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METHODS

Re-Establishing a Lost Connection: Multi-Value Logic in Causal Data Analysis in Social Science Disciplines

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ABSTRACT The main purpose of logic optimization lies in architecting integrated switching circuits. It is thus a topic well covered in electrical engineering. Some subfields of the social sciences have also employed algorithms for logic optimization since the mid-1980s to infer about cause-effect relations. Most notably, political scientists and sociologists have developed Qualitative Comparative Analysis (QCA), a configurational comparative method (CCM) that relies on the well-known Quine-McCluskey algorithm. However, while electrical engineering has progressed considerably since the advent of logic optimization in the 1950s, these advancements have not been monitored by social scientists. Nor have social scientists sought to establish interdisciplinary collaboration. The objective of our article is twofold. First, and more generally, we seek to build a bridge between electrical engineering and configurational causal inference. Secondly, and more specifically, we present Combinational Regularity Analysis (CORA), a new CCM that has been inspired by electrical engineering. In particular, we introduce one of CORA's algorithms for optimizing highly unspecified multi-value logic functions with multiple outputs. The availability of such algorithms in CORA pushes the boundaries of configurational causal inference and attests to the extent to which configurational comparative methodology could benefit from more interdisciplinary collaboration with electrical engineering.

INDEX TERMS Algorithms, causal inference, configurational comparative methods, CORA, interdisciplinarity, logic optimization, multi-value logic.

I. INTRODUCTION

Finding an optimal solution to a scientific problem with societal implications often requires collaboration between the natural, technical and social sciences. At the same time, interdisciplinary collaboration is fraught with challenges [1]. One obstacle is the increasing level of specialization in modern science, which creates growing divisions among researchers even within the same discipline. Nowadays, only few scientists have an understanding of work being conducted outside their specialty, even if that work may be pertinent to their own [2]. In addition, cultural hierarchies that privilege the technical and natural sciences over the social sciences as well as perceptions of interdisciplinarity as an impediment to career advancement act as crucial barriers [3].

When engineering and social science are discussed in an interdisciplinary context, the focus usually is on the value the latter can bring to the former. Good engineering ideas have sometimes foundered on simple failures to appropriately understand and engage with those within society meant to utilize the technology [4]. Such failures could have been avoided if engineers and social scientists had collaborated at an early stage. Yet, a disconnected mode of research can also be disadvantageous for social scientists when, for example, they import procedures that have originally been developed in engineering without a full understanding of these procedures' technicalities.

Most notably, in the mid-1980s, US sociologist Charles C. Ragin swiftly imported the Quine-McCluskey algorithm

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(QMC; [5], [7]) from electrical engineering because he believed that QMC could not only be used for optimizing logic functions, but also for discovering certain cause-effect relations in social-scientific data [8]. In that context, Ragin called his new method Qualitative Comparative Analysis (QCA; [9]). More specifically, Ragin intended QCA to become a method in which "*causes are viewed as INUS conditions: insufficient but necessary components of unnecessary but sufficient combinations of conditions*" [10], pp. 431-432. By employing an algorithm that was originally developed for a very different purpose in electrical engineering, QCA was thus meant to uncover a specific form of causal relations that analytical philosophers have subsumed under the INUS Theory of Causation [11], [12], [13], [14].[1] QCA has since joined the larger class of Configurational Comparative Methods (CCMs) and has made major inroads into many social-scientific fields of research, including sociology, political science, management, business, international relations, environmental studies and public health [15]. It would certainly not be exaggerated to say that QCA has become a revolution in social research methodology [16].

At the same time, the connection between the QCA research community and electrical engineering has always been unidirectional. While QCA has imported its core algorithmic procedure from electrical engineering and its theory of causation from analytical philosophy, none of these two fields has ever been involved in QCA's development over the last 35 years. It may, therefore, come as no surprise that, over the last ten years, researchers have revealed numerous methodological problems in QCA which an early involvement of electrical engineers—and analytical philosophers versed in theories of causation, for that matter—could have helped to prevent.

Three problems are emblematic of QCA's disconnected mode of development. First, it has been discovered that various of its associated software packages do not generate all solutions that are compatible with an understanding of cause-effect relations in terms of INUS structures [17], [18], [19], [20]. The simple reason is that the criterion of row dominance, which is used in engineering applications of QMC to directly generate only the minimal sum instead of all irredundant sums, has also been imported by Ragin. However, no QCA methodologist had ever evaluated whether compliance with the INUS Theory would require the generation of all irredundant sums or only the minimal sum.

A second, major point of criticism against QCA relates to the manipulation of don't care-terms, simply called d-terms hereafter. When Ragin imported QMC in the mid-1980s, he immediately took issue with QMC's use of d-terms, that is, terms in the function to be optimized whose output value was undetermined. For optimization proper, QMC requires unrestricted access to 1-terms and d-terms in its operations [5]. Yet, Ragin believed that whenever QMC incorporated a d-term, the algorithm would risk having to make an assumption that social scientists may find implausible, namely that this d-term was associated with the presence of the analyzed outcome, on a par with a regular 1-term. Such d-terms he thus called "difficult counterfactuals", which, he argued, should be redefined as a 0-term, that is, a term for which the function to be optimized was negative, and which QMC therefore could not use [21], p.162. Later generations of QCA methodologists devised further procedures that built on Ragin's concerns about d-terms (e.g., [22]). Instead of avoiding implausible assumptions, however, the manual relocation of d-terms to the set of 0-terms alters the original function completely, such that QCA would output causal inferences that were not only unsupported by the empirical data to be analyzed, but that, in consequence, would also lead to (extremely) high rates of false positives [23], [24]. The exposure of this algorithmic tweak in QCA has led to a bewildering array of new proposals for QCA's search target that often completely break with Ragin's original goal of identifying INUS structures (e.g., [25], [26], [27], [28], [29]). Again, the involvement of electrical engineers early on in QCA's development would have helped social scientists to fully understand the effect of manipulations of d-terms on QMC's output, and thus avoid the utter state of confusion that now prevails some 40 years after the method's introduction.

