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RESEARCH ARTICLE

Interval-Valued Pythagorean Fuzzy Information Aggregation Based on Aczel-Alsina Operations and Their Application in Multiple Attribute Decision Making

ABRAR HUSSAIN[1], KIFAYAT ULLAH[1], MUHAMMAD MUBASHER[1], TAPAN SENAPATI[2,3], AND SARBAST MOSLEM[4]

[1]Department of Mathematics, Riphah International University (Lahore Campus), Lahore 54000, Pakistan
[2]School of Mathematics and Statistics, Southwest University, Chongqing, Beibei 400715, China
[3]Department of Mathematics, Padima Janakalyan Banipith, Kukrakhupi, Jhargram 721517, India
[4]School of Architecture Planning and Environmental Policy, University College of Dublin, Dublin 4, D04 V1W8 Ireland

Corresponding authors: Tapan Senapati (math.tapan@gmail.com) and Sarbast Moslem (sarbast.moslem@ucd.ie)

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ABSTRACT Multi-attribute decision-making (MADM) technique has been widely utilized in many domains, including management, economics, and other disciplines. It plays a significant role in the mainstream of decision science. Its main process is to rearrange available choices and select the optimal option out of a group of options based on certain characteristics assigned by the decision maker. Aggregation operators (AOs) play a significant role in MADM techniques to aggregate uncertain and vague information. Aczel Alsina aggregation models are recently introduced to handle ambiguous and dubious information, Aczel Alsina aggregation models attained a lot of attention from numerous research scholars. This article aims is to generalize the concept of pythagorean fuzzy sets (PyFSs) in the framework of interval-valued pythagorean fuzzy (IVPyF) sets (IVPyFSs) by utilizing the basic operations of Aczel-Alsina aggregation models. We developed a list of certain AOs of IVPyFSs based on Aczel-Alsina aggregation models namely ''IVPyF Aczel-Alsina weighted averaging'' (IVPyFAAWA), ''IVPyF Aczel-Alsina ordered weighted averaging'' (IVPyFAAOWA), and ''IVPyF Aczel-Alsina hybrid weighted averaging'' (IVPyFAAHWA) operators. We also studied some appropriate AOs of IVPyF information such as the ''IVPyF Aczel-Alsina weighted geometric'' (IVPyFAAWG) operator. We also interpreted certain characteristics of our invented approaches. To check the effectiveness and reliability of our proposed approaches, we established an illustrative numerical example for the selection of a research scientist under the system of the MADM technique. We also explored sensitivity analysis, in which we evaluated the impact of parametric values on the result of our proposed approaches. Furthermore, we also illustrated a realistic comparison in which we compare the outcomes of exiting methodologies with the results of our invented approaches.

INDEX TERMS Aczel-Alsina operations, aggregation operator, decision support system, interval-valued Pythagorean fuzzy values, Pythagorean fuzzy values.

I. INTRODUCTION

The theory of MADM is effective for choosing the best option based on several criteria [1], [2], [3]. Historically, the majority of existing evidence often assumes that professionals provide accurate assessments of all characteristics. But because of the complexities and variations in actuality, most decisions are made in unclear or misleading circumstances. Consequently, in these situations, the decision is still expressed as information that has a slight degree of value. The outcome

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might not be optimal. Firstly, the concept of the fuzzy set was presented by Zadeh [4]. The theory of intuitionistic fuzzy (IF) sets (IFSs) was developed by Atanassov [5] to address FS. It is acknowledged as one of the most prominent and widely utilized methods for handling fuzziness. In-depth, the IFS is expressed using a membership degree (MD) and non-membership degree (NMD), and their total sum cannot exceed one [6], [7]. Yager [8] developed the idea of the PyFS by generalizing the concept of IFS, which relaxes the condition of the sum of the square of MD and NMD is restricted to less than or equal to one. The extension of Pythagorean fuzzy values (PyFVs) in the form of complex numbers was developed by Yager and Abbasov [9] in 2013. Some significant results for PyFSs were developed by Peng and Yang [10]. Furthermore, Zhang [11] expanded PyFS to IVPyFS to overcome the limitation, and handle ambiguous information. Ejegwa and Onyeke [12] enhanced the circle of the IFSs based on several distance measures and studied a list of new AOs also organized to solve MADM approaches. Ejegwa [13] presented the notions of similarities measures and developed some AOs to cope with impressions and vague information based on PyFSs. Due to a lack of information, it may be challenging for decision-makers to precisely quantify their opinions in a precise number. The IVPyFS is better prepared to describe ambiguity and apprehension in choice situations than other systems. Peng and Yang [14] provided a series of new AOs by using the IVPyF values (IVPyFVs) and used them in group decision-making. Mu et al. [15] extended the idea of an IVPyFS by using the operations of the Maclaurin symmetric mean. Yang et al. [16] extended the concepts of PyFSs by using the operations of frank power AOs. Rahman and Ali [17] developed new AOs and their special cases by using IVPyFVs. Mishra et al. [18] extended the concept of IVPyF similarity measures to evaluate the suitable technology to produce energy from waste materials. Khan et al. [19] explored the theory SFSs and overcome the complicatedness of fuzziness by using certain models of Dombi operations. Mahmood [20] discovered innovative concepts of complex bipolar FS and provided an optimal algorithm to solve a MADM problem. Hussain et al. [21] explored concepts of IFSs by utilizing the HM tools to evaluate suitable tourism destination by the MADM technique. Ejegwa et al. [22] worked on distance measures and discovered some new AOs to design arrangement and disease problem-solving under the system of PyFS. Riaz et al. [23] presented a number of approaches of polar fuzzy set based on PyF information. Ejegwa et al. [24] explored correlation measures and widespread the theory of PyFS to develop some AOs based on PyF environments.

The triangular norms play a vital role to aggregate fuzzy information. The triangular norms are robust aggregation tools to handle dubious information in fuzzy environments. The concept of TNM and TCNM was given by Menger [25] in 1942. He utilized the idea of TNM and TCNM to solve statistical metrics. Egbert [26] extended the idea of TNM and TCNM to define Cartesian products and a sum of real numbers. A lot of researchers utilized the concepts of TNM and TCNM to incorporate algebraic sum and product, minimum or Godel tools [27], Lukasiewicz tools [28], Einstein tools [29], Archimedean tools [30], Drastic tools [31], Nilpotent models [32], and Hamacher models [33]. Klement and Navara [28] worked on the Lukasiewicz TNM in a fuzzy system based on the TNM. Zeshui Xu [34] worked on IFSs and presented innovative AOs by using algebraic sum and algebraic product. Garg [35] generalized the theory of PyFSs and organized robust methodologies of Einstein models in fuzzy environments. Xu and Yager [36] extended the idea of IFS to develop the IF weighted geometric (IFWG) operator for the solution of our daily life problems. Peng [37] developed some new approaches to IVPyFS and proposed certain algorithm to solve complicatedness during aggregation of fuzzy information. The Seikh and Mandal [38] elaborated attractive methodologies of IFSs under the system of Dombi operations. Hussain et al. [39] originated some robust aggregation models and also studied an application of vendor management systems. Khan et al. [40], [41] anticipated certain methodologies of bipolar valued hesitant fuzzy system and gave an attractive algorithm to handle loss of information during aggregating process. the Jana et al. [42] proposed AOs of PyF Dombi weighted averaging operator and their properties by utilizing the operations of Dombi operations, the Zhang [43] gave some reliable aggregation tools to eliminate effects of dubious information based on Frank models, the Xia et al. [44] generalized the IFSs and gave some AOs based on Archimedean TNM and TCNM, the Huang [45] proposed AOs of IFSs and gave an application under MADM technique to select suitable objective with the help of Hammer TNM and TCNM, Senapati and Chen [46] elaborated the theory of IVPyFSs and developed some AOs of IVPyFSs based on Hamacher TNM and TCNM, and Ullah et al. [47] tried to investigate complex and uncertain information by utilizing the concepts of an attractive models of Dombi operations under the system of an interval-valued T-spherical FSs.

In 1982 János Aczél and Claudi Alsina gave a robust concept of aggregation models incorporating Aczel Alsina tools. Further, the extensions of Aczel Alsina tools were used to handle statistical information and its properties by Butnariu and Klement [48]. Farahbod and Eftekhari [49] worked on various TNM and TCNM to find suitable TNM and TCNM. He applied different TNM and TCNM based on a fuzzy classification system and observed that Aczel Alsina tools are superior to others. Babu and Ahmed [50] also studied the Aczel Alsina tools to find the flexibility and fluency of discussed TNM and TCNM. Hadžić and Pap [51] utilized the theory of Aczel Alsina tools to investigate the fixed theorem based on probabilistic metric spaces. Štefka and Holeňa [52] a generalized fuzzy integral and classifier aggregation in a statistical system. Recently, Senapati et al. [53] generalized IFSs in the framework of interval-valued IF (IVIF) based on Aczel Alsina tools. He also utilized the concepts of Aczel Alsina tools in the environment of interval-valued IFSs and studied an application to solve a MADM problem. Hussain et al. [54] developed some AOs of PyFSs by utilizing the operational laws of Aczel Alsina tools with their basic properties. Senapati [55] also discovered a new series of AOs by using the operations of Aczel Alsina tools under the environment of picture FSs.

However, all the above-discussed approaches are unable to handle insufficient information and there is a lot of chance of loss of information during the aggregation process. A decision-maker can face some difficulties during the aggregation process due to insufficient information given by the experts. An IVPyFSs is a more convenient generalization of FS, IFS, and PyFSs and contains information in a realistic form. The chance of losing information in interval form is minimum. Therefore, we utilized the information on IVPyF environments. This article aims is to generalize the concepts of PyFSs in the framework of IVPyFSs by using the basic operations of Aczel Alsina tools. An IVPyFSs contained information in such a way that each MD and NMD have upper and lower intervals. We introduced certain methodologies by using the basic operations of Aczel Alsina tools under the system of IVPyFSs. We established robust aggregation approaches of IVPyFAAWA with certain characteristics and their special cases. We also study the IVPyFAAWG operator based on Aczel Alsina operations. To find the effectiveness and reliability of our proposed AOs, we established an application to choose reasonable research scientists for a public university. We also studied a realistic comparison to compare the outcomes of existing approaches with the consequences of our current methodologies.

The structure of the presented research work is organized as follows: In section I, we explored the summary of our current research work. In section II, we explore the theory of IVPyFSs and their certain operations. In section III, we present some basic operations of Aczel Alsina tools, further we explain them with numerical examples. In section IV, we generalized IVPyFSs in the framework of the IVPyFAAWA operator based on Aczel Alsina operations with certain properties of robust proposed methodologies. In section V, we also extended IVPyFSs in the form of the IVPyFAAWG operator and its properties. In section VI, the assessment of given information by the experts under a reliable MADM approach. A practical example was also studied to choose desirable candidates for a vacant post of research scientist for a public university under our proposed methodologies. In section VII, a comprehensive comparative study is also presented here to find the efficiency and reliability of our discussed techniques with some existing approaches. In section VIII, we summarized our whole article.

II. PRELIMINARIES

First of all, we study the concepts of reliable tools of TNM, TCNM, PyFS, and IVPyFS for further development of this article. We utilized the symbol of X as a universal set in the whole article.

TABLE 1. Abbreviations and their meanings.

