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RESEARCH ARTICLE

Codebook Design for PD-SCMA NOMA Systems

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ABSTRACT The hybrid power domain sparse code non-orthogonal multiple access (PD-SCMA) scheme multiplexes multiple users on a single resource element (RE) both in power and code domains, thus achieving a superior spectral efficiency. The multiplexing performance of such a scheme can be enhanced not only through deployment of sophisticated resource allocation schemes and low complexity robust multi-user detectors but also prudent multi-user multi-dimensional codebook design. By considering the key codebook design performance indicators (KPIs) in different channel use scenarios, this work assesses various known multi-dimensional constellation (MDC) design criterion on the performance of low-rate turbo-coded PD-SCMA NOMA system. In order to strike a balance with regard to good multi-dimensional modulation (MDM) and low complex detection, a low complex codebook design based on linear error correcting block codes is proposed. Unique user-specific rotation design based on quaternion rotation matrices is proposed with the aim of maintaining users symbol uniqueness for overloaded systems. Simulation results confirm that MDCs that satisfy design KPIs outperform others. Besides, the proposed user-specific rotation design remarkably enhances the BER performance of the block code based codebook design.

INDEX TERMS PD-SCMA, key performance indicators (KPIs), multi-dimensional constellation, codebook design, user-specific rotation.

I. INTRODUCTION

The impact of non-orthogonal multiple access (NOMA) technologies, particularly on facilitating the 5G requirements of massive connectivity, ultra-high capacity, high data rates, ultra-low latency and robust interference management has been immensely appreciated. NOMA multiplexes multiple users on a single resource element (RE) in power or code domains or their integration [1]. One possible integration is the hybrid power domain sparse code multiple access (PD-SCMA) scheme whose feasibility and performance are well investigated in [2], [3], [4], and [5]. PD-SCMA employs both the power and code domains. At the code domain, PD-SCMA employs uniquely designed SCMA [6] codebooks to differentiate users using the same time-frequency resources. Proper codebook design enhances the code domain hence improves the system. The multiplexing performance of such a scheme can be enhanced not only through deployment of sophisticated resource allocation schemes and low complexity robust multi-user detectors but also prudent design of multi-user multi-dimensional codebooks [7].

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The development of PD-SCMA scheme is a multi-stage design involving; codebook design and resource allocation (RA) at the transmitter, and multi-user detector (MUD) at the receiver. In the RA stage, the design of sparse spreading signatures for the users, assignment of codebooks, user clustering and power allocation is done. The next design stage concerns designing of codebooks over the spreading signatures. At the receiver, an optimal MUD with low complexity order is required for successful detection [8].

In PD-SCMA code-domain, incoming user bits are mapped to multi-dimensional constellations (MDCs) in the code-domain. Subsequently, adequate MDCs design is instrumental in its performance. Moreover, the successful operation of a multi-user detector (MUD) in detecting the observed signal becomes more intricate when codebooks in PD-SCMA operates multiple superimposed users in the power-domain. Consequently, it is important to design, firstly, uniquely decodable symbols for users superimposed in a codebook (hereby referred to as the layer dimension) and secondly, unique symbols for users colliding in the same RE (hereby referred to as the RE dimension).

Primarily, codebooks generation is a two-fold process [8]: firstly, designing a multi-dimensional mother constellation (MC) and secondly, performing user specific operations (USO) on MC to generate codebooks, where in PD-SCMA, USO is performed on both layer and RE dimensions. The original general rule on codebook design appeared in [9] where a MC was first generated by Cartesian Product of two quadrature amplitude modulation (QAM) constellations and then USO using different permutation and complex conjugate operations [10] is performed. However, the permutation set used was not optimal leading to sub-optimality in power diversity gain among different codebooks.

Scanning through the open literature, several approaches to codebook designs are reported in [11] and [12] for downlink and [13], [14] for uplink NOMA systems. Constellation rotation-based codebooks designed by constructing sub-constellations and employing Latin generator matrix for enhanced shaping gain is proposed in [11]. In [12], interleaving is proposed to maximize the minimum Euclidean distance (MED) among codewords of a codebook. Apart from phase rotation, coordinate interleaving and permutation operations, a Trellis coded modulation (TCM) based design is proposed in [13]. The design in [14] optimizes the mutual information and cut-off rate of an equivalent multiple input multiple output (MIMO) system. The bit error rate (BER) performance of the low-density signature (LDS) employing constellation rotation-based MDC designs outdo others [8]. Contrastingly, few USOs have been designed for the uplink SCMA systems. In the uplink, USOs may lose the significance when a UE transmits a codeword through a single layer. However, while applying multi-layer superposition transmission, the superposed codewords are transmitted through the identical channels. Consequently, the layer-specific operations require to be sophistically designed to mitigate the inter-layer interferences [15].

The operation of M-point MDCs on SCMA are tied to the key performance indicators (KPIs) considered in the design process for different channel modes. The KPIs namely; Euclidean distance, Euclidean kissing number, product distance, product kissing number, modulation diversity order, number of distinct points and bit labelling are highlighted in [7] for uncoded fading, coded fast fading and coded quasi-static fading channel modes. In each of the three channel modes, two cases are considered namely, where each user observes the same or independent channel coefficients over its REs. In addition, the MUD performance at different signal to noise ratio (SNR) regions also affects MDCs behaviour. Moreover, the code rate has a significant impact on the system performance.

This work investigates existing SCMA codebook designs employed in the PD-SCMA systems. The codebook designs have been done in [11] and [12] for downlink and [13], [14] for uplink SCMA NOMA systems, [16] for various channel environments and [17] for MIMO-SCMA systems. Further, this work proposes a near-optimal PD-SCMA codebook design based on block codes and investigates its performance. To the best of our knowledge, codebook design for PD-SCMA systems has not been done. Firstly, we highlight selected studied MDCs design metrics over various channel scenarios. In particular, we consider the $M-$point d_v-dimensional complex constellations proposed for the uplink SCMA systems by authors in [14], [18], [19], [20], and [21] for $M = 4$, and $d_v = 2$ and modify to apply in PD-SCMA systems. The performance of these MDCs is evaluated for low-rate turbo-coded PD-SCMA systems over different channel conditions through extensive simulations, considering the USO proposed in [10]. To enhance the decoding experience and maintain uniqueness of users in both the power- and code- domain, an USO build on the quaternion based rotation matrices is proposed. The elements of the rotation matrix described by quaternions are quadratic polynomials and therefore relate simply with KPIs under consideration. The effectiveness of quaternions based rotation matrices is recognized in various fields such as computer vision and robotics [22]. Simulation results show significant performance enhancement for MDCs satisfying the KPIs. Moreover, the proposed USO enhances the receiver experience for the low-rate turbo-coded system. Generally, the proposed codebook design improves the system's bit error rate (BER) performance compared to the existing designs.