Third, the analytical capabilities of QCA have largely stagnated since the mid-1980s. Until today, for instance, QCA remains restricted to the analysis of one effect only. Although some tentative attempts at removing this restriction have been made [30], the possibility that data may contain evidence for the existence of more than one effect has never been put on QCA's methodological agenda. This omission cannot be due to the fact that social-scientific data do not feature more than one possible effect. Many QCA studies published as early as in the mid-1990s have listed several distinct effects as part of the same set of data, but were eventually forced to analyze each one of them in isolation (e.g., [31], [32], [33], [34], [35]). In contrast, extensions of QMC for multi-output optimization have been known in electrical engineering at least since the early 1960s (e.g., [36], [37]), yet without any attempt of interdisciplinary collaboration or at least an attempt to monitor basic developments in electrical engineering, the possibility to analyze multiple effects in QCA has never been realized.

Against the background of a missing connection between electrical engineering and configurational causal inference, we pursue two related objectives with this articles—a general one and a more specific one. First, and more generally, we seek to build an explicit bridge between electrical engineering and configurational causal inference. Secondly, and more specifically in this regard, we present Combinational Regularity Analysis (CORA; [38]), a new CCM that incorporates multi-value logic and algorithms for multi-output optimization. CORA is *combinational* because its technical core procedures have originally been developed in the area of combinational switching circuit design, a subfield of electrical engineering. In parallel, CORA contains the term

[1] An introduction to the INUS Theory of Causation is beyond the scope of our article. The best introduction remains the original work of John Mackie [13].

regularity because the INUS Theory of Causation, which also provides the epistemological basis of CORA, belongs to the larger group of regularity theories of causation [14]. In particular, we introduce one of CORA's algorithms for optimizing highly unspecified multi-value logic functions with multiple outputs. The availability of such algorithms in CORA pushes the boundaries of configurational causal inference and attests to the high potential of interdisciplinary collaboration between social research methodology and electrical engineering.

Our article is structured as follows. Section II revisits some of the advances that have already been made in the QCA literature towards the handling of multi-value problems. Section III presents a short history of multi-value logic (MVL). Section IV provides all necessary definitions. In Section V, we introduce one of CORA's optimization algorithms for handling multi-value multi-output functions. In Section VI, we provide two examples from applied research to show how this algorithm can improve configurational research. Our conclusions are presented in Section VII.

II. MULTI-VALUE LOGIC OPTIMIZATION IN CONFIGURATIONAL COMPARATIVE METHODOLOGY

At its introduction in the mid-1980s, QCA could process only binary factors, as in classical two-valued logic. Following criticism that it could not handle continuous variables, Ragin himself proposed the first extension of QCA in [39] by introducing a variant that built on fuzzy-set theory. In [21], he presented a modified version of that variant. To distinguish the original variant of QCA from its later extensions, the former has since been called crisp-set QCA (csQCA) and the latter fuzzy-set QCA (fsQCA). A third variant that could process multi-value data—called multi-value QCA [40]—joined the QCA family in the mid-2000s. A fourth variant, referred to as generalized-set QCA (gsQCA; [41]), which united all three other QCA variants as special cases, was developed in theory but has never been implemented in software. Thus, multi-value logic has, in fact, already been present in QCA for about 20 years.

Against this backdrop, it is striking that the distribution of applications of csQCA, fsQCA and mvQCA in empirical research has been heavily skewed towards csQCA and fsQCA. A comprehensive dataset of empirical QCA studies identifies only 19 applications of mvQCA among a total of 915 QCA applications that have been published in decent peer-reviewed scientific journals between 1984 and 2017 [15]. This mismatch is surprising because mvQCA is appreciably more powerful than its two cousins. Although fsQCA allows researchers to incorporate continuous variables, this advantage is lost again at the stage of constructing QCA's truth table. Both csQCA and fsQCA can only output solutions that allow inferences about binary factors. In contrast, mvQCA permits more fine-grained inferences about multi-value factors. Methodological reviews have explained mvQCA's niche existence by the absence of promotion by Ragin as QCA's most prominent representative and the outright rejection of mvQCA by many other QCA methodologists [42], [43], [44].

Against a pronounced trend of rejecting mvQCA in the mainstream QCA literature, it is also interesting that the authors of several empirical mvQCA studies not only considered the use of multi-value logic beneficial for advancing their research, but that they also already included several endogenous factors in their data [31], [33], [45]. Given that QCA has not been able to analyze more than one output, these mvQCA studies were still forced to process each output separately and independently of the others.

In summary, neither multi-value logic nor algorithms for multi-output optimization have been items on QCA's developmental agenda. In the next section, we briefly trace the appearance of multi-value logic in electrical engineering.

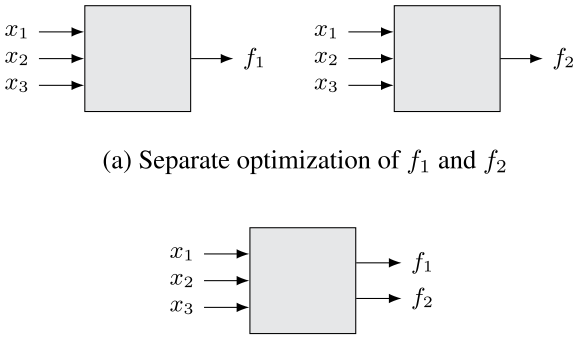
III. A BRIEF HISTORY OF MULTI-VALUE LOGIC

The roots of the development MVL can be traced back to the first two decades of the twentieth century. In 1920, Łukasiewicz argued that every concept has a truth value, as opposed to a classical binary view that every proposition is either true or false. Łukasiewicz thus extended classical binary logic to a trivalent framework, where prepositions have an intermediate truth value reflecting uncertainty. Already a year later, Emil Post developed a many-valued propositional calculus [46].[2]

The 1930s saw the flourishing of ternary logics, mainly by Bochvar, Gödel, De Finetti and Kleene. These frameworks gave different interpretations to the third value in Łukasiewicz's system. For example, while Łukasiewicz referred to the third value as *undetermined*, Bochvar named it *undecidable*, and Kleene *unknown*. The difference in the notion of this value led to different definitions of logical connectives, and in turn to important differences in their implications. For example, Bochvar connectives are more conservative and every compound proposition containing at least one undecidable is undecidable, while the rules for true and false components are exactly the same as in binary logic.

