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RESEARCH ARTICLE

Input-Output Finite-Time Sliding Mode Control of Discrete Time-Varying Systems Under an Adaptive Event-Triggered Mechanism

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ABSTRACT This paper is concerned with the issue of input-output finite-time stability (IO-FTS) for a class of nonlinear discrete time-varying systems. A time-varying observer-based sliding mode control method is proposed. In order to mitigate the transmission burden, an adaptive event-triggered mechanism is proposed by adjusting the threshold. Taking consideration of the effect of time-delay phenomenon and time-varying system matrices, a time-varying Lyapunov functional is designed. Based on the designed Lyapunov functional and IO-FTS theory, sufficient conditions are established for the error estimation system and closed-loop state estimation system. Moreover, the proposed observer-based sliding mode control method makes sure the reachability of the quasi-sliding mode surface in finite steps. And conditions in terms of recursive linear matrix inequalities (RLMIs) are attained to ensure IO-FTS during both reaching phase and sliding mode phase. An algorithm is provided to solve the RLMIs and obtain the time-varying observer gains. Finally, the effectiveness and superiority of the proposed method is demonstrated by an industrial continuous-stirred tank reactor system.

INDEX TERMS Discrete time-varying system, adaptive event-triggered mechanism, observer-based sliding mode control, input-output finite-time stability.

I. INTRODUCTION

In practice, time-varying systems represent effective tools to describe different types of dynamical systems containing some time-varying attributes, such as, periodic systems, sampled-data systems and switched systems, etc., see for instance, [1], [2], and [3]. Moreover, aperiodic sampling in networked control systems (NCSs) often results in that NCSs are transformed to discrete time-varying systems (DTVSs). Accordingly, it is of great significance to study DTVSs, and they have gained increasing attention. Observer-based finite-time H_∞ control of nonlinear DTVSs was studied in [4]. The issue of finite horizon fault detection of linear DTVSs was discussed in [5]. The consensus control problem was studied for a discrete time-varying multi-agent system in [6].

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On the other hand, one fact has been observed that, in NCSs, there are some sampled data packets that the information carried by them is less fluctuating than the last transmitted sampled data packets. This kind of transmission may introduce some unnecessary communication burden. In response to these problems, the event trigger mechanism (ETM) is proposed, for example, [7], [8], and [9]. Compared with time-triggered mechanism, ETM can reduce the network resource occupancy as well as maintaining the control performance. However, most of the proposed ETMs are designed with fixed thresholds that is not easy to give them proper values in advance, which is named as traditional ETM (TETM) in this paper. To deal with this, the adaptive event-triggered mechanisms (AETMs) with adjustable trigger conditions are proposed in [10] and [11]. It has been proved that AETM can save more network resources. Therefore, a critical issue is yet to be addressed when considering DTVSs, that is, can we propose an AETM to save more resources while preserving the desired control performance? To the author's best knowledge, there are few relevant results focus on DTVSs, which motivates our present study.

Since sliding mode control (SMC) was proposed in 1977, due to some appealing features of rapid transient response, robustness, and simplicity of implementation, it has become an effective method for suppressing disturbances in complex systems. Some SMC results of uncertain nonlinear systems, especially on advanced and optimization based algorithms were given in [12]. For continuous systems, there are many research results using SMC methods, see [13], [14], and [15]. Due to the emerging digital technologies, many practical systems are treated in the structure of discretized forms and then discrete-time SMC (DSMC) has become an increasingly hot topic, see, for instance [16] and [17]. In [18], the SMC design problem for discrete-time piecewise nonhomogeneous Markov jump nonlinear systems was discussed. In [19], dissipative-based DSMC of switched stochastic hybrid systems was discussed. And in [20], an optimal SMC approach was used to study the consensus of nonlinear discrete-time high-order multi-agent systems. In addition, there are many situations where the state variables can not be measured. In response to this issue, the dynamic and static output feedback SMC methods were studied in [21] and [22], respectively. Ma et al. discussed the observer-based adaptive sliding mode control problem for a kind of stochastic jump systems in [23]. Nevertheless, it is witnessed that the study of observer-based sliding mode control for DTVSs has not been fully spread, which becomes to the second motivation of this research.

It is noticed that most of the above mentioned literature about SMC method was considered during infinite time intervals. In fact, FTS proposed by Dorato in [24] and developed by Amato is a more practical concept [25]. It is applied in some cases where the required state or output variable does not exceed some a threshold during a fixed time interval, for example, flight control [26], terminal guidance system [27]. Another research frontier proposed based on FTS is IO-FTS, which defines that if a class of norm-bounded input signals is given within a specified time interval, the outputs of the system do not exceed the specified threshold during the time interval [28]. The issue of input-output finite-time generalized dissipative filter was studied for a kind of DTVSs in [29]. Based on the definitions of FTS and IO-FTS, an SMC design method was proposed in [30] and [31], respectively. In these two papers, FTS of the considered systems can be guaranteed during both reaching and sliding mode phase. Subsequently, FTS problem of uncertain neutral time-delay systems was discussed via the SMC approach in [32]. However, with regards to DTVSs, the research of IO-FTS based on observer-based sliding mode control method is still blank, which also motivates the following study.

Based on the above discussion, in this paper, the problem of IO-FTS is addressed for a nonlinear DTVS considering time-delay phenomenon and an AETM by the observer-based sliding mode control method. The main contributions are highlighted as: (1) Threshold of the proposed AETM can be adjusted. Thus, the network communication resources can be saved while sacrificing system performance as little as possible. (2) The reachability of quasi-sliding mode is guaranteed in finite steps and the upper bound of quasi-sliding mode surface is given under the proposed event-triggered SMC law. (3) Based on a time-varying Lyapunov functional, IO-FTS conditions during both reaching and sliding mode phase are obtained in terms of RLMIs. And the time-varying observer gains can be solved by an algorithm.