Abbreviations	Meanings
AOs	Aggregation operators
MADM	Multi-attribute decision making
PyFS	Pythagorean fuzzy set
IVPyFS	Interval-valued pythagorean fuzzy set
TNM	T-norm
TCNM	T-conorm
MD	Membership degree
NMD	Non-membership degree
IVIF	Interval-valued pythagorean fuzzy
IVPyFAAWA	Interval-valued pythagorean fuzzy Aczel Alsina weighted average
IVPyFAAOWA	Interval-valued pythagorean fuzzy Aczel Alsina order weighted average
IVPyFAAHA	Interval-valued pythagorean fuzzy Aczel Alsina hybrid average
IVPyFAAWG	Interval-valued pythagorean fuzzy Aczel Alsina weighted geometric
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
q-ROFS	q-rung orthopair fuzzy set
IVPyFWA	Interval-valued pythagorean fuzzy weighted average
IVPyFWG	Interval-valued pythagorean fuzzy weighted geometric
IVPyFEWA	Interval-valued pythagorean fuzzy Einstein weighted average
IVPyFEWG	Interval-valued pythagorean fuzzy Einstein weighted geometric
IVPyFHWA	Interval-valued pythagorean fuzzy Hamacher weighted average
IVPyFHWG	Interval-valued pythagorean fuzzy Hamacher weighted geometric
PyFAAWA	Pythagorean fuzzy Aczel Alsina weighted average
PyFAAWG	Pythagorean fuzzy Aczel Alsina weighted geometric
CPyDFWAA	Complex pythagorean Dombi fuzzy weighted arithmetic averaging
CPyDFWGA	Complex pythagorean fuzzy Dombi fuzzy weighted geometric averaging

Definition 1 [56]: A TNM is a function that lies on an interval $[0, 1]$, $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$. A TNM must satisfy the properties of symmetric, monotonicity, associative and Identity element 1 in the following form:

(i) $T(\rho, \nu) = T(\nu, \rho)$
(ii) $T(\rho, \nu) = T(\omega, \rho)$ *if* $\nu \leq \omega$
(iii) $T(\rho, T(\nu, \omega)) = T(T(\rho, \nu), \omega)$
(iv) $T(\rho, 1) = \rho$

$\forall, \rho, \nu, \omega \in [0, 1]$.

Definition 2 [57]: A TCNM is a function that lies on interval $[0, 1]$, $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$. A TCNM also satisfy the properties of symmetric, monotonic, associative and Identity element 1 in the form as:

(i) $S(\rho, \nu) = S(\nu, \rho)$
(ii) $S(\rho, \nu) \leq S(\omega, \rho)$ *if* $\nu \leq \omega$
(iii) $S(\rho, S(\nu, \omega) = S(S(\rho, \nu), \omega)$
(iv) $S(\rho, 0) = \rho$

Definition 3 [58]: The Aczel Alsina t-norm is particularized as:

$$\left(T_A^{\Omega}\right)(\rho, \nu) = \begin{Bmatrix} T_D(\rho, \nu) & \text{if } \Omega = 0 \\ \min(\rho, \nu) & \text{if } \Omega = \infty \\ e^{-\left((-\ln(\rho))^{\Omega} + (-\ln(\nu))^{\Omega}\right)^{1/\Omega}} & \text{otherwise} \end{Bmatrix},$$

and Aczel Alsina t-conorm is givens as:

$$\left(S_A^{\Omega}\right)(\rho, \nu) = \begin{Bmatrix} T_D(\rho, \nu) & \text{if } \Omega = 0 \\ \min(\rho, \nu) & \text{if } \Omega = \infty \\ 1 - e^{-\left((-\ln(1-\rho))^{\Omega} + (-\ln(1-\nu))^{\Omega}\right)^{1/\Omega}} & \text{otherwise} \end{Bmatrix}$$

respectively.

Definition 4 [59]: A PyFS P in X is in the form as:

$$P = \{(x, \gamma_P(x), \delta_P(x)) \,|x \in X\}$$

where the function $\gamma_P(x) : X \to [0, 1]$ and $\delta_P(x) : X \to [0, 1]$ denote the MD and NMD respectively. A PyFS must satisfy the following axioms:

$$0 \leq \gamma_P^2(x) + \delta_P^2(x) \leq 1$$

A hesitancy value of $x \in X$ is denoted by $\mathfrak{R}(x) = \sqrt{1 - \left(\gamma_P^2(x) + \delta_P^2(x)\right)}$ and a PyF value (PyFV) is denoted by $C = (\gamma_P(x), \delta_P(x))$.

Definition 5 [37]: An IVPyFS B is defined as:

$$B = \{(x, \gamma_B(x), \delta_B(x)) \,|x \in X\}$$

where $\gamma_B(x) = \left[\gamma_B^L(z), \gamma_B^U(x)\right] \in [0, 1]$ and $\delta_B(x) = \left[\delta_B^L(x), \delta_B^U(x)\right] \in [0, 1]$, such that:

$$0 \leq \left(\gamma_B^U(x)\right)^2 + \left(\delta_B^U(x)\right)^2 \leq 1$$

Here L and U represent the lower and upper bound respectively. A hesitancy value of $x \in X$ is denoted by (1), shown at the bottom of the next page.

Definition 6 [14]: Let $B = \left(\left[\gamma_B^L, \gamma_B^U\right], \left[\delta_B^L, \delta_B^U\right]\right)$ be an IVPyFV. Then the score function can be described as:

$$S(B) = \frac{1}{2}\left[\left(\gamma_B^L\right)^2 + \left(\gamma_B^U\right)^2 - \left(\left(\delta_B^L\right)^2 + \left(\delta_B^U\right)^2\right)\right]$$

Definition 7 [14]: Let $B = \left(\left[\gamma_B^L, \gamma_B^U\right], \left[\delta_B^L, \delta_B^U\right]\right)$ be an IVPyFV. Then the accuracy function can be described as:

$$H(B) = \frac{1}{2}\left[\left(\gamma_B^L\right)^2 + \left(\gamma_B^U\right)^2 + \left(\delta_B^L\right)^2 + \left(\delta_B^U\right)^2\right]$$

Remark 1: If B_1, B_2 be two IVPyFVs, then such conditions must be held. If $S(B_1) < S(B_2)$, then $B_1 < B_2$ and if $S(B_1) > S(B_2)$, then $B_1 > B_2$. If $S(B_1) = S(B_2)$ then some following conditions were also observed:

a. If $H(B_1) < H(B_2)$, then $B_1 < B_2$
b. If $H(B_1) > H(B_2)$, then $B_1 > B_2$
c. If $H(B_1) = H(B_2)$, then $B_1 = B_2$

Definition 8 [37]: Let $B = \left(\left[\gamma_B^L, \gamma_B^U\right], \left[\delta_B^L, \delta_B^U\right]\right)$, $B_1 = \left(\left[\gamma_{B_1}^L, \gamma_{B_1}^U\right], \left[\delta_{B_1}^L, \delta_{B_1}^U\right]\right)$ and $B_2 = \left(\left[\gamma_{B_2}^L, \gamma_{B_2}^U\right], \left[\delta_{B_2}^L, \delta_{B_2}^U\right]\right)$ be three IVPyFVs and $\lambda > 0$. Then:

(i)

$$B_1 \cup B_2 = \left(\begin{array}{c}\left[\max\left\{\gamma_{B_1}^L, \gamma_{B_2}^L\right\}, \max\left\{\gamma_{B_1}^U, \gamma_{B_2}^U\right\}\right], \\ \left[\min\left\{\delta_{B_1}^L, \delta_{B_2}^L\right\}, \min\left\{\delta_{B_1}^U, \delta_{B_2}^U\right\}\right]\end{array}\right)$$

(ii)

$$B_1 \cap B_2 = \left(\begin{array}{c}\left[\min\left\{\gamma_{B_1}^L, \gamma_{B_2}^L\right\}, \min\left\{\gamma_{B_1}^U, \gamma_{B_2}^U\right\}\right], \\ \left[\max\left\{\delta_{B_1}^L, \delta_{B_2}^L\right\}, \max\left\{\delta_{B_1}^U, \delta_{B_2}^U\right\}\right]\end{array}\right)$$

(iii)

$$B_1 \oplus B_2 = \left(\begin{array}{l}\left[\gamma_{B_1}^L + \gamma_{B_2}^L - \gamma_{B_1}^L\gamma_{B_2}^L, \gamma_{B_1}^U + \gamma_{B_2}^U - \gamma_{B_1}^U\gamma_{B_2}^U\right], \\ \left[\delta_{B_1}^L\delta_{B_2}^L, \delta_{B_1}^U\delta_{B_2}^U\right]\end{array}\right)$$

(iv)

$$B_1 \otimes B_2 = \left(\begin{array}{l}\left[\gamma_{B_1}^L\gamma_{B_2}^L, \gamma_{B_1}^U\gamma_{B_2}^U\right], \\ \left[\delta_{B_1}^L + \delta_{B_2}^L - \delta_{B_1}^L\delta_{B_2}^L, \delta_{B_1}^U + \delta_{B_2}^U - \delta_{B_1}^U\delta_{B_2}^U\right]\end{array}\right)$$

(v)

$$\lambda.B = \left(\begin{array}{l}\left[1 - \left(1 - \gamma_B^L\right)^{\lambda}, 1 - \left(1 - \gamma_B^U\right)^{\lambda}\right], \\ \left[\left(\delta_B^L\right)^{\lambda}, \left(\delta_B^U\right)^{\lambda}\right]\end{array}\right), \quad \lambda > 0$$

(vi)

$$B^{\lambda} = \left(\begin{array}{l}\left[\left(\gamma_B^L\right)^{\lambda}, \left(\gamma_B^U\right)^{\lambda}\right], \\ \left[1 - \left(1 - \delta_B^L\right)^{\lambda}, 1 - \left(1 - \delta_B^U\right)^{\lambda}\right]\end{array}\right), \quad \lambda > 0$$

III. ACZEL ALSINA OPERATIONS BASED ON IVPyFV INFORMATION

We express Aczel-Alsina operations based on IVPyFVs and their associated ideas. Assume that T be a TNM and S be a TCNM, so the Aczel-Alsina product and sum are denoted by T_A and S_A respectively. Then intersection and union over two IVPyFVs E and W based on Aczel-Alsina sum $E \oplus W$ and Aczel-Alsina product $E \otimes W$ are given as:

$$E\otimes W=\begin{pmatrix}\left[T_A\left\{\gamma_E^L,\gamma_W^L\right\},T_A\left\{\gamma_E^U,\gamma_W^U\right\}\right],\\ \left[S_A\left\{\delta_E^L,\delta_W^L\right\},S_A\left\{\delta_E^U,\delta_W^U\right\}\right]\end{pmatrix}$$

$$E\oplus W=\begin{pmatrix}\left[S_A\left\{\gamma_E^L,\gamma_W^L\right\},S_A\left\{\gamma_E^U,\gamma_W^U\right\}\right],\\ \left[T_A\left\{\delta_E^L,\delta_W^L\right\},T_A\left\{\delta_E^U,\delta_W^U\right\}\right]\end{pmatrix}$$

Definition 9: Let $B=\left(\left[\gamma_B^L,\gamma_B^U\right],\left[\delta_B^L,\delta_B^U\right]\right),B_1=\left(\left[\gamma_{B_1}^L,\gamma_{B_1}^U\right],\left[\delta_{B_1}^L,\delta_{B_1}^U\right]\right)$ and $B_2=\left(\left[\gamma_{B_2}^L,\gamma_{B_2}^U\right],\left[\delta_{B_2}^L,\delta_{B_2}^U\right]\right)$ be three IVPyFVs, $\Omega\geq 1$ and $\eta>0$. Then some fundamental Aczel-Alsina operations are given as follows:

(i) see (2), as shown at the bottom of the next page.
(ii) see (3), as shown at the bottom of the next page.
(iii)

$$\eta B=\left(\begin{bmatrix}\sqrt{1-e^{-\left(\eta\left(-ln\left(1-(\gamma_B^L)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}},\\ \sqrt{1-e^{-\left(\eta\left(-ln\left(1-(\gamma_B^U)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}}\end{bmatrix},\begin{bmatrix}e^{-\left(\eta\left(-ln\delta_B^L\right)^\Omega\right)^{\frac{1}{\Omega}}},e^{-\left(\eta\left(-ln\delta_B^U\right)^\Omega\right)^{\frac{1}{\Omega}}}\end{bmatrix}\right)$$

(iv)

$$B^\eta=\left(\begin{bmatrix}e^{-\left(\eta\left(-ln(\gamma_B^L)\right)^\Omega\right)^{\frac{1}{\Omega}}},e^{-\left(\eta\left(-ln(\gamma_B^U)\right)^\Omega\right)^{\frac{1}{\Omega}}}\end{bmatrix},\begin{bmatrix}\sqrt{1-e^{-\left(\eta\left(-ln\left(1-(\delta_B^L)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}},\\ \sqrt{1-e^{-\left(\eta\left(-ln\left(1-(\delta_B^U)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}}\end{bmatrix}\right)$$

Note: Above discussed Aczel Alsina operations are not applicable for MD and NMD if their values are 0 or 1.

