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RESEARCH ARTICLE

FMEA Assessment Under Heterogeneous Hesitant Fuzzy Preference Relations: Based on Extended Multiplicative Consistency and Group Decision Making

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ABSTRACT Failure mode and effects analysis (FMEA) is a reliability analysis method that analysis all possible failure modes for each product in a system and all possible effects of failure modes on the system, to classify each failure mode and to propose solutions and preventative measures. It is undeniable that the traditional FMEA method has been widely criticized for its simple scoring and single algorithm. To improve the usability of FMEA, hesitation fuzzy preference relation sets (HFPRs) based on hesitant fuzzy sets have been introduced into FMEA research because of their good fuzziness and uncertainty properties. Most existing consistency-based algorithms for HFPR processing, however, do not consider the possible coherence deviation of the reluctant fuzzy set itself, which includes multiplicative consistency (MC) theory, which will result in the reduced accuracy of results from such algorithms, in addition to not supporting group decision-making well in heterogeneous environments. At the same time, when building a group consensus, constantly adjusting HFPRs through group decision feedback can easily lead to conservative or radical scoring by experts. Therefore, an excellent hybrid FMEA assessment method is studied in this paper. In this approach, an extended multiplicative consistency equation is constructed by extending the applicability of MC in the treatment of HFPR, and on this basis, a mathematical model with the ability to deal with sets of heterogeneous fuzzy preference relations (H-HFPRs) is constructed. Lastly, based on the predictability principle of the occurrence level (O), a scoring correction algorithm is constructed based on group consensus theory to reduce the conservative or aggressive bias of the expert group in the results. The new method was used in the risk assessment of Change oilfield subsea pipeline engineering, and the results were compared with several existing methods to verify the effectiveness and advancement of the proposed method.

INDEX TERMS FMEA, multiplicative consistency, heterogeneous hesitation fuzzy preference relation set group decision, occurrence probability rank, risk assessment.

I. INTRODUCTION

FMEA is a technique for managing reliability and safety in a group-oriented, structured and proactive manner, in which determining the risk ranking of failure modes is a multi-faceted challenge that requires multi-criteria decision making (MCDM) analysis [1], [2]. Therefore, FMEA can be regarded as a typical multi-criteria group decision problem [3], in which the evaluation of experts plays a very important role. A typical FMEA assessment method is the risk priority number (RPN) method, where a group of experts is invited to score the three main parameters of the RPN:

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S (severity), O (occurrence rate) and D (detection rate). Simple multiplication is then used to calculate the final risk score. The traditional RPN method has some obvious defects, such as subjectivity and uncertainty, and contradictions among experts may even occur, leading to a score that does not fully capture the original information to be expressed; only a score of 0-10 results in a smaller data combination, and on the base of simple multiplication, different combinations of risk parameters may result in the same RPN value [4], [5], [6].

As the primary source of information in the FMEA risk assessment, the value of the RPN has a very large impact on the accuracy of the risk assessment results. Therefore, the improvement of the RPN evaluation is very important. Expert evaluation method is a basic method of evaluating the source of basic data. Due to their subjectivity and uncertainty, many researchers are committed to investigating optimization methods in order to improve the accuracy of expert evaluation. Representative approaches include the reduction of linguistic uncertainty, the consideration of expert fuzziness and randomness and the enhancement of the reliability of expert information by considering unknown information and expert group consensus [7], [8], [9]. Thus, the optimization of the expert evaluation method and the RPN calculation method are still mainstream research directions. Several studies have shown that the hesitant fuzzy method developed through fuzzy set theory can solve the problem of hesitant and uncertain scoring by experts. This method allows the evaluation object to belong to a set of several values [16]. However, the simple hesitant fuzzy method cannot solve the problem of expert subjectivity and the heterogeneity of the expert group. Especially when given linguistic information by the expert group, the traditional hesitant fuzzy method cannot solve the problem of randomness, which becomes increasingly obvious. In this case, the concept of heterogeneous hesitant fuzzy preference relation (H-HFPR) sets was introduced. Here, decision-makers in group decision-making often have different professional knowledge, experience and personality qualities, which will lead to different forms of evaluation information, such as complete hesitation fuzzy preference relation (HFPR), hesitation fuzzy linguistic preference relation (HFLPR), incomplete hesitation fuzzy preference relation (I-HFPR) and incomplete hesitation fuzzy linguistic preference relation (I-HFLPR), which are collectively referred to as heterogeneous hesitation fuzzy preference relation (H-HFPR) [10], [11], [12]. In the group decision problem, when there are two or more types of information in the H-HFPR, this is referred to as the H-HFPR problem. In addition to solving subjective expert uncertainty and individual heterogeneity problems, the group decision model based on H-HFPR can also solve problems of randomness and uncertainty. Existing methods based on expectation consistency or regression for dealing with fuzzy sets, however, are not fully applicable to the H-HFPR group decision problem. There are also some limitations of both the existing additive consistency algorithm and the multiplicative consistency algorithm, such as the inability to calculate the error when dealing with fuzzy sets, with respect to the H-HFPR group decision problem.

Furthermore, in engineering practice, it is possible to predict the occurrence of failure modes of a system or product through the implementation of modeling and simulation, prediction of the failure rate, and analysis of stress, similar products or historical data [13], [14]. It is possible to convert the product reliability prediction value into a failure density function and a failure rate function and obtain the failure mode occurrence rate. Therefore, the predicted value of the occurrence rate is also an important source of information in FMEA analysis. However, since there are always some improvements or changes in new products, some new potential failure modes may occur [15]. Thus, in new product FMEA, the opinions of experts are also very important information. By combining the predicted value of the occurrence rate with the opinions of experts, more efficient results can be obtained.

Motivated by the discussion above, the following goals will be achieved in this paper based on H-HFPR theory:

1) In order to make the computational results of the group H-HFPR decision more accurate, we optimize the multiplicative consistency(MC) principle.
2) A hesitant fuzzy preference matrix model and a group consensus (GC) model suitable for FMEA assessment are constructed. Expert group consensus is achieved on the basis of ensuring the consistency of expert preference information.
3) In order to rule out possible radicalness or conservativeness during fitting, the rates of S, O and D obtained by H-HFPR group decision-making are modified by the predictive value of O.

Based on the above research, we can obtain a hybrid FMEA method that can deal with ensemble heterogeneity. Not only can it better resolve uncertainty in risk assessment information, ambiguity and randomness through the high inclusivity of H-HFPR but also has the ability to adjust thresholds to make the risk ranking results more accurate and reliable.

The rest of this paper is arranged as follows. In Section II, some previous literature is reviewed. In Section III, an overview of H-HFPR theory, group consensus theory, multiplicative consistent theory, occurrence prediction theory and FMEA calculation theory, is provided. In Section IV, the proposed hybrid FMEA method is described in a step-by-step manner. In Section V, the feasibility and practicability of the hybrid FMEA method is verified by taking the ChengBei oilfield subsea pipeline as an example. In Section VI, the advancement and superiority of the hybrid method is verified by comparison with other existing methods. In Section VII, the conclusions of the entire study are drawn.

II. LITERATURE REVIEW

In this section, previous related studies will be reviewed. In the first subsection, the research on hesitant fuzzy set theory and HFPR theory is reviewed. In the second subsection, the research status of FMEA based on fuzzy sets, especially the research results of HFPR theory to evaluate FMEA, is summarized.

A. HESITANT FUZZY SET THEORY AND HFPR GROUP DECISION THEORY

In 2010, V. Torra [16] first proposed the concept of a hesitant fuzzy set, which is based on the concept of a hesitant fuzzy number and fuzzy set: the set of a single evaluation value is replaced by a set that contains all the evaluation values that the evaluator thinks is possible. It was pointed out that the hesitant fuzzy set method can better reduce the pressure of the evaluator during an evaluation and can also capture the evaluation information of the evaluator more completely. Then, Zhu et al. [17] proposed double hesitant fuzzy sets; Rodriguez et al. [18] proposed hesitant fuzzy linguistic sets. Chen et al. [19] proposed the concept of interval-valued hesitant fuzzy sets, which was extended in the paper. The theory of hesitant preference relation defines interval-valued hesitant fuzzy preference relations and applies it to group decision-making examples. Therefore, the interval fuzzy preference relationship and the intuition fuzzy preference relationship became the research hotspots [20]. Decision-making is never the responsibility of one person, especially for large-scale projects or key products. Therefore, how to perfectly integrate hesitant fuzzy set theory with group decision-making has become one of the current research directions.

The group decision theory of hesitant fuzzy preference relation sets has been considered and studied by researchers. Li et al. [21] provided advice for local government computer network system procurement by applying the multiplicative consistency theory of HFPR to group decision-making; Meng et al. [22] proposed a transformation method to deal with hesitant fuzzy preference relation group decision-making; Lin et al. [23] solved the group consensus problem by processing the multiplicative preference relationship and improving the consistency of the hesitant multiplicative preference relationship through regression methods and feedback mechanisms. In addition to multiplicative consistency, processing group decision-making based on the triangular hesitant fuzzy preference relationship is also a current research direction. Yan et al. [24] dealt with the group decision-making problem of hesitant fuzzy preference relations through triangular fuzzy theory; Qiu [25] established a set of hesitant fuzzy preference relations by introducing triangular hesitant fuzzy numbers and then dealt with the group decision-making of multi-attribute fuzzy preference information questions. At present, the mainstream research content has become the study of semantic hesitant fuzzy preference relation group decision-making. Ren et al. [26] proposed a group decision consensus model with HFLPR based on mapping and measuring the semantic modification degree of decision-makers; Liu et al. [27] proposed a group decision consensus model with HFLPR. They determined the complete HFLPR and the additive consistent relationship by setting the algorithm and model. Finally, a complete HFLPR group decision model was constructed. In this paper, the theory of MC is expanded to make it more suitable for the H-HFPR group decision problem in the FMEA environment, and a new model suitable for solving this kind of problem is established.

B. FMEA BASED ON FUZZY SETS

FMEA technology first appeared in the 1950s. It was not until 1964 that the concept of FMEA was gradually improved through the theory of ''Failure Mode Effects Analysis'' proposed by J. S. Cuntinbo. Due to its good functionality and ease of use, FMEA technology has been gradually implemented in military systems and civilian industrial fields such as machinery, automobiles and medical equipment [28]. FMEA risk assessment is mainly related to the three parameters of severity S, occurrence rate O and detection rate D. The most classic RPN calculation method is the simple multiplication of the S, O and D parameters. Because of the limitations of this method, researchers have proposed various methods to optimize the equation to improve the reliability of the RPN. Liu et al. [29] proposed a hybrid method based on fuzzy FMEA and VIKOR methods to evaluate different failure modes and determine the most important failure modes. Mandal et al. [30] combined the similarity measure of fuzzy numbers with possibility theory. The failure modes based on similar risk values can be combined with the theory of similar values to carry out FMEA risk analysis. Li et al. [31] integrated rough set and cloud model theories to process information and rank failure modes by extending TOPSIS. Xiao et al. [32] proposed a risk ranking method through the interval hesitant fuzzy interactive multi-criteria decision-making (TPDIM) model, which combined subjective expert weights and objective weights of risk factors to obtain comprehensive weights to optimize FMEA analysis.

In an actual FMEA assessment, the expert group is likely to give information about the hesitant fuzzy preference relationship or the hesitant fuzzy semantic preference relationship, which constitutes the HFPR group decision problem. In recent years, researchers have gradually paid attention to the optimization of the FMEA assessment method based on HFPR information. Yu et al. [8] established a linguistic term set through semantic triangular fuzzy numbers and the idea of evaluating cloud models, established first- and second-level factors for the S, O and D failure mode parameters, and then decomposed the S, O and D parameters into multiple factors. The RPN is calculated for each factor for risk assessment. Boral et al. [40] proposed an integrated interval type-2 fuzzy set (IT2FS)-based MCDM framework to deal with safety, economic sequence, occurrence and detection and calculated the weight of risk factors according to IT2F-AHP to analyze the FMEA of CNC machine tools. Hassan et al. [33] combined the significant advantages of hybrid FMEA with the fuzzy rule base (FRB) and gray relationship theory (GRT) to evaluate the hazards and risks of off-road oil and gas pipeline failure modes. Wang et al. [34] combined several multi-criteria decision-making (MCDM) techniques with probabilistic hesitant fuzzy linguistic term sets (PHFLTSs) to implement the risk assessment of failure modes by a panel of specialists to overcome some defects existing in conventional FMEA. At the same time, in order to make better group decisions, reduce conflicts within the group, and reach group consensus, Zhang et al. [35] proposed an optimization-based consensus model with the minimum adjustment distance to obtain the ordinal risk levels of failure modes by inputting linguistic fuzzy evaluation sets of risk assessment parameters for each failure mode from a group of experts and used in regulating the risk analysis in proton beam radiotherapy. Zhang et al. [36] construct the treatment of incomplete linguistic information by minimizing the deviation of opinions among members, and propose two different consensus optimization models based on consensus maxima and adjusted minima in order to speed up consensus reaching and improve the consensus reaching accuracy. Tang et al. [37] Optimization of the FMEA expert group decision model based on group structure detection from mesoscale perspective and fairness-oriented consensus approach, and verification of the accuracy of the method by risk evaluation of PV system. Shi et al. [38] build a new FMEA assessment by combining hesitant linguistic preference relations and an extended dynamic consensus model. Arantes et al. [39] planned patient management in the operating room of a Brazilian hospital by building a consensus reaching model with a mixture of Eliminate and select conversion trees and double hierarchy hesitant fuzzy linguistic term sets, but the results were poor.

In summary, in many related studies, there are many methods to deal with hesitant fuzzy preference relation sets based on consistency, but there are few studies on how to effectively deal with fuzzy sets in advance. Most of the existing fuzzy set preprocessing methods are based on regression or expected value. First, the resulting value, when processed using these methods, cannot effectively contain the original information of the fuzzy set, and second, the possible consistency deviation in the fuzzy set information cannot be calculated well. Therefore, in this paper, dealing with fuzzy sets in advance by using fuzzy set optimization is proposed. Moreover, in FMEA optimization research, how to effectively correct the FMEA results by using O predictors is one of the feasible research directions. Therefore, on the basis of optimizing the multiplicative consistency process of the hesitant fuzzy preference relation set, a method to modify FMEA results based on the occurrence prediction value is studied in this paper, and its availability is proven.

