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RESEARCH ARTICLE

An Optimal Simulation Configuration Method for Real-Time Simulation Based on Simulation Requirements

WENSHUAI ZHAO, XI WANG, SHUBO YANG, AND YIFU LONG
School of Energy and Power Engineering, Beihang University, Beijing 100191, China

Corresponding author: Wenshuai Zhao (m13716752318_1@163.com)

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ABSTRACT An optimal configuration method of simulation parameters, including the solver and step size of each sub-model, is proposed to realize real-time virtual simulation. The content of the presented study is threefold: firstly, the simulation requirements of three simulation modes, including Model-Exchange, Co-Simulation, and Hybrid Co-Simulation, are analyzed; secondly, the mathematical descriptions of the three simulation configuration problems are constructed, which are classified as an inequality problem and two assignment problems, where the assignment problems are decoupled; finally, an optimal simulation configuration procedure is proposed, where the analysis method and corresponding design methods for configured simulation parameters of each sub-model are given. The configuration method is applied to configure the simulation parameters of an aeroengine control system model with 6 separate components, and the simulation results verify the effectiveness of the proposed method. Specifically, the feasibility analysis results of the sub-models reveal that 4 of them meet the requirements while 2 of them are numerically unstable, which are consistent with the verification results. Meanwhile, after redesigning the infeasible sub-models using the proposed design methods, all the sub-models can meet their simulation requirements. In addition, an optimal configuration of the simulation parameters as well as several comparative configurations are provided, and the simulation results show that the time consumption of the optimal configuration is minimal and it can achieve real-time performance.

INDEX TERMS Optimal simulation configuration, real-time simulation, simulation requirements, assignment problem, analysis and design.

I. INTRODUCTION

Numerical simulation is an essential method for aeroengine control system research. For example, in the theoretical research stage, a simulation model can reveal the internal properties of the research object and help comprehending and recognizing, accelerating the research process. In the physical test stage, a simulation model can replace the real object, saving a lot of research cost [1], [2], [3]. In recent years, with the development of advanced control systems, more system characteristics are considered and more components are integrated, simulation modeling is developing toward multi-state and multi-disciplinary. Typical applications are aeroengine digital twin model [4], [5]. Besides, with the expansion of application scenarios, more stringent simulation performance requirements are also proposed. For example, an adaptive model applied in the advanced embedded control systems should meet the requirements of real-time, high precision and high stability [6], [7], [8], [9]. Hence, to follow the application requirements of the modern high-performance aero-engine,

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researching the realization method of real-time simulation to improve the practical application ability of simulation models is necessary.

To realize real-time simulation, the time consumption of the simulation process should be clarified. Some researches reveal the time consumption compositions of the unit model, which includes model initialization, integration, handling discontinuous events, etc. [10], [11]. Theoretically, the simulation speed can be improved by reducing any above item's time consumption. However, it can be found that the integration consumption almost determines the total time consumption. The integration consumption is determined by the model complexity, the simulation steps, and the number of calculations at each step [10], [12]. Among them, the model complexity determines the time consumption of model calculation once; the simulation steps and the number of calculations at each step determine the total number of calculations, and they are determined by the simulation parameters, including the solver, step size, and solving precision [12]. Therefore, there are two basic methods to reduce the integration consumption, including reducing model complexity and reducing the total number of calculations. Among them, the model complexity can be reduced by simplifying the model and reducing the order of the model, such as eliminating algebraic loops and reducing calculation modules [13], [14], [15], [16], [17]. However, these suggested techniques have limitations because the simplified model loses its fidelity and it is difficult to simplify large systems; The total number of calculations can be reduced by using advanced integrator or designing simulation step size. For example, a lightweight solver is introduced to predict discontinuous events to reduce the simulation time [18]. However, it is expensive to develop advanced integrators. Therefore, designing simulation step size by configuring the simulation parameters have better benefits.

Researches on the simulation parameters configuration of the co-simulation model to realize real-time performance are rare. On the contrary, some methods focus on dealing with the data interaction error of sub-models to improve the simulation speed [19], [20]. These methods reduce the simulation time, yet they are very complex and inefficient, and importantly, the step size is variable and the real-time performance cannot be guaranteed. The latest research proposes a configuration method to realize real-time simulation of the unit model [12]. This research does not involve the simulation parameters configuration of the co-simulation model, but it reveals the explicit relationship between the computational complexity and the simulation requirements of the unit model in real-time fixed step size simulation, and establishes explicit mathematical formulas. On this basis, this paper conducts theoretical research on the real-time performance realization method of the co-simulation model, and provides guidance measures for real-time configuration.

The novelty of this work includes:

(1) Firstly, this paper establishes explicit mathematical formulas to clarify the relationship between the model complexity and the simulation requirements of three co-simulation modes, and directly describes these abstract problems by accurate mathematical equations.

(2) Secondly, based on the precise mathematical formulas, the configuration problems are split into three separate decoupled mathematical problems that are straightforward to comprehend and solve, and the calculation theory of the optimal simulation configuration problems is constructed.

(3) Finally, an optimal simulation configuration procedure is proposed, which includes an analysis method and corresponding design methods for configured simulation parameters of each sub-model. The method is proven to complete the configuration work efficiently and accurately, and realize real-time performance.

The structure of this paper is as follows. In Section II, the simulation requirements of three simulation modes are analyzed. In Section III, the mathematical descriptions of the three simulation configuration problems are constructed. In Section IV, the assignment problems are decoupled. In Section V, an optimal simulation configuration procedure is proposed. In Section VI, the simulation parameters of an aeroengine control system model are designed to verify the effectiveness of the proposed method. In Section VII, the conclusions are presented.

