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RESEARCH ARTICLE

Reliability Analysis for k-out-of-N: F Load Sharing Systems Operating in a Shock Environment

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ABSTRACT This paper studies a reliability modeling for a k-out-of-n: F load sharing system that operates in a shock environment. Such a system consists of a protective device and n components with load sharing. The base hazard rate of the loading sharing system is affected by random shocks and the protective device. Random shocks can be classified into two types: invalid shock and valid shock. An invalid shock has no influence on the system whereas a valid shock makes the base hazard rate larger. The system fails, if the number of failed components is at least k, the system suffers at least M random shocks or the protective device fails, whichever occurs first. A Markov process is used to evaluate system reliability in this paper. A distributed computer system is given to show application of the proposed model.

INDEX TERMS Reliability analysis, load sharing system, shock environment, Markov process.

I. INTRODUCTION

Components usually work independently, and one component failure cannot affect other components' performance for most engineering systems. However, some engineering systems work with load sharing, that is, one component failure may cause higher hazard rates of the remaining working components. Such a system is usually called a load sharing system and is widely seen in distributed computer systems, gear systems and power grids.

There are most existing researches on reliability analysis for load sharing systems. For example, Liu [1] studied the reliability evaluation of the k-out-of-n: G system and considered that components are non-independent identically distributed and have arbitrary. Ye et al. [2] constructed a load sharing system reliability model where managed component degradation was considered. Liu et al. [3] proposed a load sharing system with degrading components and designed a preventive maintenance policy for it. A load sharing k-out-of-n system with consideration of discrete external load was studied by Zhang et al. [4]. Reliability analysis for degrading load sharing systems with warm standby components was studied by Ruan and Lin [5].

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External environment is a main factor that should be considered for system reliability analysis. Most researchers studied shock environment to describe system operational environment. For example, Che et al. [6] considered both degradation and random shocks and established load sharing systems. Guo et al. [7] proposed a reliability model for the consecutive k-out-of-n: F system which consists of load sharing components. In addition, Zhao et al. [8] proposed a two-stage shock model with consideration of self-healing mechanism. Wang et al. [9] studied the mixed shock model for multi-state weighted k-out-of-n: F systems with degraded resistance.

The degradation of system and external shocks will lead to system failure. Protective device is one of effective ways to improve system reliability and some related works were done. For example, Zhao et al. [10], [11] studied triggering policy for multi-state systems with a protective device. Zhao et al. [12] considered shock environment and k-out-of-n structure, and proposed a k-out-of-n: F system supported by multi-state protective device. Wang et al. [13] established a reliability model for the multi-state k-out-of-n: F system with m subsystems supported by multiple protective devices. Wang et al. [14] considered two balance concepts and proposed two reliability models for balanced systems with the protective device.

Following existing works as mentioned above, this paper proposes a load sharing system. Such the system contains n components and a protective device. One component failure may lead to high loads of the remaining working components. The base hazard rate is affected by random shocks and the protective device's state. In detail, random shocks may lead to a higher base hazard rate while the protective device can defend against some damage from shocks. The amount of shock the protective device can withstand depends on its state. The system fails, if the number of failed components is at least k, the system suffers at least M random shocks or the protective device fails, whichever occurs first.

The proposed model in this paper is motivated by practical engineering systems. Two examples of distributed computers systems and power distribution systems are given to show applications of the proposed model and motivations of this paper.

A distributed computers system contains several components to process data. Such the system is usually attacked by hackers, that is, hackers may lead computers to fail more easily. The failure of one computer causes more loads on the remaining computers. Firewall is used to protect computers and has multiple states. The distributed computers system is regarded as failure if the number of working computers is less than a determined value, firewall fails or it suffers too many attacks.

A power distribution system consists of several power stations. All power stations work to transmit electricity, and one station failure may lead to a high load at the remaining stations. High temperature or extreme weather such as thunderstorms may cause current or voltage generation in the distribution system, which in turn may cause working components to fail. To ensure the stable operation of the distribution system, protection relays are installed in the distribution system to isolate the faulty electronic components. The power distribution system is regarded as failure if the number of working stations is less than a determined value, protection relay fails or it suffers too many attacks such as high temperature or extreme weather.