In summary, the contributions of this research work features;

- Investigation of existing SCMA MDC designs and subsequent modification to hybrid PD-SCMA for performance evaluation on low-rate turbo coded systems over varied channel conditions.
- Proposing a low complex codebook design based on linear error correcting block codes.
- Proposing a unique user-specific rotation design based on quaternion rotation matrices with the aim of maintaining users symbol uniqueness for overloaded systems.
- The combination of the proposed low complex codebook design and the USO remarkably enhances the BER performance and improves the decoding experience.

A. ORGANIZATION

The rest of the paper is organized as follows: the system model is developed in Section II. Section III discusses the multi-stage design methods. Section IV outlines the modified multi-dimensional constellation designs for PD-SCMA systems. The proposed PD-SCMA codebook design is discussed in section V. In Section VI, the performance evaluation of the MDCs on PD-SCMA is presented. Finally, Section VII concludes the paper.

TABLE 1. Notation.

Notation	Meaning	Notation	Meaning
$\mathbf{x}$	Coded signal vector	$\mathbf{b}$	Incoming bits
M	Codebook size	$\mathbb{C}$	d_v−dimensional complex constellation points
J	No. of SUEs in a SBS	V	MUEs multiplexed in a Layer
F	No. of SBSs	U	Total MUEs in the system
L	No. of layers	W	Binary mapping matrix
K	Resource elements (REs)	P	Power vector
$\mathbf{y}$	The received signal vector	$\mathbf{z}$	The complex AWGN vector
$\mathbf{h}$	Channel fading gains	λ	Overloading factor
d_f	No. of layers per RE	d_v	No. of REs per layer
Δ	Constellation operator	E	Rotation Matrix
D	Total no. of user symbols in a codebook	N	Resource Elements

B. NOTATION

We denote by x, $\mathbf{x}$, $\mathbf{X}$ and $\mathcal{X}$ a scalar, vector, matrix and set respectively. A set of $M-$ary numbers is denoted by $\mathbb{M}$. What's more, $\mathbf{x}^T$ and $diag(\mathbf{x})$ represent the transpose and diagonal matrix respectively. Besides, $diag(\mathbf{X})$ is a vector of the diagonal elements of matrix $\mathbf{X}$. The summary list of all notations and variables is given in Table 1.

II. SYSTEM MODEL

An uplink PD-SCMA system on a two-tier heterogeneous network (HetNet) model is considered as presented in Fig. 1 The HetNet model comprises of a centralized macro base station (MBS) serving a set of $\mathcal{U} = \{1, \cdots, U\}$ randomly distributed macro cell users (MUEs) and underlaid set of $\mathcal{F} = \{1, \cdots, F\}$ small cells, each characterized by a centralized low power small cell base station (SBS) serving a set of $\mathcal{J} = \{1, \cdots, J\}$ uniformly distributed small cell user equipments (SUEs). Similar to the conventional SCMA, a PD-SCMA transmitter operates L layers (of set $\mathcal{L}$) and N orthogonal resource elements (REs), where $N < L$. Each layer utilizes d_v, $(d_v < N)$ REs, thus, each layer spreads its data over d_v REs. A layer is constructed by drawing select codewords from each user of SUE set $\mathcal{J}$ and clustered MUE set $\mathcal{V}_{CB}$, $(|\mathcal{V}_{CB}| = V, \mathcal{V}_{CB} \in \mathcal{U})$. This implies that a layer constitutes $D = (V + 1)$ users symbols and $L = J$. Fig. 2 illustrates the PD-SCMA block diagram with L layers, $N = 4$ REs, $d_v = 2$ in the code-domain and D multiplexed users in the power-domain. Under the constraint that no two layers should be assigned all the same RUs for an affordable complexity order, a system is fully loaded if $\lambda = D \times \binom{L}{d_v}$.

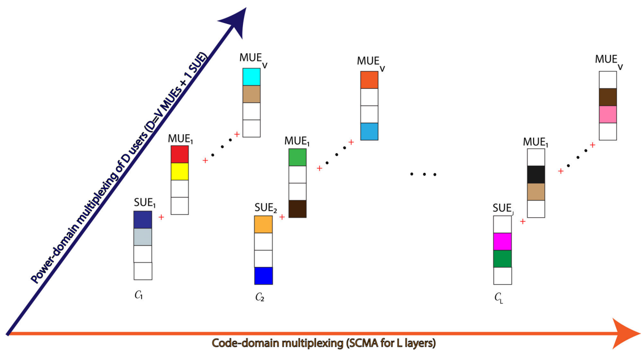


FIGURE 2. PD-SCMA block with $D = V + 1, V \in V_{CB}$ superimposed MUEs, L layers/codebooks and $N = 4$ REs.

A. PD-SCMA TRANSMITTER

A PD-SCMA transmitter combines three encoding procedures; firstly, V-BLAST encoding to obtain branch multiplexed signals; secondly, forward error correction (FEC) for correcting random error by means of introducing redundancy and thirdly, interleaving process that restrains consecutive errors by scattering the data stream. In the code-domain, symbol mapping and the sequence spreading are merged and performed together for the SCMA encoder. The incoming bits are directly mapped to the multi-dimensional complex domain codewords chosen from a predefined set of codebooks. An SCMA encoder maps $\mathbf{b}$ incoming bits such that

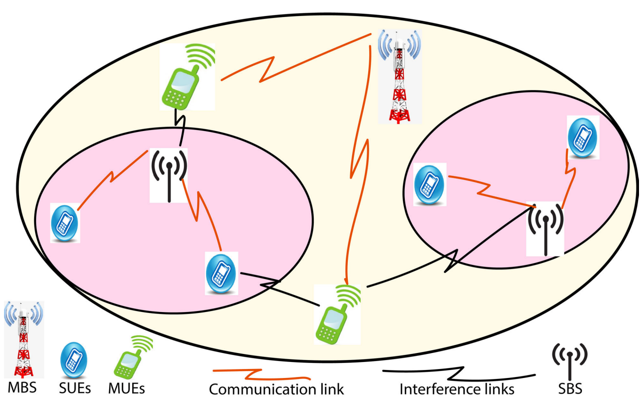


FIGURE 1. Uplink hybrid PD-SCMA HetNet model.

$\mathbb{B}^{log_2 M} \longmapsto \mathcal{S}, \mathbf{s} = g(\mathbf{b})$ with $\mathbf{s} \in \mathcal{S} \subset \mathbb{C}^N$ and $|\mathcal{S}| = M$. The N-dimensional complex codeword is sparse with $N - d_v$ non-zero entries. The encoder can be redefined to consider the mapping for only the d_v constellation points of the codeword. Let the mapping from $\mathbb{B}^{log_2 M}$ to $\mathcal{C}$ be defined by

$$\mathbb{B}^{log_2 M} \longmapsto \mathcal{C}, \quad \mathbf{c} = g(\mathbf{b}) \tag{1}$$

where $\mathbf{c}$ denotes the d_v-dimensional complex constellation point described within the mother constellation set $\mathcal{C} \subset \mathbb{C}^{d_v}$. Then an SCMA encoder can be given as

$$f :\equiv \mathbf{W}g, \quad i.e., \ \mathbf{s} = \mathbf{W}g(\mathbf{b}). \tag{2}$$

The binary mapping matrix $\mathbf{W} \in \mathbb{B}^{L \times d_v}$ maps the d_v-dimensional complex codeword of the mother constellation to a $N-$dimensional codeword of a SCMA codebook.

