IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 1 January 2023, accepted 22 January 2023, date of publication 31 January 2023, date of current version 6 February 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3241235

RESEARCH ARTICLE

Route Optimization of Customized Buses Based on Optimistic and Pessimistic Values

FAN CHEN[1], YOUPENG LU[2], LINZHONG LIU[2], AND QING ZHU[1]
[1]Faculty of Geomatics, Lanzhou Jiaotong University, Lanzhou 730070, China
[2]School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China

Corresponding author: Fan Chen (chenfan@mail.lzjtu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 51768029 and Grant 51768030; in part by the ''Double-First Class'' Major Research Programs, Educational Department of Gansu Province, under Grant GSSYLXM-04; and in part by the Joint Innovation Fund Project of Lanzhou Jiaotong University and Tianjin University under Grant 2022070.

ABSTRACT To further study the impact of passenger personality on customized bus route selection, this paper takes into account passengers' degree of optimism and studies their psychological expectation of bus arrival time and their tolerance of its untimely arrival. On this foundation, the study obtains the operation schemes of customized buses under different optimism coefficients. The results show that the operation schemes and passengers' choice will always strike a balance under different optimism coefficients, and with increasing optimism coefficients, the number of buses, operation time, operation cost and travel distance will all decrease, but the untimely rate of the buses will consequently increase. To solve the proposed model, the corresponding particle swarm optimization (PSO) algorithm is designed. In view of the former mode, in which buses can never provide service to the drop-off stations until the boarding stations are completely serviced, the proposed coding rule can provide service to the boarding stations and the drop-off stations alternately. Furthermore, based on the local road network and the actual travel demand of passengers in Lanzhou, China, the paper finds that 0.4 is the optimum value of the optimism coefficient; the operation cost associated with this value is reduced effectively, and the capacities of buses are fully utilized, with acceptable delays.

INDEX TERMS Bilevel programming, customized bus, optimistic coefficient, particle swarm optimization, route optimization, traffic engineering.

I. INTRODUCTION

Buses are an important part of the urban public transportation system. Under the human-oriented service concept, as a high-quality service mode of public transportation, customized buses would be welcomed. Differ from traditional settled route bus or on-demand transit services, customized buses achieve breakthroughs in three aspects. Firstly, buses make fixed stop only in the boarding stations and the drop-off stations, it effectively avoiding the unfriendly experience caused by multiple stops during the journey [1]. Secondly, buses can flexibly adjust the route in the existing road network, this provides sufficient possible for the operation department to reduce operating cost [2]. And finally, the operation schemes of the buses are generated on demand, it has the double advantage that neither empty buses nor crowded buses will be existed [3]. This emergent operation mode shows a prominent advantage in the period of mobile network information. As customized buses are an emerging service mode in China, there is still no unified operating mode; however, this kind of demand-oriented service mode has been extensively studied. Liu and Ceder [4] analyzed customized buses in China for the first time, including network design and timetable formulation. Lu and Pan [5] studied the service modes of customized buses in different regions and established various operation schemes. Parvasi et al. [6] established a bilevel programming model for the school bus, where the upper level minimizes the operating cost and the lower level

The associate editor coordinating the review of this manuscript and approving it for publication was Maurice J. Khabbaz.

minimizes the students' expenditure. Wang and Ding [7] established a bilevel programming model for buses in the microcirculation transportation network of historic districts and adopted TransCAD to solve it. Tong et al. [8] studied the route optimization problem of a customized bus under a time-space framework and decomposed it into a bus allocation subproblem and a route selection subproblem. Ma et al. [9] established a route optimization model for emergency buses and designed a three-segment coding genetic algorithm (GA) to solve it. Qiu [10] established a bilevel route optimization model based on the choice of passenger travel mode and designed the corresponding branch and bound algorithm.

Previous studies have put forward many schemes with different emphases, but none of them are conducted from the perspective of passengers. The main manifestations are as follows. First, these models only considered optimizing the bus operation schedule but ignored the tolerance of passengers regarding the occurrence of service within a given time window. Second, the boarding stations and the drop-off stations are always considered separately, which means that the drop-off passengers must be serviced after all the boarding passengers are aboard, but the actual situation is that several passengers may be serviced first; after they disembark, the bus goes to pick up other passengers. Third, the previous models only pursue the theoretical shortest travel time, but ignore the influence of subjective and objective factors, such as congestion and different personalities of the passengers. On the basis of the above, this paper establishes a bilevel programming model for the customized bus route optimization problem. The upper level aims to minimize the total operating cost and takes into account the time tolerance of the passengers. The lower level adopts a user equilibrium allocation model to describe the congestion and takes into account the optimistic and pessimistic characteristics of the passengers. Meanwhile, it embeds the traffic assignment algorithm of the lower-level model into a particle swarm optimization (PSO) algorithm and designs the corresponding particle coding method to make no distinction between the boarding and the drop-off stations.

The remainder of this paper is organized as follows. In Section II, the problem is described, and the model is constructed. In Section III, the corresponding PSO algorithm is described. In Section IV, the proposed model and algorithm are discussed, and their effectiveness is verified. Section V employs the proposed method in a numerical example and discusses the results. The last section concludes this paper with a brief summary.