To analyze system reliability, a Markov process is applied. The Markov process is usually applied to describe system operation and state transition. It is widely used in the field of reliability. For example, Cui et al. [15], Zhao et al. [16], [17] and Wu et al. [18], [19], [20] used the Markov process to describe the operation of balanced systems. Yu et al. [21], Zhao et al. [22] and Wu et al. [23] used the Markov process to analyze reliability for systems with common bus performance sharing.

The contributions of this paper are summarized below.

- A protective device is first considered for the loading sharing system.
- The base hazard rate is variable and depends on random shocks and the state of the protective device.
- A Markov process is used to analyze system reliability.
- A computer distributed system is illustrated to show applications of the proposed model.

This paper is organized as follows. Section II establishes a reliability model for the load sharing system with random shocks and a protective device. Section III proposes a method based on the Markov process to analyze system analysis. Section IV establishes an optimization model for the triggering policy of the protective device. Section V shows a numerical example with the distributed computer system. In addition, conclusions and further work are given in Section VI.

II. MODEL DESCRIPTIONS

A load sharing system is proposed in this section. Such a system consists of n components and a protective device. The system structure is shown in Fig. 1. Components belonging to the system are independent and identically distributed. The protective device has L states and the corresponding transfer rate is β. Such the system operates with load sharing, that is, if a component fails, the failure rates of the remaining components will increase. The hazard rate of the components is defined as a function of the number of working components, which is

$$\lambda = \left(\frac{n}{n-n_f}\right)^{\alpha} \lambda_0, \tag{1}$$

where λ_0 represents the base hazard rate when all components work, and n_f is the number of failed components in the system. In addition, α is the load factor of the system.

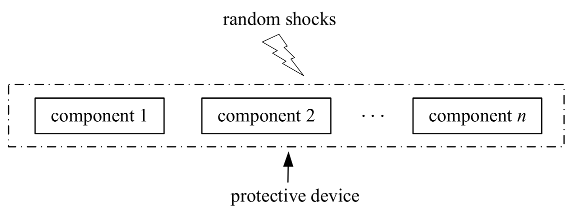


FIGURE 1. System structure of the load sharing system with a protective device.

The load sharing system works in a shock environment. The arrival rate of random shocks is η and shocks can be classified into two types: invalid shock with probability p_0, and valid shock with probability p_1. We have $p_0 + p_1 = 1$. The base hazard rate is affected by random shocks and the protective device. One valid shock can add $\Delta\lambda$ to the base hazard rate, that is, the base hazard rate is $\lambda_m = \lambda_0 + m\Delta\lambda$ with m valid shocks if the protective device is not considered. The protective device starts up if the system suffers m_s valid shocks. The protective device has L states, and the l-th state can counteract λ_l if the protective device starts up. Hence, the base hazard rate is $\lambda_{m,l} = \lambda_0 + m\Delta\lambda - \lambda_l I\,(m \geq m_s)$ with random shocks and the protective device. Equation (1) can be rewritten as

$$\lambda\left(n_f, m, l\right) = \left(\frac{n}{n-n_f}\right)^{\alpha} \lambda_{m,l} = \left(\frac{n}{n-n_f}\right)^{\alpha} \times \left(\lambda_0 + m\Delta\lambda - \lambda_l I\,(m \geq m_s)\right). \tag{2}$$

The system fails, if the number of failed components is at least k, the system suffers at least M random shocks or the protective device fails, whichever occurs first.

Example 1: An example is used to show the system operation. The corresponding parameters are set as follows. $n = 3$, $k = 1$, $m_s = 2$, $M = 4$ and $L = 4$. The base hazard rate corresponding to random shocks it suffered and the state of the protective device is shown in Table 1.