Example 1: Suppose $B=([0.35,0.61],[0.25,0.45])$, $B_1=([0.23,0.52],[0.45,0.63])$ and $B_2=([0.43,0.53],[0.41,0.49])$ be any three IVPyFVs. Then by using definition 9. For $\Omega=3,\eta=4$ we have:

(i) (4), as shown at the bottom of the next page.
(ii) (5), as shown at the bottom of the next page.
(iii)

$$4B=\left(\begin{bmatrix}\sqrt{1-e^{-\left(4\times\left(-ln\left(1-(0.35)^2\right)\right)^3\right)^{\frac{1}{3}}}},\\ \sqrt{1-e^{-\left(4\times\left(-ln\left(1-(0.61)^2\right)\right)^3\right)^{\frac{1}{3}}}}\end{bmatrix},\begin{bmatrix}e^{-\left(4\times(-\ln(0.25))^3\right)^{\frac{1}{3}}},e^{-\left(4\times(-ln(0.45))^3\right)^{\frac{1}{3}}}\end{bmatrix}\right)$$
$$=([0.4328,0.7227],[0.1107,0.2815])$$

(iv)

$$B^4=\left(\begin{bmatrix}e^{-\left(4\times(-ln(0.35))^3\right)^{\frac{1}{3}}},e^{-\left(4\times(-ln(0.61))^3\right)^{\frac{1}{3}}}\end{bmatrix},\begin{bmatrix}\sqrt{1-e^{-\left(4\times\left(-ln\left(1-(0.25)^2\right)\right)^3\right)^{\frac{1}{3}}}},\\ \sqrt{1-e^{-\left(4\times\left(-ln\left(1-(0.45)^2\right)\right)^3\right)^{\frac{1}{3}}}}\end{bmatrix}\right)$$
$$=([0.1889,0.4563],[0.3121,0.5493])$$

Theorem 1: Suppose $B=\left(\left[\gamma_B^{\mathrm{L}},\gamma_B^{\mathrm{U}}\right],\left[\delta_B^{\mathrm{L}},\delta_B^{\mathrm{U}}\right]\right),B_1=\left(\left[\gamma_{B_1}^{\mathrm{L}},\gamma_{B_1}^{\mathrm{U}}\right],\left[\delta_{B_1}^{\mathrm{L}},\delta_{B_1}^{\mathrm{U}}\right]\right)$ and $B_2=\left(\left[\gamma_{B_2}^{\mathrm{L}},\gamma_{B_2}^{\mathrm{U}}\right],\left[\delta_{B_2}^{\mathrm{L}},\delta_{B_2}^{\mathrm{U}}\right]\right)$ be any three IVPyFVs. Then the following conditions must be satisfied.

1) $B_1\oplus B_2=B_2\oplus B_1$
2) $B_1\otimes B_2=B_2\otimes B_1$
3) $\eta(B_1\oplus B_2)=\eta B_1\oplus\eta B_2,\eta>0$
4) $(\eta_1+\eta_2)B=\eta_1 B\oplus\eta_2 B,\eta_1,\eta_2>0$
5) $(B_1\otimes B_2)^\eta=B_1^\eta\otimes B_2^\eta,\eta>0$
6) $B^{\eta_1}\otimes B^{\eta_2}=B^{(\eta_1+\eta_2)},\eta_1,\eta_2>0$

Proof: Proof is straightforward.

IV. ACZEL ALSINA AGGREGATION OPERATORS BASED ON IVPyF INFORMATION

Now we established IVPyF AOs based on Aczel-Alsina operations. We also study the IVPyFAAWA operator based on Aczel Alsina operations with certain characteristics.

Definition 10: Let $B_j=\left(\left[\gamma_{B_j}^L,\gamma_{B_j}^U\right],\left[\delta_{B_j}^L,\delta_{B_j}^U\right]\right)$ $(j=1,2,\ldots,\psi)$ be a collection of IVPyFVs, and corresponding weigh vectors $\vartheta_j=(\vartheta_1,\vartheta_2,\ldots,\vartheta_\psi)^T$ of $B_j\,(j=1,2,\ldots,\psi)$ such that $\vartheta_j\in[0,1]$ and $\sum_{j=1}^{\psi}\vartheta_j=1$. Then an IVPyFAAWA is particularized as:

$$IVPyFAAWA_\vartheta\left(B_1,B_2,\ldots,B_\psi\right)=\overset{j=1}{\underset{\psi}{\oplus}}\left(\vartheta_jB_j\right)$$

Therefore, we get the associated theorem 1 that relates to AA operations on IVPyFVs.

Theorem 2: Let $B_j=\left(\left[\gamma_{B_j}^L,\gamma_{B_j}^U\right],\left[\delta_{B_j}^L,\delta_{B_j}^U\right]\right)$ $(j=1,2,\ldots,\psi)$ be a set of IVPyFVs, then aggregated value of an IVPyFAAWA operator is also an IVPyFV and given in (6), as shown at the bottom of page 7, where $\vartheta_j=\left(\vartheta_1,\vartheta_2,\ldots,\vartheta_\psi\right),(j=1,2,\ldots,\psi)$ be the set of weight vectors such that $\vartheta_{\mathrm{j}}\in[0,1],\sum_{\mathrm{j}=1}^{\psi}\vartheta_{\mathrm{j}}=1$.

$$\Re(x)=\left[\Re^L(x),\Re^U(x)\right]=\sqrt{\left[1-\left(\left(\gamma_B^L(x)\right)^2+\left(\delta_B^L(x)\right)^2\right),1-\left(\left(\gamma_B^U(x)\right)^2+\left(\delta_B^U(x)^2\right)\right)\right]}.\quad(1)$$

Proof: we prove this theorem by using the mathematical technique $\psi = 2$, we have:

$$\vartheta_1 B_1 = \left(\begin{bmatrix} \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\vartheta_1\left(-ln\delta_{B_1}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\vartheta_1\left(-ln\delta_{B_1}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix} \right),$$

$$\vartheta_2 B_2 = \left(\begin{bmatrix} \sqrt{1-e^{-\left(\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\vartheta_2\left(-ln\delta_{B_2}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\vartheta_2\left(-ln\delta_{B_2}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix} \right)$$

By using definition 9, we get (7), as shown at the bottom of page 8. Hence, (3) is true for $\psi = 2$.

$$B_1 \oplus B_2 = \left(\begin{bmatrix} \sqrt{1-e^{-\left(\left(-ln\left(1-\left(\gamma_{B_1}^L\right)^2\right)\right)^{\Omega}+\left(-ln\left(1-\left(\gamma_{B_2}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\left(-ln\left(1-\left(\gamma_{B_1}^U\right)^2\right)\right)^{\Omega}+\left(-ln\left(1-\left(\gamma_{B_2}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\left(-ln\delta_{B_1}^L\right)^{\Omega}+\left(-ln\delta_{B_2}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\left(-ln\delta_{B_1}^U\right)^{\Omega}+\left(-ln\delta_{B_2}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix} \right) \tag{2}$$

$$B_1 \otimes B_2 = \left(\begin{bmatrix} e^{-\left(\left(-ln\left(\gamma_{B_1}^L\right)\right)^{\Omega}+\left(-ln\left(\gamma_{B_2}^L\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\left(-ln\left(\gamma_{B_1}^U\right)\right)^{\Omega}+\left(-ln\left(\gamma_{B_2}^U\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}, \begin{bmatrix} \sqrt{1-e^{-\left(\left(-\ln\left(1-\left(\delta_{B_1}^L\right)^2\right)\right)^{\Omega}+\left(-ln\left(1-\left(\delta_{B_2}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\left(-ln\left(1-\left(\delta_{B_1}^U\right)^2\right)\right)^{\Omega}+\left(-ln\left(1-\left(\delta_{B_2}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix} \right) \tag{3}$$

$$\begin{aligned} B_1 \oplus B_2 &= \left(\begin{bmatrix} \sqrt{1-e^{-\left(\left(-ln(1-(0.23)^2)\right)^3+\left(-ln(1-(0.43)^2)\right)^3\right)^{\frac{1}{3}}}}, \\ \sqrt{1-e^{-\left(\left(-ln(1-(0.52)^2)\right)^3+\left(-ln(1-(0.53)^2)\right)^3\right)^{\frac{1}{3}}}} \end{bmatrix}, \begin{bmatrix} e^{-((-ln0.45)^3+(-ln0.41)^3)^{\frac{1}{3}}}, \\ e^{-((-ln0.63)^3+(-ln0.49)^3)^{\frac{1}{3}}} \end{bmatrix} \right) \\ &= \left([0.4312, 0.5780], [0.3437, 0.4617]\right) \end{aligned} \tag{4}$$

$$\begin{aligned} B_1 \otimes B_2 &= \left(\begin{bmatrix} e^{-((-ln(0.23))^3+(-ln(0.43))^3)^{\frac{1}{3}}}, \\ e^{-((-ln(0.52))^3+(-ln(0.53))^3)^{\frac{1}{3}}} \end{bmatrix}, \begin{bmatrix} \sqrt{1-e^{-\left(\left(-\ln(1-(0.45)^2)\right)^3+\left(-ln(1-(0.41)^2)\right)^3\right)^{\frac{1}{3}}}}, \\ \sqrt{1-e^{-\left(\left(-ln(1-(0.63)^2)\right)^3+\left(-ln(1-(0.49)^2)\right)^3\right)^{\frac{1}{3}}}} \end{bmatrix} \right) \\ &= \left([0.2107, 0.4439], [0.4794, 0.6420]\right) \end{aligned} \tag{5}$$

(i) Suppose that (3) is correct for $\psi = \theta$, then we have (8), as shown at the bottom of page 9.

Now for $\psi = \theta + 1$, then, we get (9), as shown at the bottom of page 9. Thus, (3) is true for $\psi = \theta + 1$.

Hence theorem is proved for all ψ.