Over the next two decades, the focus of research eventually shifted towards the extension of ternary logics to MVL proper. Researchers were increasingly working at the intersection of logic, computer science and neural science (see [48] for a detailed review). While early contributions to MVL primarily came from mathematical logic and analytical philosophy (Łukasiewicz, Bochvar, Kleene), starting from the late 1960s, advancements have mostly been driven by computer science and electrical engineering, with a focus on developing algorithms for optimizing multi-value logic functions that could improve computational technology (cf. [49], [50]).

[2] [47] notes that a decade before Łukasiewicz and Post published their respective works, Peirce already established the main concepts in his unpublished work on what he called triadic logic. Thus [47] credits the origins of ternary logic to Peirce without discrediting the works of Łukasiewicz and Post as they both were unaware of Peirce's earlier results.



(a) Separate optimization of f_1 and f_2

(b) Joint optimization of f_1 and f_2

FIGURE 1. Optimization of a system of two functions as (a) two separate systems and (b) as one system.

In sum, researchers have emphasized two main advantages of MVL over binary logic [49]. First, MVL leads to a deeper understanding of certain logic problems, and second, it can help to implement binary logic in a more efficient way. For more comprehensive introductions to the theory, definitions and properties of MVL, we refer readers to [49], [51], [52], or [53]. In the next section, we introduce the additional notion of multi-output optimization and connect it to MVL.

IV. FUSING MULTI-VALUE LOGIC WITH MULTI-OUTPUT OPTIMIZATION

Not only the development of MVL has lead to quantum leaps in logic design and optimization. In many large digital systems, it is also frequently necessary to implement several switching functions that share the same inputs. Consequently, the optimization of multi-output logic functions has become an important area of research as well. Fig. 1 shows two possible approaches to the optimization of multiple logic functions: (a) a system of two functions f_1 and f_2 that share the same inputs could be optimized via the separate optimization of each component function, or (b) f_1 and f_2 could be optimized jointly.

If x_1, x_2 or x_3 is a multi-value variable, then we speak of multi-value multi-output optimization. Below, we define those concepts that are essential for the remainder of our article.

Definition 1: An n-variable m-valued logic function is a mapping $M^n \rightarrow M$ over a finite set of values $M = 0, 1, 2, \ldots, m-1$.

Definition 2: An incompletely specified logic function f with at least one multi-value input and a single binary-value output is a mapping $f(x_1, x_2, \ldots, x_n) : P_1 \times P_2 \times \ldots \times P_n \rightarrow Q$, where x_i is a multi-value variable, $P_i = \{0, 1, \ldots, p_i - 1\}$ is a set of values that this variable may take on, and where $p_i \geq 1$ and $Q = \{0, 1, -\}$, with "$-$" denoting the value *undetermined*.

The *or*-operator "$+$" and the *and*-operator " $\cdot$ " directly carry over to multi-value functions. However, the unary *not*-operator needs to be redefined such that it can distinguish between m values of a multi-value variable [49].

Definition 3: A literal of a multi-value variable x is a unary operation defined by

$$x\{S\} = \begin{cases} 1, & \text{if } x \in S \\ 0, & \text{otherwise,} \end{cases}$$

where $S \subseteq P$. When $m = 2$, then the notation of the binary literals becomes $x^{\{1\}} = x$ and $x^{\{0\}} = x'$ as in classical binary logic. A literal is a characteristic function of type $P \rightarrow \{0, 1\}$, which implies that for multi-value functions with a binary-value single output, operations on literals are Boolean operations of type $\{0, 1\}^n \rightarrow \{0, 1\}$. Therefore, any such function can be represented in terms of *and*, *or* and multi-value literals.

Definition 4: Let $S_i \subseteq P_i$, with $i = 1, 2, \ldots, n$; then, $x_1^{S_1} \cdot x_2^{S_2} \cdots x_n^{S_n}$ is called a product term.

Definition 5: The result of an (inclusive) *or*-operation on product terms $\sum_{S_1, S_2, \ldots, S_n} x_1^{S_1} \cdot x_2^{S_2} \cdots x_n^{S_n}$ is called a sum of product (SOP).

Any multi-value function with a binary-value single output can be represented by an SOP, and there may exist many SOPs representing the same function.

A multi-value function is uniquely defined by its truth table, which contains m^n rows if each of the n inputs has the same m-base. However, inputs need not all have the same base. For example, some variables can be binary, others ternary. More generally, the size of the truth table is determined by the product of the base values of all inputs. For example, if data contains 4 binary and 2 ternary inputs, the truth table will have $2^4 * 3^2 = 144$ rows.

Definition 6: An incompletely specified n-input logic function f with at least one multi-value input and k binary-value outputs is a mapping $P^n \rightarrow Q^k$, where $Q = \{0, 1, -\}$.

The main computational distinction between the optimization of single-output and multi-output functions lies in the search process for the prime implicants (PIs). For multi-output functions, it is not sufficient to consider only the PIs of each output function in isolation, as sketched in panel (a) of Figure 1. A more economical expression will result when each function as well as each product of functions is considered jointly, as in panel (b) of Figure 1. In optimizing a system of multiple outputs, it is thus necessary to generate the PIs of each individual function in addition to the PIs of all possible products of functions. The resulting PIs are called multi-output PIs (MOPIs).

Definition 7: A multi-output prime implicant (MOPI) of a system of switching functions $\mathbf{F} = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\}$ of a set of multi-value inputs $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ is a product of literals $x_{1;i}^{\{\cdot\}} x_{2;i}^{\{\cdot\}} \cdots x_{h;i}^{\{\cdot\}}$ with $h \leq n$ and $1 \leq i_j \leq n$, which is either a PI of some $f_j \in \mathbf{F}$ with $j = 1, 2, \ldots, k$ or a PI of one of the product functions $f_1(\mathbf{x}) f_2(\mathbf{x}) \cdots f_k(\mathbf{x})$.

The notion of usefulness and uselessness of a PI can be directly applied to MOPIs as well. A MOPI that is a component of an irredundant sum is called useful, one that is no component of any irredundant sum is called useless. A PI

TABLE 1. Data on public-private partnership contracts for toll roads from [33].

