Received 30 November 2022, accepted 2 January 2023, date of publication 9 January 2023, date of current version 12 January 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3235350

RESEARCH ARTICLE

Passive Fault-Tolerant Control for NCSs Using Event-Triggered Approach

LEI FU, JIACHANG SHI, AND FEI LI
College of Electronic and Information Engineering, Hebei University, Baoding, Hebei 071002, China

Corresponding author: Fei Li (120109799@qq.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 62103126 and Grant 12171135, in part by the Natural Science Foundation of Hebei Province under Grant A2020201021 and Grant F2020201014, in part by the Foundation of Hebei Education Department under Grant QN2020224, and in part by the Advanced Talents Incubation Program of Hebei University under Grant 521000981355.

ABSTRACT In this paper, the passive event-triggered control problem for networked control systems with actuator faults is of concern. A sufficient condition is proposed by a Lyapunov-Krasovskii functional, under which such type continuous-time Networked control systems with actuator faults are asymptotically stable and passive. The designing method of controller is developed by using Wirtinger inequality and linear matrix inequalities (LMIs). The given strategy can reduce the number of transmissions, thus saving the communication resources. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

INDEX TERMS Passive fault-tolerant control, event-triggered approach, networked control systems, linear matrix inequalities (LMIs).

I. INTRODUCTION

With the development of the network, the plant, the sensor, the controller, and actuator usually located in different places. Consequently, the signals should be transmitted from one place to another. Therefore, networked control systems(NCSs), where the components are connected over networks, have been intensively studied in recent decades [1], [2], [3].

The time-triggering scheme and event-triggering scheme are the mainstream control strategies for NCSs [4], [5]. Theories and experiments have proved the event-triggered scheme can reduce the number of transmitted data and improve the utilization efficiency of limited network bandwidth to a large extent without degrading the desired system performance [6], [7]. When the network resource is limited, the time-trigger technique is inefficient. An important focus of NCSs' signal transmission and control is cost. Compared with the traditional time-triggered scheme, the event-triggered scheme allows a considerable reduction of the network resource occupancy while maintaining the control performance [8], [9], [11], [28]. In NCSs, there are schemes, such as event-triggered control based on time-delay method in [12]. It should be pointed out that the event-triggered control problems for NCSs are much more complicated. Therefore, investigating the analysis and design of NCSs is of fundamental importance in order to realize the functions of the real world.

On the other hand, actuator faults are also the distinguishing feature that can not be neglected in analysis and designing of NCSs. Because actuator faults can also lead to poor performance of the closed-loop systems and sometimes destabilizes the systems [13], [14]. Recently, a host of characters have paid attention to the study of various control problem of NCSs with actuator faults [14], [15], [16].

The event-triggered control design problem of NCSs with dynamic quantisation and fault is studied in [15], and stochastic actuator fault, which is modelled by the Bernoulli distributed white sequence is considered in this situation, and a robust feedback controller is designed. Reference [16] focuses on two novel event-triggered fault-tolerant control strategies for a class of stochastic systems with state delays.

The associate editor coordinating the review of this manuscript and approving it for publication was Haibin Sun.

Passivity is closely related to bounded realness, which has been used in control problems to deal with robust stability problems for complex uncertain dynamic systems with disturbances. A passive system utilizes the product of input and output as the energy provision and embodies the energy attenuation character, which often links the stability problems. The key point of passivity theory is that a passive system can keep itself internally stable. In fact, many applications based on passivity theory can be found in lots of areas, such as signal processing, chaos control [17], [18]. In recent years, researches on passive control of dynamic systems have achieved remarkable results [19], [20], [28]. The passive control problem for semi-Markov jump Takagi-Sugeno fuzzy systems based on an event-triggered mechanism is considered in [22]. Reference [23] proposed the asynchronous passive controller design for singular Markov jump systems.

Up to now, to the best of authors's knowledge, there are few results on the passive event-triggered control problem for NCSs with actuator faults and remains to be important and challenging. However, it is often required to reduce the transmission cost in practical engineering. Therefore, it becomes important to investigate the event-triggered controller design problems of continuous-time NCSs with actuator faults.

Motivated by the aforementioned considerations, in this paper, our attention is devoted to design the passive event-triggered controller for NCSs with actuator faults. The contributions of this paper are as follows:

- A class of more general control strategy is investigated, where we take actuator faults, event-triggered controller into consideration at the same time. The event-triggering scheme gives a unified framework that can include existing results for time-triggering scheme as special cases for NCSs. In addition, the criteria obtained in this paper are suitable for NCSs without actuator faults.
- Based on the Lyapunov-Krasovskii functional approach, a sufficient condition given in this paper makes NCSs with actuator faults and event-triggering scheme asymptotically stable and passive. Moreover, the designing method of controller is developed by using linear matrix inequalities (LMIs).
- The Wirtinger inequality is more tighter than Jensen inequality, and be used in the derivation of the sufficient criterion. In addition, the free weight matrix approach is also adopted in the criterion derivation process.

The outline of this paper is given as follows. In section II, the problem formulation and some preliminaries are presented. The controllers are given in section III. Section IV presents the numerical examples to demonstrate the proposed methodology. Conclusions are drawn in section V.

Notations: $V > 0$ means that V is positive definite; V^T and V^{-1} denote the transpose and the inverse of any square matrix V; $\|V\|$ means the spectral norm of the matrix V; $diag\{V_1, V_2, \ldots, V_N\}$ represents a block-diagonal matrix with diagonal elements $V_1, V_2, \ldots, V_N$. R^n denote the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n \times m$ real matrices; I is the identity matrix of appropriate dimensions. For asymmetric matrix $*$ denotes the matrix entries implied by symmetry.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the NCSs subject to actuator faults with the following dynamics:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu^F(t) + B_\omega \omega(t) \\ z(t) = Cx(t) + D_\omega \omega(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u^F(t) \in R^m$ is the fault control input described in (2) below. $\omega(t) \in R^q$ is the disturbance input which belongs to

$$\int_0^T \omega^T(t)\omega(t)\,dt \le \delta_0,\ \ \delta_0 \ge 0.$$

A, B, B_ω, C and D_ω are known constant matrices with appropriate dimensions.

$$u^F(t) = Fu(t) \tag{2}$$

where $F = diag\{f_1, f_2, \cdots, f_m\}$, and $0 \le f_l^- \le f_l \le f_l^+ \le 1, \forall l \in \{1, 2, \cdots, m\}$.