II. DESCRIPTION OF SIMULATION REQUIREMENTS

Firstly, to realize real-time simulation, clarifying the explicit relationship between the model computational complexity and the simulation requirements of three co-simulation modes, including Model-Exchange, Co-Simulation, and Hybrid Co-Simulation, and directly describing these abstract problems with mathematical formulas are the key and basic works. Therefore, the simulation requirements of numerical stability, solving precision, and computational complexity about the three simulation modes are analyzed as follows.

A. REQUIREMENTS OF MODEL-EXCHANGE MODE

The model based on the Model-Exchange environment is composed of sub-models generated by the Model-Exchange interface. Among them, all sub-models use the simulation configuration parameters of the main environment, which means all of them have the same solver, solving precision, and simulation step size [21], [22], [23]. Therefore, the whole model can be regarded as a unit model, and the following three basic simulation requirements are analyzed.

1) REQUIREMENT OF NUMERICAL STABILITY

Numerical stability is a basic requirement, which determines whether the calculation results are meaningful. Specifically, the requirement of numerical stability can be expressed as the following inequality [12].

$$|E\left(F(\zeta, fh)\right)| < 1 \tag{1}$$

where h is the simulation step size, f is the frequency of the eigenvalue, ζ is the damping ratio of the eigenvalue, $F(\cdot)$ is a binary function, $F(a,b) = 2\pi \cdot b \cdot (-a + j \cdot \sqrt{1-a^2})$, and $E(\cdot)$ is the function obtained by applying a certain numerical algorithm to system (2).

$$\dot{x}(t) = \lambda \cdot x(t) \tag{2}$$

where $\lambda \in \mathbb{C}$, and $\mathrm{Re}(\lambda) < 0$.

Once inequality (1) is satisfied, the requirement of numerical stability is met. In addition, the region represented by all feasible fh and ζ is called the stability region of the numerical algorithm.

However, the step size of inequality (1) is implicit and inconvenient for calculation, and an explicit formula is necessary. Let $G(\zeta)$ denote the feasible region boundary function of numerical stability, that is $fh = G(\zeta)$, which is determined by the type of numerical algorithm [12]. If the frequency f and the damping ratio ζ of an eigenvalue λ is known at one step, then the requirement of numerical stability about this eigenvalue can be expressed as

$$h < \frac{G(\zeta)}{f} \tag{3}$$

Because the stability theory requires that all the eigenvalues of a system meet numerical stability at all simulation steps [24], [25], the requirement of numerical stability can be expressed as

$$h < \min\{h_j\}, \quad j = 1, 2, \ldots, n_{\text{step}} \tag{4}$$

where n_{step} is the number of the simulation steps, $h_j = min\{G(\zeta_{i,j})/f_{i,j}\}, i = 1, 2, \ldots, m$, and m is the number of eigenvalues of the entire model. The frequency $f_{i,j}$ and the damping ratio $\zeta_{i,j}$ relate to the i-th eigenvalue λ_i at the j-th simulation step.

2) REQUIREMENT OF SOLVING PRECISION

Solving precision is relevant to the accuracy of a simulation. Generally, higher precision means more accurate simulation results. The requirement of solving precision is that the truncation error should be less than the set tolerance error, and it can be expressed as Eq. (5).

$$h < (\varepsilon)^{\frac{1}{N+1}}, \quad \varepsilon > 0 \tag{5}$$

where ε is the tolerance error, and N is the order of the numerical algorithm adopted by the main environment.

3) REQUIREMENT OF COMPUTATIONAL COMPLEXITY

Computational complexity is positively correlated with the time consumption, and higher computational complexity means longer time consumption. Assuming that there are M sub-models in the main environment, the requirement of computational complexity requires

$$T_{\text{CPU}} < T_{\text{SET}}/R \tag{6}$$

where T_{CPU} is the real CPU time consumption, T_{SET} is the set simulation time, and R is the speed coefficient. It is noted that R should be greater than 1 for real-time scenarios.

For simulations with fixed step size, T_{CPU} can be written as

$$T_{\text{CPU}} = T_{\text{step}} \cdot n_{\text{step}} \tag{7}$$

where T_{step} is the single-step calculation time of the system, and it is expressed as

$$T_{\text{step}} = K \cdot \sum_{i=1}^{M} (T_{\text{ODE}})_i \tag{8}$$

where K is the number of calculation stages of the solver used by the main environment, $(T_{\text{ODE}})_i$ is the single-stage calculation time of the i-th sub-model, and of course, M is the number of sub-models.

Meanwhile, T_{SET} is the product of the step size and the number of simulation steps, as shown in Eq. (9).

$$T_{\text{SET}} = h \cdot n_{\text{step}} \tag{9}$$

Combining Eq. (6), Eq. (7), Eq. (8), and Eq. (9), it has

$$h \geq K \cdot R \cdot \sum_{i=1}^{M} (T_{\text{ODE}})_i \tag{10}$$

B. REQUIREMENTS OF CO-SIMULATION MODE

The model based on the Co-Simulation environment is composed of sub-models generated by the Co-Simulation interface. Among them, all the sub-models have their own configuration parameters and do not use the simulation configuration parameters of the main environment, which means all of them have different solvers, solving precisions, and simulation step sizes. In addition, it requires that the step size adopted by each sub-model should be an integral multiple to the simulation step size of the main environment [21], [22], [23].

The requirements of the Co-Simulation environment are divided into two parts.

