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RESEARCH ARTICLE

An Omnichannel Retailing Operation for Solving Joint Inventory Replenishment Control and Dynamic Pricing Problems From the Perspective of Customer Experience

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ABSTRACT Customer experience, a key element in the domain of omnichannel retailing, plays a vital role in attracting consumers. Implementing a new BOPS (buy online, pick up in store) strategy will not only affect the inventory structure and pricing strategy of retail enterprises, but will also impact the original sales channel. In accordance with the consumer experience, a dynamic optimization model of a retailer inventory system was established in this research. Based on the optimal control theory, analyze the optimal control strategies under four sales modes: online, offline, BOPS, and BOPS-PLUS (BOPS and buy offline online-logistics distribution). Through numerical experiments, the inventory control strategies and optimal discounted profits under the four modes are compared and analyzed. The results show that the optimal experience investment and dynamic pricing levels gradually decrease with time, and the inventory state has a vital influence on the discounted profit of the system. The traditional dual-channel retail model is optimal when the channel inventory is not out of stock. In situations where channel inventory is in short supply (both online and offline), a deeply integrated omnichannel retail mode BOPS-PLUS is best. The numerical analysis also reveals that price sensitivity coefficient, experience service coefficient and online return rate also have significant influence on the selection of optimal channel strategy.

INDEX TERMS Omnichannel retailing, customer experience, inventory management, optimal control theory.

I. INTRODUCTION

With the rapid expansion of Internet applications and e-commerce, physical channels are gradually being combined with e-commerce channels and mobile e-commerce channels, providing a seamless consumption experience to customers [1]. A large majority of retailers have offered BOPS (buy online, pick up in store) service under omnichannel mode, such as UNIQLO, ZARA and Walmart. As an essential factor affecting consumer satisfaction, consumer experience is superior to the improvement of commodity performance, pricing and advertising in attracting buyers [2]. Simultaneously, a study shows that 79% of retailers have difficulties and problems with omnichannel inventory management [3]. The introduction of omnichannel modes such as BOPS will not only affect the existing inventory structure and pricing strategies of retail enterprises, but implementing new channel strategies will also impact retailers' original sales channels. Therefore, the objective of this study is to establish a set of omnichannel inventory system and pricing decision mechanisms that also take customer experience into account.

BOPS mode enables consumers to experience the performance and quality of goods in physical stores without waiting for product delivery. Meanwhile, picking up goods in stores

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seems to be more flexible than door-to-door delivery [4]. The global spread of COVID-19 has promoted the prosperity of e-commerce industry and put forward new requirements for the close cooperation between traditional and online channels. Due to the fact that omnichannel retail modes are designed to offer a seamless consumption experience to consumers, experience investment plays an indispensable role in omnichannel retail operation. Meanwhile, inventory costs in the omnichannel retail process are also a key factor that cannot be ignored [5]. Playing a vital role in the operation process, inventory is usually used to satisfy upstream orders and downstream demand in modern business activities. A high level of inventory can provide customers with a better experience. However, holding overmuch inventory for a long time may bring the risk of having out-of-season products and increase inventory holding costs significantly. Simultaneously, a low level of inventory may lead to an inventory shortage, resulting in sales profit losses. Losses caused by inventory overhang and inventory shortages will greatly erode the expected profits of an enterprise [6]. The motivation for this research is based on optimal control theory from a long-term perspective to optimize the experience investment, channel replenishment, and dynamic pricing of omnichannel retailing systems. This study aimed to comprehensively explore the best retail mode; traditional dual-channel or omnichannel, and how the channel inventory status affects optimal control strategies.

Recently, the issue of customer experience has become increasingly crucial and has received extensive attention. Due to the fact that a large majority of existing scholars have conducted empirical research on customer experience in omnichannel operation ([2], [7]), this study quantitatively analyzed the influence of customer experience on the optimal control strategy. To close this gap, the current study contributes by conducting dynamic optimization from the perspectives of experience investment and controlling retailing system inventory cost. This study quantitatively analyzes the influence of experience on the optimal control strategy.

Synthesizing the above realistic background and relevant literature, the following specific questions were proposed.

(1) What are the optimal control strategies and discounted profits under four retail modes (online, offline, BOPS, and BOPS-PLUS)?

(2) Which retail mode is the best, traditional dual-channel or omnichannel? What is the relationship between inventory status and optimal channel selection?

(3) What are the effects of price sensitivity coefficient, experience sensitivity coefficient, online channel return rate on optimal decision-making?

To answer the above questions, a dynamic differential equation was constructed by optimal control theory considering experience investment, channel replenishment, and pricing, to optimize the discounted profit of the inventory system. Four distribution modes were considered in this study: online, offline, BOPS, and BOPS-PLUS, which were based on an independent retail enterprise. In each distribution case, the inventory system status (in stock or out of stock) was also considered.

This study pay attention to the inventory replenishment control and dynamic pricing problems under omnichannel retailing. The research limitations are mainly reflected in the omnichannel marketing from the perspective of a retail enterprise, without considering the supply chain scenario. This study explores the BOPS model mainly from the perspective of experience, and does not consider the common showroom behavior in the omnichannel field. What's more, customer experience, as a consumer behavior, will affect consumers' purchasing decisions, which is not studied from the perspective of consumer utility theory.

The general structure is as follows: The second part introduces the literature review of this study. The third part is model hypothesis and representation. The fourth part evaluates online and offline cases. Section V discusses omnichannel modes (BOPS and BOPS-PLUS). Section VI presents numerical examples and sensitivity analysis. The last part is the conclusion and research limitations of this study.

II. LITERATURE REVIEW

To emphasize the contributions, this study reviewed the literature from three perspectives, including the influence of omnichannel retailing, inventory management and customer experience.

A. OMNICHANNEL RETAILING

The existing literature reveals that omnichannel retail originated from dual-channel and cross-channel retail. Sarkar et al. proposed an automatic inspection technology combined with an intelligent production model in the dual-channel sale of smart products [8]. In addition, a classical optimization technique was introduced to assist in obtaining decision variables. Choi et al. proposed an intelligent supply chain strategy (online to offline service) with controllable lead time and demand variability [9]. Two vital indicators, lead-time and backorder are investigated precisely by means of marginal value, which assists in providing optimal services and calculating the exact profits. Sarkar et al. investigated the issue of sustainable managerial decision of substitutable products by means of dual-channels under the carbon tax policy [10]. In accordance with green technology, a dual-channel, emission-controlled variable production system of substitutable products was established. Kar et al. proposed a cap-and-trade mechanism in a sustainable flexible production model to control the carbon emissions rate with advertisement policy and green technology for dual-channel [11]. Dey et al. studied the issue of home delivery policy in retailing, in which retailers provide free transportation services when customers order a certain quantity. The results show that the home delivery strategy based on the order quantity is the best retailing strategy of the dual-channel supply chain [12]. Kang et al. studied how risk-averse suppliers make decisions on financing and pricing via a Stackelberg game model [13].

The results reveal that the green development of supply chain will deteriorate with the increasing risk aversion of suppliers. Kang et al. investigated the effectiveness of cargo transportation insurance by establishing a supply chain interruption model of dual-channel [14]. The final results reveal that the occurrence of transportation interruption can be effectively prevented by cargo transportation insurance in supply chains. Chunming et al. investigated the demands related to sales price and credit term, by means of establishing an online retailing supply chain [15]. In addition, the usefulness of default risk, credit term and sales price on the efficiency of supply chains are also explored. Sarkar et al. proposed a sustainable online-to-offline retailing strategy under variable demand and controllable lead time to optimize the system profit [16]. Bahremand et al. explored the multi-objective pricing issue of omnichannel retail under government intervention by establishing a model [17]. Nie et al. explored the impact of cross-channels on traditional retailers when retailers have competitive relationships, using a multistage game model [18]. Additionally, prior literature has studied the impact of specific behaviors on a certain channel.

With the upgrading of marketing and consumer behavior, cross-channel and multi-channel retailing has transferred to omnichannel retailing. As for omnichannel retail, the issue of retail enterprises implementing omnichannel strategy has been discussed by some scholars from a macro point of view. Xu et al. explored the crucial indicators that affect customer perception, which creates opportunities for enterprises to identify customer requirements [19]. In addition, the topic of omnichannel retail issues have studied from a micro perspective. Singh et al. explained the different responses of customers to advertising strategies of omnichannel and advertising in other channels [20].

In addition, some articles focus in-depth on the BOPS mode. Gao et al. explored the influence of omnichannel and multi-channel sales method on offline stores, and investigated the impact of providing BOPS services on the purchase behavior of consumers [21]. Lin et al. investigated the influence of a BOPS mode on the quality and price strategies in supply chains [22]. The results reveal that both social welfare and consumer surplus will deteriorate with providing extra BOPS channels. In addition, under some simple conditions, central decision-makers can obtain higher profits by adding BOPS channels by means of increasing the quality and price. Xu et al. studied optimal experience investment and pricing strategy in an omnichannel supply chain [23]. To improve shopping experience, the supply chain provides BOPS services. A Stackelberg game model was established in this study to propose whether to choose differential pricing or unified pricing.

According to the literature, the feasibility of omnichannel retailing has been analyzed from both micro and macro perspectives. Omnichannel retail is a business pattern that allow retailers to provide seamless consumption experience to enable consumers to have access to all available channels. However, few of the existing literature examines customer experience from a quantitative perspective. Furthermore, no decision guidance is given to enterprises related to the amount of experience investment needed.

B. INVENTORY MANAGEMENT

The problem of inventory coordination among multiple channels in the omnichannel era is a hot research topic. Some scholars have studied the supply chain inventory control model under random demand when different channels allow out-of-stock substitution. Yang et al. analyzed the pricing and order quantity when inventory is shared and not shared between channels [24]. Lei et al. proposed two heuristic algorithms for joint pricing and order fulfillment optimization [25]. Through a heuristic algorithm with the goal of minimizing transportation cost, Acimovic and Graves studied the optimal shipment adjustment problem of retailers at multiple inventory points [26]. In an omnichannel situation, the integration of multiple channels makes inventory preparation more complicated. Kumar et al. proposed the store participation effect and store return effect, believing that offline channels enable customers to evaluate products in stores and increase their sense of participation, thus leading to an increase in offline purchases [27]. Xu et al. investigated the ordering decisions of nonperishable products [5]. The nonperishable products mentioned in this research refer to goods stored either in self-owned warehouses or in both the self-owned and the leased warehouses. A real inventory case, Heilan Home (HLA), was introduced in this study to indicate that the unit rental cost and the storage capacity of self-owned warehouses have substantial influence on determining whether to use leased warehouses. Kusuda et al. explored the information effect of BOPS mode in store replenishment of omnichannel retailing [28]. The results indicated that an optimal omnichannel strategy affected by several factors, including delivery, hassle cost, and customer behavior. Essentially, this research proposed a relationship between customers and retailers based on a game-theoretical omnichannel structure.