Let $f_l^{middle} = \frac{f_l^+ + f_l^-}{2}$, $f_l^d = \frac{f_l^+ - f_l^-}{f_l^+ + f_l^-}$, $f_l^s = \frac{f_l - f_l^{middle}}{f_l^{middle}}$, then we can obtain that

$$F = F^{middle}\left(I + F^s\right),\ \ \|F^s\| \le F^d \le I$$

where

$$\begin{aligned} F^{middle} &= diag\left\{f_1^{middle}, f_2^{middle}, \cdots, f_m^{middle}\right\}, \\ F^s &= diag\left\{f_1^s, f_2^s, \cdots, f_m^s\right\}, \\ F^d &= diag\left\{f_1^d, f_2^d, \cdots, f_m^d\right\}. \end{aligned}$$

Then the control input can be described as

$$u^F(t) = F^{middle}\left(I + F^s\right)u(t). \tag{3}$$

There is an event generator in the network systems, which is constructed between sensor and controller, which is visualized in Fig 1. The event generator can determine which sampled signal should be sent out or not by utilizing the following judgement algorithm (4).

$$t_{i+1} = \inf\left\{t : t > t_i + \tau,\ e^T(t)\,\Theta e(t) \ge \delta x^T(t)\,\Theta x(t)\right\} \tag{4}$$

where

$$e(t) = x(t) - x(t_i).$$

The event-triggered instant is denoted as $\{t_i\}_{i=0}^{\infty}$ with $0 = t_0 < t_1 < t_2 = \cdots$. Θ is a positive-definite matrix. $t_{i+1} - t_i > \tau > 0$, and $0 \le \delta < 1$.

In this paper, we consider an event-triggered controller as

$$u(t) = Kx(t_i) \tag{5}$$

where K is the matrix to be sought.

Denote $\tau(t) = t - t_i$, $t \in [t_i,\ t_i + \tau)$ with $0 < \tau(t) < \tau$.

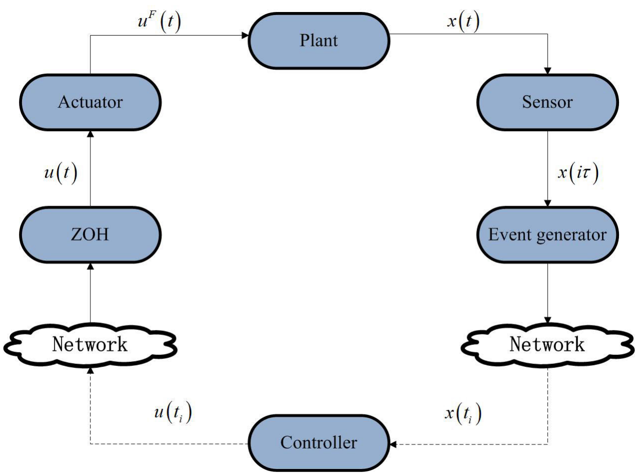

FIGURE 1. The structure of an event-triggered NCSs.

Then, the closed-loop system can be described as

$$\dot{x}(t) = \begin{cases} Ax(t) + BF^{middle}(I + F^s)Kx(t - \tau(t)) \\ \quad + B_\omega \omega(t), \quad when\ t \in [t_i, t_i + \tau) \\ \bar{A}x(t) - BF^{middle}(I + F^s)Ke(t) \\ \quad + B_\omega \omega(t), \quad when\ t \in [t_i + \tau,\ t_{i+1}) \end{cases} \tag{6}$$

where

$$\bar{A} = A + BF^{middle}\left(I + F^s\right)K.$$

Remark 1: According to the above description of actuator failure, the following three special cases can be drawn, and the conclusions obtained in this paper are applicable to the following three special cases, which illustrates the universality of the model in this paper.

- When $f_l{}^- = f_l{}^+ = 0$, the actuator fails completely.
- When $0 < f_l{}^- \leq f_l{}^+ < 1$, the actuator partially fails;.
- When $f_l{}^- = f_l{}^+ = 1$, the actuator has not failed.

Fault tolerant control in this paper means that the NCSs with actuator faults can still remain asymptotically stable, and can meet the performance indicator of passive. We are ready to state the passive fault-tolerant event-triggered control problem for system (1).

Definition 1: A linear control law of the form (5) is passive fault-tolerant event-triggered control for the NCSs (1) with the actuator faults represented by (2) and the event-triggering scheme (4), if the following two requirements are satisfied.

- The system (6) with $\omega(t) = 0$ is asymptotically stable.
- if there exists $\beta > 0$, such that

$$2\int_0^{T_0} \omega^T(t) z(t)\, dt \geq \quad -\beta \int_0^{T_0} \omega^T(t)\omega(t)\, dt, \qquad \forall T_0 > 0,$$

holds for all trajectories with zero initial condition.

Lemma 1 [24]: For given constants $r_1,\ \ r_2\ (0 \leq r_1 < r_2)$ and an $n \times n$ real matrix $Q = Q^T > 0$, a scalar continuous function $r(t)$ satisfying $r_1 r(t) r_2$ and a vector-valued function $\dot{x} : [t - r_2, t - r_1] \to R^n$ such that the integrations concerned below are well defined, then the following inequality is true for any matrices $Z_j \in R^{m_j \times m_j}$ and $S_j \in R^{m_j \times n}$ satisfying $\begin{bmatrix} Z_j & S_j \\ * & Q \end{bmatrix} \geq 0$ and any vectors $c_j \in R^{m_j}, j = 1, 2, 3, 4.$

$$\begin{aligned} &-(r_2 - r_1)\int_{t-r_2}^{t-r_1} \dot{x}^T(s) Q\dot{x}(s)\, ds \\ &\quad \leq \sum_{j=1}^{4} 2c_j^T S_i \vartheta_i(t) + \beta\left(c_1^T Z_1 c_1 + c_3^T Z_3 c_3\right) \\ &\qquad + (1 - \beta)\left(c_2^T Z_2 c_2 + c_4^T Z_4 c_4\right) \end{aligned}$$

where $\beta = (r_2 - r(t))/(r_2 - r_1)$ and

$$\begin{aligned} \vartheta_1(t) &= x(t - r(t)) - x(t - r_2), \\ \vartheta_2(t) &= x(t - r_1) - x(t - r(t)), \\ \vartheta_3(t) &= \frac{\pi}{2}\left(x(t - r(t)) + x(t - r_2) - 2\upsilon_1\right), \\ \vartheta_4(t) &= \frac{\pi}{2}\left(x(t - r_1) + x(t - r(t)) - 2\upsilon_2\right), \\ \upsilon_1 &= \frac{1}{r_2 - r(t)}\int_{t-r_2}^{t-r(t)} x(s)\ ds, \\ \upsilon_2 &= \frac{1}{r(t) - r_1}\int_{t-r(t)}^{t-r_1} x(s)\ ds. \end{aligned}$$