1) REQUIREMENT OF COMPUTATIONAL COMPLEXITY FROM THE MAIN ENVIRONMENT

For Co-Simulation environment, the single-step calculation time of the i-th sub-model can be expressed as $(T_{\text{ODE}})_i \cdot K_i$, then the single-step calculation time of the entire model T_{step} is expressed as

$$T_{\text{step}} = \sum_{i=1}^{M} ((T_{\text{ODE}})_i \cdot K_i) \tag{11}$$

where K_i is the number of calculation stages of the solver used by the i-th sub-model, and of course, M is the number of sub-models.

Combining Eq. (6), Eq. (7), Eq. (9), and Eq. (11), the requirement of computational complexity is expressed as

$$h > R \cdot \sum_{i=1}^{M} ((T_{\text{ODE}})_i \cdot K_i) \tag{12}$$

2) REQUIREMENT OF SOLVING PRECISION AND NUMERICAL STABILITY FROM THE SUB-MODELS

Assuming the step size adopted by the main environment is h, and the step size adopted by the i-th sub-model is $n_i \cdot h, n_i \in Z, n_i > 0$. Since each sub-model should meet the requirements of solving precision and numerical stability, there are

$$\begin{cases} n_i \cdot h < (h_p)_i \\ n_i \cdot h < (h_s)_i \end{cases} \tag{13}$$

where $(h_p)_i$ and $(h_s)_i$ represent the feasible collection boundary values of the step size of the i-th sub-model, corresponding to solving precision and numerical stability [12]. According to Eq. (4) and Eq. (5), there are

$$\begin{cases} (h_s)_i = \min\{h_j\}, \quad j = 1, 2, \ldots, n_{\text{step}} \\ (h_p)_i = (\varepsilon_i)^{\frac{1}{N+1}} \end{cases} \tag{14}$$

Then, the requirement of the i-th sub-model can be expressed as

$$n_i < \min\{(h_p)_i/h, (h_s)_i/h\}, \quad i = 1, 2, \ldots, M \tag{15}$$

C. REQUIREMENTS OF HYBRID CO-SIMULATION MODE

The model based on the Hybrid Co-Simulation environment is composed of sub-models generated by the Co-Simulation interface and the Model-Exchange interface. It can be regarded as a special case of the Co-Simulation mode, because its sub-models generated by the Model-Exchange interface use the simulation configuration parameters of the main environment [21], [22], [23].

Therefore, the requirements of the Hybrid Co-Simulation mode are the same as the Co-Simulation mode, except:

a) For Eq. (12), if the i-th sub-model is generated by the Model-Exchange interface, there is $K_i = K$.

b) For Eq. (15), if the i-th sub-model is generated by the Model-Exchange interface, there is $n_i = 1$.

III. MATHEMATICAL DESCRIPTION OF SIMULATION CONFIGURATION PROBLEMS

The simulation configuration problem of each simulation mode can be described as a specific mathematical problem that is straightforward to comprehend and solve. The analysis process is as follows.

A. MATHEMATICAL DESCRIPTION OF MODEL-EXCHANGE MODE

An inequality problem can be used to describe the simulation configuration problem of the Model-Exchange mode. The requirements of the Model-Exchange environment can be represented by the feasible collection of the step size.

$$\begin{cases} h \geq h_v \\ h < h_s \\ h < h_p \end{cases} \tag{16}$$

where h_v, h_s and h_p represent the feasible collection boundary values of the step size, corresponding to computational complexity, numerical stability, and solving precision respectively, and they are expressed as

$$\begin{cases} h_v = K \cdot R \cdot \sum_{i=1}^{M} (T_{\text{ODE}})_i \\ h_s = \min\{h_j\}, \quad j = 1, 2, \ldots, n_{\text{step}} \\ h_p = (\varepsilon)^{\frac{1}{N+1}} \end{cases} \tag{17}$$

After determining the simulation requirements and the solver, the feasible collection boundary values of the step size h_v, h_s and h_p can be calculated. If there is a feasible collection of the step size when calculating inequality Eq. (16), the simulation configuration is feasible. If not, the simulation configuration is infeasible.

B. MATHEMATICAL DESCRIPTION OF CO-SIMULATION MODE

An assignment problem can be used to describe the simulation configuration problem of the Co-Simulation environment. The diagram of the assignment problem is shown in Fig. 1.

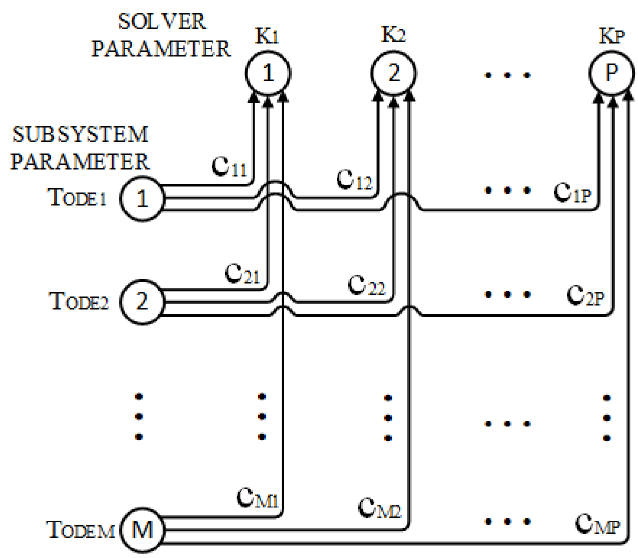

FIGURE 1. Mathematical description of simulation configuration for the Co-Simulation mode.

There are P solvers and M sub-models, and the design task is to assign the i-th sub-model to use the j-th solver to implement simulation. Further, an optimal simulation configuration problem can be derived, and the optimization objective is to minimize the single-step calculation time T_{step} of the system. The analysis process is as follows.