Example 2: Let $B_1 = ([0.31, 0.52], [0.28, 0.42])$, $B_2 = ([0.09, 0.35], [0.43, 0.61])$, $B_3 = ([0.29, 0.34], [0.27, 0.45])$ and $B_4 = ([0.17, 0.19], [0.08, 0.71])$ are four IVPyFVs, with corresponding weight vectors $\vartheta_j = (0.15, 0.35, 0.30, 0.20)^T$. Then IVPyFAAWA operator for $\Omega = 3$ is given as in (10), shown at the bottom of the page 10. Now we will study some properties of the IVPyFAAWA operator by using the basic operations of Aczel Alsina tools.

Theorem 3: If $B_j = \left(\left[\gamma_{B_j}^L, \gamma_{B_j}^U\right], \left[\delta_{B_j}^L, \delta_{B_j}^U\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be the set of all equal IVPyFVs, then IVPyFAAWA is defined as:

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = B$$

Proof: Since $B_j = \left(\left[\gamma_{B_j}^L, \gamma_{B_j}^U\right], \left[\delta_{B_j}^L, \delta_{B_j}^U\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be the set of all same IVPyFVs. Then Equation (6) can be written as:

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(\begin{bmatrix} \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\delta_{B_j}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\delta_{B_j}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} \sqrt{1 - e^{-\left(\left(-ln\left(1-\left(\gamma_B^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1 - e^{-\left(\left(-ln\left(1-\left(\gamma_B^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\left(-ln\delta_B^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\left(-ln\delta_B^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right) = B$$

Thus,

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = B$$

Theorem 4: Let $B_j = \left(\left[\gamma_{B_j}^L, \gamma_{B_j}^U\right], \left[\delta_{B_j}^L, \delta_{B_j}^U\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be the collection of IVPyFVs such that $B^- = min_j\left(B_1, B_2, \ldots, B_{\psi}\right)$ and $B^+ = max_j\left(B_1, B_2, \ldots, B_{\psi}\right)$. Then we have:

$$B^- \leq IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) \leq B^+.$$

Proof: Let $B_j = \left(\left[\gamma_{B_j}^L, \gamma_{B_j}^U\right], \left[\delta_{B_j}^L, \delta_{B_j}^U\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be a set of IVPyFVs such that $B^- = min(B_1, B_2, \ldots, B_{\psi}) = \left(\left[\gamma_B^{L-}, \gamma_B^{U-}\right], \left[\delta_B^{L-}, \delta_B^{U-}\right]\right)$ and $B^+ = max\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(\left[\gamma_B^{L+}, \gamma_B^{U+}\right], \left[\delta_B^{L+}, \delta_B^{U+}\right]\right)$. We have $\gamma_B^{L-} = min_j\left\{\gamma_{B_j}^L\right\}$, $\gamma_B^{U-} = min_j\left\{\gamma_{B_j}^U\right\}$, $\delta_B^{L-} = max_j\left\{\delta_{B_j}^L\right\}$, $\delta_B^{L-} = max_j\left\{\delta_{B_j}^U\right\}$, $\gamma_B^{L+} = max_j\left\{\gamma_{B_j}^L\right\}$, $\gamma_B^{U+} = max_j\left\{\gamma_{B_j}^U\right\}$, $\delta_B^{L+} = min_j\left\{\delta_{B_j}^L\right\}$ and $\delta_B^{L+} = min_j\left\{\delta_{B_j}^U\right\}$. As a result, we get the inequalities as in (11), shown at the bottom of page 11. Therefore

$$B^- \leq IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) \leq B^+.$$

Theorem 5: Consider B_j and B_j' $(j = 1, 2, \ldots, \psi)$ are two sets of IVPyFVs. If $B_j \leq B_j'$, $\forall\,(j = 1, 2, \ldots, \psi)$ then:

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) \leq IVPyFAAWA_{\vartheta} \times \left(B_1', B_2', \ldots, B_{\psi}'\right)$$

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(\begin{bmatrix} \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\delta_{B_j}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\sum_{j=1}^{\psi} \vartheta_j\left(-ln\delta_{B_j}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right) \tag{6}$$

Proof: Consider $B_j = \left(\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right], \left[\delta_{B_j}^{L}, \delta_{B_j}^{U}\right]\right)(j = 1, 2, \ldots, \psi)$ and $B_j' = (\left[\gamma_{B_j}^{L'}, \gamma_{B_j}^{U'}\right], \left[\delta_{B_j}^{L'}, \delta_{B_j}^{U'}\right])(j = 1, 2, \ldots, \psi)$ be two sets of IVPyFVs.

First, we will show $\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right] \leq \left[\gamma_{B_j}^{L'}, \gamma_{B_j}^{U'}\right]$.

$$\left[1-\left(\gamma_{B_j}^{L}\right)^2, 1-\left(\gamma_{B_j}^{U}\right)^2\right] \leq \left[1-\left(\gamma_{B_j}^{L'}\right)^2, 1-\left(\gamma_{B_j}^{U'}\right)^2\right]$$

Then, we get (12), as shown at the bottom of page 12. Hence $\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right] \leq \left[\gamma_{B_j}^{L'}, \gamma_{B_j}^{U'}\right]$.

Similarly, we can show $\left[\delta_{B_j}^{L}, \delta_{B_j}^{U}\right] \leq \left[\delta_{B_j}^{L'}, \delta_{B_j}^{U'}\right]$. Thus from $\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right] \leq \left[\gamma_{B_j}^{L'}, \gamma_{B_j}^{U'}\right]$ and $\left[\delta_{B_j}^{L}, \delta_{B_j}^{U}\right] \leq \left[\delta_{B_j}^{L'}, \delta_{B_j}^{U'}\right]$.
We can conclude:

$$\left(\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right], \left[\delta_{B_j}^{L}, \delta_{B_j}^{U}\right]\right) \leq \left(\left[\gamma_{B_j}^{L'}, \gamma_{B_j}^{U'}\right], \left[\delta_{B_j}^{L'}, \delta_{B_j}^{U'}\right]\right),$$

That is:

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) \leq IVPyFAAWA_{\vartheta} \times \left(B_1', B_2', \ldots, B_{\psi}'\right)$$

$$\begin{aligned}
IVPyFAAWA_{\vartheta}\left(B_1, B_2\right) &= \vartheta_1 B_1 \oplus \vartheta_2 B_2 \\
&= \left(\begin{bmatrix} \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\vartheta_1\left(-ln\left(-\delta_{B_1}^{L}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\vartheta_1\left(-ln\left(-\delta_{B_1}^{U}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right) \\
&\oplus \left(\begin{bmatrix} \sqrt{1-e^{-\left(\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\vartheta_2\left(-ln\delta_{B_2}^{L}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\vartheta_2\left(-ln\delta_{B_2}^{U}\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right) \\
&= \left(\begin{bmatrix} \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^{L}\right)^2\right)\right)^{\Omega}+\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_1\left(-ln\left(1-\left(\gamma_{B_1}^{U}\right)^2\right)\right)^{\Omega}+\vartheta_2\left(-ln\left(1-\left(\gamma_{B_2}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\vartheta_1\left(-ln\left(1-\delta_{B_1}^{L}\right)\right)^{\Omega}+\vartheta_2\left(-ln\left(1-\delta_{B_2}^{U}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\ e^{-\left(\vartheta_1\left(-ln\left(1-\delta_{B_1}^{U}\right)\right)^{\Omega}+\vartheta_2\left(-ln\left(1-\delta_{B_2}^{U}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right) \\
&= \left(\begin{bmatrix} \sqrt{1-e^{-\left(\sum_{j=1}^{2}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\sum_{j=1}^{2}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \end{bmatrix}, \begin{bmatrix} e^{-\left(\sum_{j=1}^{2}\vartheta_j\left(-ln\delta_{B_j}^{L}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{2}\vartheta_j\left(-ln\delta_{B_j}^{U}\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \end{bmatrix}\right)
\end{aligned} \tag{7}$$

Now we will study an IVPyFAA in the form of an order-weighted averaging (IVPyFAAOWA) operator and its basic properties.

Definition 11: Let $B_j = \left(\left[\gamma^L_{B_j}, \gamma^U_{B_j}\right], \left[\delta^L_{B_j}, \delta^U_{B_j}\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be a set of IVPyFVs, with corresponding weight vectors $\vartheta_j = \left(\vartheta_1, \vartheta_2, \ldots, \vartheta_\psi\right), (j = 1, 2, \ldots, \psi)$ such that $\vartheta_{\mathrm{j}} \in [0, 1]$ and $\sum_{\mathrm{j}=1}^{\psi} \vartheta_{\mathrm{j}} = 1$. A ψ-dimensional IVPyFAAOWA operator is a function of an $IVPyFAAOWA : (L^*)^\psi \to L^*$. Then we have:

$$IVIFAAOWA_{\vartheta_j}\left(B_1, B_2, \ldots, B_\psi\right) = \oplus_{j=1}^{\psi}\left(\vartheta_j B_{\gimel(j)}\right)$$

where $(\gimel(1), \gimel(2), \ldots, \gimel(\psi))$ are the permutations of $(j = 1, 2, \ldots, \psi)$, with the property of $B_{\gimel(j-1)} \geq B_{\gimel(j)}, \forall j = 1, 2, \ldots, \psi$.

$$IVPyFAAWA_\vartheta\left(B_1, B_2, \ldots, B_\theta\right) = \overset{\theta}{\underset{j=1}{\oplus}}\left(\vartheta_j B_j\right) = \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\left(1-\left(\gamma^L_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\left(1-\left(\gamma^U_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\delta^L_{B_j}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\delta^U_{B_j}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right). \quad (8)$$

$$\begin{aligned} IVPyFAAWA_\vartheta\left(B_1, B_2, \ldots, B_\theta, B_{\theta+1}\right) &= \overset{\theta}{\underset{j=1}{\oplus}}\left(\vartheta_j B_j\right) \oplus \left(\vartheta_{\theta+1} B_{\theta+1}\right) \\ &= \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\left(1-\left(\gamma^L_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\left(1-\left(\gamma^U_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\delta^L_{B_j}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\theta}\vartheta_j\left(-ln\delta^U_{B_j}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right) \\ &\oplus \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-e^{-\left(\vartheta_{\theta+1}\left(-ln\left(1-\left(\gamma^L_{B_{\theta+1}}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\vartheta_{\theta+1}\left(-ln\left(1-\left(\gamma^U_{B_{\theta+1}}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\vartheta_{\theta+1}\left(-ln\delta^L_{B_{\theta+1}}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\vartheta_{\theta+1}\left(-ln\delta^U_{B_{\theta+1}}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right) \\ &= \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-e^{-\left(\sum_{j=1}^{\theta+1}\vartheta_j\left(-ln\left(1-\left(\gamma^L_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1-e^{-\left(\sum_{j=1}^{\theta+1}\vartheta_j\left(-ln\left(1-\left(\gamma^U_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\sum_{j=1}^{\theta+1}\vartheta_j\left(-ln\left(\delta^L_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\theta+1}\vartheta_j\left(-ln\left(\delta^U_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right) \quad (9)\end{aligned}$$

Theorem 6: Let $B_j = \left(\left[\gamma^L_{B_j}, \gamma^U_{B_j}\right], \left[\delta^L_{B_j}, \delta^U_{B_j}\right]\right)(j = 1, 2, \ldots, \psi)$ be a collection of IVPyFVs, then an IVPyFAAOWA operator is given in (13), as shown at the bottom of page 13, where $(\daleth(1), \daleth(2), \ldots, \daleth(\psi))$ are the permutations of $(j = 1, 2, \ldots, \psi)$, and must be satisfied such property $B_{\daleth(j-1)} \geq B_{\daleth(j)}, \forall j = 1, 2, \ldots, \psi$.