From equations (1) and (2), a given SCMA codebook for one chosen user j can be expressed as;

$$\mathbf{s}_j = \mathbf{W}_j(\Delta_j \cdot g)(\mathbf{b})_j \tag{3}$$

where Δ_j is defined as the constellation operator for the j-th user, while $(\cdot)$ denotes a composition operation. We note that $\Delta_j = \mathbf{E}_j\mathbf{I}$ with $\mathbf{E}_j$ as the rotation matrix defined in [6] and $\mathbf{I}$ is N-dimensional all one vector, then $\mathbf{s}_j$ is an LDS codeword which is a special case of SCMA. Following this definition, an encoded SUE and MUE vector symbols can be respectively be given by

$$\mathbf{s}^{SUE_j} = \left[s_1^{SUE_j}, \cdots, s_K^{SUE_j}\right]^T, \\ \mathbf{s}^{MUE_v} = \left[s_1^{MUE_v}, \cdots, s_K^{MUE_v}\right]^T \tag{4}$$

with entries $s_k^{SUE_j} = 0$ and $s_k^{MUE_v} = 0$ at $N - d_v$ constellation points. Each codebook can be utilized by one user, like in conventional SCMA or several users superimposed in the codebook, as with PD-SCMA system by allocating users with distinct power levels.

After codebook assignment of the SUEs and power allocation, the MUEs symbols in (4) are then clustered to get $\mathcal{V}_{CB}$ for pairing with a SUE assigned to a codebook using schemes in [5]. At each transmission period, a codeword vector is selected from each user codebook to constitute a layer $\mathbf{X}_l$ which can be expressed as

$$\mathbf{X}_l = \left[\mathbf{x}_l^{SUE_j}\mathbf{x}_l^{MUE_v} \cdots \mathbf{x}_l^{MUE_V},\right] \in \mathbb{C}^{K \times M} \tag{5}$$

The entries $\mathbf{x}_l^{SUE_j} = \sqrt{P_l^{SUE_j}} \cdot \mathbf{s}^{SUE_j}$ and $\mathbf{x}_l^{MUE_v} = \sqrt{P_l^{MUE_v}} \cdot \mathbf{s}^{MUE_v}$ and $P_l^{SUE_j}$ and $P_l^{MUE_v}$ are the normalized SUE and MUE power levels respectively. The users in $\mathbf{X}_l$ are then multiplexed in power domain resulting to the layer message vector $\mathbf{x}_l \in \mathbb{C}^{K \times 1}$.

The received signal vector can be expressed as $\mathbf{y} = \left[y_1 y_2 \cdots y_N\right]^T \in \mathbb{C}^{N \times 1}$ over the PD-SCMA block after the synchronous layer multiplexing, where y_n as the received symbol on the n^{th} RE is given as

$$y_n = \sum_{l=1}^{L} h_{l,SUE_j}^n \mathbf{x}_l^{SUE_j} + \sum_{l=1}^{L}\sum_{m=1}^{V} h_{l,MUE_v}^n \mathbf{x}_l^{MUE_v} + \mathbf{z}^n \tag{6}$$

where, looking through the n^{th} resource element, h_{l,SUE_j}^n and h_{l,MUE_v}^n denote SUE and MUE channel coefficients averaged over the d_v REs in each layer l respectively.

In SCMA, the matrices $\mathbf{W}_j$ maps each user to the resources in a sparse allocation. Given that the matrices are fixed and independent of the selected codewords, the layer - RE assignment is also fixed during the transmissions. As a result, the SCMA structure can then precisely be described by a graph structure as illustrated in Fig. 3 and its respective indicator matrix defined by $\mathbf{F} = (f_1, \cdots, f_L)$. The layer node l and resource node n are connected if and only if $(\mathbf{F})_{n,l} = 1$. Using matrix $\mathbf{F}$, the graph structure of SCMA can easily be described. Let $\vartheta_l = \{n : f_{n,l} = 1\}$ define the sets of the resources where layer l spreads its codeword, and $\varphi_n = \{l : f_{n,l} = 1\}$ define the sets of users colliding in resource n. The relationships $d_v = |\vartheta_l|$ and $d_f = |\varphi_n|$ are respectively named the spreading degree of layer l and collision degree of resource n. The indicator matrix of Fig. 3 is can be given as

$$\mathbf{F_{4\times 6}} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{7}$$

B. PD-SCMA RECEIVER

The MUD for a PD-SCMA system involves user signal detection in both the code and power domains. In [4] and [5], practical joint MUDs based on logarithmic message passing algorithm (log-MPA) in the code-domain and successive interference cancellation (SIC) in the power domain is employed. A low complex modified joint expectation propagation algorithm (EPA) with SIC receiver is proposed for the uplink system. Compared to the MPA, EPA detection algorithm enjoys a salient advantage of linear complexity that scales M and d_f on a given RE. EPA exhibits enhanced error rate performances dependent on the codebook size. Consequently, this work employs the joint MUD proposed in [23] in a single input single output (SISO) environment.

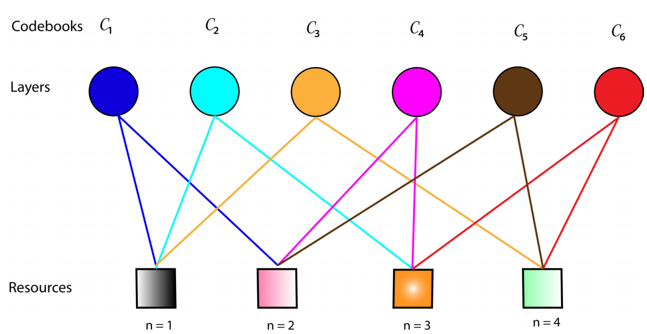


FIGURE 3. PD-SCMA factor graph representation with $L = 6$ and $N = 4$ REs.