Let c_{ij} represents the single-step consumption time when the i-th sub-model uses the j-th solver to implement simulation. Assuming that the number of solver calculation stages of the j-th solver is K_j, and the single-stage calculation time of the i-th sub-model is $(T_{\text{ODE}})_i$, then c_{ij} can be expressed as

$$c_{\text{ij}} = (T_{\text{ODE}})_i \cdot K_j \tag{18}$$

Import variable $x_{\rm ij}$, let

$$x_{\rm ij} = \begin{cases} 1, & \text{the } i\text{-th subsystem uses the } j\text{-th solver} \\ 0, & \text{other case} \end{cases} \tag{19}$$

The optimization objective is expressed as

$$\min \sum_{i=1}^{M} \sum_{j=1}^{P} c_{\rm ij} \cdot x_{\rm ij} \tag{20}$$

The constraints of the assignment problem are analyzed as follows.

Since a sub-model uses only one solver when simulating, some of the constraints are

$$\sum_{j=1}^{P} x_{\rm ij} = 1, \quad i = 1, 2, \ldots, M \tag{21}$$

In addition, the i-th sub-model can use the j-th solver for simulation only if the requirements Eq. (15) is met. Let $(h_p)_{\rm ij}$ and $(h_s)_{\rm ij}$ represent the feasible collection boundary of the step size when the i-th sub-model uses the j-th solver for simulation, then defining the judge coefficient

$$d_{\rm ij} = \min\{(h_p)_{\rm ij}/h, (h_s)_{\rm ij}/h\} \tag{22}$$

Considering Eq. (15) and the available range of n, that is $n \in Z, n > 0$, other constraints are

$$\begin{cases} x_{\rm ij} \in \{0, 1\}, & \text{if } \mathrm{d_{ij}} \geq 1 \\ x_{\rm ij} = 0, & \text{if } \mathrm{d_{ij}} < 1 \end{cases} \tag{23}$$

Therefore, the mathematical model for the optimal simulation configuration problem of the Co-Simulation environment is expressed as

$$\begin{aligned} &\textit{minimize} \sum_{i=1}^{M} \sum_{j=1}^{P} c_{\rm ij} \cdot x_{\rm ij} \\ &\textit{subject to} \sum_{j=1}^{P} x_{\rm ij} = 1, i = 1, 2, \ldots, M \\ &\qquad x_{\rm ij} = 0, \quad \text{if } \mathrm{d_{ij}} < 1, \ i = 1, 2, \ldots, M, \\ &\qquad j = 1, 2, \ldots, P \\ &\qquad x_{\rm ij} \in \{0, 1\}, \quad \text{if } \mathrm{d_{ij}} \geq 1, \ i = 1, 2, \ldots, M, \\ &\qquad j = 1, 2, \ldots, P \end{aligned} \tag{24}$$

C. MATHEMATICAL DESCRIPTION OF HYBRID CO-SIMULATION MODE

As mentioned in Section II, the model based on the Hybrid Co-Simulation mode is a special case of the Co-Simulation mode. Evidently, the mathematical model Eq. (24) can be used to describe the optimal configuration problem of the Hybrid Co-Simulation environment, except:

Because the sub-models generated by the Model-Exchange interface all use the simulation configuration parameters of the main environment, there are additional constraints expressed as

$$x_{ij} = x_{rj}, i, r \in B \tag{25}$$

where B is the collection of the serial number of sub-models generated by the Model-Exchange interface, and $B \in \{1, 2, \ldots, M\}$.

IV. DECOUPLING OF THE OPTIMAL SIMULATION CONFIGURATION PROBLEM

The optimal simulation configuration problem expressed by Eq. (24) can be decoupled into a series of optimal simulation configuration problems of the sub-models, and the decoupling processes are as follows.

A. DECOUPLING ANALYSIS

Eq. (24) can be converted as

$$\begin{aligned} &\textit{minimize} \sum_{i=1}^{M} \left(\sum_{j=1}^{P} c_{\rm ij} \cdot x_{\rm ij} \right) \\ &\textit{subject to} \sum_{j=1}^{P} x_{\rm ij} = 1, i = 1, 2, \ldots, M \\ &\qquad x_{\rm ij} = 0, \quad \text{if } \mathrm{d_{ij}} < 1, \ j = 1, 2, \ldots, P, \\ &\qquad i = 1, 2, \ldots, M \\ &\qquad x_{\rm ij} \in \{0, 1\}, \quad \text{if } \mathrm{d_{ij}} \geq 1, j = 1, 2, \ldots, P, \\ &\qquad i = 1, 2, \ldots, M \end{aligned} \tag{26}$$

The constraints of each sub-model are linearly independent, then the optimal configuration problem can be decoupled.

Defining

$$y_i = \sum_{j=1}^{P} c_{ij} \cdot x_{ij} \tag{27}$$

Then the optimal configuration problem of the i-th sub-model is expressed as

$$\begin{aligned} &\textit{minimize } y_i \\ &\textit{subject to} \sum_{j=1}^{P} x_{\rm ij} = 1 \\ &\qquad x_{\rm ij} = 0, \quad \text{if } \mathrm{d_{ij}} < 1, j = 1, 2, \ldots, P \\ &\qquad x_{\rm ij} \in \{0, 1\}, \quad \text{if } \mathrm{d_{ij}} \geq 1, j = 1, 2, \ldots, P \end{aligned} \tag{28}$$

And accordingly, the original optimal configuration problem is converted as

$$\textit{minimize} \sum_{i=1}^{M} y_i \tag{29}$$

B. OPTIMAL CONFIGURATION PROBLEM OF THE I-TH SUBMODEL

Noting that the single-step consumption time $c_{\rm ij}$ of Eq. (18), then the optimization objective of the i-th sub-model is

$$y_i = (T_{\rm ODE})_i \cdot \sum_{j=1}^{P} \left(K_j \cdot x_{ij}\right) \tag{30}$$

Obviously, the simulation configuration is optimal when a minimum value of K_j is selected within the feasible solution which is determined by Eq. (23).