A. NOTATIONS

Throughout this paper, $\mathbb{N}$ is the set of natural numbers. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. For a scalar $N > 0$, denote a set as $\mathcal{T} = \{0, 1, 2, \dots, N\}$. For a symmetric matrix A, $A > 0$ ($A \geq 0$) means that A is a symmetric positive-definite (semi-positive-definite) matrix. $\mathbb{M}^+$ denotes the set of positive matrix-valued sequence. $\mathcal{C}^+$ denotes the set of positive scalars. For $R_k \in \mathbb{M}^+$, define $\mathbf{W}_2(\mathcal{T}, R_k) := \{\omega_k \in \mathcal{L}_{2,\mathcal{T}} : \|\omega_k\|^2_{\mathcal{T},R_k} \leq \alpha\}$, $\alpha \in \mathcal{C}^+$ and $\|\omega\|^2_{\mathcal{T},R_k} = \sum_{k\in\mathcal{T}} \omega_k^T R_k \omega_k$. I and 0 denote the identity matrix and null matrix of the appropriate dimension, respectively. The symbol $*$ means the symmetric terms in a symmetric matrix. Matrices without special explanation are considered to have proper dimensions.

II. PROBLEM FORMULATION

Consider a nonlinear DTVS as:

$$\begin{aligned} x_{k+1} &= A_k x_k + B(u_k + f_k) + D_k \omega_k \\ y_k &= C_k x_k \end{aligned} \tag{1}$$

where A_k, D_k and C_k are time-varying matrix-valued sequences, B is the input matrix with rank(B)=m. $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^r$ stands for the system measured output, $u_k \in \mathbb{R}^m$ is the controlled input, and ω_k is the disturbance input satisfying $\omega_k \in \mathbf{W}_2(\mathcal{T}, R_k)$.

Assumption 1: $f_k \triangleq f(k, x_k) \in \mathbb{R}^m$ is a given nonlinear vector function, which satisfies:

$$\|f_k\| \leq \sigma_k \|y_k\| \tag{2}$$

where $\sigma_k \in \mathcal{C}^+$ is a series of known scalars.

Assumption 2: For $\forall k \in \mathcal{T}$, it is assumed that (A_k, B) is controllable and (C_k, A_k) is observable.

Then, we design an observer in the following form to estimate the unmeasurable states:

$$\begin{aligned} \hat{x}_{k+1} &= A_k \hat{x}_k + B(u_k + \hat{f}_k) + L_k(\bar{y}_k - \hat{y}_k) \\ \hat{y}_k &= C_k \hat{x}_k \end{aligned} \tag{3}$$

where $\hat{x}_k$ is the estimation of the state, $\hat{y}_k$ is the output of the observer, $\bar{y}_k$ is the plant output after the network, $\hat{f}_k \triangleq f(k, \hat{x}_k)$ and L_k is the time-varying observer gain to be designed.

A. MODELING BASED ON THE AETM

To alleviate waste of resources and avoid unnecessarily frequent transmission, an event detector is adopted between the sensor and observer as shown in Fig. 1 that describes the control process.

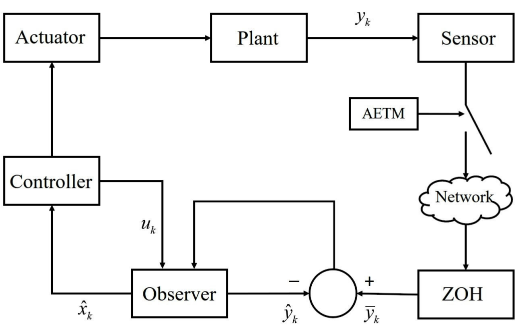

FIGURE 1. The schematic of the NCS with an AETM-based observer.

It is assumed that sensors are time-triggered and the measured instant is denoted as $l \in \mathbb{N}$. The measurement output is only transmitted when the following condition is satisfied:

$$(y_{k_s+l} - y_{k_s})^T \Omega_k (y_{k_s+l} - y_{k_s}) \geq \gamma_k y_{k_s}^T \Omega_k y_{k_s} \tag{4}$$

where $s \in \mathbb{N}$, $k_s + l$ is the current measurement instant, $y_{k_s+l} = y(k_s + l)$ is the current measurement output, k_s is the instant when an event happens and y_{k_s} is the transmitted state output. $\Omega_k \in \mathbb{M}^+$ are weighting matrices to be designed. γ_k is a series of scalars determined by the following condition:

$$\gamma_{k+1} = \begin{cases} \lambda_1 \gamma_k, & \text{if } \|y_e\|^2 > \epsilon \\ \lambda_2 \gamma_k, & \text{if } \|y_e\|^2 < \epsilon \\ \gamma_k, & \text{if } \|y_e\|^2 = \epsilon \end{cases} \tag{5}$$

where $y_e = y_{k_s+l} - y_{k_s}$, $\lambda_1 = 1 - \frac{2}{\pi} atan(\varsigma \|y_e\|^2)$, $\lambda_2 = 1 - \frac{2}{\pi} atan(-\varsigma \|y_e\|^2)$, $atan(\cdot)$ is the arctangent function. It is assumed that $\gamma_k \in [\gamma_m, \gamma_M]$, γ_m and γ_M are the lower and upper bounds of γ_k, respectively, and $\epsilon, \varsigma \in \mathcal{C}^+$.

Remark 1: In (5), when γ_k is a constant, the adaptive event-triggered condition (4) is reduced to a TETM. When $\gamma_k = 0$, it is the time-triggered case.

Remark 2: Using the property of arctangent function, we can see that if $\|y_e\|^2 > \epsilon$, it holds that $\gamma_{k+1} = \lambda_1 \gamma_k$. Since $0 < \lambda_1 < 1$, thus $\gamma_{k+1} < \gamma_k$. It means that smaller γ_k will be used in the next event judgement, which leads to high tranmission frequency. Oppositely, if $\|y_e\|^2 < \epsilon$, it holds that $\gamma_{k+1} = \lambda_2 \gamma_k$. Since $\lambda_2 > 1$, thus $\gamma_{k+1} > \gamma_k$. Then, larger γ_k will be selected in the next judgement, the communication bandwidth usage can be saved.

Remark 3: It should be noticed that introducing an ETM may sacrifice some SMC performance. However, the proposed AETM is adjusted by the transmission error, which can help improving the performance while reducing the network utilization.