III. PRELIMINARIES

In this section, the basic theory that supports the optimization algorithm proposed in the paper is summarized. The related theory of H-HFPR, the basic theory of consistency-based group consensus and the basic theory of how to predict the failure rate are introduced.

A. HFPR THEORY

As the basic structure of H-HFPR, a hesitant fuzzy set is a means to better read expert information. The definitions are as follows.

Definition 1 [41]: Let X be the given domain of discourse; then, $H = \{< x, h_H(x) >\}$ is a hesitant fuzzy set on X, where $h_H(x)$ is a set composed of different numbers in the interval [0, 1]. The possible membership degree of x belonging to the set $H : h = h_H(x) = \{\gamma|\gamma \in h_H(x)\} = H\{\gamma^1, \gamma^2, \gamma^3, \ldots, \gamma^l\}$ is a hesitant fuzzy number, where $\gamma^\alpha \in [0, 1], \alpha = 1, 2, \ldots, l$ and l represents the number of elements in the hesitant fuzzy number h.

To better solve the group decision problem, the preference relationship is introduced. The main objective of this study is H-HFPR. H-HFPR consist of four preference relationships: HFPR, HFLPR, I-HFPR and I-HFLPR. The definitions of HFPR and HFLPR are as defined in Definition 2 and Definition 3:

Definition 2 [42], [43]: A set of schemes X is represented by a matrix P to represent its hesitant fuzzy relation set, i.e., $P = (P_{ij})_{n\times n} \subset X \times X, P_{ij} \in [0, 1]$. The elements in the matrix satisfy complementarity, i.e., $P_{ij} = 1 - P_{ji}(\forall i.j \in N)$, where P_{ij} represents the preference of method i relative to j. If $P_{ij} > 0.5$, method i is superior to method j. If $P_{ij} = 0.5$, there is no difference between method i and method j. $P_{ij} = \{r_{ij}^{\sigma(l)}|l = 1, 2 \ldots, \#P_{ij}\}$, where $\#P_{ij}$ represents the number of elements in P_{ij}, and the following formula is satisfied.

$$\begin{aligned} &r_{ij}^{\sigma(l)} + r_{ji}^{\sigma(\#p_{ij}-l+1)} = 1; \\ &r_{ij} = \{0.5\}; \quad \#P_{ij} = \#P_{ji};\ i, j = 1, 2, \ldots, n \end{aligned} \tag{1}$$

where $r_{ij}^{\sigma(l)}$ represents the first element of P_{ij}, $r_{ij}^{\sigma(l)} \in [0, 1]$.

Definition 3 [44], [45]: Given a term set S in advance, the term set-to-symbol representation is shown in Table 1. If a scheme set X is represented by the matrix R:$R = (R_{ij})_{n*n} \subset X * X$, the matrix element $r_{ij} \in S$ in R represents the preference strength that experts believe that option i is better than option j, and the matrix elements satisfy complementarity, i.e., $\oplus r_{ij} r_{ji} = S_0, \forall i.j \in N$. Let $R_{ij} = \{r_{ij}^{\sigma(l)}|l = 1, 2, \ldots, \#R_{ij}\}$, where $\#R_{ij}$ represents the linguistic item contained in R_{ij}. If $i < j$, then

$$\begin{aligned} &r_{ij}^{\sigma(l)} \oplus r_{ji}^{\sigma(\#p_{ij}-l+1)} = S_0; \quad r_{ii} = \{S_0\}; \\ &\qquad \#R_{ij} = \#R_{ji}; \quad r_{ij}^{\sigma(l)} < r_{ij}^{\sigma(l+1)}; \\ &\qquad r_{ji}^{\sigma(l)} < r_{ji}^{\sigma(l+1)} \end{aligned} \tag{2}$$

In this study, the subscript symmetric linguistic term set $S = \{S_\alpha|\alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ defined by Reference [46], whereτ is a positive integer and S satisfies $S_\alpha > S_\beta$ (if $\alpha > \beta$), is followed. There is a negative operator Neg, and $\text{Neg}(S_\alpha) = S_{-\alpha}$.

Xu [44] defined the basic operation rules for two linguistic words as follows:

$$\begin{cases} s_a \oplus s_\beta = s_{a+\beta};\ s_a \oplus s_\beta = s_\beta \oplus s_a \\ \lambda s_a = s_{\lambda a}; \\ (\lambda_1+\lambda_2)s_a = \lambda_1 s_a + \lambda_2 s_a;\ \lambda, \lambda_1, \lambda_2 \geq 0 \\ \lambda(s_a \oplus s_\beta) = \lambda s_a \oplus \lambda s_\beta \end{cases} \tag{3}$$

In this study, when dealing with HFPR and HFLPR, MC and additive consistency (AC) theory are introduced to solve the problem of consistency. The theoretical definitions are as follows:

TABLE 1. Linguistic set S and symbolic expression examples.

Linguistic term	Symbol
Extremely low	S_{-4}
Very low	S_{-3}
Low	S_{-2}
Relatively low	S_{-1}
Medium	S_0
Relatively high	S_1
High	S_2
Very high	S_3
Extremely high	S_4

Definition 4 [47]: Let $H = (h_{ij})_{n*n}$ be a HFPR. If $h_{ik}h_{kj}h_{ji} = h_{ki}h_{jk}h_{ij}$ $(i, j, k = 1, 2, \ldots, n)$, then H is said to be a multiplicative consistent fuzzy preference relation.

Reference [48] set $\delta(h_{ij}^{\sigma(l)})$ as the exact number of the processed hesitant fuzzy set and proposed $\delta(h_{ij}^{\sigma(l)}) = h_{ij}^1$ or h_{ij}^2 or $h_{ij}^3 \ldots$ or $h_{ij}^{\#h_{ij}}$.

Reference [49] proposed an expectation (multiplicative) consistency theory on the basis of Definition 4 to deal with hesitant fuzzy sets. If $e(h_{ik})e(h_{kj})e(h_{ji}) = e(h_{ki})e(h_{jk})e(h_{ij})$ $(i, j, k = 1, 2, \ldots, n)$, then H is expected to be (multiplicative) consistent.

$$\begin{aligned} e(h_{ij}) &= \frac{\omega_i}{\omega_i+\omega_j}, \quad \forall i,\ h = 1, 2, \ldots, n; \\ e(h_{ij}) &= \delta(h_{ij}^{\sigma(l)}) = \frac{\omega_i}{\omega_i+\omega_j} \\ &= h_{ij}^1 \text{ or } h_{ij}^2 \text{ or } h_{ij}^3 \ldots \text{ or } h_{ij}^{\#h_{ij}} \end{aligned} \tag{4}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weight vector of preference matrix H, and $\sum_{i=1}^n \omega_i = 1$.

In most studies, the hesitant fuzzy set usually needs to be preprocessed before processing the hesitant fuzzy preference matrix based on MC theory; in other words, the hesitant fuzzy set is replaced by some precise representative number. However, the existing studies usually use the method based on expectation or regression to preprocess fuzzy sets, which ignores the consistency deviation caused by hesitant fuzzy sets. Therefore, in the fourth section, a new method to obtain a more accurate deviation between the expert's original information is proposed, and it is made more consistent by extending the scope of the multiplicative consistency applied to the hesitant fuzzy set.

The Multiplicative Consistency theory is used in different group decision environments to calculate the degree of consistency of the experts in the HFPR matrix, and the Additive Consistency theory is used to calculate the expert coherence procedure in the HFLPR matrix, where each expert is assigned a weight according to the degree of coherence. In this study, we propose a formula for converting HFLs to HFs to better handle the heterogeneous group decision environment, which requires a partial Additive Consistency theory for the support, and the theory is defined as follows:

Definition 5 [45]: The HFLPR $H = (h_{ij})_{n*n}$ and its normalized HFLPR $\overline{H} = (\overline{h_{ij}})_{n*n}$ are confirmed to be AC if the following formula is satisfied.

$$\overline{h}_{ij}^{\sigma(l)} = \overline{h}_{ik}^{\sigma(l)} \oplus \overline{h}_{kj}^{\sigma(l)}; \quad (i, j, k = 1, 2 \ldots, n; i \neq j \neq k) \tag{5}$$

The AC-HFLPR $\check{H} = (\widetilde{h_{ij}})_{n*n}$ can be determined by the following formula:

$$\widetilde{h_{ij}} = \oplus \frac{1}{n} \sum_{k=1}^{n} (\overline{h}_{ik}\overline{h}_{kj}) \tag{6}$$

AC calculation is generally based on the consistency index of the HFLPR distance measurement, which is defined as follows.

Definition 6 [50]: Suppose S is a semantic set with length $L(S) = T : S = \{S_{-\frac{T-1}{2}}, \ldots, S_0, \ldots, S_{\frac{T-1}{2}}\}$. Let $S_a, S_\beta \in S$; *then*, the distance between S_a and S_β is defined as $d(S_a, S_\beta) = \frac{|\alpha-\beta|}{T}$. If h^α and h^β are fuzzy linguistic sets and satisfy $h^\alpha, h^\beta \in S$, $L(h^\alpha) = L(h^\beta)$ =l, then the distance between h^α and h^β, denoted as $d(h^\alpha, h^\beta)$, is calculated by the following formula.

$$d(h^\alpha, h^\beta) = \frac{\sum_{i=1}^{l} |h_i^\alpha - h_i^\beta|}{l * T} \tag{7}$$

If B_α and B_β are the hesitant fuzzy linguistic relationship matrices, then the distance between B_α and $B_\beta : D(B_\alpha, B_\beta)$ is calculated by the following formula.

$$D(B_\alpha, B_\beta) = \sqrt{\frac{2}{n*(n-1)} \sum_{i<j}^{n} (d((b_{ij}^N)_\alpha, (b_{ij}^N)_\beta))^2} \tag{8}$$

When solving the linguistic set distance problem, the semantic quantity of the two sets needs to be equal. Sometimes, the linguistic items contained in each element in the hesitant fuzzy linguistic preference relation matrix given by experts are not necessarily the same. Therefore, we first need to process the linguistic entry. Reference [45] proposed two normalization methods, α-normalization and β-normalization. For two sets, α-normalization reduces a fuzzy set of data with many elements to the minimum number of elements between the two sets, and β-normalization adds to a fuzzy set of data with few elements to obtain the maximum number of elements between the two sets. There are three ways to add elements to β-normalization, pessimistic, optimistic, and average.

When experts give the HFPR matrix, sometimes the I-HFPR matrix is given instead due to limited information or other external reasons. The following are definitions of the I-HFPR and I-HFLPR matrices.

Definition 7 [48]: Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set. I-HFPR on X are represented by matrix $H = (h_{ij})_{n\times n} \subset X \times X$. For all known HFEs, $H_{ij} = \{\gamma_{ij}^l | l = 1, 2, \ldots, \#h_{ij}\}$, where $\#h_{ij}$ is the number of values in h_{ij}, represents all possible values for the preference degree and should satisfy the following conditions.

$$\begin{cases} \gamma_{ij}^{\sigma(l)} + \gamma_{ji}^{\sigma(l)} = 1, \\ h_{ii} = \{0.5\} \\ \#h_{ij} = \#h_{ji} \end{cases} \tag{9}$$

I-HFPR allows single or multiple vacancies in H (undetermined). There is a minimum acceptable situation for I-HFPR; that is, at least one row and one column for determining elements must be satisfied. An I-HFPR is an I-HFLPR when HFEs are linguistic sets.

B. CONSISTENCY-BASED GROUP CONSENSUS THEORY

Expert group decision-making is a heterogeneous GDM decision, which requires more reasonable attention to experts who provide consistent information. Chiclana et al. [52] introduced the induced importance ordered weighted average (I-IOWA) operator to handle expert weights in complex environments where it is difficult to characterize expert attitudes. I-IOWA is defined as follows.

Definition 8 [53], [54]: Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ is an information vectors. Introduce an n-dimensional correlation weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ and a set of inducers $u = (u_1, u_2, \ldots, u_n)$. The IOWA operator of an n-dimensional importance weight mapping $R^n \rightarrow R$ follows the formula:

$$IOWA(< u_1, a_1 >, < u_2, a_2 >, \ldots, < u_n, a_n >) = \sum_{j=1}^{n} \omega_j b_j \tag{10}$$

where $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. The I-IOWA calculation is based on pairing u_i and a_i, then sorting a_i by u_i, where b_j is the jth a_i after sorting a_i by u_i. Reference [52] introduced the regular increment operator (RIM) concept of the Q algorithm into the group weight vector to make the individual weight vector aggregated by the I-IOWA operator more reasonable: $Q(0) = 0$; $Q(1) = 1$; $Q(X) > Q(Y)$ if $x > y$.

The calculation of the individual weights in the group follows the formula:

$$\omega_k = Q(\frac{S(k)}{S(n)}) - Q(\frac{S(k-1)}{S(n)}) \tag{11}$$

where $S(k) = \sum_{l=1}^{k} u_{\sigma(l)}$, u_k is the importance of the k-th expert. In HFPR group decision-making, $\sigma(l)$ is the l-th value of u after ranking the degree of consistency.

C. FAILURE RATE AND RELIABILITY CONVERSION THEORY

The failure rate O can be a time-based failure rate $\lambda(t)$ or a quantity-based failure rate $\lambda(t)$. $\lambda(t)$ is calculated by the following formula:

$$\lambda(t) = \frac{\mathrm{d}r(t)}{N_s(t) * \mathrm{d}t} \tag{12}$$

where $\mathrm{d}r(t)$ is the number of faulty products within dt after time t and $N_s(t)$ is the number of remaining products that have not failed at time t.

The formula can be transformed as follows:

$$\lambda(t) = \frac{\mathrm{d}r(t)}{N_s(t) * \mathrm{d}t} = \frac{\mathrm{d}r(t)}{N_s(t) * \mathrm{d}t} * \frac{N_0(t)}{N_0(t)} = \frac{f(t)}{R(t)} \tag{13}$$

$$\lambda(t)dt = -\frac{dR(t)}{R(t)} \rightarrow R(t) = e^{-\int_0^t \lambda(t)\mathrm{d}t} \tag{14}$$

where $R(t)$ is the product reliability, $f(t)$ is the failure density function, and $N_0(t)$ is the number of products initially worked. According to the above formula, the conversion relationship between the failure rate O and the reliability is obtained. Between them, if O is the time-based failure rate, Eq. (14) can also be obtained by generalizing the number of products to the time series.