V. OPTIMAL SIMULATION CONFIGURATION METHOD

To realize optimal simulation configuration, the following procedures should be carried out, as shown in Fig. 2, and the details are as follows.

a) Determine simulation requirements and numerical algorithms.

b) Calculate the judge coefficient of each sub-model.

c) Analysis the feasibility.

d) Design the optimal configured parameters.

e) Guide the redesign.

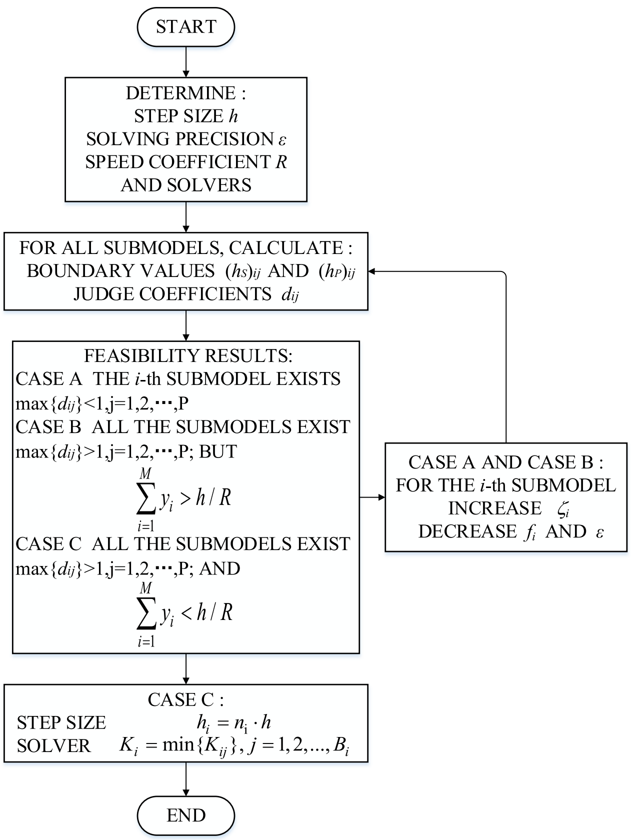

FIGURE 2. Diagram of optimal simulation configuration procedure.

A. DETERMINE SIMULATION REQUIREMENTS AND NUMERICAL ALGORITHMS

Firstly, the simulation requirements of the main environment should be determined according to the actual research objective and application scenario, including the simulation step size h, the solving precision ε, and the speed coefficient R.

Secondly, the available solvers and their simulation parameters should be determined according to different simulation platforms and actual applications, including the number of calculation stages K, the order N, and the feasible region boundary function $G(\zeta)$ of numerical stability.

B. CALCULATE THE JUDGE COEFFICIENT OF EACH SUBMODEL

Firstly, the following parameters of the i-th sub-model should be determined:

a) The single-stage calculation time $(T_{\mathrm{ODE}})_i$.

b) The core max frequency f_i and the corresponding damping ratio ζ_i during simulation.

Then, the following parameters should be calculated when using different solvers:

a) The feasible collection boundary values $(h_s)_{ij}$ and $(h_p)_{ij}$.

b) The parameter d_{ij}, which is used to determine whether the value of the variable x_{ij} is zero, and determine the collection D_i of the available solvers.

The above processes should be executed for all sub-models.

C. ANALYSIS THE FEASIBILITY

There are three analysis results:

a) Case A: Infeasible. Specifically, there is a sub-model, presented as the i-th sub-model, does not satisfy Eq. (15) even if it tests all solvers, and the collection D_i of the available solvers is empty. It has

$$\max\{d_{ij}\} < 1, \quad j = 1, 2, \ldots, P \tag{31}$$

It indicates that the i-th sub-model cannot satisfy the requirements of solving precision and numerical stability simultaneously within the workable solvers. And of course, there is no suitable simulation configuration to meet the simulation requirements and realize real-time performance.

b) Case B: Infeasible. Interpretatively, each sub-model can satisfy Eq. (15) after it tests all solvers, which indicates that there is at least one solver can be used for each sub-model, and the collection D_i of the available solvers is nonempty, but the real-time performance cannot be realized. There are

$$\begin{cases} \max\{d_{ij}\} > 1, \quad j = 1, 2, \ldots, P, \ for\ i = 1, 2, \ldots, M \\ \sum_{i=1}^{M} y_i > h/R \end{cases} \tag{32}$$

c) Case C: Feasible. Illustratively, there is at least one solver can be used for each sub-model, and the collection D_i of the available solvers is nonempty, and the real-time performance can be realized. There are

$$\begin{cases} \max\{d_{ij}\} > 1, \quad j = 1, 2, \ldots, P, \ for\ i = 1, 2, \ldots, M \\ \sum_{i=1}^{M} y_i < h/R \end{cases} \tag{33}$$

D. DESIGN THE OPTIMAL CONFIGURED PARAMETERS

For case C, the simulation parameters of each sub-model can be designed as:

a) The solver with a minimum number of calculation stages within the collection D_i of the available solvers is optimal. For $i = 1, 2, \ldots, M$, it has

$$K_i = \min\{K_{ij}\}, \quad j = 1, 2, \ldots, j \in D_i \tag{34}$$

b) The step size is designed as

$$h_i = n_{\mathrm{i}} \cdot h \tag{35}$$

where n_{i} can be any integer between 1 and d_{ij}, and $j \in D_i$.