B. INDUCED-TIME DELAY MODELING

Let d_{k_s} denote the transmission delay at triggered instant k_s with the assumption $d_{k_s} \in [0, d_m]$. d_m is a positive scalar. Considering the triggered condition (4) and during the holding interval $[k_s + d_{k_s}, k_{s+1} + d_{k_{s+1}})$, it holds that,

$$(y_{k_s+l} - y_{k_s})^T \Omega_k (y_{k_s+l} - y_{k_s}) < \gamma_k y_{k_s}^T \Omega_k y_{k_s} \tag{6}$$

Based on [8] and [33], denoting $\mathcal{I} = [k_s + d_{k_s}, k_{s+1} + d_{k_{s+1}})$, the following two cases should be discussed.

Case I. If $k_s + d_m + 1 \geq k_{s+1} + d_{k_{s+1}}$, define the time delay function d_k as:

$$d_k = k - k_s, \quad k \in \mathcal{I}$$

Thus, $d_{k_s} \leq d_k \leq k_{s+1} - k_s + d_{k_{s+1}} \leq d_m + 1$.

Case II. If $k_s + d_m + 1 < k_{s+1} + d_{k_{s+1}}$, two intervals $[k_s + d_{k_s}, k_s + d_m]$ and $[k_s + d_m + l, k_s + d_m + l + 1]$ are considered. Since $d_{k_s} \leq d_m$, an integer 'q' can be found satisfying:

$$k_s + q + d_m < k_{s+1} + d_{k_{s+1}} - 1 \leq k_s + q + d_m + 1$$

Denote

$$\mathcal{I}_0 = [k_s + d_{k_s}, k_s + d_m + 1),$$
$$\mathcal{I}_l = [k_s + d_m + l, k_s + d_m + l + 1), \quad l = 1, 2, \ldots, q - 1,$$
$$\mathcal{I}_{d_m} = [k_s + d_m + q, k_{s+1} + d_{k_{s+1}}).$$

Based on the above discussion, d_k and δ_k are given as:

$$d_k = \begin{cases} k - k_s, & k \in \mathcal{I}_0 \\ k - k_s - l, & k \in \mathcal{I}_l, l = 1, 2, \ldots, q - 1 \\ k - k_s - q, & k \in \mathcal{I}_{d_m} \end{cases}$$

$$\delta_k = \begin{cases} 0, & k \in \mathcal{I}_0 \\ y_{k_s+l} - y_{k_s}, & k \in \mathcal{I}_l, l = 1, 2, \ldots, q - 1 \\ y_{k_s+q} - y_{k_s}, & k \in \mathcal{I}_{d_m} \end{cases}$$

Then, it holds that $0 \leq d_1 \leq d_k \leq d_m + 1 \triangleq d_2$. d_1 and d_2 are the lower and upper bounds of d_k, respectively.

Thus, y_{k_s} can be rewritten as

$$\bar{y}_k = y_{k_s} = y_{k-d_k} - \delta_k \tag{7}$$

where $y_{k-d_k} \triangleq y(k - d_k) = C_k x_{k-d_k}$.

Then, the error estimation system is obtained as follows:

$$\begin{aligned} e_{k+1} &= A_k e_k - L_k C_k e_{k-d_k} + L_k C_k \hat{x}_k - L_k C_k \hat{x}_{k-d_k} \\ &\quad + B(f_k - \hat{f}_k) + D_k \omega_k + L_k \delta_k \end{aligned} \tag{8}$$

where $e_{k-d_k} = x_{k-d_k} - \hat{x}_{k-d_k}$.

C. SLIDING SURFACE AND SLIDING MODE CONTROLLER

We design the following sliding function as follows:

$$S_k = G\hat{x}_k \tag{9}$$

where $G = B^T X$, $X > 0$ thus GB is nonsingular.

To this end, an appropriate observer-based sliding mode control law is given as follows:

$$\begin{aligned} u_k &= -(GB)^{-1} G A_k \hat{x}_k - (GB)^{-1} G L_k (\bar{y}_k - \hat{y}_k) \\ &\quad - \sigma_k sgn(S_k) \end{aligned} \tag{10}$$

Using SMC law (10), the closed-loop state estimation system can be obtained as follows:

$$\hat{x}_{k+1} = \mathcal{G}[(A_k - L_kC_k)\hat{x}_k + L_kC_k\hat{x}_{k-d_k} + L_kC_ke_{k-d_k} - L_k\delta_k] + B\hat{f}_k - B\mu_k \tag{11}$$

where $\mathcal{G} = I - B(GB)^{-1}G$ and $\mu_k = \sigma_k sgn(S_k)$.

In the following, the definition of IO-FTS proposed in [28] is extended for the overall closed-loop system composed of (8) and (11).

Definition 1: (IO-FTS) For a given positive scalar $N \in \mathbb{N}$, a class of input signal $\mathbf{W}_2$ defined on $\mathcal{T}$, a matrix-valued sequence $R_k \in \mathbb{M}^+$, and an output constraint scalar $\beta > 0$, the overall system (8) and (11) is IO-FTS with respect to $(\mathbf{W}_2, R_k, \mathcal{T})$, if for all $k \in \mathcal{T}$, it holds that:

$$\omega_k \in \mathbf{W}_2 \Longrightarrow \hat{y}_k^T R_k \hat{y}_k + (y_k - \hat{y}_k)^T R_k (y_k - \hat{y}_k) < \beta$$

III. MAIN RESULTS

In this section, based on the discrete-time sliding surface constructed in (9) and an event-triggered SMC law in (10), IO-FTS conditions are given in Theorem 1 and reachability for the quasi-sliding mode surface is analyzed in Theorem 2. Synthesis of the time-varying observer is given in Theorem 3.

