases when consensus tracking is achieved. In this manner, the proposed approach can reduce the number of unnecessary network transmissions among agents while achieving favorable consensus tracking performance. Thus, compared with the static threshold approach [29], the dynamic triggering condition (3) helps balance the consensus tracking performance and the cost of communication resources.

B. DESIGN OF DISTRIBUTED ADAPTIVE CONTROLLER USING THE EVENT-TRIGGERED OUTPUT INFORMATION OF NEIGHBORS

In this section, we design a distributed adaptive consensus control scheme based on dynamic event-triggered inter-agent communication, in which the dynamic surface design method [39] is used to overcome the non-differentiability problem of the communicated output triggering signals.

The error variables for the ith follower are defined as

$$s_{i,1} = y_i - \nu_i y_r - (1-\nu_i)\hat{y}_{i,r} \tag{6}$$

$$s_{i,p} = x_{i,p} - \bar{\alpha}_{i,p-1} \tag{7}$$

$$z_{i,p-1} = \bar{\alpha}_{i,p-1} - \alpha_{i,p-1} \tag{8}$$

where $i \in \mathcal{V}$, $p = 2, \ldots, n_i$, $\hat{y}_{i,r}$ is the estimated leader signal for the ith follower without any connection links to the leader, $z_{i,p-1}$ is the boundary layer error, $\alpha_{i,p-1}$ is the virtual controller, and $\bar{\alpha}_{i,p-1}$ is the filtered signal obtained through the low-pass filter

$$\tau_{i,p-1}\dot{\bar{\alpha}}_{i,p-1} + \bar{\alpha}_{i,p-1} = \alpha_{i,p-1} \tag{9}$$

with $\bar{\alpha}_{i,p-1}(0) = \alpha_{i,p-1}(0)$ and a design constant $\tau_{i,p-1} > 0$.

Step 1: From (6), $s_{i,1}$ is rewritten as $s_{i,1} = y_i - \check{y}_{i,r}$ with $\check{y}_{i,r} = \nu_i y_r + (1-\nu_i)\hat{y}_{i,r}$ and the time derivative of $s_{i,1}$ is

$$\dot{s}_{i,1} = h_{i,1}(x_{i,1}, x_{i,2}) + d_{i,1} - \dot{\check{y}}_{i,r}. \tag{10}$$

Owing to $\partial v_{i,1}/\partial x_{i,2} = 0$ with $v_{i,1} = -\dot{\check{y}}_{i,r}$, $\partial(h_{i,1}(x_{i,1}, x_{i,2}) + v_{i,1})/\partial x_{i,2} > \xi_{i,10}$. The implicit function theorem [30] implies the existence of an ideal virtual control input $x_{i,2} = \alpha^*_{i,1}(x_{i,1}, v_{i,1})$ satisfying $h_{i,1}(x_{i,1}, \alpha^*_{i,1}) + v_{i,1} = 0$. By applying the mean value theorem, we have

$$h_{i,1}(x_{i,1}, x_{i,2}) = h_{i,1}(x_{i,1}, \alpha^*_{i,1}) + \xi_{\lambda_{i,1}}(x_{i,2} - \alpha^*_{i,1}) \tag{11}$$

where $\xi_{\lambda_{i,1}} = \xi_{i,1}(x_{i,1}, x_{i,2\lambda_1})$ and $x_{i,2\lambda_1} = \lambda_{i,1}x_{i,2} + (1-\lambda_{i,1})\alpha^*_{i,1}$ with a constant $0 < \lambda_{i,1} < 1$.

From (11), (10) becomes

$$\dot{s}_{i,1} = \xi_{\lambda_{i,1}}(x_{i,2} - \alpha^*_{i,1}) + d_{i,1}. \tag{12}$$

A Lyapunov function is defined as

$$V_{s_{i,1}} = \frac{1}{2\xi_{i,10}} s_{i,1}^2. \tag{13}$$

Using (7), (8), and Young's inequality, $\dot{V}_{s_{i,1}}$ is obtained as

$$\begin{aligned} \dot{V}_{s_{i,1}} &= \frac{s_{i,1}}{\xi_{i,10}}(\xi_{\lambda_{i,1}}(x_{i,2} - \alpha^*_{i,1}) + d_{i,1}) \\ &\le \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}(s_{i,2} + \alpha_{i,1}) + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1} z_{i,1} \\ &\quad - \varrho_i^2 s_{i,1}^2 + |s_{i,1}| M_{i,1}(\zeta_{i,1}) + \frac{\varepsilon_{i,1}}{2} \end{aligned} \tag{14}$$

where $\varepsilon_{i,1} > 0$ is a constant and $M_{i,1}(\zeta_{i,1}) = |s_{i,1}|(\xi_{i,11}^2(\alpha^*_{i,1})^2 + \bar{d}_{i,1}^2)/(\xi_{i,10}^2\varepsilon_{i,1}) + \varrho_i^2|s_{i,1}|$ is an unknown nonlinear function with $\zeta_{i,1} = [x_{i,1}, s_{i,1}, \dot{\check{y}}_{i,r}]^\top$ and $\varrho_i = 1.1997\xi_{i,11}/\xi_{i,10}$.

To approximate $M_{i,1}(\zeta_{i,1})$, the RBFNN is employed as follows:

$$M_{i,1}(\zeta_{i,1}) = \theta_{i,1}^\top W_{i,1}(\zeta_{i,1}) + \delta_{i,1}. \tag{15}$$

Substituting (15) into (14), we obtain

$$\begin{aligned} \dot{V}_{s_{i,1}} &\le \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}(s_{i,2} + \alpha_{i,1}) + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1} z_{i,1} + |s_{i,1}|\theta_{i,1}^\top W_{i,1} \\ &\quad - \varrho_i^2 s_{i,1}^2 + |s_{i,1}|\delta_{i,1} + \frac{\varepsilon_{i,1}}{2}. \end{aligned} \tag{16}$$

Then, the first virtual control law is given by

$$\alpha_{i,1} = -\frac{s_{i,1}\beta_{i,1}^2}{\sqrt{s_{i,1}^2\beta_{i,1}^2 + \kappa_{i,1}^2}} \tag{17}$$

$$\beta_{i,1} = c_{i,1}|s_{i,1}| + \hat{\theta}_{i,1}^\top W_{i,1} + \phi_i \tanh\left(\frac{|s_i|\phi_i}{k_i}\right) \tag{18}$$

where $\kappa_{i,1} > 0$, $c_{i,1} > 0$, and $k_i > 0$ are constants, $\hat{\theta}_{i,1}$ is the estimate of $\theta_{i,1}$, and

$$\phi_i = \sum_{j=1}^{N} a_{ij}(y_i - \bar{y}_j) + \nu_i(y_i - y_r). \tag{19}$$

Here, the tuning laws of $\tilde{\theta}_{i,1}$ and $\tilde{y}_{i,r}$ are constructed as

$$\dot{\hat{y}}_{i,r} = -r_i\phi_i - r_i\kappa_{y_r}\hat{y}_{i,r} \tag{20}$$

$$\dot{\hat{\theta}}_{i,1} = \Gamma_{i,1}(|s_{i,1}|W_{i,1} - \kappa_{\theta_{i,1}}\hat{\theta}_{i,1}) \tag{21}$$

where $\hat{\theta}_{i,1}(0) > 0$, r_i, κ_{y_r}, and $\kappa_{\theta_{i,1}}$ are positive design constants and $\Gamma_{i,1} > 0$ is a diagonal matrix.

Remark 2: In (17), when the tracking error $s_{i,1}$ is not zero, $\beta_{i,1}$ may be zero. It leads to an undesirable input value $\alpha_{i,1} = 0$ when $s_{i,1} \neq 0$. In (18), we design $\beta_{i,1}$ to overcome this problem. From (18) and (21) with $\hat{\theta}_{i,1}(0) > 0$, $\beta_{i,1} > 0$ is ensured when $s_{i,1} \neq 0$. Thus, the proposed virtual controller $\alpha_{i,1}$ is zero only when the tracking error is zero (i.e, $s_{i,1} = 0$). Accordingly, the proposed virtual control (17) can be well implemented.