Proof: Proof is straightforward.

Theorem 7: If $B_j = \left(\left[\gamma^L_{B_j}, \gamma^U_{B_j}\right], \left[\delta^L_{B_j}, \delta^U_{B_j}\right]\right)$ $(j = 1, 2, \ldots, \psi)$ are equal, that is $B_j = B$ for all j then

$$IVPyFAAOWA_\vartheta\left(B_1, B_2, \ldots, B_\psi\right) = B.$$

Theorem 8 (Bounded Property): Let $B_j = \left(\left[\gamma^L_{B_j}, \gamma^U_{B_j}\right], \left[\delta^L_{B_j}, \delta^U_{B_j}\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be a collection of IVPyFVs. Let $B^- = min_j\left(B_1, B_2, \ldots, B_\psi\right)$ and $B^+ = max_j$

$$\begin{aligned}
&IVPyFAAWA_\vartheta\left(B_1, B_2, B_3, B_4\right)\\
&= \left(\begin{bmatrix}\sqrt{1 - e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma^L_{B_j}\right)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}},\\ \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma^U_{B_j}\right)^2\right)\right)^\Omega\right)^{\frac{1}{\Omega}}}}\end{bmatrix},
\begin{bmatrix} e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta^L_{B_j}\right)^\Omega\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta^U_{B_j}\right)^\Omega\right)^{\frac{1}{\Omega}}}\end{bmatrix}\right)\\
&= \left(\begin{bmatrix}\sqrt{1 - e^{-\begin{pmatrix}\vartheta_1\left(-ln\left(1-\left(\gamma^L_{B_1}\right)^2\right)\right)^\Omega + \vartheta_2\left(-ln\left(1-\left(\gamma^L_{B_2}\right)^2\right)\right)^\Omega + \\ \vartheta_3\left(-ln\left(1-\left(\gamma^L_{B_3}\right)^2\right)\right)^\Omega + \vartheta_4\left(-ln\left(1-\left(\gamma^L_{B_4}\right)^2\right)\right)^\Omega\end{pmatrix}^{\frac{1}{\Omega}}}},\\
\sqrt{1 - e^{-\begin{pmatrix}\vartheta_1\left(-ln\left(1-\left(\gamma^U_{B_1}\right)^2\right)\right)^\Omega + \vartheta_2\left(-ln\left(1-\left(\gamma^U_{B_2}\right)^2\right)\right)^\Omega + \\ \vartheta_3\left(-ln\left(1-\left(\gamma^U_{B_3}\right)^2\right)\right)^\Omega + \vartheta_4\left(-ln\left(1-\left(\gamma^U_{B_4}\right)^2\right)\right)^\Omega\end{pmatrix}^{\frac{1}{\Omega}}}}\end{bmatrix},\right.\\
&\qquad\left.\begin{bmatrix} e^{-\begin{pmatrix}\vartheta_1\left(-ln\delta^L_{B_1}\right)^\Omega + \vartheta_2\left(-ln\delta^L_{B_2}\right)^\Omega + \\ \vartheta_3\left(-ln\delta^L_{B_3}\right)^\Omega + \vartheta_4\left(-ln\delta^L_{B_4}\right)^\Omega\end{pmatrix}^{\frac{1}{\Omega}}},\\
e^{-\begin{pmatrix}\vartheta_1\left(-ln\delta^U_{B_1}\right)^\Omega + \vartheta_2\left(-ln\delta^U_{B_2}\right)^\Omega + \\ \vartheta_3\left(-ln\delta^U_{B_3}\right)^\Omega + \vartheta_4\left(-ln\delta^U_{B_4}\right)^\Omega\end{pmatrix}^{\frac{1}{\Omega}}}\end{bmatrix}\right)\\
&= \left(\begin{bmatrix}\sqrt{1 - e^{-\begin{pmatrix}(0.15)\left(-ln\left(1-(0.31)^2\right)\right)^3 + (0.35)\left(-ln\left(1-(0.09)^2\right)\right)^3 + \\ (0.30)\left(-ln\left(1-(0.29)^2\right)\right)^3 + (0.20)\left(-ln\left(1-(0.17)^2\right)\right)^3\end{pmatrix}^{\frac{1}{3}}}},\\
\sqrt{1 - e^{-\begin{pmatrix}(0.15)\left(-ln\left(1-(0.52)^2\right)\right)^3 + (0.35)\left(-ln\left(1-(0.35)^2\right)\right)^3 + \\ (0.30)\left(-ln\left(1-(0.34)^2\right)\right)^3 + (0.20)\left(-ln\left(1-(0.19)^2\right)\right)^3\end{pmatrix}^{\frac{1}{3}}}}\end{bmatrix},\right.\\
&\qquad\left.\begin{bmatrix} e^{-\begin{pmatrix}(0.15)\left(-ln(0.28)\right)^3 + (0.35)\left(-ln(0.43)\right)^3 + \\ (0.30)\left(-ln(0.27)\right)^3 + (0.20)\left(-ln(0.08)\right)^3\end{pmatrix}^{\frac{1}{3}}},\\
e^{-\begin{pmatrix}(0.15)\left(-ln(0.42)\right)^3 + (0.35)\left(-ln(0.61)\right)^3 + \\ (0.30)\left(-ln(0.45)\right)^3 + (0.20)\left(-ln(0.71)\right)^3\end{pmatrix}^{\frac{1}{3}}}\end{bmatrix}\right)\\
&= ([0.2624, 0.4081], [0.1939, 0.5116])
\end{aligned} \tag{10}$$

$\left(B_1, B_2, \ldots, B_\psi\right)$. Then,

$$B^{-} \le IVPyFAAOWA_{\vartheta}\left(B_1, B_2, \ldots, B_\psi\right) \le B^{+}.$$

Theorem 9: Let B_j and B_j' $(j = 1, 2, \ldots, \psi)$ be two sets of IVPyFVs, if $B_j \le B_j'$ for all j, then

$$IVPyFAAOWA_{\vartheta}\left(B_1, B_2, \ldots, B_\psi\right) \le IVPyFAAOWA_{\vartheta} \times \left(B_1', B_2', \ldots, B_\psi'\right).$$

Theorem 10: Let B_j and B_j' $(j = 1, 2, \ldots, \psi)$ are two of IVPyFSs, if $B_j \le B_j'$ for all j, then:

$$IVPyFAAOWA_{\vartheta}\left(B_1, B_2, \ldots, B_\psi\right) \le IVPyFAAOWA_{\vartheta} \times \left(B_1', B_2', \ldots, B_\psi'\right)$$

where B_j' $(j = 1, 2, \ldots, \psi)$ be any permutation of B_j $(j = 1, 2, \ldots, \psi)$.

Only IVPyFVs are investigated by the IVPyFAAWA operator, as stated in definition 10, and only the ordered position of IVPyFVs is investigated by the IVPyFAAOWA operator, as defined in definition 11. So, as a result, different components of the IVPyFAAWA and IVPyFAAOWA operators are characterized in different ways. Whatever the case may be, both operations consider only one of them. To solve this problem, we define the IVPyFAAHA operator by considering the IVPyFV and its ordered location.

Definition 12: Consider B_j $(j = 1, 2, \ldots, \psi)$ be the family of IVPyFVs. A ψ-dimensional IVPyFAAHA operator is given as:

$$IVPyFAAHA_{\vartheta, v_j}\left(B_1', B_2', \ldots, B_\psi'\right) = \overset{\psi}{\underset{j=1}{\oplus}}\left(\vartheta_j \dot{B}_{\lambda(j)}\right)$$

where $\vartheta = \left(\vartheta_1, \vartheta_2, \ldots, \vartheta_\psi\right)^T$ be the vectors directly associated with the IVPyFAAHA operator with $\vartheta_j \in [0, 1], \sum_{j=1}^{\psi}\vartheta_j = 1, \dot{B}_j = \psi\vartheta_j B_j, j = 1, 2, \ldots, \psi$, $\left(\dot{B}_{\lambda(1)}, \dot{B}_{\lambda(2)}, \ldots, \dot{B}_{\lambda(\psi)}\right)$ be the set of any permutation of the weighted of IVPyFVs $\left(\dot{B}_{\lambda(1)}, \dot{B}_{\lambda(2)} \ldots, \dot{B}_{\lambda(\psi)}\right)$, so $\dot{B}_{\lambda(j-1)} \ge \dot{B}_{\lambda(j)}$, $(j = 1, 2, \ldots, \psi)$ and $v = (v_1, v_2, \ldots, v_\psi)^T$ represent weight vectors of $B_j (j = 1, 2, \ldots, \psi)$ with $v_j \in [0, 1] (j = 1, 2, \ldots, \psi)$ and $\sum_{j=1}^{\psi} v_j = 1$. Since ψ is the balancing coefficient.

Theorem 11: Suppose that B_j $(j = 1, 2, \ldots, \psi)$ be a collection of IVPyFVs. Then the aggregated value of the IVPyFAAHA operator is given in (14), as shown at the bottom of page 13

Theorem 12: The IVPyFAAWA and IVPyFAAOWA AOs are special cases of the IVPyFAAHA operator.

Proof:

(1) Supposing $v = (1/\varphi, 1/\varphi, \ldots, 1/\varphi)^T$. Then,

$$\begin{aligned}
&IVPyFAAHA_{\vartheta, v}\left(B_1', B_2', \ldots, B_\varphi'\right)\\
&= \overset{\psi}{\underset{j=1}{\oplus}}\left(\vartheta_j \dot{B}_{\lambda(j)}\right) \vartheta_1 \dot{B}_{(1)} \oplus \vartheta_2 \dot{B}_{(2)} \oplus \ldots \oplus \vartheta_\psi \dot{B}_{(\psi)}\\
&= \frac{1}{\varphi}\left(\dot{B}_{\lambda(1)} \oplus \dot{B}_{\lambda(2)} \oplus \ldots \oplus \dot{B}_{\lambda(\varphi)}\right)\\
&= \vartheta_1 \dot{B}_{\lambda(1)} \oplus \vartheta_2 \dot{B}_{\lambda(2)} \oplus \ldots \oplus \vartheta_\psi \dot{B}_{\lambda(\varphi)}\\
&= IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_\varphi\right).
\end{aligned}$$

(2) Suppose $v = (1/\varphi, 1/\varphi, \ldots, 1/\varphi)^T$. Then $\dot{B}_h = B_j$ $(j = 1, 2, \ldots, \psi)$ and

$$\begin{aligned}
&IVPyFAAHA_{\vartheta, v}\left(B_1, B_2, \ldots, B_\varphi\right)\\
&= \vartheta_1 \dot{B}_{\lambda(1)} \oplus \vartheta_2 \dot{B}_{\lambda(2)} \oplus \ldots \oplus \vartheta_\psi \dot{B}_{\lambda(\varphi)}\\
&= \frac{1}{\varphi}\left(\dot{B}_{\lambda(1)} \oplus \dot{B}_{\lambda(2)} \oplus \ldots \oplus \dot{B}_{\lambda(\varphi)}\right)\\
&= \vartheta_1 \dot{B}_{\lambda(1)} \oplus \vartheta_2 \dot{B}_{\lambda(2)} \oplus \ldots \oplus \vartheta_\varphi \dot{B}_{\lambda(\varphi)}\\
&= \boldsymbol{IVPyFAAOWA_{\vartheta}\left(B_1, B_2, \ldots, B_\varphi\right)}
\end{aligned}$$