For the following proof process, we give some denotations as:

$$\varrho_i = [\underbrace{0\cdots0}_{i-1}\ \ I\ \ \underbrace{0\cdots0}_{13-i}],\ i = 1, 2, \cdots, 13,$$

$$d_{12} = d_2 - d_1,\ \eta_k^T = [\hat{x}_k^T\ \ e_k^T],$$

$$\xi_k^T = [\eta_k^T\ \ \eta_{k-d_1}^T\ \ \eta_{k-d_k}^T\ \ \eta_{k-d_2}^T\ \ \delta_k^T\ \ \omega_k^T\ \ \mu_k^T\ \ \hat{f}_k^T\ \ f_k^T - \hat{f}_k^T].$$

A. IO-FTS ANALYSIS OF SLIDING MODE DYNAMICS

The IO-FTS of the overall closed-loop system composed of (8) and (11) is given in the following theorem.

Theorem 1: For given scalars $d_1, d_2, \alpha, \beta, N \in \mathcal{C}^+$, matrix-valued sequence $R_k \in \mathbb{M}^+$, and input disturbance in $\mathbf{W}_2$, the overall system (8) and (11) is IO-FTS under AETM (4) and SMC law (10), if there exist matrix-valued sequences $P_k, Q_{1k}, Q_{2k}, Q_{3k}, \Omega_k \in \mathbb{M}^+$, and a scalar $\nu \in \mathcal{C}^+$, such that following conditions hold:

$$\begin{bmatrix} \Lambda_k + \Lambda_{3k} + \Lambda_{4k} + \Lambda_{5k} & \Lambda_{1k}^T & \Lambda_{2k}^T \\ * & -P_{k+1}^{-1} & 0 \\ * & * & -P_{k+1}^{-1} \end{bmatrix} < 0 \tag{12}$$

$$diag\{C_k^T R_k C_k, C_k^T R_k C_k\} < \mathcal{P}_k \tag{13}$$

where

$$\begin{aligned}\Lambda_k = diag\{&Q_{1k} + Q_{2k} + Q_{3k} + d_{12}Q_{1k} - P_k,\\ &Q_{1k} + Q_{2k} + Q_{3k} + d_{12}Q_{1k} - P_k, -Q_{2(k-d_1)},\\ &-Q_{2(k-d_1)}, -Q_{1(k-d_k)} + \gamma_k C_k^T \Omega_k C_k,\\ &-Q_{1(k-d_k)} + \gamma_k C_k^T \Omega_k C_k,\\ &-Q_{3(k-d_2)}, -Q_{3(k-d_2)}, (\gamma_k - 1)\Omega_k,\\ &-\tilde{\beta} R_k, -\nu^{-1} I, 0, 0\},\end{aligned}$$

$$\begin{aligned}\Lambda_{1k} &= [\mathcal{G}(A_k - L_kC_k)\ \ 0\ \ 0\ \ 0\ \ \mathcal{G}L_kC_k\ \ \mathcal{G}L_kC_k\\ &\quad\ 0\ \ 0\ \ -\mathcal{G}L_k\ \ 0\ \ -B\ \ B\ \ 0],\\ \Lambda_{2k} &= [L_kC_k\ \ A_k\ \ 0\ \ 0\ \ -L_kC_k\ \ -L_kC_k\\ &\quad\ 0\ \ 0\ \ L_k\ \ D_k\ \ 0\ \ 0\ \ B],\\ \Lambda_{3k} &= 2\gamma_k\varrho_5^T C_k^T \Omega_k C_k \varrho_6 - 2\gamma_k \varrho_5^T C_k^T \Omega_k \varrho_9\\ &\quad - 2\gamma_k \varrho_6^T C_k^T \Omega_k \varrho_9,\\ \Lambda_{4k} &= \sigma_k^2 \varrho_1^T C_k^T C_k \varrho_1 - \varrho_{12}^T \varrho_{12},\\ \Lambda_{5k} &= 2\sigma_k^2 \varrho_1^T C_k^T C_k \varrho_1 + 2\sigma_k^2 \varrho_1^T C_k^T C_k \varrho_2\\ &\quad + \sigma_k^2 \varrho_2^T C_k^T C_k \varrho_2 - \varrho_{13}^T \varrho_{13}.\end{aligned}$$

Proof: Define a time-varying Lyapunov functional as follows:

$$V(k) = \sum_{i=1}^{5} V_i(k) \tag{14}$$

where

$$\begin{aligned}V_1(k) &= \eta_k^T \mathcal{P}_k \eta_k,\\ V_2(k) &= \sum_{i=k-d_k}^{k-1} \eta_i^T \mathcal{Q}_{1i} \eta_i\\ V_3(k) &= \sum_{i=k-d_1}^{k-1} \eta_i^T \mathcal{Q}_{2i} \eta_i\\ V_4(k) &= \sum_{i=k-d_2}^{k-1} \eta_i^T \mathcal{Q}_{3i} \eta_i\\ V_5(k) &= \sum_{j=-d_2}^{-d_1-1} \sum_{i=k+j}^{k-1} \eta_i^T \mathcal{Q}_{1i} \eta_i\end{aligned}$$

with $\mathcal{P}_k = diag\{P_k, P_k\}$ and $\mathcal{Q}_{\iota k} = diag\{Q_{\iota k}, Q_{\iota k}\}$, $\iota = 1, 2, 3$.

Defining $\Delta V(k) = V(k+1) - V(k)$, it can be obtained that

$$\begin{aligned}\Delta V_1(k) &= \eta_{k+1}^T \mathcal{P}_{k+1} \eta_{k+1} - \eta_k^T \mathcal{P}_k \eta_k\\ &= \hat{x}_{k+1}^T P_{k+1} \hat{x}_{k+1} + e_{k+1}^T P_{k+1} e_{k+1}\\ &\quad - \hat{x}_k^T P_k \hat{x}_k - e_k^T P_k e_k\\ &= \xi_k^T \Lambda_{1k}^T P_{k+1} \Lambda_{1k} \xi_k + \xi_k^T \Lambda_{2k}^T P_{k+1} \Lambda_{2k} \xi_k\\ &\quad - \hat{x}_k^T P_k \hat{x}_k - e_k^T P_k e_k \end{aligned} \tag{15}$$