III. MULTI-STAGE DESIGN METHODS

A SCMA scheme $\mathcal{I}$ with L layers, N REs and $\mathbf{W}_l$ layer-to-RE mapping matrix of each layer, where each layer occupies d_v REs and employs an $M-$point $N-$dimensional codebook, can be represented as [24]

$$\mathcal{I}(\mathbf{W}, \mathcal{C}; M, L, d_v, N) \tag{8}$$

where $\mathcal{C} = \mathcal{C}_1 \times \mathcal{C}_2 \times \mathcal{C}_3 \times \cdots \times \mathcal{C}_L$ and $\mathbf{W} = [\mathbf{W}_1 \cdots \mathbf{W}_L]$. The design of an SCMA systems two-fold; 1) finding the optimum layer-to-RE mapping matrix, $\mathbf{W}^*$ and 2), design of optimum MDC, $\mathcal{C}^*$. Considering a design criterion $\aleph$ to be maximized in the SCMA system, an optimized design would consider the following problem

$$\mathbf{W}^*, \mathcal{C}^* = \underset{\mathbf{W}, \mathcal{C}}{\operatorname{argmax}} \aleph\left(\mathcal{I}(\mathbf{W}, \mathcal{C}; M, L, d_v, N)\right) \tag{9}$$

To the best of our knowledge, there is no practical proposed design criterion that achieves optimality. Several SCMA designs have been proposed through multi-stage sub-optimal optimization approaches [24] that involves finding $\mathbf{W}^*$ and then $\mathcal{C}^*$. The layer-to-RE mapping matrix, W for SCMA systems can be obtained by employing proposed resource allocation techniques. Besides, there is a unique solution for $\mathbf{W}^* = \mathbf{W}$ when the parameters L, d_v and N are fully loaded. Consequently, the design problem reduces to

$$\mathcal{C}^* = \underset{\mathcal{C}}{\operatorname{argmax}} \aleph\left(\mathcal{I}(\mathbf{W}^*, \mathcal{C} : M, L, d_v, N)\right) \tag{10}$$

Now the problem reduces to find the L different $M-$point d_v- dimensional complex constellations. The solution to the optimization problem can further be considered in two steps. Firstly is to design a d_v- dimensional complex mother constellation of size M, $\mathcal{C}^+$, with each element of $\mathcal{C}^+$ representing one constellation point of the complex mother constellation, i.e.,

$$c_m^+ = \left(c_{1,m}^+, \cdots, c_{d_v,m}^+\right)^T, \quad m \in \{1, \cdots, M\} \tag{11}$$

Secondly is to perform designed layer-specific unitary rotations on the mother constellation to generate layer - specific multi-dimensional constellations (e.g., [11]). The proposition of rotations is particularly motivated by additive white Gaussian noise (AWGN) channels without fading [25]. The two steps can be represented as

$$\mathcal{C}_l = \{\Delta_l c_m^+ | c_m^+ \in \mathcal{C}^+\} \tag{12}$$

where Δ_l represents a $d_v \times d_v$ layer-specific rotation matrix. Consequently, $C = \Delta_l c_m^+$. As a result, the optimization problem (10) becomes

$$\{\Delta_l^*\}\mathcal{C}^{+^*} = \underset{\{\Delta_l\}, \mathcal{C}^+}{\operatorname{argmax}} \aleph\left(\mathcal{I}(\mathbf{W}^*, \{\Delta_l, \mathcal{C}^+\}; M, L, d_v, N)\right) \tag{13}$$

In order to maintain user symbol uniqueness at the layer-specific rotations stage, user specific rotations should be done at both the code-domain dimension and the power-domain dimension. This can be achieved by first performing user-specific rotation similar to [24] for the code-domain. Then, the power-domain user uniqueness can be achieved by employing the user-specific rotations proposed in Section V-C.

The PD-SCMA system users spread their data symbols over d_v, ($d_v < N$) REs. Classification of the channel is based on either; 1) The fading coefficients correlations for the different REs at each layer, or 2) The fading coefficients temporal correlation at each RE at different times [7]. In the previous scenario, in one case we can have completely correlated coefficients, achieving no diversity, which predominantly happens when the resource elements are adjacent. On the other case, there can have completely independent coefficients, in which case predominantly occurs when the REs are isolatedly apart. The later scenario is explicitly important to systems that employ channel coding. At one extreme there is "fast" fading, in which the channels observe completely independent coefficients with each channel use. At the other extreme, there is slow "quasi-static" fading, in which the channels observe constant coefficients during the transmission period of an entire codeword.

The scenarios lead to channel mode cases namely; Uncoded fading where each user observes the same channel coefficients over its REs (FSC), Uncoded fading where each user observes independent channel coefficients over its REs (FIC), Coded fast fading where each user observes the same channel coefficients over its REs (FFSC), Coded fast fading where each user observes independent channel coefficients over its REs (FFIC), Coded quasi-static fading where each user observes the same channel coefficients over its REs (SFSC) and lastly, Coded quasi-static fading where each user observes independent channel coefficients over its REs (SFIC).

The design process of $M-$point d_v- dimensional complex constellations $\mathcal{C}^+$ considers several key performance indicators (KPIs) applicable under the channel scenarios of transmission. The authors in [7] highlight these KPIs namely; Euclidean distance, Euclidean Kissing Number, Product Distance, Product Kissing Number, Modulation Diversity Order, Number of Distinct Points and Bit-Labeling. Table 2 represent the consideration KPI in each channel mode.

In the PD-SCMA transmission, users (MUEs and SUEs) are superimposed in power domain and transmitted using d_v REs associated with the assigned codebook for that cluster. The different users experience different channel conditions characterised by temporal correlation of the fading coefficients at each RE at different times. Consequently, the users' experience coded fast fading where each user observes independent channel coefficients over its REs in which most of the KPIs should be put into consideration for efficient MDC design.

IV. MODIFIED MULTI-DIMENSIONAL CONSTELLATION DESIGNS FOR PD-SCMA SYSTEMS

This section highlights the state of the art $M-$point d_v-dimensional constellations proposed for SCMA systems and modified for application in PD-SCMA systems. We consider a system with $M = 4$ and $d_v = 2$ REs.

TABLE 2. KPIs considered in different channel modes.

KPI	FSC	FIC	FFSC	FFIC	SFSC	SFIC
Euclidean distance, $d^2_{E,min}$	✓		✓		✓	
Euclidean Kissing Number, τ_E	✓		✓		✓	
Product Distance, $d^2_{P,min}$		✓		✓		✓
Product Kissing Number, τ_P		✓		✓		✓
Modulation Diversity Order, L		✓		✓		✓
Number of Distinct Points, N_d	✓	✓	✓	✓	✓	✓
Bit-Labeling	✓	✓	✓	✓	✓	✓

A. PD-SCMA LDS CODEBOOKS

Taking a symbol x from a set of Quadrature-Amplitude Modulation (QAM) of size M, the LDS codebooks spreads the symbol over some sparse signature $\mathbf{f}$. This can be observed as repeating the $M-$QAM constellation point over the 2 REs [18]. An LDS codebook with parameters $(M, N) = (4, 2)$ can be described by (14), where E denotes a normalization factor. The complex projections in each dimension of this LDS codebook in illustrated in Figure 4.

$$\mathcal{C} = \frac{1}{\sqrt{E}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ +1+j & +1-j & -1+j & -1-j \\ 0 & 0 & 0 & 0 \\ +1+j & +1-j & -1+j & -1-j \end{bmatrix} \quad (14)$$

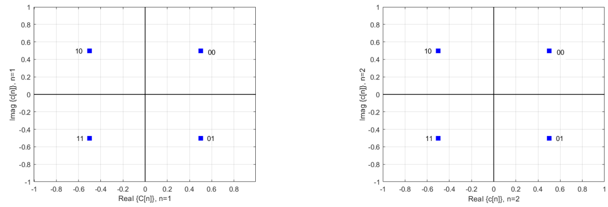
B. PD-SCMA TM-QAM CODEBOOKS

Proposed in [10] and later patented in [19], the TM-QAM MDC design for multiple layers involves two steps. Firstly, a d_v dimensional complex MDC with a good Euclidean distance profile is designed. Secondly, a unitary rotation matrix is applied to the base constellation to maximize the minimum product distance.