In this literature stream, the research perspective mainly focuses on omnichannel inventory complementarity and coordination. Different from the existing literature, the current study dynamically optimizes the omnichannel inventory system based on the optimal control theory, considering the scenario of online and offline inventory shortage.

C. CUSTOMER EXPERIENCE

According to existing literature, scholars have conducted various researches around the theme of consumer experience. Iyer and Kuksov explored the influence of retailers' advantages on price advertising strategies and balanced pricing considering consumer shopping experience [29]. The results revealed that the impact of experience on retailers' profits depends on shopping experience level. Hong and Han conducted a research to reveal the influence of shopping

TABLE 1. A relevant summary of the existing literature.

Representative papers	Omnichannel retailing	BOPS strategy	customer experience	dynamic optimization	inventory management
Hong et al. (2020)	No	No	Yes	No	No
Bourg et al. (2021)	No	No	Yes	No	No
Fan et al. (2022)	Yes	Yes	No	No	No
Kusuda et al. (2022)	Yes	Yes	No	No	Yes
Jiang et al. (2022)	Yes	Yes	No	No	No
Gao et al. (2022)	Yes	Yes	No	No	Yes
Choi et al. (2022)	No	No	No	Yes	No
Lin et al. (2021)	Yes	Yes	No	No	No
Xu et al. (2021)	No	No	No	Yes	Yes
Xu et al. (2022)	Yes	Yes	Yes	No	No
Our article	Yes	Yes	Yes	Yes	Yes

experience in augmented reality and virtual reality on customer decision making [7]. Under the O2O environment, to keep the brand image of companies via the life cycle of products, manufacturers provide clients with various services [30]. Applying different service strategies in the O2O retail model can obtain about 71% more profits than the traditional supply chain. Fan et al. investigated the issue of optimal service decisions by means of the omnichannel BOPS mode [31]. An analytical model, composed of independent retailers and manufacturers with online channels, was established to analyze the interaction between BOPS strategies and service decisions. Bourg et al. studied smart retailing and proposed a SMARTBUY ecosystem, which is a hybrid retail model combining the appeal of traditional shopping with the benefits of online shopping [2]. This study outlined the functional aspects and the main architectural components of SMARTBUY. Cai and Lo explored the issue of omnichannel retailing in the new shopping era [32]. This review adopted the Citation Network Analysis (CNA) and proposed a framework for All Channel Management (PFOM), which is beneficial to provide management inspiration for enterprises and guidance for future researches.

Thus, to explore the structure and attributes of consumer experience, a large majority of existing studies are empirical research. However, few scholars pay attention to the node enterprises' product operation in omnichannel retailing supply chains.

Table 1 includes references closely related to this issue. The current research will benefit the existing literature from three aspects. To begin with, in terms of omnichannel retail enterprises strategy setting, this study quantifies the investment of consumer experience in operation ([2], [7]). Secondly, combined with the optimal control theory, this study analyzes the optimal replenishment control and dynamic pricing strategies of omnichannel retail enterprises from a long-term perspective. The previous studies are conducted based on a short-term Stackelberg game ([23], [33], [34]). Finally, the BOPS-PLUS mode is deeply integrated. While it is a relatively common retailing mode in practice, BOPS-PLUS is rarely cited in the existing researches ([28], [31]). The purpose of this study is to provide some insights for the decision-making of participants in the omnichannel supply chain enterprise.

III. PROBLEM DEFINITION AND MODEL ASSUMPTIONS

A. PROBLEM DEFINITION

Based on an omnichannel retailer who opens both online and offline channels, this study investigates the optimal control strategies under online, offline, and two omnichannel cases (BOPS and BOPS-PLUS) and analyzes the characteristics of the optimal solutions of decision makers. This study considers the experience investment strategy and constructs a dynamic differential equation model, aiming to maximize the present profits of the inventory system in the whole inventory cycle by solving the optimal dynamic price and replenishment rate. Additionally, it compares the optimal discounted profit level of the four models. As Fig.1 illustrates, omnichannel retailers sell products by integrating online and physical channels.

B. MODEL NOTATIONS

The notations and related definitions are shown in Table 2. For convenience, subscripts "o" and "n" denote online and offline case, respectively. Add "*" to denote the optimal value. In addition, subscripts "I_o" and "I_n" denote online and offline inventory levels, respectively. In addition, "$V_o^*(x)$" and "$V_n^*(y)$" denote online optimal discounted profit and offline optimal discounted profit.

C. MODEL ASSUMPTIONS

- The retail enterprises have both online channels and physical channels, assuming that all consumers are rational and only physical channels need to experience investment [23].
- For the omnichannel cases, to eliminate price discrimination, the online and offline pricing is set to be consistent ([35], [36]), and the relevant exogenous parameters in the model are known.

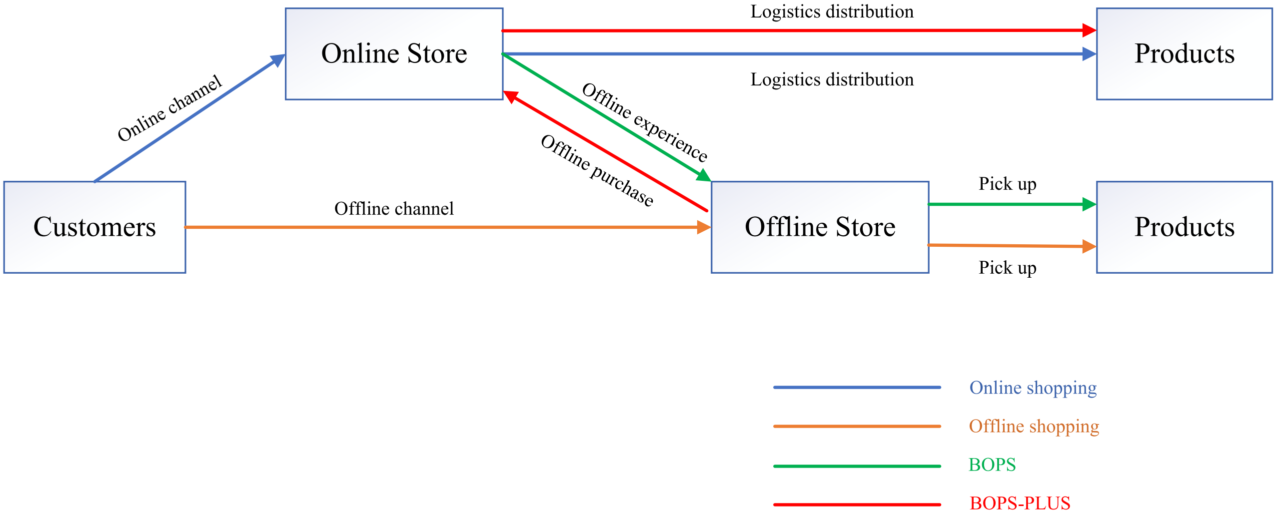


FIGURE 1. The Omnichannel retailing case.

TABLE 2. The notations and descriptions.

Notation	Description
Decision variable	
t	The time point of the inventory status
$\xi(t)$	The experience input level
$\mu_o(t)$	The online replenishment rate
$\mu_n(t)$	The offline replenishment rate
$p_0(t)$	The online retail price
$p_1(t)$	The offline retail price
Parameters	
α	The offline purchase fraction
β_0	The online price coefficient
β_1	The offline price coefficient
m	The customer experience coefficient
k	The experience input coefficient
c	basic cost per unit
h	The holding cost per unit
ρ	The discounted rate
A	The online purchase return fraction
N	The omnichannel customers still pick up by logistics distribution fraction
M	The omnichannel customers who do not pick up after offline experience fraction
P	The omnichannel customers who return products through online channel fraction
Q	The offline customers still insist on selecting products by shopping experience fraction
Outputs	
$I_o(t)$	The online inventory level
$I_n(t)$	The offline inventory level
$V_o(x)$	The online inventory discounted profit
$V_n(y)$	The offline inventory discounted profit

- According to the literature [5], the demand depends on experience service level and price level, and the online and offline demand functions are assumed to be the following: online case demand function: $D_o(t) = (1-\alpha)D(t) - \beta_0 p_0(t)$; offline case demand function: $D_n(t) = \alpha D(t) - \beta_1 p_1(t) - m\xi(t)$.
- Based on the literature [37], the experience investment level is assumed to be $m(\xi) = \frac{1}{2}k\xi(t)^2$.
- The initial online and offline inventory levels are assumed to be $I_o(0) = x_0$ and $I_n(0) = y_0$. The replenishment cost is assumed to be $c\mu_o^2(t)$. The inventory cost is simplified as $h\,(I_n(t))$.
- According to the previous research ([38], [39]), assume the linear programming of the discounted profit inventory system is $V_o(x) = ax + b$.

