IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 29 December 2022, accepted 5 February 2023, date of publication 14 February 2023, date of current version 17 February 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3245026

RESEARCH ARTICLE

New Stabilization Conditions for Fuzzy-Based Sampled-Data Control Systems Using a Fuzzy Lyapunov Functional

DANTE J. S. ONCOY[1], RODRIGO CARDIM[1], MARCELO C. M. TEIXEIRA[1], (Member, IEEE), FLÁVIO A. FARIA[2], EDVALDO ASSUNÇÃO[1], AND ADALBERTO Z. N. LAZARINI[1,3]

[1]Department of Electrical Engineering, São Paulo State University (UNESP), Ilha Solteira 15385-000, Brazil
[2]Department of Mathematics, School of Engineering, São Paulo State University (UNESP), Ilha Solteira 15385-000, Brazil
[3]Computer Engineering Course Coordination (COENC), Federal University of Technology-Paraná (UTFPR), Apucarana-PR 86812-460, Brazil

Corresponding author: Dante J. S. Oncoy (dante.solis-oncoy@unesp.br)

This work was supported in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior–Brazil (CAPES) under Grant 001; and in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico–Brazil (CNPq) under Grant 312065/2020-5, Grant 303637/2021-8, and Grant 309872/2018-9.

ABSTRACT This paper focuses on the stabilization problem of Takagi-Sugeno fuzzy systems via a sampled-data controller using a fuzzy dependent functional. By employing a property of convex combination, a new approach to deal with the time derivatives of the fuzzy membership functions (FMFs) in the stabilization conditions is proposed, and less conservative conditions are derived in the form of linear matrix inequalities (LMIs). Moreover, the proposed approach introduces a switching function which opens up possibilities to use a switched controller and take advantage of its benefits well known in the literature. Therefore, two sampled-data control strategies are proposed, where the first one is a fuzzy controller and the second is a robust switched controller, that does not require the expressions of the FMFs to implement the control law, which guarantees the robustness of the controlled system in cases where the FMFs depend on uncertain parameters. Finally, the effectiveness of the proposed strategies is verified by two examples.

INDEX TERMS Takagi-sugeno (T-S) fuzzy systems, fuzzy dependent Lyapunov–Krasovskii functional, switched sampled-data control, linear matrix inequalities (LMIs).

I. INTRODUCTION

Since their introduction in [1], Takagi-Sugeno (T-S) fuzzy systems have been extensively studied over the past few decades, owing to these systems can exactly represent in a local sector, a nonlinear dynamics of complex systems by using a quantity of IF-THEN rules with fuzzy sets, resulting in a convex combination between linear systems and FMFs. In the extensive literature related to the control of T-S fuzzy systems, several strategies have been applied, including fuzzy control [2], [3], [4], [5], switched control [6], [7], [8], [9], [10], sliding mode control [11], etc. Moreover, in these methods, the stabilization problem has been addressed in different contexts, such as systems without full-state measurements [3], presence of uncertainties and disturbances [12], sampled-data control [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], etc.

On the other hand, with the increasing development of digital technologies and communication networks, digital controllers play an important role in the design of new control strategies. The sampled-data control is a type of digital control which feedback the state variables obtained from the system at its sampling instants to the controller during a certain sampling interval, generating a control signal by a zero-order-hold (ZOH) function. In fact, sampled-data systems are a type of hybrid system, where the plant is a continuous-time system while the controller is discrete-time system. The main goal of the sampled-data controller design criterion is to obtain the maximum permissible sampling interval to keep the

The associate editor coordinating the review of this manuscript and approving it for publication was Alessandra Bertoldo.

sampled-data system stable. This fact follows due to at higher sampled intervals, the efficiency in the system is increased, as for example, bandwidth resources are economised and relaxed the communication capacity between the controller and the system.

Three main approaches have been used to obtain the design criteria for sampled-data controllers: a discrete-time, a input delay, and an impulsive system approach, among which input delay approach [17], is usually employed due to it can be used in systems with nonuniform sampling intervals. Based on the input-delay approach, many strategies have been proposed in the literature in order to reduce the conservatism of the design conditions, as in [16] where was proposed a time-dependent Lyapunov functional, later based on Wirtinger's inequality a discontinuous term in the Lyapunov functional was introduced in [18] considerably improving the results achieved until then. Another strategy is the use of looped-functional-based method [19], [20], which has the benefit of not needing the Lyapunov functional to be positive within the sampling intervals and can be improved by taking into account entire information of the intervals $x(t_k)$ to $x(t)$ and $x(t)$ to $x(t_{k+1})$, [21], [22], [23], [24], [25]. To further obtain improved stability conditions of sampled-data control systems, in [26] was introduced a free-matrix-based time-dependent discontinuous Lyapunov functional which is based on free-matrix-approach in time delay systems [27], but modifies the term $\int_{\alpha}^{\beta} x(s)\,ds$ instead of $\frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} x(s)\,ds$, applying to to sampled data systems by the convex combination technique. Among other recent approaches to address the design of sampled-data systems include, by transforming them into discrete-time switched polytopic system [28], by use the triple integral terms in the Lyapunov functional [29], or by include a delay term in the signal successfully transmitted from the sampler to the controller and to the ZOH, obtaining a memory sampled-data controller [30].

From the above techniques, the Lyapunov functional was derived from constant symmetric matrices, which can be improved using fuzzy dependent matrices instead of constant symmetric matrices. However, the main difficulty of using a fuzzy dependent Lyapunov functional is to deal with the combination of the time derivative of FMFs with the symmetric matrices, which can not be introduced directly in the conditions given in the LMIs. Focused on that issue, in [39], was proposed a new method, by using the information of the time derivative of FMFs at each instant of time, a switching idea was applied to ensure the time derivative of the Lyapunov functional is negative. Therefore, based on [39], a fuzzy dependent Lyapunov functional was used in other recent works [21], [23], [31], [32], [33], [34], obtaining less conservative conditions. However, three principal disadvantages of this method can be mentioned; it is necessary to assume that the derivative of FMFs can be measured, which in some systems can be complicated; the number of possible switching cases increase exponentially with respect to the rules obtained, which can be large for systems with many rules; and the method requires that all the terms of the combination of the time derivatives of FMFs with symmetric matrices be negative, which leads to conservatism in the LMIs.

On the other hand, switched control based on T-S fuzzy model is another research field, which involves a state dependent switching function leading the switching of controllers. As for example, in [6] was introduce a type of switched control which focused on T-S fuzzy systems with parametric uncertainties in FMFs, which achieves the stabilisation of the system without the need to measure the FMFs in the controller. Furthermore, in [8] and [10], was included the minimization of the $\mathcal{H}_\infty$ norm in the stability conditions, and in [7] was addressed the problem of chattering and sliding mode in the switching of the controllers.

Motivated by the above discussions, in this paper, we consider the stabilization problem with sampled-data control for fuzzy systems, using a fuzzy dependent Lyapunov-Krasovskii functional and a switching function proposed in [6]. Briefly, the main contributions of this work are summarized as follows:

- In order to use a fuzzy Lyapunov functional in the controller design conditions, a new method to deal with time derivatives of FMFs is proposed.
- The proposed method does not need to obtain the derivative of FMFs in each instant of time and only requires to know its maximum value.
- Based on a switching function, a switched sampled-data controller is proposed, which does not require to obtain the membership functions, being suitable for systems with parametric uncertainties.

Finally, the mentioned contributions are verified in two simulation examples.

Notations: The notation used throughout is standard. $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes the sets of real matrices with $n \times m$ dimension. $\mathbb{K}_r$ represents the set $\{1, 2, \ldots, r\}$, $\forall r \in \mathbb{N}$. For a matrix $X \in \mathbb{R}^{m\times n}$, X^T means its transpose. The identity and zero matrices are represented by I and 0 with appropriate dimensions, respectively. For any symmetric matrix, $X > 0 (X \geq 0)$ denotes a positive (semi) definite matrix, $(*)$ represents the elements below the main diagonal, and Sym$\{X\}$ is defined as $X + X^T$. diag{B7 B7 B7} denotes block diagonal matrix. Finally, $\Lambda_r \in \mathbb{R}^r$ is considered as a general convex set with dimension r, i.e. $\Lambda_r = \{\alpha \in \mathbb{R}^r / \alpha_i \geq 0, i \in \mathbb{K}_r, \sum_{i=1}^{r} \alpha_i = 1\}$.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following Takagi-Sugeno fuzzy model:

Rule i: **IF** $z_1(t)$ is M_1^i, $z_2(t)$ is M_2^i, ..., and $z_p(t)$ is M_p^i, **THEN**

$$\dot{x}(t) = A_i x(t) + B_i u(t), \tag{1}$$

where $M_1^i, M_2^i, \ldots, M_p^i$ are the fuzzy sets of the rule i, $i \in \mathbb{K}_r$; $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input vector; $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are known constant matrices; $z_1(t), z_2(t), \ldots, z_p(t)$ are called premise variables, that are smooth nonlinear functions available in real time, whose values are used to determine which Rule i is active at time t. By construction, $z_p(t)$ can depend on state variables, external disturbances, and/or time [2]. Hence, applying the center average defuzzifier, product interferences and singleton fuzzifier, the form of T-S fuzzy model is given below, more details can be found in [1].

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z(t))[A_i x(t) + B_i u(t)], \tag{2}$$

where $\alpha_i(z(t))$ are the weights of each local model i defined as

$$\alpha_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \quad w_i(z(t)) = \prod_{j=1}^{p} M_j^i(z(t)), \tag{3}$$

and satisfies

$$\sum_{i=1}^{r} \alpha_i(z(t)) = 1 \;\; \alpha_i(z(t)) \geq 0, \quad \forall t \geq 0. \tag{4}$$

Similar to the fuzzy model (1), the fuzzy sampled-data controller is designed as follows:

Rule j: **IF** $z_1(t)$ is M_1^i, $z_2(t)$ is M_2^i , . . . , and $z_p(t)$ is M_p^i, **THEN**

$$u(t) = K_j x(t_k), \quad t_k \leq t < t_{k+1}, j \in \mathbb{K}_r, \tag{5}$$

where $K_j \in \mathbb{R}^{m\times n}$ is the control gain matrix to be determined and the state vector $x(t)$ is assumed to be generated by a ZOH function at the sampling instant t_k, satisfying:

$$0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\to\infty} t_k = +\infty. \tag{6}$$

In order to work with the time delay approach [17], we consider the function $\tau(t) = t - t_k$, such that from (5) satisfies:

$$0 < \tau(t) \leq \tau_k = t_{k+1} - t_k \leq \tau, \; k = 0, 1, 2, \ldots, \infty, \tag{7}$$

where τ denotes the upper bound of the time interval length between any two sampling instants. Thus, the fuzzy sampled data controller is inferred by

$$u(t) = \sum_{j=1}^{r} \alpha_j(z(t_k)) K_j x(t - \tau(t)), \quad t \in [t_k, t_{k+1}), \tag{8}$$

and the closed-loop system T-S fuzzy sampled-data system is obtained

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z(t))\alpha_j(z(t_k))[A_i x(t) + B_i K_j x(t - \tau(t))], \quad t \in [t_k, t_{k+1}). \tag{9}$$

A. SWITCHED SAMPLED-DATA CONTROLLER

In the sampled-data control, the states $x(t)$ are feedback only at sampling instants t_k, hence, we have only the information of membership functions $\alpha_j(z(t_k))$ for the controller design.