TABLE 1. Example for shocks and the protective device's influence on components.

m \ l	1	2	3 (L-1)
0	$\lambda_{0,1}=\lambda_0$	$\lambda_{0,2}=\lambda_0$	$\lambda_{0,3}=\lambda_0$
1	$\lambda_{1,1}=\lambda_0+\Delta\lambda$	$\lambda_{1,2}=\lambda_0+\Delta\lambda$	$\lambda_{1,3}=\lambda_0+\Delta\lambda$
2	$\lambda_{2,1}=\lambda_0+2\Delta\lambda-\lambda_1$	$\lambda_{2,2}=\lambda_0+2\Delta\lambda-\lambda_2$	$\lambda_{2,3}=\lambda_0+2\Delta\lambda-\lambda_3$
3	$\lambda_{3,1}=\lambda_0+3\Delta\lambda-\lambda_1$	$\lambda_{3,2}=\lambda_0+3\Delta\lambda-\lambda_2$	$\lambda_{3,3}=\lambda_0+3\Delta\lambda-\lambda_3$

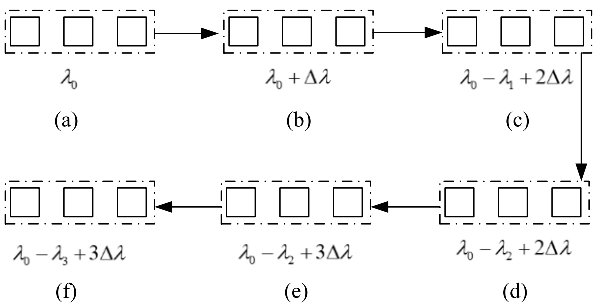


FIGURE 2. A possible operation solution of the load sharing system.

Fig. 2 shows one of the possible operation solutions of the proposed load sharing system. At the initial time, all components work and the protective device is forced down. Thus, the base hazard rate is λ_0, as shown in Fig.2 (a). Then, the system suffers a valid shock and the hazard rate changes to be $\lambda_0 + \Delta\lambda$ as shown in Fig. (b). Similarly, the base hazard rate is affected by valid shock and the state of protective device when the protective device starts up. Fig.2 (c)-(f) show different cases. Note that the system still works in Fig.2 (f). If it suffers a valid shock, the protective device changes to the fourth state or one component fails, and the system will fail.

III. SYSTEM ANALYSIS

A Markov process model is constructed to analyze system reliability in this section. Two random variables N^s, N^f and S are defined as follows: N^s and N^f represent the number of valid shocks and the number of failed components, respectively. S denotes the state of the protective device. The state space of the Markov process is

$$\Omega = \mathbf{W} \cup \mathbf{F} = \left\{n^s < M, n^f < k \ and \ s \neq L\right\} \cup \left\{n^s \geq M, n^f \geq k \ or \ s = L\right\}.$$

The transition rate rule among states of the Markov process is given as follows.

(1) If $n^s < M-1, n^f \leq k-1$ and $s < L-1$, $(n^s, n^f, s) \to (n^s+1, n^f, s)$ with transition rate $p_1\eta$.

(2) If $n^s \leq M-1$, $n^f < k-1$ and $s < L-1$, $(n^s, n^f, s) \to (n^s, n^f+1, s)$ with transition rate $\left(\frac{n}{n-n_f}\right)^\alpha (\lambda_0 + n^s\Delta\lambda - \lambda_s I(n^s \geq m_s))$.

(3) If $m_s \leq n^s \leq M-1$, $n^f \leq k-1$ and $S < L-1$, $(n^s, n^f, s) \to (n^s, n^f, s+1)$ with transition rate β.

(4) If $n^s = M-1, n^f \leq k-1$ and $s < L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $p_1\eta$.

(5) If $n^s < M-1$, $n^f = k-1$ and $s < L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $\left(\frac{n}{n-k+1}\right)^\alpha (\lambda_0 + n^s\Delta\lambda - \lambda_s I(n^s \geq m_s))$.

(6) If $n^s < M-1, n^f < k-1$ and $s = L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate β.

(7) If $n^s = M-1$, $n^f = k-1$ and $s < L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $p_1\eta + \left(\frac{n}{n-k+1}\right)^\alpha (\lambda_0 + (M-1)\Delta\lambda - \lambda_s I(n^s \geq m_s))$.

(8) If $n^s = M-1, n^f < k-1$ and $s = L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $p_1\eta + \beta$.

(9) If $n^s < M-1, n^f = k-1$ and $s = L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $\left(\frac{n}{n-k+1}\right)^\alpha (\lambda_0 + n^s\Delta\lambda - \lambda_{L-1}) + \beta$.