IV. BASIC CASE

A. ONLINE CASE

In this subsection, an online retailing case is investigated. Let's assume that a few online customers will return goods because of sometimes receive products that do not achieve expectations [23]. For convenience, let's assume that the returned products can continue to be sold by online retailers. Thus, any variation in the inventory level of the online retailers can be attributed to the replenishment level of the online retailers as well as the actual purchase level of the online customers. The rate of change in the inventory level is expressed by:

$$\begin{aligned} &\mathrm{d}I_o(t) = [\mu_o(t) - (1-A)\left((1-\alpha)D(t) - \beta_0 p_0(t)\right)]\,\mathrm{d}t, \\ &I_o(0) = x_0. \end{aligned} \tag{1}$$

The total discounted profit of the system in $[0, +\infty)$ is expressed by:

$$\int_t^{+\infty} e^{-\rho t}\{\underbrace{p_0(t)(1-A)[(1-\alpha)D(t) - \beta_0 p_0(t)]}_{\text{total revenue}}$$

$$- \underbrace{c\mu_o^2(t)}_{\text{replenishment cost}} - \underbrace{h(I_o(t))}_{\text{holding or shortages cost}} \}dt. \tag{2}$$

Therefore, the optimization problems for an online-only case can be written as:

$$\begin{aligned} \max_{\mu_o(\cdot),p_0(\cdot)} \int_0^{+\infty} & e^{-\rho t}\{p_0(t)(1-A)[(1-\alpha)D(t) \\ & - \beta_0 p_0(t)] - c\mu_o^2(t) - h(I_o(t))\}dt, \\ \text{s.t. } & dI_o(t) = [\mu_o(t) - (1-A)((1-\alpha)D(t) \\ & + \beta_0 p_0(t))]dt, \\ & I_o(0) = x_0. \end{aligned} \tag{3}$$

The aim of this research is to maximize the present profit of the inventory system in the whole inventory cycle by solving the optimal dynamic price and replenishment rate. Defining the value function of online retailers in $[0, +\infty)$ by $V_o(x)$, model the Hamilton-Jacobi-Bellman (HJB) equation, satisfied by the value function.

$$\begin{aligned} \rho V_o(x) = \max_{\mu_o(\cdot),p_0(\cdot)} & p_0(t)(1-A)[(1-\alpha)D(t) - \beta_0 p_0(t)] \\ & - c\mu_o^2(t) - h(x) + [\mu_o(t) - (1-A)((1-\alpha)D(t) \\ & + \beta_0 p_0(t))]V_o'(x). \end{aligned} \tag{4}$$

Taking the first-order conditions for right-hand side of equation (4) with respect to $\mu_o(t)$ and $p_o(t)$, obtain:

$$\mu_o^*(t) = \frac{V_o'(x)}{2c} \tag{5}$$

and

$$p_0^*(t) = \frac{(1-\alpha)D(t) + \beta_0 V_o'(x)}{2\beta_0}. \tag{6}$$

From equation (4), easily observe that value function $V_o(x)$ can be linearly formulated as:

$$V_o(x) = ax + b, \tag{7}$$

where a and b are unknown parameters.

Next, the two unknown parameters are determined: a and b. By substituting equation (5) to (7) into equation (4) and equating the parameters of x on both sides of the resulting equation, it leads to:

$$\begin{cases} a = -\frac{h'(x)}{\rho}; \\ b = \frac{(1-A)[\rho(1-\alpha)D(t) - \beta_0 h'(x)]^2}{4\beta_0\rho^2} + \frac{(h'(x))^2}{4c\rho^2}. \end{cases} \tag{8}$$

The optimal theorem can be obtained by substituting equation (8) with equation (5) to (7), correspondingly.

Theorem 1: For an online-only retail channel, the optimal control strategies are gained as follows.

(1) The optimal replenishment rate is:

$$\mu_o^*(t) = \begin{cases} 0, & x > 0, \\ h_1^o/2\rho c, & x \le 0. \end{cases} \tag{9}$$

(2) The optimal retail price is:

$$p_0^*(t) = \begin{cases} \frac{\rho(1-\alpha)D(t) - h_0^o\beta_0}{2\rho\beta_0}, & x > 0, \\ \frac{\rho(1-\alpha)D(t) + h_1^o\beta_0}{2\rho\beta_0}, & x \le 0. \end{cases} \tag{10}$$

(3) The optimal discounted profit of system is:

$$V_o^*(x) = \begin{cases} \frac{(1-A)[\rho(1-\alpha)D(t) + \beta_0 h_0^o]^2}{4\beta_0\rho^3} - \frac{h_0^o x}{\rho}, & x > 0, \\ \frac{(1-A)[\rho(1-\alpha)D(t) - \beta_0 h_1^o]^2}{4\beta_0\rho^3} + \frac{(h_1^o)^2}{4c\rho^3} + \frac{h_1^o x}{\rho}, & x \le 0. \end{cases} \tag{11}$$

Theorem 1 shows that for an online retail system, when the instantaneous inventory level is positive, the optimal replenishment strategy is not to replenish goods in any way; and when the inventory level is negative, the optimal replenishment strategy is to replenish them at a fixed rate. The optimal retail price in the positive inventory case is lower than that in the negative inventory case. The optimal discounted profit is expressed in the form of linear feedback of the instantaneous inventory level.

B. OFFLINE CASE

In this subsection, consider an offline retailing case, different from the online case. To receive the positive effect of customer demand, retailers often attract customers by creating a pleasurable shopping environment. This research considers customer demand rate $D_n(t) = \alpha D(t) - \beta_1 p_1(t) + m\xi(t)$, where $\xi(t)$ denotes the investment level in the shopping experience. Furthermore, to distinguish the online retail case, the returning of goods behavior was not considered here. Thus, the rate of change of the inventory level is given by:

$$\begin{aligned} & \mathrm{d}I_n(t) = (\mu_n(t) - \alpha D(t) + \beta_1 p_1(t) - m\xi(t))\,\mathrm{d}t, \\ & I_n(0) = y_0. \end{aligned} \tag{12}$$

Therefore, the optimization problems for the online-only case can be written as:

$$\begin{aligned} \max_{\mu_n(\cdot),p_n(\cdot),\xi(\cdot)} \int_0^{+\infty} & e^{-\rho t}[p_1(t)(\alpha D(t) - \beta_1 p_1(t) \\ & + m\xi(t)) - c\mu_n^2(t) - h(y) - k\xi^2(t)/2]dt. \\ \text{s.t. } & dI_n(t) = (\mu_n(t) - \alpha D(t) + \beta_1 p_1(t) - m\xi(t))dt, \\ & I_n(0) = y_0. \end{aligned} \tag{13}$$

To gain the optimal control strategy for the offline retail case, model the HJB equation is expressed by the value function $V_n(y)$.

$$\begin{aligned} \rho V_n(y) = \max_{\mu_n(\cdot),p_1(\cdot),\xi(\cdot)} & p_1(t)[\alpha D(t) - \beta_1 p_1(t) + m\xi(t)] \\ & - c\mu_n^2(t) - h(y) - k\xi^2(t)/2 + [\mu_n(t) - \alpha D(t) \\ & + \beta_1 p_1(t) - m\xi(t)]V_n'(y). \end{aligned} \tag{14}$$

Similar to the optimality analysis of the online case, have the following control strategies.

Theorem 2: For the offline retail channel, the optimal control results are gained as follows.

(1) The optimal replenishment rate is:

$$\mu_n^*(t) = \begin{cases} 0, & x > 0, \\ \dfrac{h_1^n}{2\rho c}, & x \le 0. \end{cases} \tag{15}$$

(2) The optimal price is:

$$p_1^*(t) = \begin{cases} \dfrac{k\rho\alpha D(t) - h_0^n(\beta_1 k - m^2)}{\rho(2\beta_1 k - m^2)}, & x > 0, \\ \dfrac{k\rho\alpha D(t) + h_1^n(\beta_1 k - m^2)}{\rho(2\beta_1 k - m^2)}, & x \le 0. \end{cases} \tag{16}$$

(3) The optimal investment level of retailers is:

$$\xi^*(t) = \begin{cases} \dfrac{\rho\alpha D(t) + h_0^n\beta_1}{\rho(2\beta_1 k - m^2)}, & x > 0, \\ \dfrac{\rho\alpha D(t) - h_1^n\beta_1}{\rho(2\beta_1 k - m^2)}, & x \le 0. \end{cases} \tag{17}$$

(4) The optimal discounted profit of inventory system is:

$$V_n^*(y) = \begin{cases} \dfrac{k}{2} \cdot \dfrac{(\alpha\rho D(t) + \beta_1 h_0^n)^2}{(2\beta_1 k - m^2)\rho^3} - \dfrac{h_0^n y}{\rho}, & y > 0, \\ \dfrac{k}{2} \cdot \dfrac{(\alpha\rho D(t) - \beta_1 h_1^n)^2}{(2\beta_1 k - m^2)\rho^3} + \dfrac{(h_1^n)^2}{4c\rho^3} + \dfrac{h_1^n y}{\rho}, & y \le 0. \end{cases} \tag{18}$$

Theorem 2 indicates that for the offline retail system, the optimal price and experience investment level increase with market demand. The experience investment level in the positive inventory case is higher than that in the negative inventory case. The optimal discounted profit function is the linear decreasing function of instantaneous inventory level.

V. OMNICHANNEL CASE

In this section, discuss two omnichannel cases, BOPS and BOPS-PLUS.

A. BOPS CASE

Omnichannel retailer sells products by integrating their online and offline channels at unified dynamic price $p(t)$. In the online retail channel, a fraction of $N \in (0, 1)$ of online customers still pick up goods through logistics distribution, and the return rate is $A \in (0, 1)$. Another fraction, $1 - N$, choose BOPS, in which the pickup is an offline experience. This research refer to the former as "pure online customers", and the latter as "omnichannel customers". The proportion of omnichannel customers who do not pick up offline is $M \in (0, 1)$. While in the offline channel, offline customers select products by shopping in the store, thus the return of goods is not considered in this channel.