A important disadvantage occurs when the premise variables $z(t_k)$ include complex nonlinear terms or parametric uncertainties that are difficult to know. Since $\alpha_j(z(t_k))$ depends on the $z(t_k)$, it leads to complexity in obtaining the membership functions and then increases the difficulty in the implementation of the controller. Focused on that problem, in [6] was proposed a switched controller that is designed for T-S fuzzy systems, and does not require the computation of membership functions, by using the switched controller defined as follows:

$$u(t) = K_\sigma x(t), \tag{10}$$

$$\sigma(t) = \arg\min_{j\in\mathbb{K}_r}\{x^T(t)\bar{Q}_j x(t)\}, \tag{11}$$

where $\bar{Q}_j \in \mathbb{R}^{n\times n}$ is a symmetric matrix and is designed to obtain the index $\sigma(t) \in \mathbb{K}_r$ and $\arg\min_{j\in\mathbb{K}_r}\{x^T(t)Q_j x(t)\}$ returns the index j at which the function $x^T(t)Q_j x(t)$ assumes its minimum value at instant t. The control gain $K_\sigma \in \mathbb{R}^{m\times n}$ belongs to a set of linear controllers $\{K_1, \ldots, K_r\}$, which are switching leaded by the function $\sigma(t)$, and are obtained by solving optimisation problems formulated by LMIs. Substituting (10) in (2), gives the following closed-loop system:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z(t))[A_i x(t) + B_i K_\sigma x(t)], \quad t \geq 0. \tag{12}$$

Therefore, in order to obtain the LMIs that guarantee the stability of the system using the switching function (11), the following property of convex combination is used:

$$\min_{j\in\mathbb{K}_r}\{x^T(t)\bar{Q}_j x(t)\} \leq \sum_{i=1}^{r} \alpha_i(t) x^T(t)\bar{Q}_i x(t), \tag{13}$$

where $\alpha_i(t) \in \Lambda_r$ is the membership function and $i, j \in \mathbb{K}_r$.

In the literature of switched control, the function $\sigma(t)$ is designed to lead the switching of a set of gains. However, the approach proposed in this paper uses the definition (11) to deal with the time derivative of FMFs in the Lyapunov functional candidate, independently of the controller chosen. Thus, in the case where a switched controller is not used (Example 1), inequality (13) will be satisfied $\forall t \geq 0$, but when the sampled-data controller (10) is implemented (Example 2) we will only have the information of the states at sampling instants t_k, and the state dependent switching function will be:

$$\sigma(t_k) = \arg\min_{j\in\mathbb{K}_r}\{x^T(t_k)\bar{Q}_j x(t_k)\}, \tag{14}$$

as shown in Figure 1. Therefore, (13) will be satisfied only at the time instants t_k.

Remark 1: Different from the Lyapunov functions for systems without sampled-data control discussed in [6], the Lyapunov-Krasovskii functionals include several state-dependent vectors such as: $x(t)$, $\dot{x}(t)$, $x(t_k)$, etc. For each of them, a different switching function can be obtained, so it is important to guarantee a unique function $\sigma(t)$ that satisfies (13) for all vectors.

B. DISCUSSION OF THE TIME DERIVATIVE OF MEMBERSHIP FUNCTIONS

The use of FMFs in the Lyapunov function can considerably reduces the conservatism of the LMIs, due to FMFs

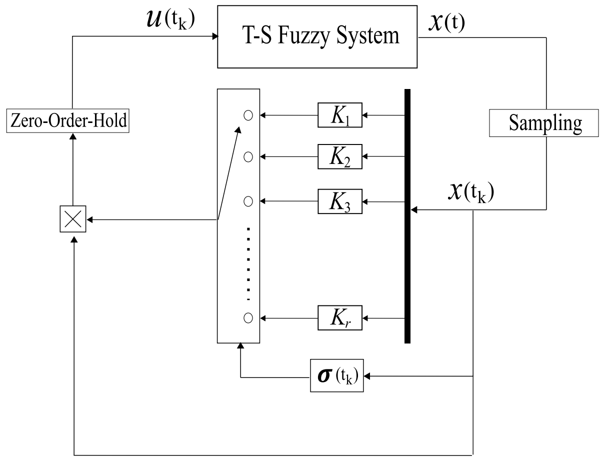

FIGURE 1. Schematic diagram of the proposed switched controller.

includes membership informations on the Lyapunov function candidate which reduces the conservatism of the stability conditions in comparison to quadratic Lyapunov functions. However, it generates the difficulty of introducing the time derivative of the FMFs in the stability conditions. For instance, let the Lyapunov function as follows:

$$V(t) = x^T(t)\sum_{i=1}^{r}\alpha_i(t)P_ix(t), \tag{15}$$

where, for $i \in \mathbb{K}_r$, P_i is a positive definite matrix and $\alpha_i(t)$ are the FMFs. Considering the following notation $P_\alpha = \sum_{i=1}^{r}\alpha_i(t)P_i$ for the remainder of the paper, the time derivative of (15) will be:

$$\dot{V}(t) = 2x^T(t)P_\alpha\dot{x}(t) + x^T(t)\dot{P}_\alpha x(t), \tag{16}$$

where $\dot{P}_\alpha = \sum_{i=1}^{r}\dot{\alpha}_i(t)P_i$.

In the recent literature on sampled-data control [21], [23], [31], [32], [33], [34], switched rules have been introduced to guarantee that $\dot{P}_\alpha \leq 0$ hold. Due the property of the membership functions, $\sum_{i=1}^{r}\dot{\alpha}_i(t) = 0$, it yields that:

$$\dot{P}_\alpha = \sum_{i=1}^{r}\dot{\alpha}_i(t)P_i = \sum_{l=1}^{r-1}\dot{\alpha}_l(t)(P_l - P_r). \tag{17}$$

Then by [39], it is proposed an **IF-THEN** approach to guarantee $\dot{P}_\alpha \leq 0$:

$$\begin{cases} \text{If } \dot{\alpha}_l(t) < 0, & \text{then } P_l - P_r > 0, \\ \text{If } \dot{\alpha}_l(t) \geq 0, & \text{then } P_l - P_r \leq 0. \end{cases} \tag{18}$$

From (18), the number of possible cases are 2^{r-1}. Denote $\psi \in \mathcal{R} = \{1, 2, \ldots, 2^{r-1}\}$ and

$$\begin{aligned} \mathcal{U}_\psi &= \{\psi : \text{ The possible permutations of } \dot{\alpha}_l(t) \text{ for } \psi \in \mathcal{R}\}, \\ \mathcal{G}_\psi &= \{\psi : \text{ The possible constraints of } P_i\}. \end{aligned} \tag{19}$$

Thus, the **IF-THEN** rules in (18) can be rewritten in a more compact form as follows:

$$\text{if } \mathcal{U}_\psi, \text{ then } \mathcal{G}_\psi. \tag{20}$$

Based on the above discussions, some remarks are important to mention.

If is required to satisfy additional conditions as $\dot{Q}_\alpha \leq 0$, the number of rules remains 2^{r-1} but is necessary to include the conditions $Q_l - Q_r > 0$ or $Q_l - Q_r \leq 0$, in each **IF-THEN** rule. Moreover, in order to satisfied $\dot{P}_\alpha \leq 0$, all the terms $\dot{\alpha}_1(t)(P_1 - P_r), \dot{\alpha}_2(t)(P_2 - P_r), \dot{\alpha}_3(t)(P_3 - P_r), \ldots,$ are required to be negative which leads to a conservatism in conditions.

In addition, the above procedure can requires assuming that the continuous time derivative of $\alpha_i(t)$ can be measured for $t \in [t_k, t_{k+1})$. However, when the system present complex nonlinearities, the time derivative can be difficult to obtain or could present some parametric uncertainties in $\alpha_i(t)$, it leads to increased difficulty in implementation.

III. PRELIMINARY RESULTS

In this section, we present a method, which assists in including the term $\dot{P}_\alpha$ in the LMIs. Before proceeding further, is necessary to introduce the following assumption to derive the preliminaries results.

Assumption 1: [37] It is assumed that the continuous first-order derivative of $\alpha_i(t)$ exist for $t \in [t_k, t_{k+1})$, and there are known scalars ρ_i, that satisfies $|\dot{\alpha}_i(t)| \leq \rho_i$, $i \in \mathbb{K}_r$.

For the system (2), based on (4) follows:

$$\sum_{i=1}^{r}\dot{\alpha}_i(t) = 0, \quad \forall t \geq 0. \tag{21}$$

Let the set

$$\Upsilon_r = \left\{\left(\frac{\dot{\alpha}_i(t)}{r\rho} + \frac{1}{r}\right) \in \mathbb{R}/\sum_{i=1}^{r}\dot{\alpha}_i(t) = 0, i \in \mathbb{K}_r\right\}, \tag{22}$$

which contains r elements, and $\rho = \max_{i\in\mathbb{K}_r}\{\rho_i\}$. It is easy to verify that due to Assumption 1 and (21), Υ_r satisfy (4). Therefore, we can apply the property (13) on the set Υ_r, obtaining

$$\min_{j\in\mathbb{K}_r}\{\varsigma^T(t)X_j\varsigma(t)\} \leq \mathcal{X} \leq \max_{j\in\mathbb{K}_r}\{\varsigma^T(t)X_j\varsigma(t)\}, \tag{23}$$

where $X_j \in \mathbb{R}^{n\times n}$ is a symmetric matrix, $\varsigma(t) \in \mathbb{R}^n$ is any time dependent continuous vector, and

$$\mathcal{X} = \varsigma^T(t)\left[\sum_{i=1}^{r}\dot{\alpha}_i(t)\frac{X_i}{r\rho} + \sum_{i=1}^{r}\frac{X_i}{r}\right]\varsigma(t), i \in \mathbb{K}_r. \tag{24}$$

We define the following switching functions:

$$\overline{\sigma}(t) = \arg\max_{j\in\mathbb{K}_r}\{\varsigma^T(t)X_j\varsigma(t)\}, \tag{25}$$

$$\underline{\sigma}(t) = \arg\min_{j\in\mathbb{K}_r}\{\varsigma^T(t)X_j\varsigma(t)\}. \tag{26}$$

Thus, since $\overline{\sigma}$ and $\underline{\sigma}$ are independent of i, multiplying (23) by $\alpha_i(t)$ and adding for all $i \in \mathbb{K}_r$, gives

$$\sum_{i=1}^{r}\alpha_i(t)\varsigma^T(t)X_{\underline{\sigma}}\varsigma(t) \leq \mathcal{X} \leq \sum_{i=1}^{r}\alpha_i(t)\varsigma^T(t)X_{\overline{\sigma}}\varsigma(t), \tag{27}$$

substituting (24) in (27), we obtain

$$\sum_{i=1}^{r}\alpha_i(t)\varsigma^T(t)\left(r\rho X_{\underline{\sigma}}-\rho\sum_{i=1}^{r}X_i\right)\varsigma(t)$$
$$\leq\sum_{i=1}^{r}\dot{\alpha}_i(t)\varsigma^T(t)X_i\varsigma(t)$$
$$\leq\sum_{i=1}^{r}\alpha_i(t)\varsigma^T(t)\left(r\rho X_{\overline{\sigma}}-\rho\sum_{i=1}^{r}X_i\right)\varsigma(t). \qquad (28)$$