II. PREPARATION PROBLEM DESCRIPTION AND MODEL ESTABLISHMENT

A. PROBLEM DESCRIPTION

Suppose there is a parking lot and several boarding stations and drop-off stations in the road network. Customized buses depart from the parking lot, provide service to each station and finally return to the parking lot. Each station can be serviced by any bus, but can only be serviced once. Each bus can service multiple stations, but each origin–destination (OD) station must be serviced by the same bus. As Figure 1 shows, the left is the structure of the road network, the solid and dashed lines indicate different routes between the stations, Origin (O) represents the boarding stations and Destination (D) represents the drop-off stations. On the right is the operation scheme. A bus departs from the parking lot. First, it services the boarding station O_1 and transits the passengers to the drop-off station D_1. Then, it services the boarding stations O_2 and O_3 in turn. After transiting the passengers to the corresponding drop-off stations, the bus continues to service the next OD stations O_4D_4, and after that, it returns to the parking lot.

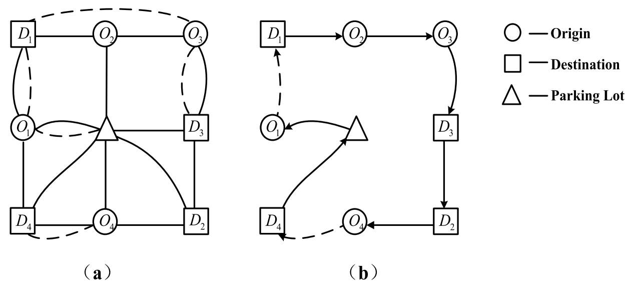

FIGURE 1. The road network.

B. MODEL ESTABLISHMENT

Define a road network $G = (M, N)$; the collection of boarding stations is $A_1 = \{i|\, i = 1, 2, 3, \cdots, m\}$, the collection of drop-off stations is $A_2 = \{j|\, j = 1, 2, 3, \cdots, n\}$, and the boarding stations and the drop-off stations are in pairs. Set the parking lot as A_3; then, the stations in the network can be described as $M = A_1 \cup A_2 \cup A_3$, and the road sections are $N = \{(i, j)|\, i \in M, j \in M\}$. Assume there is a certain initial passenger flow q_{ij}. The collection of the customized buses is $K = \{k|\, k = 1, 2, 3, \cdots, k_{\max}\}$, where $k_{\max}$ is the highest numbered customized bus, $h_{ij}^{(k)}$ represents the number of passengers on the $k - th$ customized bus from station i to station j, p_j represents the number of passengers getting on or off in station j. If the passengers get on the bus, $p_j = p_j$, and if they get off it, $p_j = -p_j$. C is the fixed cost of the bus, c is the unit mileage cost of operation, d_{ij} is the length between stations (i, j), Q is the passenger capacity of the bus, $x_{ij}^{(k)} = 1$ denotes the $k - th$ bus transit from station i to station j; otherwise, $x_{ij}^{(k)} = 0$, and $y_i^{(k)} = 1$ denotes the $k - th$ bus give service to station i; otherwise, $y_i^{(k)} = 0$. The travel time of the buses is determined by the passenger flows; we have $t_{ij} = t_0 \left[1 + \alpha (q_{ij}/Q_{ij})^{\beta}\right]$, where t_0 is the zero flow time of the road, Q_{ij} is the designed flow of passengers, and both α, β are impedance coefficients. T_k^l and T_k^b are the time when the $k - th$ bus departs from the parking lot and the time when it returns to it, respectively. $T_{\max}$ is the longest running time of the bus, $D_{\max}$ is the longest running mileage of the bus, $T_i^{(k)}$ is the actual time of the $k - th$ bus that arrives at station i, and $[E_i, L_i]$ is the passengers' tolerance time window at station

i. If the bus arrives outside the time window, corresponding penalty costs will be incurred, where φ and ψ are the penalty costs per unit time for early and late arrivals, respectively.

1) UPPER LEVEL MODEL

Considering the tolerance of passengers for the untimely arrival of buses, with the goal of minimizing operating cost, we can establish the following upper-level programming model:

$$\min Z = \sum_{i,j \in A_1 \cup A_2} \sum_{k \in K} x_{ij}^{(k)} c d_{ij} + \sum_{i \in A_1 \cup A_2, k \in K} C y_i^{(k)} + \sum_{i \in A_1 \cup A_2} \sum_{k \in K} [\varphi \max(E_i - T_i^{(k)}, 0) + \psi \max(T_i^{(k)} - L_i, 0)] \tag{1}$$

$$s.t. \sum_k y_i^{(k)} = 1; \quad \forall i \in A_1 \cup A_2, \ \forall k \in K \tag{2}$$

$$0 \le T^{(kb)} - T^{(kl)} \le T_{\max}, \quad \forall k \in K \tag{3}$$

$$\sum_{i,j} x_{ij}^{(k)} d_{ij} \le D_{\max}; \quad \forall i,j \in A_1 \cup A_2, \ \forall k \in K \tag{4}$$