Let $\tau_1 = N(1-A)$ and $\tau_2 = (1-N)(1-M)$, then changes of inventory level in the omnichannel situation can be described as:

$$\begin{aligned} &dI_o(t) = (\mu_o(t) - \tau_1 D_o(t))dt, \\ &I_o(0) = x_0; \end{aligned} \tag{19}$$

and

$$\begin{aligned} &dI_n(t) = (\mu_n(t) - D_n(t) - \tau_2 D_o(t))dt, \\ &I_n(0) = y_0. \end{aligned} \tag{20}$$

The total discounted profit of the inventory system in $[0, +\infty)$ is shown by:

$$\begin{aligned} \int_t^{+\infty} e^{-\rho t}\{ & \underbrace{p(t)[D_n(t) + (\tau_1 + \tau_2)D_o(t)]}_{\text{total revenue}} \\ & - \underbrace{c[\mu_o^2(t) + \mu_n^2(t)]}_{\text{replenishment cost}} \\ & - \underbrace{[h(I_o(t)) + h(I_n(t))]}_{\text{holding or shortages cost}} \\ & - \underbrace{k\xi^2(t)/2}_{\text{investment cost in experience}} \}dt. \end{aligned} \tag{21}$$

Therefore, the optimization problems for the omnichannel cases can be written as:

$$\begin{aligned} \max_{\mu_o(\cdot),\mu_n(\cdot),p(\cdot),\xi(\cdot)} & \int_t^{+\infty} e^{-\rho t}\{p(t)[D_n(t) + (\tau_1 + \tau_2)D_o(t)] \\ & - c[\mu_o^2(t) + \mu_n^2(t)] - h(x) - h(y) - k\xi^2(t)/2\}dt. \\ \text{s.t. } & dI_o(t) = (\mu_o(t) - \tau_1 D_o(t))dt, \\ & dI_n(t) = (\mu_n(t) - D_n(t) - \tau_2 D_o(t))dt, \\ & I_o(0) = x_0, I_n(0) = y_0. \end{aligned} \tag{22}$$

The aim of this research is to maximize the present profit of the system in the whole inventory cycle by solving the optimal replenishment rate, dynamic price, and customer experience investment level. Letting $x = I_o(t)$, $y = I_n(t)$, and defining the value function of online retailers in $[0, +\infty)$ by $V_{\text{omni}}(x, y)$, model the Hamilton-Jacobi-Bellman (HJB) equation satisfied by the value function.

$$\begin{aligned} &\rho V_{\text{omni}}(x, y) \\ &\quad = \max_{\mu_o(\cdot),\mu_n(\cdot),p(\cdot),\xi(\cdot)} p(t)[D_n(t) + (\tau_1 + \tau_2)D_o(t)] \\ &\qquad - c[\mu_o^2(t) + \mu_n^2(t)] - h(x) - h(y) - k\xi^2(t)/2 \\ &\qquad + (\mu_o(t) - \tau_1 D_o(t))\frac{\partial V_{\text{omni}}(x, y)}{\partial x} + (\mu_n(t) \\ &\qquad - D_n(t) - \tau_2 D_o(t))\frac{\partial V_{\text{omni}}(x, y)}{\partial y}. \end{aligned} \tag{23}$$

Taking the first-order conditions for the right-hand side of equation (23) with respect to $\mu_o(t)$, $\mu_n(t)$, $p(t)$, and $\xi(t)$, get:

$$\mu_o^*(t) = \frac{\frac{\partial V_{\text{omni}}(x,y)}{\partial x}}{2c}, \tag{24}$$

$$\mu_n^*(t) = \frac{\frac{\partial V_{\text{omni}}(x,y)}{\partial y}}{2c}, \tag{25}$$

$$\xi^*(t) = \frac{m(p(t) - \frac{\partial V_{\text{omni}}(x,y)}{\partial x})}{k}, \tag{26}$$

and

$$p^*(t) = \frac{\alpha D(t) + (\tau_1 + \tau_2)(1-\alpha)D(t) + k\xi(t)}{2\left(\beta_0(\tau_1+\tau_2)+\beta_1\right)} + \frac{\frac{\partial V_{\text{Ominix}}(x,y)}{\partial x}(\beta_0\tau_2+\beta_1) + \frac{\partial V_{\text{Omni}}(x,y)}{\partial y}\beta_0\tau_1}{2\left(\beta_0(\tau_1+\tau_2)+\beta_1\right)}. \tag{27}$$

From equation (23), easily find that value function $V_{\text{omni}}(x, y)$ can be linearly formulated as:

$$V_{\text{omni}}(x, y) = a_1x + a_2y + b_1, \tag{28}$$

where a_1, a_2 and b_1 are unknown parameters.

Similar to the proof procedure in the online retail case, by substituting equation (24) to (28) into equation (23) and equating the parameters of x and y on both sides of the resulting equation, can determine unknown parameters a_1, a_2, and b_1. Furthermore, combining with the value of $h(I(t))$ in different regions, following theorem is derived.

Theorem 3: In the omnichannel retail setting, let $F_1 = \rho[\alpha+(1-\alpha)(\tau_1+\tau_2)], F_2 = \beta_1+(\tau_1+\tau_2)\beta_0, F_3 = \beta_1+\beta_0\tau_2$, $F_4 = \beta_0\tau_1$, and $F_5 = 2k[\beta_1 + (\tau_1 + \tau_2)\beta_0] - m^2$, then the optimal control results are gained as follows.

(1) The optimal online and offline replenishment rate are:

$$\mu_o^*(t) = \begin{cases} 0, & x > 0, \\ h_1/2\rho c, & x \le 0. \end{cases} \tag{29}$$

and

$$\mu_n^*(t) = \begin{cases} 0, & y > 0, \\ h_1/2\rho c, & y \le 0. \end{cases} \tag{30}$$

(2) The optimal retail price is:

$$p^*(t) = \begin{cases} \frac{k[F_1D(t)-h_0F_2]+m^2h_0}{\rho F_5}, & x > 0, y > 0, \\ \frac{k[F_1D(t)-h_0F_3+h_1F_4]+m^2h_0}{\rho F_5}, & x > 0, y \le 0, \\ \frac{k[F_1D(t)+h_1F_3-h_0F_4]-m^2h_1}{\rho F_5}, & x \le 0, y > 0, \\ \frac{k[F_1D(t)+h_1F_2]-m^2h_1}{\rho F_5}, & x \le 0, y \le 0. \end{cases} \tag{31}$$

(3) The optimal investment level of retailers is:

$$\xi^*(t) = \begin{cases} \frac{m[F_1D(t)+h_0F_2]}{\rho F_5}, & x > 0, y > 0, \\ \frac{m[F_1D(t)-h_0F_3+h_1F_4+2h_0F_2]}{\rho F_5}, & x > 0, y \le 0, \\ \frac{m[F_1D(t)+h_1F_3-h_0F_4-2h_1F_2]}{\rho F_5}, & x \le 0, y > 0, \\ \frac{m[F_1D(t)-h_1F_2]}{\rho F_5}, & x \le 0, y \le 0. \end{cases} \tag{32}$$

From Theorem 3, similar to the online and offline situation, the experience investment level and the optimal price steadily grow with the increase of basic market demand, but the omnichannel case is much more complex. For example, when the system has an online inventory shortage but available offline inventory (i.e., $x \le 0, y > 0$), the optimal price and experience investment level will be affected by the holding cost, shortage cost, and experience effect factor, simultaneously.

Theorem 4: In the omnichannel retail environment, let $F_6 = F_1D(t)-h_0F_2, F_7 = F_1D(t)+h_0F_2, F_8 = k[F_1D(t)-h_0F_3+h_1F_4]+m^2h_0, F_9 = F_1D(t)-h_0F_3+h_1F_4+2h_0F_2$, $F_{10} = [\alpha + \tau_2(1-\alpha)]D(t) - \frac{F_3F_8-m^2F_9}{\rho F_5}$, $F_{11} = \tau_1[(1-\alpha)D(t) - \frac{\beta_0F_8}{\rho F_5}]$, $F_{12} = k(mF_9)^2$, $F_{13} = k[F_1D(t) + h_1F_3 - h_0F_4] - m^2h_1$, $F_{14} = F_1D(t) + h_1F_3 - h_0F_4 - 2h_1F_2$, $F_{15} = [\alpha + \tau_2(1-\alpha)]D(t) - \frac{F_3F_{13}-m^2F_{14}}{\rho F_5}$, $F_{16} = \tau_1[(1-\alpha)D(t) - \frac{\beta_0F_{13}}{\rho F_5}]$, $F_{17} = k(mF_{14})^2$, $F_{18} = F_1D(t)+h_1F_2$, and $F_{19} = F_1D(t) - h_1F_2$, then the optimal discounted profit of inventory system is:

$$V_{\text{omni}}(x, y) = \begin{cases} \frac{2k[kF_6+(m^2+F_5)h_0]F_2F_7-k(mF_7)^2}{2\rho^3F_5^2} \\ -\frac{h_0(x+y)}{\rho}, x > 0, y > 0, \\ \frac{2F_8(F_1F_5D(t)-F_2F_8+F_9m^2)-F_{12}}{2\rho^3F_5^2} + \frac{h_1^2}{4c\rho^3} \\ +\frac{h_0(F_{10}-\rho x)+h_1(\rho y-F_{11})}{\rho^2}, x > 0, y \le 0, \\ \frac{2F_{13}(F_1F_5D(t)-F_2F_{13}+F_{14}m^2)-F_{17}}{2\rho^3F_5^2} + \frac{h_1^2}{4c\rho^3} \\ +\frac{h_0(F_{16}-\rho y)+h_1(\rho x-F_{15})}{\rho^2}, x \le 0, y > 0, \\ \frac{2k[kF_{18}-(m^2+F_5)h_1]F_2F_{19}-k(mF_{19})^2}{2\rho^3F_5^2} \\ +\frac{h_1(x+y)}{\rho} + \frac{h_1^2}{2c\rho^3}, x \le 0, y \le 0. \end{cases} \tag{33}$$

From Theorem 4, the optimal present value profit in the omnichannel retail case is the linear feedback form of the offline and online inventory level. Interestingly, under an omnichannel retail environment, regardless of whether the channel is online or offline, the optimal present value profit increases as the out-of-stock inventory decreases; and it decreases as the in-stock inventory level increases.

B. BOPS-PLUS CASE

In this subsection, the BOPS-PLUS omnichannel retailing case is considered further. The online channel retail operation is the same as that in the BOPS case. For the offline retail channel, however, a fraction, $Q \in (0, 1)$, of offline customers still insist on selecting products through in-person shopping experiences. Another fraction, $1 - Q$, of offline customers choose BOPS-PLUS. Similar to the BOPS case, the return of goods is only considered in the online channel due to lack of in-person shopping experience.