Adding and subtracting $|s_{i,1}|\beta_{i,1}$ into (16) yields

$$\begin{aligned} \dot{V}_{s_{i,1}} &\le \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}(s_{i,2} + \alpha_{i,1}) + |s_{i,1}|\beta_{i,1} - |s_{i,1}|\beta_{i,1} \\ &\quad - \varrho_i^2 s_{i,1}^2 + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1} z_{i,1} + s_{i,1}\theta_{i,1}^\top W_{i,1} \\ &\quad + s_{i,1}\delta_{i,1} + \frac{\varepsilon_{i,1}}{2}. \end{aligned} \tag{22}$$

Then, using (17), we have

$$\begin{aligned} &\frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}\alpha_{i,1} + |s_{i,1}|\beta_{i,1} \\ &\quad = -\frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} \frac{s_{i,1}^2\beta_{i,1}^2}{\sqrt{s_{i,1}^2\beta_{i,1}^2 + \kappa_{i,1}^2}} + |s_{i,1}|\beta_{i,1} \\ &\quad \le -\frac{s_{i,1}^2\beta_{i,1}^2}{|s_{i,1}||\beta_{i,1}| + \kappa_{i,1}} + |s_{i,1}|\beta_{i,1} \\ &\quad \le \kappa_{i,1}. \end{aligned} \tag{23}$$

By substituting (23) into (22), we obtain

$$\dot{V}_{s_{i,1}} \le \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}s_{i,2} + \kappa_{i,1} - |s_{i,1}|\beta_{i,1} + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}} s_{i,1}z_{i,1}$$

$$-\varrho_i^2 s_{i,1}^2 + |s_{i,1}|\theta_{i,1}^\top W_{i,1} + |s_{i,1}|\delta_{i,1} + \frac{\varepsilon_{i,1}}{2}. \quad (24)$$

Define the Lyapunov function

$$V_{i,1} = V_{s_{i,1}} + \frac{1}{2}\tilde{\theta}_{i,1}^\top \Gamma_{i,1}^{-1}\tilde{\theta}_{i,1} + \frac{(1-\nu_i)}{2r_i}\tilde{y}_{i,r}^2 + \varpi_i \quad (25)$$

where $\tilde{\theta}_{i,1} = \hat{\theta}_{i,1} - \theta_{i,1}$ and $\tilde{y}_{i,r} = y_r - \hat{y}_{i,r}$.

From (4) and (18), differentiating $V_{i,1}$ yields

$$\begin{aligned}\dot{V}_{i,1} \le& -c_{i,1}s_{i,1}^2 + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}s_{i,2} + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}z_{i,1} \\ &+\tilde{\theta}_{i,1}^\top(\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} - |s_{i,1}|W_{i,1}) - |s_{i,1}|\phi_i \tanh\left(\frac{|s_i|\phi_i}{k_i}\right) \\ &+\kappa_{i,1} + |s_{i,1}|\delta_{i,1} + \frac{\varepsilon_{i,1}}{2} + \frac{(1-\nu_i)}{r_i}\tilde{y}_{i,r}(\dot{y}_r - \dot{\hat{y}}_{i,r}) \\ &-\varrho_i^2 s_{i,1}^2 - \mu_i\varpi_i - \rho_i\varpi_i^2 \\ &+m_i \exp\left(-\frac{|\phi_i|}{n_i}\right). \end{aligned} \quad (26)$$

According to Lemma 1, we have

$$\begin{aligned} -|s_{i,1}|\phi_i \tanh\left(\frac{|s_{i,1}|\phi_i}{k_i}\right) &= -s_{i,1}\phi_i + s_{i,1}\phi_i - s_{i,1}\phi_i\tanh\left(\frac{s_{i,1}\phi_i}{k_i}\right) \\ &\le -s_{i,1}\phi_i + |s_{i,1}\phi_i| - s_{i,1}\phi_i\tanh\left(\frac{s_{i,1}\phi_i}{k_i}\right) \\ &\le -\phi_i s_{i,1} + 0.2785k_i. \end{aligned} \quad (27)$$

Using $s_{i,1} = \varphi_i + (1-\nu_i)\tilde{y}_{i,r}$ with $\varphi_i = y_i - y_r$ and defining

$$\epsilon_i = \sum_{j=1}^N a_{ij}(y_i - y_j) + \nu_i(y_i - y_r), \quad (28)$$

the term $-\phi_i s_{i,1}$ can be written as

$$\begin{aligned} -\phi_i s_{i,1} &= -s_{i,1}\epsilon_i - s_{i,1}\sum_{j=1}^N a_{ij}(y_j - \bar{y}_j) \\ &\le -\varphi_i\epsilon_i - (1-\nu_i)\tilde{y}_{i,r}\epsilon_i \\ &\quad + |s_{i,1}|\sum_{j=1}^N a_{ij}|y_j - \bar{y}_j|. \end{aligned} \quad (29)$$

By applying (20), (21), and (29) to (26), it is obtained that

$$\begin{aligned}\dot{V}_{i,1} \le& -\varphi_i\epsilon_i - c_{i,1}s_{i,1}^2 + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}s_{i,2} + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}z_{i,1} \\ &-\kappa_{\theta_{i,1}}\tilde{\theta}_{i,1}^\top\hat{\theta}_{i,1} + |s_{i,1}|\sum_{j=1}^N a_{ij}|y_j - \bar{y}_j| + \kappa_{i,1} \\ &+|s_{i,1}|\delta_{i,1} + (1-\nu_i)|\tilde{y}_{i,r}|\sum_{j=1}^N a_{ij}|y_j - \bar{y}_j| \\ &+\frac{(1-\nu_i)}{r_i}\tilde{y}_{i,r}\dot{y}_r + (1-\nu_i)\kappa_{y_r}\tilde{y}_{i,r}\hat{y}_{i,r} + 0.2785k_i \\ &-\varrho_i^2 s_{i,1}^2 + \frac{\varepsilon_{i,1}}{2} - \mu_i\varpi_i - \rho_i\varpi_i^2 + m_i. \end{aligned} \quad (30)$$

Step p ($p = 2, \ldots, n_i - 1$): The time derivative of $s_{i,p}$ is

$$\begin{aligned}\dot{s}_{i,p} =& h_{i,p}(\bar{x}_{i,p}, x_{i,p+1}) + d_{i,p} - \frac{\xi_{i,p0}\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}}s_{i,p-1} \\ &+\frac{\xi_{i,p0}\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}}s_{i,p-1} - \dot{\bar{\alpha}}_{i,p-1}. \end{aligned} \quad (31)$$

From Assumption 1, it is satisfied that $\partial h_{i,p}(\bar{x}_{i,p}, x_{i,p+1})/\partial x_{i,p+1} > \xi_{i,p0} > 0$ for all $\bar{x}_{i,p+1} \in \mathbb{R}^{p+1}$. Let $v_{i,p} = (\xi_{i,p0}\xi_{\lambda_{i,p-1}}/\xi_{i,(p-1)0})s_{i,p-1} - \dot{\bar{\alpha}}_{i,p-1}$. Based on the fact that $\partial v_{i,p}/\partial x_{i,p+1} = 0$, it holds that $\partial(h_{i,p}(\bar{x}_{i,p}, x_{i,p+1}) + v_{i,p})/\partial x_{i,p+1} > \xi_{i,p0} > 0$. Applying the implicit function theorem gives an ideal virtual control input $x_{i,p+1} = \alpha_{i,p}^*(\bar{x}_{i,p}, v_{i,p})$ satisfying $h_{i,p}(\bar{x}_{i,p}, \alpha_{i,p}^*) + v_{i,p} = 0$. By applying the mean value theorem, we have

$$h_{i,p}(\bar{x}_{i,p}, x_{i,p+1}) = h_{i,p}(\bar{x}_{i,p}, \alpha_{i,p}^*) + \xi_{\lambda_{i,p}}(x_{i,p+1} - \alpha_{i,p}^*) \quad (32)$$

where $\xi_{\lambda_{i,p}} = \xi_{i,p}(\bar{x}_{i,p}, x_{i,(p+1)\lambda_p})$ and $x_{i,(p+1)\lambda_p} = \lambda_{i,p}x_{i,p+1} + (1-\lambda_{i,p})\alpha_{i,p}^*$ with $0 < \lambda_{i,p} < 1$.