(10) If $n^s = M-1$, $n^f = k-1$ and $s = L-1$, $(n^s, n^f, s) \to \mathbf{F}$ with transition rate $p_1\eta + \left(\frac{n}{n-k+1}\right)^\alpha (\lambda_0 + (M-1)\Delta\lambda - \lambda_{L-1}) + \beta$.

(11) All other transition rates are zero.

Once the transition rate rules among states are obtained, the transition matrix is

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{WW} & \mathbf{Q}_{WF} \\ \mathbf{Q}_{FW} & \mathbf{Q}_{FF} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{WW} & \mathbf{Q}_{WF} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{3}$$

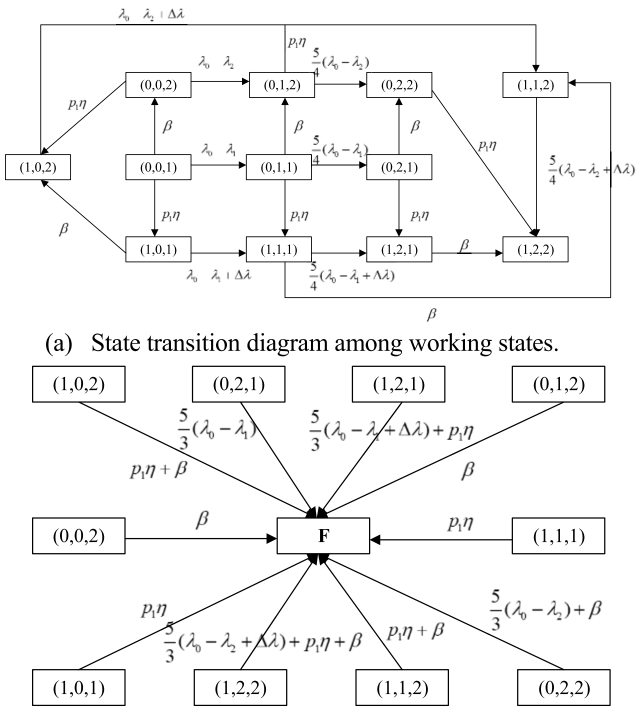
where the matrix $\mathbf{Q}_{WW}$ is a square matrix and represents transition rates among working states whereas $\mathbf{Q}_{FF}$ denotes transition rates among failed states. $\mathbf{Q}_{WF}$ and $\mathbf{Q}_{FW}$ consist of transition rates from working states to failed states and from failed states and working states, respectively.

Example 2: An example is given to show the construction of the Markov process. The load sharing system consists of 5 ($n = 5$) components and the protective device has 3 ($L = 3$) states. The base hazard rate of components is λ_0 and the load factor is set as 1 ($\alpha = 1$). Additionally, the hazard rate of the protective device is β. Such the system works in a shock environment and random shocks arrive following a homogeneous Poisson process with rate η. External shocks can be classified into two types: invalid shock with probability p_0, and valid shock with probability p_1. The protective device is triggered in the initial time, that is $m_s = 0$. The system fails if the number of failed components is no less than 3 ($k = 3$), the system suffers 2 ($M = 2$) valid shocks, or the protective device is in the third state, whichever occurs first.

The state space of the constructed Markov process for the load sharing system is

$$\begin{aligned}\Omega &= \mathbf{W} \cup \mathbf{F} \\ &= \{(0,0,1),(0,0,2),(1,0,1),(0,1,1),(1,0,2), \\ &\quad (0,1,2),(1,1,1),(0,2,1),(1,1,2),(0,2,2), \\ &\quad (1,2,1),(1,2,2)\} \cup \mathbf{F}\end{aligned}$$

Based on transition rules, the state transition diagrams among states are shown in Fig. 3. Fig. 3 (a) and (b) give transition diagrams from working states to working states and the failed state respectively.



(a) State transition diagram among working states.

(b) State transition diagram from working states to failed states.

FIGURE 3. State transition diagrams of the Markov process in Example 2.