Lemma 2 [25]: Assume Σ_1, Σ_2 and Υ are constant matrices. Then, for any $l(t) \in [l_m, l_M]$,

$$(l(t) - l_m)\Sigma_1 + (l_M - l(t))\Sigma_2 + \Upsilon < 0$$

is true if and only if

$$(l_M - l_m)\Sigma_1 + \Upsilon < 0, (l_M - l_m)\Sigma_2 + \Upsilon < 0.$$

Lemma 3 [26]: Let M, F, N and P be real matrices of appropriate dimensions with $P > 0$, $F^{\mathrm{T}}F \leq I$ and a scalar $\eta > 0$. Then

$$MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \leq \eta MP^{-1}M^{\mathrm{T}} + \frac{1}{\eta}N^{\mathrm{T}}F^{\mathrm{T}}PFN.$$

Lemma 4 [27]: For a matrix $R > 0$ and a differentiable signal x in $[\alpha,\ \beta] \to R^n$,the following inequality holds:

$$\begin{aligned} &-(\beta - \alpha)\int_\alpha^\beta x^T(s) Rx(s)\, ds \\ &\quad \leq -\left(\int_\alpha^\beta x(s)\, ds\right)^T R\left(\int_\alpha^\beta x(s)\, ds\right) - 3\varpi_0{}^T R\varpi_0, \end{aligned}$$

where

$$\varpi_0 = \int_\alpha^\beta x(s)\, ds - \frac{2}{\beta - \alpha}\int_\alpha^\beta \int_\alpha^s x(u)\, duds.$$

III. MAIN RESULTS

In this section, we first give a condition to guarantee the system (6) with $\omega(t) = 0$ is asymptotically stable.

Theorem 1: For the given scalars τ, a_1, and a_2, the closed-loop system (6) with $\omega(t) = 0$ is asymptotically stable, if there exist symmetric matrices $X > 0$, $\bar{P}_0 > 0$, $\bar{Q} > 0$, $\bar{\Theta} > 0$, and matrices Y, $\bar{S}_j$, $\bar{Z}_j$, such that

$$\begin{bmatrix} \bar{Z}_j & \bar{S}_j \\ * & \bar{Q} \end{bmatrix} \geq 0, \tag{7}$$

$$\bar{\Sigma}_{1+3} = \begin{bmatrix} \bar{\Sigma}^1_{1+3} & \bar{\Sigma}^2 & \varepsilon_1 \bar{\Sigma}^3 \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{8}$$

$$\bar{\Sigma}_{2+4} = \begin{bmatrix} \bar{\Sigma}^1_{2+4} & \bar{\Sigma}^2 & \varepsilon_1 \bar{\Sigma}^3 \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{9}$$

$$\bar{\Pi} = \begin{bmatrix} \bar{\Pi}^1 & \bar{\Pi}^2 & \bar{\Pi}^3 \\ * & -\varepsilon_2 I & 0 \\ * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{10}$$

where

$$\bar{\Sigma}^1_{1+3} = \bar{\Sigma} + \bar{S} + \bar{Z}_1 + \bar{Z}_3,$$
$$\bar{\Sigma}^1_{2+4} = \bar{\Sigma} + \bar{S} + \bar{Z}_2 + \bar{Z}_4,$$
$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{11} & Y_B & 0 & a_1 X A^T & 0 & 0 \\ * & 0 & 0 & a_1 Y_B^T & 0 & 0 \\ * & * & -\bar{P}_0 & 0 & 0 & 0 \\ * & * & * & \tau^2 \bar{Q} - 2a_1 X & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix},$$
$$\bar{\Sigma}_{11} = AX + XA^T + \bar{P}_0,$$
$$Y_B = BF^{middle} Y,$$
$$\bar{\Sigma}^2 = \left[\left(BF^{middle}\right)^T \ \ 0 \ \ 0 \ \ a_1 \left(BF^{middle}\right)^T \ \ 0 \ \ 0 \right]^T,$$
$$\bar{\Sigma}^3 = \left[0 \ \ F^d Y \ \ 0 \ \ 0 \ \ 0 \ \ 0 \right]^T,$$
$$\bar{\Pi}^1 = \begin{bmatrix} \bar{\Pi}_{11} & -2\bar{Q} & \bar{\Pi}_{13} & Y_B & 6\bar{Q} \\ * & -\bar{P}_0 - 4\bar{Q} & 0 & 0 & 6\bar{Q} \\ * & * & \tau^2 \bar{Q} - 2a_2 X & a_2 Y_B & 0 \\ * & * & * & -\bar{\Theta} & 0 \\ * & * & * & * & -12\bar{Q} \end{bmatrix},$$
$$\bar{\Pi}_{11} = AX + XA^T + Y_B + {Y_B}^T + \bar{P}_0 - 4\bar{Q} + \delta \bar{\Theta},$$
$$\bar{\Pi}_{13} = a_2 \left(XA^T + {Y_B}^T \right),$$
$$\bar{\Pi}^2 = \left[\left(BF^{middle}\right)^T \ \ 0 \ \ a_2 \left(BF^{middle}\right)^T \ \ 0 \ \ 0 \right]^T,$$
$$\bar{\Pi}^3 = \left[F^d Y \ \ 0 \ \ 0 \ \ F^d Y \ \ 0 \right]^T,$$
$$\begin{aligned} \bar{S} = {} & \bar{S}_1 (e_2 - e_3) + (e_2 - e_3)^T \bar{S}_1^T \\ & + \bar{S}_2 (e_1 - e_2) + (e_1 - e_2)^T \bar{S}_2^T \\ & + \frac{\pi}{2} \bar{S}_3 (e_2 + e_3 - 2e_5) + \frac{\pi}{2} (e_2 + e_3 - 2e_5)^T \bar{S}_3^T \\ & + \frac{\pi}{2} \bar{S}_4 (e_1 + e_2 - 2e_6) + \frac{\pi}{2} (e_1 + e_2 - 2e_6)^T \bar{S}_4^T, \end{aligned}$$

and

$$e_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\cdots$$
$$e_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and the controller gain is taken as $K = YX^{-1}$.