$$h_{ij}^{(k)} + p_j \le Q, \quad \forall i,j \in A_1 \cup A_2, \ \forall k \in K \tag{5}$$

$$\sum_{i,j} \sum_k h_{ij}^{(k)} = \sum_j |p_j|/2, \quad \forall i,j \in A_1 \cup A_2, \ \forall k \in K \tag{6}$$

$$\sum_{i,j} x_{ij}^{(k)} - \sum_{i,j} x_{ji}^{(k)} = 0; \quad \forall i \in A_1, \ \forall j \in A_2, \ \forall k \in K \tag{7}$$

$$x_{ij}^{(k)} \in \{0, 1\}; \quad \forall i \in A_1, \ \forall j \in A_2, \ \forall k \in K \tag{8}$$

$$y_i^{(k)} \in \{0, 1\}; \quad \forall i \in A_1 \cup A_2, \ \forall k \in K \tag{9}$$

Among them, (1) is the objective function, which means minimizing the operating cost of the bus and the waiting cost of the passengers, (2) means that each station will be serviced and can only be serviced by one bus, (3) is the transit time constraints of the bus, (4) is the mileage constraints of the bus, (5) is the bus capacity constraints, (6) represents the relationship between the numbers of passengers in the road sections and at the stations, (7) is the flow balance constraints of the passengers, and (8) and (9) are the 0-1 constraints.

2) LOWER LEVEL MODEL

Regardless of the influence of two collinear bus lines, assume that the passengers will consider the fluctuation of travel time when they pursue the shortest time; that is, the travel time includes the normal travel time and the fluctuation time. The fluctuation time obeys a normal distribution and has different effects on passengers with different optimism levels. The ρ optimistic value of the travel time is $\min \lambda_{ij} = t_{ij}^{(k)} + s_{\text{sup}}$, the ϑ pessimistic value is $\max \lambda_{ij} = t_{ij}^{(k)} - s_{\inf}$, and $t_{ij}^{(k)}$ is the time impedance of road section (i, j), which is calculated by the function shown in (10). $s_{\text{sup}} = \sigma_{ij}^{(k)} \theta^{-1}(\rho)$ is denoted as the optimistic fluctuation time, $s_{\inf} = \sigma_{ij}^{(k)} \theta^{-1}(\vartheta)$ is the pessimistic fluctuation time, and $\sigma_{ij}^{(k)}$ is the variance of the fluctuation time distribution function, which is calculated by (11), in which $0 \le \xi_{ij} \le 1$ refers to the utilization rate of the capacity under congestion, $\theta(x)$ represents the standard normal distribution function, and ρ, ϑ are given confidence levels.

$$t_{ij}^{(k)} = t_0 \left\{ 1 + \alpha \left[\left(h_{ij}^{(k)} + q_{ij} \right) \Big/ Q_{ij} \right]^{\beta} \right\} \tag{10}$$

$$\left[\sigma_{ij}^{(k)}\right]^2 = \alpha^2 (t_0)^2 \left[h_{ij}^{(k)} + q_{ij}\right]^{2\beta} \cdot \left\{ \frac{1 - \xi_{ij}^{1-2\beta}}{(Q_{ij})^{2\beta} (1 - \xi_{ij})(1 - 2\beta)} - \left[\frac{1 - \xi_{ij}^{1-\beta}}{(Q_{ij})^{\beta} (1 - \xi_{ij})(1 - \beta)} \right]^2 \right\} \tag{11}$$

To further establish a compromise between extreme optimism and extreme pessimism, according to the Hurwicz optimism coefficient criterion, we assign weights λ and $1 - \lambda$ to the optimistic and pessimistic values. The result is shown in (12):

$$h = \lambda \left(t_{ij}^{(k)} + s_{\text{sup}} \right) + (1 - \lambda) \left(t_{ij}^{(k)} - s_{\inf} \right) \tag{12}$$

λ is the optimism coefficient, and $1 \ge \lambda \ge 0$; $\lambda = 0$ indicates extremely pessimistic, and $\lambda = 1$ indicates extremely optimistic.

Starting from the parking lot, we assume that there are $R = \{r | r = 1, 2, 3, \cdots, l\}$ routes to return to it, f_r is the passenger flows on route r, and $\delta_{ij}^{(r)}$ indicates whether route r passes through road section (i, j); if it does, $\delta_{ij}^{m} = 1$; otherwise, $\delta_{ij}^{m} = 0$. By setting the compromise value h in the flow assignment model, the lower-level model can be obtained:

$$\min Z = \sum_{(i,j)} \int_0^{q_{ij}} \left[\begin{array}{l} \lambda \left(t_{ij}^{(k)} + s_{\text{sup}} \right) \\ + (1 - \lambda) \left(t_{ij}^{(k)} - s_{\inf} \right) \end{array} \right] d\omega, \quad \forall k \in K, (i,j) \in N \tag{13}$$

$$s.t. \sum_r f_r = \sum_j |p_j|/2, \quad \forall j \in A_1 \cup A_2, \ \forall r \in R \tag{14}$$