E. GUIDE THE REDESIGN

For Case A, the judge coefficient is less than 1, so the optimization methods are increasing the judge coefficient. The methods are increasing $(h_s)_{ij}$ and $(h_p)_{ij}$, which can be realized by optimizing the core max frequency and the damping as well as adjusting the simulation requirements of solving precision of the sub-model which has no available solver, and ensuring that there is one or more solvers can be used. According to Eq. (3) and Eq. (5), the design methods include:

a) Reducing the abnormally high frequency f, and improving the abnormally small damping ratio ζ.

b) Reducing the solving precision ε appropriately.

Specifically, to meet the simulation requirement of numerical stability, the designed core max frequency and the damping should meet

$$f < \frac{G(\zeta)}{h} \tag{36}$$

For Case B, the optimization methods are adjusting simulation requirements or optimizing the core max frequency and the damping characteristic of the sub-model which has a small collection D of the available solvers, to increase the number of available solvers. The optimization methods are same as Case A.

VI. APPLICATION TO AN AERO-ENGINE CONTROL SYSTEM MODEL

The proposed method is used to configure the simulation parameters of an aeroengine control system model in this section, which is established based on the Co-Simulation environment. As mentioned in Section II, the Model-Exchange mode and the Hybrid Co-Simulation mode are special cases of the Co-Simulation mode, thus the model based on the Co-Simulation environment can be fully used for the verification works. The composition of the aeroengine control system is shown in Fig. 3. Complementarily, this linear model consists of six components, which can be used to design the dynamic control laws of the control system.

A. DETERMINE SIMULATION REQUIREMENTS AND NUMERICAL ALGORITHMS

1) DETERMINE SIMULATION REQUIREMENTS OF THE MAIN ENVIRONMENT

The step size h and the speed coefficient R of the main environment are 0.001 and 2 respectively, and the solving precision ε is 10^{-6}.

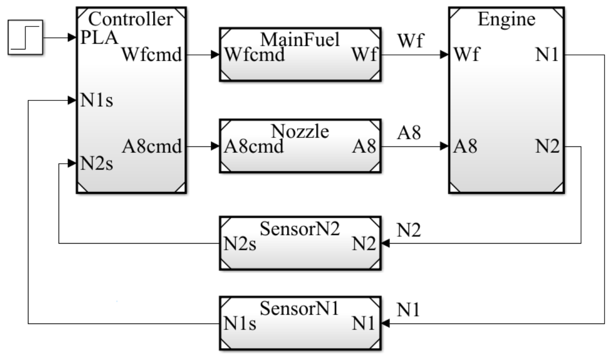

FIGURE 3. The compositions of the aeroengine control system.

2) DETERMINE THE SIMULATION PARAMETERS OF THE NUMERICAL ALGORITHMS

The simulation parameters of the adopted numerical algorithms are shown in Table 1 [12], corresponding to the algorithms of Runge-Kutta1 to Runge-Kutta4.

TABLE 1. Simulation parameters of the adopted solvers.

Algorithm type	N	K	$G(\zeta)$
RK1	1	1	ζ/π
RK2	2	2	$-0.55 \cdot \zeta^2 + 0.78 * \zeta + 0.08 * \zeta^{1/5}$
RK3	3	3	$1.207 \cdot \zeta^3 - 2.135 * \zeta^2 + 1.1 * \zeta + 0.24$
RK4	4	4	$0.02 \cdot \sin(2.5 \cdot \pi \cdot \zeta) + 0.44$

B. CALCULATE THE JUDGE COEFFICIENT OF EACH SUBMODEL

The core eigenvalues and the calculated simulation parameters of each sub-model are shown in Table 2.

TABLE 2. Simulation parameters of each sub-model.

Sub-model	Eigenvalues	Core f /Hz and ζ	T_{ODE}/s
Controller	(0)	-	$5.1*10^{-5}$
Main Fuel	(-5, -2000)	(318.3099, 1)	$4.7*10^{-5}$
Nozzle	(-10, -2500)	(397.8874, 1)	$4.7*10^{-5}$
Engine	(-2.8375+0.7602i, -2.8375-0.7602i)	(0.4675, 0.9659)	$4.8*10^{-5}$
SensorN1	(-50, -5000)	(795.7747, 1)	$4.5*10^{-5}$
SensorN2	(-50, -5000)	(795.7747, 1)	$4.5*10^{-5}$

1) CALCULATE THE FEASIBLE COLLECTION BOUNDARY VALUES OF THE STEP SIZE

Firstly, calculating the feasible collection boundary values of the step size $(h_p)_{\mathrm{ij}}$ and $(h_s)_{\mathrm{ij}}$ according to Eq. (14), as shown in Table 3.

TABLE 3. The feasible collection boundary values of the step size.

$((h_p)_{ij},\ (h_s)_{ij})$	1 RK1	2 RK2	3 RK3	4 RK4
1 Controller	(0.001, -)	(0.01, -)	(0.0316, -)	(0.0631, -)
2 Main Fuel	(0.001, 0.0010)	(0.01, 0.0010)	(0.0316, 0.0013)	(0.0631, 0.0014)
3 Nozzle	(0.001, 0.0008)	(0.01, 0.0008)	(0.0316, 0.0010)	(0.0631, 0.0012)
4 Engine	(0.001, 0.6577)	(0.01, 0.6839)	(0.0316, 0.8520)	(0.0631, 0.9825)
5 SensorN1	(0.001, 0.0004)	(0.01, 0.0004)	(0.0316, 0.0005)	(0.0631, 0.0006)
6 SensorN2	(0.001, 0.0004)	(0.01, 0.0004)	(0.0316, 0.0005)	(0.0631, 0.0006)

TABLE 4. Values of the judge coefficient.