$$\Delta V_2(k) \le \eta_k^T \mathcal{Q}_{1k} \eta_k - \eta_{k-d_k}^T \mathcal{Q}_{1(k-d_k)} \eta_{k-d_k} + \sum_{i=k+1-d_2}^{k-d_1} \eta_i^T \mathcal{Q}_{1i} \eta_i \tag{16}$$

$$\Delta V_3(k) = \eta_k^T \mathcal{Q}_{2k} \eta_k - \eta_{k-d_1}^T \mathcal{Q}_{2(k-d_1)} \eta_{k-d_1} \tag{17}$$

$$\Delta V_4(k) = \eta_k^T \mathcal{Q}_{3k} \eta_k - \eta_{k-d_2}^T \mathcal{Q}_{3(k-d_2)} \eta_{k-d_2} \tag{18}$$

$$\Delta V_5(k) = d_{12} \eta_k^T \mathcal{Q}_{1k} \eta_k - \sum_{i=k+1-d_2}^{k-d_1} \eta_i^T \mathcal{Q}_{1i} \eta_i \tag{19}$$

Based on Assumption 1, it holds that

$$\begin{aligned} \hat{f}_k^T\hat{f}_k &\leq \sigma_k^2\hat{x}_k^T C_k^T C_k\hat{x}_k \\ (f_k^T-\hat{f}_k^T)(f_k-\hat{f}_k) &\leq \sigma_k^2(\hat{x}_k^T+e_k^T)C_k^T C_k(\hat{x}_k+e_k) \\ &\quad +\sigma_k^2\hat{x}_k^T C_k^T C_k\hat{x}_k \end{aligned} \tag{20}$$

Considering (6), $\forall k \in [k_s+d_{k_s}, k_{s+1}+d_{k_{s+1}})$, it holds that,

$$\gamma_k y_{k_s}^T\Omega_k y_{k_s} - \delta_k^T\Omega_k\delta_k > 0 \tag{21}$$

Define

$$J_k = \Delta V(k) - \tilde{\beta}\omega_k^T R_k\omega_k - \nu^{-1}\mu_k^T\mu_k \tag{22}$$

where $\nu \in \mathcal{C}^+$ and $\tilde{\beta} = \beta\alpha^{-1} - N\nu^{-1}m\sigma_k^2\alpha^{-1}$.

Combining (15)-(21) and (7), it can be deduced that,

$$\begin{aligned} J_k \leq &\xi_k^T(\Lambda_k + \Lambda_{3k} + \Lambda_{1k}^T P_{k+1}\Lambda_{1k} + \Lambda_{2k}^T P_{k+1}\Lambda_{2k} \\ &+ \Lambda_{4k} + \Lambda_{5k})\xi_k \end{aligned} \tag{23}$$

Then, by Schur complement lemma and (12), it can be guaranteed that

$$\Delta V(k) - \tilde{\beta}\omega_k^T R_k\omega_k - \nu^{-1}\mu_k^T\mu_k < 0 \tag{24}$$

Summing (24) over $\{0, 1, 2, \cdots, N-1\}$, considering zero initial condition and $\mu_k^T\mu_k \leq m\sigma_k^2$, $\forall\omega_k \in \mathbf{W}_2$, it holds that:

$$\begin{aligned} V(k) &< \tilde{\beta}\sum_{i=0}^{N-1}\omega_i^T R_i\omega_i + N\nu^{-1}m\sigma_k^2 \\ &= \tilde{\beta}\|\omega\|_{0,1,\cdots,N-1}^2 + N\nu^{-1}m\sigma_k^2 \\ &< \tilde{\beta}\alpha + N\nu^{-1}m\sigma_k^2 \\ &< \beta \end{aligned} \tag{25}$$

Then, it is true that

$$V_1(k) < \beta \tag{26}$$

From (13) and (26), one can see that

$$\begin{aligned} &\hat{y}_k^T R_k\hat{y}_k + (y_k-\hat{y}_k)^T R_k(y_k-\hat{y}_k) \\ &\quad = \hat{x}_k^T C_k^T R_k C_k\hat{x}_k + e_k^T C_k^T R_k C_k e_k \\ &\quad < \eta_k^T\mathcal{P}_k\eta_k \\ &\quad = V_1(k) < \beta \end{aligned} \tag{27}$$

To sum up, the overall system (8) and (11) is IO-FTS with respect to $(\mathbf{W}_2, R_k, \mathcal{T})$. ■

Remark 4: It is noticed that, in Lyapunov functional (14), the Lyapunov matrices P_k, Q_{1k}, Q_{2k}, Q_{3k} are time-varying matrix-valued sequences. This design method greatly improves the solvability of obtained conditions.

B. ANALYSIS OF REACHABILITY

In this section, we focus on the reachability of the quasi-sliding mode surface of the overall closed-loop system (8) and (11)'s trajectories in finite steps.

Theorem 2: Considering the overall system (8) and (11), the state trajectories can be driven into a sliding region $\mathcal{S}$ around the sliding mode surface (9) within finite steps N^* by SMC law (10), and IO-FTS during the reaching phase is guaranteed, if there exist matrix-valued sequences $P_k, Q_{1k}, Q_{2k}, Q_{3k}, \Omega_k \in \mathbb{M}^+$, and a scalar $\nu \in \mathcal{C}^+$, such that (13) and the following conditions hold and the sliding region $\mathcal{S}$ is given as follows:

$$\begin{bmatrix} \bar{\Lambda}_k + \Lambda_{3k} + \Lambda_{4k} + \Lambda_{5k} & \Lambda_{1k}^T & \Lambda_{2k}^T \\ * & -P_{k+1}^{-1} & 0 \\ * & * & -P_{k+1}^{-1} \end{bmatrix} < 0 \tag{28}$$

$$\mathcal{S} \triangleq \{S_k \mid \|S_k\| \leq S_k^*\} \tag{29}$$

where

$$\begin{aligned} \bar{\Lambda}_k = &\, diag\{Q_{1k} + Q_{2k} + Q_{3k} + d_{12}Q_{1k} - P_k \\ &+\sigma_k^2 C_k^T(GB)^T C_k, Q_{1k} + Q_{2k} + Q_{3k} + d_{12}Q_{1k} \\ &- P_k, -Q_{2(k-d_1)}, -Q_{2(k-d_1)}, \\ &- Q_{1(k-d_k)} + \gamma_k C_k^T\Omega_k C_k, \\ &- Q_{1(k-d_k)} + \gamma_k C_k^T\Omega_k C_k, \\ &- Q_{3(k-d_2)}, -Q_{3(k-d_2)}, \\ &(\gamma_k - 1)\Omega_k, -\tilde{\beta}R_k, -\nu^{-1}I, 0, 0\}, \end{aligned}$$