	Inputs					Outputs	
Cases	*pric*	*leng*	*upsi*	*dosi*	*risk*	*frfl*	*tort*
ROUTE460	2	0	1	0	1	0	0
I595	1	1	0	0	0	1	0
I495	1	2	1	0	0	1	0
SKYWAY	2	2	0	0	0	0	0
SR91	1	1	1	0	0	1	0
SH121	1	2	1	1	0	1	1
WARNOW1	0	1	0	0	1	0	1
WARNOW2	2	2	0	0	1	0	1
HERREN	2	1	0	0	1	0	0
SKYE	0	0	0	1	1	0	1
LISBON	0	0	0	0	0	0	1
MADRID	0	1	0	0	0	0	1
ELMELON	2	1	1	0	1	0	0
SANTIAGO	2	0	0	1	0	0	1
407ETR1	1	1	0	0	1	1	0
407ETR2	2	2	0	0	0	0	0
CONFED	0	1	0	1	1	0	1
CROSCITY	2	1	0	0	1	0	0

that is a necessary component of an irredundant sum is called essential, one that is no necessary component inessential.

For configurational causal inference, the objective function of optimization algorithms must be set to sum irredundancy instead of sum minimality. with single-output data the optimization target is the set of all irredundant SOPs that are equivalent to the original function. With multi-output functions, the concept of irredundancy needs to be generalized to a system level. The optimization target becomes the set of irredundant systems [54].

Definition 8: An **F**-equivalent system of switching functions $\mathbf{S_F}$ is called an irredundant system $\mathbf{S^*_F}$ if it is impossible to cancel any literal in the writing of its MOPIs and any MOPI in the writing of its switching functions f_j and still be able to ensure **F**-equivalence.

In the literature on CCMs, the existence of multiple models has been referred to as model ambiguity [17], [18], [20]. As CORA works on the higher level of systems of functions, this concept is generalized to system ambiguity. System ambiguity implies that there may exist many systems that can potentially explain the causal structure behind a set of data. Each system comprises as many functions as there are outputs, but these functions are not alternatives to each other, whereas different systems are.

In the next section, we present one of CORA's optimization algorithms for generating the set of all irredundant systems from a system of multi-value logic functions.

V. CORA: A NEW CONFIGURATIONAL METHOD FOR MULTI-VALUE MULTI-OUTPUT LOGIC OPTIMIZATION

To derive the set of irredundant systems for multi-value multi-output functions, CORA draws on but extends McCluskey's algorithm for highly unspecified functions [6]. For the sake of brevity, in the remainder of our article, we refer to the original version of this algorithm as MC and to our extended version as MC′. The choice of MC is based on the considerations that it has been developed for optimization problems with relatively large d-sets, which represent the rule rather than the exception for multi-value functions. We first discuss our generalization to multi-value single-output functions, subsequently to systems of multi-value functions.

The flowcharts in Fig. 2 and Fig. 3 visualize the procedural protocol for MC′ for deriving essential and inessential PIs for single functions. The flowchart in Fig. 4 shows the procedural protocol for deriving MOPIs. We illustrate each step in these charts in the next section with two empirical examples. Essentially, there are three main differences between MC and MC′.

1) To derive the essential PIs in MC, the rows of the on-set (the set of 1-terms)—called C-matrix or just C, for short—and the rows of the off-set (the set of 0-terms)—called N-matrix or just N, for short—are grouped according to the number of 1s. In MC′, the data are grouped by the number of non-zero entries (Step 2 in the flowchart in Fig. 2).
2) To derive the essential PIs in MC, each row in C is combined with each row in N belonging to one higher or one lower weight group and having a difference on one input only. In MC′, however, because the same weight groups in C and N can also have rows with a difference on only one input (because of multi-value entries), in addition to one more and one less weights, each row in C is combined also with a row in N of the same weight and having a difference on one input (Step 3 in the flowchart in Fig. 2).
3) In MC, to derive the PIs for systems of functions, every matrix derived from combining a row C with all rows of N is first reduced and then multiplied out. This reduction process involves two components: (1) when two or more rows are identical, the identical rows are removed, and (2) if one row is dominated by another row, it is removed from the matrix. In MC′, the reduction process

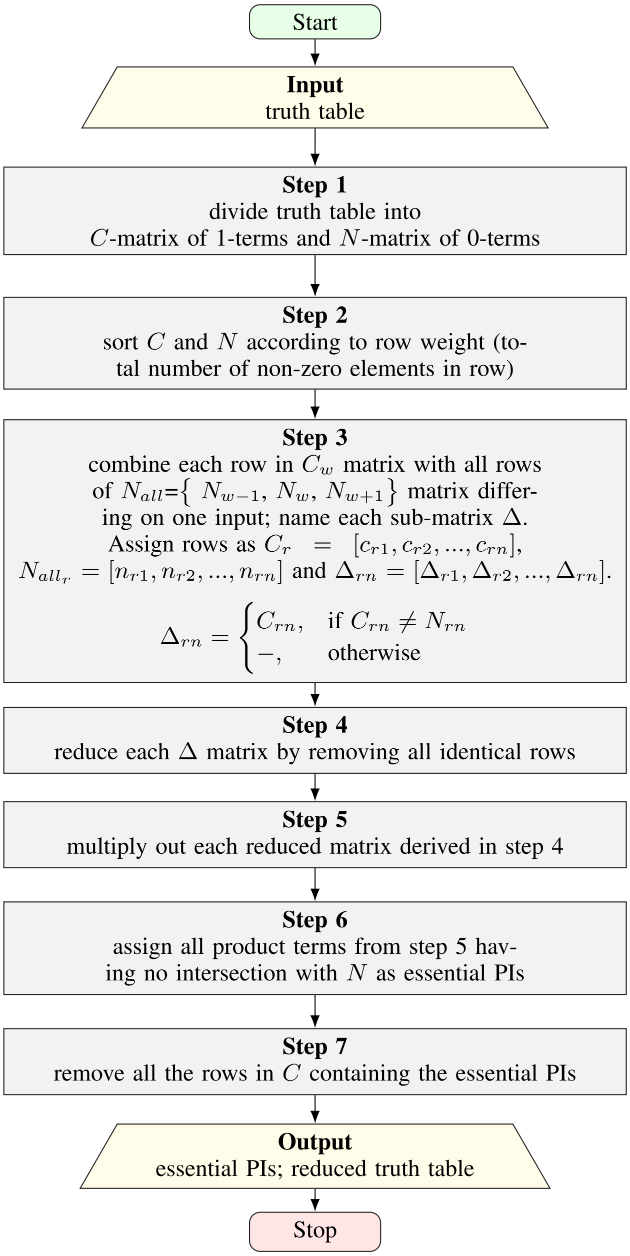


FIGURE 2. Procedural protocol to derive essential PIs from multi-value single-output truth table.