$$h_{ij}^{(k)} = \sum_r f_r \delta_{ij}^{(r)}, \quad \forall i,j \in A_1 \cup A_2, \ \forall r \in R \tag{15}$$

$$f_r \ge 0, \quad \forall r \in R \tag{16}$$

$$\delta_{ij}^{(r)} \in \{0, 1\}, \quad \forall i,j \in A_1 \cup A_2, \ \forall r \in R \tag{17}$$

where (13) is the lower-level objective function, which evaluates integrals of the travel time in each road section, and its goal is to minimize the sum of the integrals; (14) indicates that the sum of the passenger flows on all the routes is equal to the total passenger flows in the network; (15) represents that the passenger flows on each road section equal the sum of the passenger flows on the route where the route passes through the road section; and (16) and (17) are nonnegative constraints and 0-1 constraints, respectively.

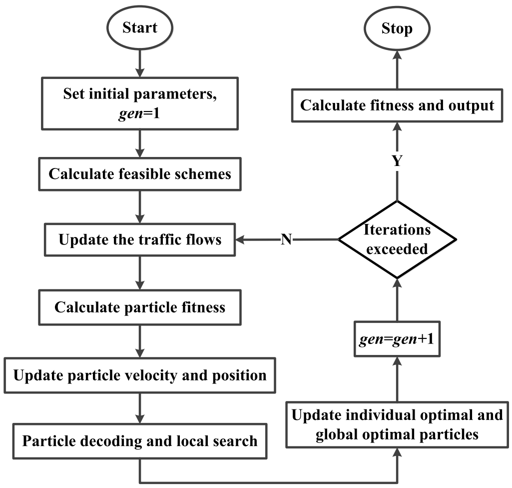

FIGURE 2. Flowchart of the algorithm.

III. SOLVING ALGORITHM

A. STEPS OF THE ALGORITHM

It has been verified that the customized bus route optimization problem is an NP-hard problem [11], and heuristic algorithms are the main methods. We embed the all-or-nothing assignment algorithm into the PSO algorithm. To solve the bilevel programming model, the algorithm flow chart is shown in Figure 2.

Step 1: Set the particle swarm size $N_{\max}$, learning factors c_1, c_2, inertia weight ω and inertia weight decay rate ω_d, and initialize the particle velocity and position. Set the initial passenger flows q_{ij}, and let $gen = 1$; then, calculate the feasible scheme.

Step 2: Feed back the calculation results to the lower level and adopt the all-or-nothing assignment method to obtain the new passenger flows.

Step 3: Calculate the particle fitness and update the particle speed and position according to (18) and (19).

$$X^{l+1} = X^l + V^{l+1} \tag{18}$$

$$V^{l+1} = \omega V^l + c_1 r_1 \left(P^l_{id} - X^l\right) + c_2 r_2 \left(P^l_{gd} - X^l\right) \tag{19}$$

l represents the algebraic term; X^l and V^l are the position and velocity of the lth generation particles, respectively; P^l_{id} is the individual optimal particle; and P^l_{gd} is the global optimal particle.

Step 4: Use the relocate method and the GENE method to perform a local search on the decoded route, reduce the solutions that violate the constraints [12], and calculate the new particle fitness value.

Step 5: Update the positions of the individual optimal and global optimal particles, compare the fitness of the current particle with the historical individual optimal fitness and the global optimal fitness, and replace them with the greater value.

Step 6: Update the inertia weight $\omega = \omega \times \omega_d$. The decay rate of the inertia weight is used to control the dynamic change of the inertia weight to prevent it from falling into local optima.

Step 7: If gen is less than the maximum number of iterations, update algebra $gen = gen + 1$ and return to Step 2; otherwise, proceed to Step 8.

Step 8: Update the global optimal particle and obtain the optimal solution.

TABLE 1. Example of particles.

OD	1 6	2 7	3 9	4 10
X_v	2	2	1	3
X_r	3	2	2	2
X_{rr}	3	0	0	1

B. PARTICLE ENCODING AND DECODING

Assume that there are L pairs of ODs, the number of buses is k, and the particles are constructed as a matrix with 3 rows and L columns. X_v in the first row adopts a random integer not greater than k, and it indicates the bus number. X_r in the second row indicates the service order of the boarding stations, and its value is represented by a random integer not greater than L; the higher the value of X_r is, the higher the service order [13]. X_{rr} in the third row represents the service order of the drop-off stations, and its value is also a random integer but not greater than the corresponding boarding station X_r. As shown in Table 1, the OD pairs 1-6 and 2-7 are serviced by the second bus, and the service order of each point is arranged in descending order. We obtain 3-3-2-0. When the orders are the same, the boarding point will be serviced first; then, the actual service order is 1→6→2→7, and the overall scheme is as follows:

Bus 1: 0→3→9→0;
Bus 2: 0→1→6→2→7→0;
Bus 3: 0→4 →10→0.