IV. PROPOSED HYBRID FMEA METHOD

This section will describe in detail the implementation steps for FMEA opyimization by building a hybrid algorithm based on the group decision method MC processing H-HFPR for O predictable failure modes will be described in detail. The method proposed in this paper is divided into five key steps. First, the FMEA expert group is required to determine the failure mode and risk factors for the project product. Second, the expert group is required to establish HFPR for the S, O, D parameter preference relationship under each failure mode and the possible scores of the S, O, D parameters to establish HFPR. In the third step, the level of consistency and weighting of each expert is determined based on the MC and I-IOWA operator. Next, by establishing a group consensus, the weights and the possible scores of the three parameters S, O, and D under the failure mode are obtained. Final parameters scores can be determined according to the weights. Lastly, based on the predicted value of O, the final score of the three parameters is adjusted to calculate the risk priority number of the failure mode. After the risk priority number of each failure mode is determined by the same method, each failure mode can be prioritized.

Fig. 1 shows a flow chart of the steps for the hybrid FMEA.

Suppose a product has n failure modes, which are denoted as $\mathrm{FM} = \{\mathrm{fm}_1, \mathrm{fm}_2, \mathrm{fm}_3, \ldots, \mathrm{fm}_n\}$. An introduction to the computational stages of product risk assessment via the hybrid algorithm proposed in this paper is provided below.

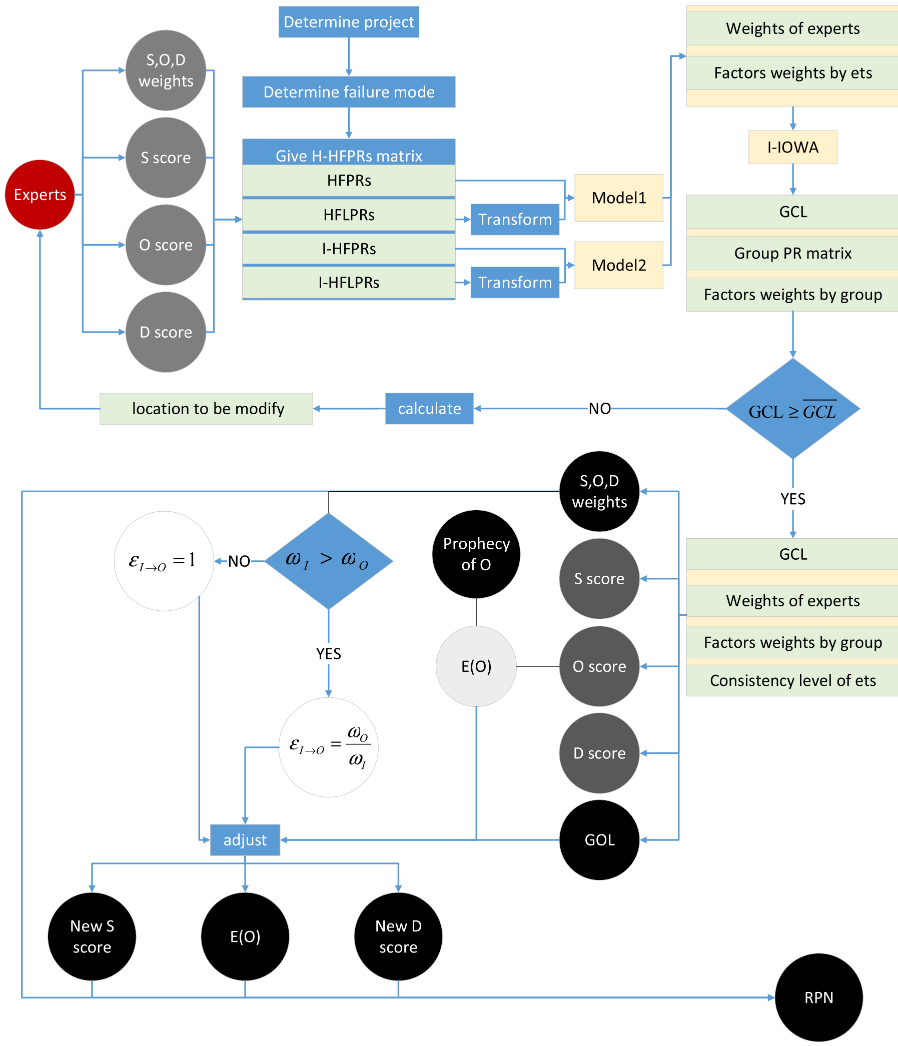

FIGURE 1. Workflow of Hybrid FMEA.

A. STEP I: DETERMINE THE H-HFPR MATRIX FOR FMEA

In this paper, failure mode risk assessment occurs primarily through the determination of the weights and scores for the three parameters S, O and D as well as the mixed evaluation based on the predictable nature of the occurrence rate O. Therefore, a preference relation matrix that evaluates the S, O and D weights is first determined. The preference relation matrix is shown in Table 1, where $h_{ij}(i, j = 1, 2, 3)$ is the hesitant fuzzy preference set or the exact value.

It is often difficult to directly give exact S, O and D scores. As a result, the project team needs to establish a possible score set from 1-9 points through experience and historical data and then use this set as the evaluation element of the HFPR matrix. Each score in the score set is determined a weight based on the HFPR group decision, and the final score can be determined by a simple weighted summation. The score can be determined according to the S, O and D score table given in the new version of the FMEA handbook [55].

B. STEP II: DETERMINE THE CONSISTENCY LEVEL CL FOR EACH EXPERT AND THE WEIGHT OF THE EVALUATION METRICS

The determination of the expert level can be assessed by the consistency level (CL) [56]. In actual decision-making, it is difficult for experts in heterogeneous group decision-making environments to provide completely consistent HFPR. Experts who were able to provide more consistent responses were considered to have a higher professional level. As a result, the group decision-making weights will be correspondingly higher, which represents a positive correlation.

From the previous theories, the decision-making of HFPR can be handled by MC theory; the decision-making of HFLPR can be handled by AC theory. However, in a heterogeneous group decision-making environment, if the processing method is different, the evaluation results of expert who give HFPR and those who give HFLPR will be different, which will cause the calculated expert weights to deviate from the

TABLE 2. HFPR matrix for determining S, O, and D weights.

	(S)	(O)	(D)
(S)	0.5	h_{12}	h_{13}
(O)	h_{21}	0.5	h_{23}
(D)	h_{31}	h_{32}	0.5

actual weights. Therefore, in order to establish weights in a heterogeneous environments, MC is theoretically extended to solve the HFLPR problem. It is also extended to the heterogeneous group decision-making environment with I-HFPR.

The processing flow chart of H-HFPR is shown in the Fig. 2.

It can be seen from Fig. 2 that the processing is slightly different for different evaluation forms in heterogeneous environments, and processing methods and model construction for different evaluation forms are described in detail below.

1) HESITANT FUZZY PREFERENCE RELATIONSHIP

According to Definition 4 and Eq. (3), in the existing research [57], the MC formula $\delta(h_{ij}^{\sigma(l)}) = \frac{w_i}{w_i+w_j}$ is transformed to obtain the following formula:

$$(\delta(h_{ij}^{\sigma(l)}) - 1)w_i + \delta(h_{ij}^{\sigma(l)})w_j = 0 \tag{15}$$

Set $z_{ij}^{\sigma(l)} \in (0, 1)$, let $\delta(h_{ij}^{\sigma(l)}) = z_{ij}^{\sigma(l)} h_{ij}^{\sigma(l)}$. Then, a model is directly built for the preprocessed preference relationship matrix, and the total deviation value is calculated by setting the positive and negative operators d_{ij}^{+} and d_{ij}^{-}. Throughthe aforementioned study, it was found that the expected (multiplicative) consistency theory for HFPR did not calculate the bias of dealing with hesitant fuzzy sets into the consistency level CL, so we extended it further on the basis of Song's research.

To obtain the bias of processing fuzzy sets, we treat the fuzzy set as an interval. The theoretical construction of $\delta(h_{ij}^{\sigma(l)})$ in Extended Definition 4 is as follows:

$$\begin{aligned} \delta(h_{ij}^{\sigma(l)}) &= \frac{\omega_i}{\omega_i+\omega_j} = h_{ij}^{\sigma(l)}, \quad h_{ij}^{\sigma(l)} \in [h_{ij}^{1}, h_{ij}^{\#h_{ij}}]; \\ h_{ij}^{\sigma(l)} &= z_{ij}^{1}h_{ij}^{1} + z_{ij}^{2}h_{ij}^{2} + \ldots + z_{ij}^{\#h_{ij}}h_{ij}^{\#h_{ij}} \\ &\quad (z_{ij}^{1}, z_{ij}^{2}, \ldots, z_{ij}^{\#h_{ij}} \in [0,1], \sum\nolimits_{m=1}^{\#h_{ij}} z_{ij}^{m} = 1) \end{aligned} \tag{16}$$

After processing $h_{ij}^{\delta(l)}$, H^{δ} remains consistent with the definition of HFPR, where $h_{ij} = \{r_{ij}^{l} | l = 1, 2, \ldots, \#h_{ij}\}$, as shown in the following proof.

Proof 1: Let $\#h_{ij} = \#h_{ji}$, $r_{ij}^{l} = 1 - r_{ji}^{l}$;

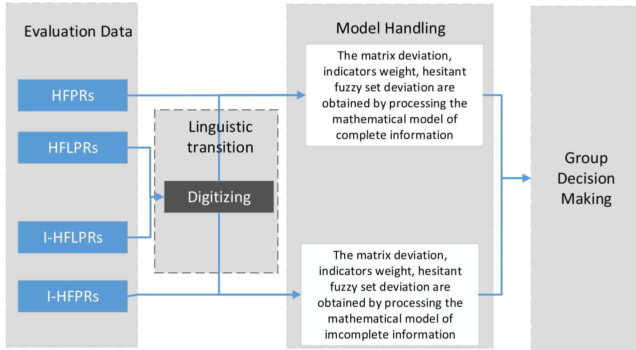

FIGURE 2. Processing flow chart of H-HFPR.

then:

$$\begin{aligned} h_{ij}^{\delta(l)} &= z_{ij}^{1}h_{ij}^{1} + z_{ij}^{2}h_{ij}^{2} + \ldots + z_{ij}^{\#h_{ij}}h_{ij}^{\#h_{ij}} \\ &= z_{ij}^{1}(1-h_{ji}^{1}) + z_{ij}^{2}(1-h_{ji}^{2}) + \ldots + z_{ij}^{\#h_{ij}}(1-h_{ji}^{\#h_{ij}}) \\ &= 1 - z_{ij}^{1}h_{ji}^{1} - z_{ij}^{2}h_{ji}^{2} - \ldots - z_{ij}^{\#h_{ij}}h_{ji}^{\#h_{ij}} \\ &= 1 - h_{ji}^{\delta(l)} \end{aligned}$$

where H^{δ} is the fuzzy preference relation.

Next, Eq. (15) is further treated using Eq. (16). To determine the optimal weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ closest to the expert information through the preference relationship, the maximum deviation value of Eq. (15) needs to be minimized, so the following mathematical model can be established:

$$\min f = \sum\nolimits_{m=1}^{\frac{n*(n-1)}{2}} D_m$$

$$s.t. \begin{cases} (\sum\limits_{l=1}^{\#h_{ij}} (z_{ij}^{l}h_{ij}^{l}) - 1)\omega_i + (\sum\limits_{l=1}^{\#h_{ij}} (z_{ij}^{l}h_{ij}^{l}))\omega_j - D_m \leq 0, \\ (1 - \sum\limits_{l=1}^{\#h_{ij}} (z_{ij}^{l}h_{ij}^{l}))\omega_i - (\sum\limits_{l=1}^{\#h_{ij}} (z_{ij}^{l}h_{ij}^{l}))\omega_j - D_m \leq 0 \\ \sum\limits_{i=1}^{n} \omega_i = 1, \\ \sum\limits_{l=1}^{\#h_{ij}} z_{ij}^{l} = 1, \\ z_{ij}^{l} \in [0, 1], \\ \omega_i > 0, \\ i, j \subseteq \{1, 2, \ldots, n\}, \\ i < j. \end{cases} \tag{17}$$

where D_m represents the deviation value of the m-th element of the triangular part on the hesitant fuzzy preference relation

$$cl(H) = 1 - \frac{2(f + \sum_{i=1}^{n}\sum_{j>i}^{n} \min\{|\sum_{l=1}^{\#h_{ij}} z_{ij}^{l} * h_{ij}^{l} - h_{ij}^{m}||m = 1, 2, \ldots, \#h_{ij}\})}{n*(n-1)} \tag{18}$$

set matrix. z_{ij}^l is the weight value of element h_{ij}^l in the hesitant fuzzy set h_{ij}. Here, with the idea of assuming that the fuzzy set is an interval number, to optimise the fuzzy set, the variable z_{ij}^l is added to determine the optimal representative number of the hesitant fuzzy set h_{ij}. The consistency level cl(H) contains the matrix deviation f and the deviation between the initial fuzzy set and the final value after multiplicative consistency processing because the mathematical programming also addresses fuzzy sets. Therefore, the formula for cl(H) is as in (8), shown at the bottom of the previous page.

2) HESITANT FUZZY LINGUISTIC PREFERENCE RELATIONSHIP

Reference [51] explored the scheme of additive consistent processing of HFLPR. However, when there are HFPR in group decision-making, if different methods are used to process HFLPR and HFPR, obviously, it will lead to deviation in the results. Therefore, the HFLPR are addressed to make them applicable to the mathematical model built in this paper based on the extended MC theory to solve the H-HFPR group decision-making problem.