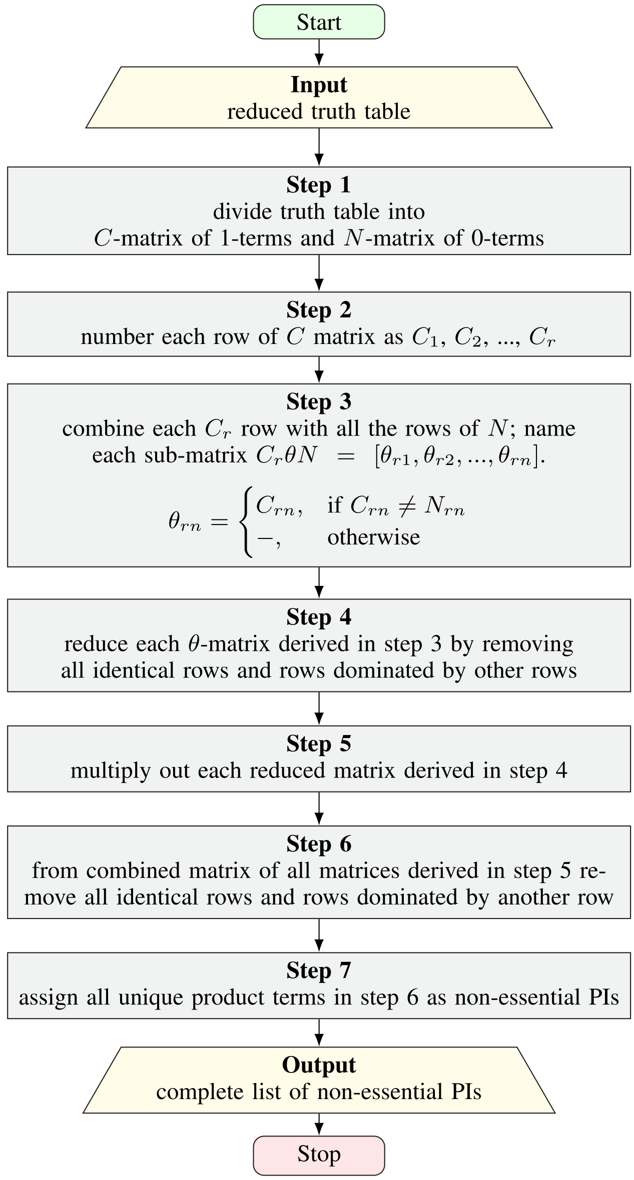


FIGURE 3. Procedural protocol to derive PIs from multi-value, single-output reduced truth table.

includes only the first component because of CORA's objective function of sum irredundancy.

VI. APPLYING CORA TO MULTI-VALUE DATA

In this section, we reanalyze empirical data on 18 public-private partnership projects for toll roads in order to demonstrate the usage of MC′ in CORA. These data were originally analyzed with mvQCA in [33] and are shown in Table 1. Each case represents a project on which the following data on input factors hypothesized to influence the public pricing objectives of the projects are available:

- *pric*: toll rate approach (0: average cost pricing; 1: marginal social-cost pricing; 2: revenue-maximizing pricing)
- *leng*: concession length (0: variable; 1: short; 2: long)
- *upsi*: upside revenue sharing (0: absent; 1: present)
- *dosi*: downside risk sharing (0: absent; 1: present)
- *risk*: traffic-demand risk (0: low; 1: high)

From the three output factors in the original study, we use only the following two (as our primary aim is a methodological demonstration rather than a substantive reanalysis):

- *frfl*: managing congestion or maximizing throughput (0: low; 1: high)
- *tort*: achieving an affordable / specific toll rate (0: low, 1: high)

Section VI-A starts with one output, following which Section VI-B extends the example to a simultaneous analysis of both outputs.

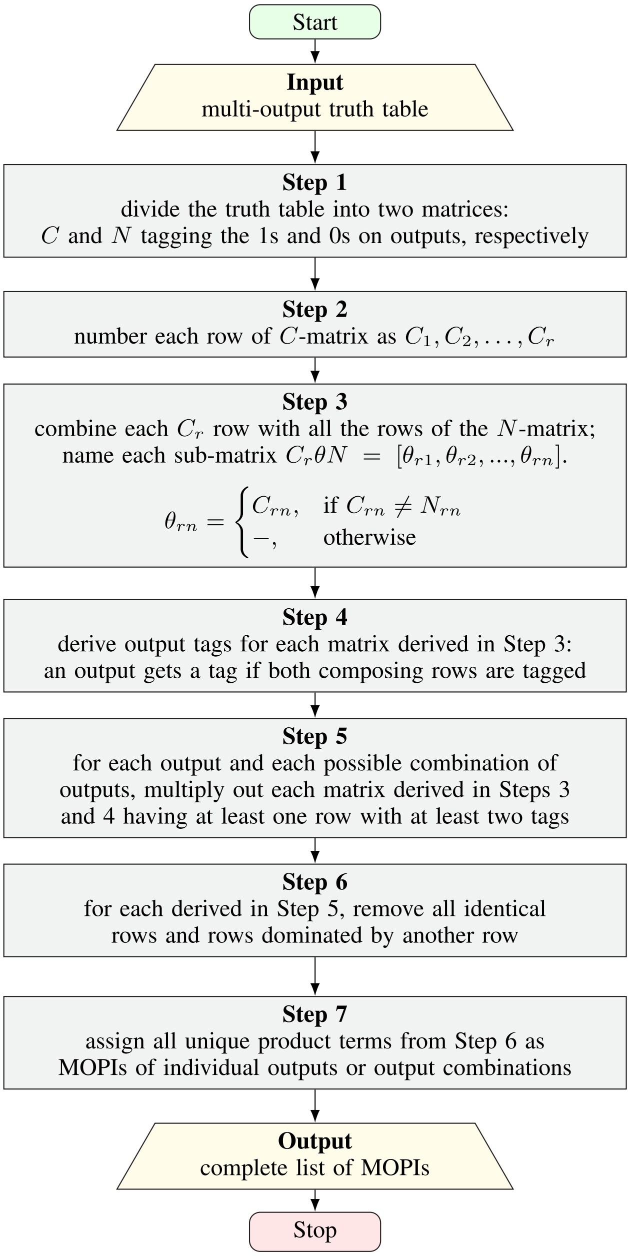


FIGURE 4. Procedural protocol for deriving multi-output prime implicants from multi-output truth table.