V. ACZEL ALSINA GEOMETRIC AGGREGATION OEPATROS BASED ON IVPyF INFORMATION

In this section, we will study the geometric aggregation operator based on Aczel Alsina operations under IVPyFVs and its basic properties.

$$\begin{aligned}
&\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{L-}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \le \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\\
&\qquad \le \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{L+}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\\
&\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{U-}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}} \le \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\\
&\qquad \le \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{U+}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\\
&e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{L+}\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \le e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\delta_{B_j}^{L}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \le e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{L-}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\\
&e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{U+}\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \le e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\delta_{B_j}^{U}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}} \le e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{U-}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}
\end{aligned} \tag{11}$$

Definition 13: Let $B_j = \left(\left[\gamma_{B_j}^{L}, \gamma_{B_j}^{U}\right], \left[\delta_{B_j}^{L}, \delta_{B_j}^{U}\right]\right)$ $(j = 1, 2, \ldots, \psi)$ be a collection of IVPyFVs, and corresponding weigh vectors $\vartheta_j = \left(\vartheta_1, \vartheta_2, \ldots, \vartheta_\psi\right)^T$ of $B_j\,(j = 1, 2, \ldots, \psi)$ such that $\vartheta_j \in [0, 1]$ and $\sum_{j=1}^{\psi} \vartheta_j = 1$.

$$
\begin{aligned}
&\left[\begin{array}{l}
\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}, \\
\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}
\end{array}\right] \\
&\quad \leq \left[\begin{array}{l}
\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}, \\
\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}
\end{array}\right] \\
&\left[\begin{array}{l}
e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\
e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}
\end{array}\right] \\
&\quad \leq \left[\begin{array}{l}
e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\
e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}
\end{array}\right] \\
&\left[\begin{array}{l}
1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\
1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}
\end{array}\right] \\
&\quad \leq \left[\begin{array}{l}
1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, \\
1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}
\end{array}\right] \\
&\left[\begin{array}{l}
\sqrt{1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\
\sqrt{1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}
\end{array}\right] \\
&\quad \leq \left[\begin{array}{l}
\sqrt{1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\
\sqrt{1-e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1-\left(\gamma_{B_j}^{U'}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}
\end{array}\right]
\end{aligned}
\tag{12}
$$

Then an IVPyFAAWG operator is particularized as:

$$IVPyFAAWG_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(B_j^{\vartheta_j}\right)$$

Theorem 13: If $B_j = \left(\left[\gamma_{B_j}^L, \gamma_{B_j}^U\right], \left[\delta_{B_j}^L, \delta_{B_j}^U\right]\right)(j = 1, 2, \ldots, \psi)$ be a collection of IVPyFVs, so the IVPyFAAGA operation is also IVPyFVs, which is given by (15), as shown at the bottom of page 13. where the weight vector of $B_j\,(j = 1, 2, \ldots, \psi)$ is denoted by the $\vartheta = \left(\vartheta_1, \vartheta_2, \ldots, \vartheta_{\psi}\right)$ in a way that permits $\vartheta_j \in [0, 1]$ and $\sum_{j=1}^{\psi} \vartheta_j = 1$.

Example 3: Consider $B_1 = ([0.43, 0.52], [0.28, 0.42])$, $B_2 = ([0.09, 0.35], [0.43, 0.61])$, $B_3 = ([0.29, 0.34], [0.27, 0.45])$ and $B_4 = ([0.17, 0.19], [0.08, 0.71])$ are four IVPyFVs, with corresponding weigh vectors $\vartheta_j = (0.15, 0.35, 0.30, 0.20)^T$. Then IVPyFAAWA operator for $\Omega = 3$ is given in (16), as shown at the bottom of the next page.

Remark 2: The IVPyFAAWG aggregation operator is satisfy the basic properties such as idempotency, monotonicity, and boundedness.

VI. ASSESSMENT OF A MADM TECHNIQUE UNDER THE SYSTEM OF IVPyF INFORMATION

In this section, we find out the suitable alternative by using the MADM technique is used to find the best appropriate solution from all possible alternatives that are evaluated against different attributes. Let $\beta_j = \left\{\beta_1, \beta_2, \ldots, \beta_{\psi}\right\}$ and $J = \left\{J_1, J_2, \ldots, J_{\psi}\right\}$ are described as a distinct arrangement of alternative and the set of attributes respectively, and the weight vector of attributes is represented by $\theta = (\theta_1, \theta_2, \ldots, \theta_{\psi})^T$ such that $\sum_{j=1}^{n} \theta_j = 1$ where $\theta_j \in [0, 1]$. Consider the ranking of alternatives $\beta_l\,(l = 1, 2, 3, \ldots, \tau)$ on attributes $J_j\,(j = 1, 2, 3, \ldots, \psi)$ are IVPyFVs in $N_{lj} = \left([\gamma_{B_{lj}}^L, \gamma_{B_{lj}}^U], [\delta_{B_{lj}}^L, \delta_{B_{lj}}^U]\right)(l = 1, 2, 3, \ldots, \tau, j = 1, 2, 3, \ldots, \psi)\left[\gamma_{B_{lj}}^L, \gamma_{B_{lj}}^U\right]$ denote the MD which β_l satisfies the attributes J_j, and $\left[\delta_{B_{lj}}^L, \delta_{B_{lj}}^U\right]$ represents the degree of NMD of an alternative β_l fails to fulfil the attributes J_j and $\left[\gamma_{B_{lj}}^L, \gamma_{B_{lj}}^U\right] \subseteq D[0, 1]$ and $[\delta_{B_{lj}}^L, \delta_{B_{lj}}^U] \subseteq D[0, 1]$ which satisfies the condition $0 \leq \gamma_{B_{lj}}^L + \delta_{B_{lj}}^U \leq 1 (l = 1, 2, 3, \ldots, \tau, j = 1, 2, 3, \ldots, \psi)$. A MADM issue can be represented by the IVPyF decision matrix $R = \left(N_{lj}\right)_{\tau\times\psi}$ and written in as in (17), shown at the bottom of the next page.

Now, we will apply the IVPyFAAWA AO to the MADM problem. The decision-making steps are given below which are followed to handle sufficiently, the mentioned MADM issue with Pythagorean fuzzy information. The decision maker follows the following steps of the algorithm to select a suitable alternative.

A. ALGORITHM

Step 1: First of all, the decision maker collects the information and depicts it in a decision matrix $R = \left(N_{lj}\right)_{\tau\times\psi}$.

Step 2: If types of attributes are two types including benefit and cost type then we have to need to transform given decision matrices into normalised matrices by using the following technique:

$$N_{lj} = \begin{cases} N_{lj} & \textit{for benefit attributes } J_j \\ N_{lj} & \textit{for cost attributes } J_j \end{cases}$$

$$IVPyFAAWA_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(\begin{array}{l}\left[\begin{array}{l}\sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1 - \left(\gamma_{\daleth(j)}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1 - \left(\gamma_{\daleth(j)}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\delta_{\daleth(j)}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\delta_{\daleth(j)}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right), \quad (13)$$

$$IVPyFAAHA_{\vartheta, \nu_j}\left(B'_1, B'_2, \ldots, B'_{\psi}\right) = \left(\begin{array}{l}\left[\begin{array}{l}\sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1 - \left(\dot{\gamma}_{\daleth(j)}^L\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \\ \sqrt{1 - e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\left(1 - \left(\dot{\gamma}_{\daleth(j)}^U\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right], \\ \left[e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\dot{\delta}_{\daleth(j)}^L\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi} \vartheta_j \left(-ln\dot{\delta}_{\daleth(j)}^U\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right) \quad (14)$$

where $(N_{lj})^c$ is complement of N_{lj}, such that $(N_{lj})^c = \left([\delta^L_{B_{lj}}, \delta^U_{B_{lj}}], [\gamma^L_{B_{lj}}, \gamma^U_{B_{lj}}]\right)$. Hence decision matrix converts into normalized IVPyF decision matrix $R = (N_{lj})_{\tau\times\psi}$.

Step 3: Put the values in the IVPyFAAWA and IVPyFAAWG operators for alternatives β_l, we get (18) and (19), shownv at the bottom of the next page.

$$IVPyFAAWG_\vartheta\left(B_1, B_2, \ldots, B_\psi\right) = \left(\begin{array}{l}\left[e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma^L_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma^U_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right], \\ \left[\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta^L_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta^U_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\right]\end{array}\right) \tag{15}$$

$$\begin{aligned} & IVPyFAAWG_\vartheta\left(B_1, B_2, B_3, \ldots, B_n\right) \\ &= \left(\begin{array}{l}\left[e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma^L_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma^U_{B_j}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right], \\ \left[\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta^L_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}, \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta^U_{B_j}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\right]\end{array}\right) \\ & IVPyFAAWG_\vartheta\left(B_1, B_2, B_3, B_4\right) \\ &= \left(\begin{array}{l}\left[\begin{array}{l} e^{-\left(\begin{array}{l}\left((0.15)\left(-ln\,(0.43)\right)^3\right)+\left((0.35)\left(-ln\,(0.09)\right)^3\right)\\ +\left((0.30)\left(-ln\,(0.29)\right)^3\right)+\left((0.20)\left(-ln\,(0.17)\right)^3\right)\end{array}\right)^{\frac{1}{3}}}, \\ e^{-\left(\begin{array}{l}\left((0.15)\left(-ln\,(0.52)\right)^3\right)+\left((0.35)\left(-ln\,(0.35)\right)^3\right)\\ +\left((0.30)\left(-ln(0.34\right)^3\right)+\left((0.20)\left(-ln\,(0.19)\right)^3\right)\end{array}\right)^{\frac{1}{3}}}\end{array}\right], \\ \left[\begin{array}{l}\sqrt{1-e^{-\left(\begin{array}{l}\left((0.15)\left(-ln\left(1-(0.28)^2\right)\right)^3\right)+\left((0.35)\left(-ln\left(1-(0.43)^2\right)\right)^3\right)\\ +\left((0.30)\left(-ln\left(1-(0.27)^2\right)\right)^3\right)+\left((0.20)\left(-ln\left(1-(0.08)^2\right)\right)^3\right)\end{array}\right)^{\frac{1}{3}}}}, \\ \sqrt{1-e^{-\left(\begin{array}{l}\left((0.15)\left(-ln\left(1-(0.42)^2\right)\right)^3\right)+\left((0.35)\left(-ln\left(1-(0.61)^2\right)\right)^3\right)\\ +\left((0.30)\left(-ln\left(1-(0.45)^2\right)\right)^3\right)+\left((0.20)\left(-ln\left(1-(0.20)^2\right)\right)^3\right)\end{array}\right)^{\frac{1}{3}}}}\end{array}\right]\end{array}\right) \\ &= \left([0.4237, 0.5241], [0.4179, 0.4483]\right) \end{aligned} \tag{16}$$

$$R = (N_{lj})_{\tau\times\psi} = \begin{array}{c} \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_\tau \end{array}\left(\begin{array}{cccc} J_1 & J_2 & \cdots & J_\psi \\ \left([\gamma^L_{B_{11}}, \gamma^U_{B_{11}}], [\delta^L_{B_{11}}, \delta^U_{B_{11}}]\right) & \left([\gamma^L_{B_{12}}, \gamma^U_{B_{12}}], [\delta^L_{B_{12}}, \delta^U_{B_{12}}]\right) & \cdots & \left([\gamma^L_{B_{1\psi}}, \gamma^U_{B_{1\psi}}], [\delta^L_{B_{1\psi}}, \delta^U_{B_{1\psi}}]\right) \\ \left([\gamma^L_{B_{21}}, \gamma^U_{B_{21}}], [\delta^L_{B_{21}}, \delta^U_{B_{21}}]\right) & \left([\gamma^L_{B_{22}}, \gamma^U_{B_{22}}], [\delta^L_{B_{22}}, \delta^U_{B_{22}}]\right) & \cdots & \left([\gamma^L_{B_{2\psi}}, \gamma^U_{B_{2\psi}}], [\delta^L_{B_{2\psi}}, \delta^U_{B_{2\psi}}]\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left([\gamma^L_{B_{\tau1}}, \gamma^U_{B_{\tau1}}], [\delta^L_{B_{\tau1}}, \delta^U_{B_{\tau1}}]\right) & \left([\gamma^L_{B_{\tau2}}, \gamma^U_{B_{\tau2}}], [\delta^L_{B_{\tau2}}, \delta^U_{B_{\tau2}}]\right) & \cdots & \left([\gamma^L_{B_{\tau\psi}}, \gamma^U_{B_{\tau\psi}}], [\delta^L_{B_{\tau\psi}}, \delta^U_{B_{\tau\psi}}]\right) \end{array}\right) \tag{17}$$