In Fig 3 (a), the system is in (0,0,1) at the initial time. Then, if a valid shock arrives, the system changes to state (1,0,1) from (0,0,1). Because shock arrival follows a homogeneous Poisson process with rate η and the probability of valid shock is p_1, the transition rate is $p_1\eta$. The state (0,0,1) transfers to state (0,1,1) due to failure of one component, and the corresponding transition rate is $\lambda_0 - \lambda_1$. If the protective device changes to the second state from the first state, the state (0,0,1) transfers to state (0,0,2) with transition rate β. All transition rates among working states can be obtained according to transition rules. Fig. 3 (b) gives all transition rates from working states to the failed state. For example, state (0,0,2) transfers to the failure state with transition rate β due to failure of the protective device. State (1,0,1) transfers to failure state with transition rate $p_1\eta$, because one valid shock causes system failure. Once one failed condition is satisfied, state (1,2,2) changes to the failure state. Thus, the corresponding transition rate is $\frac{5}{3}(\lambda_0 - \lambda_2 + \Delta\lambda) + p_1\eta + \beta$.

Based on the transition rules above, the transition rate matrix is, as shown in the equation at the bottom of the next page, where $a_{11}, a_{22}, \ldots, a_{1212}$ represent the inverse of the sum of the respective rows. For example, $a_{11} = -\beta - p_1\eta - \lambda_0 + \lambda_1$.

Once the transition matrix among all components is obtained, the reliability can be derived as

$$R(t) = \sigma e^{Q_{ww}t} I, \tag{4}$$

where $\sigma = [1, 0, \ldots, 0]$ represents the initial state and $I = [1, 1, \ldots, 1]^T$.Note that Equation (4) can be derived by basic theory of the Markov process.

IV. OPTIMAL TRIGGERING POLICY OF THE PROTECTIVE DEVICE

As mentioned above, the protective device is triggered when the system suffers m_s valid shocks. In this section, an optimization model is constructed to obtain the optimal triggering threshold. Consider that the system should perform a mission at time $[0, T]$, thus it should keep working until $t = T$. Taking $R(T)$ as the objective function, the optimization model can be established as

$$\begin{aligned}&\max R(T) \\ &s.t. 0 \le m_s < M\end{aligned} \tag{5}$$

To solve the optimization model as Equation (5), genetic algorithm is used when the calculation is large while enumeration method is applied when the calculation is small. An example of solution when the calculation is small is given below.

(1) Give all possible values which should satisfy $0 \le m_s < M$.
(2) Calculate $R(T)$.
(3) Select maximum $R(T)$.

V. NUMERICAL EXAMPLE

A. BACKGROUND

Load sharing systems are widely seen in engineering systems, such as distributed computer systems, gear systems and power grids. This section takes the distributed computer system as an example. The distributed computer system consists of 5 ($n = 5$) servers which work together to complete the workload imposed on the system. The base hazard rate of components belonging to the system is 1 ($\lambda_0 = 1$). The failure of one server causes more workloads on the remaining working severs and the load factor is 1 ($\alpha = 1$). A firewall is to protect severs and has 3 ($L = 3$) states. Its hazard rate is 1 ($\beta = 1$). Attacks of hawkers can be regarded as shocks and classified into two types: invalid shock with probability 0.9 ($p_0 = 0.9$) and valid shock with probability

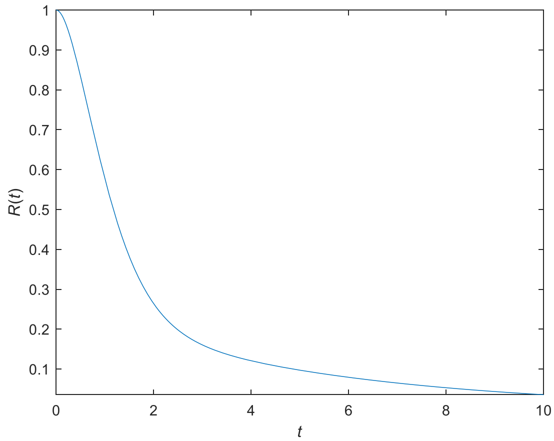


FIGURE 4. Reliability of the proposed load sharing system.