Eq. (3) gives the basic rules for operations between linguistic words. According to the rules, we found that HFLPR can show similar properties to HFPR by some processing of linguistic word sets. Let $H = (h_{ij})_{n*n}$ be a given HFLPR matrix and $S = \{S_{-\frac{T-1}{2}}, \ldots, S_0, \ldots, S_{\frac{T-1}{2}}\}$ be its linguistic term set; then, the processing formula is as follows:

$$\check{h}_{ij}^m = \frac{h_{ij}^m \oplus \frac{(T-1)}{2}}{T-1} \tag{19}$$

At this point, S_0 becomes $S_{0.5}$, $S_{-\frac{T-1}{2}} = S_0$, and $S_{\frac{T-1}{2}} = S_1$. According to the rules of Eq. (3), it is consistent with the HFPR definition and can be used to solve for CL and metric weight through multiplicative consistency in the additive computing environment. Eqs. (15)-(16) become the following formulas:

$$\begin{aligned}
&(\delta(h_{ij}^{\sigma(l)}) - 1)\omega_i \oplus \delta(h_{ij}^{\sigma(l)})\omega_j = 0,\\
&\quad h_{ij}^{\sigma(l)} \in [h_{ij}^1, h_{ij}^{\#h_{ij}}];\\
&\quad h_{ij}^{\sigma(l)} = z_{ij}^1 h_{ij}^1 \oplus z_{ij}^2 h_{ij}^2 \oplus \ldots \oplus z_{ij}^{\#h_{ij}} h_{ij}^{\#h_{ij}},\\
&\qquad (z_{ij}^1, z_{ij}^2, \ldots, z_{ij}^{\#h_{ij}} \in [0,1], \textstyle\sum_{m=1}^{\#h_{ij}} z_{ij}^m = 1)
\end{aligned} \tag{20}$$

Next, the HFLPR can be solved by the MC mathematical programming proposed in Eq. (17) and the expert CL and the metric weight vector ω can be obtained.

3) INCOMPLETE - HESITANT FUZZY PREFERENCE RELATIONSHIP AND INCOMPLETE - HESITANT FUZZY LINGUISTIC PREFERENCE RELATIONSHIP

For acceptable I-HFPR and I-HFLPR, receiving incomplete information may result in a deviation from the information that the expert wants to provide. Based on this, the "soft consistency" theory [60], [61] is proposed to represent approximate consistency. Correspondingly, to make the calculation results closer to the original information of experts, the maximum deviation in the preference matrix is transformed into the multiplicative consistency matrix and becomes the expert's deviation value, not the average value as shown in Eq. (18). After processing the I-HFLPRs through Eq. (19), following mathematical model (Eq. (22)) is proposed in this paper to solve the cl(H) of the H-HFPR group decision problem under incomplete conditions, (21) and (22), as shown at the bottom of the page, where b is the number of incomplete elements in the matrix. By solving the obtained

$$\begin{aligned}
&\min f = \xi\\
&s.t. \begin{cases}
(\sum_{l=1}^{\#h_{ij}} (z_{ij}^l h_{ij}^l) - 1)\omega_i + (\sum_{l=1}^{\#h_{ij}} (z_{ij}^l h_{ij}^l))\omega_j - \xi \le 0\\
(1 - \sum_{l=1}^{\#h_{ij}} (z_{ij}^l h_{ij}^l))\omega_i - (\sum_{l=1}^{\#h_{ij}} (z_{ij}^l h_{ij}^l))\omega_j - \xi \le 0\\
\sum_{i=1}^{n} \omega_i = 1,\\
\sum_{l=1}^{\#h_{ij}} z_{ij}^l = 1,\\
z_{ij}^l \in [0,1],\\
\omega_i > 0,\\
i, j \subseteq \{1, 2, \ldots, n\},\\
i < j.
\end{cases}
\end{aligned} \tag{21}$$

$$\text{cl}(H) = 1 - \xi - \frac{2(\sum_{i=1}^{n}\sum_{j=i+1}^{m} \min\{|\delta(h_{ij}) - h_{ij}^l||l = 1, 2, \ldots, \#h_{ij}\})}{n(n-1) - b} \tag{22}$$

weight vector ω, the ambiguous elements in the I-HFPR can be solved by using Eq. (16).

C. STEP 3: ESTABLISH A GROUP CONSENSUS OF H-HFPR AND DETERMINE THE FINAL WEIGHT

The idea for the I-IOWA algorithm is given in Definition 8. Reference [58] proposed the following application for calculate priority weight of the group.

Suppose the HFPR of m experts are obtained: $H^{e(k)} = (h_{ij})^{e(k)}_{n*n}(k = 1, 2, \ldots, m)$. Let $\omega^{e(k)} = (\omega_1^{e(k)}, \omega_2^{e(k)}, \ldots, \omega_n^{e(k)})^T (k = 1, 2, \ldots, m)$ be the individual priority weight vector according to proposed IOWA-based operator aggregation. The group priority weight vector $\omega^c = (\omega_1^c, \omega_2^c, \ldots, \omega_n^c)^T$ is obtained, where n is the number of metrics, m is the number of experts, $e(k)$ represents the kth expert, and c represents the group. The formula is as follows:

$$\begin{aligned}\omega_i^c &= IOWA_{Q^c}(\omega_i^{e(1)}, \omega_i^{e(2)}, \ldots, \omega_i^{e(m)}) \\ &= \varphi_\omega(<cl^1, \omega_i^{e(1)}>, <cl^2, \omega_i^{e(2)}>, \ldots, <cl^m, \omega_i^{e(m)}>) \\ &= \sum_{\gamma=1}^{m} \lambda_\gamma * \omega_i^{e(m)}\end{aligned} \tag{23}$$

where λ is calculated by Eq. (11). $S(k) = \sum_{l=1}^{k} cl^{\sigma(l)}$, $cl^{\sigma(l)}$ is the lth largest value in $\{cl^1, cl^2, \ldots, cl^k\}$. The ability of the expert is positively correlated with the consistency level given by the expert. At this time, $Q(x)$ should represent the regular increasing monotonic RIM quantifier.

Reference [52] provided the Q(x) function under RIM as follows:

$$Q(x) = x^\lambda \text{ or } Q(x) = 1 - (1 - x)^\lambda \tag{24}$$

$Q(x)$ is used to express whether the consistency level can meet the conceptual degree of expert linguistic quantifiers. Reference [59] studied the results caused by the value of λ. Generally, λ is set as 0.9.

After determining the group weight $\omega_c = (\omega_c^1, \omega_c^2, \ldots, \omega_c^m)$ of each expert, calculate the priority weight vector ω^c and group decision matrix $H^c = (h_{ij})^c_{n*n}$ of each expert group by using the following formulas:

$$\omega_i^c = \sum_{k=1}^{m} \omega_i^{e(k)} \omega_c^k \tag{25}$$

$$h_{ij}^c = \sum_{k=1}^{m} \omega_i^{e(k)} h_{ij}^k \tag{26}$$

where h_{ij}^k is an exact value that is the representative number of hesitant fuzzy sets determined by Eq. (17). At this time, the individual priority weight $\omega^{(k)}$ and the group priority weight ω^c have been obtained, so the group consensus degree (GCD) of each expert can be determined by the following formula:

$$\text{GCD}(e(k)) = 1 - \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\omega_i^{e(k)} - \omega_i^c\right)^2} \tag{27}$$

Eq. (27) is used to determine the group consensus level for group decision-making for complete HFPR. Then, the average consistency degree of the expert group can be solved by the following formula:

$$\text{GCL} = \frac{\sum_{k=1}^{m} \text{GCD}(e(k))}{m} \tag{28}$$

If GCL=1, then all experts agree with the group.

In addition, the GCD can be calculated based on the de-fuzzy evaluation matrix in addition to the metric weights.

Assume that there are m experts' preference matrices: $H^k = (h_{ij})^k_{n*n}(k = 1, 2, \ldots, m)$. The group decision matrix $H^c = (h_{ij})^c_{n*n}$ can be solved by Eq. (26) after the weight vector $\omega = (\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(m)})$ is determined by Eq. (21). Because the initial information of I-HFPR is incomplete, the index weight based on soft consistency cannot define the original information of experts well. Therefore, the expert group consensus is determined by the distance between the completed scoring matrix H^k and the group decision matrix H^c; that is, the calculation of GCD is shown in following formula:

$$\begin{aligned}&GCD(e(k)) \\ &\quad = 1 - \frac{2}{n*(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} |h_{ij}^{e(k)} - h_{ij}^c|\end{aligned} \tag{29}$$

A comparison is made with the group consensus threshold $\overline{\text{GCD}}$ set by the project team. If $\text{GCD}(e(k)) < \overline{\text{GCD}}$, feedback adjustment is needed.

Reference [12] provided a method based on an interaction mechanism. According to the different representations of preference structures, we propose modifications to the expert preference matrix with the smallest GCD, which is only suitable for AC environments. To solve the group decision problem of solving H-HFPR through MC, the interaction mechanism method is partially modified in this paper. The modified method is as follows:

First, determine where each expert needs to adjust $p_{ij,k}^{(t)}$:

$$p_{ij,k}^{(t)} = max_{(i,j)}|h_{ij,k}^{(t)} - h_{ij,c}^{(t)}| \tag{30}$$

Next, according to the following relationship feedback to the expert to modify the elements of $p_{ij,k}^{(t)}$, and according to the conditions that $H = (h_{ij})_{n*n}$ was HFPR and $L = (l_{ij})_{n*n}$ was HFLPR,

HFPR:

$$\begin{aligned}&\overline{h_{ij,k}^{(t)}} \in [\min\{h_{ij,k}^{(t)}, h_{ij,c}^{(t)}\}, \max\{h_{ij,k}^{(t)}, h_{ij,c}^{(t)}\}]; \\ &\overline{h_{ji,k}^{(t)}} = 1 - \overline{h_{ij,k}^{(t)}}; \quad (i < j)\end{aligned} \tag{31}$$

HFLPR:

$$\begin{aligned}&l_{ij,k}^{(t)} \le \overline{l_{ij,k}^{(\#l_{ij,k}^{(t)})}} \le h_{ij,c}^{(t)}, \quad (l_{ij,k}^{(t)} < h_{ij,c}^{(t)}), (i < j); \\ &h_{ij,c}^{(t)} \ge \overline{l_{ij,k}^{(\#l_{ij,k}^{(t)})}} \ge l_{ij,k}^{(t)}, \quad (l_{ij,k}^{(t)} > h_{ij,c}^{(t)}), (i < j)\end{aligned} \tag{32}$$

These calculations need to be recalculated from the beginning of step 1 after the expert modifies the original preference matrix. Iterations continued until $GCD \ge \overline{GCD}$.

D. STEP 4: SOLVING FOR OCCURRENCE PREDICTABLE RPN VALUES

We obtain the parameter weight S_W, O_W, D_W and score S_v, O_v, D_v of the failure mode as well as the consistent level $\mathrm{CL}_S^{e(k)}$, $\mathrm{CL}_O^{e(k)}$, $\mathrm{CL}_D^{e(k)}$ and group consensus GCL_S, GCL_O, GCL_D of each expert when scoring.

To solve the conservative or radical problems that may arise among experts when building group consensus, an expected value $\overline{O}$ is established through the occurrence prediction value O_{pre} and the expert rating value O_v to obtain a more objective evaluation result. At the same time, based on this O_v, S_v and D_v are adjusted.

Based on the consensus loss information and Bayesian decision theory, the results of a group consensus are usually affected by the group influence of the decision-maker group itself, the scope brought about by the problem orientation and the relationship and exchange of opinions among decision-makers [62], [63], [64]. In our view, scores from the same group of experts on all three parameters under the same failure mode (same scope definition) are always positively correlated; that is, their scores for S and D will be too conservative or radical if the scores of the expert group for O are also too conservative or radical.

First, $\overline{O}$ is solved by the mean formula: $\overline{O} = \frac{O_{\text{pre}}+O_v}{2}$. At the same time, the adjustment value of O can be obtained: $O_v^d = \overline{O} - O_v$. Then, S_v and D_v are optimized and adjusted by the following method.

Next, we need to solve the group offset level (GOL). It is know that all three metrics are scored by the same group of experts, the original HFPR have more or less deviation from the multiplicative consistent HFPR due to the problem of expert ability or subjective factors, which will lead to a certain offset between the final result and the ability of the expert group itself. Therefore, GOL is related to the consistent level CL and the group consensus degree GCL, which can be solved by the following formula:

$$\mathrm{GOL}_i = \frac{\sum_{k=1}^{m}(1-\mathrm{cl}_i^{e(k)})}{m*\mathrm{GCL}_i} \tag{33}$$

where GOL_i is the group offset level of the HFPR group decision for metric i and m is the number of experts. From this, the adjustment values S_v^d and D_v^d of S_v and D_v, respectively, for group decision-making can be derived as follows:

$$\begin{aligned} S_v^d\mathrm{GOL}_S &= O_v^d\mathrm{GOL}_O = D_v^d\mathrm{GOL}_D \\ &\Rightarrow S_v^d = \frac{O_v^d\mathrm{GOL}_O}{\mathrm{GOL}_S};\quad D_v^d = \frac{O_v^d\mathrm{GOL}_O}{\mathrm{GOL}_D} \\ &\Rightarrow S_v^d = \frac{(\overline{O}-O_V)\mathrm{GCL}_S\sum_{k=1}^{m}(1-cl_O^{e(k)})}{\mathrm{GCL}_O*\sum_{k=1}^{m}(1-cl_S^{e(k)})} \\ D_v^d &= \frac{(\overline{O}-O_V)\mathrm{GCL}_D\sum_{k=1}^{m}(1-cl_O^{e(k)})}{\mathrm{GCL}_O\sum_{k=1}^{m}(1-cl_D^{e(k)})} \end{aligned} \tag{34}$$

In actual work, the weight of S is likely to exceed the weight of O too much. Similarly, for certain failure modes, the weight of D may also account for the highest proportion. This may cause our adjustment of S and D to result in a high variance in the resulting value of the final RPN. Therefore, a cautious parameter $\gamma_{I\to O}$ is set in this paper for tuning values here if $S_W > O_W$ or $D_W > O_W$:

$$\gamma_{I\to O} = \frac{O_W}{I_W}\quad (I\text{ is S or D}) \tag{35}$$

Eq. (34) becomes the following:

$$\begin{aligned} S_v^d &= \frac{\varepsilon_{S\to O}(\overline{O}-O_V)\mathrm{GCL}_S\sum_{k=1}^{m}(1-cl_O^{e(k)})}{\mathrm{GCL}_O\sum_{k=1}^{m}(1-cl_S^{e(k)})}; \\ D_v^d &= \frac{\varepsilon_{D\to O}(\overline{O}-O_V)\mathrm{GCL}_D\sum_{k=1}^{m}(1-cl_O^{e(k)})}{\mathrm{GCL}_O\sum_{k=1}^{m}(1-cl_D^{e(k)})}; \end{aligned} \tag{36}$$

Finally, after modifying S_v^d and D_v^d, the risk of each failure mode can be quantitatively evaluated and ranked by solving the risk priority number $\widetilde{RPN}$ through the following formula:

$$\widetilde{RPN} = S_\omega S_v + O_\omega O_v + D_\omega D_v \tag{37}$$

The improved risk priority number $\widetilde{RPN}$ is prioritized. At this time, the larger the $\widetilde{RPN}$, the higher the priority of the failure mode, the greater the harm, and the greater the number of improvement measures and attention that needs to be paid. When a failure mode with a large hazard cannot be eliminated, its hazard degree must be reduced as much as possible from the aspects of design, use and maintenance. Then, the product is reevaluated.