The complex projections onto two dimensions for $M = 4$ are shown in Figure 5. Each UE maps its incoming bitstreams onto the constellation points with green circles in Figure 5 in order to transmit them over RE1 and RE2. With the rotated constellation, various sets of operators such as phase rotations [19], Latin square rotation criterion [26], structured approach design of spread sequences [11] and diversity order based rotation technique [25] can be applied to build multiple TM-QAM based sparse codebooks for multiple layers of SCMA.

C. PD-SCMA ML-QAM CODEBOOKS

The $M-$point constellation design proposed in [14] is based on the [10] and [19] however, with a reduced number of projections. The design process, similar to TM-QAM involves a shuffling method to establish a d_v-dimensional

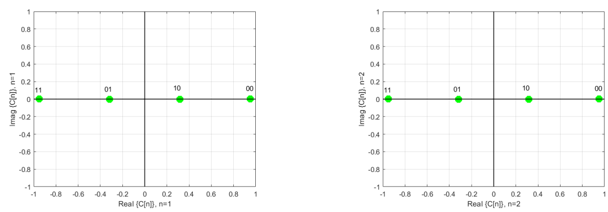


(a) LDS, n=1 (b) LDS, n=2

FIGURE 4. PD-SCMA LDS codebook with $(M, N) = (4, 2)$ [18].

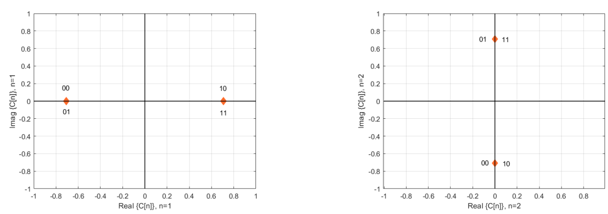
complex constellations from two d_v-dimensional real constellations Cartesian product that have a desired Euclidean distance ($d^2_{E,min}$) profile. However, the unitary rotation in ML-QAM is applied to minimize the number of projected points, N_d over each RE. This has the advantage of minimizing the detector's computational complexity.

The projections of ML-QAM for $M = 4$ on two dimensions over RE1 and RE2 are shown in Figure 6, where four (4) points of the complex constellation are mapped to only $N_d = 2$ points. As can be seen from Figure 6, both 00 and 01 are mapped to $(\frac{-\sqrt{2}}{2}, 0)$, and both 10 and 11 are mapped to $(\frac{\sqrt{2}}{2}, 0)$ over RE1. Besides, both 00 and 10 are mapped to $(0, \frac{-\sqrt{2}}{2})$, while 01 and 11 are mapped to $(0, \frac{\sqrt{2}}{2})$ over RE2. The lowered number of projections results in reduced computational complexity from 4^{d_f} to 2^{d_f}.



(a) T4-QAM, n=1 (b) T4-QAM, n=2

FIGURE 5. PD-SCMA TM-QAM codebook with $(M, N) = (4, 2)$ [10].



(a) 4L-QAM, n=1 (b) 4L-QAM, n=2

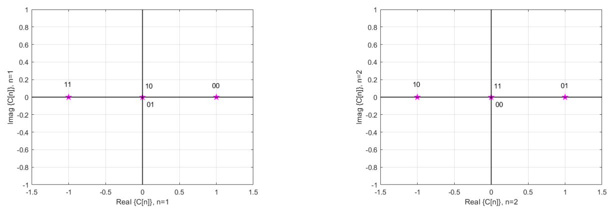
FIGURE 6. PD-SCMA ML-QAM codebook with $(M, N) = (4, 2)$.

D. PD-SCMA MC-QAM CODEBOOKS

The $M-$point circular QAM constellation (MC-QAM), proposed in [14] bases its design on the investigation of the signal space diversity (SSD) for the multiple input multiple output (MIMO) systems over Rayleigh fading channels in [13], [15].

Similar to ML-QAM, MC-QAM targets to obtain a reduced number of projections per each complex dimension.

The projections of ML-QAM for $M = 4$ on two dimensions over RE1 and RE2 are shown in Figure 7. The four (4) points of the constellation are mapped to only $N_d = 3$ points, therefore minimizing the complexity from 4^{d_f} to 3^{d_f}. By employing Gray labelling, both the points 01 and 10 are mapped to $(0, 0)$, the point 00 mapped to $(1, 0)$ while 11 is mapped to $(-1, 0)$ for transmission over RE1. Considering transmission over RE2, both points 00 and 11 are mapped to $(0, 0)$, the point 01 is mapped to $(1, 0)$, while the point 10 is mapped to $(-1, 0)$.



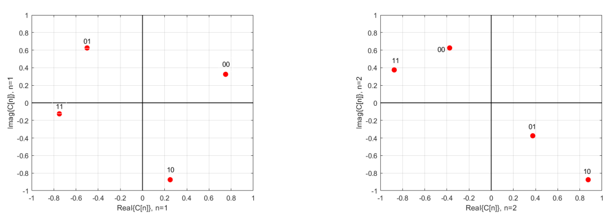
(a) 4C-QAM, n=1 (b) 4C-QAM, n=2

FIGURE 7. **PD-SCMA MC-QAM codebook with $(M, N) = (4, 2)$.**

E. PD-SCMA AVERAGE SYMBOL ENERGY BASED CODEBOOKS

The $M-$point constellation proposed in [21] and [20] bases its design on minimizing the average symbol energy (ASE) (M-ASE) for a given minimum Euclidean distance $d_{E,min}$ between the constellation points. The design process problem is formulated as a non-convex optimization problem. The main problem can then be decomposed into sub-problems which are solved by applying a sequence of convex optimization techniques.

The projections of M-ASE for $M = 4$ on two dimensions over RE1 and RE2 are shown in Figure 8. It can be observed that, as an example, for transmission of 11, the UE maps the corresponding bitstreams to $(0.7543, 0.3852)$ and $(-0.3993, 0.3509)$ over RE1 and RE2, respectively.