Define the Lyapunov function

$$V_{i,p} = \frac{1}{2\xi_{i,p0}}s_{i,p}^2 + \frac{1}{2}\tilde{\theta}_{i,p}^\top\Gamma_{i,p}^{-1}\tilde{\theta}_{i,p} \quad (33)$$

where $\tilde{\theta}_{i,p} = \hat{\theta}_{i,p} - \theta_{i,p}$ and $\Gamma_{i,p} > 0$ is a diagonal matrix. From (32), $\dot{V}_{i,p}$ is obtained as

$$\begin{aligned}\dot{V}_{i,p} =& \frac{s_{i,p}}{\xi_{i,p0}}\Big(\xi_{\lambda_{i,p}}(s_{i,p+1} + \alpha_{i,p} + z_{i,p} - \alpha_{i,p}^*) \\ &-\frac{\xi_{i,p0}\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}}s_{i,p-1} + d_{i,p}\Big) + \tilde{\theta}_{i,p}^\top\Gamma_{i,p}^{-1}\dot{\hat{\theta}}_{i,p} \\ \le& \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}}s_{i,p}(s_{i,p+1} + \alpha_{i,p}) + \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}}s_{i,p}z_{i,p} \\ &+|s_{i,p}|M_{i,p}(\zeta_{i,p}) + \frac{\varepsilon_{i,p}}{2} - \frac{\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}}s_{i,p-1}s_{i,p} \\ &+\tilde{\theta}_{i,p}^\top\Gamma_{i,p}^{-1}\dot{\hat{\theta}}_{i,p} \end{aligned} \quad (34)$$

where $\varepsilon_{i,p} > 0$ is a constant and $M_{i,p}(\zeta_{i,p}) = |s_{i,p}|(\xi_{i,p1}^2(\alpha_{i,p}^*)^2 + \bar{d}_{i,p}^2)/(\xi_{i,p0}^2\varepsilon_{i,p})$ with $\zeta_{i,p} = [(\alpha_{i,p-1} - \bar{\alpha}_{i,p-1})/\tau_{i,p-1}, \bar{x}_{i,p}, s_{i,p-1}, s_{i,p}]^\top$.

To approximate $M_{i,p}(\zeta_{i,p})$, the RBFNN is employed as follows:

$$M_{i,p}(\zeta_{i,p}) = \theta_{i,p}^\top W_{i,p}(\zeta_{i,p}) + \delta_{i,p}. \quad (35)$$

Substituting (35) into (34), we obtain

$$\begin{aligned}\dot{V}_{s_{i,p}} \le& \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}}s_{i,p}(s_{i,p+1} + \alpha_{i,p}) + \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}}s_{i,p}z_{i,p} \\ &+|s_{i,p}|\theta_{i,p}^\top W_{i,p} + |s_{i,p}|\delta_{i,p} + \frac{\varepsilon_{i,p}}{2} \\ &-\frac{\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}}s_{i,p-1}s_{i,p} + \tilde{\theta}_{i,p}^\top\Gamma_{i,p}^{-1}\dot{\hat{\theta}}_{i,p}. \end{aligned} \quad (36)$$

Then, we present the virtual controller $\alpha_{i,p}$ as

$$\alpha_{i,p} = -\frac{s_{i,p}\beta_{i,p}^2}{\sqrt{s_{i,p}^2\beta_{i,p}^2 + \kappa_{i,p}^2}} \quad (37)$$

$$\beta_{i,p} = c_{i,p}|s_{i,p}| + \hat{\theta}_{i,p}^\top W_{i,p} \quad (38)$$

where $c_{i,p}$ and $\kappa_{i,p}$ are positive constants, and $\hat{\theta}_{i,p}$ is the updated parameter of $\theta_{i,p}$ and is updated by

$$\dot{\hat{\theta}}_{i,p} = \Gamma_{i,p}(|s_{i,p}|W_{i,p} - \kappa_{\theta_{i,p}}\hat{\theta}_{i,p}) \tag{39}$$

with $\hat{\theta}_{i,p}(0) > 0$ and a constant $\kappa_{\theta_{i,p}} > 0$.

By applying a technique similar to that for deriving (23), we have

$$\frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}} s_{i,p}\alpha_{i,p} + |s_{i,p}|\beta_{i,p} \le \kappa_{i,p}. \tag{40}$$

Substituting (38)-(40) into (36), it follows that

$$\begin{aligned}\dot{V}_{s_{i,p}} \le& \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}} s_{i,p}s_{i,p+1} + \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}} s_{i,p}z_{i,p} \\ &-|s_{i,p}|\beta_{i,p} + |s_{i,p}|\theta_{i,p}^\top W_{i,p} + |s_{i,p}|\delta_{i,p} + \frac{\varepsilon_{i,p}}{2} \\ &-\frac{\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}} s_{i,p-1}s_{i,p} + \tilde{\theta}_{i,p}^\top \Gamma_{i,p}^{-1}\dot{\hat{\theta}}_{i,p} + \kappa_{i,p} \\ \le& -c_{i,p}s_{i,p}^2 + \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}} s_{i,p}s_{i,p+1} + \frac{\xi_{\lambda_{i,p}}}{\xi_{i,p0}} s_{i,p}z_{i,p} \\ &-\frac{\xi_{\lambda_{i,p-1}}}{\xi_{i,(p-1)0}} s_{i,p-1}s_{i,p} + |s_{i,p}|\delta_{i,p} + \frac{\varepsilon_{i,p}}{2} \\ &-\kappa_{\theta_{i,p}}\tilde{\theta}_{i,p}^\top\hat{\theta}_{i,p} + \kappa_{i,p}. \end{aligned} \tag{41}$$

Step n_i: Differentiating s_{i,n_i} with respect to time gives

$$\begin{aligned}\dot{s}_{i,n_i} =& h_{i,n_i}(\bar{x}_{i,n_i}, u_i) + d_{i,n_i} - \frac{\xi_{i,n_i0}\xi_{\lambda_{i,n_i-1}}}{\xi_{i,(n_i-1)0}} s_{i,n_i-1} \\ &+\frac{\xi_{i,n_i0}\xi_{\lambda_{i,n_i-1}}}{\xi_{i,(n_i-1)0}} s_{i,n_i-1} - \dot{\bar{\alpha}}_{i,n_i-1}. \end{aligned} \tag{42}$$

From Assumption 1, we know $\partial h_{i,n_i}(\bar{x}_{i,n_i}, u_i)/\partial u_i > \xi_{i,n_i0} > 0$. Let $v_{i,n_i} = (\xi_{i,n_i0}\xi_{\lambda_{i,n_i-1}}/\xi_{i,(n_i-1)0})s_{i,n_i-1} - \dot{\bar{\alpha}}_{i,n_i-1}$. Because $\partial v_{i,n_i}/\partial u_i = 0$, $\partial(h_{i,n_i}(\bar{x}_{i,n_i}, u_i) + v_{i,n_i})/\partial u_i > \xi_{i,p0}$. The implicit function theorem indicates the existence of an ideal control input $u_i^* = \alpha_{i,n_i}^*(\bar{x}_{i,n_i}, v_{i,n_i})$ satisfying $h_{i,n_i}(\bar{x}_{i,n_i}, \alpha_{i,n_i}^*) + v_{i,n_i} = 0$. Using the mean value theorem yields

$$\begin{aligned}h_{i,n_i}(\bar{x}_{i,n_i}, u_i) =& h_{i,n_i}(\bar{x}_{i,n_i}, \alpha_{i,n_i}^*) \\ &+\xi_{\lambda_{i,n_i}}(u_i - \alpha_{i,n_i}^*)\end{aligned} \tag{43}$$

with $\xi_{\lambda_{i,n_i}} = \xi_{i,n_i}(\bar{x}_{i,n_i}, x_{i,(n_i+1)\lambda_{n_i}})$ and $x_{i,(n_i+1)\lambda_{n_i}} = \lambda_{i,n_i}u_i + (1-\lambda_{i,n_i})\alpha_{i,n_i}^*$ with $0 < \lambda_{i,n_i} < 1$.

The Lyapunov function is considered as

$$V_{i,n_i} = \frac{1}{2\xi_{i,n_i0}} s_{i,n_i}^2 + \frac{1}{2}\tilde{\theta}_{i,n_i}^\top \Gamma_{i,n_i}^{-1}\tilde{\theta}_{i,n_i} \tag{44}$$

where $\tilde{\theta}_{i,n_i} = \hat{\theta}_{i,n_i} - \theta_{i,n_i}$ and $\Gamma_{i,n_i} > 0$ is a diagonal matrix. Differentiating V_{i,n_i} with respect to time and using (43) give

$$\begin{aligned}\dot{V}_{i,n_i} \le& \frac{\xi_{\lambda_{i,n_i}}}{\xi_{i,n_i0}} s_{i,n_i}u_i + |s_{i,n_i}|M_{i,n_i}(\zeta_{i,n_i}) + \frac{\varepsilon_{i,n_i}}{2} \\ &+\tilde{\theta}_{i,n_i}^\top \Gamma_{i,n_i}^{-1}\dot{\hat{\theta}}_{i,n_i} - \frac{\xi_{\lambda_{i,n_i-1}}}{\xi_{i,(n_i-1)0}} s_{i,n_i-1}s_{i,n_i}\end{aligned} \tag{45}$$

where $\varepsilon_{i,n_i} > 0$ is a constant and $M_{i,n_i}(\zeta_{i,n_i}) = |s_{i,n_i}|(\xi_{i,n_i1}^2(\alpha_{i,n_i}^*)^2 + \bar{d}_{i,n_i}^2)/(\xi_{i,n_i0}^2\varepsilon_{i,n_i})$ with $\zeta_{i,n_i} = [(\alpha_{i,n_i-1} - \bar{\alpha}_{i,n_i-1})/\tau_{i,n_i-1}, \bar{x}_{i,n_i}, s_{i,n_i-1}, s_{i,n_i}]^\top$.