C. LOCAL SEARCH OPERATION

We adopt the relocate method to conduct local research, determine the pair of boarding and drop-off stations that have the largest time penalty cost, remove them and search for the new insertion points in all the routes [14]. The insertion principle is that the increased cost is the lowest and the constraints are met. As shown in Figure 3, the particles in Table 1 are decoded into 3 routes, and the pairs 1-6 of route 0→1→6→2→7→0 have the highest time penalty cost; these pairs are removed from the original route, and then the insertion position with the least increased cost is sought among the three routes; then, pairs 1-6 are reinserted to obtain a new route.

If there is no time penalty cost in the routes, the GENE method is used to ensure the diversity of the particles. Select two routes randomly, select any pair of boarding stations and drop-off stations in one route, and then insert them into the

TABLE 2. Result comparison.

Comparative indicators	PSO	GA
Bus route	Bus 1:0→2→8→6→12→0 Bus 2:0→4→11→5→10→3→9→0 Bus 3:0→1→7→0	Bus 1:0→6→2→7→9→0 Bus 2:0→3→12→0 Bus 3:0→4→8→0 Bus 4:0→5→1→11→10→0
Number of buses	3	4
Average passenger carrying coefficient	1.2	0.9
Total mileage (km)	78.8	79.5
Operation time (s)	46	21

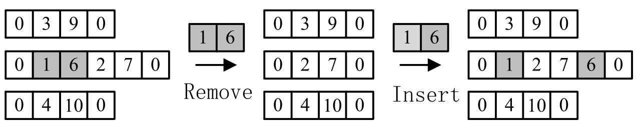

FIGURE 3. Relocate/exchange.

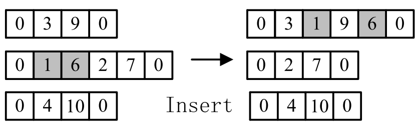

FIGURE 4. GENE.

other route. The insertion principle is that the increased cost is the lowest and the constraints are met. As shown in Figure 4, the above two routes are randomly selected, and points 1-6 in route 2 are randomly selected and then inserted into route 1 to obtain a new route.

IV. VALIDATION OF THE PROPOSED MODEL AND ALGORITHM

To verify the effectiveness of the model and algorithm, comparative experiments are carried out by using the numerical examples in the literature [15], selecting the parking lot a in the literature and setting the coefficients $\varphi = \psi = \rho = \vartheta = 0$ and $c = 1$. We can obtain the same scenario as in the literature. Each algorithm runs 20 times, and the optimal solution is selected for comparison. The results are shown in Table 2. Compared with the results in the literature, the proposed approach provides similar or better results: the total mileage is reduced by 0.7 km, the number of buses is reduced by 1, and the average passenger load factor is increased by 0.3. Due to the iterative feedback process of the bilevel algorithm, the overall calculation time is longer, but as Figure 5 shows, the convergence algebra is less than the three-stage coding genetic algorithm (GA) in the literature. Further analysis shows that buses service all the boarding stations first in the literature; after all the passengers on the boarding stations are collected in the bus, the bus starts to deliver passengers to the corresponding drop-off stations. However, according to the coding method proposed above, boarding stations and drop-off stations can be serviced in an interspersed manner. Taking bus 2 in Table 2 as an

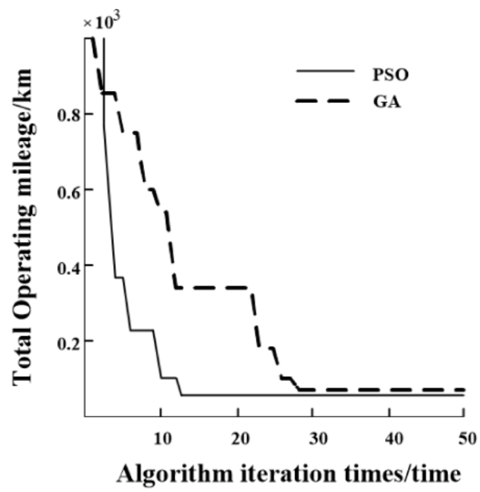

FIGURE 5. Actual network and stations.

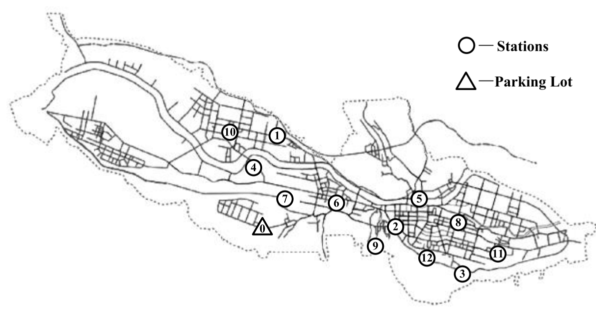

FIGURE 6. The algorithm compares the results.

example, starting from the parking lot, it gives service to boarding station 4. Although the capacity constraint has not been reached, the bus chooses to deliver passengers to the corresponding drop-off station 11; after that, it goes to give service to the next boarding station 5. Benefiting from the reasonable allocation, the capacity of one bus is saved.

V. CASE ANALYSIS

Taking the urban road network in Lanzhou, China, as Figure 6 shows, point 0 is the parking lot of customized buses, points 1-6 are boarding stations, and points 7-12 are the corresponding drop-off stations. The coordinates of the stations are shown in Table 3. The number of passengers at

TABLE 3. Coordinate of the stations.