(a) M-ASE, n=1 (b) M-ASE, n=2

FIGURE 8. **PD-SCMA M-ASE codebook with $(M, N) = (4, 2)$ [21].**

V. PROPOSED PD-SCMA CODEBOOK DESIGN

Most of the designs that result to the codebooks discussed previously involve complex procedures in the extension from 1 to N dimensions. The main motivation in codebook designs is to strike-a-balance regarding good multi-dimensional modulation (MDM) properties, low-complexity codebook design method, and low-complexity detection.

A. CODEBOOK DESIGN BASED ON BLOCK CODES

In this work, we propose a codebook design over alphabets of q number of projections less than the number of codewords M, (i.e., $q < M$) using linear error correcting codes, that can exhibit a significantly reduced complexity. The design not only considers maximizing the cut off rate approximation [27], minimum (squared) Euclidean distance $min_{i,j} d_E^2(\mathbf{c}_i, \mathbf{c}_j)$ and the minimum $L-$product distance $min_{i,j} d_{p,L}(\mathbf{c}_i, \mathbf{c}_j)$ but also the signal space diversity (SSD). The SSD [25] is associated to the Hamming distance $d_H(\mathbf{c}, \mathbf{c}')$ of two codewords $\mathbf{c}, \mathbf{c}' \in \mathcal{C}$. It represents the number of different complex entries between two codeword vectors that are compared dimension-wise.

There is a close correlation between the forward error correction (FEC) codes and SCMA codebooks and the error metrics of Singleton bound and Hamming distance can be used correspondingly to measure performance. Similar to FECs, the SCMA codebooks will only have the largest minimum Hamming distance if the codewords have M unique projections in each complex projection. In other words, if the projections of the codewords are designed such that $c_n \neq c_n'$ for $n = 1, \cdots, N$ for all pairs $(\mathbf{c}, \mathbf{c}' \in \mathcal{C})$ such that $\mathbf{c} \neq \mathbf{c}'$.

Consider the Singleton bound for FEC codes [28] given as $M \leq q^{N-d_{min}+1}$, which can be reordered as $d_{min} \leq 1+N-b$, where $M = q^b$ for all integer value of $b > 0$. For the low projection ($q < M$) SCMA codebooks, the Singleton bound for the minimum Hamming distance holds true and is bounded by $d_{min} \leq 1 + N - b$ with $b > 1$. The codebooks that achieve the singleton bound are classified as Maximum Distance Separable (MDS) codes, with which the feasibility to realize a near-maximum minimum SSD is possible. At $b > 2$, MDS codes exhibit $d_{min} = N - 1$. Therefore, the minimum SSD L is upper bounded at $L \leq N - 1$.

Apart from the Hamming distance and SSD, another important consideration in this codebook design is the concept of the underlying vector - space structure. The $M-$length integer permutations $\pi_1, \cdots, \pi_N$ of codebook space $\mathcal{A}_q$ used to build the SCMA codebook in [27] can be seen as a list of M $N-$dimensional vectors whose entries are taken over an alphabet $\mathbb{Z}_q$. Indeed, a (N, b, d) block code can be observed as a stack of N $M-$length permutations over $\mathbb{Z}_q$ or rather $\mathbb{F}_q$, if q is a power of prime. d denotes the minimum Hamming distance of the code while q denotes the alphabet length. In summary, an $M-$ary $N-$dimensional PD-SCMA codebook $\mathcal{C}$, each with complex-dimensions designed with complex-projections over a $q-$ary complex valued alphabet $\mathcal{A}_q$, for any q power of a prime number has an underlying vector-subset of the vector-space $\mathbb{F}_q^N$, represented by a matrix $\Pi_{N,M}^q \in \mathbb{F}_q^{N \times M}$ whose columns are the elements of the subset.

Motivated by the concepts of underlying vector-space structure and the SSD figure of merit, this work proposes

SCMA codebooks design over small alphabets based on the fundamental structure of a generally applied error-correcting block codes. The proposed codebooks rely on good MDM properties. Indeed, MDMs with large SSD exhibit better performance especially at higher SNR values.

Consider a $q-$ary finite field $\mathbb{F}_q = \{\alpha_1, \cdots, \alpha_q\}$. A one-to-one mapping between each vector $\mathbf{u}$ of the message-space $\mathbb{F}_q^b$ and the elements $\mathbf{v}$ of the vector-subspace $\mathbf{V} \subset \mathbb{F}_q^b$ generated by the generator matrix $\mathbf{G} \in \mathbb{F}_q^{b\times N}$ results to a linear block code. It is proposed that the $N-$dimensional $M-$ary PD-SCMA codebook $\mathcal{C}$ be structured based on the vector-subspace generated by $\mathbf{G}$ by related the field elements $\{\alpha_1, \cdots, \alpha_q\}$ with the set $\mathcal{A}_q = \{x_1, \cdots, x_q\}, x_i \in \mathrm{C}$, $i = 1 : q$. The chosen elements of $\mathcal{A}_q$ are unique and the mapping $\mathbb{F}_q \longrightarrow \mathcal{A}_q$ should be a one to one. Additionally, this work considers generator matrix of MDS linear codes whose resultant Hamming distance is given as $d_{mds} = N - b + 1$ and consequently SCMA codewords with SSD $L = d_{mds}$. Given that $b = 1$ corresponds to repetition codes, again this work considers SCMA codebooks design based on MDS block codes with $b \geq 2$. The steps involved the design are as follows;

1) **Step 1**: Selecting the Block-Code
   a) Select b and q, taking into consideration that $\mathcal{C} = M \leq q^b$.
   b) Select a Reed-Solomon (RS) [29] generator matrix given as

$$\mathbf{G}_{b\times N} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ g_1 & g_2 & \cdots & g_q & \vdots \\ \vdots & \vdots & \ddots & \vdots & 0 \\ g_1^{b-1} & g_2^{b-1} & \cdots & g_q^{b-1} & 1 \end{bmatrix} \tag{15}$$

   c) Employ G to generate a $(N, b, d_{mds})_q$ code,
   d) For each of the codewords, replace the field elements $\{\alpha_1, \cdots, \alpha_q\}$ by the elements $\{x_1, \cdots, x_q\}$ respectively.