Let $\tau_3 = (1 - P)(1 - Q)$ and $\tau_4 = Q$, then changes of inventory level in the BOPS-PLUS situation can be described as:

$$dI_o(t) = (\mu_o(t) - \tau_1D_o(t) - \tau_3D_n(t))dt, \quad I_o(0) = x_0; \tag{34}$$

and

$$dI_n(t) = (\mu_n(t) - \tau_2D_o(t) - \tau_4D_n(t))dt, \quad I_n(0) = y_0; \tag{35}$$

The total discounted profit of the system in $[0, +\infty)$ is shown by:

$$\int_t^{+\infty} e^{-\rho t}\{\underbrace{p(t)[(\tau_1+\tau_2)D_o(t)+(\tau_3+\tau_4)D_n(t)]}_{\text{total revenue}} - \underbrace{c[\mu_o^2(t)+\mu_n^2(t)]}_{\text{replenishment cost}} - \underbrace{[h(I_o(t))+h(I_n(t))]}_{\text{holding or shortages cost}} - \underbrace{k\xi^2(t)/2}_{\text{investment cost in experience}}\}dt. \tag{36}$$

Therefore, the optimization problems for the BOPS-PLUS case can be written as:

$$\begin{aligned} &\max_{\mu_o(\cdot),\mu_n(\cdot),p(\cdot),\xi(\cdot)} \int_t^{+\infty} e^{-\rho t}\{p(t)[(\tau_1+\tau_2)D_o(t)+(\tau_3+\tau_4)\\ &\quad D_n(t)] - c[\mu_o^2(t)+\mu_n^2(t)] - h(x)\\ &\quad - h(y) - k\xi^2(t)/2\}dt.\\ &\text{s.t. } dI_o(t) = (\mu_o(t) - \tau_1 D_o(t) - \tau_3 D_n(t))dt,\\ &\quad dI_n(t) = (\mu_n(t) - \tau_2 D_o(t) - \tau_4 D_n(t))dt,\\ &\quad I_o(0) = x_0, I_n(0) = y_0. \end{aligned} \tag{37}$$

Similar to the solution procedure in the BOPS case, have the following property theorem for the optimal solution to equation (37).

Theorem 5: In BOPS-PLUS setting, let $G_1 = \rho[(1-\alpha)(\tau_1+\tau_2)+\alpha(\tau_3+\tau_4)]$, $G_2 = \beta_0\tau_1+\beta_1\tau_3$, $G_3 = \beta_0\tau_2+\beta_1\tau_4$, $G_4 = \tau_3+\tau_4$, $G_5 = h_1\tau_4 - h_0\tau_3$, $G_6 = h_1\tau_3 - h_0\tau_4$, and $G_7 = 2k(G_2+G_3)-(mG_4)^2$, then the optimal control results are gained as follows.

(1) The optimal replenishment rate are:

$$\mu_0^*(t) = \begin{cases} 0, & x > 0,\\ h_1/2\rho c, & x \le 0. \end{cases} \tag{38}$$

and

$$\mu_1^*(t) = \begin{cases} 0, & y > 0,\\ h_1/2\rho c, & y \le 0. \end{cases} \tag{39}$$

(2) The optimal retail price is:

$$p^*(t) = \begin{cases} \frac{k[G_1D(t)-h_0(G_2+G_3)]+(mG_4)^2h_0}{\rho G_7}, & x > 0, y > 0,\\ \frac{k[G_1D(t)-h_0G_2+h_1G_3]-m^2G_4G_5}{\rho G_7}, & x > 0, y \le 0,\\ \frac{k[G_1D(t)+h_1G_2-h_0G_3]-m^2G_4G_6}{\rho G_7}, & x \le 0, y > 0,\\ \frac{k[G_1D(t)+h_1(G_2+G_3)]-(mG_4)^2h_1}{\rho G_7}, & x \le 0, y \le 0. \end{cases} \tag{40}$$

(3) The optimal investment level of retailers is:

$$\xi^*(t) = \begin{cases} \frac{m[G_1D(t)+h_0G_4(G_2+G_3)]}{\rho G_7}, x > 0, y > 0,\\ \frac{m[G_1D(t)+G_4(h_1G_3-h_0G_2)-2G_5(G_2+G_3)]}{\rho G_7},\\ x > 0, y \le 0,\\ \frac{m[G_1D(t)+G_4(h_1G_2-h_0G_3)-2G_6(G_2+G_3)]}{\rho G_7},\\ x \le 0, y > 0,\\ \frac{m[G_1D(t)-h_0G_4(G_2+G_3)]}{\rho G_7}, x \le 0, y \le 0. \end{cases} \tag{41}$$

Theorem 6: In BOPS-PLUS environment, let $G_8 = k[G_1D(t) - h_0(G_2+G_3)] + (mG_4)^2h_0$, $G_9 = m[G_1D(t) + h_0G_4(G_2+G_3)]$, $G_{10} = k[G_1D(t)-h_0G_2+h_1G_3]-m^2G_4G_5$, $G_{11} = m[G_1D(t) + G_4(h_1G_3 - h_0G_2) - 2G_5(G_2+G_3)]$, $G_{12} = G_1G_7D(t)-(G_2+G_3)G_{10}+mG_4G_{11}$, $G_{13} = [\tau_1(1-\alpha)+\tau_3\alpha]D(t)+\frac{m\tau_3G_{11}-G_2G_{10}}{\rho G_7}$, $G_{14} = [\tau_2(1-\alpha)+\tau_4\alpha]D(t)+\frac{m\tau_3G_{11}-G_3G_{10}}{\rho G_7}$, $G_{15} = k[G_1D(t)+h_1G_2-h_0G_3]-m^2G_4G_6$, $G_{16} = m[G_1D(t) + G_4(h_1G_2 - h_0G_3) - 2G_6(G_2+G_3)]$, $G_{17} = G_1G_7D(t)-(G_2+G_3)G_{15}+mG_4G_{16}$, $G_{18} = [\tau_1(1-\alpha)+\tau_3\alpha]D(t)+\frac{m\tau_3G_{16}-G_2G_{15}}{\rho G_7}$, $G_{19} = [\tau_2(1-\alpha)+\tau_4\alpha]D(t)+\frac{m\tau_3G_{16}-G_3G_{15}}{\rho G_7}$, $G_{20} = k[G_1D(t)+h_1(G_2+G_3)]-(mG_4)^2h_1$, and $G_{21} = m[G_1D(t) - h_0G_4(G_2+G_3)]$, then the optimal discounted profit of inventory system is:

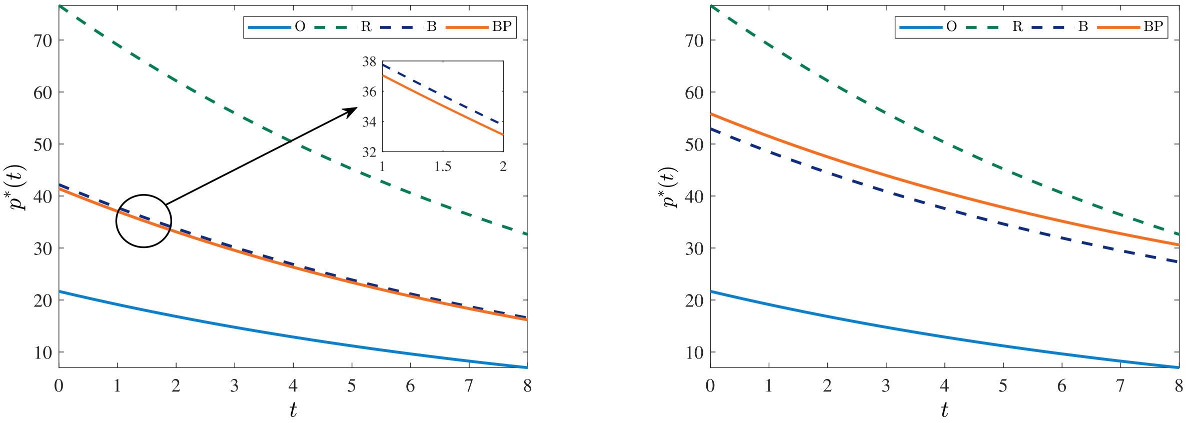
$$V_{\text{omni}}(x,y) = \begin{cases} \frac{2[(G_1G_7D(t)-(G_2+G_3)G_8+mG_4G_9)(G_8+G_7h_0)]}{2\rho^3G_7^2}\\ -\frac{k(\rho G_7G_9)^2}{2\rho^3G_7^2} - \frac{h_0(x+y)}{\rho}, x > 0, y > 0,\\ \frac{2G_{10}G_{12}-k(\rho G_{11})^2}{2\rho^3G_7^2} + \frac{h_0(G_{13}-\rho x)+h_1(\rho y-G_{14})}{\rho^2}\\ +\frac{h_1^2}{4c\rho^3}, x > 0, y \le 0,\\ \frac{2G_{15}G_{17}-k(\rho G_{16})^2}{2\rho^3G_7^2} + \frac{h_0(G_{19}-\rho y)+h_1(\rho x-G_{18})}{\rho^2}\\ +\frac{h_1^2}{4c\rho^3}, x \le 0, y > 0,\\ \frac{2[(G_1G_7D(t)-(G_2+G_3)G_{20}+mG_4G_9)(G_{20}-G_7h_1)]}{2\rho^3G_7^2}\\ -\frac{k(\rho G_7G_{21})^2}{2\rho^3G_7^2}+| \frac{h_1(x+y)}{\rho} + \frac{h_1^2}{2c\rho^3}x \le 0, y \le 0. \end{cases} \tag{42}$$

VI. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

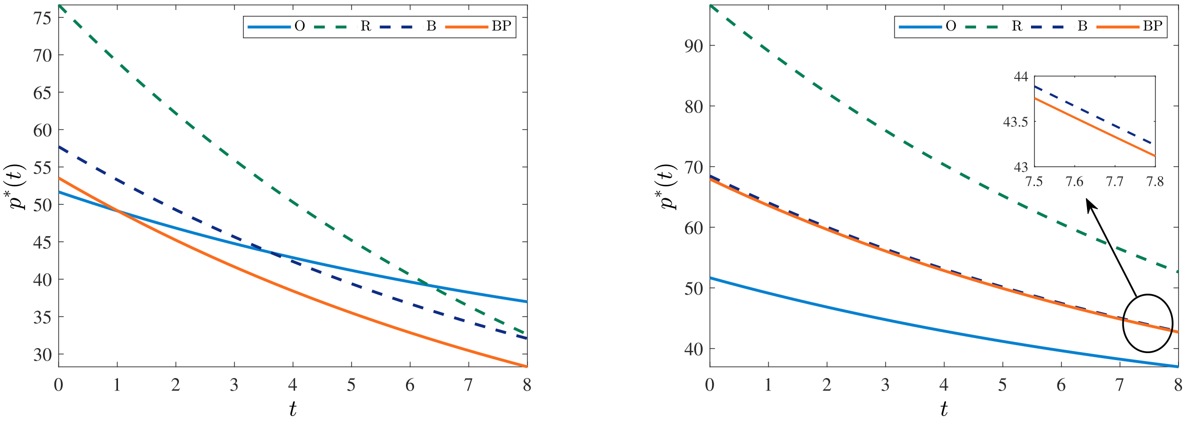
A. NUMERICAL EXAMPLES

In this subsection, the optimal control strategy under the basic and omnichannel cases is further illustrated. Following previous research ([38], [39]), assuming parameter $\rho = 0.1, \alpha = 0.6, \beta_0 = 0.15, \beta_1 = 0.1, m = 0.2, h_o(0) = 1, h_n(0) = 5, c = 0.1, A = 0.2, M = 0.1, N = 0.6, Q = 0.8, P = 0.2, k = 0.8, I_o(0) = 100, I_n(0) = 100$, and market demand follows an exponential distribution, $20 \times \exp(-0.1 \times \text{t})$.