Boarding stations				Drop off stations			
num	name	longitude	latitude	num	name	longitude	latitude
1	Lanxing Garden	103.7615	36.10059	7	Lanzhou West Railway Station	103.7601	36.07668
2	Century Square	103.8327	36.06061	8	Donghu Squsre	103.8799	36.05761
3	Lanzhou Railway Station	103.8571	36.04044	9	Provincial Opera House	103.8237	36.05438
4	Bamboo slips museum	103.7371	36.08273	10	Provincial Science and Technology Museum	103.7112	36.10461
5	Convention and Exhibition Center	103.8560	36.07499	11	Eastern market	103.8813	36.04889
6	Wenzhou mall	103.8011	36.07010	12	Jinlun Square	103.8480	36.04517

TABLE 4. *OD* demand information.

OD	1 7	2 8	3 9	4 10	5 11	6 12
Number of passengers (per)	15	13	21	23	10	13
Time window	8:00-8:20	8:05-8:15	8:00-8:15	8:10-8:15	8:10-8:20	8:05-8:20

TABLE 5. Results with different optimism coefficients.

λ	Bus route	T_0 (h)	$\overline{T}$ (h)	Total time (h)	Total cost (yuan)	Total distance (km)	$s_{\sup}^{0}$	$s_{\inf}^{0}$	h_0
0	Bus 1：0→2→5→8→11→0	[0.89,0.95]	0.94	2.58	578.02	95.51	0.11	0.09	0.16
	Bus 2：0→4→10→0	[0.46,0.51]	0.46						
	Bus 3：0→3→9→0	[0.38,0.43]	0.38						
	Bus 4：0→1→6→7→12→0	[0.80,0.86]	0.80						
0.2	Bus 1：0→5→2→8→11→0	[0.79,0.87]	0.82	2.46	560.06	95.10	0.17	0.12	0.31
	Bus 2：0→4→10→0	[0.38,0.54]	0.46						
	Bus 3：0→3→9→0	[0.30,0.51]	0.38						
	Bus 4：0→1→6→7→12→0	[0.76,0.89]	0.80						
0.4	Bus 1：0→4→1→7→10→0	[0.58,0.75]	0.68	2.13	498.82	88.13	0.12	0.15	0.68
	Bus 2：0→5→3→9→11→0	[0.61,0.83]	0.77						
	Bus 3：0→2→6→8→12→0	[0.69,0.85]	0.71						
0.6	Bus 1：0→4→1→10→7→0	[0.52,0.73]	0.60	1.92	483.56	80.34	0.16	0.21	0.88
	Bus 2：0→6→3→5→12→9→11→0	[0.73,0.90]	0.81						
	Bus 3：0→2→8→0	[0.32,0.59]	0.51						
0.8	Bus 1：0→4→1→7→10→0	[0.42,0.77]	0.68	1.98	433.88	79.54	0.24	0.28	1.24
	Bus 2：0→5→11→0	[0.45,0.75]	0.56						
	Bus 3：0→6→3→12→2→9→8→0	[0.68,0.93]	0.74						
1	Bus 1：0→4→1→10→7→0	[0.58,0.75]	0.60	1.76	429.76	76.26	0.26	0.25	1.31
	Bus 2：0→5→11→0	[0.61,0.83]	0.56						
	Bus 3：0→6→3→12→2→9→8→0	[0.69,0.85]	0.74						

each station and their service time windows are shown in Table 4. Assume the service time at each station is 1 minute, and the capacity of the customized bus is 40 people/bus. The operating cost of the bus is 1.8 yuan/km, and the utilization rate of the road section capacity ξ_{ij} is 0.9 under congestion conditions.

The model is solved by MATLAB 2018a programming. Set particle swarm size $N_p = 50$, maximum number of iterations $gen = 50$, learning factor $c_1 = 1.5$, $c_2 = 2.0$, inertia weight $\omega = 1$, inertia weight decay rate $\omega_d = 0.9$, violation capacity coefficient $L_d = 10$, unit penalty cost $\varphi = \psi = 100$, impedance coefficient $\alpha = 0.15$, $\beta = 4$, confidence

$\rho = \vartheta = 0.95$. The optimal solutions under different optimism coefficients are shown in Table 5.

T_0 represents the estimated travel time, where the upper limit is $T_{\sup}(\rho) = \max\left[T_k^b(\rho) - T_k^l(\rho)\right]$, the lower limit is $T_{\inf}(\vartheta) = \min\left[T_k^b(\vartheta) - T_k^l(\vartheta)\right]$, $\overline{T}$ is the average travel time, $s_{\sup}^0 = \frac{T_{\sup}(\rho)-\overline{T}}{\overline{T}} \times 100\%$ is the optimistic time risk index, $s_{\inf}^0 = \frac{\overline{T}-T_{\inf}(\vartheta)}{\overline{T}} \times 100\%$ is the pessimistic time risk index, and $h_0 = \frac{h}{\overline{T}} \times 100\%$ is the time compromise index.