2) **Step 2**: Designing the Complex Projections, $\mathbf{A}_q$.
   a) Since a MUD calculates the messages independently in each resource, the codebooks can have a different set $\mathcal{A}_q$ in each of the N dimensions, i.e., $\mathcal{A}q^{(n)}, n = 1 : N$.
   b) The choice of the N sets $\mathcal{A}q^{(n)}$ should optimize the codebook's figure of merit which is of interest.
   c) The codebooks with infinitesimal values of q, $\mathcal{A}q^{(n)}$ is simplified to $\mathcal{A}q, \forall n$ i.e., $\mathcal{A}q^{(n)} = \mathcal{A}q, \forall n$. This can be done by choosing arbitrary q complex projections among studied constellations, for example QAM and APSK.
   d) For codebooks with $q^b > M$ and once $\mathcal{A}q$ is explicitly defined, we may consider other optimizations, including for arbitrary $\mathcal{A}q$. In this way, it is possible to purge $q^b - M$ vectors for purposes of optimizing some pre-determined figure of merit that depends on $\mathcal{C}$. In this work, we optimize the figure of merit based on the cut off rate approximation $\Psi(\mathcal{C})$ and given by

$$\Psi(\mathcal{C}) = \log_2 M - \log\left[1 + \frac{1}{M}\sum_{\mathbf{c}\in\mathcal{C}}\sum_{\substack{\mathbf{c}'\in\mathcal{C}\\ \mathbf{c}'\neq\mathbf{c}}} \Pi_{n=1}^{N} \times\left(1 + \frac{|c_n - c_n'|^2}{4N_0}\right)^{-1}\right] \tag{16}$$

3) **Step 3**: Designing the Binary Labeling.
   a) When considering classical one-dimensional (complex) rectangular QAM constellations, labelling based on Gray-code labeling is employed.
   b) When considering the general MDMs, the binary switching algorithm (BSA) is proposed in [30] with the aim of finding a good labeling. This is achieved through optimizing a cost function based on the Euclidean distance between codeword pairs.

The codebook design based on block codes can be summarised as presented in Algorithm 1.

B. COMPLEXITY ANALYSIS OF THE PROPOSED CODEBOOK DESIGN

Compared to [27] in which the codebook design exhibits a complexity of $\mathcal{O}\big((M!)^{N-1}\big)$, and further compared to [31] whose design achieves a complexity $\mathcal{O}\big(M^{d_f}\big)$, the achieved complexity order of this codebook design is $\mathcal{O}\big(M\big)$. The complexity order reduces via obtaining a set of integer permutations in a straightforward manner, using $\mathbf{G}_{b\times N}^T$ and linear operations to span a vector space. The low complex design procedure is also desirable in the design of design of high-order constellations.

C. USER-SPECIFIC ROTATION FOR THE POWER-DOMAIN SUPERIMPOSED USERS

Primarily, user-specific rotations (USRs) serve the purpose to introduce and maintain uniquely detectable symbols for UEs that transmit within the same RE in the RE dimension. However, in the uplink SCMA scenarios, the channels additionally introduces random rotations to the complex constellations. As a result, the optimization of USR does not necessarily influence the performance of the system. Separately, for the PD-SCMA NOMA system, multiplexed UEs also collide through superposition in the layer dimension. It is therefore imperative to design sophisticated USR that maintains uniqueness of the multiplexed user symbols, mitigate intra-layer interferences and enhance the decoding experience.

Denote by $\left[n_x, n_y, n_z\right]^T$ the rotation axis whose norm is equal to 1. A quaternion is a vector q defined by

$$q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^T, \tag{17}$$

Algorithm 1 Block Codes Based Codebook Design

Input variables: Input M, q, b.

Step 1: Selecting the Block-Code
Select b and q as per 1 (a) and Generate G according to 1(b).
Use G to generate code 1(c) and perform through step 1 (d).
Step 2: Designing the Complex Projections, A_q.
Choose N that sets $\mathcal{A}q^{(n)}$ to optimize the codebook's figure of merit which is of interest.
for For infinitesimal values of q **do**
  set $\mathcal{A}q^{(n)}$ to $\mathcal{A}q$ by choosing q complex projections from either QAM or APSK.
end for
for Codebooks with $q^b > M$ **do**
  Optimize the figure of merit $\Psi(\mathcal{C})$, eqn. 16.
end for
Step 3: Designing the Binary Labeling.
for classical one-dimensional (complex) rectangular QAM constellations **do**
  Perform Gray code labelling.
end for
for General MDMs **do**
  Perform binary switching algorithm (BSA) [30].
end for

where $q_0 = \cos\frac{\theta}{2}$, $q_1 = n_x \sin\frac{\theta}{2}$, $q_2 = n_y \sin\frac{\theta}{2}$ and $q_3 = n_z \sin\frac{\theta}{2}$, and $\theta \in [0, \pi]$ denotes the rotation angle. Then, the 3-D rotation matrix described by the quaternion is given as below [22].

$$R_q = 2\begin{bmatrix} q_0^2 + q_1^2 - 0.5 & q_1q_2 - q_0q_3 & 2q_1q_3 + q_0q_2 \\ q_1q_2 + q_0q_3 & q_0^2 + q_2^2 - 0.5 & q_2q_3 - q_0q_1 \\ q_1q_3 - q_0q_2 & q_2q_3 + q_0q_1 & q_0^2 + q_3^2 - 0.5 \end{bmatrix} \quad (18)$$

Since the constellation points span two dimensions (i.e.,n_x, n_y), the quaternion matrix in 2-D is used by setting $z = 0$. A constellation point (x, y, 0) can be rotated by the quaternion q using the following three steps:

1) **Step 1:** Convert the constellation point p to be rotated into a quaternion as follows

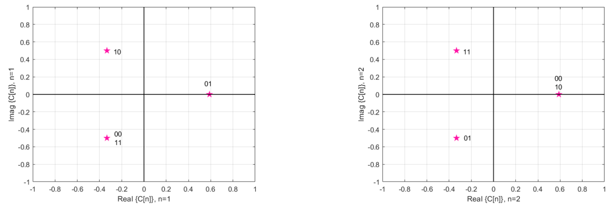
$$p = (p_0, p_1, p_2, p_3) = (0, x, y, 0) \quad (19)$$

2) **Step II:** Perform the quaternion active rotation with respect to the coordinate system to obtain coordinates of the rotated point $\hat{p}$ as follows

$$\hat{p} = q^{-1}pq \quad (20)$$

3) **Step III:** Extract the rotated coordinates from $\hat{p}$; $\hat{p} = (0, \hat{x}, \hat{y}, 0)$.

The steps are repeated with each $\hat{p}$ being the new p for the next constellation point to be rotated.



(a) Proposed CB, n=1 (b) Proposed CB, n=2

FIGURE 9. Proposed codebook based on block code $(M, N) = (4, 2)$ generated with RS.

TABLE 3. KPIs considered in different channel modes.