As shown in Fig.2, with the change of time, the optimal dynamic price decreases gradually is observed. In most cases, regardless of whether the inventory system is shortage, the offline, omnichannel and online price stay at high, moderate and low level, respectively. However, over time, if the online inventory is shortage, the online dynamic pricing



(a) $p^*(t) - x > 0, y > 0$ (b) $p^*(t) - x > 0, y < 0$



(c) $p^*(t) - x < 0, y > 0$ (d) $p^*(t) - x < 0, y < 0$

FIGURE 2. The optimal dynamic pricing strategy under different inventory status and retail channels.

becomes higher than the optimal pricing in the other three cases.

Looking at the comparative analysis in Fig.3, as time continues, the optimal level of investment in the experience decreases. Compared to the omnichannel scenario, the offline independent channel experience has the largest investment. When the online inventory system is not shortage, the optimal experience level investment of the more fully integrated omnichannel, BOPS-PLUS, is the lowest, which improves the customer's choice of shopping convenience and reduces the experience investment. From the direction of investment cost, it is advantageous to offline retail. If the physical channel inventory is shortage, the more complete the channel integration is, the higher the optimal experience investment level is.

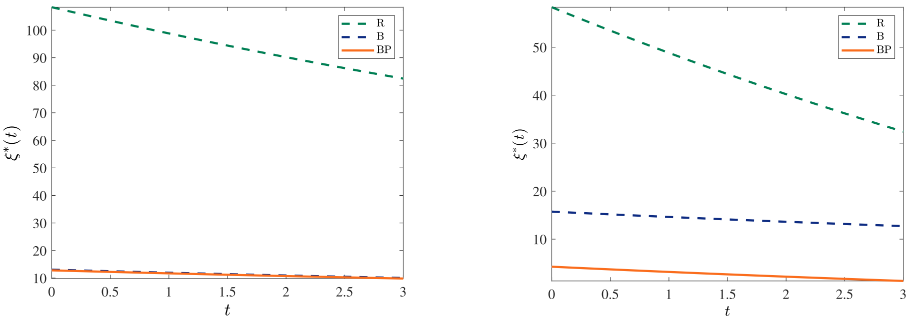
From the four graphs in Fig.4, observe that as time continues, when the inventory system is stocked, the total online and physical profit is higher than that in the omnichannel case; and the more integrated the channel, the greater the channel profit. Once the inventory system is out of stock, no matter whether it is online or offline, the profit advantage of the omnichannel becomes obvious, and it is higher than the integration of the online and offline cases. As time continues, the stronger the integrated channel, the higher the profitability. When the inventory system is not shortage, the cost of the omnichannel mode is relatively large.

Through numerical examples, the optimal control strategy and discount profit under different inventory conditions are compared. It is found that the current status of the inventory system has a significant influence on the optimal experience service level, pricing level and optimal profit. When the inventory system is shortage, the more complete the channel integration, the greater the customer experience investment, the higher the online channel pricing level, and the profit advantage of the omni channel is obvious.

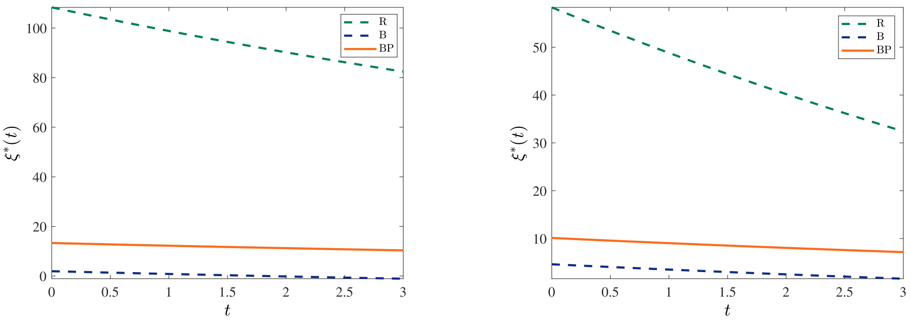
B. SENSITIVITY ANALYSIS

In this part, to evaluate the stability of model and gain management insights, same parameter are adopted, as that in subsection A.

Fig.5 indicates that as online price sensitivity coefficient β_0 increases, the profit level of the online channel, BOPS, and BOPS-PLUS decrease. When the online and offline

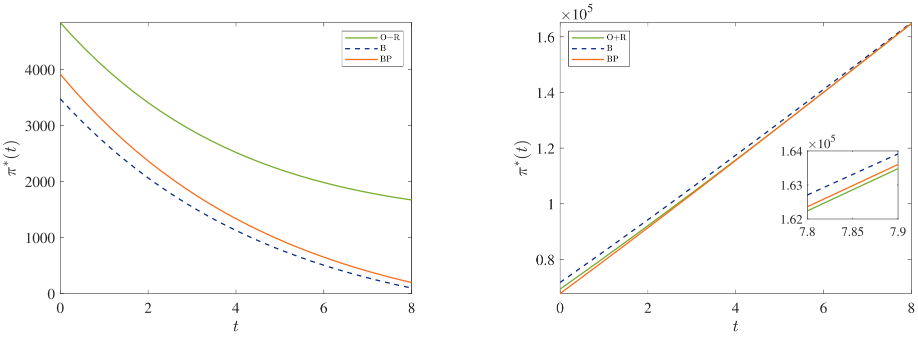


(a) $\xi^*(t) - x > 0, y > 0$ (b) $\xi^*(t) - x > 0, y < 0$

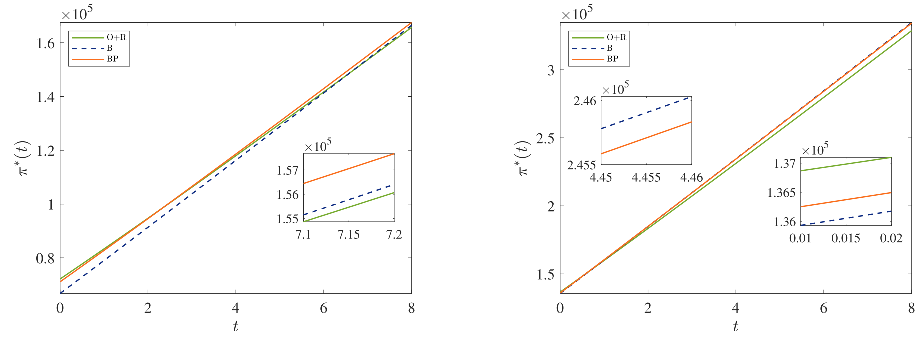


(c) $\xi^*(t) - x < 0, y > 0$ (d) $\xi^*(t) - x < 0, y < 0$

FIGURE 3. **The optimal experience investment strategy under different inventory status and retail channels.**

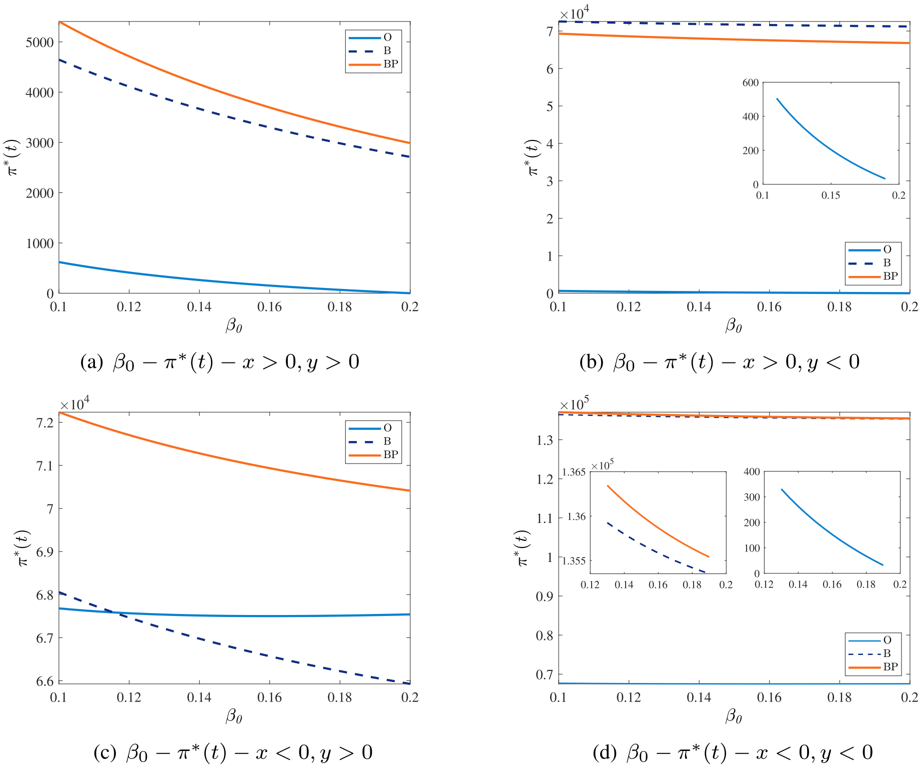


(a) $\pi^*(t) - x > 0, y > 0$ (b) $\pi^*(t) - x > 0, y < 0$



(c) $\pi^*(t) - x < 0, y > 0$ (d) $\pi^*(t) - x < 0, y < 0$

FIGURE 4. **The optimal discounted profit under different inventory status and retail channels.**



(a) $\beta_0 - \pi^*(t) - x > 0, y > 0$

(b) $\beta_0 - \pi^*(t) - x > 0, y < 0$

(c) $\beta_0 - \pi^*(t) - x < 0, y > 0$

(d) $\beta_0 - \pi^*(t) - x < 0, y < 0$

FIGURE 5. **The impact of online price sensitivity coefficient on the channel profit.**

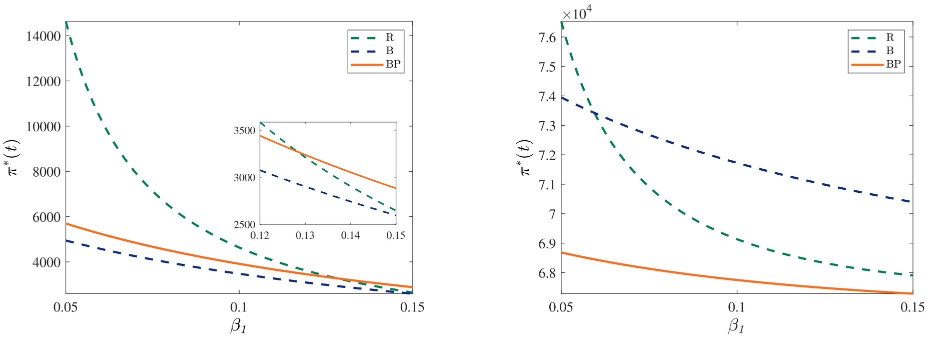
inventory system is not out of stock, the more sufficient the channel integration, the greater the channel profit. When online and offline inventory is out of stock at the same time, the discount profit of the BOPS mode is still the highest. With the increase of sensitivity coefficient β_0, the profit level of the online independent channel exceeds that of the omnichannel when online channel is shortage and offline channel is in stock.