Therefore, in order to guarantee $\dot{V}(t) < 0$, for $x(t) \neq 0$, it will be possible to replace the expression $\pm\varsigma^T(t)\sum_{i=1}^{r}\dot{\alpha}_i(t)X_i\varsigma(t)$ by the right or left hand side of (28), according to required in the inequalities. The above discussion was developed for a single matrix X_i but can be extended for more matrices. Therefore, the following Lemma is proposed:

Lemma 1: Let any $s \in \mathbb{N}$ and the scalar ρ that satisfies $|\dot{\alpha}_i(t)| \leq \rho, \forall i \in \mathbb{K}_r$. Thus, for any symmetric matrices $X_j^m \in \mathbb{R}^{n\times n}$, $j \in \mathbb{K}_r$, $m \in \mathbb{K}_s$, and functions $\alpha_i(t) \in \mathbb{R}$ that satisfies (4), the following inequality is satisfied:

$$\sum_{m=1}^{s}\sum_{i=1}^{r}\varsigma_m^T(t)\dot{\alpha}_i(t)X_i^m\varsigma_m(t)$$
$$\leq\rho\sum_{m=1}^{s}\sum_{i=1}^{r}\alpha_i(t)\varsigma_m^T(t)(rX_\sigma^m-\sum_{i=1}^{r}X_i^m)\varsigma_m(t), \qquad (29)$$

where $\sigma \in \mathbb{K}_r$ represents the switching function:

$$\sigma(t)=\arg\max_{j\in\mathbb{K}_r}\{\varsigma^T(t)\text{diag}\{X_j^1, X_j^2, \ldots, X_j^s\}\varsigma(t)\}, \qquad (30)$$

and $\varsigma(t)=[\varsigma_1^T(t), \varsigma_2^T(t), \ldots, \varsigma_s^T(t)]^T$, $\varsigma_m(t) \in \mathbb{R}^n$.

Proof: Since matrices X_j^m are symmetric, we define diagonal block matrices X_i =diag$\{X_i^1, X_i^2, \ldots, X_i^s\}$. Therefore, by the right hand side of the inequality (28) and the switching function (30), it follows:

$$\sum_{i=1}^{r}\dot{\alpha}_i(t)\varsigma^T(t)X_i\varsigma(t)$$
$$\leq\rho\sum_{i=1}^{r}\alpha_i(t)\varsigma^T(t)\left(rX_\sigma-\sum_{i=1}^{r}X_i\right)\varsigma(t). \qquad (31)$$

Substituting $\varsigma(t) = [\varsigma_1^T(t), \varsigma_2^T(t), \ldots, \varsigma_s^T(t)]^T$, and X_i =diag$\{X_i^1, X_i^2, \ldots, X_i^s\}$ in (31), as shown in the equation at the bottom of the page. ■

In Lemma 1, is generalised the previous result, for the sum of several fuzzy dependent matrices, which is usually found in the design conditions of the sampled-data controller using the input delay approach. In addition, the switching function (30) is unique for all matrices X_i^s.

Remark 2: Since the matrices X_j^m only require to be symmetric, if there is an term $-\varsigma_m^T(t)X_\alpha^m\varsigma_m(t)$ on the left side of

$$\sum_{i=1}^{r}\dot{\alpha}_i(t)\varsigma^T(t)X_i\varsigma(t)$$
$$=\sum_{i=1}^{r}\dot{\alpha}_i(t)\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\\ \vdots\\ \varsigma_s(t)\end{bmatrix}^T\begin{bmatrix}X_i^1 & 0 & \ldots & 0\\ 0 & X_i^2 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \vdots & X_i^s\end{bmatrix}\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\\ \vdots\\ \varsigma_s(t)\end{bmatrix}$$
$$=\sum_{m=1}^{s}\sum_{i=1}^{r}\varsigma_m^T(t)\dot{\alpha}_i(t)X_i^m\varsigma_m(t)$$
$$\leq\rho\sum_{i=1}^{r}\alpha_i(t)\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\\ \vdots\\ \varsigma_s(t)\end{bmatrix}^T$$
$$\times\begin{bmatrix}rX_\sigma^1-\sum_{i=1}^{r}X_i^1 & 0 & \ldots & 0\\ 0 & rX_\sigma^2-\sum_{i=1}^{r}X_i^2 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \vdots & rX_\sigma^s-\sum_{i=1}^{r}X_i^s\end{bmatrix}\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\\ \vdots\\ \varsigma_s(t)\end{bmatrix}$$
$$=\rho\sum_{m=1}^{s}\sum_{i=1}^{r}\alpha_i(t)\varsigma_m^T(t)(rX_\sigma^m-\sum_{i=1}^{r}X_i^m)\varsigma_m(t)$$

(29), from the left side to (28) follows:

$$-\sum_{i=1}^{r}\dot{\alpha}_i(t)\varsigma_m^T(t)X_i^m\varsigma_m(t)$$
$$\leq \sum_{m=1}^{s}\sum_{i=1}^{r}\alpha_i(t)\varsigma_m^T(t)(\rho\sum_{i=1}^{r}X_i^m - r\rho X_{\underline{\sigma}}^m)\varsigma_m(t). \quad (32)$$

As is known that

$$\arg\max_{j\in\mathbb{K}_r}\{\varsigma_m^T(t)X_j^m\varsigma_m(t)\} = \arg\min_{j\in\mathbb{K}_r}\{-\varsigma_m^T(t)X_j^m\varsigma_m^T(t)\}, \quad (33)$$

just replace with $(\rho X_i^m - r\rho X_\sigma^m)$ a m-term on the right-hand side of (29) and change the sign of X_j^m in the diagonal block matrix X_j.

Remark 3: In Lemma 1, the conditions does not focus on ensuring $\sum_{m=1}^{s}\varsigma_m^T(t)\dot{P}_\alpha^m\varsigma_m(t) \leq 0$ or $\dot{P}_\alpha^m \leq 0$, which can be a restrictive condition. In fact, the expression of the right side of (29) is always positive, for example due to the property (13), yields:

$$r\varsigma_m^T(t)X_\sigma^m\varsigma_m(t) \geq \varsigma_m^T(t)\sum_{i=1}^{r}X_i^m\varsigma_m(t), \forall m \in \mathbb{K}_s. \quad (34)$$

Therefore, Lemma 1 introduces a positive term in the conditions, which is increased proportionally to the value of ρ as a consequence of using a fuzzy depended Lyapunov functional. However, this restriction does not require to know the time derivative of FMFs in order to lead with fuzzy depended matrices, and is less conservative than used in [21], [22], [23], [32], [33], and [34], as will be seen later.

IV. MAIN RESULTS

In this section we present the design conditions, first for the closed loop system (9) and then using the controller (10). For the sake of simplicity, let us define $\varepsilon_\ell = [0_{n,(\ell-1)n} \ \ I_n \ \ 0_{n,(6-\ell)n}], (\ell = 1, 2, \ldots, 6)$ as block entry matrices, and vectors:

$$\eta_1(t) = \begin{bmatrix} x^T(t) - x^T(t_k) & \int_{t_k}^{t} x^T(s)ds \end{bmatrix}^T,$$
$$\eta_2(t) = \begin{bmatrix} x^T(t_{k+1}) - x^T(t) & \int_{t}^{t_{k+1}} x^T(s)ds \end{bmatrix}^T,$$
$$\eta_3(t) = \begin{bmatrix} x^T(t) & x^T(t_k) & x^T(t_{k+1}) \end{bmatrix}^T,$$
$$\eta_4(t) = \begin{bmatrix} x^T(t) & x^T(t_k) & \int_{t_k}^{t} x^T(s)ds \end{bmatrix}^T,$$
$$\varsigma_1(t) = \begin{bmatrix} x^T(t) & \eta_1^T \end{bmatrix}^T,$$
$$\varsigma_2(t) = \begin{bmatrix} x^T(t) & \eta_2^T \end{bmatrix}^T,$$
$$\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t_k) & x^T(t_{k+1}) & \int_{t_k}^{t} x^T(s)ds & \int_{t}^{t_{k+1}} x^T(s)ds \end{bmatrix}^T.$$

Theorem 1: Let scalars $\tau > 0$, $\rho > 0$ that satisfies $|\dot{\alpha}_i(t)| \leq \rho$, $\forall i \in \mathbb{K}_r$, and gain matrices K_j. The T-S fuzzy system (9) is asymptotically stable, $\forall \tau_k \in (0, \tau]$, if there exist positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, $Q_{11} \in \mathbb{R}^{n\times n}$, $\mathcal{M} \in \mathbb{R}^{n\times n}$, symmetric matrices

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}, S_i \in \mathbb{R}^{2n\times 2n},$$
$$R_i \in \mathbb{R}^{2n\times 2n}, F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ * & F_{22} & F_{23} \\ * & * & F_{33} \end{bmatrix} \in \mathbb{R}^{3n\times 3n},$$

and any matrices $\mathcal{J}, \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ with appropriate dimensions, such that the following inequalities hold, $\forall i, j, l_1, l_2 \in \mathbb{K}_r$:

$$\begin{bmatrix} \Psi_{(\tau_k,0)} & \frac{\tau_k}{2}\mathcal{F}_1 \\ * & -\mathcal{M} \end{bmatrix} < 0, \quad (35)$$
$$\begin{bmatrix} \Psi_{(\tau_k,\tau_k)} & \sqrt{\tau_k}\mathcal{J}^T & \frac{\tau_k}{2}\mathcal{F}_1 \\ * & -Q_{11} & 0 \\ * & * & -\mathcal{M} \end{bmatrix} < 0, \quad (36)$$