	$d^2_{E,min}$	τ_E	$d^2_{P,min}$	τ_P	L	N_d	Gray-Labelled
T4-QAM	2	2	0.64	2	2	4	Yes
4L-QAM	2	2	2	2	1	2	Yes
4C-QAM	2	2	1	2	1	3	Yes
4-ASE	2.67	3	0.29	0.5	2	4	No
LDS	2	2	1	2	2	4	Yes
Proposed Method	2.25	2	0.6495	2	3	4	Yes

VI. PERFORMANCE EVALUATION OF THE MDCs ON PD-SCMA

Through extensive Monte Carlo simulations, the performance of the constellations applied on PD-SCMA scheme over different channel models is evaluated and presented in this section. This work compares the proposed codebook design based on block codes with the existing MDCs considered. For simplicity and consistency in performance comparison, QAM based $A_{M,q}$ with $(M, q) = (4, 3)$ is employed. Since this work considers the model in the uplink, every UE does observe independent channel coefficients while transmitting over its REs. Consequently, simulation results for the FIC, FFIC and SFIC channel models are presented.

Achieving the optimal decoding complexity is not practical even for moderate values of M and d_f. The low-complexity joint SIC-log-MPA receiver based on [5] is used with 100 iterations. The KPIs corresponding to the considered MDCs are provided in [7] and given in Table 3. As observed in Table 2, most of the KPIs have significant impact on the BER performance of uncoded systems. Moreover, the behaviour of the MUD at different SNRs affects the performance. For clarity, we define $\frac{E_b}{N_0} = \frac{SNR}{\log_2 M}$. In the Figures 10, 11 and 12, simulations are done considering $(L, N, D) = (28, 8, 4)$, which is withing the multiplexing capacity bounds obtained in [5].

The BER performance of uncoded PD-SCMA system in FIC is illustrated in Fig. 10. It is observed that the proposed codebook design based on block codes outperforms the existing MDCs with larger diversity. At low SNRs, 4-LDS and 4C-QAM outperform the rest of the MDCs. The M-ASE achieves improved performance at high SNRs than the other considered MDCs at the same KPIs. M-ASE exhibit the highest $d^2_{E,min}$ compared with other 4−point constellations since

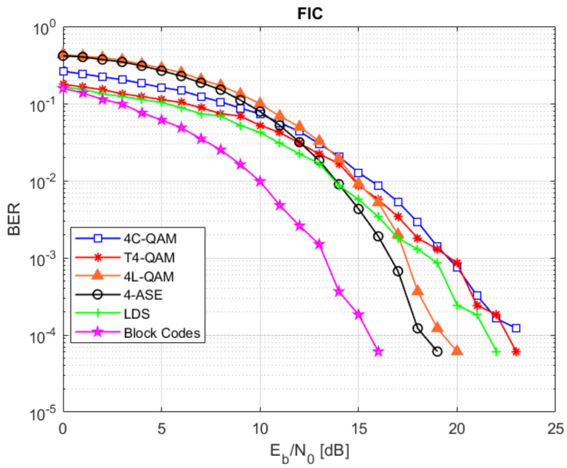


FIGURE 10. BER performance of uncoded SCMA systems with 4-point constellations over FIC.

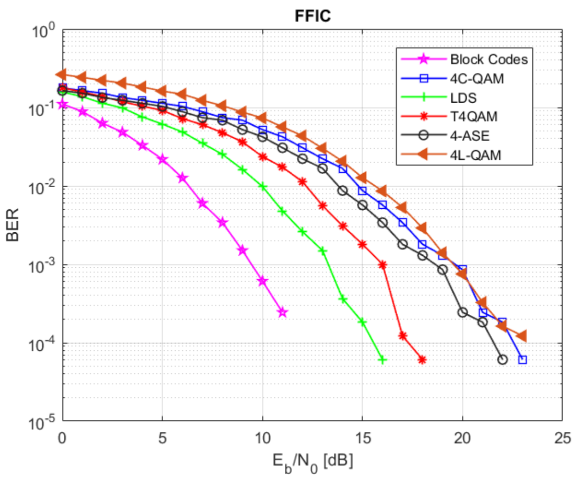


FIGURE 11. BER performance of coded SCMA systems with 4-point constellations over FFIC.

it is designed for AWGN channels. This notwithstanding, 4−ASE also have the highest τ_E that affects its performance especially at low SNRs.

In Fig. 11, the BER performance of coded PD-SCMA system in FFIC channel scenario is illustrated. The proposed method in the coded QAM yields better performance compared to the other MDCs, followed by the 4-LDS and 4C-QAM in the FFIC channel scenario. The constellations with low $d^2_{P,min}$ exhibit a significant higher performance across the SNR region, with the proposed codebook design method outperforming the other MDCs. Moreover, compared with the other constellations with τ_P, the performance of 4L-QAM is deteriorated by its lowest L.

Lastly, we compare the BER performance of coded PD-SCMA system in SFIC channel scenario as illustrated in Fig. 12. The same scenario is exhibited with the proposed codebook design exhibiting better performance compared to the rest at both low and high SNR values. In overall, the

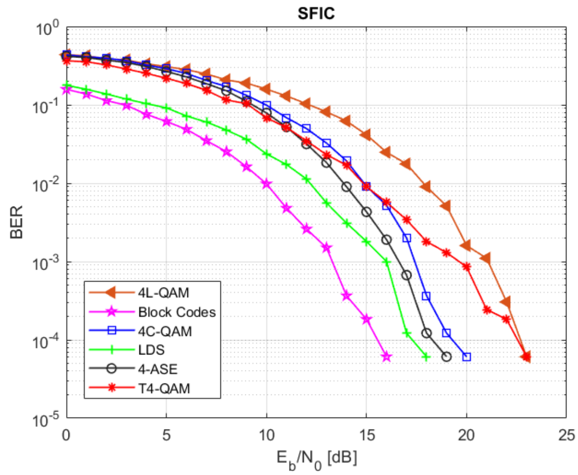


FIGURE 12. BER performance of coded SCMA systems with 4-point constellations over SFIC.

block-code and quarternion rotation based design significantly results in improved BER performance under similar loading and KPIs.

VII. CONCLUSION

In this work, a new codebook design method based on block-codes and quarternion rotation matrices is proposed for PD-SCMA NOMA systems and compared with existing designs. The behavior of MDCs on the performance of PD-SCMA NOMA systems is analysed for low-rate coded systems. It is observed that different constellations perform differently over various channel scenarios. Consequently, in order to optimize the performance in a certain channel scenario, a proper MDC need be designed considering the specific KPIs for that scenario. The proposed codebook design enhances the BER performance compared to the existing designs for the PD-SCMA NOMA systems. In a PD-SCMA scenario, where multiple users traverse the code and power domain simultaneously, an optimal and dynamic design of MDCs and the associated user-specific rotation design to be applied in both domains is under consideration as part of our future work. The MDC block codes structure exhibit linear complexity and algebraic properties. Consequently, the structure provides a new direction for a new family of SCMA codebooks based on classical code designs. The limitations of the work is that it considers selected generator matrices such as the Reed-Solomon (RS) [30]. However, other generator matrices can be considered. Additionally, other figures of merit to be optimized can be used to improve the system. Besides, the performance of the MDCs for high rate coded systems is a possible research direction.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

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