$S_k^* \triangleq \sqrt{\frac{2\lambda_{max}[(GB)^T]m}{\lambda_{min}[(GB)^{-1}]}}\sigma_k + \sqrt{\frac{2\varepsilon}{\lambda_{min}[(GB)^{-1}]}}$, and $\varepsilon > 0$ is a specified scalar satisfying $\varepsilon = \frac{\lambda_{max}[(GB)^{-1}]}{2N}\|G\hat{x}_0\|^2$ with $\hat{x}_0$ is the initial value of $\hat{x}_k$.

Proof: From (9) and (10), it can be obtained that

$$S_{k+1} = GB\hat{f}_k - GB\mu_k$$

Construct a Lyapunov functional as

$$V_{sk} = V(k) + \frac{1}{2}S_k^T(GB)^{-1}S_k \tag{30}$$

where $V(k)$ is defined in (14).

Then, it can be derived that

$$\begin{aligned} \Delta V_{sk} &= \Delta V(k) + \frac{1}{2}S_{k+1}^T(GB)^{-1}S_{k+1} \\ &\quad - \frac{1}{2}S_k^T(GB)^{-1}S_k \\ &= \Delta V(k) + \frac{1}{2}(\hat{f}_k - \mu_k)^T(GB)^T(\hat{f}_k - \mu_k) \\ &\quad - \frac{1}{2}S_k^T(GB)^{-1}S_k \\ &\leq \Delta V(k) + \hat{f}_k^T(GB)^T\hat{f}_k + \mu_k^T(GB)^T\mu_k \\ &\quad - \frac{1}{2}S_k^T(GB)^{-1}S_k \end{aligned} \tag{31}$$

Considering J_k defined in (22), one can obtain that

$$\begin{aligned} &J_k + \hat{f}_k^T(GB)^T\hat{f}_k + \mu_k^T(GB)^T\mu_k - \frac{1}{2}S_k^T(GB)^{-1}S_k \\ &\quad \leq \xi_k^T(\bar{\Lambda}_k + \Lambda_{3k} + \Lambda_{1k}^T P_{k+1}\Lambda_{1k} + \Lambda_{2k}^T P_{k+1}\Lambda_{2k} \\ &\qquad + \Lambda_{4k} + \Lambda_{5k})\xi_k + \mu_k^T(GB)^T\mu_k - \frac{1}{2}S_k^T(GB)^{-1}S_k \end{aligned} \tag{32}$$

Since outside the region $\mathcal{S}$, it is true that

$$\|S_k\| > S_k^* \tag{33}$$

Then, considering (31), (32) and (33), one can deduce that

$$\Delta V_{sk} < -\varepsilon \tag{34}$$

Thus, it means that state trajectories are strictly decreasing outside the region $\mathcal{S}$ defined in (29).

Summing both sides of (34) from 0 to N^*-1, denoting V_{s0} as the initial value of V_{sk}, it can be seen that

$$N^* < \frac{V_{s0}}{\varepsilon} \leq \frac{\frac{1}{2}\lambda_{max}[(GB)^{-1}]}{\varepsilon}\|G\hat{x}_0\|^2 \tag{35}$$

Combining the definition of ε and (35), one can derive $N^* < N$. It means that, for a finite-time interval $[0, N]$, under the SMC law (10), the estimation state trajectory can be driven onto the quasi-sliding mode surface of S_k within finite step N^*. The reachability is proved.

On the other hand, by condition (28), it is easy to see that $J_k < 0$ can be guaranteed. Based on the proof in Theorem 1, the overall closed-loop system composed of (8) and (11) is IO-FTS during the reaching phase. ■

Remark 5: According to Theorem 1 and 2, it can be concluded that trajectories of overall closed-loop system (8) and (11) can be driven into the quasi-sliding mode region $\mathcal{S}$ under the SMC law (10) and IO-FTS can be guaranteed with respect to $(\mathbf{W}_2, R_k, \mathcal{T})$ by conditions (12), (13) and (28) during both the reaching and sliding mode phase.

C. SYNTHESIS OF THE TIME-VARYING OBSERVER

Obviously, inequality (28) ensures (12). Thus (28) and (13) can guarantee Theorems 1 and 2. In what follows, design of corresponding observer-based sliding mode controller will be discussed.

Theorem 3: Given scalars $d_1, d_2, \alpha, \beta \in \mathcal{C}^+$, matrix-valued sequence $R_k \in \mathbb{M}^+$, any scalars $\iota_k \in \mathcal{C}^+$, and input disturbance in $\mathbf{W}_2$, select a matrix $X > 0$ such that B^TXB is nonsingular. The overall closed-loop system (8) and (11) is IO-FTS during both the reaching and sliding mode phase with respect to $(\mathbf{W}_2, R_k, \mathcal{T})$, if there exist matrices $P_k, Q_{1k}, Q_{2k}, Q_{3k}, \Omega_k \in \mathbb{M}^+$, such that following conditions hold:

$$\begin{bmatrix} \bar{\Lambda}_k + \Lambda_{3k} + \Lambda_{4k} + \Lambda_{5k} & \Lambda_{1k}^T & \Lambda_{2k}^T \\ * & \Lambda_{22} & 0 \\ * & * & \Lambda_{22} \end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix} -P_k & 0 & C_k^T & 0 \\ * & -P_k & 0 & C_k^T \\ * & * & -R_k^{-1} & 0 \\ * & * & * & -R_k^{-1} \end{bmatrix} < 0 \tag{37}$$

where

$$\Lambda_{22} = -2\iota_k^2 I + \iota_k P_{k+1}.$$

Proof: For $\iota_k \in \mathcal{C}^+$ and $P_{k+1} \in \mathbb{M}^+$, it holds that

$$(\frac{1}{\iota_k} - P_{k+1})P_{k+1}^{-1}(\frac{1}{\iota_k} - P_{k+1}) \geq 0$$

Thus, it is true that

$$-P_{k+1}^{-1} < -2\iota_k I + \iota_k^2 P_{k+1}$$

Using Schur complement lemma, conditions (36) and (37) can ensure Theorem 1 and Theorem 2, simultaneously. And matrix inequalities (36) and (37) are RLMIs. Thus, the time-varying observer gain L_k and triggered weighting matrix-valued sequence Ω_k can be designed by solving (36) and (37) using Matlab Toolbox. ■

In the following, Algorithm 1 is given to solve RLMIs (36) and (37).