where

$$\begin{aligned}\Psi_{(\tau_k,\tau(t))} = {} & \mathrm{Sym}\{\varepsilon_1 P_i \varepsilon_2^T - [\varepsilon_1 - \varepsilon_3]Q_{12}\varepsilon_3^T + \mathcal{J}^T(\varepsilon_1^T \\ & - \varepsilon_3^T) + (\tau_k - \tau(t))([\varepsilon_1 - \varepsilon_3 \ \varepsilon_5]R_i[\varepsilon_2 \ \varepsilon_1]^T) \\ & - \tau(t)([\varepsilon_4 - \varepsilon_1 \ \varepsilon_6]S_i[\varepsilon_2 \ \varepsilon_1]^T)\} \\ & + (\tau_k - \tau(t))\Big([\varepsilon_1 \ \varepsilon_3 \ \varepsilon_4]F[\varepsilon_1 \ \varepsilon_3 \ \varepsilon_4]^T \\ & \quad + \varepsilon_1\mathcal{P}_{l_1}\varepsilon_1^T \\ & \quad + [\varepsilon_1 - \varepsilon_3 \ \varepsilon_5]\mathcal{R}_{l_1}[\varepsilon_1 - \varepsilon_3 \ \varepsilon_5]^T \\ & + [\varepsilon_2 \ \varepsilon_3]Q[\varepsilon_2 \ \varepsilon_3]^T\Big) \\ & - \tau(t)\Big(\varepsilon_3 Q_{22}\varepsilon_3^T \\ & \quad + \varepsilon_1\mathcal{P}_{l_2}\varepsilon_1^T + [\varepsilon_4 - \varepsilon_1 \ \varepsilon_6]\mathcal{S}_{l_2}[\varepsilon_4 - \varepsilon_1\varepsilon_6]^T \\ & \quad + [\varepsilon_1 \ \varepsilon_3 \ \varepsilon_4]F[\varepsilon_1 \ \varepsilon_3 \ \varepsilon_4]^T\Big) \\ & - [\varepsilon_1 - \varepsilon_3 \ \varepsilon_5]R_i[\varepsilon_1 - \varepsilon_3 \ \varepsilon_5]^T \\ & + [\varepsilon_4 - \varepsilon_1 \ \varepsilon_6]S_i[\varepsilon_4 - \varepsilon_1 \ \varepsilon_6]^T \\ & + \mathrm{Sym}\{(\varepsilon_1\mathcal{G}_1 + \varepsilon_2\mathcal{G}_2 + \varepsilon_3\mathcal{G}_3)[A_i\varepsilon_1^T \\ & + B_iK_j\varepsilon_3^T - \varepsilon_2^T]\} + \frac{\tau_k^2}{4}\varepsilon_2\mathcal{M}\varepsilon_2^T,\end{aligned}$$

with

$$\mathcal{F}_1 = [F_{11} \ \ 0 \ \ F_{12} \ \ F_{13} \ \ 0 \ \ 0]^T,$$
$$\mathcal{P}_{(l_1,l_2)} = r\rho P_{(l_1,l_2)} - \rho\sum_{i=1}^{r}P_i,$$
$$\mathcal{R}_{l_1} = r\rho R_{l_1} - \rho\sum_{i=1}^{r}R_i,$$
$$\mathcal{S}_{l_2} = r\rho S_{l_2} - \rho\sum_{i=1}^{r}S_i.$$

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V(t) = \sum_{\ell=1}^{4} V_\ell(t),\ t \in [t_k, t_{k+1}), \tag{37}$$

where

$$\begin{aligned} V_1(t) &= x^T(t)P_\alpha x(t),\\ V_2(t) &= (\tau_k - \tau(t)) \int_{t_k}^{t} \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds,\\ V_3(t) &= (\tau_k - \tau(t))\eta_1^T(t)R_\alpha \eta_1(t) + \tau(t)\eta_2^T(t)S_\alpha \eta_2(t),\\ V_4(t) &= (\tau_k - \tau(t))\tau(t)\eta_3^T(t)F\eta_3(t) \end{aligned}$$

Since

$$\lim_{t\to t_k^-} V_\ell(t) = \lim_{t\to t_k^+} V_\ell(t) = 0,\ \ell = 2, 3, 4;$$

and $\lim_{t\to t_k} V(t) = V_1(t_k)$. Thus, the Lyapunov-Krasovskii functional $V(t)$ is continuous.

Taking the first time derivative of $V(t)$, along the trajectory of the system (9) yields:

$$\dot{V}_1(t) = 2x^T(t)P_\alpha \dot{x}(t) + x^T(t)\dot{P}_\alpha x(t), \tag{38}$$

$$\begin{aligned} \dot{V}_2(t) &= -\int_{t_k}^{t} \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds\\ &\quad + (\tau_k - \tau(t)) \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}\\ &= -\int_{t_k}^{t} \dot{x}^T(s)Q_{11}\dot{x}(s)\, ds - \tau(t)x^T(t_k)Q_{22}x(t_k)\\ &\quad - 2(x(t) - x(t_k))^T Q_{12}x(t_k)\\ &\quad + (\tau_k - \tau(t)) \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}. \end{aligned} \tag{39}$$

Since $Q_{11} > 0$, by Schur complement is obtained

$$\begin{bmatrix} Q_{11} & I \\ I & Q_{11}^{-1} \end{bmatrix} \geq 0,$$

which mean that for any matrix $\mathcal{J} \in \mathbb{R}^{n\times 6n}$, is obtained

$$\int_{t_k}^{t} \begin{bmatrix} \dot{x}(s) \\ \mathcal{J}\xi(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & I \\ I & Q_{11}^{-1} \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ \mathcal{J}\xi(t) \end{bmatrix} ds \geq 0.$$

Thus, we obtain

$$\begin{aligned} -\int_{t_k}^{t} \dot{x}^T(s)Q_{11}\dot{x}(s)\, ds &\leq \tau(t)\xi(t)\mathcal{J}^T Q_{11}^{-1}\mathcal{J}\xi(t)\\ &\quad + 2\xi^T(t)\mathcal{J}^T(x(t) - x(t_k)). \end{aligned} \tag{40}$$

$$\begin{aligned} \dot{V}_3(t) &= -\eta_1^T(t)R_\alpha \eta_1(t) + \eta_2^T(t)S_\alpha \eta_2(t)\\ &\quad + 2[(\tau_k - \tau(t))\eta_1^T(t)R_\alpha - \tau(t)\eta_2^T(t)S_\alpha] \begin{bmatrix} \dot{x}(t) \\ x(t) \end{bmatrix}\\ &\quad + (\tau_k - \tau(t))\eta_1^T(t)\dot{R}_\alpha \eta_1(t) + \tau(t)\eta_2^T(t)\dot{S}_\alpha \eta_2(t). \end{aligned} \tag{41}$$

$$\begin{aligned} \dot{V}_4(t) &= -\tau(t)\eta_3^T(t)F\eta_3(t) + (\tau_k - \tau(t))\eta_3^T(t)F\eta_3(t)\\ &\quad + 2(\tau_k - \tau(t))\tau(t)\eta_3^T(t)\mathcal{F}_1^T\dot{x}(t), \end{aligned} \tag{42}$$

where $\mathcal{F}_1 = [F_{11}, F_{12}, F_{13}]$. Therefore, for any matrix $\mathcal{M} > 0$, based on Schur complement, we have

$$\begin{bmatrix} \mathcal{M} & I \\ I & \mathcal{M}^{-1} \end{bmatrix} \geq 0,$$

which means that for a matrix $\mathcal{F}_1$, is satisfied

$$\begin{bmatrix} \dot{x}(t) \\ -\mathcal{F}_1\eta_3(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{M} & I \\ I & \mathcal{M}^{-1} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ -\mathcal{F}_1\eta_3(t) \end{bmatrix} \geq 0.$$

Thus, is obtained

$$\begin{aligned} &2(\tau_k - \tau(t))\tau(t)\eta_3(t)^T\mathcal{F}_1^T\dot{x}(t)\\ &\quad \leq \frac{\tau_k^2}{4}(\eta_3^T(t)\mathcal{F}_1\mathcal{M}^{-1}\mathcal{F}_1\eta_3(t) + \dot{x}^T(t)\mathcal{M}\dot{x}(t)). \end{aligned} \tag{43}$$

Now, we discuss about fuzzy dependent matrix P_α, R_α and S_α. By summing the fuzzy dependent terms in (38) and (41) yields:

$$\begin{aligned} &\left(\frac{\tau_k - \tau(t)}{\tau_k}\right)(x^T(t)\dot{P}_\alpha x(t) + \eta_1^T(t)\dot{R}_\alpha \eta_1(t))\\ &\quad + \left(\frac{\tau(t)}{\tau_k}\right)(x^T(t)\dot{P}_\alpha x(t) + \eta_2^T(t)\dot{S}_\alpha \eta_2(t)). \end{aligned} \tag{44}$$

Therefore, applying the Lemma 1 is obtained

$$\begin{aligned} &x^T(t)\dot{P}_\alpha x(t) + \eta_1^T(t)\dot{R}_\alpha \eta_1(t)\\ &\quad \leq x^T(t)\left(r\rho P_{\sigma_1} - \rho\sum_{i=1}^{r} P_i\right)x(t)\\ &\qquad + \eta_1^T(t)\left(r\rho R_{\sigma_1} - \rho\sum_{i=1}^{r} R_i\right)\eta_1(t), \end{aligned} \tag{45}$$

$$\begin{aligned} &x^T(t)\dot{P}_\alpha x(t) + \eta_2^T(t)\dot{S}_\alpha \eta_2(t)\\ &\quad \leq x^T(t)\left(r\rho P_{\sigma_2} - \rho\sum_{i=1}^{r} P_i\right)x(t)\\ &\qquad + \eta_2^T(t)\left(r\rho S_{\sigma_2} - \rho\sum_{i=1}^{r} S_i\right)\eta_2(t), \end{aligned} \tag{46}$$

where the functions $\sigma_1(t)$ and $\sigma_2(t)$ are defined by

$$\begin{aligned} \sigma_1(t) &= \arg\max_{l_1\in\mathbb{K}_r}\{\varsigma_1^T(t)\mathrm{diag}\{P_{l_1}, R_{l_1}\}\varsigma_1(t)\},\\ \sigma_2(t) &= \arg\max_{l_2\in\mathbb{K}_r}\{\varsigma_2^T(t)\mathrm{diag}\{P_{l_2}, S_{l_2}\}\varsigma_2(t)\}, \end{aligned}$$

where the switching functions $\sigma_1(t), \sigma_2(t) \in \{1, 2, \ldots, r\}$.

Then, from the system (9), we have

$$\begin{aligned} &2\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(t)\alpha_j(t_k)[x^T(t)\mathcal{G}_1 + \dot{x}^T(t)\mathcal{G}_2 + x(t_k)^T\mathcal{G}_3]\\ &\quad \times [-\dot{x}(t) + A_i x(t) + B_i K_j x(t_k)] = 0 \end{aligned} \tag{47}$$

Then, from (38)-(47), and replacing $\sigma_1(t)$ and $\sigma_2(t)$ by l_1 and l_2 respectively, it obtains

$$\dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(t)\alpha_j(t_k)\xi^T(t)\Xi_{(\tau_k,\tau(t))}\xi(t)$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(t)\alpha_j(t_k)\xi^T(t)\left[\frac{\tau_k-\tau(t)}{\tau_k}\Xi_{(\tau_k,0)} + \frac{\tau(t)}{\tau_{t_k}}\Xi_{(\tau_k,\tau_k)}\right]\xi(t), \quad (48)$$

where

$$\Xi_{(\tau_k,\tau(t))} = \Psi_{(\tau_k,\tau(t))} + \tau(t)\mathcal{J}^T Q_{11}^{-1}\mathcal{J} + \frac{\tau_k^2}{4}\mathcal{F}_1\mathcal{M}^{-1}\mathcal{F}_1^T.$$

Finally, according to the convex combination technique, $\Xi_{(\tau_k,\tau(t))} < 0, t \in [t_k, t_{k+1})$ is equivalent to $\Xi_{(\tau_k,0)} < 0$ and $\Xi_{(\tau_k,\tau_k)} < 0$. Thus, by Schur complement, one has from (35) and (36) that

$$\dot{V}(t) < 0, \ t \in [t_k, t_{k+1}). \quad (49)$$

In order to prove that $V(t) > 0$, from $\lim_{t\to t_k^-} V_\ell(t) = \lim_{t\to t_k^+} V_\ell(t) = 0$, $\ell = (2,3,4)$ and $P_i > 0$, follows:

$$V(t) > V(t_{k+1}) > 0, \ t \in [t_k, t_{k+1}), \ k = 0, 1, 2, \ldots \quad (50)$$

which means that $V_\ell(t)$, $\ell = (2,3,4)$ are not be positive definite on the sampling intervals, and $V(t)$ is only required to be positive definite at sampling times.