Proof: First we choose a Lyapunov−Krasovskii functional candidate as

$$\begin{aligned} V(x(t), t) = {} & x^T(t) P x(t) + \int_{t-\tau}^{t} x^T(s) P_0 x(s)\, ds \\ & + \tau \int_{-\tau}^{0} \int_{t+\mu}^{t} \dot{x}^T(s) Q \dot{x}(s)\, ds d\mu. \end{aligned} \tag{11}$$

Taking the time derivative along with the trajectory of the system (6), it yields

$$\begin{aligned} \dot{V}(x(t), t) = {} & 2x^T(t) P \dot{x}(t) \\ & + x^T(t) P_0 x(t) - x^T(t-\tau) P_0 x(t-\tau) \\ & + \tau^2 \dot{x}^T(t) Q \dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s) Q \dot{x}(s)\, ds. \end{aligned}$$

To prove $\dot{V}(x(t), t) < 0$, $\forall t \in [t_i, t_{i+1})$, we now consider the following two cases:

- Case 1 $t \in [t_i, t_i + \tau)$

$$\begin{aligned} & \dot{V}(x(t), t) \\ & = 2x^T(t) PAx(t) \\ & \quad + 2x^T(t) P \left[BF^{middle} \left(I + F^s\right) Kx(t - \tau(t)) \right] \\ & \quad + x^T(t) P_0 x(t) - x^T(t-\tau) P_0 x(t-\tau) \\ & \quad + \tau^2 \dot{x}^T(t) Q \dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s) Q \dot{x}(s)\, ds. \end{aligned} \tag{12}$$

Consider the relaxation factor as follows

$$2a_1 \dot{x}^T(t) P [Ax(t) + BFKx(t - \tau(t)) - \dot{x}(t)] = 0. \tag{13}$$

Apply Lemma 1 to get the fact that

$$\begin{aligned} & -\tau \int_{t-\tau}^{t} \dot{x}^T(s) Q \dot{x}(s)\, ds \\ & \leq \zeta^T(t) \left(S + \frac{\tau - \tau(t)}{\tau} (Z_1 + Z_3) + \frac{\tau(t)}{\tau} (Z_2 + Z_4) \right) \zeta(t), \end{aligned} \tag{14}$$

where

$$\begin{aligned} S = {} & S_1 (e_2 - e_3) + (e_2 - e_3)^T {S_1}^T \\ & + S_2 (e_1 - e_2) + (e_1 - e_2)^T {S_2}^T \\ & + \frac{\pi}{2} S_3 (e_2 + e_3 - 2e_5) + \frac{\pi}{2} (e_2 + e_3 - 2e_5)^T {S_3}^T \\ & + \frac{\pi}{2} S_4 (e_1 + e_2 - 2e_6) + \frac{\pi}{2} (e_1 + e_2 - 2e_6)^T {S_4}^T, \end{aligned}$$
$$\zeta^T(t) = \left[x^T(t), x^T(t - \tau(t)), x^T(t - \tau), \dot{x}^T(t) \right.$$

$$\times \frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)} x^T(s)\,ds,\quad \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} x^T(s)\,ds\Big].$$

Then, we can obtain that

$$\dot V(x(t),t) \le \zeta^T(t)\Sigma_0\zeta(t), \tag{15}$$

where

$$\Sigma_0 = \Sigma+S+\frac{\tau-\tau(t)}{\tau}(Z_1+Z_3)+\frac{\tau(t)}{\tau}(Z_2+Z_4),$$

$$\Sigma=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & 0 & a_1A^TP & 0 & 0\\ * & 0 & 0 & a_1\Sigma_{12}{}^T & 0 & 0\\ * & * & -P_0 & 0 & 0 & 0\\ * & * & * & \tau^2Q-2a_1P & 0 & 0\\ * & * & * & * & 0 & 0\\ * & * & * & * & * & 0\end{bmatrix},$$

$$\Sigma_{11}=PA+A^TP+P_0,$$
$$\Sigma_{12}=PBF^{middle}\left(I+F^s\right)K.$$

Then, pre-and post multiply inequality Σ_0 by $X_O = diag\{X,X,X,X,X,X\}$ and its transposition. Let $X = P^{-1}, Y = KX, \bar P_0 = XP_0X, \bar Q = XQX, \bar\Theta = X\Theta X, \bar S = X_OSX_O, Z_j = X_OZ_jX_O\ (j=1,2,3,4)$. considering the fact

$$X_O\Sigma X_O = \bar\Sigma+\bar\Sigma^2F^s\bar\Sigma^{3T}+\bar\Sigma^3F^s\bar\Sigma^{2T},$$

and

$$\left\|F^s\right\| \le F^d.$$

By Lemma 2 and 3, the LMIs (7),(8) and (9) mean $\dot V(x(t),t)<0, \forall t\in[t_i,t_i+\tau)$.

- Case 2 $t\in[t_i+\tau,\ t_{i+1})$

$$\begin{aligned}\dot V(x(t),t) &= 2x^T(t)P\left[\bar Ax(t)-BFKe(t)\right]\\ &\quad + x^T(t)P_0x(t)-x^T(t-\tau)P_0x(t-\tau)\\ &\quad + \tau^2\dot x^T(t)Q\dot x(t)-\tau\int_{t-\tau}^{t}\dot x^T(s)Q\dot x(s)\,ds.\end{aligned}\tag{16}$$

We also consider the relaxation factor as follows

$$2a_2\dot x^T(t)P\left[\bar Ax(t)-BFKe(t)-\dot x(t)\right]=0. \tag{17}$$

Apply Lemma 4 to get the fact that

$$-\tau\int_{t-\tau}^{t}\dot x^T(s)Q\dot x(s)\,ds \le \psi^T(t)\Pi_Q\psi(t), \tag{18}$$

where

$$\Pi_Q=\begin{bmatrix}-4Q & -2Q & 6Q\\ * & -4Q & 6Q\\ * & * & -12Q\end{bmatrix},$$

$$\psi^T(t)=\left[x^T(t),x^T(t-\tau),\frac{1}{\tau}\int_{t-\tau}^{t}x^T(s)\,ds\right],$$

Considering the event-triggered condition, we can get

$$\dot V(x(t),t) \le \xi^T(t)\Pi^1\xi(t), \tag{19}$$

where

$$\Pi^1=\begin{bmatrix}\Pi_{11} & -2Q & a_2\bar A^TP & \Sigma_{12} & 6Q\\ * & -P_0-4Q & 0 & 0 & 6Q\\ * & * & \Pi_{33} & a_2\Sigma_{12} & 0\\ * & * & * & -\Theta & 0\\ * & * & * & * & -12Q\end{bmatrix},$$

$$\Pi_{11}=P\bar A+\bar A^TP+P_0-4Q+\delta\Theta,$$
$$\Pi_{33}=\tau^2Q-2a_2P,$$
$$\xi^T(t)=\Big[x^T(t),x^T(t-\tau),\dot x^T(t),e^T(t),\ \times\frac{1}{\tau}\int_{t-\tau}^{t}x^T(s)\,ds\Big].$$

Then, pre-and post multiply inequality Π^1 by $X_O^0 = diag\{X,X,X,X,X\}$ and its transposition, considering the fact

$$X^0{}_O\Pi^1X^0{}_O=\bar\Pi+\bar\Pi^2F^s\bar\Pi^{3T}+\bar\Pi^3F^s\bar\Pi^{2T}.$$

By Lemma 3, the condition (10) means $\dot V(x(t),t)<0, \forall t\in[t_i+\tau,t_{i+1})$.