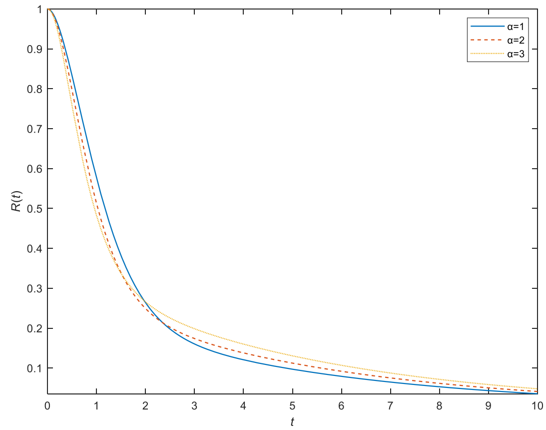


FIGURE 5. Sensitivity analysis of the load factor.

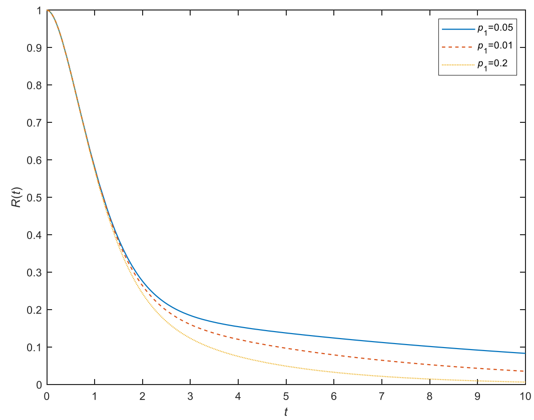


FIGURE 6. Sensitivity analysis of valid shock probability.

0.1 ($p_1 = 0.1$). If the attack is small and has almost no influence on severs, it can be regarded as an invalid shock. A valid shock adds 0.1 ($\Delta\lambda = 0.1$) to the base hazard rate. The firewall is triggered in the initial time ($m_s = 0$). If the firewall is in state 1 and state 2, it can counteract 0.5 and 0.3($\lambda_1 = 0.5$, $\lambda_2 = 0.3$) respectively. The system fails, if the number of failed components is at least 3 ($k = 3$), the system suffers at least ($M = 5$) random shocks or the protective device fails, whichever occurs first.

B. SYSTEM ANALYSIS

Fig. 4 shows the system reliability for the load sharing system with consideration of random shocks and a protective device. In the initial time $t = 0$, the system works perfectly. As time goes by, the system decreases and tends to 0. The reason is that, the performance of the components and the protective device worsen with time, and the system fails at the end.

C. SENSITIVITY ANALYSIS

Some sensitivity analyses are given to find the main factors which affect system reliability. Fig. 5 gives sensitivity analysis of the load factor and three cases in which $\alpha = 1$, $\alpha = 2$ and $\alpha = 3$ are considered. From Fig. 5, it can be seen that the reliability is larger when α is smaller in the initial stage. However, in the later stage, the reliability is larger when α is larger.

Fig. 6 shows the sensitivity analysis of the probability of valid shock. In the initial stage, there is almost no difference in reliability between different probabilities of valid shock. The reason is that the system works and shocks may not arrive in the initial stage, and the probability of a valid shock does not affect the system. As time goes by, the reliability is larger

$$\mathbf{Q} = \left[\begin{array}{cccccccccccc:c}
a_{11} & \beta & p_1\eta & \lambda_0-\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{22} & 0 & 0 & p_1\eta & \lambda_0-\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & \beta \\
0 & 0 & a_{33} & 0 & \beta & 0 & \lambda_0-\lambda_1+\Delta\lambda & 0 & 0 & 0 & 0 & 0 & p_1\eta \\
0 & 0 & 0 & a_{44} & 0 & \beta & p_1\eta & \frac{5}{4}(\lambda_0-\lambda_1) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 & \lambda_0-\lambda_2+\Delta\lambda & 0 & 0 & 0 & p_1\eta+\beta \\
0 & 0 & 0 & 0 & 0 & a_{66} & 0 & 0 & p_1\eta & \frac{5}{4}(\lambda_0-\lambda_2) & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & 0 & 0 & a_{77} & 0 & \beta & 0 & \frac{5}{4}(\lambda_0-\lambda_1+\Delta\lambda) & 0 & p_1\eta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} & 0 & \beta & p_1\eta & 0 & \frac{5}{3}(\lambda_0-\lambda_1) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{99} & 0 & 0 & \frac{5}{4}(\lambda_0-\lambda_2+\Delta\lambda) & p_1\eta+\beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1010} & 0 & p_1\eta & \frac{5}{3}(\lambda_0-\lambda_2)+\beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1111} & \beta & \begin{array}{l}\frac{5}{3}(\lambda_0-\lambda_1+\Delta\lambda)\\+p_1\eta\end{array} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1212} & \begin{array}{l}\frac{5}{3}(\lambda_0-\lambda_2+\Delta\lambda)\\+p_1\eta+\beta\end{array} \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