According to the calculation results, when $\lambda \leq 0.2$, the number of deployed buses is 4, which incurs an obvious waste of transportation capacity. When $\lambda = 0.4$, the number of deployed buses decreases to 3, and the transportation capacity of the 3 buses is equivalent. When $\lambda \geq 0.6$, the number of buses remains 3, and the transportation capacity of the third bus is not exhausted. The overall transportation time and distance monotonically decrease with increasing optimism coefficient. In fact, the greater the optimism coefficient is, the more operation schemes are available, and the operation department can choose better schemes with fewer buses, shorter distance, and less time to achieve the purpose of reducing costs. However, the excessive pursuit of the lowest cost will cause confusion among riders and lower occupancy of the buses, which is also undesirable. In the above case, the scheme with $\lambda = 0.4$ is most recommended for the operation department.

For passengers, when $\lambda = 0$, its optimism risk index and pessimism risk index are the smallest, and with the increase in λ, the risk borne by passengers will also increase, which is more likely to cause travel delays; surely, the spending by passengers will also be reduced accordingly. Because different risk indices have different evaluation angles, when the risk index is insufficient to provide a judgment basis for passengers, we adopt the time compromise index h_0. Taking Table 5 as an example, when $\lambda = 0.4$, the optimism risk index is the smallest, and when $\lambda = 0.2$, the pessimism risk index is the smallest; they are both 0.12. Further comparing the time compromise index h_0, it can be seen that when $\lambda = 0.2$, the corresponding h_0 is smaller; therefore, the corresponding bus operation scheme is more reliable.

In addition, to investigate the specific value of the optimistic coefficient, we divide travelers into four attributes namely work, life, back, and entertainment according to their travel purposes, set the corresponding optimistic coefficients as 0.2, 0.4, 0.6 and 0.8 respectively, because the proportion of various travel purposes is completely unknown, we adopt the decision method based on information entropy [16]. Figure 7 shows the distribution of the travelers with different attributes on each station, calculate the entropy weight of each attribute, and then use the weighted summation method, we can obtain the optimistic coefficient as 0.4937.

In fact, assume that the service frequency is 20min, according to their own description of travelers, the distribution of acceptable time is shown in Figure 8, where the blue area indicates the acceptable time range, and it accounts 48% of the total area. This also reflects that the optimistic coefficient of 0.4937 selected by the above method is effective.

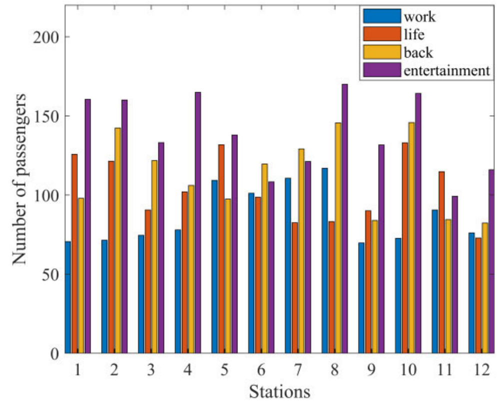

FIGURE 7. Distribution of the travelers.

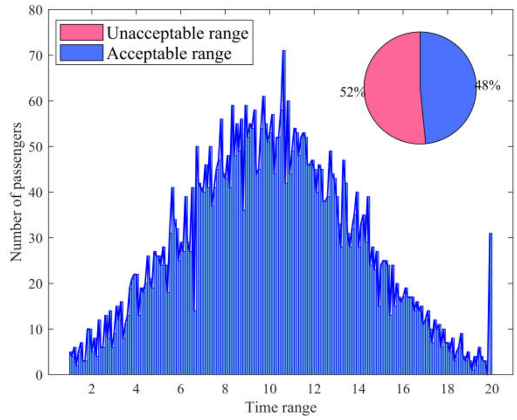

FIGURE 8. Distribution of the acceptable time.

VI. CONCLUSION

To save transportation capacity resources, this paper proposes a new coding method that enables customized buses to provide services to boarding stations and drop-off stations alternately. On this basis, the relationship between passengers' optimism and customized buses' operation scheme is studied, and it is found that with different levels of passenger optimism, different Nash equilibria will be reached between passengers and the operation department; the higher the passengers' optimism is, the lower the bus operation cost. However, with the reduction in bus operation cost, the probability of delay will also increase, which must be considered by bus operation departments and passengers. And it also gives a method to adopt the optimistic coefficient by a numerical example, which provides reference for actual operation.

It shows the practical significance that from the perspective of customized bus operation departments, they can determine the optimism coefficient according to the lifestyle and values of passengers in different regions to achieve the balance between passengers' optimism and the operation cost. For example, passengers in an enterprise park have high requirements for punctuality and are not very sensitive to ticket prices; then, the scheme with an optimistic coefficient of 0.2-0.6 is suitable, whereas passengers in the urban fringe do not have high requirements for punctuality. The scheme with an optimism coefficient of 0.4-0.8 can be adopted to reduce operation costs. From the perspective of passengers, on the premise of the operation scheme, they should choose the route with a smaller risk index and compromise index to obtain more reliable travel.