Still now, this completes the proof.

Remark: The event-triggering scheme in this paper can avoid the Zeno phenomenon. In the special case $\delta = 0$, it comes into a time-triggering method.

Next, a sufficient condition is provided to guarantee asymptotical stability and passivity of system (1).

Theorem 2: For the given scalars τ, a_1, and a_2, the system (1) is asymptotically stable and passive, if there exist symmetric matrices $X>0$, $\bar P_0>0$, $\bar Q>0$, $\bar\Theta>0$, and matrices $Y, \bar S_j^{th2}, \bar Z_j^{th2}$, such that

$$\begin{bmatrix}\bar Z_j^{th2} & \bar S_j^{th2}\\ * & \bar Q\end{bmatrix}\ge 0, \tag{20}$$

$$\bar\Sigma_{1+3}^{th2}=\begin{bmatrix}\bar\Sigma_{1+3}^{1th2} & \bar\Sigma^{2th2} & \varepsilon_1\bar\Sigma^{3th2}\\ * & -\varepsilon_1I & 0\\ * & * & -\varepsilon_1I\end{bmatrix}<0, \tag{21}$$

$$\bar\Sigma_{2+4}^{th2}=\begin{bmatrix}\bar\Sigma_{2+4}^{1th2} & \bar\Sigma^{2th2} & \varepsilon_1\bar\Sigma^{3th2}\\ * & -\varepsilon_1I & 0\\ * & * & -\varepsilon_1I\end{bmatrix}<0, \tag{22}$$

$$\bar\Pi=\begin{bmatrix}\bar\Pi^{1th2} & \bar\Pi^{2th2} & \varepsilon_2\bar\Pi^{3th2}\\ * & -\varepsilon_2I & 0\\ * & * & -\varepsilon_2I\end{bmatrix}<0, \tag{23}$$

where

$$\bar\Sigma_{1+3}{}^{1th2}=\bar\Sigma^{th2}+\bar S^{th2}+\bar Z_1^{th2}+\bar Z_3^{th2},$$
$$\bar\Sigma_{2+4}{}^{1th2}=\bar\Sigma^{th2}+\bar S^{th2}+\bar Z_2^{th2}+\bar Z_4^{th2},$$
$$\bar\Sigma^{th2}=\begin{bmatrix}\bar\Sigma & \bar\Sigma^{12th2}\\ * & -\beta I-D_\omega-D_\omega^T\end{bmatrix},$$
$$\bar\Sigma^{12th2}=\left[B_\omega^T-CX\ \ 0\ \ 0\ \ a_1B_\omega^T\ \ 0\ \ 0\right]^T,$$
$$\bar\Sigma^{2th2}=\left[\bar\Sigma^{2T}\ \ 0\right]^T,$$

$$\bar{\Sigma}^{3th2} = \begin{bmatrix} 0 & F^d Y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\bar{\Pi}^{1th2} = \begin{bmatrix} \bar{\Pi}^1 & \Pi^{12}th2 \\ * & -\beta I - D_\omega - D_\omega^T \end{bmatrix},$$
$$\Pi^{12th2} = \begin{bmatrix} B_\omega^T - CX & 0 & a_2 B_\omega^T & 0 & 0 \end{bmatrix}^T,$$
$$\bar{\Pi}^{2th2} = \begin{bmatrix} \bar{\Pi}^{2T} & 0 \end{bmatrix}^T,$$
$$\bar{\Pi}^{3th2} = \begin{bmatrix} F^d Y & 0 & 0 & F^d Y & 0 & 0 \end{bmatrix}^T,$$
$$\begin{aligned}\bar{S}^{th2} = {} & \bar{S}_1^{th2}\left(e_2{}^{th2} - e_3{}^{th2}\right) + \left(e_2{}^{th2} - e_3{}^{th2}\right)^T \bar{S}_1^{th2\,T} \\ & + \bar{S}_2^{th2}\left(e_1{}^{th2} - e_2{}^{th2}\right) + \left(e_1 - e_2{}^{th2}\right)^T \bar{S}_2^{th2\,T} \\ & + \frac{\pi}{2}\bar{S}_3^{th2}\left(e_2{}^{th2} + e_3{}^{th2} - 2e_5{}^{th2}\right) \\ & + \frac{\pi}{2}\left(e_2{}^{th2} + e_3{}^{th2} - 2e_5{}^{th2}\right)^T \bar{S}_3^{th2\,T} \\ & + \frac{\pi}{2}\bar{S}_4^{th2}\left(e_1{}^{th2} + e_2{}^{th2} - 2e_6{}^{th2}\right) \\ & + \frac{\pi}{2}\left(e_1{}^{th2} + e_2{}^{th2} - 2e_6{}^{th2}\right)^T \bar{S}_4^{th2\,T},\end{aligned}$$

and

$$e_1{}^{th2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\cdots$$
$$e_6{}^{th2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

and the controller gain is taken as $K = YX^{-1}$.

Proof: First we also choose the Lyapunov−Krasovskii functional candidate (11).