To approximate $M_{i,n_i}(\zeta_{i,n_i})$, the RBFNN is employed as follows:

$$M_{i,n_i}(\zeta_{i,n_i}) = \theta_{i,n_i}^\top W_{i,n_i}(\zeta_{i,n_i}) + \delta_{i,n_i}. \tag{46}$$

Applying (46) to (45), it follows that

$$\begin{aligned}\dot{V}_{i,n_i} \le& \frac{\xi_{\lambda_{i,n_i}}}{\xi_{i,n_i0}} s_{i,n_i}u_i + |s_{i,n_i}|\delta_{i,n_i} \\ &+|s_{i,n_i}|\theta_{i,n_i}^\top W_{i,n_i} + \tilde{\theta}_{i,n_i}^\top \Gamma_{i,n_i}^{-1}\dot{\hat{\theta}}_{i,n_i} \\ &-\frac{\xi_{\lambda_{i,n_i-1}}}{\xi_{i,(n_i-1)0}} s_{i,n_i-1}s_{i,n_i} + \frac{\varepsilon_{i,n_i}}{2}.\end{aligned} \tag{47}$$

We design the actual control law as

$$u_i = -\frac{s_{i,n_i}\beta_{i,n_i}^2}{\sqrt{s_{i,n_i}^2\beta_{i,n_i}^2 + \kappa_{i,n_i}^2}} \tag{48}$$

$$\beta_{i,n_i} = c_{i,n_i}|s_{i,n_i}| + \hat{\theta}_{i,n_i}^\top W_{i,n_i} \tag{49}$$

$$\dot{\hat{\theta}}_{i,n_i} = \Gamma_{i,n_i}(W_{i,n_i}|s_{i,n_i}| - \kappa_{\theta_{i,n_i}}\hat{\theta}_{i,n_i}) \tag{50}$$

where $\hat{\theta}_{i,n_i}(0) > 0$, c_{i,n_i} and $\kappa_{\theta_{i,n_i}}$ are positive constants and $\hat{\theta}_{i,n_i}$ is the adaptive tuning parameter of θ_{i,n_i}.

Adding and subtracting $|s_{i,n_i}|\beta_{i,n_i}$ to (47), applying (48)-(50) into it, and using the inequality $(\xi_{\lambda_{i,n_i}}/\xi_{i,n_i0})s_{i,n_i}u_i + |s_{i,n_i}|\beta_{i,n_i} \le \kappa_{i,n_i}$, we obtain

$$\begin{aligned}\dot{V}_{i,n_i} \le& -c_{i,n_i}s_{i,n_i}^2 - \frac{\xi_{\lambda_{i,n_i-1}}}{\xi_{i,(n_i-1)0}} s_{i,n_i-1}s_{i,n_i} + |s_{i,n_i}|\delta_{i,n_i} \\ &+\frac{\varepsilon_{i,n_i}}{2} - \kappa_{\theta_{i,n_i}}\tilde{\theta}_{i,n_i}^\top\hat{\theta}_{i,n_i} + \kappa_{i,n_i}.\end{aligned} \tag{51}$$

Remark 3: In (17) and (18), the proposed virtual controller $\alpha_{i,1}$ is designed using the event-triggered inter-agent communication information $\bar{y}_j$ in ϕ_i. Because $\bar{y}_j$ is piecewise continuous, $\alpha_{i,1}$ is not differentiable. To circumvent this problem, the dynamic surface control technique using filtered virtual control is used to design the proposed actual controller u_i in (49) in a recursive manner. Unlike the conventional dynamic surface design, the non-differentiability of the boundary layer error $z_{i,1}$ on $\alpha_{i,1}$ must be addressed to perform the closed-loop stability analysis. To this end, we present a novel stability analysis strategy, as stated in the following section.

C. CLOSED-LOOP STABILITY ANALYSIS

The virtual control law $\alpha_{i,1}$ in (17) is non-differentiable owing to the discontinuous event-triggered communication-based error signals ϕ_i in (19). Thus, the error dynamics of the boundary layer error $z_{i,1}$ cannot be used in the stability analysis, compared to the existing dynamic surface designs. To circumvent this problem, we first analyze the stability of the following boundary error $\breve{z}_{i,1}$ using the continuous communication-based error signal ϵ_i in (28):

$$\breve{z}_{i,1} = \bar{\alpha}_{i,1} - \breve{\alpha}_{i,1} \tag{52}$$

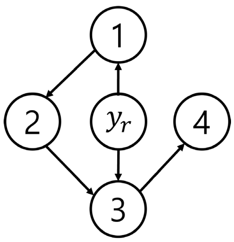


FIGURE 1. **Directed graph.**

where

$$\breve{\alpha}_{i,1} = -\frac{s_{i,1}\breve{\beta}_{i,1}^2}{\sqrt{s_{i,1}^2\breve{\beta}_{i,1}^2 + \kappa_{i,1}^2}} \tag{53}$$

$$\breve{\beta}_{i,1} = c_{i,1}|s_{i,1}| + \hat{\theta}_{i,1}^{\top} W_{i,1} + \epsilon_i \tanh\left(\frac{|s_{i,1}|\epsilon_i}{k_i}\right). \tag{54}$$

Then, the boundedness of $z_{i,1}$ is proved from the fact that $\breve{z}_{i,1}$ is bounded. To this end, the total Lyapunov function is

$$V = \sum_{i=1}^{N}\left[\sum_{p=1}^{n_i} V_{i,p} + \frac{1}{2}\breve{z}_{i,1}^2 + \sum_{p=2}^{n_i-1}\frac{1}{2}z_{i,p}^2\right]. \tag{55}$$

We develop the necessary lemma for the stability analysis as follows.

Lemma 4: There exist any constant $k > 0$ and any ψ_1, $\psi_2 \in \mathbb{R}$ satisfying

$$\left|\frac{\psi_1^2}{\sqrt{\psi_1^2 + k^2}} - \frac{\psi_2^2}{\sqrt{\psi_2^2 + k^2}}\right| < 2|\psi_1 - \psi_2|. \tag{56}$$

Proof: Consider the function $G(\psi) = \psi^2/\sqrt{\psi^2 + k^2}$ with $k > 0$ and $\psi \in \mathbb{R}$. The derivative of $G(\psi)$ for ψ is

$$\frac{dG(\psi)}{d\psi} = \frac{\psi}{\sqrt{\psi^2 + k^2}}\left(2 - \frac{\psi^2}{\psi^2 + k^2}\right) < 2 \tag{57}$$

Using (57), we get

$$G(\psi_1) - G(\psi_2) = \int_{\psi_2}^{\psi_1}\frac{\psi}{\sqrt{\psi^2 + k^2}}\left(2 - \frac{\psi^2}{\psi^2 + k^2}\right)d\psi,$$

$$|G(\psi_1) - G(\psi_2)| < \left|\int_{\psi_2}^{\psi_1} 2d\psi\right| = 2|\psi_1 - \psi_2|. \tag{58}$$

The above conclusion leads to (56). This completes the proof of Lemma 4. ■

Theorem 1: Consider the closed-loop system composed of the nonaffine nonlinear multiagent system (1), the distributed adaptive controller (48), distributed estimators (20), adaptation laws (21), (39), (50), and dynamic event-triggering conditions (3). For initial conditions such that $V(0) \le \gamma$ with any constant $\gamma > 0$, all the signals of the subsystem (1) are semiglobally uniformly ultimately bounded, and the consensus tracking errors converge to an adjustable bound around zero. Furthermore, Zeno behavior is excluded.