Algorithm 1 The Procedure for Computing L_k

1: For given scalars d_1, d_2, ι_k α, β, σ_k, $N \in \mathcal{C}^+$ and matrix-valued sequence $R_k \in \mathbb{M}^+$, and select a matrix X such that B^TXB is nonsingular. Select initial values for matrices $P_0, Q_{10}, Q_{20}, Q_{30}, Q_{1(-d_k)}, Q_{2(-d_1)}, Q_{3(-d_2)}$, Ω_0 satisfying (36), (37) and set $k = 0$;
2: Obtain P_{k+1}, $Q_{1(k+1)}$, $Q_{2(k+1)}$, $Q_{3(k+1)}$, $Q_{1(k-d_k)}$, $Q_{2(k-d_1)}$, $Q_{3(k-d_2)}$, Ω_{k+1} and L_k by solving (36) and (37);
3: For given ς, γ_m, γ_M, calculate λ_1, λ_2 and judge $E_\mu = \|y_e\|^2 - \epsilon$. If $E_\mu > 0$, $\gamma_{k+1} = \lambda_1\gamma_k$; else if $E_\mu < 0$, $\gamma_{k+1} = \lambda_2\gamma_k$; else $\gamma_{k+1} = \gamma_k$. Set $k = k+1$;
4: If $k \leq N$, then go to Step 2, else stop.

IV. NUMERICAL EXAMPLE

In this section, we consider an industrial continuous-stirred tank reactor system (CSTRS) in network environment [22], [34]. In the CSTRS, chemical species A reacts to form species B. Denote C_{ai} and C_a as the input and output concentration of a key reactant A, T and T_c being the reaction and the cooling medium temperature, respectively. According to [22], both parameter uncertainties and system nonlinearity should be taken into consideration when model the CSTRS. Since the system is in a network environment, we attempt to use the proposed AETM in this paper to save communication resources.

Take the state variable as $x_k^T = [C_a^T \ \ T^T]$, control input as $u_k^T = [T_c^T \ \ C_{ai}^T]$. Considering the influence of disturbance, the discrete model of CSTRS can be represented as system (1) with the following system matrices:

$$A_k = \begin{bmatrix} 0.9719 + 0.05sin(k) & -0.0013 + 0.02cos(k) \\ -0.0340 + 0.01e^{-k} 0.9328 & \end{bmatrix},$$

$$B = \begin{bmatrix} -0.0839 & 0.0232 \\ 0.0761 & 0.4144 \end{bmatrix}, D_k = \begin{bmatrix} 0.1e^{-k} & 0 \\ 0 & 0.1e^{-k} \end{bmatrix},$$

$$f_k = \begin{bmatrix} 0.01 + sin(k) \\ 0.5 + cos(x_{1k}^2) \end{bmatrix}, \omega_k = \begin{bmatrix} 0.5e^{-k} \\ 0.4sin(k) \end{bmatrix}, C_k = I.$$

For simulation purposes, under the influence of time-varying parameters, disturbance and nonlinearity, our goal is to design a time-varying observer-based sliding mode controller such that the estimation of C_a and T can be bounded by a given level. Give the parameters as $\gamma_m = 0.47$, $\gamma_M = 0.74$, $d_1 = 1$, $d_2 = 3$, $\alpha = 1$, $R_k = I$, $N = 20$. Let $sgn(S_k) = \frac{S_k}{\|S_k\|+10^{-3}}$ for avoiding the chattering of signals.

Solving the RLMIs (36) and (37), some of the time-varying observer gain sequence L_k can be tabulated in Table 1. The minimum value of β, denoted as β_{min}, is obtained as $\beta_{min} = 4.40$. Fig.2 depicts the trajectories of control signal u_k. Fig.3 simulates the sliding mode surface function S_k. The simulation of state estimation errors is shown in Fig.4. Fig.5 depicts the trajectories of $\hat{y}_k^T R_k \hat{y}_k + (y_k - \hat{y}_k)^T R_k (y_k - \hat{y}_k)$ and β_{min}, from which we can see that $\hat{y}_k^T R_k \hat{y}_k + (y_k - \hat{y}_k)^T R_k (y_k - \hat{y}_k) < \beta$ can be guaranteed. That is, IO-FTS is ensured by the proposed control method.

TABLE 1. Some of the observer gain sequence L_k.

k	L_k
1	$\begin{bmatrix} 0.2985 & 0.2983 \\ 0.3622 & 0.4167 \end{bmatrix}$
2	$\begin{bmatrix} -0.0014 & -0.0360 \\ 0.2052 & 0.2295 \end{bmatrix}$
$\vdots$	$\vdots$
$N-1$	$\begin{bmatrix} 0.2449 & 0.2567 \\ 0.2973 & 0.3570 \end{bmatrix}$
N	$\begin{bmatrix} 0.2438 & 0.2567 \\ 0.2958 & 0.3570 \end{bmatrix}$

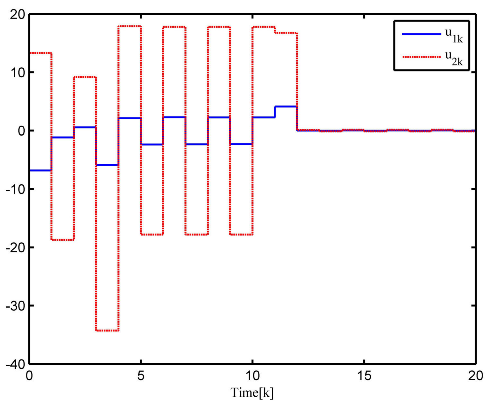

FIGURE 2. Trajectories of control signal u_k.