V. APPLICATION OF METHODOLOGY: CASE STUDY

In this section, a case of failure risk assessment for oil and gas pipeline systems in Chengbei Oilfield, Bohai Sea, is presented. The failure risk of the oil and gas pipeline system will be evaluated through the optimization algorithm proposed in this paper, which will serve as a reference for improving the safety factor of the oil and gas pipeline system and at the same time proving the feasibility and effectivency of the method.

The Bohai Oilfield is the largest offshore oilfield in China and the second largest crude oil production base in the country. The total oilfield resources are approximately 12 billion cubic meters. Its geologic reservoirs are characterized by broken structures, developed fractures and complex reservoirs. The Chengbei Oilfield is one of the Bohai Oilfields. The reservoirs in the oilfield are sedimentary facies of discriminative river delta facies, and the sedimentary structure has an obvious vertical positive rhythm [65]. The oilfield system consists of three platforms, namely, the wellhead platform in area A, the wellhead platform in area B, and the comprehensive processing platform [66]. Their geographic locations are shown in Fig. 3.

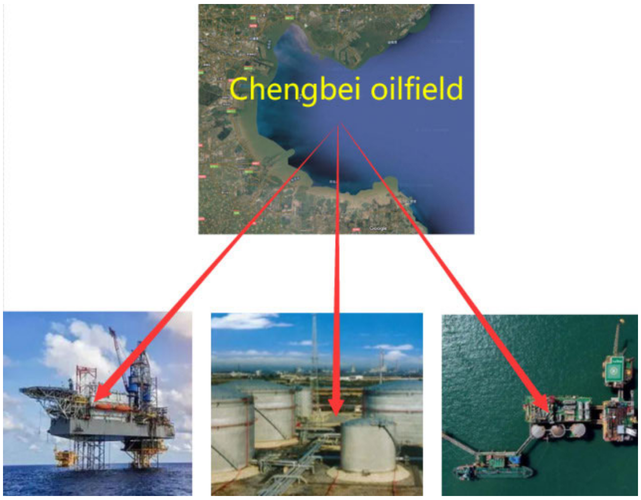

FIGURE 3. Illustration of the Chenbei oilfield in the Bohai Sea.

The oilfield has an average water depth of 15.8 m and the three platforms are connected by submarine pipelines, including an oil-gas-water mixing pipeline, a water injection pipeline and a submarine cable. The submarine pipeline is divided into a horizontal pipe section and riser section, with a total length of 1.64 km and a wall thickness of 9.5 mm. The pipe is covered with a 50 mm-thick rock wool insulation layer, and the outer wall has a 3.2 mm-thick polyethylene layer. The historical operating conditions of the pipeline are as follows: an inlet pressure of 0.15 MPa, an inlet temperature of 95 °C, hoop stress of 3 MPa, total axial stress of 204 MPa, equivalent stress of 203 MPa and allowable stress of 197 MPa [67].

In order to conduct a risk assessment of the oil-gas-water mixing pipeline at Chengbei Oilfield, 4 reliability engineers were specially invited to form an FMEA team. The team evaluated the pipeline environmental conditions, operating status, basic parameters, and historical operating information in order to analyze the potential risk of pipeline failure and determine the failure mode. Based on the database statistics of OGP [68], the analysis results of references [69], [70] and the combined experience of the FMEA team members, the seven typical types of oil and gas pipeline systems in the Chengbei Oilfield were assessed according to the causes of submarine pipeline failure, including corrosion (FM1), suspended span (FM2), external loads (FM3), natural disasters (FM4), material defects (FM5), weld defects (FM6) and auxiliary failures (FM7). The expert group set the group consensus threshold at 90% through discussion.

A. DETERMINING THE H-HFPR MATRIX

For seven typical failure modes, the expert FMEA group was invited to establish an HFPR matrix of S, O, and D metrics and to establish an HFPR matrix for each metric score set under the failure mode. The experts were allowed to use either HFPR, HFLPR, I-HFPR or I-HFLPR.

The term set of HFLPR is shown in Table 1, and the HFPR set of FM1 and FM2 from all experts is given in Table 3.[1]

At the same time, after the reliability of each unit was predicted, the project team estimated the occurrence rate O of seven failure modes according to the failure rate of other submarine pipeline products that are similar to the product and have historical data, as shown in Table 4.

The seven typical failure modes of oil field pipelines were evaluated and ranked by the optimization method proposed in the fourth section of this paper. Using expert-provided HFPR, we found that, for FM1, experts 1 and 4 provided HFPR matrices, and experts 2 and 3 used HFLPR matrices. For FM2, Expert 1 and Expert 4 gave the I-HFPR matrix, and Expert 2 and Expert 3 used the I-HFLPR matrix. Therefore, the risk assessment of FM1 and FM2 are taken as an example in this section to give the detailed calculation process.

B. DETERMINING EXPERT CL AND METRIC WEIGHTS

FM1 and FM2 are both H-HFPR group decision problems, so the HFLPR matrices for FM1 and FM2 need to be processed by using Eq. (19). Some of the processing results are shown in Table 5.[2]

Next, each HFPR is solved by using Eq. (17). For example, the following mathematical model (Eq. (38)) can be established for the HFPR of the three metrics of FM1 for Expert 1.

$$\min f = \sum_{m=1}^{3} D_m$$
$$s.t.\begin{cases} ((0.6z_{12}^1 + 0.7z_{12}^2) - 1) * \omega_1 + (0.6z_{12}^1 + 0.7z_{12}^2)\omega_2 \\ \quad - D_1 \le 0, \\ (1 - (0.6z_{12}^1 + 0.7z_{12}^2)) * \omega_1 - (0.6z_{12}^1 + 0.7z_{12}^2)\omega_2 \\ \quad - D_1 \le 0, \\ ((0.4z_{23}^1 + 0.5z_{23}^2) - 1) * \omega_2 + (0.4z_{23}^1 + 0.5z_{23}^2)\omega_3 \\ \quad - D_3 \le 0, \\ (1 - (0.4z_{23}^1 + 0.5z_{23}^2)) * \omega_2 - (0.4z_{23}^1 + 0.5z_{23}^2)\omega_3 \\ \quad - D_3 \le 0, \\ -0.2\omega_1 + 0.8\omega_3 - D_2 \le 0, \\ 0.2\omega_1 - 0.8\omega_3 - D_2 \le 0, \\ \sum_{i=1}^{3} \omega_i = 1, \\ z_{12}^1 + z_{12}^2 = 1, z_{23}^1 + z_{23}^2 = 1, \\ z_{12}^1, z_{12}^2, z_{23}^1, z_{23}^2 \in [0, 1], \\ \omega_1, \omega_2, \omega_3 > 0 \end{cases} \tag{38}$$

Using the LINGO 18.0 software to solve the mathematical model in Eq. (38), the metric weight $\omega_{\text{fm1}}^{\text{e(1)}} = \{S_W, O_W, D_W\} = \{0.600, 0.200, 0.200\}$ can be solved. The fuzzy

[1]The evaluation matrix for all failure modes is in Table 1 of the Appendix.
[2]The results of all treatments are shown in Appendix Table 2.

TABLE 3. FM1 and FM2 Evaluation results of the four experts.

Failure modes	Experts	Hesitant fuzzy preference relation matrix: S&O&D-HFPR	S_v-HFPR	O_v-HFPR	D_v-HFPR
FM1	Expert1	{0.5} {0.6,0.7} {0.8} {0.4,0.3} {0.5} {0.4,0.5} {0.2} {0.6,0.5} {0.5}	score 5 4 3 5 {0.5} {0.4,0.5} {0.4} 4 {0.6,0.5} {0.5} {0.3} 3 {0.6} {0.7} {0.5}	*score* 5 4 3 5 {0.5} {0.5,0.6} {0.9} 4 {0.5,0.4} {0.5} {0.9} 3 {0.1} {0.1} {0.5}	score 5 4 3 2 5 {0.5} {0.1,0.2} {0.1} {0.4,0.5} 4 {0.9,0.8} {0.5} {0.5,0.6} {0.7} 3 {0.9} {0.5,0.4} {0.5} {0.6,0.7} 2 {0.6,0.5} {0.3} {0.4,0.3} {0.5}
	Expert2	$\{S_0\}$ $\{S_0,S_1,S_2\}$ $\{S_3\}$ $\{S_0,S_{-1},S_{-2}\}$ $\{S_0\}$ $\{S_{-1},S_0,S_1\}$ $\{S_{-3}\}$ $\{S_1,S_0,S_{-1}\}$ $\{S_0\}$	score 5 4 3 5 $\{S_0\}$ $\{S_{-3}\}$ $\{S_{-2},S_{-1},S_0\}$ 4 $\{S_3\}$ $\{S_0\}$ $\{S_2,S_3\}$ 3 $\{S_2,S_1,S_0\}$ $\{S_{-2},S_{-3}\}$ $\{S_0\}$	*score* 5 4 3 5 $\{S_0\}$ $\{S_{-3},S_{-2}\}$ $\{S_2\}$ 4 $\{S_3,S_2\}$ $\{S_0\}$ $\{S_3,S_4\}$ 3 $\{S_{-2}\}$ $\{S_{-3},S_{-4}\}$ $\{S_0\}$	score 5 4 3 2 5 $\{S_0\}$ $\{S_1,S_2,S_3\}$ $\{S_2,S_3\}$ $\{S_{-1},S_0,S_1\}$ 4 $\{S_{-1},S_{-2},S_{-3}\}$ $\{S_0\}$ $\{S_{-1},S_0\}$ $\{S_{-4},S_{-3}\}$ 3 $\{S_{-2},S_{-3}\}$ $\{S_1,S_0\}$ $\{S_0\}$ $\{S_{-3},S_{-2}\}$ 2 $\{S_1,S_0,S_{-1}\}$ $\{S_4,S_3\}$ $\{S_3,S_2\}$ $\{S_0\}$
	Expert3	$\{S_0\}$ $\{S_0,S_1,S_2\}$ $\{S_2\}$ $\{S_0,S_{-1},S_{-2}\}$ $\{S_0\}$ $\{S_1,S_2\}$ $\{S_{-2}\}$ $\{S_{-1},S_{-2}\}$ $\{S_0\}$	score 5 4 3 5 $\{S_0\}$ $\{S_3\}$ $\{S_3,S_4\}$ 4 $\{S_{-3}\}$ $\{S_0\}$ $\{S_2,S_3\}$ 3 $\{S_{-3},S_{-4}\}$ $\{S_{-2},S_{-3}\}$ $\{S_0\}$	*score* 5 4 3 5 $\{S_0\}$ $\{S_{-2}\}$ $\{S_{-1},S_0,S_1\}$ 4 $\{S_2\}$ $\{S_0\}$ $\{S_2,S_3\}$ 3 $\{S_1,S_0,S_{-1}\}$ $\{S_{-2},S_{-3}\}$ $\{S_0\}$	score 5 4 3 2 5 $\{S_0\}$ $\{S_{-2},S_{-1}\}$ $\{S_1\}$ $\{S_1,S_2,S_3\}$ 4 $\{S_2,S_1\}$ $\{S_0\}$ $\{S_2\}$ $\{S_3,S_4\}$ 3 $\{S_{-1}\}$ $\{S_{-2}\}$ $\{S_0\}$ $\{S_0,S_1\}$ 2 $\{S_{-1},S_{-2},S_{-3}\}$ $\{S_{-3},S_{-4}\}$ $\{S_0,S_{-1}\}$ $\{S_0\}$
	Expert4	{0.5} {0.8} {0.9} {0.2} {0.5} {0.5,0.6} {0.1} {0.5,0.4} {0.5}	score 5 4 3 5 {0.5} {0.4,0.6} {0.5,0.6} 4 {0.4,0.6} {0.5} {0.7} 3 {0.5,0.4} {0.3} {0.5}	*score* 5 4 3 5 {0.5} {0.6,0.7} {0.8} 4 {0.4,0.3} {0.5} {0.5,0.6} 3 {0.2} {0.5,0.4} {0.5}	score 5 4 3 2 5 {0.5} {0.4,0.5} {0.2,0.4} {0.1} 4 {0.6,0.5} {0.5} {0.4} {0.3} 3 {0.8,0.6} {0.6} {0.5} {0.5,0.6} 2 {0.9} {0.7} {0.5,0.4} {0.5}
FM2	Expert1	{0.5} {0.5,0.6} {0.8} {0.5,0.4} {0.5} {0.6,0.7} {0.2} {0.4,0.3} {0.5}	score 6 5 4 3 6 {0.5} – {0.4} {0.2,0.3} 5 – {0.5} – {0.4,0.5} 4 {0.6} – {0.5} {0.4,0.5} 3 {0.8,0.7} {0.6,0.5} {0.6,0.5} {0.5}	score 6 5 4 3 6 {0.5} – {0.3} – 5 – {0.5} {0.3,0.4} {0.4} 4 {0.7} {0.7,0.6} {0.5} {0.6} 3 – {0.6} {0.4} {0.5}	score 6 5 4 3 6 {0.5} {0.6,0.8} – {0.4} 5 {0.4,0.2} {0.5} – {0.3} 4 – – {0.5} {0.3,0.4} 3 {0.7} {0.7} {0.7,0.6} {0.5}
	Expert2	$\{S_0\}$ $\{S_1,S_2\}$ $\{S_2\}$ $\{S_{-1},S_{-2}\}$ $\{S_0\}$ $\{S_0,S_1\}$ $\{S_{-2}\}$ $\{S_0,S_{-1}\}$ $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ – $\{S_{-1},S_0\}$ $\{S_{-2}\}$ 5 – $\{S_0\}$ – $\{S_{-4},S_{-3}\}$ 4 $\{S_1,S_0\}$ – $\{S_0\}$ $\{S_{-2}\}$ 3 $\{S_2\}$ $\{S_4,S_3\}$ $\{S_2\}$ $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ $\{S_1,S_2\}$ $\{S_0,S_1,S_2\}$ $\{S_{-1},S_0\}$ 5 $\{S_{-1},S_{-2}\}$ $\{S_0\}$ – $\{S_{-2}\}$ 4 $\{S_0,S_{-1},S_{-2}\}$ – $\{S_0\}$ – 3 $\{S_1,S_0\}$ $\{S_2\}$ – $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ $\{S_1,S_2,S_3\}$ $\{S_1,S_2\}$ $\{S_{-1},S_0\}$ 5 $\{S_{-1},S_{-2},S_{-3}\}$ $\{S_0\}$ $\{S_{-1}\}$ $\{S_{-3}\}$ 4 $\{S_{-1},S_{-2}\}$ $\{S_1\}$ $\{S_0\}$ $\{S_{-2}\}$ 3 $\{S_1,S_0\}$ $\{S_3\}$ $\{S_2\}$ $\{S_0\}$
	Expert3	$\{S_0\}$ $\{S_{-1},S_0\}$ $\{S_1\}$ $\{S_1,S_0\}$ $\{S_0\}$ $\{S_2,S_3\}$ $\{S_{-1}\}$ $\{S_{-2},S_{-3}\}$ $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ – $\{S_0,S_1,S_2\}$ $\{S_{-3}\}$ 5 – $\{S_0\}$ $\{S_{-3},S_{-2},S_{-1}\}$ $\{S_{-3},S_{-2}\}$ 4 $\{S_0,S_{-1},S_{-2}\}$ $\{S_3,S_2,S_1\}$ $\{S_0\}$ – 3 $\{S_3\}$ $\{S_3,S_2\}$ – $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ – – $\{S_{-3}\}$ 5 – $\{S_0\}$ $\{S_{-1},S_0,S_1\}$ $\{S_{-3}\}$ 4 – $\{S_1,S_0,S_{-1}\}$ $\{S_0\}$ $\{S_{-3},S_{-2}\}$ 3 $\{S_3\}$ $\{S_3\}$ $\{S_3,S_2\}$ $\{S_0\}$	score 6 5 4 3 6 $\{S_0\}$ – $\{S_{-4},S_{-3}\}$ $\{S_{-3}\}$ 5 – $\{S_0\}$ – $\{S_{-3}\}$ 4 $\{S_4,S_3\}$ – $\{S_0\}$ $\{S_{-1},S_0,S_1,S_2\}$ 3 $\{S_3\}$ $\{S_3\}$ $\{S_1,S_0,S_{-1},S_{-2}\}$ $\{S_0\}$
	Expert4	{0.5} {0.6} {0.5,0.6} {0.4} {0.5} {0.4,0.5} {0.4,0.5} {0.6,0.5} {0.5}	score 6 5 4 3 6 {0.5} {0.3,0.5} {0.3,0.4} {0.2} 5 {0.7,0.5} {0.5} {0.4,0.5} {0.4} 4 {0.7,0.6} {0.6,0.5} {0.5} {0.4,0.5} 3 {0.8} {0.6} {0.6,0.5} {0.5}	score 6 5 4 3 6 {0.5} – – {0.1,0.2} 5 – {0.5} {0.3} {0.1,0.2} 4 – {0.3} {0.5} {0.4,0.6} 3 {0.9,0.8} {0.9,0.8} {0.4,0.6} {0.5}	score 6 5 4 3 6 {0.5} {0.6} {0.7,0.8} {0.8} 5 {0.4} {0.5} {0.7} – 4 {0.3,0.2} {0.3} {0.5} – 3 {0.2} – – {0.5}