$(\{(h_p)_{ij}/h, (h_s)_{ij}/h\}, d_{ij})$	1 RK1	2 RK2	3 RK3	4 RK4
1 Controller	({1, -}, 1)	({10, -}, 10)	({31.6, -}, 31.6)	({63.1, -}, 63.1)
2 Main Fuel	({1, 1}, 1)	({10, 1}, 1)	({31.6, 1.3}, 1.3)	({63.1, 1.4}, 1.4)
3 Nozzle	({1, 0.8}, 0.8)	({10, 0.8}, 0.8)	({31.6, 1}, 1)	({63.1, 1.2}, 1.2)
4 Engine	({1, 657.7}, 1)	({10, 683.9}, 10)	({31.6, 852}, 31.6)	({63.1, 982.5}, 63.1)
5 SensorN1	({1, 0.4}, 0.4)	({10, 0.4}, 0.4)	({31.6, 0.5}, 0.5)	({63.1, 0.6}, 0.6)
6 SensorN2	({1, 0.4}, 0.4)	({10, 0.4}, 0.4)	({31.6, 0.5}, 0.5)	({63.1, 0.6}, 0.6)

2) CALCULATE THE JUDGE COEFFICIENT

Then, calculating the judge coefficient d_{ij} according to Eq. (22) to determine the parameter values of x_{ij}, as shown in Table 4.

C. ANALYSIS THE FEASIBILITY

Firstly, the requirement of solving precision is reasonable, because all the sub-models can meet the solving precision when they use all the adopted solvers.

Secondly, for the sensor sub-models of N1 and N2, there are

$$\max\{d_{ij}\} = 0.6 < 1 \tag{37}$$

TABLE 5. Design range of the frequency and the damping ratio.

Solver	Constraint of f and ζ	Range of f when $\zeta = 1$
RK1	$(0, (\zeta/\pi)/h)$	(0, 318) Hz
RK2	$(0, (-0.55\cdot\zeta^2 + 0.78*\zeta + 0.08*\zeta^{\frac{1}{5}})/h)$	(0, 318) Hz
RK3	$(0, (1.207\cdot\zeta^3 - 2.135*\zeta^2 + 1.1*\zeta + 0.24)/h)$	(0, 412) Hz
RK4	$(0, (0.02\cdot\sin(2.5\cdot\pi\cdot\zeta) + 0.44)/h)$	(0, 460) Hz

TABLE 6. The feasible collection boundary values of the step size.

$(h_s)_{ij}$	1 RK1	2 RK2	3 RK3	4 RK4
5 SensorN1	0.0013	0.0013	0.0017	0.0019
6 SensorN2	0.0013	0.0013	0.0017	0.0019

The sensor sub-models of N1 and N2 cannot satisfy the requirement of numerical stability within the workable solvers under the set step size of the main environment. And of course, there is no suitable simulation configuration to meet the simulation requirements and realize real-time simulation. Correspondingly, the simulation configuration status is classified as Case A.

Setting the feasible minimum simulation step to 1e-3, which is also the simulation step size of the main environment, the simulation results of the sensor sub-models of N1 and N2 are shown in Fig. 4.

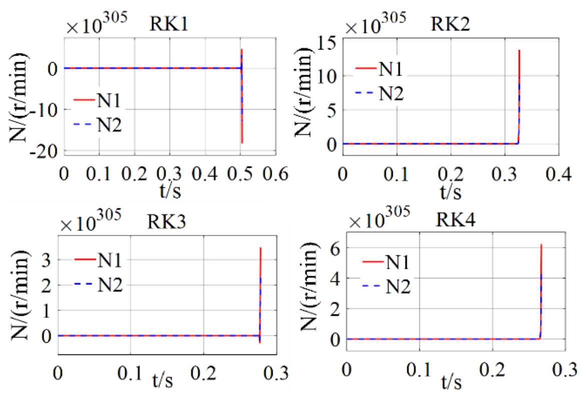

FIGURE 4. Simulation results of the sensor sub-models of N1 and N2.

Evidently, the simulation results are numerically unstable, and the feasibility analysis results are correct.

D. GUIDE THE REDESIGN

The researched frequency range of the aeroengine control system is generally within 100 Hz. However, the high frequency characteristics of the sensor sub-models are far beyond the researched frequency range, which are

TABLE 7. Design of the simulation parameters of each sub-model under different schemes.

	Scheme 1		Scheme 2		Scheme 3		Scheme 4	
	Algorithm	h	Algorithm	h	Algorithm	h	Algorithm	h
1 Controller	RK1	0.001	RK2	0.005	RK3	0.01	RK4	0.05
2 Main Fuel	RK1	0.001	RK2	0.001	RK3	0.001	RK4	0.001
3 Nozzle	RK3	0.001	RK3	0.001	RK4	0.001	RK4	0.001
4 Engine	RK1	0.001	RK2	0.005	RK3	0.01	RK4	0.05
5 SensorN1	RK1	0.001	RK2	0.001	RK3	0.001	RK4	0.001
6 SensorN2	RK1	0.001	RK2	0.001	RK3	0.001	RK4	0.001

unnecessary. Therefore, the method of optimizing the high frequency characteristics of the sensor sub-models is adopted [26], [27].

According to Eq. (36), the design range of the frequency and the damping ratio for eigenvalues are obtained, as shown in Table 5.

The calculation results indicate that, by limiting the high frequency characteristics within about 300Hz, the sub-models can satisfy the requirements of solving precision and numerical stability simultaneously within the workable solvers under the set step size of the main environment.