Finally, from (49) and (50), the system (9) is asymptotically stable. ■

Remark 4: In Theorem 1, a Lyapunov functional dependent on the FMFs was constructed. In contrast to most of the existing work on the sampled-data control [21], [23], [31], [32], [33], [34], [40], in this case only the Assumption 1 is required, and it is not necessary to obtain the time derivatives of the FMFs at each instant time.

Remark 5: Moreover, the functions $\sigma_1(t)$ and $\sigma_2(t)$ do not need to be considered as switching functions. In fact, since $\sigma_1(t), \sigma_1(t) \in \mathbb{K}_r$, just by replacing by index terms l_1 and l_2 respectively is obtain stability conditions of the system.

Based on Theorem 1, the sampled-data controllers design method for system (9) is provided by Theorem 2.

Theorem 2: Let scalars $\tau > 0, \upsilon_1 > 0, \upsilon_2 > 0, \rho > 0$ that satisfies $|\dot{\alpha}_i(t)| \le \rho, \forall i \in \mathbb{K}_r$. The T-S fuzzy system (9) is asymptotically stabilized by the controller (8), $\forall \tau_k \in (0, \tau]$, if there exist positive definite matrices $\overline{P}_i \in \mathbb{R}^{n\times n}$, $\overline{Q}_{11} \in \mathbb{R}^{n\times n}$, $\overline{\mathcal{M}} \in \mathbb{R}^{n\times n}$, symmetric matrices

$$\overline{Q} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ * & \overline{Q}_{22} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}, \overline{S}_i \in \mathbb{R}^{2n\times 2n}$$
$$\overline{R}_i \in \mathbb{R}^{2n\times 2n}, \overline{F} = \begin{bmatrix} \overline{F}_{11} & \overline{F}_{12} & \overline{F}_{13} \\ * & \overline{F}_{22} & \overline{F}_{23} \\ * & * & \overline{F}_{33} \end{bmatrix} \in \mathbb{R}^{3n\times 3n},$$

and any matrices $\overline{\mathcal{J}}, \mathcal{G}$ with appropriate dimensions, such that the following inequalities hold, $\forall i, j, l_1, l_2 \in \mathbb{K}_r$

$$\begin{bmatrix} \overline{\Psi}_{(\tau_k,0)} & \frac{\tau_k}{2}\overline{\mathcal{F}}_1 \\ * & -\overline{\mathcal{M}} \end{bmatrix} < 0, \quad (51)$$

$$\begin{bmatrix} \overline{\Psi}_{(\tau_k,\tau_k)} & \sqrt{\tau_k}\overline{\mathcal{J}}^T & \frac{\tau_k}{2}\overline{\mathcal{F}}_1 \\ * & -\overline{Q}_{11} & 0 \\ * & * & -\overline{\mathcal{M}} \end{bmatrix} < 0, \quad (52)$$

where the controller gain matrix $K_j = \overline{K}_j\mathcal{G}^{-1}$, and

$$\begin{aligned}\overline{\Psi}_{(\tau_k,\tau(t))} = {} & \mathrm{Sym}\{\varepsilon_1\overline{P}_i\varepsilon_2^T - [\varepsilon_1 - \varepsilon_3]\overline{Q}_{12}\varepsilon_3^T + \overline{\mathcal{J}}^T(\varepsilon_1^T \\ & - \varepsilon_3^T) + (\tau_k - \tau(t))([\varepsilon_1 - \varepsilon_3\ \varepsilon_5]\overline{R}_i[\varepsilon_2\ \varepsilon_1]^T) \\ & - \tau(t)([\varepsilon_4 - \varepsilon_1\ \varepsilon_6]\overline{S}_i[\varepsilon_2\ \varepsilon_1]^T)\} \\ & + (\tau_k - \tau(t))\Big([\varepsilon_1\ \varepsilon_3\ \varepsilon_4]\overline{F}[\varepsilon_1\ \varepsilon_3\ \varepsilon_4]^T \\ & \quad + \varepsilon_1\overline{\mathcal{P}}_{l_1}\varepsilon_1^T \\ & \quad + [\varepsilon_1 - \varepsilon_3\ \varepsilon_5]\overline{\mathcal{R}}_{l_1}[\varepsilon_1 - \varepsilon_3\varepsilon_5]^T \\ & \quad + [\varepsilon_2\ \varepsilon_3]\overline{Q}[\varepsilon_2\ \varepsilon_3]^T\Big) \\ & - \tau(t)\Big(\varepsilon_3\overline{Q}_{22}\varepsilon_3^T + \varepsilon_1\overline{\mathcal{P}}_{l_2}\varepsilon_1^T \\ & \quad + [\varepsilon_4 - \varepsilon_1\ \varepsilon_6]\overline{\mathcal{S}}_{l_2}[\varepsilon_4 - \varepsilon_1\ \varepsilon_6]^T \\ & \quad + [\varepsilon_1\ \varepsilon_3\ \varepsilon_4]\overline{F}[\varepsilon_1\ \varepsilon_3\ \varepsilon_4]^T\Big) \\ & - [\varepsilon_1 - \varepsilon_3\ \varepsilon_5]\overline{R}_i[\varepsilon_1 - \varepsilon_3\ \varepsilon_5]^T \\ & + [\varepsilon_4 - \varepsilon_1\ \varepsilon_6]\overline{S}_i[\varepsilon_4 - \varepsilon_1\ \varepsilon_6]^T \\ & + \mathrm{Sym}\{(\varepsilon_1 + \upsilon_1\varepsilon_2 + \upsilon_2\varepsilon_3)[A_i\mathcal{G}\varepsilon_1^T \\ & + B_i\overline{K}_j\varepsilon_3^T - \mathcal{G}\varepsilon_2^T]\} + \frac{\tau_k^2}{4}\varepsilon_2\overline{\mathcal{M}}\varepsilon_2^T,\end{aligned}$$

with

$$\begin{aligned}\overline{\mathcal{F}}_1 &= [\mathcal{G}^T F_{11}\mathcal{G}\ \ 0\ \ \mathcal{G}^T F_{12}\mathcal{G}\ \ \mathcal{G}^T F_{13}\mathcal{G}\ \ 0\ \ 0]^T, \\ \overline{\mathcal{P}}_{(l_1,l_2)} &= r\rho\overline{P}_{(l_1,l_2)} - \rho\sum_{i=1}^{r}\overline{P}_i, \\ \overline{\mathcal{R}}_{l_1} &= r\rho\overline{R}_{l_1} - \rho\sum_{i=1}^{r}\overline{R}_i, \\ \overline{\mathcal{S}}_{l_2} &= 2r\rho\overline{S}_{l_2} - \rho\sum_{i=1}^{r}\overline{S}_i.\end{aligned}$$

Proof: Define

$$\begin{aligned}\mathcal{G}_1 &= \mathcal{G}^{-1}, \mathcal{G}_2 = \upsilon_1\mathcal{G}^{-1}, \mathcal{G}_3 = \upsilon_2\mathcal{G}^{-1}, \overline{\mathcal{M}} = \mathcal{G}^T\mathcal{M}\mathcal{G}, \\ \overline{P}_i &= \mathcal{G}^T P_i\mathcal{G}, \overline{R}_i = \mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T\}R_i\mathrm{diag}\{\mathcal{G}, \mathcal{G}\}, \\ \overline{S}_i &= \mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T\}S_i\mathrm{diag}\{\mathcal{G}, \mathcal{G}\}, \\ \overline{Q} &= \mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T\}Q\mathrm{diag}\{\mathcal{G}, \mathcal{G}\}, \\ \overline{F} &= \mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T\}F\mathrm{diag}\{\mathcal{G}, \mathcal{G}, \mathcal{G}\}, \\ \overline{\mathcal{J}} &= \mathcal{G}^T\mathcal{J}\mathrm{diag}\{\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}\}.\end{aligned}$$

Therefore, multiplying on the left and right side of (49) by $\mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T\}$ and $\mathrm{diag}\{\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}\}$ respectivly, and similar to the prove of Theorem 1, applying the Schur complement we obtain (51) and (52). ■

Remark 6: In Theorem 2 the scalars υ_1 and υ_2 are introduced to conditions to provide more degrees of freedom in the LMIs. For every value of υ_1 and υ_2 we can obtain a different maximum value of τ. Focus on this discussion, a grid search Algorithm [21], [32], [33], [40], is used to obtain the highest value of τ in a bounded interval. Therefore, for each value of υ_1, υ_2, and τ, (51) and (52) become LMIs, that are easy to solve with any LMIs toolbox.

Now, in order to obtain a switched controller which does not depend on FMFs in the controller implementation, we will obtain the stability conditions for the system (2) using the controller (10) as shown in Figure 1.

In Theorem 1 and Theorem 2 the fuzzy dependent matrices P_α, R_α, and S_α were considered. However, this leads to two switching functions, which increases the number of controllers to be switched. Therefore, in order to reduce the number of controllers, matrices R_α and S_α will be considered as constant matrices.