The triggered instants of the proposed AETM are simulated in Fig.6, from which we see that 50% communication resources are used and then the network resources are economized effectively. On the other hand, we suppose $\gamma_k = \gamma_M$ such that the AETM is reduced to TETM. The release instants are simulated in Fig.7, from which we can see that 75% communication resources are used under the TETM. From Fig.6 and Fig.7, it can been seen that when the norm of state estimation error tends to zero, the proposed AETM can save more communication resources. Therefore, in terms of saving network resources, the proposed AETM is more effective than TETM.

In the end, to investigate the influence of event-triggered parameters γ_k, values of γ_k are given as some constants. We aim to obtain the value of β_{min} when γ_k takes different values. The number of trigger instants N_t and values of β_{min}

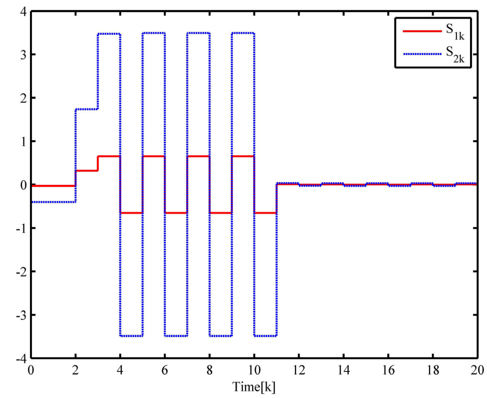

FIGURE 3. Sliding mode surface S_k.

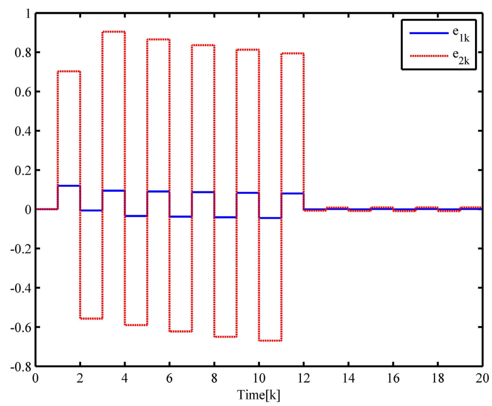

FIGURE 4. Trajectories of state estimation error e_k.

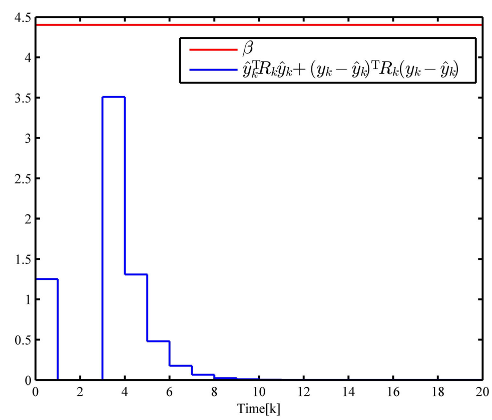

FIGURE 5. The trajectories of $\hat{y}_k^T R_k \hat{y}_k + (y_k - \hat{y}_k)^T R_k (y_k - \hat{y}_k)$ and β_{min}.

are given in Table 2. From Table 2, it can be seen that larger triggered parameter γ_k can save more network resources but derive larger value of β_{min}. That is to say, saving network resources may sacrifice system performance to some extent. In conclusion, the AETM method proposed in this paper can economize the network resources meanwhile sacrifice system performance as little as possible.

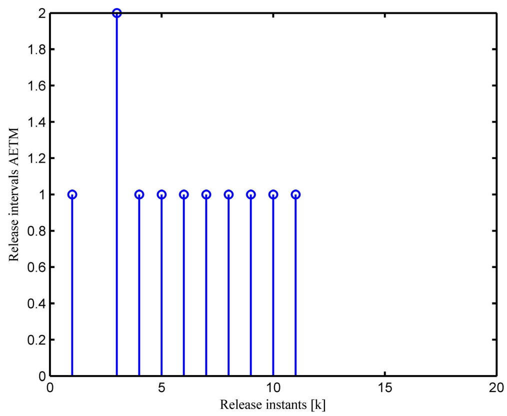

FIGURE 6. Release intervals and instants of the AETM.

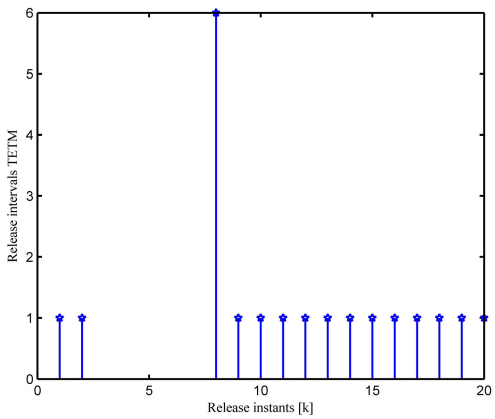

FIGURE 7. Release intervals and instants of the TETM.

TABLE 2. Values of N_t and β_{min} by different γ_k.

γ_k	0.47	0.55	0.63	0.74
β_{min}	4.17	4.30	4.37	4.39

V. CONCLUSION

This paper has studied the issue of IO-FTS for a kind of nonlinear DTVSs by an observer-based sliding mode control method. An AETM with an adaptive law has been proposed to save network resources. The reachability of the quasi-sliding mode surface has been guaranteed in finite steps and the upper bound of the quasi-sliding mode surface is obtained. Moreover, IO-FTS during both reaching and sliding mode phase has been guaranteed by a series of RLMIs. And a time-varying observer has been designed by solving the RLMIs. A CSTRS has been given to verify the effectiveness and superiority of the proposed method.

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