TABLE 4. Predicted O values for the seven failure modes.

Failure mode	FM1	FM2	FM3	FM4	FM5	FM6	FM7
Estimated O	4	5	5	4	6	3	5

TABLE 5. Preprocessed HFLPR.

Form	S&O&D-HFPR of FM2 from Expert2		
HFL PR	$\{S_{0.5}\}$	$\{S_{0.61},S_{0.72}\}$	$\{S_{0.72}\}$
	$\{S_{0.39},S_{0.28}\}$	$\{S_{0.5}\}$	$\{S_{0.5},S_{0.61}\}$
	$\{S_{0.28}\}$	$\{S_{0.5},S_{0.39}\}$	$\{S_{0.5}\}$

Form	S_v-HFPR of FM2 from Expert2				
I-HF LPR	score	6	5	4	3
	6	$\{S_{0.5}\}$	–	$\{S_{0.39},S_{0.5}\}$	$\{S_{0.28}\}$
	5	–	$\{S_{0.5}\}$	–	$\{S_{0.06},S_{0.17}\}$
	4	$\{S_{0.61},S_{0.5}\}$	–	$\{S_{0.5}\}$	$\{S_{0.28}\}$
	3	$\{S_{0.72}\}$	$\{S_{0.94},S_{0.83}\}$	$\{S_{0.72}\}$	$\{S_{0.5}\}$

set processing parameters $\{z_{12}^1, z_{12}^2, z_{23}^1, z_{23}^2\} = \{0.000, 1.000, 0.000, 1.000\}$ and total offset $F_{fm1}^{e(1)} = 0.04$ are obtained, and consistency level $CL_{fm1,sod}^{e(1)} = 0.987$ can be obtained by using Eq. (18).

The I-HFPR matrix proposed by Expert 2 for the severity S score of FM2 can be solved by using Eq. (21) to establish the following model.

$$\min f = \zeta$$
$$s.t.\begin{cases} ((0.39z_{13}^1 + 0.5z_{13}^2) - 1)\omega_1 + (0.39z_{13}^1 + 0.5z_{13}^2) \\ \quad \times \omega_3 - \zeta \le 0, \\ (1 - (0.39z_{13}^1 + 0.5z_{13}^2))\omega_1 - (0.39z_{13}^1 + 0.5z_{13}^2) \\ \quad \times \omega_3 - \zeta \le 0, \\ ((0.06z_{24}^1 + 0.17z_{24}^2) - 1)\omega_2 + (0.06z_{24}^1 + 0.17z_{24}^2) \\ \quad \times \omega_4 - \zeta \le 0, \\ (1-(0.06z_{24}^1+0.17z_{24}^2)) * \omega_2-(0.06z_{24}^1 + 0.17z_{24}^2) \\ \quad \times \omega_4 - \zeta \le 0, \\ -0.72\omega_1 + 0.28\omega_4 - \zeta \le 0, \\ 0.72\omega_1 - 0.28\omega_4 - \zeta \le 0, \\ -0.72\omega_3 + 0.28\omega_4 - \zeta \le 0, \\ 0.72\omega_3 - 0.28\omega_4 - \zeta \le 0, \\ \sum_{i=1}^{4} \omega_i = 1, \\ z_{13}^1 + z_{13}^2 = 1, z_{24}^1 + z_{24}^2 = 1, \\ z_{12}^1, z_{12}^2, z_{23}^1, z_{23}^2 \in [0, 1], \\ \omega_1, \omega_2, \omega_3 > 0 \end{cases} \tag{39}$$

TABLE 6. Consistency level CL and metric weight before establishing a group consensus.

Failure mode	Expert	S&O&D-HFPR		S_v-HFPR		O_v-HFPR		D_v-HFPR	
		Weights	CL	Weights	CL	Weights	CL	Weights	CL
FM1	Expert1	{0.600,0.200,0.200}	0.987	{0.318,0.205,0.477}	0.981	{0.474,0.473,0.053}	0.999	{0.098,0.374,0.373,0.155}	0.999
	Expert2	{0.656,0.210,0.124}	0.991	{0.144,0.705,0.151}	0.995	{0.253,0.649,0.098}	0.987	{0.407,0.083,0.087,0.423}	0.996
	Expert3	{0.578,0.197,0.225}	0.989	{0.779,0.159,0.062}	0.995	{0.243,0.624,0.133}	0.988	{0.278,0.454,0.169,0.899}	0.996
	Expert4	{0.735,0.184,0.081}	0.992	{0.318,0.477,0.205}	0.998	{0.596,0.255,0.149}	0.996	{0.075,0.183,0.343,0.399}	0.995
FM2	Expert1	{0.522,0.348,0.130}	0.998	{0.140,0.325,0.213,0.322}	1.000	{0.169,0.175,0.394,0.262}	0.999	{0.241,0.156,0.241,0.362}	0.999
	Expert2	{0.562,0.219,0.219}	1.000	{0.163,0.253,0.164,0.420}	1.000	{0.307,0.119,0.267,0.307}	0.999	{0.372,0.081,0.153,0.394}	0.997
	Expert3	{0.315,0.493,0.192}	0.999	{0.134,0.112,0.152,0.602}	0.991	{0.124,0.124,0.145,0.607}	0.993	{0.066,0.066,0.547,0.321}	0.997
	Expert4	{0.428,0.286,0.286}	1.000	{0.103,0.253,0.253,0.391}	0.996	{0.121,0.121,0.298,0.460}	0.995	{0.454,0.303,0.130,0.113}	0.996

The weights of the ratings {4, 5, 6, 7} are obtained for S: $\omega^{e(2)}_{\text{fm}2,S} = \{0.163, 0.253, 0.164, 0.420\}$, fuzzy set processing parameters $\{z^1_{12}, z^2_{12}, z^1_{23}, z^2_{23}\} = \{0.000, 1.000, 1.000, 0.000\}$, minimum offset $\zeta^{e(2)}_{\text{fm}2,S} = 0.83 \times 10^{-7}$ and consistency level $cl^{e(2)}_{\text{fm}2,S} = 1 - \zeta^{e(2)}_{\text{fm}2,S}$. The uncertain element is solved by used Eq. (16), and the results are $h_{12} = \frac{0.163}{0.163+0.253} = 0.391$, $h_{21} = 0.609$, $h_{23} = 0.607$, $h_{32} = 0.393$.

At this time, the consistency level CL and metric weight of all experts in FM1 and FM2 before the establishment of a group consensus is obtained, as shown in Table 6.

C. DETERMINE GROUP CONSENSUS

The HFPR matrix CL given by the four experts for three-metric FM1 is as follows:

$\text{CL}^{e(1)}_{\text{fm}1,\text{sod}} = 0.987$, $\text{CL}^{e(2)}_{\text{fm}1,\text{sod}} = 0.991$, $\text{CL}^{e(3)}_{\text{fm}1,\text{sod}} = 0.989$, and $\text{CL}^{e(4)}_{\text{fm}1,\text{sod}} = 0.992$.

Using Eq. (24), $\lambda = 0.9$ [12]; then,

$$\text{CL}^{e(4)}_{\text{fm}1,\text{sod}} > \text{CL}^{e(2)}_{\text{fm}1,\text{sod}} > \text{CL}^{e(3)}_{\text{fm}1,\text{sod}} > \text{CL}^{e(1)}_{\text{fm}1,\text{sod}}$$

The following formulas are calculated:

$$\omega^{e(4)}_{\text{fm}1,\text{sod}} = Q\left(\frac{0.992}{\sum_{i=1}^{4} \text{CL}^{e(i)}_{\text{fm}1,\text{sod}}}\right) = 0.288;$$

$$\omega^{e(2)}_{\text{fm}1,\text{sod}} = Q\left(\frac{0.992+0.991}{\sum_{i=1}^{4} \text{CL}^{e(i)}_{\text{fm}1,\text{sod}}}\right) - Q\left(\frac{0.992}{\sum_{i=1}^{4} \text{CL}^{e(i)}_{\text{fm}1,\text{sod}}}\right)0.288 = 0.249;$$

$$\omega^{e(3)}_{\text{fm}1,\text{sod}} = Q\left(\frac{0.992+0.991+0.989}{\sum_{i=1}^{4} \text{CL}^{e(i)}_{\text{fm}1,\text{sod}}}\right) - Q\left(\frac{0.992+0.991}{\sum_{i=1}^{4} \text{CL}^{e(i)}_{\text{fm}1,\text{sod}}}\right) = 0.236;$$

$$\omega^{e(1)}_{\text{fm}1,\text{sod}} = 0.227.$$

Then, the weight vector of each expert for the S, O, D weights is obtained:

$$\{e_1, e_2, e_3, e_4\} = \{0.227, 0.249., 0.236, 0.288\}$$

Next, the relative weights of the S, O, and D metrics of the group can be obtained by using Eq. (25):

$$\omega^c_{fm1,sod} = \{0.6476, 0.1972, 0.1552\}$$

Then, the GCD of each expert can be obtained by using Eq. (27):

$\text{GCD}^{e(1)}_{\text{fm}1,\text{sod}} = 0.9612$, $\text{GCD}^{e(2)}_{\text{fm}1,\text{sod}} = 0.9812$, $\text{GCD}^{e(3)}_{\text{fm}1,\text{sod}} = 0.9420$, and $\text{GCD}^{e(4)}_{\text{fm}1,\text{sod}} = 0.9343$.

The GCD of each expert is greater than the threshold of 0.9, so the relevant weight of the metric is $\omega^c_{\text{fm}1,\text{sod}}$.

For group decision-making of the FM1 S-score HFPR, $\text{GCD}^{e(1)}_{\text{fm}1,S} = 0.849$, $\text{GCD}^{e(2)}_{\text{fm}1,S} = 0.787$, $\text{GCD}^{e(3)}_{\text{fm}1,S} = 0.697$, and $\text{GCD}^{e(4)}_{\text{fm}1,S} = 0.929$. It was found that except for Expert 4, the GCD was less than the initial set threshold. The preference matrix for Expert 1, Expert 2 and Expert 3 needs to be adjusted. The group consensus HFPR matrix of the FM1 S-score is shown in Table 7.

TABLE 7. Group consensus HFPR matrix of the FM1 S-score.