Moreover, as mentioned previously, the switching function will be (14) and the inequality (13) will only be satisfied at sampling instants. Therefore, the expression $x^T(t)\dot{P}_\alpha x(t)$ can not be reduced by using the Lemma 1, and matrix P_α will be considered as constant. However, will be added to the Lyapunov functional the following expression:

$$V_5(t) = \sum_{i=1}^{r} \alpha_i(t) x^T(t_k) X_i x(t_k), \quad i \in \mathbb{K}_r, \tag{53}$$

where X_i are positive definite matrices and the Lyapunov functional given in (37) reduces to the following:

$$\begin{aligned} V(t) = {} & V_1(t) + V_2(t) + V_4(t) + (\tau_k - \tau(t))\eta_1^T(t) R \eta_1(t) \\ & + \tau(t)\eta_2^T(t) S \eta_2(t) + V_5(t). \end{aligned} \tag{54}$$

Corollary 1: Let scalars $\tau > 0$, $\upsilon_1 > 0$, $\upsilon_2 > 0$, $\rho > 0$ that satisfies $|\dot{\alpha}_i(t)| \leq \rho$, $\forall i \in \mathbb{K}_r$, the T-S fuzzy system (2) is asymptotically stabilized by the switching controller (10) and the switching function $\sigma(t) = \arg\max_{j \in \mathbb{K}_r}\{x^T(t) X_j x(t)\}$, $\forall \tau_k \in (0, \tau]$, if there exist positive definite matrices $\overline{X}_j \in \mathbb{R}^{n\times n}$, $\overline{P} \in \mathbb{R}^{n\times n}$, $\overline{Q}_{11} \in \mathbb{R}^{n\times n}$, $\overline{\mathcal{M}} \in \mathbb{R}^{n\times n}$, symmetric matrices

$$\overline{Q} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ * & \overline{Q}_{22} \end{bmatrix} \in \mathbb{R}^{2n\times 2n},$$

$$\overline{S}, \overline{R} \in \mathbb{R}^{2n\times 2n}, \overline{F} = \begin{bmatrix} \overline{F}_{11} & \overline{F}_{12} & \overline{F}_{13} \\ * & \overline{F}_{22} & \overline{F}_{23} \\ * & * & \overline{F}_{33} \end{bmatrix} \in \mathbb{R}^{3n\times 3n},$$

and any matrices $\overline{\mathcal{J}}, \mathcal{G}$ with appropriate dimensions, such that the following inequalities hold, $\forall i, j \in \mathbb{K}_r$

$$\begin{bmatrix} \overline{\Psi}_{(\tau_k,0)} & \frac{\tau_k}{2}\overline{\mathcal{F}}_1 \\ * & -\overline{\mathcal{M}} \end{bmatrix} < 0, \tag{55}$$

$$\begin{bmatrix} \overline{\Psi}_{(\tau_k,\tau_k)} & \sqrt{\tau_k}\,\overline{\mathcal{J}}^T & \frac{\tau_k}{2}\overline{\mathcal{F}}_1 \\ * & -\overline{Q}_{11} & 0 \\ * & * & -\overline{\mathcal{M}} \end{bmatrix} < 0, \tag{56}$$

where the controller gain matrix $K_j = \overline{K}_j \mathcal{G}^{-1}$, and

$$\begin{aligned} \overline{\Psi}_{(\tau_k,\tau(t))} = {} & \mathrm{Sym}\{\varepsilon_1 \overline{P} \varepsilon_2^T - [\varepsilon_1 - \varepsilon_3]\overline{Q}_{12}\varepsilon_3^T + \overline{\mathcal{J}}^T(\varepsilon_1^T \\ & - \varepsilon_3^T) + (\tau_k - \tau(t))([\varepsilon_1 - \varepsilon_3\ \varepsilon_5]\overline{R}[\varepsilon_2\ \varepsilon_1]^T) \\ & - \tau(t)([\varepsilon_4 - \varepsilon_1\ \varepsilon_6]\overline{S}[\varepsilon_2\ \varepsilon_1]^T)\} \\ & + (\tau_k - \tau(t))\Big([\varepsilon_1\ \varepsilon_3\ \varepsilon_4]\overline{F}[\varepsilon_1\ \varepsilon_3\ \varepsilon_4]^T \\ & \qquad + [\varepsilon_2\ \varepsilon_3]\overline{Q}[\varepsilon_2\ \varepsilon_3]^T\Big) \\ & - \tau(t)\Big(\varepsilon_3\overline{Q}_{22}\varepsilon_3^T \\ & \qquad + [\varepsilon_1\ \varepsilon_3\ \varepsilon_4]\overline{F}[\varepsilon_1\ \varepsilon_3\ \varepsilon_4]^T\Big) \\ & + \varepsilon_3\overline{\mathcal{X}}_j\varepsilon_3^T - [\varepsilon_1 - \varepsilon_3\ \varepsilon_5]\overline{R}[\varepsilon_1 - \varepsilon_3\ \varepsilon_5]^T \\ & + [\varepsilon_4 - \varepsilon_1\ \varepsilon_6]\overline{S}[\varepsilon_4 - \varepsilon_1\ \varepsilon_6]^T \\ & + \mathrm{Sym}\{(\varepsilon_1 + \upsilon_1\varepsilon_2 + \upsilon_2\varepsilon_3)[A_i\mathcal{G}\varepsilon_1^T \\ & + B_i\overline{K}_j\varepsilon_3^T - \mathcal{G}\varepsilon_2^T]\} + \frac{\tau_k^2}{4}\varepsilon_2\overline{\mathcal{M}}\varepsilon_2^T, \end{aligned}$$

with

$$\begin{aligned} \overline{\mathcal{F}}_1 &= [\mathcal{G}^T F_{11}\mathcal{G}\ \ 0\ \ \mathcal{G}^T F_{12}\mathcal{G}\ \ \mathcal{G}^T F_{13}\mathcal{G}\ \ 0\ \ 0]^T, \\ \overline{\mathcal{X}}_j &= r\rho\overline{X}_j - \rho\sum_{i=1}^{r}\overline{X}_i. \end{aligned}$$

Proof: Since (54), the values of $\dot{P}_\alpha$, $\dot{R}_\alpha$ and $\dot{S}_\alpha$ are null. Then in order to lead with the fuzzy dependent matrix X_α, applying the Lemma 1, yields:

$$\begin{aligned} \dot{V}_5(t) &= x^T(t_k)\dot{X}_\alpha x(t_k) \\ &\leq x^T(t_k)\Big(r\rho X_\sigma - \rho\sum_{i=1}^{r}X_i\Big)x(t_k), \end{aligned} \tag{57}$$

where

$$\sigma(t_k) = \arg\max_{j\in\mathbb{K}_r}\{x^T(t_k)X_j x(t_k)\}.$$

Following the same procedure as in the proof of Theorem 1, and replacing σ by j we obtain

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{r}\alpha_i(t)\xi^T(t)\Xi_{(\tau_k,\tau(t))}\xi(t) \\ &= \sum_{i=1}^{r}\alpha_i(t)\xi^T(t)\left[\frac{\tau_k - \tau(t)}{\tau_k}\Xi_{(\tau_k,0)} + \frac{\tau(t)}{\tau_{t_k}}\Xi_{(\tau_k,\tau_k)}\right]\xi(t), \end{aligned} \tag{58}$$

where

$$\Xi_{(\tau_k,\tau(t))} = \Psi_{(\tau_k,\tau(t))} + \tau(t)\mathcal{J}^T Q_{11}^{-1}\mathcal{J} + \frac{\tau_k^2}{4}\mathcal{F}_1\mathcal{M}^{-1}\mathcal{F}_1^T.$$

Therefore, multiplying on the left and right side of (58) by $\mathrm{diag}\{\mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T, \mathcal{G}^T\}$ and $\mathrm{diag}\{\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{G}\}$ respectively, applying the Schur complement, and taking the same definitions of the proof of Theorem 2, we obtain (55) and (56). ■

Based on Lemma 1 a design conditions of switched sampled-data controller was obtained, which does not require to feedback the FMFs to implement the controller.

Remark 7: Procedures to lead with controllers (8) and (10) are similar, due to from (4) the switched controller can also be expressed as $u(t_k) = \sum_{i=1}^{r} \alpha_i(t) K_\sigma x(t-\tau(t)),\ t \in [t_k, t_{k+1})$, where the function $\sigma(t)$ is replaced by j because not depend of i.

V. SIMULATION EXAMPLES

In this section, two simulation examples are given to show the effectiveness and less conservatism of our results.

A. EXAMPLE 1

In this subsection is presented the chaotic Rossler's system with an input signal [41]:

$$\begin{cases} \dot{x}_1 = -x_2(t) - x_3(t) \\ \dot{x}_2 = x_1(t) + a x_2(t) \\ \dot{x}_3 = b x_1(t) - (c - x_1(t)) x_3(t) + u(t). \end{cases} \quad , \qquad (59)$$

where $x(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$ is the state vector and $u(t)$ is the input control. In order to obtain the T-S fuzzy model, let $|c - x_1(t)| \leq d$ where d is a know constant value. Therefore, for $x_1(t) \in [c-d, c+d]$ following the fuzzy rules:

$$\text{Rule 1}: \ \textbf{IF}\ x_1(t)\ \text{is}\ M_1^1,\ \textbf{THEN}\ \dot{x}(t) = A_1 x(t) + B_1 u(t),$$
$$\text{Rule 2}: \ \textbf{IF}\ x_1(t)\ \text{is}\ M_1^2,\ \textbf{THEN}\ \dot{x}(t) = A_2 x(t) + B_2 u(t),$$

where

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad (60)$$

with FMFs as $\alpha_1(t) = \frac{c+d-x_1(t)}{2d}$ and $\alpha_2(t) = 1 - \alpha_1(t)$. For this example the parameters values will be $a = 0.3$, $b = 0.5$, $c = 5$, $d = 10$, and the initial conditions $x(0) = [-1, 2, 3]$.

For $u(t) \equiv 0$, the time derivative of $x_1(t)$ and the state variables $x(t)$ are shown in the Figure 2, where the lower and upper bound of $|\dot{x}(t)| \leq 6$. Thus, in order to satisfy the Assumption 1, we can choose $\rho = 0.3$. Moreover, solving the LMIs conditions of Theorem 2 for $\upsilon_1 = 3.2$ and $\upsilon_2 = 21.7$, we obtain $\tau = 0.1412$ s as the upper bound of the time interval length, and the following gains:

$$K_1 = \begin{bmatrix} 14.9450 & 4.3837 & -13.5467 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 15.3410 & 4.3839 & -14.1431 \end{bmatrix}.$$

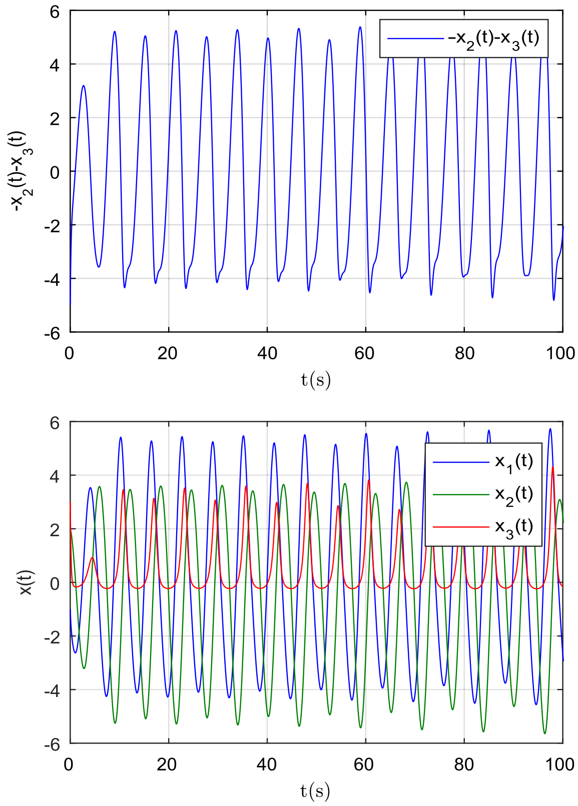

FIGURE 2. Time derivative of $x_1(t)$ and open-loop system state response.

TABLE 1. The admissible upper bound τ of time interval sampled.

Methods:	[35]	[32]	[36]	[22]	[33]	Theorem 2
τ:	0.0736	0.1107	0.1147	0.1165	0.1188	0.1412

The state variables of the system and the input control using the obtained gains are shown in the Fig. 3, where it is verified the asymptotic stability of the system, and we can see that the system achieves the stability in a time of approximately 50 seconds. In addition, in the Table 1 it is shown a comparison of the maximum sampled time obtained with respect to [22] and [33], which use the rule (20) to deal with the fuzzy dependent matrices.