The result is shown in Fig.6, with the increase of physical price coefficient β_1, the optimal online channel, BOPS, and BOPS-PLUS profit levels decrease. When the online and offline inventory is in-stock, sensitivity coefficient β_1 has a great influence on the profit of the offline channel. When $\beta_1 = 0.13$, the profit of the BOPS-PLUS exceeds that of the offline channel. When there is a shortage of offline inventory, the profit of the offline channel falls between BOPS and BOPS-PLUS. When the online channel inventory is shortage, the BOPS case has obvious profit advantages compared with the offline channel, and the more fully integrated the channel, the higher the profit level.

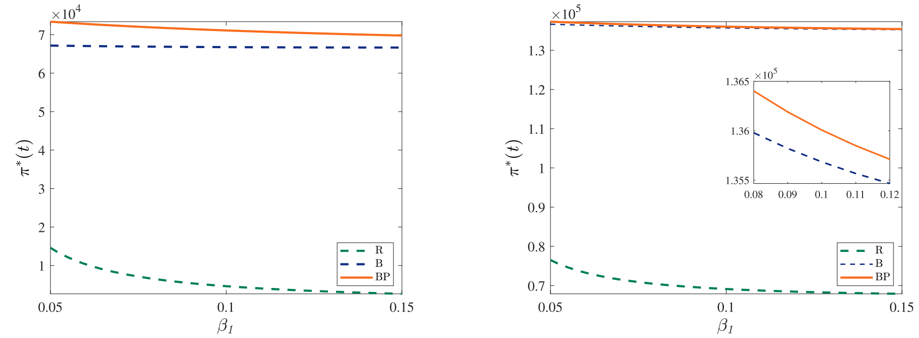
Fig.7 suggests that with an increase in experience sensitive coefficient k, the optimal experience input level gradually decreases. In addition, the offline experience investment level is greater than that in the BOPS case. As the k experience coefficient changes, the optimal offline experience investment level continues to be influenced by online inventory status; when there is an online inventory shortage, the fuller the channel integration, the lower the experience input level. When the physical inventory is shortage, the fuller the channel integration, the higher the optimal experience investment level.

In Fig.8, observe that as experience sensitivity coefficient k increases, the optimal channel profit level gradually decreases; and as experience factor k changes, the more the optimal profit is affected by the inventory status online and offline. When both channel inventories are not out of stock, the basic offline channel has a more obvious profit advantage than the omnichannel case. In the case of online inventory shortage, the omnichannel case has obvious advantages, and the more fully integrated the channels, the greater the profit advantage.

As shown in Fig.9, when the inventory system is not shortage, with the increase of online return rate A, the channel profit shows a downward trend; and the more fully integrated the channel, the higher the profit. The profit is the lowest in the online case. When the offline inventory is short, the profit of is greater in the omnichannel case than the integrated omnichannel mode, and the profit of the online channel is the lowest. When the offline inventory is insufficient, it can be seen by comparing the profits of the three models that the online model has the lowest profit level, while the omni-channel model has higher profits than the comprehensive omni channel model. When the online inventory is insufficient, the profit level of comprehensive omnichannel cases is the highest, the profit level of omnichannel cases is the lowest, and the profit

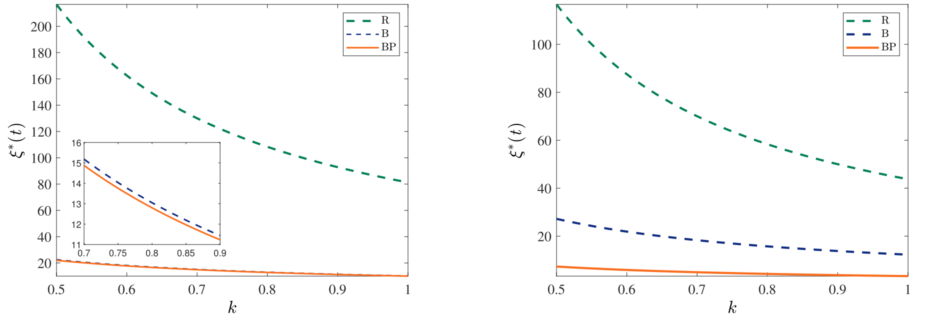


(a) $\beta_1 - \pi^*(t) - x > 0, y > 0$ (b) $\beta_1 - \pi^*(t) - x > 0, y < 0$

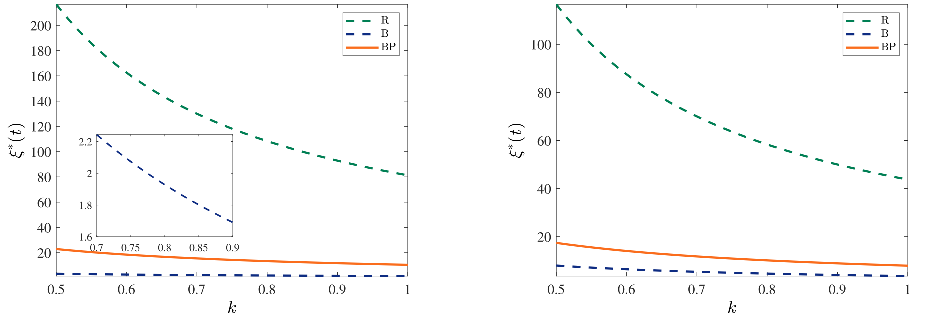


(c) $\beta_1 - \pi^*(t) - x < 0, y > 0$ (d) $\beta_1 - \pi^*(t) - x < 0, y < 0$

FIGURE 6. **The impact of offline price sensitivity coefficient on the channel profit.**



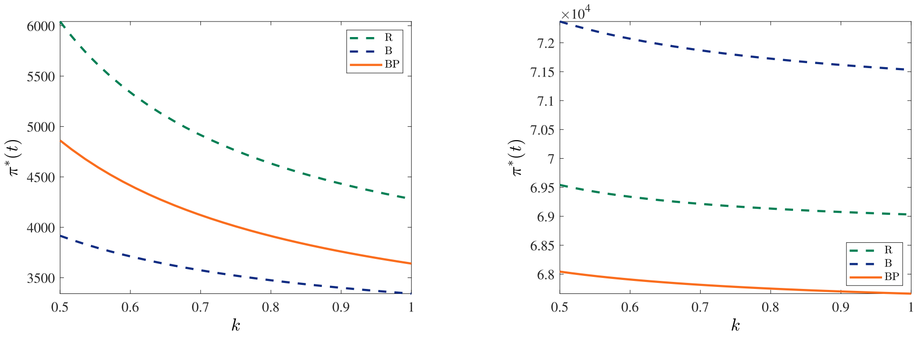
(a) $k - \xi^*(t) - x > 0, y > 0$ (b) $k - \xi^*(t) - x > 0, y < 0$



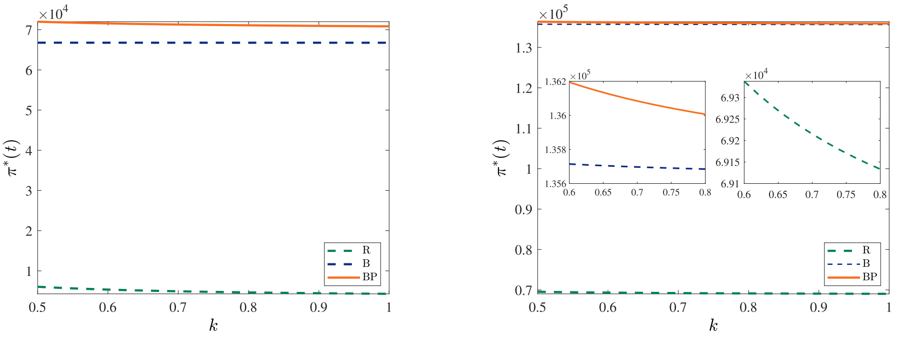
(c) $k - \xi^*(t) - x < 0, y > 0$ (d) $k - \xi^*(t) - x < 0, y < 0$

FIGURE 7. **The impact of experience sensitivity coefficient on the optimal experience input level.**

level of online channels is medium. When both offline and online channels are out of stock, channel profits show a downward trend with the increasing of online return rate A. In addition, the profit level of omnichannel is higher than that