	5	4	3
5	0.5	0.491	0.600
4	0.509	0.5	0.593
3	0.400	0.407	0.5

The modified positions of Expert 1, Expert 2 and Expert 3 are found through Eq. (30). The modified positions are $h_{2,3}$, $h_{1,2}$ and $h_{1,3}$, and the modification interval for each expert is found by Eqs. (31)-(32). The results show that this interval is [0.3, 0.593] for Expert 1, [0.17, 0.491] for Expert 2, and [0.491, 0.83] for Expert 3. After expert consideration, Expert 1 changed $h_{2,3}$ to 0.55, Expert 2 changed $h_{1,2}$ to 0.45, and Expert 3 changed $h_{1,3}$ to 0.5. The HFPR matrix of the four experts is recalculated. Then,

$$\omega^{e(1)}_{\text{fm}1,S} = \{0.355, 0.355, 0.290\}, \text{CL}^{e(1)}_{\text{fm}1,S} = 0.968;$$

$$\omega^{e(2)}_{\text{fm}1,S} = \{0.383, 0.468, 0.149\}, \text{CL}^{e(2)}_{\text{fm}1,S} = 0.951;$$

$$\omega^{e(3)}_{\text{fm}1,S} = \{0.474, 0.474, 0.052\}, \text{CL}^{e(3)}_{\text{fm}1,S} = 0.969;$$

$$\omega^{e(4)}_{\text{fm}1,S} = \{0.342, 0.512, 0.146\}, \text{CL}^{e(4)}_{\text{fm}1,S} = 0.984.$$

The weight of each expert is obtained as $\omega^{\{e_1,e_2,e_3,e_4\}}_{\text{fm}1,S} = \{0.291, 0.249, 0.279, 0.181\}$, and the weight of each metric is $\omega^c_{\text{fm}1,S} = \{0.393, 0.445, 0.162\}$.

The GCD are $\text{GCD}^{\text{e}(1)}_{\text{fm}1,\text{S}} = 0.907$, $\text{GCD}^{\text{e}(2)}_{\text{fm}1,\text{S}} = 0.983$, $\text{GCD}^{\text{e}(3)}_{\text{fm}1,\text{S}} = 0.919$, and $\text{GCD}^{\text{e}(4)}_{\text{fm}1,\text{S}} = 0.950$.

The group consensus of the four experts is greater than N, which meets the standard. The final S-score is $S_{\text{fm}1} = \omega^c_{\text{fm}1,S} \times [5, 4, 3]^T = 4.231$.

D. MODIFIED SCORES BASED ON OCCURRENCE PREDICTIONS

The above method is repeated to obtain the other results, which are $O_{\text{fm}1} = 4.306$ and $D_{\text{fm}1} = 3.411$. It can be seen from Table 4 that $O^{\text{pre}}_{\text{FM}1} = 4$. Therefore, it is believed that the evaluation of FM1 by the expert group is radical. The group offset level of FM1 can be obtained by Eq.(33):

$\text{GOL}_{\text{fm}1,S} = 0.034$, $\text{GOL}_{\text{fm}1,O} = 0.016$, $\text{GOL}_{\text{fm}1,D} = 0.035$.

The cautious parameter can be obatined by Eq.(35): $\varepsilon_{S\to O} = 0.333$. Then, $O_v^d = -0.153$, $S_v^d = -0.022$, and $D_v^d = -0.07$ can be calculated by using Eq. (36). Therefore, the final scores are

$D_{\text{fm}1} = 3.341$, $S_{\text{fm}1} = 4.209$, $O_{\text{fm}1} = 4.153$, and $\widetilde{RPN}_{\text{fm}1} = \omega^c_{\text{fm}1,\text{sod}} \times [S_{\text{fm}1}, O_{\text{fm}1}, D_{\text{fm}1}]^T = 4.063$.

In the risk assessment of FM2, the $\omega^{e(2)}_{\text{fm}2,S}$ determined in Section V-B needs to be iteratively calculated because the group consensus is not met. At this time, the weights determined by the experts who changed the matrix have changed: $\omega^{e(2)}_{\text{fm}2,S} = \{0.154, 0.097, 0.154, 0.595\}$. However, the original matrix should remain unchanged except for the part to be modified in the calculation.

After the group consensus has been established in FM1 and FM2, the above steps are repeated to obtain the experts' weights, the group weight, the metrics weight given by each expert, the final CL, and the GCL, as shown in Table 8.[3] Further determination of the three-parameter scores for the remaining six remaining failure modes FM2, ..., FM7 and the final $\widetilde{RPN}_{\text{fm}2} \ldots, \widetilde{RPN}_{\text{fm}7}$ scores, respectively, can be made, as shown in Table 9. Finally, according to Table 4, a determination is made on whether there is a conservative scoring situation in the expert group, and the score and the final score are further modified, as shown in Table 10.

VI. VALIDATION AND DISCUSSION

A. RESULT ANALYSIS

The seven failure modes can be ranked from the final $\widetilde{RPN}$ of each failure mode calculated in Section V: FM5> FM7> FM6> FM2> FM1> FM3> FM4. Based on the ranking results of the seven failure modes according to the method proposed in this study, this allows the project team to further determine the improvement needs of the project and to implement stricter measures and supervision for the failure modes with high severity. Appropriate control measures are also taken for other less critical failure modes in order to maximize the reliability of the project based on cost savings. FM5, for example, has the highest risk priority. It has low detection efficiency, while its severity and frequency are moderately high. Therefore, it is necessary to give priority (the greatest attention) to preventive measures in the safety management of the project team, from preventive control to detection control.

It can be seen from Table 3 that when establishing the HFPR matrix for the three-metrics scores of each failure mode, the project team gave possible score sets based on experience and previous historical data. After these score sets are counted and averaged, the average value of the failure mode score is calculated. The statistical results are shown in Table 9. It is found that the ranking of failure modes by this average result is somewhat similar to the final ranking of risk priorities calculated by the method proposed in this paper. For example, the rankings of F5 and F7 are both in the first and second positions. The reason for this is that there are various evaluation criteria for all three metrics under the same failure mode. For example, the criticality S can be evaluated in terms of casualties, structural damage, environmental hazards, and progress delays. It is possible that different experts recognize different emphases, leading to great differences in the preference matrix results given by the experts. When performing group consensus fitting, the average score will naturally be approached. it can be seen, However, that the results obtained by the project team based on experience and historical data are actually quite different from the final results. For example, although the average score of FM6 is lower than that of FM4, the expert group believes that the severity is particularly important in the failure mode risk assessment of FM6 and FM4, but the weight of the occurrence rate is not high. The severity score of FM6 is much higher than that of FM4, resulting in the RPN value of FM6 being higher than that of FM4 in the final score of the method proposed in this paper. Attention should be given to FM6. Although FM4 has a high occurrence rate, it is easy to detect and is not harmful. After combining O_{pro}, its risk rating is further reduced, so it is placed at the bottom of the list. Although the occurrence rate and detection rate of FM5 are higher than those of FM7, the risk assessment of the two failure modes focuses more on the severity. Because the severity of FM5 is much lower than that of FM7, the initial assessment of the risk of FM5 is lower than that of FM7. However, after modifing the FM5 score by the failure occurrence prediction value, FM5 was given a higher score than FM7, so FM5 was assigned the first rank.

B. COMPARATIVE ANALYSIS

In order to verify the efficiency and advanced nature of the proposed method, In what follows, the results of the methods of this study will be compared in two aspects, respectively: the first aspect is to compare the gap values of the matrix obtained by the original processing method [12] without the theory of extended MC and the method of the present paper; the second aspect is to compare the results of the failure mode ranking obtained by the methods in different papers with the results obtained by the method in this study.

1) DEVIATIONS COMPARISON

The evaluation matrices of 12 groups of different forms (HFPR, HFLPR, I-HFPR, I-HFLPR) were input into the original method as well as the method of this study, respectively, and 48 comparisons were obtained in total.[4]

By averaging the internal deviations of the data for each different assessment form, a line graph can be drawn comparing the deviations of the assessment matrix obtained based on the MC theory with expanded application with the deviations of the assessment matrix obtained from the MC theory

[3]The results of all failure modes are shown in Table 3 in the Appendix.

[4]See Table 4 in Appendix.

TABLE 8. E-metric weight, CL, GCL, expert weight, and G-metric weight of FM1 AND FM2.

Failure mode	Expert	S&O&D-HFPR			S_v-HFPR			O_v-HFPR			D_v-HFPR		
		Weights	CL	GCL	Weights	CL	GCL	Weights	CL	GCL	Weights	CL	GCL
FM1	Expert1	{0.600,0.200,0.200}	0.987	0.961	{0.355,0.355,0.290}	0.968	0.907	{0.474,0.473,0.053}	0.999	0.927	{0.181,0.277,0.271,0.271}	0.971	0.984
	Expert2	{0.656,0.210,0.134}	0.991	0.981	{0.383,0.468,0.149}	0.951	0.983	{0.377,0.566,0.057}	0.989	0.984	{0.204,0.240,0.250,0.306}	0.948	0.968
	Expert3	{0.578,0.197,0.225}	0.989	0.942	{0.474,0.474,0.052}	0.969	0.919	{0.250,0.643,0.107}	0.968	0.907	{0.246,0.405,0.157,0.192}	0.972	0.915
	Expert4	{0.735,0.184,0.081}	0.992	0.934	{0.342,0.512,0.146}	0.984	0.950	{0.405,0.494,0.101}	0.983	0.963	{0.143,0.214,0.321,0.322}	0.975	0.935
Experts weight		{0.227,0.249.,0.236,0.288}			{0.291,0.249,0.279,0.181}			{0.291,0.236,0.225,0.248}			{0.237,0.224,0.289,0.250}		
Group weight		{0.6476,0.1972,0.1552}			{0.393,0.445,0.162}			{0.384,0.538,0.078}			{0.195,0.290,0.246,0.269}		
FM2	Expert1	{0.522, 0.348, 0.13}	0.996	0.948	{0.127, 0.17, 0.283, 0.42}	0.994	0.920	{0.169,0.175,0.394,0.262}	0.999	0.912	{0.237,0.368,0.237,0.158}	0.999	0.947
	Expert2	{0.562,0.219,0.219}	0.999	0.923	{0.154,0.097,0.154,0.595}	0.991	0.921	{0.307,0.119,0.267,0.307}	0.999	0.913	{0.173,0.412,0.281,0.134}	0.993	0.925
	Expert3	{0.371,0.453,0.176}	0.987	0.901	{0.106,0.122,0.144,0.628}	0.997	0.904	{0.136 0.137,0.218,0.509}	0.999	0.910	{0.276,0.393,0.181,0.150}	0.990	0.923
	Expert4	{0.428,0.286,0.286}	0.988	0.941	{0.111,0.254,0.296,0.339}	0.996	0.922	{0.121,0.121,0.298,0.46}	0.996	0.920	{0.147,0.346,0.286,0.221}	0.992	0.940

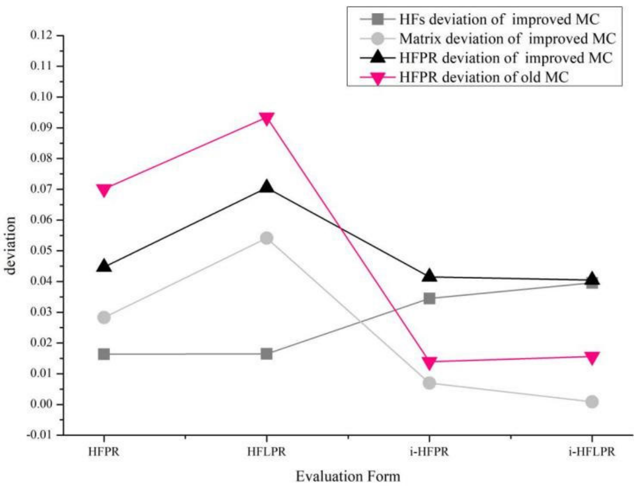

FIGURE 4. Comparison of the degree of deviation of the original MC and extended MC treatment H-HFPR.

without expanded scope for different assessment forms, as shown in the following Fig. 4.

According to TAB, it can be found that the deviation values of the matrices obtained based on this paper are 100% lower or equal to the deviation values of the hesitant fuzzy sets processed by the original MC, regardless of the form of evaluation. Lower values of matrix deviations tend to represent that the obtained metric weights as well as the defuzzification matrix are closer to the consistency matrix, i.e., they are more accurate.

Meanwhile, since the original MC does not consider the possible consistency deviation in the hesitant fuzzy set, we can find that after incorporating the hesitant fuzzy set deviation values into the expert consistency level deviation, there exists 12.5% matrix consistency deviation greater than the original MC consistency deviation in the evaluation matrix with complete information, while in the evaluation matrix with incomplete information, all the matrix consistency deviations with hesitant deviation are greater than the This is precisely because after the incomplete information is brought into the model, the matrix consistency level decreases significantly due to the reduced information, and more consistent level deviations are included in the hesitation ambiguity set, which leads to the HFPR deviation of extended MC being larger than the HFPR deviation of original MC. This is one of the evidences that the method in this paper can be used in the heterogeneous hesitant fuzzy group decision environment.

TABLE 9. First score.

Failure mode	S_vHFPR	O_v-HFPR	D_v-HFPR	$\widetilde{RPN}$
FM1	4.231	4.306	3.411	4.119
FM2	3.908	4.139	4.632	4.138
FM3	3.029	3.048	4.253	3.523
FM4	4.706	4.707	3.973	4.430
FM5	4.415	4.335	6.205	5.048
FM6	5.922	3.504	4.984	5.045
FM7	6.154	5.295	4.425	5.168

TABLE 10. Final score.

Failure mode	S_vHFPR	O_v-HFPR	D_v-HFPR	$\widehat{RPN}$
FM1	4.205	4.153	3.341	4.063
FM2	4.008	4.570	4.75	4.352
FM3	3.619	4.024	4.482	4.053
FM4	4.108	4.354	3.275	3.844
FM5	5.156	5.168	6.782	5.750
FM6	5.775	3.252	4.866	4.876
FM7	6.081	5.148	4.347	5.356

TABLE 11. Possible score means of the three metrics under each failure mode.

Failure mode	S	O	D	MEAN
FM1	4	4	3.5	3.833
FM2	4.5	4.5	4.5	4.5
FM3	3	3.5	4.5	3.667
FM4	5.5	5	4.5	5
FM5	4.5	4.5	6.5	5.167
FM6	6	3	5	4.667
FM7	6.5	4	4.5	5

2) COMPARISON OF ASSESSMENT RESULTS

Compare five different FMEA assessment methods: the traditional FMEA method, the improved CEV-FMEA method based on the cloud model, the ZFB-FMEA method based on Z-MOORA and the fuzzy BWM, fuzzy FMEA and nonpreset O threshold data of the MHHG-FMEA method based on MC and H-HFPR group decisions to evaluate failure modes, and the finalized failure mode ranking are compared with the research results.