Step 4: We rearrange all the alternatives in descending order. Finally, choose the suitable alternative which has the maximum overall ranking value.

Step 5. End

B. APPLICATION

The process of conducting research is essential for the development of scientific thinking. There are researcher scientists in a variety of fields, including mathematics, ecology, chemistry, biology, software engineering, medicine, materials science, history of human science, and political science. To find out more about individuals and the shared environment, they speculate, acquire analyze and interpret information. Researchers are employed by a variety of institutions, including universities and colleges, government agencies, nonprofits and private manufacturing and innovation enterprises. Most researchers have master's or doctoral degrees in the field in which they conduct their research. The majority of the researchers are experts in their field after completing postgraduate studies. Master's degrees can usually be sufficient for general employment in the commercial sector, while doctoral degrees are subsequent evaluations for employment as research scientists in colleges and universities. Generally speaking, researchers are inquisitive. Their work necessitates the implementation of a predictable approach as well as careful, caring attention to appreciate achievements precisely. A researcher's ability to talk and write is required if they want to publish their findings in papers and make oral presentations.

C. NUMERICAL EXAMPLE

In this part, we established an example is given to summarize the application of the proposed tactic for researcher scientist selection for a public university. Let's consider a university that is looking to hire a research scientist for available places. After the preliminary selection five candidates β_l (l = 1, 2, 3, 4, 5) were selected for additional testing. The four attributes listed below must be used to define your decision.

a) J_1: Research publications and their impact.
b) J_2: Research experience /teaching experience.
c) J_3: Interview:
d) J_4: Non-university grants, (journal editorial boards).

The committee of selection interviews the candidate and examines some aspects, such as subject knowledge and understanding, ability to understand, oral communication skills, general character/attitude/presentation, knowledge of institute/position, educational background, and so on. Suppose that $\theta = (0.29, 0.23, 0.30, 0.18)^T$ denotes the attribute weight vector. The five applicants β_l $(l = 1, 2, 3, 4, 5)$ are as decision makers in connection with IVPyF data according to four attributes $J_j (j = 1, 2, 3, 4)$, as said in Table 2.

To construct a MADM theory with IVPyF data the IVPyFAAWA operator is used to select the most preferred candidate $\beta_l (l = 1, 2, 3, \ldots, \tau)$, which is frequently used and we carry out the following steps:

Step 1: Consider a decision maker who collects information about various research scientists which is depicted in the following Table 2.

Step 2: We utilized our proposed AOs of IVPyFAAWA and IVPyFAAWG operator to compute information which is depicted in Table 1. The results of our discussed AOs shown in the following Table 2.

Step 3. In this step we compute the score values by using the information of alternatives given by the decision maker

$$IVPyFAAWA_{\theta}\left(N_{l1}, N_{l2}, \ldots, N_{l\psi}\right) = \left(\begin{array}{l}\left[\begin{array}{l}\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}},\\ \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\gamma_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right],\\ \left[e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{L}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\delta_{B_j}^{U}\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right]\end{array}\right) \tag{18}$$

$$IVPyFAAWG_{\vartheta}\left(B_1, B_2, \ldots, B_{\psi}\right) = \left(\begin{array}{l}\left[e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma_{B_j}^{L}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}, e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(\gamma_{B_j}^{U}\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}\right],\\ \left[\begin{array}{l}\sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta_{B_j}^{L}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}},\\ \sqrt{1-e^{-\left(\sum_{j=1}^{\psi}\vartheta_j\left(-ln\left(1-\left(\delta_{B_j}^{U}\right)^2\right)\right)^{\Omega}\right)^{\frac{1}{\Omega}}}}\end{array}\right]\end{array}\right) \tag{19}$$

TABLE 2. Shows a decision matrix has information in the form of IVPyFVs.

	J_1	J_2
β_1	([0.43,0.52], [0.39,0.42])	([0.47,0.53], [0.41,0.45])
β_2	([0.35,0.43], [0.45,0.51])	([0.32,0.42], [0.42,0.51])
β_3	([0.29,0.43], [0.51,0.55])	([0.55,0.56], [0.13,0.31])
β_4	([0.67,0.73], [0.11,0.21])	([0.13,0.22], [0.56,0.64])
β_5	([0.51,0.61], [0.35,0.41])	([0.49,0.51], [0.39,0.46])
	J_3	J_4
β_1	([0.39,0.61], [0.25,0.35])	([0.41,0.45], [0.49,0.51])
β_2	([0.48,0.55], [0.31,0.38])	([0.65,0.68], [0.29,0.32])
β_3	([0.65,0.67], [0.15,0.21])	([0.68,0.71], [0.16,0.25])
β_4	([0.55,0.66], [0.23,0.32])	([0.43,0.48], [0.23,0.39])
β_5	([0.52,0.75], [0.13,0.17])	([0.31,0.36], [0.49,0.59])

TABLE 3. Shows the results of IVPyFAAWA and IVPyFAAWG operators.

IVPyFAAWA
$([0.4298, 0.5546], [0.3386, 0.4100])$
$([0.5305, 0.5707], [0.3539, 0.4121])$
$([0.6122, 0.6366], [0.1769, 0.2807])$
$([0.5886, 0.6601], [0.1876, 0.3020])$
$([0.4952, 0.6676], [0.2312, 0.2832])$
IVPyFAAWG
$([0.4203, 0.5249], [0.413, 0.4427])$
$([0.3939, 0.479], [0.402, 0.4701])$
$([0.4236, 0.5346], [0.4252, 0.4622])$
$([0.2754, 0.3808], [0.4542, 0.5279])$
$([0.4418, 0.5104], [0.4012, 0.4840])$

TABLE 4. Shows the ranking and score values OF IVPyFVs.

	$S(\beta_1)$	$S(\beta_2)$	$S(\beta_3)$	$S(\beta_4)$	$S(\beta_5)$	Ranking and ordering
IVPyFAAWA	0.1048	0.1561	0.3350	0.3279	0.2787	$\beta_3 > \beta_4 > \beta_5 > \beta_2 > \beta_1$
IVPyFAAWG	0.0428	0.0010	0.0354	−0.1321	0.0302	$\beta_1 > \beta_3 > \beta_5 > \beta_2 > \beta_4$

which are depicted in following Table 3 and the results of the score values are shown in following Table 4.

Step 4. Ranking score values of all five candidates β_l $(l = 1, 2, 3, 4, 5)$ using the results of IVPyFAAWA and IVPyFAAWG operators are $\beta_3 > \beta_4 > \beta_5 > \beta_2 > \beta_1$ and $\beta_1 > \beta_3 > \beta_5 > \beta_2 > \beta_4$ respectively.

Step 4. For vacant position β_3 is selected as the most suitable research scientist.

We show the results of score values in a graphical representation in the following Figure 1. These are obtained by the IVPyFAAWA and IVPyFAAWG operators and depicted in above Table 4.

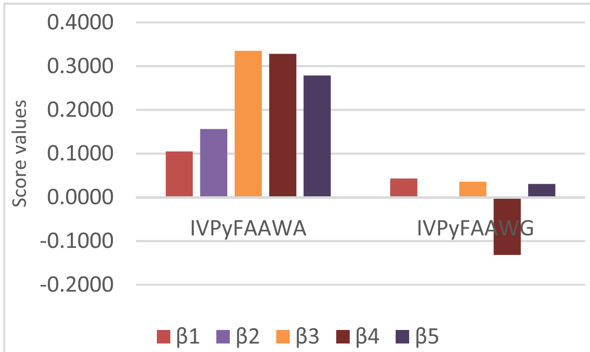

FIGURE 1. Shows the score values of our proposed AOs.

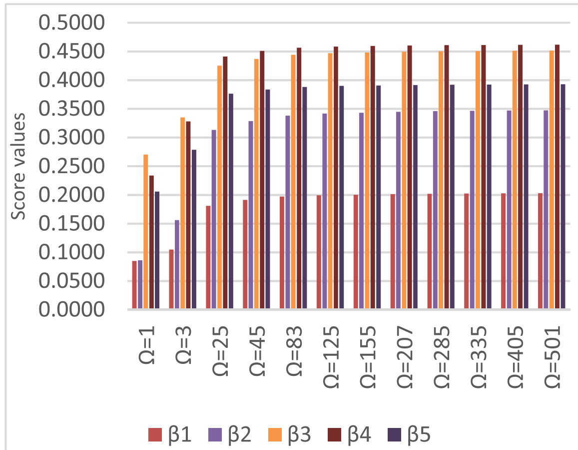

FIGURE 2. Score values of IVPyFAAWA operator for the variation of £.

D. IMPACT OF DIFFERENT PARAMETRIC VALUES ON THE CONSEQUENCES OF OUR PROPOSED METHODOLOGIES

To illustrate the effects of various parameter magnitudes, we characterize the alternatives under our given strategies using various parametric values. Table 5 and Table 6 display the ranking effects of the proposed technique on the IVPyFAAWA and IVPyFAAWG operators respectively. They are also depicted graphically in Figure 2 and Figure 3 respectively. It is noticeable that as the magnitude maximizes then the consequences of the IVPyFAAWA and IVPyFAAWG operator begin to increase and the score values of the different variables are gradually increased but the corresponding ranking does not change, showing the fact that the isotonicity property was a constant in optimization techniques depending on their preferences decision-makers should select the appropriate value. In addition, Figure 2 and Figure 3 displayed that when the values of Ω are altered throughout the example. The results obtained from the options continue to be the same, illustrating the validity of the prescribed IVPyFAAWA and IVPyFAAWG operators.

VII. COMPARATIVE STUDY

In this section, to find the validity and feasibility of our proposed approaches, we contrast our proposed techniques with existing other approaches. The AOs of IVPyFWA

TABLE 5. Ranking and ordering score values by an IVPyFAAWA operator for variation of £.