Proof: From (30), (41), and (51), differentiating V with respect to time gives

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Bigg[& -\varphi_i\epsilon_i + \sum_{p=1}^{n_i}\Big\{ - c_{i,p}s_{i,p}^2 + |s_{i,p}|\delta_{i,p} + \frac{\varepsilon_{i,p}}{2} \\ & -\kappa_{\theta_{i,p}}\tilde{\theta}_{i,p}^{\top}\hat{\theta}_{i,p} + \kappa_{i,p}\Big\} + \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}(z_{i,1} - \breve{z}_{i,1}) \\ & -\frac{1}{\tau_{i,1}}\breve{z}_{i,1}^2 + \breve{z}_{i,1}\Pi_{i,1} + \sum_{p=2}^{n_i-1}\left(-\frac{1}{\tau_{i,p}}z_{i,p}^2 + z_{i,p}\Pi_{i,p}\right) \\ & +|s_{i,1}|\sum_{j=1}^{N}a_{ij}|y_j - \bar{y}_j| + (1-\nu_i)|\tilde{y}_{i,r}|\sum_{j=1}^{N}a_{ij}|y_j - \bar{y}_j| \\ & +\frac{(1-\nu_i)}{r_i}\tilde{y}_{i,r}\dot{y}_r + (1-\nu_i)\kappa_{y_r}\tilde{y}_{i,r}\hat{y}_{i,r} + 0.2785k_i \\ & -\varrho_i^2 s_{i,1}^2 - \mu_i\varpi_i - \rho_i\varpi_i^2 + m_i\Bigg]\end{aligned} \tag{59}$$

where $\Pi_{i,1} = (\xi_{\lambda_{i,1}}/\xi_{i,10})s_{i,1} + (\alpha_{i,1} - \breve{\alpha}_{i,1})/\tau_{i,1} - \dot{\breve{\alpha}}_{i,1}$ and $\Pi_{i,p} = (\xi_{\lambda_{i,p}}/\xi_{i,p0})s_{i,p} - \dot{\alpha}_{i,p}$.

Using (8), (52), the triggering condition (3), and Lemma 4, we have

$$\begin{aligned}&\frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}(z_{i,1} - \breve{z}_{i,1}) \\ &\quad= \frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}\left(\frac{s_{i,1}^2\beta_{i,1}^2}{\sqrt{s_{i,1}^2\beta_{i,1}^2 + \kappa_{i,1}^2}} - \frac{s_{i,1}^2\breve{\beta}_{i,1}^2}{\sqrt{s_{i,1}^2\breve{\beta}_{i,1}^2 + \kappa_{i,1}^2}}\right) \\ &\quad\le \frac{2\xi_{i,11}}{\xi_{i,10}}|s_{i,1}\beta_{i,1} - s_{i,1}\breve{\beta}_{i,1}|.\end{aligned} \tag{60}$$

Then, using (18), (54), and Lemma 2, we obtain

$$\begin{aligned}&|s_{i,1}\beta_{i,1} - s_{i,1}\breve{\beta}_{i,1}| \\ &\quad\le |s_{i,1}||\beta_{i,1} - \breve{\beta}_{i,1}| \\ &\quad= \left| |s_{i,1}|\phi_i \tanh\left(\frac{|s_i|\phi_i}{k_i}\right) - |s_{i,1}|\epsilon_i \tanh\left(\frac{|s_{i,1}|\epsilon_i}{k_i}\right)\right| \\ &\quad\le 1.1997|s_{i,1}||\phi_i - \epsilon_i|.\end{aligned} \tag{61}$$

Using (61), (60) becomes

$$\frac{\xi_{\lambda_{i,1}}}{\xi_{i,10}}s_{i,1}(z_{i,1} - \breve{z}_{i,1}) \le 2\varrho_i|s_{i,1}|\sum_{j=1}^{N}a_{ij}|y_j - \bar{y}_j|. \tag{62}$$

From Young's Inequality, Assumption 3, $a_{ij} = 1$, and the triggering condition (3), the following inequalities hold:

$$\tilde{y}_{i,r}\hat{y}_{i,r} \le -\frac{3}{4}\tilde{y}_{i,r}^2 + H_1^2 \tag{63}$$

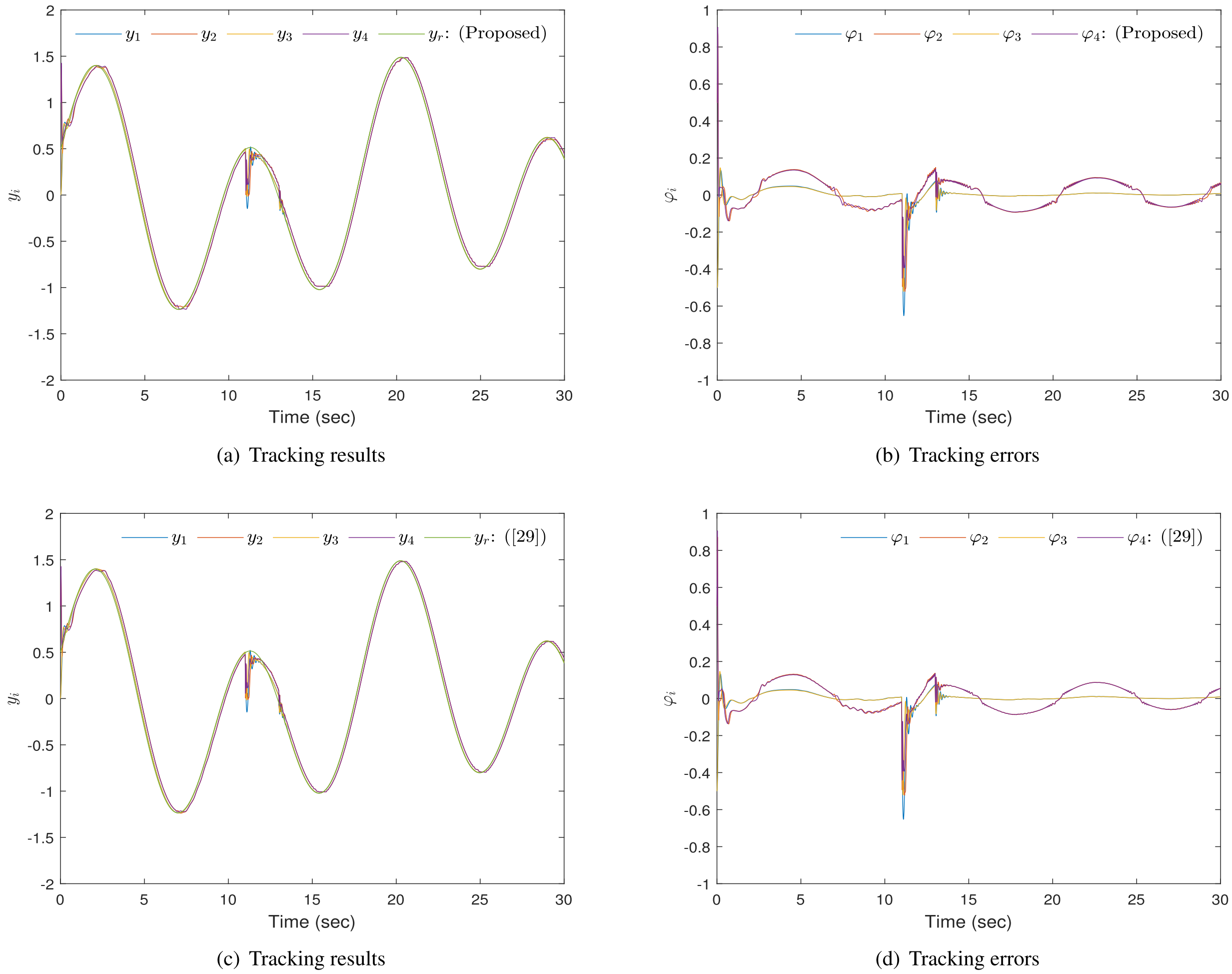
$$-\tilde{\theta}_{i,p}^{\top}\hat{\theta}_{i,p} \le -\frac{1}{2}\tilde{\theta}_{i,p}^{\top}\tilde{\theta}_{i,p} + \frac{1}{2}\theta_{i,p}^{\top}\theta_{i,p} \tag{64}$$

$$|s_{i,p}|\delta_{i,p} \le s_{i,p}^2 + \frac{1}{4}\bar{\delta}_{i,p}^2 \tag{65}$$

$$|s_{i,1}|\sum_{j=1}^{N}a_{ij}|y_j - \bar{y}_j| \le \frac{1}{4}s_{i,1}^2 + (\sum_{j=1}^{N}a_{ij}\varpi_j)^2$$

TABLE 1. Comparison of the number of events.