Each of the five methods has different characteristics, as shown in Table 13.

The traditional FMEA method first solves the mean value by calculating the possible scores of the metrics under each

TABLE 12. Comparison of the results of different methods.

Failure mode	Traditional FMEA		CEV-FMEA		ZFB-FMEA			Fuzzy FMEA		MHHG-FMEA		Proposed method	
	AIAG and VDA[55]		Yu Jiangxing[8]		Ghoushch and Yousefi[7]			Yazdi[71]		-			
	RPN	Ranking	$\underline{Q}_i$	Ranking	$\tilde{y}_i$	$\bar{y}_i$	Ranking	RPN	Ranking	$\widetilde{RPN}$	Ranking	$\widetilde{RPN}$	Ranking
FM1	56	6	0.253	2	[0.078, 0.092, 0.098]	0.268	7	0.614	4	4.119	6	4.119	5
FM2	91.13	4	0.681	6	[0.09, 0.117, 0.152]	0.359	6	0.518	6	4.138	5	4.895	4
FM3	47.25	7	1.000	7	[0.094, 0.129, 0.166]	0.389	5	0.384	7	3.523	7	4.807	6
FM4	123.75	2	0.582	5	[0.193, 0.177, 0.152]	0.552	2	0.536	5	4.430	4	4.430	7
FM5	131.63	1	0.444	4	[0.147, 0.175, 0.278]	0.6	1	0.693	2	5.048	2	6.179	1
FM6	90	5	0.350	3	[0.19, 0.099, 0.152]	0.441	4	0.652	3	5.045	3	5.045	3
FM7	117	3	0.000	1	[0.215, 0.143, 0.154]	0.512	3	0.715	1	5.168	1	5.168	2

TABLE 13. Different characteristics of methods.

Method	Case study	Specification- characteristics					
		Uncertainty	Full-ranking	Indicator -Weights	Expert-weights	Group-consensus	Objective-numerical-correction
Traditional-FMEA	/		√				
CEV-FMEA	submarine pipeline	√	√		√(Subjective)		
ZFB-FMEA	Automotive industry	√	√	√			
Fuzzy-FMEA	Construction period of a refinery	√	√	√			√
MHHG-FMEA	submarine pipeline	√	√	√	√	√	
Proposed	submarine pipeline	√	√	√	√	√	√

failure mode given by the project team, as shown in Table 11. Next, the mean results of the three metrics in each failure mode in Table 11 are multiplied to obtain the RPN value under each failure mode, and each failure mode is sorted according to the RPN value, as shown in Table 12.

The CEV-FMEA method decomposes the primary factors of failure mode occurrence, severity, detection and maintenance into nine secondary factors. Through the evaluation standard cloud constructed in this paper, the expert panel sets the semantic label of the secondary factors and the trust of the experts and obtains the expert rating interval for each secondary factor. The expert evaluation interval is solved, and finally, the ranking of each failure mode after the expert weight is determined by each professional factor of the expert is determined. In this study, the weight of the experts is measured through the level of expert consistency. Therefore, the HFPR matrix given by the experts can be transformed, and the preference value can be broken down into the range of assessment of the seven failure mode reviews, which are required by the CEV-FMEA method. In parallel, by consulting the data and according to the original data of the paper [8], the evaluation interval of the two second-level hazardous metrics under the first-level metric M of the failure mode is determined in order to calculate the ranking result for the CEV-FMEA method, as shown in Table 12.

The scoring index of the FMEA method using Z-MOORA and fuzzy BWM is established by linguistic interval [VL, L, ML, M, MH, H, VH]. Each expert needs to give the credibility of each score. The credibility is determined from the interval [VL, L, M, H, VH]. The triangular fuzzy number indicators are given in that paper for different semantic scores and credibility semantic combinations. Therefore, the score from 0-10 in this study can be divided into 7 equal parts, and the CI can be divided into 5 equal parts starting from 0.90, that is, by dividing it into five intervals [0-0.92), [0.92, 0.94), [0.94, 0.96). All the result values of the HFPR matrix in Table 3 when the group consensus is not established are converted into the linguistic expression in the above paper according to the interval to equivalently convert the consistent degree into the credibility degree. The S, O, D preference matrix of each expert is processed and converted into the semantic preference of the best and worst S, O, D metrics that each expert believes in order to calculate the ranking result using the ZFB-FMEA method, as shown in Table 12.

Linguistic variables based on fuzzy set numbers are utilized in the fuzzy-FMEA method to deal with uncertain and complex situations. The linguistic review set $S = 9$ defined in this study, which contains semantic repartitioning, S_{-4} is merged with S_{-3}, and S_4 is merged with S_3 to correspond to the linguistic variable $T = 7$ of the fuzzy-FMEA method. In the fuzzy-FMEA method, fuzzy AHP is used to calculate the experts' respective abilities and weights. In addition, through the fuzzy AHP and entropy methods, considering subjective and objective weights, the comprehensive weight $\omega_j (j = 1, 2, \ldots, n)$ of risk factors is calculated. Fuzzy RPN is defined as the sum of multiple products: $\text{RPN} = (\text{RF}_1 \times \omega_1) + (\text{RF}_2 \times \omega_2) + \ldots + (\text{RF}_n \times \omega_n)$. According to the HFPR matrix given by this study for the S, O, and D metrics, the

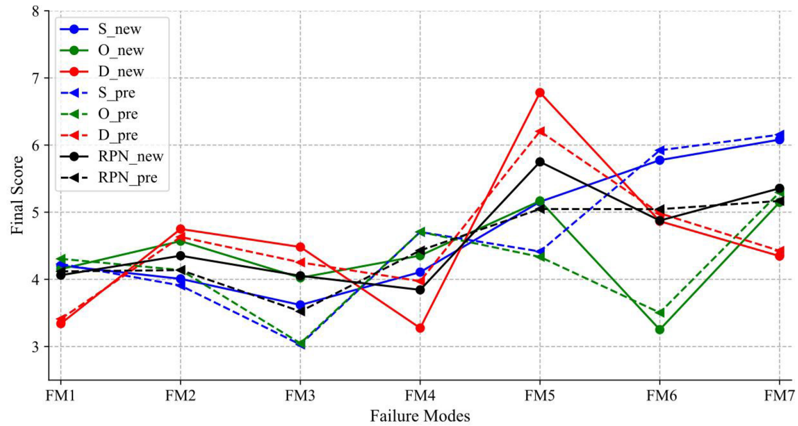

FIGURE 5. MHG-FMEA vs. Hybrid FMEA score line chart.

fuzzy RPN and ranking results are calculated by appropriate transformation, as shown in Table 12.

To visualize the differences between the MHHG-FMEA and hybrid FMEA results, a three-metric score line graph and a risk score line graph for seven failure modes between the two methods are drawn, as shown in Fig. 5. The Figure shows that the hybrid method, which further processes the risk priority operator through the predicted value of O, has greatly changed in FM3, FM4 and FM5. From the ranking of the results, we can also find that the main changes are reflected in FM4 and FM5, especially FM4, which was down from fourth to last place, indicating that the objective prediction of the historical data proves that FM4 is not of much concern. The hybrid FMEA method combines the results of expert scoring with the predicted value, which fully removes the impact of subjectivity on the results and makes the risk ranking more objective. In Table 11, although FM3 attaches great importance to the detection rate, its maintenance cost is not high. However, the detection difficulty of FM3 is underestimated in the actual scoring, leading it to be ranked last. This can be adjusted by estimating O. Afterward, its final ranking shifted from last to third to last, which reduces the risk of FM3 occurring due to insufficient attention of the project team. The assessment of FM2 is the same as that of FM5.

As shown in Fig. 6, the results of the research method in this paper are compared with the CEV-FMEA method, and it is found that except for the FM6 ranking, the rankings of the other failure modes are different. This is because the combination of expert score and the predicted O value weakens the subjectivity of the expert group, which has a certain impact on the results. Comparing the MHHG-FMEA method to the CEV-FMEA method, we find that FM6 is the same, with FM7 ranks first and FM3 ranks last. It should be noted that the ranking from the MHHG-FMEA method is still different from the CEV-FMEA method because the research content of this paper considers not only the fuzzy set, but also the incomplete hesitation fuzzy set matrix, this further improves the accuracy of the experts' preference information, as well as extending the uncertainty handling function. Compared with the ability of the CEV-FMEA, ZFB-FMEA and fuzzy-FMEA methods to only read linguistic information, the research method in this paper can read heterogeneous information, and by compatibility with incomplete preference information, the restriction on experts to better receive expert information is alleviated. Therefore, the final result is closer to the original expert information. In addition, the CEV-FMEA method requires experts to give the reliability of semantic information while determining semantic information. The degree of subjectivity is too high. The method in this paper determines the reliability of the preference matrix through multiplicative consistency, which is determined by objective data. This method is obviously more reliable.

Similarly, Fig. 6 shows that the results obtained by this method proposed in this paper and the fuzzy-FMEA method are only consistent in their ranking of FM6. The comparison of the results of the MHHG-FMEA and fuzzy-FMEA methods also shows that the rankings of FM1, FM2 and FM4 are different. This is because the traditional hesitant fuzzy set method only has a good function when dealing with fuzzy sets but cannot deal with uncertainty and the H-HFPR matrix. In addition, the semantic scope of this method is only seven, which is smaller than the nine scopes utilized in the method proposed in this paper. These deficiencies cause the method's acceptance rate of expert original information to be lower than the acceptance rate of method proposed in this paper, and when receiving semantic information, it is easy to confuse the similar original information of experts, which eventually lead to deviation in the results. Thus, the research approach of this paper involving a heterogeneous group decision environment and a study of objective data modify scores based on the consensus of the group is clearly more reasonable and reliable.

By comparing the research results for the method proposed in this paper with the ZFB-FMEA method results, it can be found that the results obtained by the ZFB-FMEA method are more similar to the results obtained using the research

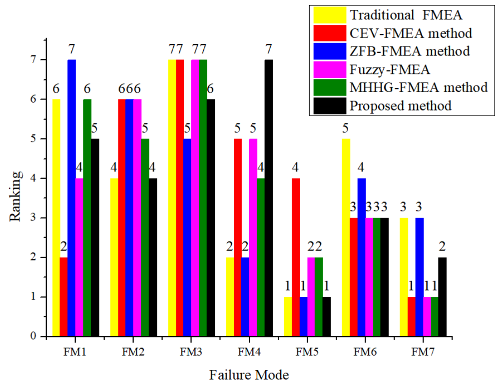

FIGURE 6. Comparison of risk priority ranking results of different methods.

method in this paper but are completely inconsistent with the results obtained using the MHHG-FMEA method. This can be explained by the following reasons: in the ZFB-FMEA method, experts determine the weights of S, O, and D metrics directly by linguitic variables (rating or reliability) and do not establish group consensus. This, coupled with the fact that the weights of experts are directly equalized, which leads to the fact that the final result is not a good group consensus result; that is, some experts who give high scores are weighted too high. The research method in this paper still has a certain feedback connection with the expert when iteratively processing the preference matrix, which further eliminates the arbitrariness of an expert's one-time decision. However, repeated group consensus building is likely to lead to more conservative or radical experts. Therefore, it is clear that the final results are most reliable when the expert scores are corrected by the predicted occurrence values with the group consensus.

The above comparative analysis shows that this research method can obtain a more accurate, more realistic and more reliable failure mode risk ranking.

VII. SUMMARY

In this paper, a group decision theory for H-HFPR based on extended MC is proposed, and a hybrid FMEA method is constructed through this improved theory combined with the principle of occurrence predictable. In addition to optimizing the original FMEA method, which further eliminates ambiguity and randomness as well as the possible influence of uncertainty on FMEA results, this method relaxes the constraints of expert scoring, supporting the H-HFPR group decision environment. To verify the effectiveness and advanced nature of the method, an example of a submarine pipeline in the Chengbei oilfield was presented, and the result of risk ranking after failure mode analysis was obtained to compare with other feasible methods. This result confirmed that the risk priority results provided by this research method are more reasonable. The main contributions of the method proposed in this paper are obtained through the analysis and verification described above as follows:

1) Through the extended MC theory to deal with fuzzy sets, the HFPR matrix can allow the existence of fuzzy sets with different numbers of evaluation values and can calculate the possible consensus offset value caused by fuzzy sets to make the obtained expert consistency level more reliable.
2) A new mathematical model is established to calculate the weights and CL, which supports the solution of H-HFPR group decisions. The model does not limit the number of metrics in the fuzzy set, which will make it easier for the expert group to give scoring preferences. The model can more effectively obtain the original information of experts, which enhances the flexibility and applicability of FMEA in complex environments.
3) The model is applied to FMEA assessment, which improves the accuracy and reliability of FMEA assessment.
4) The hybrid FMEA assessment method is established through occurrence prediction theory, which can effectively remove the subjectivity of the expert group and make the results more objective, efficient and reliable.

In conclusion, while this research method has been validated in the case of the ChengBei submarine pipeline, attention must still be given to the limitations of the method. These will be our future studies and topics.

1) In Many linguistic GDM problems which decision makers may have personalized individual semantic (PIS) [72], [73]. One direction that can be explored in the future is how to improve the hesitant fuzzy linguistic conversion algorithm proposed in this study in conjunction with current research related to the PIS, so that the method can be better applied to complex heterogeneous group decision-making environments.
2) For occurrence prediction problems, if there is no similar products or historical products, and the products are too innovative or complex, the accuracy of occurrence prediction values for the products will be less than perfect, which may limit the method of correcting FMEA based on occurrence prediction values to occurrence predictable products. Future research directions, such as deep learning and accelerated test models, will be pursued with the goal of breaking such limitations.
3) The structure of our consensus feedback mechanism is a traditional mechanism for ''central'' purposes, which does not take into account the demands of subgroups and the cohesion among subgroups, which may lead to the modified original opinion of each member and increase the cost of consensus. The existing group consensus framework based on the social relationship networks of the members can be a future research direction to address this issue.

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