Of course, the limited high frequency characteristic is reasonable, because it is beyond the researched frequency range. Therefore, the eigenvalues related to high frequency of the sensor sub-models is designed as -1500, and the feasible collection boundary values $(h_s)_{ij}$ of the step size are shown in Table 6.

According to Table 6, the feasible simulation step size of the two sub-models is still 1e-3, and the simulation results of the sensor sub-models of N1 and N2 after redesign are shown in Fig. 5.

Evidently, the simulation results of the sensor sub-models of N1 and N2 after redesign are numerically stable and correct.

E. DESIGN THE OPTIMAL CONFIGURED PARAMETERS

The simulation parameters of each sub-model under different schemes are designed, as shown in Table 7. And from the theory of optimal simulation configuration in Section IV, the Scheme 1 is optimal.

F. SIMULATION RESULTS AND DISCUSSION

The theoretical calculation consumption time is

$$T_{\mathrm{CPU}} = \left(\sum_{i=1}^{M} (T_{\mathrm{ODE}})_i \cdot \sum_{j=1}^{P} (K_j \cdot x_{ij}) \right) \cdot \frac{T_{\mathrm{SET}}}{h} \tag{38}$$

Setting the simulation time T_{SET} to 10s, the actual CPU consumption and the theoretical calculation consumption are shown in Table 8.

TABLE 8. Results of the time consumption under different schemes.

No.	Theoretical time /s	Actual CPU time /s	$T_{\mathrm{ODE}} < 5$?
Scheme 1	3.77	3.88	Yes
Scheme 2	6.13	6.35	No
Scheme 3	8.96	9.05	No
Scheme 4	11.32	11.52	No

The simulation results show that:

(1) The actual CPU consumption is nearly identical to the theoretical calculation consumption, indicating the accuracy of the established mathematical calculation formulas.

(2) The Scheme 1 is optimal, and it can achieve real-time performance and satisfy the simulation requirements, which is consistent with the conclusion of the theoretical analysis. It indicates that the constructed mathematical problems are appropriate and accurate, and they can completely represent and solve the optimal simulation configuration problems.

(3) Using the proposed design methods, all the sub-models can meet their simulation requirements after redesigning the model, demonstrating the feasibility and effectiveness of the proposed design methods.

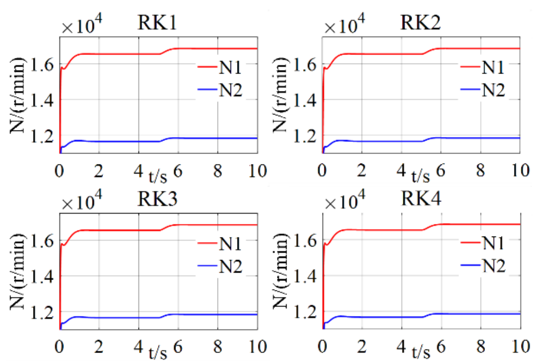

FIGURE 5. Simulation results of the sensor sub-models of N1 and N2 after redesign.

However, the simulation step size of each sub-model is a trade-off between the simulation requirements of precision, stability, and simulation speed, which are in opposition to one another. There is no specific method for making tradeoffs; instead, it relies on the research purpose and the application scenarios, thus the application of the proposed redesign method has limitations.

VII. CONCLUSION

To realize real-time simulation and optimal configuration, this paper constructs the mathematic description of the optimal simulation configuration problem and converts it to an assignment problem, and decouples it into a series of optimal simulation configuration problems of the sub-models. On this basis, an optimal configuration method for simulation parameters is proposed, which includes an analysis method and corresponding design methods. Simulations on an aeroengine control system model are implemented, and the conclusions are as follows.

(1) The analysis method can obtain the feasibility of the simulation configuration accurately, and the design methods can redesign the sub-models effectively to realize real-time simulation.

(2) The proposed method can solve the optimal simulation configuration problem by theoretical calculation. The method is more accurate and effective because it avoids the trouble and inaccuracy of selecting simulation parameters through trial-and-error method, and it can be widely used.

The method is based on the real-time theory, then the simulation step size is fixed and the solver keeps unchanged during a simulation. Thus, this method is not applicable to handle simulations adopting variable-solvers or iterative-solvers. In the future research, the theory of real-time simulation for the model applying iterative solvers will be researched to improve the practical application ability of the aeroengine control system model.

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WENSHUAI ZHAO received the B.S. degree in flight vehicle propulsion engineering from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 2019, where he is currently pursuing the Ph.D. degree. His research interests include aeroengine control system modeling, simulation, and design.

XI WANG was born in 1961. He received the Ph.D. (Engineering) degree in aeroengine control from Northwestern Polytechnic University, in 1998. He is currently a Professor with the School of Energy and Power Engineering, Beijing University of Aeronautics and Astronautics, and a doctoral supervisor in aero-engine control. His current research interests include aeroengine control, mathematical model and simulation of aeroengine power control systems, mathematical model and simulation of hydro mechanical regulation systems, fault diagnosis and health management of aeroengine control systems, and robust multivariable control of aeroengine.

SHUBO YANG was born in Ürümqi, China. He received the Ph.D. degree in engineering from the Beijing University of Aeronautics and Astronautics, in 2020. He is currently a Postdoctoral Researcher with the Beijing University of Aeronautics and Astronautics. His current research interests include control design, dynamic system modeling, and simulation.

YIFU LONG received the B.S. degree in flight vehicle propulsion engineering and the M.S. degree in aeroengine control from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 2014 and 2017, respectively, where he is currently pursuing the Ph.D. degree. His research interests include aero-engine control system modeling, simulation, and design.