A. MULTI-VALUE LOGIC OPTIMIZATION WITH CORA: SINGLE OUTPUTS

To demonstrate how CORA implements MC′ along the flowcharts in Fig. 2 and Fig. 3, we first analyze the output *frfl*, and more specifically, $frfl^1$ as an outcome. Given the number of levels for each input, $3^2 \cdot 2^3 = 72$ configurations are theoretically possible, out of which 56 are undetermined and 16 are empirically observed.

MC′ generates the essential PIs first, which comes with two advantages. First, the number of iterations is reduced because the input of the optimization algorithm is not the initial truth table but the reduced truth table, where all 1-terms contained in the essential PIs have been removed. Second, a subset of useless PIs are not generated, thereby increasing computational efficiency in generating the irredundant sums.

TABLE 2. Data representation of Table 1 for deriving the essential PIs.

	pric	*leng*	*upsi*	*dosi*	*risk*
C_2	1	1	0	0	0
C_3	1	2	1	0	0
	1	1	1	0	0
	1	1	0	0	1
C_4	1	2	1	1	0
N_0	0	0	0	0	0
N_1	0	1	0	0	0
N_2	2	2	0	0	0
	0	1	0	0	1
	0	0	0	1	1
	2	0	0	1	0
N_3	2	0	1	0	1
	2	2	0	0	1
	2	1	0	0	1
	0	1	0	1	1
N_4	2	1	1	0	1

TABLE 3. Combination of all C_w with N_{w-1}, N_{w-1} and N_{w+1} of Table 2 with a difference on one position for at least one 0-term.

	pric	*leng*	*upsi*	*dosi*	*risk*
$\Delta_{C_{2_1},N_{1_1}}$	1	-	-	-	-
$\Delta_{C_{3_3},N_{2_2}}$	1	-	-	-	-
$\Delta_{C_{3_3},N_{3_3}}$	1	-	-	-	-

To derive the essential PIs with MC′, we follow the flowchart in Fig. 2. The truth table is divided into two sub-tables according to the number of non-zero values in C and N, respectively. For convenience, each set of rows in C is labelled as C_w and each set of rows in N as N_w, where w is the number of non-zero entries as shown in Table 2.

In the next step, each row in C is compared with each row in N having the same, one lower, or one higher weight group and differing on only one position to check whether they can be combined. If they can be combined, the new row takes the value of the position of the 1-term if the two rows differ in their values, but assigns a dash, "-", if the values are identical.

Table 3 shows all possible combinations of C_w with $\{N_{w-1}, N_w, N_{w+1}\}$ for the data in Table 2. The Δ sub-matrices contain only one row. Therefore, the operations in Step 4 and 5 of Fig. 2 are not applicable.

In Table 3, all product terms are identical. Thus, there is only one candidate for an essential PI. To decide if that candidate PI is really essential, it is compared with each row in N in Table 2 to check if both terms have at least one

TABLE 4. Multi-output representation of Table 1 with 1-terms and 0-terms.

Inputs					Outputs		
pric	*leng*	*upsi*	*dosi*	*risk*	*frfl*	*tort*	
1	1	0	0	0	✓		C_1
1	2	1	0	0	✓		C_2
1	1	1	0	0	✓		C_3
1	2	1	1	0	✓	✓	C_4
0	1	0	0	1		✓	C_5
2	2	0	0	1		✓	C_6
0	0	0	1	1		✓	C_7
0	0	0	0	0		✓	C_8
0	1	0	0	0		✓	C_9
2	0	0	1	0		✓	C_{10}
1	1	0	0	1	✓		C_{11}
0	1	0	1	1		✓	C_{12}
2	0	1	0	1	✓	✓	N
1	1	0	0	0		✓	
1	2	1	0	0		✓	
2	2	0	0	0	✓	✓	
1	1	1	0	0		✓	
0	1	0	0	1	✓		
2	2	0	0	1	✓		
2	1	0	0	1	✓	✓	
0	0	0	1	1	✓		
0	0	0	0	0	✓		
0	1	0	0	0	✓		
2	1	1	0	1	✓	✓	
2	0	0	1	0	✓		
1	1	0	0	1		✓	
0	1	0	1	1	✓		

conflicting coordinate. As no 0-term in Table 2 shows a 1 on input *pric*, $pic^{\{1\}}$ is an essential PI.

To derive the non-essential PIs, we follow the steps of the flowchart in Fig. 3. As an input, MC′ now takes the reduced truth table, where all the rows in C containing the essential PI have been removed. As each row in C contains $pric^{\{1\}}$, all rows are eliminated and the process aborted. CORA's solution for *frfl* contains only one path, which is the essential PI $pric^{\{1\}}$.

B. MULTI-VALUE LOGIC OPTIMIZATION WITH CORA: MULTIPLE OUTPUTS

In this section, we demonstrate how CORA implements MC′ with multi-value systems of functions. To generate all MOPIs, we follow the procedural protocol in Fig. 4.

As a first step, the table in Table 1 is again rearranged into two sub-tables, one containing the 1-terms and the other the 0-terms. If a row in a truth table is associated with positive outputs only, it is marked in the first sub-table. If both outputs are negative, the row is assigned to the second sub-table and if the outputs are mixed, the row appears in both sub-tables with corresponding tags in the outputs. For example, the case *SH*121 is assigned to the 1-terms sub-table only, the case *ROUTE*460 appears in the second sub-table only, and the case *I*595 is written in both sub-tables tagged for output *frfl* in the 1-terms sub-table and for output *tort* in the 0-terms sub-table.