REFERENCES

[1] C. Ma, C. Wang, and X. Xu, "A multi-objective robust optimization model for customized bus routes," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 4, pp. 2359–2370, Apr. 2021.

[2] C. X. Ma and X. C. Xu, "Providing spatial–temporal priority control strategy for BRT lanes: A simulation approach," *J. Transp. Eng., A, Syst.*, vol. 146, no. 7, pp. 2473–2907, Jul. 2020.

[3] C. X. Ma and D. Yang, "Public transit network planning in small cites considering safety and convenience," *Adv. Mech. Eng.*, vol. 12, no. 1, pp. 1–12, Jan. 2020.

[4] T. Liu and A. Ceder, "Analysis of a new public-transport-service concept: Customized bus in China," *Transp. Policy*, vol. 39, pp. 63–76, Apr. 2015.

[5] X. L. Lu and S. L. Pan, "An optimization method of hybrid flexible feeder transit service to rail stations," *J. Transp. Syst. Eng. Info. Tech.*, vol. 19, no. 4, pp. 155–163, Apr. 2019.

[6] S. P. Parvasi, M. Mahmoodjanloo, and M. Setak, "A bi-level school bus routing problem with bus stops selection and possibility of demand outsourcing," *Appl. Soft Comput.*, vol. 61, pp. 222–238, Dec. 2017.

[7] Q. P. Wang and M. Ding, "Bi-level programming model of traffic microcirculation optimization for historic districts," *J. Traffic Transp. Eng.*, vol. 16, no. 3, pp. 125–132, Jun. 2016.

[8] L. Tong, L. S. Zhou, J. T. Liu, and X. S. Zhou, "Customized bus service design for jointly optimizing passenger-to-vehicle assignment and vehicle routing," *Transp. Res. C, Emerg. Technol.*, vol. 85, pp. 451–475, Dec. 2017.

[9] C. Ma, C. Wang, W. Hao, J. Liu, and Z. Zhang, "Emergency customized bus route optimization under public health emergencies," *J. Traffic Transp. Eng.*, vol. 20, no. 3, pp. 89–99, Jun. 2020.

[10] G. Qiu, "Research on optimizing design method of customized bus route based on passenger travel mode choice," Ph.D. dissertation, School Traffic Transp., Beijing Jiaotong Univ., Beijing, China, 2019.

[11] R. G. Guo, W. Guan, W. Y. Zhang, and M. Y. Duan, "Customized electric bus routing optimization considering multi-path selection," *J. Transp. Syst. Eng. Info. Tech.*, vol. 21, no. 2, pp. 133–138, Apr. 2021.

[12] C. Ma, P. Liu, and X. Xu, "Vehicles robust scheduling of hazardous materials based on hybrid particle swarm optimisation and genetic algorithm," *IET Intell. Transp. Syst.*, vol. 14, no. 14, pp. 1955–1966, Dec. 2020.

[13] L. Tao and C. X. Ma, "Multiobjective robust optimization model and algorithm for customized bus," *Western China Commun. Sci. Tech.*, vol. 126, no. 1, pp. 174–180, Jan. 2018.

[14] R. Liu and Z. Jiang, "A hybrid large-neighborhood search algorithm for the cumulative capacitated vehicle routing problem with time-window constraints," *Appl. Soft Comput.*, vol. 80, pp. 18–30, Jul. 2019.

[15] C. Wang and C. X. Ma, "Optimization of parking lot and multi-vehicle route of customized buses based on genetic algorithm," *J. Transp. Info. Saf.*, vol. 37, no. 3, pp. 109–117, Jun. 2019.

[16] Z. S. Xu, *Uncertain Multiple Attribute Decision Making: Methods and Applications*. Beijing, China: Tsinghua Univ. Press, 2004.

FAN CHEN received the B.S. degree in architecture from Lanzhou Jiaotong University, Lanzhou, in 2011, and the M.S. degree in architecture from the Beijing University of Technology, Beijing, in 2015. She is currently pursuing the Ph.D. degree with Lanzhou Jiaotong University. Her current research interests include urban traffic planning and urban spatial planning.

YOUPENG LU received the B.S. degree in engineering management from Chang'an University, Xi'an, in 2011, and the M.S. degree in transportation planning and management and the Ph.D. degree in management science and engineering from Lanzhou Jiaotong University, Lanzhou, China, in 2014 and 2020, respectively. He is currently a Lecturer with Lanzhou Jiaotong University. His current research interests include operation research, transportation organization, uncertain modeling, and optimization.

LINZHONG LIU received the B.S. degree in mathematics from Northwest Normal University, Lanzhou, in 1986, the M.S. degree in applied mathematics from Lanzhou Jiaotong University, Lanzhou, in 1993, and the Ph.D. degree in operational research and cybernetics from Tsinghua University, Beijing, China, in 2006. He is currently a Professor with Lanzhou Jiaotong University. His current research interests include operation research and uncertainty theory.

QING ZHU is currently a Professor with Southwest Jiaotong University, Chengdu. He is within Lanzhou Jiaotong University, Lanzhou. His current research interest includes geo information science.