Agent i	1	2	3	4	Total
Proposed	382	474	400	480	1736
Previous [29]	576	579	561	559	2275



(a) Tracking results (b) Tracking errors

(c) Tracking results (d) Tracking errors

FIGURE 2. Comparison of consensus tracking results and errors.

$$\leq \frac{1}{4}s_{i,1}^2 + N\sum_{j=1}^{N}\varpi_j^2 \tag{66}$$

$$2\varrho_i|s_{i,1}|\sum_{j=1}^{N}a_{ij}|y_j-\bar{y}_j| \leq \varrho_i^2 s_{i,1}^2 + N\sum_{j=1}^{N}\varpi_j^2 \tag{67}$$

$$|\tilde{y}_{i,r}|\sum_{j=1}^{N}a_{ij}|y_j-\bar{y}_j| \leq \frac{\kappa_{y_r}}{4}\tilde{y}_{i,r}^2 + \frac{1}{\kappa_{y_r}}(\sum_{j=1}^{N}a_{ij}\varpi_j)^2$$

$$\leq \frac{\kappa_{y_r}}{4}\tilde{y}_{i,r}^2 + \frac{1}{\kappa_{y_r}}N\sum_{j=1}^{N}\varpi_j^2 \tag{68}$$

$$\frac{(1-\nu_i)}{r_i}\tilde{y}_{i,r}\dot{y}_r \leq \frac{H_2^2}{r_i^2\kappa_{y_r}} + \frac{(1-\nu_i)\kappa_{y_r}}{4}\tilde{y}_{i,r}^2 \tag{69}$$

$$\breve{z}_{i,1}\Pi_{i,1} \leq \frac{\breve{z}_{i,1}^2\Pi_{i,1}^2}{\varepsilon_{i,1}} + \frac{\varepsilon_{i,1}}{4} \tag{70}$$

$$z_{i,p}\Pi_{i,p} \leq \frac{z_{i,p}^2\Pi_{i,p}^2}{\varepsilon_{i,p}} + \frac{\varepsilon_{i,p}}{4} \tag{71}$$

where H_1 and H_2 are positive constants satisfying $|y_r| \leq H_1$ and $|\dot{y}_r| \leq H_2$, respectively.

Define compact sets as $\Lambda_1 = \{\sum_{i=1}^{N}[\sum_{p=1}^{n_i}(s_{i,p}^2 + \tilde{\theta}_{i,p}^\top\Gamma_{i,p}^{-1}\tilde{\theta}_{i,p}) + \breve{z}_{i,1}^2 + \sum_{p=2}^{n_i-1}z_{i,p}^2 + (1-\nu_i)\tilde{y}_{i,r}^2/r_i + \varpi_i] \leq 2\gamma\}$ and $\Lambda_2 = \{y_r^2 + \dot{y}_r^2 \leq y^*\}$ with a constant $y^* > 0$. For a constant $\Pi_{i,p}^* > 0$, it is ensured that $|\Pi_{i,p}| \leq \Pi_{i,p}^*$ on $\Lambda_1 \times \Lambda_2$.

By substituting the above inequalities and choosing $c_{i,1} = c_{i,1}^* + 5/4$, $c_{i,p} = c_{i,p}^* + 1$, $1/\tau_{i,p-1} = \tau_{i,p-1}^* + (\Pi_{i,p-1}^*)^2/\varepsilon_{i,p-1}$, $p = 2, \ldots, n_i$ and $\rho_i \geq 2N^2 + N^2/\kappa_{y_r}$ into (59), we have

$$\dot{V} \leq \sum_{i=1}^{N}\Big[-\varphi_i\epsilon_i - c_{i,1}^* s_{i,1}^2 - \sum_{p=2}^{n_i}c_{i,p}^* s_{i,p}^2 - \frac{(1-\nu_i)\kappa_{y_r}}{4}\tilde{y}_{i,r}^2$$

$$
\begin{aligned}
&-\kappa_{\theta_{i,p}}\sum_{p=1}^{n_i}\frac{\tilde{\theta}_{i,p}^{\top}\tilde{\theta}_{i,p}}{2}-\mu_i\varpi_i-\tau_{i,1}^{*}\breve{z}_{i,1}^{2}\\
&-\left(1-\frac{\Pi_{i,1}^{2}}{(\Pi_{i,1}^{*})^{2}}\right)\frac{(\Pi_{i,1}^{*})^{2}\breve{z}_{i,1}^{2}}{\varepsilon_{i,1}}-\sum_{p=2}^{n_i}\tau_{i,p}^{*}z_{i,p}^{2}\\
&-\sum_{p=2}^{n_i}\left(1-\frac{\Pi_{i,p}^{2}}{(\Pi_{i,p}^{*})^{2}}\right)\frac{(\Pi_{i,p}^{*})^{2}z_{i,p}^{2}}{\varepsilon_{i,p}}\Bigg]+\sigma_2\\
\le&-\frac{1}{2}\lambda_{min}(Q)\|\varphi\|^{2}-\sigma_1V+\sigma_2\\
&-\sum_{i=1}^{N}\left(1-\frac{\Pi_{i,1}^{2}}{(\Pi_{i,1}^{*})^{2}}\right)\frac{(\Pi_{i,1}^{*})^{2}\breve{z}_{i,1}^{2}}{\varepsilon_{i,1}}\\
&-\sum_{i=1}^{N}\sum_{p=2}^{n_i}\left(1-\frac{\Pi_{i,p}^{2}}{(\Pi_{i,p}^{*})^{2}}\right)\frac{(\Pi_{i,p}^{*})^{2}z_{i,p}^{2}}{\varepsilon_{i,p}}
\end{aligned}
\tag{72}
$$

where $\varphi = [\varphi_1, \ldots, \varphi_N]^\top$; $\varphi_i = y_i - y_r$, $i = 1, \ldots, N$, $\sigma_1 = \min_{i=1,\ldots,N,p=1,\ldots,n_i}\{2\xi_{i,p0}c_{i,p}^*, \kappa_{\theta_{i,p}}/\lambda_M(\Gamma_{i,p}^{-1}), 2\tau_{i,1}^*, \ldots, 2\tau_{i,n_i-1}^*, \mu_i, \kappa_{y_r}r_i/2\}$ and $\sigma_2 = \sum_{i=1}^N[\sum_{p=1}^{n_i}(\kappa_{i,p} + \theta_{i,p}^\top\theta_{i,p}\kappa_{\theta_{i,p}}/2 + \bar{\delta}_{i,p}^2/4 + \varepsilon_{i,p}/2) + \sum_{p=1}^{n_i-1}(\varepsilon_{i,p}/4) + H_2^2/(r_i^2\kappa_{y_r}) + (1-\nu_i)H_1^2\kappa_{y_r} + m_i + 0.2785k_i]$.

From $|\Pi_{i,p}| \le \Pi_{i,p}^*$ on $V = \gamma$, (72) can be written as $\dot{V} \le -\sigma_1 V + \sigma_2$, which leads to $V(t) \le V(0)\exp(-\sigma_1 t) + (\sigma_2/\sigma_1)(1-\exp(-\sigma_1 t))$. Thus, all the closed-loop signals are semiglobally uniformly ultimately bounded. Owing to $s_{i,1} = \varphi_i + (1-\nu_i)\tilde{y}_{i,r}$, $\|\varphi\|^2 \le \sum_{i=1}^N 2(s_{i,1}^2 + (1-\nu_i)\tilde{y}_{i,r}^2) \le \sigma_3 V$ with $\sigma_3 = \max\{4, 4r_1, \ldots, 4r_N\}$. This means that φ converges to the compact set $\Psi = \{\varphi \,|\, \|\varphi\|^2 \le \sigma_3(\sigma_2/\sigma_1)\}$ which can be made arbitrarily small by increasing σ_1 (i.e., the design parameters $c_{i,p}$, $\tau_{i,1}$, μ_i, $\Gamma_{i,p}$, r_i, κ_{y_r}, and $\kappa_{\theta_{i,p}}$).

To demonstrate the avoidance of Zeno behavior, we prove that the inter-event times $(t_{k+1}^j - t_k^j)$ are lower bounded by a positive constant, where $j \in \mathcal{V}$ and $k \in \mathbb{Z}^+$. To this end, we first define the measurement error as $\iota_j = y_j - \bar{y}_j$ for $t \in [t_k^j, t_{k+1}^j)$ and check the boundedness of the change rate of ι_j. Because $\bar{y}_j$ is a constant for $t \in [t_k^j, t_{k+1}^j)$, differentiating $|\iota_j|$ gives $d|\iota_j|/dt \le |\dot{y}_j| = |h_{j,1}(x_{j,1}, x_{j,2}) + d_{j,1}|$. Because the boundness of $x_{j,1}$, $x_{j,2}$, and $d_{j,1}$ is guaranteed, there exists a constant $\Delta > 0$ such that $d|\iota_j|/dt \le \Delta$. Then, integrating $d|\iota_j|/dt \le \Delta$ with respect to time and using the event-triggered condition (3) and the property $\varpi_j > 0$ presented in Lemma 3, $t_{k+1}^j - t_k^j \ge \varpi_j/\Delta > 0$ is ensured. In other words, Zeno behavior is excluded. ■

Remark 4: To analyze the effect of the graph structure on the tracking performance, we consider (72). Because $|\Pi_{i,p}| \le \Pi_{i,p}^*$ on $V = \gamma$, (72) can be written as $\dot{V} \le -(\lambda_{min}(Q)/2)\|\varphi\|^2 + \sigma_2$ where $Q = (\mathcal{L} + \Upsilon) + (\mathcal{L} + \Upsilon)^\top$. This implies that when $\|\varphi\| > \sqrt{2\sigma_2/\lambda_{min}(Q)}$, $\dot{V} < 0$ is ensured. φ is finally bounded as $\|\varphi\| \le \sqrt{2\sigma_2/\lambda_{min}(Q)}$. Thus, increasing $\lambda_{min}(Q)$ (i.e., the link connectivity of the graph structure $\mathcal{L} + \Upsilon$) helps reduce the bound of the consensus tracking error.