Ω	$S(\beta_1)$	$S(\beta_2)$	$S(\beta_3)$	$S(\beta_4)$	$S(\beta_5)$	Ranking and Ordering
1	0.0850	0.0859	0.2701	0.2338	0.2058	$\beta_3 > \beta_4 > \beta_5 > \beta_2 > \beta_1$
3	0.1048	0.1561	0.3350	0.3279	0.2787	$\beta_3 > \beta_4 > \beta_5 > \beta_2 > \beta_1$
25	0.1808	0.3131	0.4251	0.4410	0.3763	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
45	0.1911	0.3286	0.4369	0.4507	0.3836	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
83	0.1970	0.3379	0.4441	0.4563	0.3880	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
125	0.1994	0.3417	0.4470	0.4585	0.3899	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
155	0.2003	0.3431	0.4481	0.4593	0.3906	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
207	0.2012	0.3447	0.4493	0.4602	0.3913	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
285	0.2020	0.3459	0.4502	0.4609	0.3919	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
335	0.2023	0.3464	0.4506	0.4612	0.3922	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
407	0.2026	0.3469	0.4510	0.4615	0.3924	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$
501	0.2028	0.3473	0.4513	0.4617	0.3926	$\beta_4 > \beta_3 > \beta_5 > \beta_2 > \beta_1$

TABLE 6. Ranking and ordering score values by an IVPyFAAWG operator for variation of £.

Ω	$S(\beta_1)$	$S(\beta_2)$	$S(\beta_3)$	$S(\beta_4)$	$S(\beta_5)$	Ranking and Ordering
1	0.0669	0.0415	0.1680	0.0602	0.1192	$\beta_3 > \beta_5 > \beta_1 > \beta_4 > \beta_2$
3	0.0428	0.0010	0.0354	−0.1321	0.0302	$\beta_1 > \beta_3 > \beta_5 > \beta_2 > \beta_4$
25	−0.0413	−0.0723	−0.1221	−0.3057	−0.1474	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
45	−0.0548	−0.0801	−0.1331	−0.3161	−0.1625	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
83	−0.0629	−0.0851	−0.1394	−0.3220	−0.1711	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
125	−0.0662	−0.0873	−0.1419	−0.3243	−0.1745	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
155	−0.0675	−0.0882	−0.1428	−0.3252	−0.1758	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
207	−0.0688	−0.0891	−0.1438	−0.3262	−0.1772	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
285	−0.0699	−0.0899	−0.1446	−0.3269	−0.1783	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
335	−0.0704	−0.0902	−0.1450	−0.3272	−0.1787	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
407	−0.0708	−0.0905	−0.1453	−0.3275	−0.1792	$\beta_1 > \beta_2 > \beta_3 > \beta_5 > \beta_4$
501	−0.0712	0.0103	−0.1456	−0.3278	−0.1796	$\beta_2 > \beta_1 > \beta_3 > \beta_5 > \beta_4$

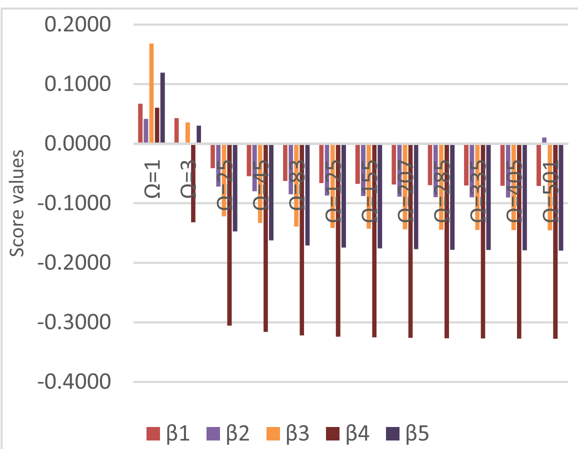

FIGURE 3. Shows the Score values of the IVPyFAAWG operator for the variation of £.

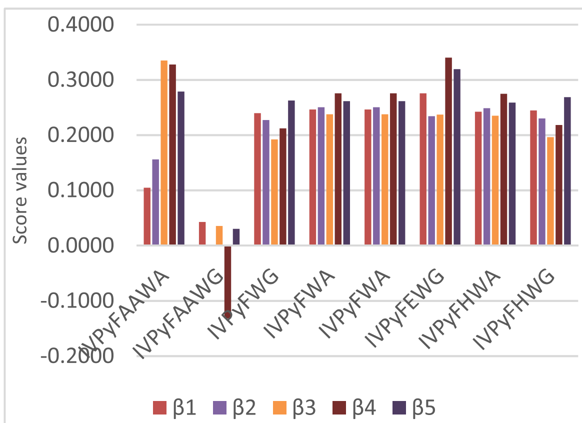

FIGURE 4. Comparison of our proposed work with some existing AOs.

proposed by Wang et al. [60], AOs of IVPyFWG by Xu and Chen [61], AOs of IVPyFWA and AOs of IVPyFWG by Peng and Yang [14], AOs of PyFAAWA and PyFAAWG by the Hussain et al. [54] and AOs of CPDFWAA and CPDFWG Akram et al. [62]. We applied all the above-discussed operators to the given decision matrix shown in Table 1. The consequences of applied AOs shown in the following Table 6. We observe that AOs discussed in [54] and [62] are failed to aggregate information which is displayed in Table 1. Furthermore, results obtained from AOs given by Wang et al. [60], and Xu and Chen [61] are depicted in

TABLE 7. Comparison of our proposed work with some existing approaches.

Operators	Environment	Ranking and ordering
IVPyFAAWA	IVPyFSs	$\beta_3 > \beta_4 > \beta_5 > \beta_2 > \beta_1$
IVPyFAAGA	IVPyFSs	$\beta_1 > \beta_3 > \beta_5 > \beta_2 > \beta_4$
IVPyFWA[60]	IVIFSs	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
IVPyFWG [61]	IVIFSs	$\beta_5 > \beta_4 > \beta_3 > \beta_1 > \beta_2$
IVPyFWA[14]	IVPyFSs	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
IVPyFWG[14]	IVPyFSs	$\beta_5 > \beta_4 > \beta_3 > \beta_1 > \beta_2$
PyFAAWA[54]	PyFSs	Failed
PyFAAWG [54]	PyFSs	Failed
CPDFWAA[62]	CPyFSs	Failed
CPDFWGA [62]	CPyFSs	Failed

Table 6. We observe that the outcomes from applied AOs are near to similar $\beta_5 > \beta_4 > \beta_2 > \beta_1 > \beta_3$. In this comparative study, we observe that our proposed methodology is more flexible and comprehensive than other existing AOs due to significant of log natural and its wider range of parametric values of parameters. We show all results which are shown in the following Table 6 in the following graphical representation in Figure 4.

VIII. CONCLUSION

In a few decades, the MADM technique gained a lot of attention and has a great capacity to solve complex and complicated situations faced. A decision maker faces different challenges due to insufficient information and loss of information during the aggregation process. To handle dubious information and human opinions, several researchers utilized the concepts of the MADM technique. The theory of the MADM approach is utilized in several fields of life including networking, social science, biotechnology, chemistry and computational environments. The main goal of this article is to extend the concepts of PyFSs in the framework of IVPySs and explored their appropriate operations. Recently, Aczel Alsina aggregation models attained a lot of attention from numerous research scholars. By considering the significance of Aczel Alsina aggregation models, we developed a list of certain AOs based on IVPyF information namely, IVPyFAAWA, IVPyFAAOWA, IVPyFAAHA and IVPyFAAWG operators. Some prominent characteristics of our invented approaches are also discussed. To check the effectiveness and flexibility of our proposed AOs, we establish an application with the help of a numerical example for the selection of research scientists. To find the effectiveness and reliability of our proposed methodologies We illustrated a comprehensive comparative analysis to compare the results of our proposed work with the results of existing AOs.

However, our invented methodologies are unable to handle, when information in terms of four memberships values or our invented approach cannot handle the electoral situations. To cope with this situation, we will enlarge our proposed methodology in the framework of picture fuzzy Maclaurin symmetric mean operators [63]. We will explored our invented approaches to solve MADM issues in the framework of statistical models [64]. Furthermore, we will utilize our developed approaches in the framework of bipolar soft set [65].

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ABRAR HUSSAIN received the M.Phil. degree in mathematics from Riphah International University Islamabad (Lahore Campus), Lahore, Punjab, Pakistan, where he is currently pursuing the Ph.D. degree in mathematics with the Department of Mathematics, Riphah Institute of Computing and Applied Sciences, under the supervision of Dr. Kifayat Ullah. He is also with the School Education Department, Pakpattan, Punjab. His research interest includes aggregation operators of extended fuzzy frameworks and their applications under multi-attribute decision-making techniques.

KIFAYAT ULLAH received the bachelor's, master's, and Ph.D. degrees in mathematics from International Islamic University Islamabad, Pakistan, in 2020. He is currently an Assistant Professor with the Department of Mathematics, Riphah International University Islamabad (Lahore Campus), Lahore, Pakistan. He was a Research Fellow with the Department of Data Analysis and Mathematical Modeling, Ghent University, Ghent, Belgium. He has more than 82 international publications to his credit. His research interests include fuzzy aggregation operators, information measures, fuzzy relations, fuzzy graph theory, and soft set theory. He has supervised 19 master's students. Currently, five Ph.D. and seven master's students are under his supervision.

MUHAMMAD MUBASHER received the M.Phil. degree in mathematics from Riphah International University Islamabad (Lahore Campus), Punjab, Pakistan. His research interest includes the aggregation operators of extended fuzzy frameworks and their applications under multi-attribute decision-making techniques.

TAPAN SENAPATI received the B.Sc., M.Sc., and Ph.D. degrees in mathematics from Vidyasagar University, India, in 2006, 2008, and 2013, respectively. He was a Postdoctoral Fellow with the School of Mathematics and Statistics, Southwest University, Chongqing, China. He is currently an Assistant Teacher of mathematics under the Government of West Bengal, India. He has published three books and more than 80 articles in reputed international journals. His research results have been published in Fuzzy Sets and Systems, IEEE Transactions on Fuzzy Systems, *Expert Systems with Applications*, *Applied Soft Computing*, *Engineering Applications of Artificial Intelligence*, *International Journal of Intelligent Systems*, and *International Journal of General Systems*. His main research interests include concentrating on fuzzy sets, fuzzy optimization, soft computing, multi-attribute decision-making, and aggregation operators. He is a reviewer of several international journals and an Academic Editor of *Computational Intelligence and Neuroscience* (SCIE, Q1), *Discrete Dynamics in Nature and Society* (SCIE), and *Mathematical Problems in Engineering* (SCIE). Recently, his has been listed in the ''World's Top 2% Scientists' list'' (Stanford University), from 2020 to 2021. He is the Editor of the book titled *Real Life Applications of Multiple Criteria Decision-Making Techniques in Fuzzy Domain* (Springer).

SARBAST MOSLEM received the B.Sc. and M.Sc. degrees (Hons.) in civil engineering and the Ph.D. degree (Hons.) in transportation and vehicle engineering from the Budapest University of Technology and Economics, Hungary, in 2012, 2015, and 2020, respectively. He currently holds a Postdoctoral Research Fellow position with University College Dublin, Ireland. He has published more than 30 articles in refereed top journals, such as *Applied Soft Computing*, *Expert Systems With Applications*, *European Transport Research Review*, and *Sustainable Cities and Society*, with more than 915 citations. His current research interest includes decision-making methods to solve engineering complex problems.