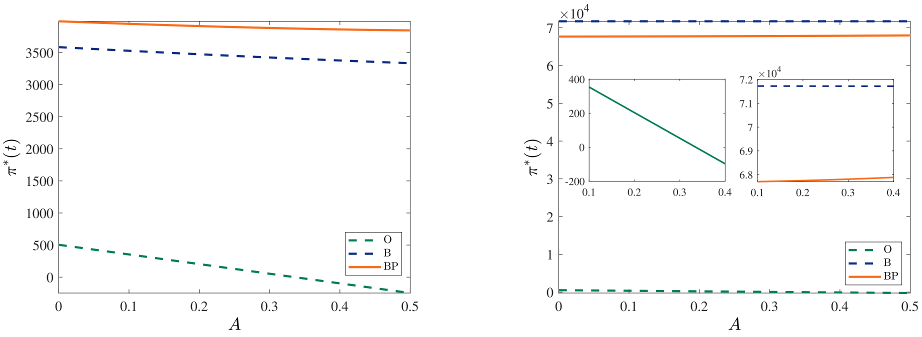


(a) $k - \pi^*(t) - x > 0, y > 0$ (b) $k - \pi^*(t) - x > 0, y < 0$

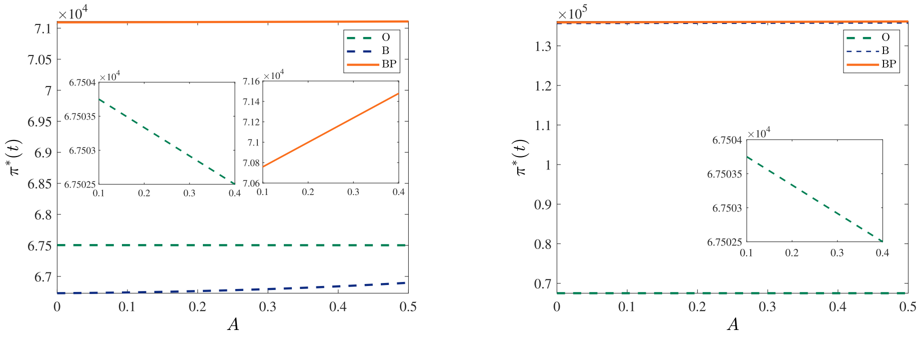


(c) $k - \pi^*(t) - x < 0, y > 0$ (d) $k - \pi^*(t) - x < 0, y < 0$

FIGURE 8. The impact of experience sensitivity coefficient on the channel profit.



(a) $A - \pi^*(t) - x > 0, y > 0$ (b) $A - \pi^*(t) - x > 0, y < 0$



(c) $A - \pi^*(t) - x < 0, y > 0$ (d) $A - \pi^*(t) - x < 0, y < 0$

FIGURE 9. The impact of online return rate on the channel profit.

of comprehensive omni-channel, and the profit level of online channel is the lowest.

C. A REAL INVENTORY CASE

WUMART is one of the well-known supermarket with more than 791 stores, which was founded in 1994 in Beijing, China. WUMART Group adheres to the strategy of integrating digitalization; online and offline. It builds a digital supply chain with fresh as the core, and provides standardized goods and services through multiple channels, including B2B business led by food service distribution, rapid growth of APP online sales to home business, and member stores and community supermarkets dominated by its own brands.

To verify the usability of the inventory model in practice, this research investigated a WUMART supermarket located in Tianjin, China. After contacting the staff of WUMART supermarket, a field visit was conducted and it was found that the omnichannel retail mode is extremely suitable for the integration of online and physical channels of this retail store. Especially when there is a shortage of channel inventory, consumers are more inclined to choose the "buy online, and pick up in store" mode. Over time, the profitability of omnichannel retail has stabilized within a range. Therefore, the inventory cost control model under the omnichannel retail mode is suitable for such large retailers.

In accordance with interviews of WUMART supermarket staffs, the following retail inventory operation data has been obtained. According to consumer needs and sales data, WUMART managers made a rough estimation. The segmentation function of customer demand rate is shown below.

$$D(t)=\begin{cases}20+3\,t, & 0\le t\le 2,\\ 26, & 2\le t\le 6,\\ 56-5\,t, & 6\le t\le 8.\end{cases}$$

In a retail scenario where the online and offline initial inventory of an individual product are both 26, how can managers formulate the best replenishment strategy to regulate the retail inventory of WUMART stores?

According to the model provided in this study and its solution procedures, have $\pi^*(t) = 1250$ Chinese Yuan(CNY) and $\pi^*_{\mathrm{Omni}}(t) = 1150$ CNY, where $x > 0, y > 0$. $\pi^*(t) = 1620$ CNY and $\pi^*_{\mathrm{Omni}}(t) = 1660$ CNY, where $x > 0, y < 0$. Therefore, it is a relatively optimal choice to adopt the omnichannel strategy when the inventory system exceeds expectations for WUMART managers. In addition, the results of inventory observations can be easily obtained, similar to subsection A.

VII. CONCLUSION AND LIMITATIONS

In this study, from the perspective of experience combined with optimal control theory, the experience investment level, replenishment rate, and retail price of omnichannel systems are dynamically optimized through inventory cost control.

A. PRIMARY CONCLUSION

Based on the omnichannel retailers that open offline and online channels simultaneously, this study explored the optimal decision-making behavior of online cases, offline cases and two omnichannel retail modes (BOPS and BOPS-PLUS). The characteristics of the optimal control strategy were also analyzed. The optimal discounted profit of the four cases was compared. The detailed conclusions are as follows.

The control strategy of optimal experience investment and pricing is affected by channel inventory level. The optimal experience investment gradually decreases with time. Compared with the omnichannel situation, the offline channel experience investment is the largest. When the online inventory is not out of stock, the best experience investment of the more fully integrated BOPS-PLUS is the lowest, which will improve the customer's choice of shopping convenience while also reducing the experience investment. From the scale of investment, this is advantageous to the offline retail channel. When there is an online inventory shortage, the BOPS-PLUS experience input is higher than BOPS; the fuller the channel integration, the higher the experience input level. The optimal price gradually decreases and changes slowly over time. In most cases, the offline optimal price is at a higher level, regardless of stock status. The omnichannel price is at an intermediate level, and the online price is the lowest. However, when the online inventory is shortage, the online dynamic pricing is higher than the optimal pricing of the other three states. When both online and offline inventory systems have stock, it is not certain that the omnichannel model would be profitable.

When the omnichannel inventory system has no online or offline stock shortage, the omnichannel retail model is not necessarily profitable. When the omnichannel inventory system is in a state of stock shortage, the advantages of the omnichannel retail model gradually appear, and the more fully integrated the channel, the higher the system profit level. For the omnichannel retail model, regardless of online and offline inventory status, the integrated omnichannel is always more profitable than the unintegrated omnichannel model. When the omnichannel inventory system has no stock shortage, the investment cost of omnichannel creation is relatively large. When the inventory system has a stock shortage, the profit advantage of omnichannel mode is reflected.

With the increase of offline price coefficient, the profits of offline and omnichannel modes are reduced. Regardless of whether the online and offline inventory systems are out of stock, the more fully integrated the channel, the greater the overall profit. As the offline price coefficient increases, the profit of the offline channel and omnichannel modes decrease. When the online and offline inventory system has no shortage, the more fully integrated the channel is, the greater the overall profit of the channel; and the sensitivity coefficient has a greater influence on the profit of the physical channel. As the experience sensitivity coefficient increases, the optimal profit level gradually decreases, and the offline experience sensitivity coefficient is negatively correlated with the retailer's optimal profit. For the case in which the inventory is not shortage, the profit shows a downward trend as the online return rate increases. For cases in which both online and offline are out of stock, the profit shows a downward trend as the online return rate increases. The omnichannel profit level is the highest, while the online channel profit level is the lowest.

B. MANAGEMENT INSIGHTS

This work can provide implications for the practice of omnichannel retail enterprise operation optimization. The specific management implications are as follows.

The optimal price and experience investment strategies of omnichannel retailers are closely related to the channel inventory level. In most cases, the physical price is at a high level, the online price is at a low level, and the omnichannel price is at an intermediate level. Managers of enterprises that have opened omnichannel retail operations can adjust pricing strategies in real time according to the inventory level of the online and physical channels. The best experience investment level under the omnichannel mode is lower than that under the offline independent operation. When the inventory system is not shortage, the experience investment of the more fully integrated omnichannel mode is the lowest, which reflects the ''experience-centric'' concept of omnichannel operation. This also indicates that the integration of channels will not only improve the convenience of shopping but also reduce the input of experience cost. From the perspective of investment cost, it is meaningful to integrate offline channels into an omnichannel system.

The optimal profit of omnichannel retail enterprises is related to the inventory level. If the inventory system is shortage, the omnichannel model will be superior to the integration of traditional and online channels. However, facing a situation of inventory shortage, in most healthy and mature markets, the profit advantage of an omnichannel system is obviously higher than integrated online and offline channels. Over time, the stronger the integration ability, the higher the profitability level. This also reflects that when the inventory system is not shortage, the cost of carrying out the omnichannel mode is higher; and when the inventory system is shortage, the omnichannel mode shows a profit advantage. Thus, the omnichannel system will have a wide range of application prospects.

The impact of online and offline price coefficients, experience factors, and return rate on optimal profit is also affected by inventory status. When online and offline channels are not shortage, the cost of omnichannel integration will be higher and the profit will be lower. When the inventory system is shortage, cross-channel experience still accounts for a large proportion of consumers' purchasing factors, thus the advantage of the omnichannel mode starts to appear again, and the profit level exceeds that of independent operation. Therefore, when the inventory system is shortage, it can play the advantage of the omnichannel mode by adjusting the product inventory between online and physical channels to realize a seamless link between online and physical channels. As UNIQLO, IKEA, and other open omnichannel retailers, the greater the flow of products across channels, the higher the omnichannel input costs, including offline experiences and channel integration costs. As the channels integrate more fully, the convenience effect and information effect increase simultaneously, and thus the corresponding experience will cost more. This can affect channel pricing, replenishment, experience investment levels, and other control strategies.

C. LIMITATION AND FUTURE RESEARCH

In this study, investigated the impact of consumer experience on inventory optimization and dynamic pricing on two omnichannel cases, BOPS and BOPS-PLUS. However, it did not examine the impact of the behavior preferences of customers or the impact of a new omnichannel mode. First, customer experience, as a type of consumer behavior, will affect consumers' purchasing decisions, which is not studied from the perspective of utility theory [40]. Second, the research object of this study was a retail enterprise with online and pyhsical channels. The scenario of omnichannel supply chains is the development direction of future research [31]. In addition, this study only considered BOPS and BOPS-PLUS modes in omnichannel retailing, but other modes, such as showrooming was not considered in this research [41].

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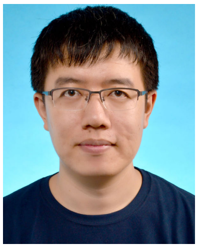

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