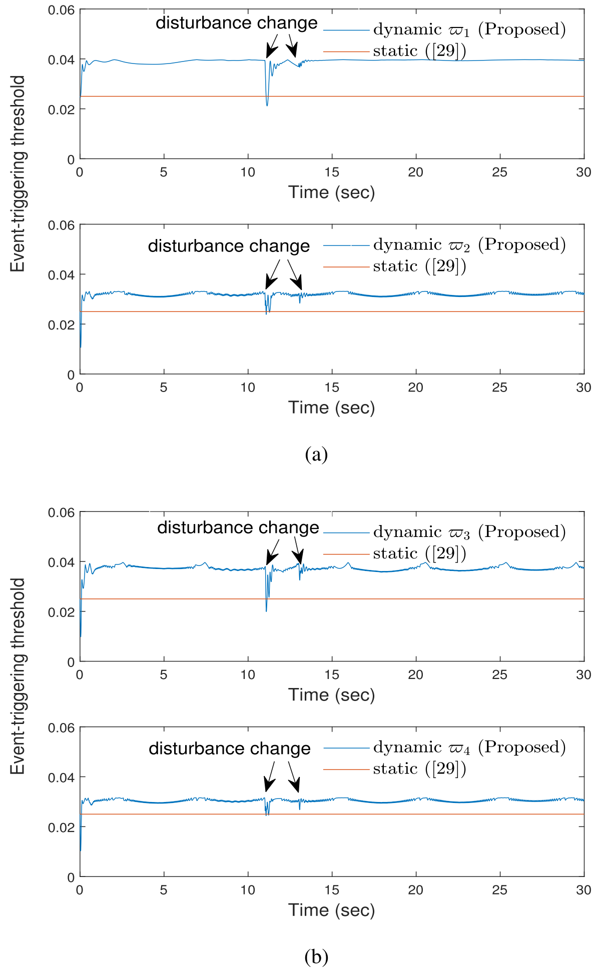


FIGURE 3. Comparison of event-triggering thresholds (a) agents 1 (upper figure) and 2 (lower figure) (b) agents 3 (upper figure) and 4 (lower figure).

Remark 5: In this paper, RBFNNs are employed to approximate the unknown nonlinear functions $M_{i,p}(\zeta_{i,p})$, $p = 1, \ldots, N$. The bounds $\bar{\delta}_{i,p}$ of the approximation errors $\delta_{i,p}$ are treated by decreasing the semiglobal uniform ultimate bound (i.e., the compact set Ψ can be made arbitrarily small by increasing σ_1). The RBFNNs can also be replaced with fuzzy logic systems, as reported in [31]. Unlike the basis functions of RBFNNs, which are defined as Gaussian functions, the basis functions of fuzzy logic systems are constructed to include a fuzzifier, fuzzy inference engine, and defuzzifier [40].

IV. SIMULATION EXAMPLES

Consider the heterogeneous nonlinear followers described by

$$
\begin{aligned}
\dot{x}_{i,1} &= (K_{i,1} + 0.2e^{x_{i,1}})x_{i,2} + 0.5\cos(x_{i,1}^2) + d_{i,1}\\
\dot{x}_{i,2} &= (K_{i,2} + 0.2\sin(x_{i,1}x_{i,2}))u_i + 0.5\sin(u_i) + d_{i,2}\\
y_i &= x_{i,1}
\end{aligned}
$$

where $i = 1, \ldots, 4$, $K_{1,p} = 1$, $K_{2,p} = 1.1$, $K_{3,p} = 1.2$, and $K_{4,p} = 1.3$, $p = 1, 2$. The leader output is $y_r(t) = \sin(0.7t) + 0.5\cos(0.3t)$. The directed topology for multiagent

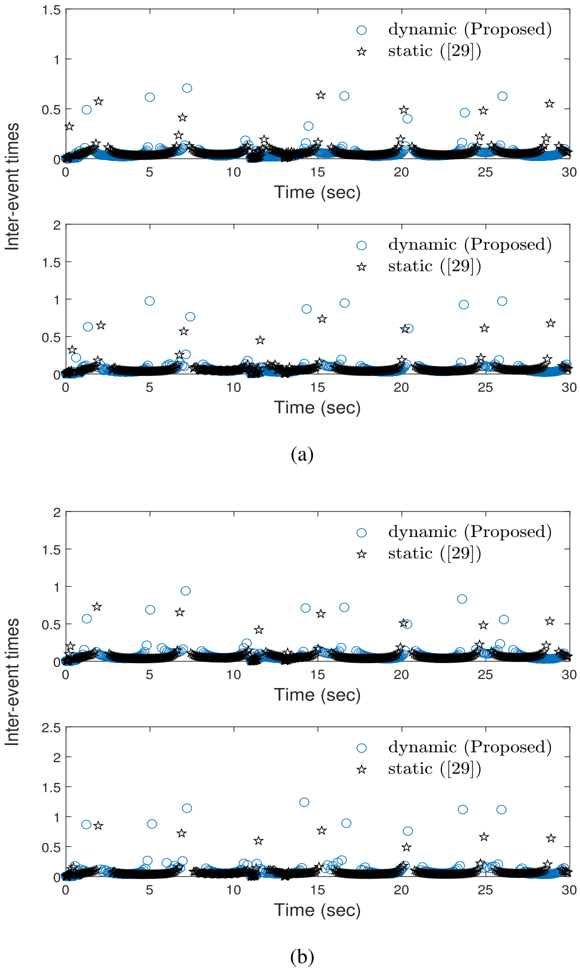


FIGURE 4. Comparison of inter-event times (a) agents 1 (upper figure) and 2 (lower figure) (b) agents 3 (upper figure) and 4 (lower figure).

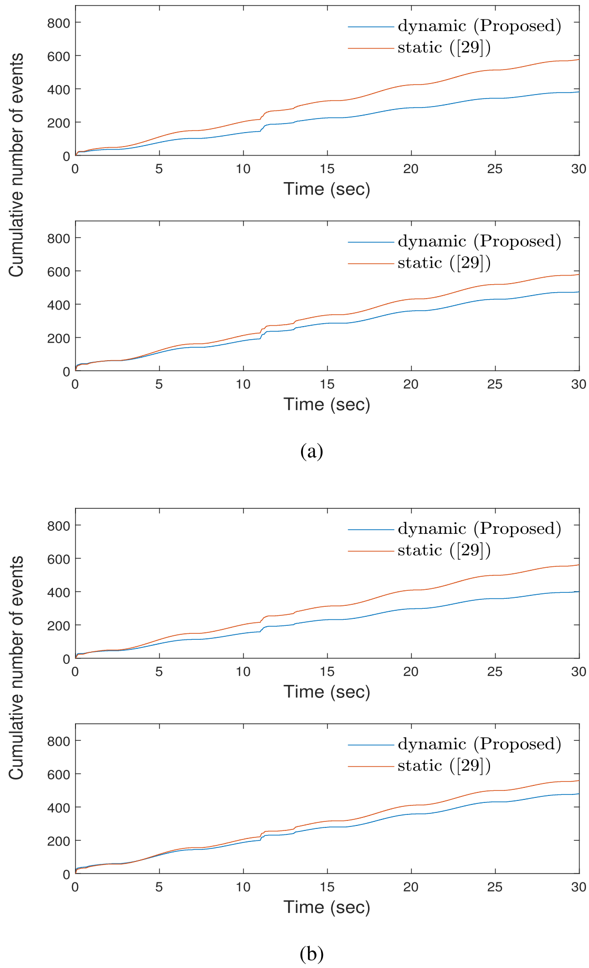


FIGURE 5. Comparison of the cumulative number of events (a) agents 1 (upper figure) and 2 (lower figure) (b) agents 3 (upper figure) and 4 (lower figure).