B. EXAMPLE 2

In this case, we consider the inverted pendulum [37], [38], which the model is shown in Fig. 4 and the dynamic model is given by (61), as shown at the bottom of the page, where $x_1(t)$ represent the angle of the pendulum from vertical and $x_2(t)$ is the angular velocity. Therefore the space state model

$$\begin{cases} \dot{x}_1(t) = -x_2(t) \\ \dot{x}_2(t) = \dfrac{g\sin(x_1(t)) - amlx_2^2(t) sin(2x_1(t))/2 - a\cos(x_1(t))u(t)}{4l/3 - aml\cos^2(x_1(t))} \end{cases} \qquad (61)$$

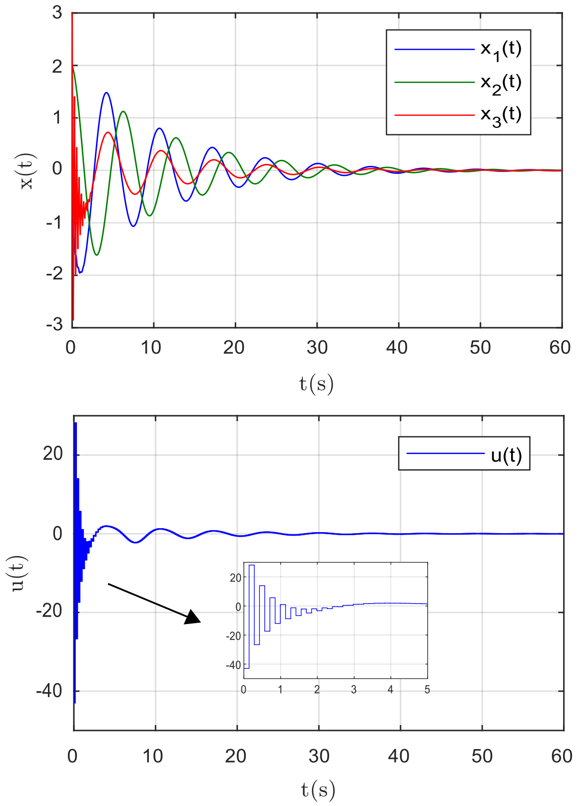

FIGURE 3. Close-loop system state response and input control of Example 1.

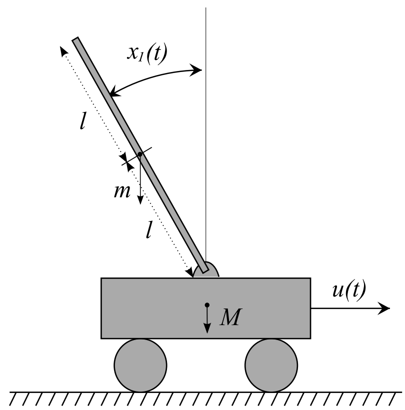

FIGURE 4. Inverted pendulum control system model.

will be:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ f_1(t) & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ f_2(t) \end{bmatrix} u(t).$$

with

$$f_1(t) = \frac{g\sin(x_1(t)) - amlx_2^2(t)\sin(2x_1(t))/2}{4l/3x_1(t) - aml\cos^2(x_1(t))x_1(t)}, \quad \text{and}$$

$$f_2(t) = \frac{a\cos(x_1(t))u(t)}{aml\cos^2(x_1(t)) - 4l/3},$$

as two non linear terms. Therefore, by achieving the T-S fuzzy model using the Taniguchi's method [2], it could generate

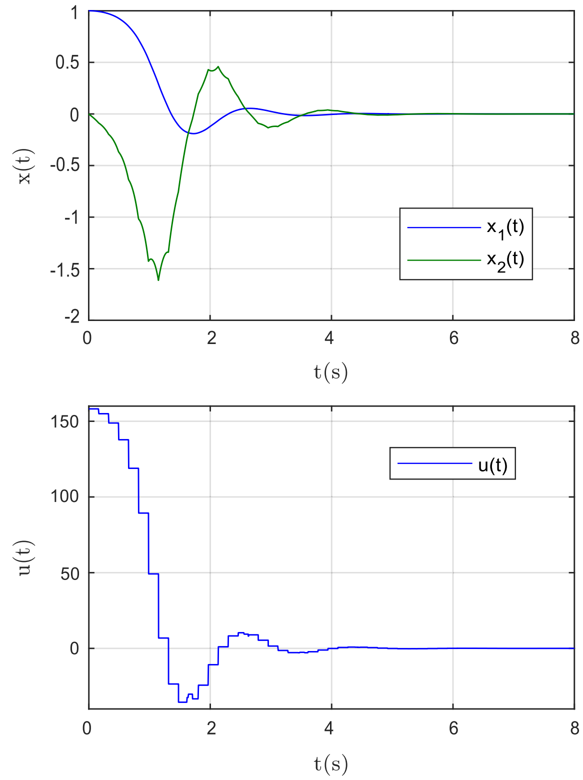

FIGURE 5. Close-loop system state response and input control of Example 2.

FMFs with complex expressions, which generate a difficulty in the implementation of the fuzzy controller. For this reason, in most of the literature, the system is reduced to only two fuzzy rules considering that $x_1(t)$ is close to $\pm\frac{\pi}{2}$ or 0, obtaining two membership functions of triangular shape. Focusing on this issue, a switched controller that does not require to obtain the FMFs for the implementation will be designed using Corollary 1.

Choose $M = 8$ kg, $m = 2$ kg, $l = 0.5$ m, $a = \frac{1}{(m+M)}$, $g = 9.8$ m/s^2, and β =cos(88°). We consider $x_1(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and applying the T-S fuzzy model as [37] and [38], system (61) can be rewritten as

$$\text{Rule 1}: \ \textbf{IF}\ x_1(t) \text{ is } 0,\ \textbf{THEN}\ \dot{x}(t) = A_1x(t) + B_1u(t),$$
$$\text{Rule 2}: \ \textbf{IF}\ x_1(t) \text{ is } \pm\frac{\pi}{2},\ \textbf{THEN}\ \dot{x}(t) = A_2x(t) + B_2u(t),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-aml\beta^2)} & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3-aml} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3-aml\beta^2} \end{bmatrix}.$$

The FMFs are

$$\begin{cases} \alpha_1(t) = 1 - \frac{2}{\pi}x_1(t), & \text{if } 0 \le x_1(t) < \frac{\pi}{2} \\ \alpha_2(t) = 1 + \frac{2}{\pi}x_1(t), & \text{if } -\frac{\pi}{2} \le x_1(t) < 0, \end{cases} \tag{62}$$

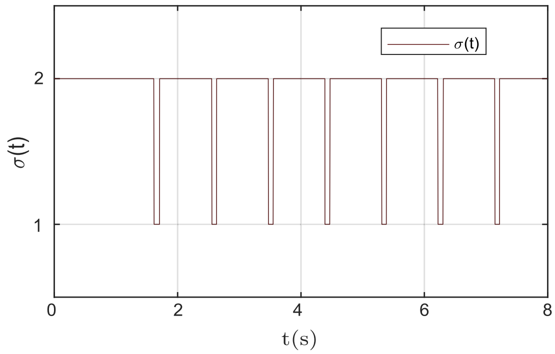

FIGURE 6. Switching function $\sigma(t)$.

and $\alpha_2(t) = 1 - \alpha_1(t)$. As in [37] and [38], choose $|x_2(t)| \leq 10$ rad/sec. Then, we obtain $|\dot{\alpha}_i(t)| \leq \rho = 20/\pi$.

Solving the LMI's of the Corollary 1 for $\upsilon_1 = 2.3$ and $\upsilon_2 = 23.7$, we obtain $\tau = 0.1445$ s as the upper bound of the time interval length, and the following gains

$$K_1 = \begin{bmatrix} 146.1636 & 22.2314 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 158.1663 & 24.2612 \end{bmatrix},$$

and the matrices

$$X_1 = \begin{bmatrix} 1.0023 & -0.0044 \\ -0.0044 & 2.9184 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.9982 & 0.0042 \\ 0.0042 & 1.8281 \end{bmatrix}.$$

The maximum sampled interval obtained is greater than those obtained in [37] and [38], which confirms the less conservatism of the conditions. By use the control strategy shown in the Fig. 1, the state variables of the system and the input control are shown in the Fig. 5, where the system achieves the stability in approximately 5 seconds. Moreover, in the Fig. 6 is shown the switched index $\sigma(t)$, which change the value between 1 and 2, achieving the stability of system.

VI. CONCLUSION

Relaxed design conditions of sampled-data controller using fuzzy dependent Lyapunov functional was proposed in this work. By using the convex combination inequality, we derive new stability conditions to deal with the derivatives of the FMFs, which does not require to obtain the derivative of the membership functions to guarantee the stability of the system. Moreover, the convex combination property allows us to design a switched controller, that not depend of the FMFs for implemented the controller. The design conditions obtained are applied to two classic examples in the literature, verifying the improvement of the proposed strategy.

Finally, this paper introduces a switched control approach based on state-dependent switching functions of type, *argmin* and *argmax* functions; expanding the possibilities of taking the benefits of switched systems that have been widely studied in the literature. Moreover, the Lemma 1 is not limited to being applied only in sampled-data control, but in several applications which involve Lyapunov functions dependent on FMFs.

REFERENCES

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985.

[2] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. Hoboken, NJ, USA: Wiley, 2004.

[3] M. C. M. Teixeira, E. Assuncao, and R. G. Avellar, "On relaxed LMI-based designs for fuzzy regulators and fuzzy observers," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 613–623, Oct. 2003.

[4] R. Gaino, M. R. Covacic, R. Cardim, M. A. A. Sanches, A. A. De Carvalho, A. R. Biazeto, and M. C. M. Teixeira, "Discrete Takagi–Sugeno fuzzy models applied to control the knee joint movement of paraplegic patients," *IEEE Access*, vol. 8, pp. 32714–32726, 2020.

[5] M. R. Covacic, M. C. M. Teixeira, A. A. De Carvalho, R. Cardim, E. Assunção, M. A. A. Sanches, H. S. Fujimoto, M. S. Mineo, A. R. Biazeto, and R. Gaino, "Robust T–S fuzzy control of electrostimulation for paraplegic patients considering norm-bounded uncertainties," *Math. Problems Eng.*, vol. 2020, pp. 1–28, Aug. 2020.

[6] W. A. De Souza, M. C. M. Teixeira, R. Cardim, and E. Assuncao, "On switched regulator design of uncertain nonlinear systems using Takagi–Sugeno fuzzy models," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 6, pp. 1720–1727, Dec. 2014.

[7] U. N. L. Alves, M. C. M. Teixeira, D. R. De Oliveira, R. Cardim, E. Assunção, and W. A. De Souza, "Smoothing switched control laws for uncertain nonlinear systems subject to actuator saturation," *Int. J. Adapt. Control Signal Process.*, vol. 30, nos. 8–10, pp. 1408–1433, Feb. 2016.