- Case 1 $t \in [t_i, t_i + \tau)$
  Taking the time derivative along with the trajectory of the system (6), it yields

$$\begin{aligned}&\dot{V}(x(t), t) \\ &\quad = 2x^T(t) PAx(t) + 2x^T(t) PB_\omega \omega(t) \\ &\qquad + 2x^T(t) P[BFKx(t - \tau(t))] \\ &\qquad + x^T(t) P_0 x(t) - x^T(t-\tau) P_0 x(t-\tau) \\ &\qquad + \tau^2 \dot{x}^T(t) Q\dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s) Q\dot{x}(s)\, ds.\end{aligned}$$

The relaxation factor is considered as

$$\begin{aligned}2a_1\dot{x}^T(t) P[Ax(t) + BFKx(t - \tau(t)) + B_\omega\omega(t)] \\ = 2a_1\dot{x}^T(t) P\dot{x}(t).\end{aligned}$$

We can get the following condition by Lemma 1,

$$\begin{aligned}\dot{V}(x(t), t) - 2z^T(t)\,\omega(t) - \beta\omega^T(t)\,\omega(t) \\ \leq \zeta^{th2T}(t)\, \Sigma_0{}^{th2}\zeta^{th2}(t),\end{aligned} \tag{24}$$

where

$$\begin{aligned}\zeta^{th2T}(t) = \Big[& x^T(t), x^T(t - \tau(t)), x^T(t-\tau), \dot{x}^T(t) \\ & \times \frac{1}{\tau - \tau(t)} \int_{t-\tau}^{t-\tau(t)} x^T(s)\, ds, \\ & \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x^T(s)\, ds, \omega(t)\Big],\end{aligned}$$

$$\begin{aligned}\Sigma_0{}^{th2} = {} & \Sigma^{th2} + S^{th2} + \frac{\tau - \tau(t)}{\tau}\left(Z_1{}^{th2} + Z_3{}^{th2}\right) \\ & + \frac{\tau(t)}{\tau}\left(Z_2{}^{th2} + Z_4{}^{th2}\right),\end{aligned}$$
$$\Sigma^{th2} = \begin{bmatrix} \Sigma & \Sigma^{12} \\ * & -\beta I - D_\omega - D_\omega^T \end{bmatrix},$$
$$\Sigma^{12} = \begin{bmatrix} B_\omega^T P - C & 0 & 0 & a_1 B_\omega^T P & 0 & 0 \end{bmatrix}^T.$$

By integrating (24) over the time period 0 to T_0,

$$\begin{aligned}\int_0^{T_0} 2z^T(t)\,\omega(t)dt + \beta \int_0^{T_0} \omega^T(t)\,\omega(t)dt \\ \geq V(x(t), T_0) - V(x(t), 0)\end{aligned}$$

Then,

$$\int_0^{T_0} 2z^T(t)\,\omega(t)dt + \beta \int_0^{T_0} \omega^T(t)\,\omega(t)dt \geq 0$$

Pre- and post multiply $\Sigma_0{}^{th2}$ by $X_O{}^{th2} = diag\{X, X, X, X, X, X, I\}$ and its transposition. Then, the same proof process can be obtained for inequalities (20), (21) and (22).

- Case 2 $t \in [t_i + \tau,\ t_{i+1})$
  Based on the condition

$$\begin{aligned}&\dot{V}(x(t), t) \\ &\quad = 2x^T(t) P\left[\bar{A}x(t) - BFKe(t) + B_\omega\omega(t)\right] \\ &\qquad + x^T(t) P_0 x(t) - x^T(t-\tau) P_0 x(t-\tau) \\ &\qquad + \tau^2\dot{x}^T(t) Q\dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s) Q\dot{x}(s)\, ds,\end{aligned}$$

and

$$2a_2\dot{x}^T(t) P\left[\bar{A}x(t) - BFKe(t) + B_\omega\omega(t) - \dot{x}(t)\right] = 0.$$

The same proof process can be obtained for inequality (23). This completes the proof.

IV. NUMERICAL EXAMPLE

In this section, we provide two numerical examples to show the effectiveness of the proposed conditions.

Example 1: Consider the NCSs system with $\omega(t) = 0$ and the following coefficient matrices:

$$A = \begin{bmatrix} -1 & 0.5 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 \\ -1 \end{bmatrix}.$$

Let $\varepsilon_1 = 0.1, \varepsilon_2 = 0.2, a_1 = 1.5, a_2 = 1.5, \tau = 0.2, \delta = 0.4$, the fault function $0.08 \sin t + 0.8$. Solving the LMIs (7-10) in Theorem 1, it is obtained that

$$X = \begin{bmatrix} 88.7768 & 29.7882 \\ * & 82.0759 \end{bmatrix},$$
$$Y = \begin{bmatrix} 31.4391 & -38.3420 \end{bmatrix},$$
$$\Theta = \begin{bmatrix} 70.5113 & -1.9400 \\ * & 74.4384 \end{bmatrix}.$$

Then,

$$K = \begin{bmatrix} 0.5817 & -0.6783 \end{bmatrix}.$$

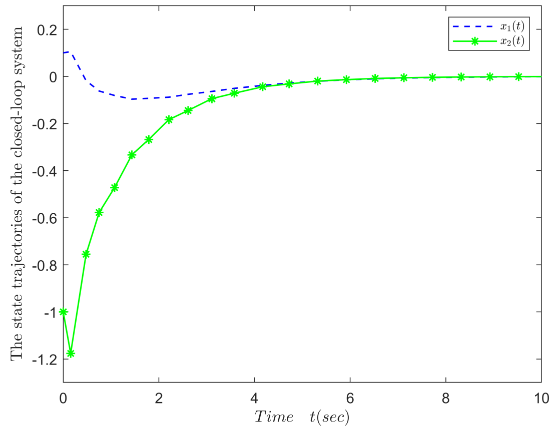

FIGURE 2. The state trajectories of the closed-loop system (Example 1).

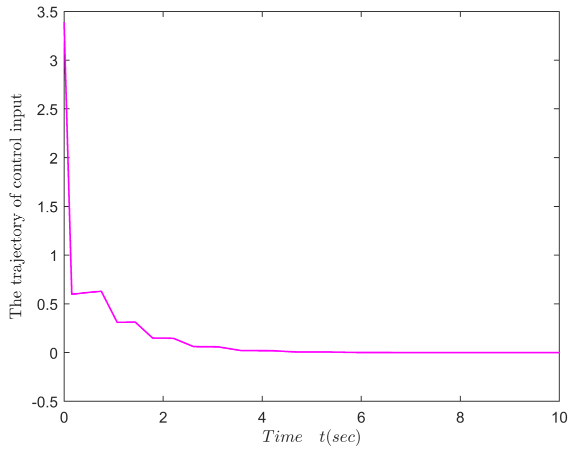

FIGURE 3. The trajectory of control input (Example 1).

Now, we assume that the initial condition is $x_0 = [0.1 - 1]^T$.

In the following, we provide some simulation results. Fig 2, Fig 3, and Fig 4 show the plots of the closed-loop system state trajectories, control input signal and the triggering time intervals. And the average release interval can be calculated as 1.11 s.