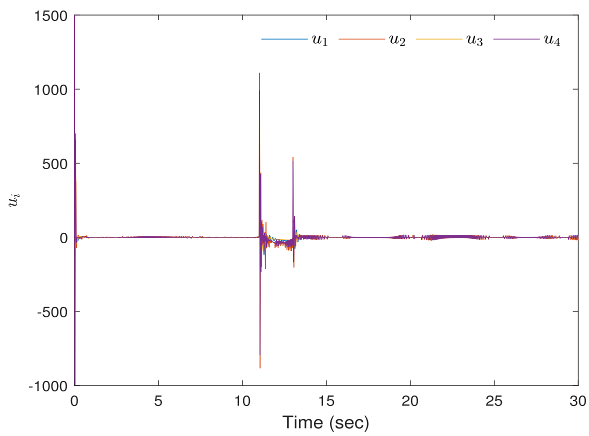


FIGURE 6. Control inputs of the proposed approach.

systems is described in Fig 1. The external disturbances are set as follows:

$$d_{1,1} = d_{3,1} = \begin{cases} 0.1\sin(t), & 0 \le t < 11 \text{ s} \\ 12\sin(t), & 11 \le t < 13 \text{ s} \\ 0.1\sin(t), & t \ge 13 \text{ s} \end{cases}$$

$$d_{2,1} = d_{4,1} = \begin{cases} 0.1\sin(t), & 0 \le t < 11 \text{ s} \\ 30\sin(t), & 11 \le t < 13 \text{ s} \\ 0.1\sin(t), & t \ge 13 \text{ s} \end{cases}$$

$$d_{1,2} = d_{3,2} = \begin{cases} 0.1\cos(t), & 0 \le t < 11 \text{ s} \\ 12\cos(t), & 11 \le t < 13 \text{ s} \\ 0.1\cos(t), & t \ge 13 \text{ s} \end{cases}$$

$$d_{2,2} = d_{4,2} = \begin{cases} 0.1\cos(t), & 0 \le t < 11 \text{ s} \\ 30\cos(t), & 11 \le t < 13 \text{ s} \\ 0.1\cos(t), & t \ge 13 \text{ s}. \end{cases}$$

We compare the proposed dynamic threshold approach with the existing static threshold approach [29]. The static triggering condition presented in [29] is defined as

$$t^j_{k+1} = \inf\{t > t^j_k \mid |x_{j,n}(t) - \bar{x}_{j,n}(t)| \ge m^j_n\} \tag{73}$$

with $n = 1, 2$ and static thresholds $m^1_n = m^2_n = m^3_n = m^4_n = 0.025$. The parameters of the proposed dynamic threshold (4) are set as follows: $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 50$, $\mu_4 = 63$, $\rho_i = 10$, $m_i = 2$, and $n_i = 1$. The initial

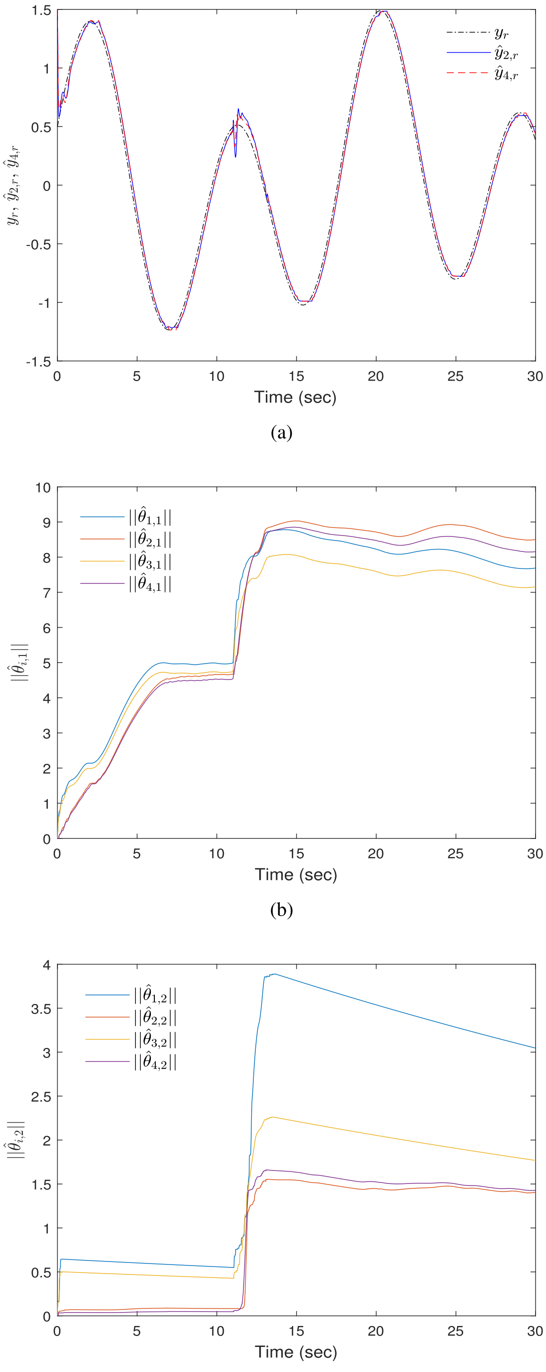


FIGURE 7. Estimated parameters for the proposed approach (a) y_r, $\hat{y}_{i,r}$, $\hat{y}_{i,r}$ (b) $||\hat{\theta}_{i,1}||$ (c) $||\hat{\theta}_{i,2}||$.

values for the proposed controller are $x_{1,1}(0) = x_{3,1}(0) = 0$, $x_{2,1}(0) = x_{4,1}(0) = 1$, $x_{i,2}(0) = 0.1$, $\hat{y}_{2,r}(0) = \hat{y}_{4,r}(0) = 1.5$, and $\varpi_i(0) = 0.025$. To ensure a fair comparison, the initial values $\varpi_i(0)$ of the dynamic thresholds are set to be identical to those of the static thresholds m_n^i. Then, we choose the design parameters as $c_{1,1} = 20$, $c_{1,2} = 12$, $c_{2,1} = 80$, $c_{2,2} = 50$, $c_{3,1} = 20$, $c_{3,2} = 12$, $c_{4,1} = 80$, $c_{4,2} = 50$, $\tau_{i,1} = 0.001$, $r_2 = r_4 = 12$, $\kappa_{1,1} = \kappa_{3,1} = \kappa_{i,2} = 1$, $\kappa_{2,1} = \kappa_{4,1} = 2$, $\Gamma_{i,1} = \text{diag}[20]$, $\Gamma_{1,2} = \Gamma_{3,2} = \text{diag}[15]$, $\Gamma_{2,2} = \Gamma_{4,2} = \text{diag}[20]$, and $\kappa_{y_r} = \kappa_{\theta_{i,1}} = \kappa_{\theta_{i,2}} = 0.001$. The sampling time is $0.001\ s$, and thus, the event-triggering condition (3) is monitored every $0.001\ s$.

In Fig. 2, the consensus tracking results and errors of the proposed approach and the previous approach [29] are compared, which exhibit that the outputs of all followers follow $y_r(t)$ with similar tracking errors. In Fig. 3, the proposed dynamic thresholds for the event-triggering conditions are compared with the static thresholds reported in [29]. The inter-event times are compared in Fig. 4. As shown in Figs. 3 and 4, the proposed dynamic thresholds can adjust the number of triggered events according to the distributed consensus tracking errors. When large external disturbances influence the agents at $t = 11\ s$, the thresholds are reduced to recover the consensus tracking performance by increasing the number of events. In contrast, when distributed consensus tracking is achieved at $t = 0.5\ s$, the thresholds are increased to prevent the unnecessary transmission of the inter-agent information. Thus, the number of triggered events for the proposed approach is smaller than that for the existing approach [29], as indicated in Table 1 and Fig. 5. Figs. 6 and 7 display the control inputs and estimated parameters of the proposed approach, respectively. The proposed dynamic event-triggered controller consumes less network resources than the static event-triggered controller [29], regardless of external disturbances and unknown nonaffine nonlinearities.

V. CONCLUSION

We has developed a dynamic event-triggered inter-agent communication-based adaptive consensus control approach for uncertain nonlinear multiagent systems in the pure-feedback form. The novel dynamic event-triggering mechanism has used the distributed consensus tracking error to flexibly adjust the inter-event times according to the consensus tracking performance. The dynamic surface consensus tracking scheme using event-triggered neighbors' outputs has been constructed to resolve the non-differentiability problem of the first virtual control law associated with event-triggered inter-agent communication. By using the Lyapunov stability theorem and developing technical lemmas, we have presented the closed-loop stability strategy although the boundary layer error is non-differentiable. Finally, the efficiency of the theoretical approach is proved by comparative simulation results with the existing static threshold approach.

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