TABLE 5. Combination of rows from Table 4.

Inputs					Outputs		
pric	*leng*	*upsi*	*dosi*	*risk*	*frfl*	*tort*	
1	1	0	-	0	✓		$C_1 \theta N$
1	1	-	-	-	✓		
1	-	-	-	0	✓		
1	1	-	-	0	✓		
1	-	-	-	0	✓		
1	1	-	0	0	✓		
1	1	-	-	-	✓		
1	-	-	-	-	✓		
1	-	0	-	0	✓		
1	1	-	0	-	✓		
1	-	-	0	0	✓		
1	2	-	1	0	✓	✓	$C_4 \theta N$
-	2	1	1	-		✓	
-	-	-	1	-		✓	
1	-	1	1	-	✓	✓	
-	2	-	1	-		✓	
1	2	1	1	0	✓		
1	-	1	1	0	✓		
1	2	1	1	0	✓	✓	
1	2	1	-	0	✓		
1	2	1	1	-	✓		
1	2	1	1	-	✓		
1	2	-	1	0	✓	✓	
1	2	1	-	-	✓		
-	2	1	1	0		✓	
1	2	1	-	0	✓		
0	1	0	-	-		✓	$C_5 \theta N$
0	-	-	-	1		✓	
0	1	0	-	1		✓	
0	1	-	-	1		✓	
0	-	0	-	1		✓	
0	-	-	-	-		✓	
0	-	0	-	-		✓	
0	-	-	-	-		✓	

TABLE 6. Multiplying out of each sub-matrix containing output *frfl* in Table 5.

pric	*leng*	*upsi*	*dosi*	*risk*	
1	-	-	-	-	$C_1 \theta N$
1	-	-	-	-	$C_4 \theta N$
-	2	1	-	-	
-	2	-	1	-	
-	-	1	1	-	

TABLE 7. Multiplying out of each sub-matrix containing output *tort* in Table 5.

$pric$	$leng$	$upsi$	$dosi$	$risk$	
-	-	-	1	-	$C_4 \theta N$
0	-	-	-	-	$C_5 \theta N$

TABLE 8. Multiplying out of each sub-matrix containing outputs *frfl* and *tort* in Table 5.

$pric$	$leng$	$upsi$	$dosi$	$risk$	
1	-	-	1	-	$C_4 \theta N$
-	2	-	1	-	
-	-	1	1	-	

TABLE 9. The final list of MOPIs for the data in Table 1.

$pric$	$leng$	$upsi$	$dosi$	$risk$	Output
1	-	-	-	-	$frfl$
-	2	1	-	-	
-	-	1	-	0	
-	0	-	-	0	$tort$
-	2	-	-	1	
-	0	0	-	-	
0	-	-	-	-	
-	-	-	1	-	
-	-	1	1	-	$frfl$ and $tort$
1	-	-	1	-	
-	2	-	1	-	

TABLE 10. The list of PIs for the data in Table 1 under separate optimization.

$pric$	$leng$	$upsi$	$dosi$	$risk$	Output
1	-	-	-	-	$frfl$
0	-	-	-	-	$tort$
-	-	-	1	-	
-	2	-	-	1	
-	-	-	-	-	$frfl$ and $tort$

The second step is the same as in the previous section, with one additional rule for the output tags. In Table 5, each row in the 1-terms sub-table in Table 4 is combined with all rows of the 0-terms sub-table. The new row takes the value of the 1-term if the corresponding row in the 0-terms sub-table has a different value, and is cancelled out if both rows have identical values. The output tag is assigned only if both composing rows in the 1-terms and 0-terms sub-tables have tags in that output, otherwise no tag is given to the new row for the output. As can be seen from Table 5, some rows are

TABLE 11. The list of PIs for the data in Table 1 under separate minimization, including useless PIs.

$pric$	$leng$	$upsi$	$dosi$	$risk$	Output
1	-	-	-	-	$frfl$
-	2	-	1	-	
-	2	1	-	-	
-	-	1	1	-	
-	-	1	-	0	
0	-	-	-	-	$tort$
-	-	-	1	-	
-	2	-	-	1	
-	0	-	-	0	
-	0	0	-	-	
-	-	-	-	-	$frfl$ and $tort$

TABLE 12. The irredundant sums generated through separate optimization.

$$S = \begin{cases} pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ pric^{\{0\}} + dosi^{\{1\}} + leng^{\{2\}} \cdot risk^{\{1\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

associated with the first or second output only, and some other rows have tags in both outputs.

The next step involves the multiplication for all sub-matrices in Table 5. Note that for single-output optimization, before this step, each sub-matrix is reduced considering the dominance relation of the rows in the sub-matrix. For multi-output problems, this step is omitted, as we do not define dominance relations for multi-output data. Thus, each row is multiplied out even if the sub-matrix has only one row. Table 6 and Table 7 show the results of multiplication for outputs *frfl* and *tort*, respectively, while Table 8 lists the results for the product of both outputs. If a PI is present in an individual output matrix and in the matrix for the product of the outputs, it is assigned to the product of the outputs in the final list of MOPIs. For example, both Table 6 and Table 8 contain the PIs $upsi^{\{1\}} \cdot dosi^{\{1\}}$ and $leng^{\{2\}} \cdot dosi^{\{1\}}$. However, in the final list of PIs, they belong to the list of MOPIs only.

The final list of MOPIs per output and product of outputs is listed in Table 9. To compare the approach of separate optimization with that of multi-output optimization, we list the PIs generated by separate optimization in Table 10. As can be seen from Table 9 and Table 10, the two outputs do not share any PIs under separate optimization, whereas three shared MOPIs result under the more comprehensive approach of multi-output optimization.