Example 2: Consider the NCSs system (1) with $\omega(t) = 0.2 * e^{-0.1t} * \sin(0.5t)$,and

$$A = \begin{bmatrix} -1.2 & 0.9 \\ 0.1 & -0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ B_\omega = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 \ 0 \end{bmatrix}, \quad D = [0.1]$$

Let $\varepsilon_1 = 10, \varepsilon_2 = 2, a_1 = 1.5, a_2 = 1.25, \tau = 0.2, \delta = 0.4$, the fault function $0.09 \sin t + 0.9$. Solving the LMIs (20-23) in Theorem 2, it is obtained that

$$X = \begin{bmatrix} 1.2530 & 0.6230 \\ * & 2.6193 \end{bmatrix},$$
$$Y = \begin{bmatrix} -0.2491 & -1.2195 \end{bmatrix},$$
$$\Theta = \begin{bmatrix} 1.5525 & 1.7763 \\ * & 7.3106 \end{bmatrix}.$$

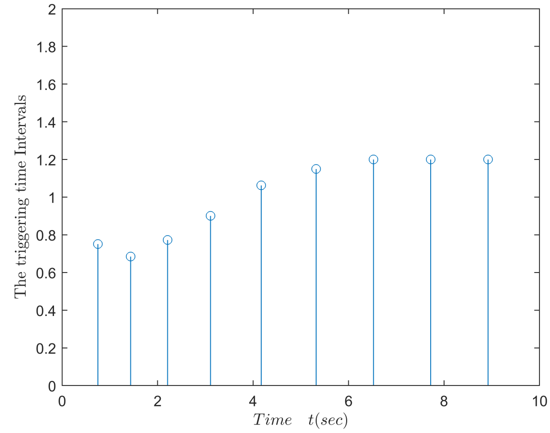

FIGURE 4. The triggering time intervals (Example 1).

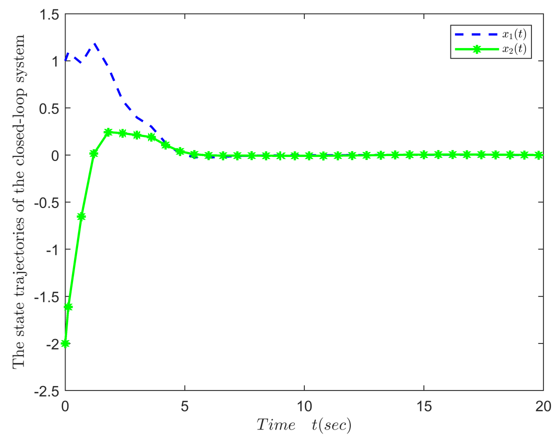

FIGURE 5. The state trajectories of the closed-loop system (Example 2).

TABLE 1. The maximum allowable value of τ for different β.

β	0.40	0.45	0.50	0.55
τ	0.8089	1.4378	2.3215	3.5450

Then, the feedback gain

$$K = \begin{bmatrix} 0.0371 & -0.4744 \end{bmatrix}.$$

Initial state has been chosen as $x_0 = [1 \ -2]^T$. Fig 5 shows the state responses of the closed-loop system, which shows that the system stability is guaranteed. Fig 6 displays the trajectory of control input. Based on the judgement algorithm (4), the release instants and the time intervals between any two adjacent release instants are presented in Fig 7. The average release interval can be calculated as 1.25 s. It is easy to obtain that event-triggered control can efficiently reduce the number of control task execution. The communication resources can be saved significantly while retaining satisfactory closed-loop performance.

Table 1 shows the maximum allowable value of τ for different β.

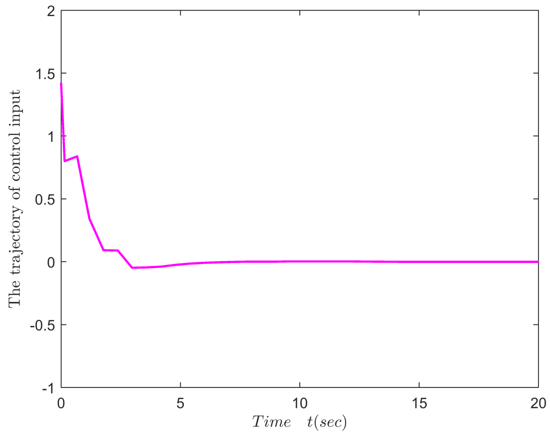

FIGURE 6. The trajectory of control input (Example 2).

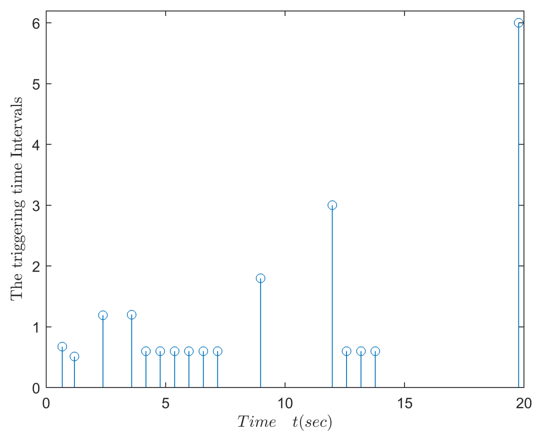

FIGURE 7. The triggering time intervals (Example 2).

V. CONCLUSION

Stability analysis and passive event-triggered controller designing method for continuous-time NCSs with actuator faults are investigated in this paper. A sufficient condition is proposed by a Lyapunov-Krasovskii functional, which can guarantee the NCSs with actuator faults are asymptotically stable and passive. The event-triggering scheme has been used in the designing of the feedback controller. The effectiveness of the proposed theorem is illustrated by two numerical examples. The closed-loop systems are modeled as dynamics with time-varying delay, so event-triggered fault-tolerant control in this paper can be extended to NCSs with time delay. In addition, the observer-based event-triggered fault-tolerant control of NCSs with time-delay is of great interest and will be studied in the future [28]. Besides, the event-triggered control for stochastic nonlinear systems can be considered in future work.

REFERENCES

[1] L. Su and G. Chesi, "Robust stability analysis and synthesis for uncertain discrete-time networked control systems over fading channels," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 1966–1971, Apr. 2017.

[2] F.-L. Lian, J. Moyne, and D. Tilbury, "Modelling and optimal controller design of networked control systems with multiple delays," *Int. J. Control*, vol. 76, no. 6, pp. 591–606, Jan. 2003.