[8] D. R. De Oliveira, M. C. M. Teixeira, U. N. L. T. Alves, W. A. De Souza, E. Assunção, and R. Cardim, "On local H_∞ switched controller design for uncertain T–S fuzzy systems subject to actuator saturation with unknown membership functions," *Fuzzy Sets Syst.*, vol. 344, pp. 1–26, Aug. 2018.

[9] L. J. Elias, F. A. Faria, R. F. Q. Magossi, and V. A. Oliveira, "Switched control design for nonlinear systems using state feedback," *J. Control, Autom. Electr. Syst.*, vol. 33, no. 3, pp. 733–742, Jan. 2022.

[10] D. J. S. Oncoy, R. Cardim, M. C. M. Teixeira, and E. Assuncao, "Robust switched controller based on Takagi–Sugeno fuzzy model for DC–DC converters against disturbances," in *Proc. 16th Int. Workshop Variable Struct. Syst. (VSS)*, Sep. 2022, pp. 1–7.

[11] H. Li, J. Wang, H. Du, and H. R. Karimi, "Adaptive sliding mode control for Takagi–Sugeno fuzzy systems and its applications," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 531–542, Apr. 2018.

[12] M. Chadli, H. R. Karimi, and P. Shi, "On stability and stabilization of singular uncertain Takagi–Sugeno fuzzy systems," *J. Franklin Inst.*, vol. 351, no. 3, pp. 1453–1463, Mar. 2014.

[13] X.-L. Zhu, H. Lin, and X.-P. Xie, "Sampled-data fuzzy stabilization of nonlinear systems under nonuniform sampling," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 6, pp. 1654–1667, Dec. 2016.

[14] W. Cheng, D. Wang, X.-L. Zhu, and Y. Wang, "H_∞ stabilization for sampling fuzzy systems with asynchronous constraints on membership functions," *IEEE Access*, vol. 5, pp. 1524–1533, 2017.

[15] A. N. D. Lopes, K. Guelton, L. Arcese, V. J. S. Leite, and F. Bourahala, "Finsler-based sampled-data controller design for Takagi–Sugeno systems," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 7965–7970, 2020.

[16] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, Feb. 2010.

[17] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: An input delay approach," *Automatica*, vol. 40, no. 8, pp. 1441–1446, 2004.

[18] K. Liu and E. Fridman, "Wirtinger's inequality and Lyapunov-based sampled-data stabilization," *Automatica*, vol. 48, no. 1, pp. 102–108, Jan. 2012.

[19] A. Seuret, "A novel stability analysis of linear systems under asynchronous samplings," *Automatica*, vol. 48, no. 1, pp. 177–182, 2012.

[20] H.-B. Zeng, K. L. Teo, and Y. He, "A new looped-functional for stability analysis of sampled-data systems," *Automatica*, vol. 82, pp. 328–331, Aug. 2017.

[21] J. Zhao, S. Xu, and J. H. Park, "Improved criteria for the stabilization of T–S fuzzy systems with actuator failures via a sampled-data fuzzy controller," *Fuzzy Sets Syst.*, vol. 392, pp. 154–169, Aug. 2020.

[22] C. Hua, S. Wu, and X. Guan, "Stabilization of T–S fuzzy system with time delay under sampled-data control using a new looped-functional," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 400–407, Feb. 2020.
[23] R. Saravanakumar, R. Datta, and Y. Cao, "New insights on fuzzy sampled-data stabilization of delayed nonlinear systems," *Chaos, Solitons Fractals*, vol. 154, Jan. 2022, Art. no. 111654.
[24] V. Gandhi and Y. H. Joo, "T–S fuzzy sampled-data control for nonlinear systems with actuator faults and its application to wind energy system," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 2, pp. 462–474, Feb. 2022.
[25] Z. Sheng, C. Lin, B. Chen, and Q.-G. Wang, "Stability analysis of sampled-data systems via novel Lyapunov functional method," *Inf. Sci.*, vol. 585, pp. 559–570, Mar. 2022.
[26] T. Lee and J. H. Park, "Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3653–3657, Jul. 2017.
[27] H.-B. Zeng, Y. He, M. Wu, and J. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2768–2772, Oct. 2015.
[28] J. Sun, G. Chen, and J. Chen, "Stability analysis of aperiodic sampled-data systems: A switched polytopic system method," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 67, no. 6, pp. 1054–1058, Jun. 2020.
[29] X.-C. Shang-Guan, Y. He, W.-J. Lin, and M. Wu, "Improved synchronization of chaotic Lur'e systems with time delay using sampled-data control," *J. Franklin Inst.*, vol. 354, no. 3, pp. 1618–1636, Feb. 2017.
[30] Y. Liu, B.-Z. Guo, J. H. Park, and S.-M. Lee, "Nonfragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 1, pp. 118–128, Jan. 2018.
[31] X. Wang, J. H. Park, H. Yang, G. Zhao, and S. Zhong, "An improved fuzzy sampled-data control to stabilization of T–S fuzzy systems with state delays," *IEEE Trans. Cybern.*, vol. 50, no. 7, pp. 3125–3135, Jul. 2020.
[32] X. Wang, J. H. Park, K. She, S. Zhong, and L. Shi, "Stabilization of chaotic systems with T–S fuzzy model and nonuniform sampling: A switched fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 6, pp. 1263–1271, Jun. 2019.
[33] Z. You, H. Yan, H. Zhang, S. Chen, and M. Wang, "Fuzzy-dependent-switching control of nonlinear systems with aperiodic sampling," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 11, pp. 3349–3359, Nov. 2021.
[34] J. Luo, M. Li, X. Liu, W. Tian, S. Zhong, and K. Shi, "Stabilization analysis for fuzzy systems with a switched sampled-data control," *J. Franklin Inst.*, vol. 357, no. 1, pp. 39–58, 2020.
[35] Z.-P. Wang and H.-N. Wu, "On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 819–829, Apr. 2015.
[36] R. Zhang, D. Zeng, J. H. Park, Y. Liu, and S. Zhong, "A new approach to stabilization of chaotic systems with nonfragile fuzzy proportional retarded sampled-data control," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3218–3229, Sep. 2019.
[37] M. Wang, J. Qiu, M. Chadli, and M. Wang, "A switched system approach to exponential stabilization of sampled-data T–S fuzzy systems with packet dropouts," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3145–3156, Dec. 2016.
[38] X.-L. Zhu, B. Chen, D. Yue, and Y. Wang, "An improved input delay approach to stabilization of fuzzy systems under variable sampling," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 330–341, Apr. 2012.
[39] L. Wang and H.-K. Lam, "New stability criterion for continuous-time Takagi–Sugeno fuzzy systems with time-varying delay," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1551–1556, Apr. 2019.
[40] L. Yang, J. Zhang, C. Ge, W. Li, and Z. Zhao, "Stability and stabilization for uncertain fuzzy system with sampled-data control and state quantization," *Int. J. Speech Technol.*, vol. 51, no. 10, pp. 7469–7483, Mar. 2021.
[41] K. Tanaka, T. Ikeda, and H. O. Wang, "A unified approach to controlling chaos via an LMI-based fuzzy control system design," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 45, no. 10, pp. 1021–1040, Oct. 1998.

DANTE J. S. ONCOY was born in Huaraz, Peru, in 1996. He received the B.Sc. degree in electronic engineering from the National University of Engineering (UNI), Lima, Peru, in 2021. He is currently pursuing the M.S. degree in electrical engineering with São Paulo State University (UNESP), Ilha Solteira, Brazil. His research interests include control theory and its applications, linear matrix inequalities-based designs, sampled-data control, fuzzy systems, and switched systems.

RODRIGO CARDIM was born in Adamantina, Brazil, in 1981. He received the B.Sc. and Doctoral degrees in electrical engineering from São Paulo State University (UNESP), Ilha Solteira, Brazil, in 2004 and 2009, respectively. In 2012, he held a postdoctoral position. He is currently a Professor with the Department of Electrical Engineering, UNESP. His research interests include control theory and applications, linear matrix inequalities-based designs, variable structure control, fuzzy systems, and switched systems. He received the São Paulo Engineering Institute Award for the Best Undergraduate Student of the year 2004 at UNESP.

MARCELO C. M. TEIXEIRA (Member, IEEE) was born in Campo Grande, Brazil, in 1957. He received the B.Sc. degree in electrical engineering from the Escola de Engenharia de Lins, Lins, Brazil, in 1979, the M.Sc. degree in electrical engineering from the Instituto Alberto Luiz Coimbra de Pós-graduação e Pesquisa de Engenharia, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil, in 1982, and the D.Sc. degree in electrical engineering from Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, in 1989. In 1982, he joined the Department of Electrical Engineering, Universidade Estadual Paulista (UNESP), Ilha Solteira (FEIS), Brazil, where he is currently a Professor. From 1996 to 1997, he was a Visiting Scholar with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, USA. He was a Coordinator, from 1991 to 1993, and a Vice Coordinator, from 1989 to 1991 and from 1993 to 1995, of the undergraduate program in electrical engineering with FEIS-UNESP. He was also a Coordinator, from 2005 to 2007, and a Vice Coordinator, from 2000 to 2005 and from 2007 to 2010, of the postgraduate program in electrical engineering. His research interests include control theory and applications, neural networks, variable-structure systems, linear-matrix-inequality-based designs, fuzzy systems, nonlinear systems, adaptive systems, and switched systems. From 2009 to 2013, he was a member of the Brazilian Evaluation Committee of Postgraduate Courses IV Engineering CAPES. From 2016 to 2020, he was an Associate Editor of the IEEE Transactions on Fuzzy Systems.

FLÁVIO A. FARIA received the B.Math., M.Sc., and Ph.D. degrees in electrical engineering from São Paulo State University (UNESP), in 2003, 2005, and 2009, respectively. Since 2013, he has been an Assistant Professor with UNESP. His research interests include LMI-based control designs, TS fuzzy modeling, and its applications.

EDVALDO ASSUNÇÃO was born in Andradina, Brazil, in 1965. He received the B.Sc. degree from São Paulo State University (UNESP), Ilha Solteira, Brazil, in 1989, the M.Sc. degree from the Instituto Tecnológico de Aeronáutica, Brazil, in 1991, and the D.Sc. degree from the Universidade Estadual de Campinas, Brazil, in 2000, all in electrical engineering. In 1992, he joined the Department of Electrical Engineering, UNESP, where he is currently an Assistant Professor. His research interests include control theory and applications, linear matrix inequalities-based design, optimal and robust control. He received the Instituto de Engenharia de São Paulo Award from FEIS-UNESP, in 1989.

ADALBERTO Z. N. LAZARINI was born in Ilha Solteira, Brazil, in 1991. He received the B.Sc., M.Sc., and D.Sc. degrees in electrical engineering from São Paulo State University (UNESP), Ilha Solteira, in 2015, 2017, and 2021, respectively. He is currently an Associate Professor in the Computer Engineering Course Coordination with Federal University of Technology–Paraná (COENC-UTFPR-AP). His research interests include control theory and its applications, fuzzy control using TS models, and LMI-based design for optimal and robust control.