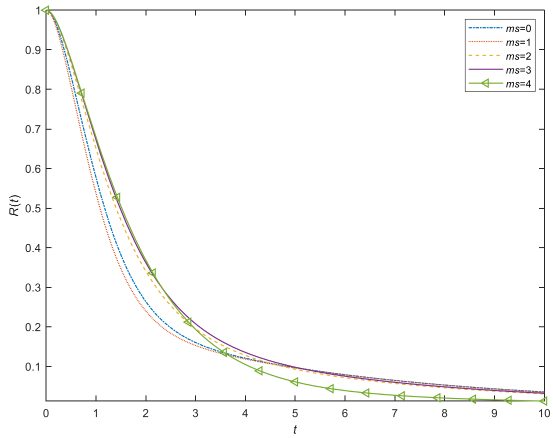


FIGURE 7. Triggering policy analysis of the protective device.

when the probability is less, because larger probability of valid shock causes large hazard rates of working components.

D. TRIGGERING POLICY ANALYSIS

Fig. 7 shows the system reliability in the case that m_s takes values from 0 to 4. In the initial stage, the reliability of the system will be greater if the protection device is triggered later, and in the later stage, the reliability of the system will be greater if the protection device is triggered earlier. The reason is that the system is most likely to avoid failure due to protection failure when the protection device is activated late in the initial phase. However, in the later stage, the system is in poor condition due to the late activation of the protection device, which accelerates the failure.

When the system performs a mission at a prescribed time, the time when to trigger the protective device is particularly important. Let T take a value of 2, and reliability in cases of different m_s can be derived as

$$R(2) = \begin{cases} 0.2638, m_s = 0 \\ 0.2400, m_s = 1 \\ 0.3406, m_s = 2 \\ 0.3622, m_s = 3 \\ 0.3683, m_s = 4 \end{cases}$$

Therefore, the protective device should be triggered when it suffers 4 valid shocks in this case. Let T take a value of 4, and reliability in cases of different m_s can be derived as

$$R(4) = \begin{cases} 0.1206, m_s = 0 \\ 0.1185, m_s = 1 \\ 0.1279, m_s = 2 \\ 0.1354, m_s = 3 \\ 0.1048, m_s = 4 \end{cases}$$

Therefore, the protective device should be triggered when it suffers 3 valid shocks in this case.

As mentioned above, when to trigger the protective device depends on the mission time. It should make decisions with consideration of system performance and mission time.

In conclusion, load factor, probability of shock type and triggering policy have influence on system reliability. Sensitivity analyses are provided to obtain main factors affecting system reliability. The results obtained by this paper can provide supports for engineers.

VI. CONCLUSION

This paper proposed a load sharing system with consideration of random shocks and a protective device. Such a system contains n components and a protective device. Components belonging to the system work with load sharing and the system works in a shock environment. Shocks can be classified into two types: invalid shock and valid shock. An invalid shock has no influence on the system whereas a valid shock can increase the failure rate of the components. The protective device has multiple states and different states can decrease different rates. The Markov process is used to analyze system reliability. Reliability analysis and sensitivity analysis of the distributed computers system are given to show applications of the proposed model.

Future investigations could be studied by extending the model proposed in this paper. For example, other system structures such as consecutive-k-out-of-n structures based on actual engineering systems, may be studied. A maintenance policy will be considered for the load sharing system in further work. In addition, the same components are considered in this paper, and the different components in the system will be investigated in the future.

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