[3] Y. Xu, H. Su, and Y.-J. Pan, "Output feedback stabilization for Markov-based nonuniformly sampled-data networked control systems," *Syst. Control Lett.*, vol. 62, no. 8, pp. 656–663, Aug. 2013.

[4] R. Sakthivel, S. Santra, and B. Kaviarasan, "Resilient sampled-data control design for singular networked systems with random missing data," *J. Franklin Inst.*, vol. 355, no. 3, pp. 1040–1072, Feb. 2018.

[5] H. Li, Z. Chen, L. Wu, H.-K. Lam, and H. Du, "Event-triggered fault detection of nonlinear networked systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1041–1052, Apr. 2017.

[6] J. Liu, "Event-based reliable H_∞ control for networked, control system with probabilistic actuator, faults," *Chin. J. Electron.*, vol. 22, no. 4, pp. 843–848, Sep. 2013.

[7] Z. Wu, J. Xiong, and M. Xie, "Improved event-triggered control for networked control systems subject to deception attacks," *J. Franklin Inst.*, vol. 358, no. 4, pp. 2229–2252, Mar. 2021.

[8] L. Zha, J.-A. Fang, and J. Liu, "Two channel event-triggering communication schemes for networked control systems," *Neurocomputing*, vol. 197, pp. 45–52, Jul. 2016.

[9] H. Wang, P. Shi, C.-C. Lim, and Q. Xue, "Event-triggered control for networked Markovian jump systems," *Int. J. Robust Nonlinear Control*, vol. 25, no. 17, pp. 3422–3438, Nov. 2014.

[10] J. Liu and D. Yue, "Event-triggering in networked systems with probabilistic sensor and actuator faults," *Inf. Sci.*, vol. 240, pp. 145–160, Aug. 2013.

[11] Y. Tan, Q. Li, D. Du, B. Niu, and S. Fei, "Observer-based distributed hybrid-triggered H_∞ control for sensor networked systems with input quantisation," *IET Control Theory Appl.*, vol. 14, no. 19, pp. 3240–3250, Dec. 2020.

[12] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.

[13] X. Li, H. R. Karimi, Y. Wang, D. Lu, and S. Guo, "Robust fault estimation and fault-tolerant control for Markovian jump systems with general uncertain transition rates," *J. Franklin Inst.*, vol. 355, no. 8, pp. 3508–3540, May 2018.

[14] G. Chen, Y. Song, and F. L. Lewis, "Distributed fault-tolerant control of networked uncertain Euler–Lagrange systems under actuator faults," *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1706–1718, Jul. 2017.

[15] K. Duan and W. Zhang, "Event-triggered fault-tolerant control for networked systems with dynamic quantiser," *IET Control Theory Appl.*, vol. 10, no. 9, pp. 1088–1096, Jun. 2016.

[16] Y. Gao, Y. Li, L. Peng, and J. Liu, "Design of event-triggered fault-tolerant control for stochastic systems with time-delays," *Sensors*, vol. 18, no. 6, p. 1929, Jun. 2018.

[17] R. Rakkiyappan, A. Chandrasekar, S. Laksmanan, and J. H. Park, "State estimation of memristor-based recurrent neural networks with time-varying delays based on passivity theory," *Complexity*, vol. 19, no. 4, pp. 32–43, Nov. 2013.

[18] H. Li, C. Wu, P. Shi, and Y. Gao, "Control of nonlinear networked systems with packet dropouts: Interval type-2 fuzzy model-based approach," *IEEE Trans. Cybern.*, vol. 45, no. 11, pp. 2378–2389, Nov. 2015.

[19] H. Shen, L. Su, and J. H. Park, "Reliable mixed H_∞/passive control for T–S fuzzy delayed systems based on a semi-Markov jump model approach," *Fuzzy Sets Syst.*, vol. 314, pp. 79–98, May 2017.

[20] L. Su and D. Ye, "Mixed H_∞ and passive event-triggered reliable control for T–S fuzzy Markov jump systems," *Neurocomputing*, vol. 281, pp. 96–105, Mar. 2018.

[21] L. Wu and W. X. Zheng, "Passivity-based sliding mode control of uncertain singular time-delay systems," *Automatica*, vol. 45, no. 9, pp. 2120–2127, Sep. 2009.

[22] H. Shen, M. Chen, Z.-G. Wu, J. Cao, and J. H. Park, "Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1708–1722, Aug. 2020.

[23] H. Wang, Y. Wang, G. Zhuang, and J. Lu, "Asynchronous passive dynamic event-triggered controller design for singular Markov jump systems with general transition rates under stochastic cyber-attacks," *IET Control Theory Appl.*, vol. 14, no. 16, pp. 2291–2302, Sep. 2020.

[24] X. Zhang and Q. Han, "Event-based H_∞ filtering for sampled-data systems," *Automatica*, vol. 51, pp. 55–69, Jan. 2015.

[25] D. Yue, E. Tian, Y. Zhang, and C. Peng, "Delay-distribution-dependent stability and stabilization of T–S fuzzy systems with probabilistic interval delay," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 39, no. 2, pp. 503–516, Apr. 2009.

[26] I. R. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Syst. Control Lett.*, vol. 8, no. 4, pp. 351–357, Mar. 1987.

[27] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.

[28] Y. Tan, Q. Liu, J. Liu, X. Xie, and S. Fei, "Observer-based security control for interconnected semi-Markovian jump systems with unknown transition probabilities," *IEEE Trans. Cybern.*, vol. 52, no. 9, pp. 9013–9025, Sep. 2022.

LEI FU received the B.S. degree from the Hebei Normal University of Science and Technology, Qinhuangdao, China, in 2012, and the M.S. and Ph.D. degrees from Yanshan University, Qinhuangdao, in 2015, and 2019, respectively.

She is currently a Lecturer with the College of Electronic and Information Engineering, Hebei University, China. Her research interests include stability and control theory of the singular systems and stochastic systems.

JIACHANG SHI received the bachelor's degree in information engineering from Hebei University, in 2021, where he is currently pursuing the master's degree with the College of Electronic and Information Engineering.

His main research interests include dissipative control and fault tolerant control of T-S fuzzy systems.

FEI LI received the Ph.D. degree from the Beijing University of Posts and Telecommunications (BUPT), China, in 2019.

He is currently a Lecturer with the College of Electronic and Information Engineering, Hebei University. He has accumulated rich experience in the field of 5G. His research interests include wireless communication, such as new waveform and channel estimation, and communication problems in control systems.