With MC′, useless PIs are eliminated in the process of generating the essential PIs and reducing the truth table to generate the inessential PIs. Table 11 shows that for both outputs, separate optimization under QMC, as so far practiced in QCA research, generates many useless PIs. For example,

TABLE 13. CORA's solution, consisting of seven irredundant systems.

$$S_1 = \begin{cases} upsi^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ upsi^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} + leng^{\{0\}} \cdot risk^{\{0\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_2 = \begin{cases} pric^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ pric^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} + leng^{\{0\}} \cdot upsi^{\{0\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_3 = \begin{cases} leng^{\{2\}} \cdot dosi^{\{1\}} + pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ leng^{\{2\}} \cdot dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} + leng^{\{0\}} \cdot risk^{\{0\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_4 = \begin{cases} leng^{\{2\}} \cdot dosi^{\{1\}} + pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ leng^{\{2\}} \cdot dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} + leng^{\{0\}} \cdot upsi^{\{0\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_5 = \begin{cases} pric\,\{1\} \cdot dosi\,\{1\} + pric\,\{1\} \Leftrightarrow frfl^{\{1\}} \\ pric\,\{1\} \cdot dosi\,\{1\} + pric\,\{0\} + leng\,\{2\} \cdot risk\,\{1\} + leng\,\{0\} \cdot risk\,\{0\} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_6 = \begin{cases} upsi^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ upsi^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} + leng^{\{0\}} \cdot upsi^{\{0\}} + pric^{\{0\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

$$S_7 = \begin{cases} pric^{\{1\}} \Leftrightarrow frfl^{\{1\}} \\ dosi^{\{1\}} + pric^{\{0\}} + leng^{\{2\}} \cdot risk^{\{1\}} \Leftrightarrow tort^{\{1\}} \end{cases}$$

for the output *frfl*, QMC generates four inessential PIs and all these PIs are useless and not part of the final solution space.

The difference in the PIs generated with separate and multi-output optimization directly influences the final results for both outputs. Table 12 shows the irredundant sums per output generated with separate optimization, and Table 13 shows the irredundant systems of the two outputs when optimized jointly. With separate optimization, one solution per output is generated, whereas under joint optimization, seven systems are generated, six of which contain shared paths, and one system is identical to the one generated under separate optimization.

Note again that CORA's notion of system irredundancy is not the same as sum irredundancy in QCA. For example, if we take the system S_2 and take each function separately, the first function contains a redundancy because $pric^{\{1\}} \cdot dosi^{\{1\}} + pric^{\{1\}} = pric^{\{1\}} \cdot (dosi^{\{1\}} + 1) = pric^{\{1\}} \cdot 1 = pric^{\{1\}}$. However, when considering the whole system, the product term $pric^{\{1\}} \cdot dosi^{\{1\}}$ cannot be eliminated because if it is removed from the first function, it must also be removed from the second, which will result in a non-equivalent solution for *tort*.

Comparing the results generated with separate optimization in Table 12 and multi-output optimization in Table 13, it becomes evident that, with multi-output optimization, the whole spectrum of causal paths shared between the two outputs is discovered. In fact, under separate optimization, no shared path is discovered, whereas six systems in Table 13 contain a common MOPI.

From a policy perspective, the results of CORA's MC′ give more insights into the potential causal structures operating behind different pricing objectives. First, the two outputs contain essential prime implicants. In all seven systems, the toll rate approach is part of each output being in average cost pricing for outcome *tort* and in marginal social-cost pricing for outcome *frfl*. Second, decision-makers have a possibility to understand the combinations of conditions which may lead to the simultaneous presence of both outcomes, not only the presence of each outcome alone.

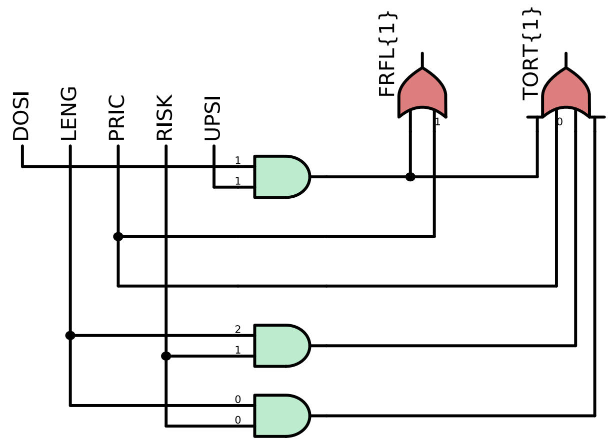


FIGURE 5. Logigram of System 1 in Table 13.

For communication purposes, CORA's solutions can be visualized as standardized logic diagrams, called logigrams in CORA. For example, Fig. 5 shows the system S_1 (using upper-case letters instead of lower-case letters).

VII. CONCLUSION

Logic optimization is a topic well covered in electrical engineering since at least the 1950s. Some subfields of the social sciences have also employed logic optimization algorithms since the mid-1980s for purposes of causal inference. Most prominently, the well-known Quine-McCluskey optimization algorithm has been imported by sociologists and political scientists to uncover complex causal relations by means of a method called Qualitative Comparative Analysis (QCA). However, while electrical engineering has progressed considerably since the advent of logic optimization, these advancements have not been monitored by social scientists. Nor have social scientists attempted to establish any interdisciplinary collaboration. Several methodological publications have recently revealed several serious problems in the use of QMC that social scientists have not been able to detect earlier due to their swift import of QMC without any consultation with electrical engineers (or analytical philosophers).

Against this background, the objective of our article has been twofold. First, and more generally, we have sought to reinvigorate the lost connection between electrical engineering and configurational causal inference. Secondly, and more specifically, we have presented Combinational Regularity Analysis (CORA), a new configurational method that incorporates multi-value logic and multi-output optimization. In particular, we have introduced one of CORA's algorithms for optimizing highly unspecified multi-value functions with multiple outputs. The availability of such algorithms in CORA will push the boundaries of configurational causal inference.

Future research directions in CORA will seek to further connect the advances electrical engineering has made on algorithmic procedures to questions of causal inference. For example, one promising territory of electrical engineering that has not yet been charted by analytical philosophers or social research methodologists is that of sequential circuit design. Other possible avenues include heuristic algorithms for the analysis of very large data sets. Irrespective of where configurational methodology and electrical engineering may meet in the coming years, CORA should have clearly demonstrated that there is a lot to gain for causal theorists and applied researchers from more, not less interdisciplinarity.

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