# An Observer/Predictor-Based Model of the User for Attaining Situation Awareness

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***Abstract*—Situation awareness (SA) is essential for the safe operation of systems involving human-automation interaction. In this paper, using the theory of functional observers, we model SA for the user interacting with a continuous-time linear time-invariant dynamical system. For systems under human control or shared control, we use the proposed model to determine the required information to be displayed in the user interface for achieving SA. The user interface provides the user with the ability to observe the continuous-time outputs of the system, as well as the ability to enter continuous-time control inputs. In some systems, due to inadequacy of the displayed information, the user may not be able to accomplish the desired task. To determine the required information to be displayed and the necessary states to be tracked, we propose a model of attaining SA for the users by modeling the user as a specific type of estimator (i.e., the extended delayed functional observer/predictor). We then evaluate what information is needed for such an estimator and how the desired functional of the states have to be expanded so that the user can precisely reconstruct and accurately predict the desired task. As an application example, we investigate the problem of controlling the depth of anesthesia during surgery and determine whether there exists a feasible combination of the expanded task and the displayed information that allows the anesthetist to precisely predict the depth of anesthesia of the patient.**

***Index Terms*—Functional delayed observers, functional delayed predictors, linear time-invariant (LTI) systems, situation awareness, user modeling.**

## I. Introduction

DESPITE successful efforts to increase the autonomy of systems and devices, many systems function under the shared control of a human operator and a computer. In these cases, the system should provide the required information for the user to maintain situation awareness (SA) [1]–[3]. Therefore, it is of interest to develop techniques to evaluate and determine the minimum information required for the user to support SA. More specifically, the display design requires a careful selection and clear presentation of the information for the user to perform the task properly.

In [4], we introduced two novel subspaces for linear time-invariant (LTI) systems, the user-observable and the user-predictable subspaces, to assess the correctness of the displayed information based on SA requirements. Having these subspaces, we could evaluate the correctness of the displayed information by checking whether the task space is user-observable and user-predictable. Not having a user-observable and a user-predictable task would be a rejection condition for the displayed information.

Researchers have investigated conceptual models of human information processing for attaining SA in various applications [5]–[7]. An early attempt to model the user estimating the states of the system and acting on the system based on these estimates was the optimal control model [8], [9]. Other researchers modeled the human as an observer-based fault detector with the focus on modeling the decision-making process [10], [11]. Rather than modeling the user as a full-order Kalman-filter/observer, in [12] and [13], we modeled the user as a functional observer under a set of given assumptions about the actual behavior of the user. The suggested observer's model could be personalized based on the capabilities of the operator.

A functional observer is an observer designed to only reconstruct a desired functional of the states of the system. In most cases, a functional observer has a lower order than that of a full-order observer. Existence of the observers to estimate single and multifunctionals of the states of a system has been studied extensively for systems with known inputs [14]–[19] as well as systems with partially or fully unknown inputs [20]–[25]. Design procedures have also been developed for cases, where functional observers exist. The effect of the availability of the delayed output on the size of the desired functional has also been evaluated [26]–[28]. In [12], we studied the existence of and design of a functional observer for a system with known and/or unknown inputs and with the delayed inputs and the outputs (or the higher order derivatives). A discussion of functional observers can be found in [29].

In this paper, we introduce a more detailed model of a human attaining SA. We take into account the user's limitations and capabilities regarding the information presented to them and estimated by them. The process of attaining SA includes observations as well as predictions by the user. Processing of information generally introduces a delay [30]–[32]. Assuming the derivatives of the inputs and the outputs might be available, we design (and evaluate the existence of) a novel estimator for LTI systems generating delayed estimates of the current and upcoming desired functional of states. In addition, in state-space representation of the system, we consider the model to be affected by the low-level input from the human, as well as the desired trajectory of the automation. In addition, we consider the existence of an unknown set of reference trajectories, which might be partially or entirely measured in the user interface. Since we consider the user to 1) only reconstruct and predict the desired set of states rather than the entire state space, 2) make delayed

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estimations, and 3) possibly have knowledge of the derivatives of the inputs and outputs, we model the process of attaining SA as an extended delayed functional observation/prediction.

We introduce a theorem providing conditions needed for an estimator with human specifications to exist, i.e., the required conditions on the system, the display, and the task so that the user can attain SA toward specific goals. If these conditions are not satisfied for a triplet of dynamics, measurements, and task, then it is not possible for the user to attain SA regarding the specific task through the available information.

Our main contributions can be summarized as follows:

1) SA modeling with specific consideration of the capabilities of the user.
2) Evaluating the existence of and then designing the novel delayed observer/predictors.

In Section II, we present the conceptual idea and the proposed mathematical framework. In Section III, we present the existence conditions and the design procedure of an extended delayed functional observer/predictor considered to be the model of attaining SA by the user. We include an example of the existence and design of the delayed/nondelayed functional observer/predictor. Finally, in Section IV, we investigate a safety critical application, prediction of the depth of anesthesia during surgery.

### A. Common Notations

The identity matrix of size $n$ is denoted as $I_{n \times n}$ and $e_i$ represents the $i$th basis vector of the corresponding space.

## II. Problem Statement

The fundamental stages of human information processing include 1) understanding the situation, 2) decision making, and 3) action implementation. In this paper, we focus on the first stage of information processing—that is, understanding the situation [2], [33]–[35]. Our main goal here is to evaluate the displayed information and to obtain the required information, which has to be displayed so that the user can accomplish a desired task.

Attaining SA [2] necessitates three stages of processing the information, 1) perception of the information, 2) comprehension of the information, and 3) projection or prediction of the information and can be done using *working memory or long-term memory.* Factors such as learning and attention can clearly affect the perception of the information (the first stage of attaining SA). Here, as stated in Assumption 1, we focus on the latter two stages of attaining SA to help us evaluate and determine the information content of the display.

*Assumption 1:* The user can perceive all the displayed information.

For a fully experienced and trained user with a well-developed mental model about the behavior of the system, comprehension and prediction are achieved using long-term memory. This process of pattern-matching takes place without loading the working memory and is almost instantaneous. We limit the role of the mental model to only providing the user with an internal representation of the system and knowledge about system dynamics.

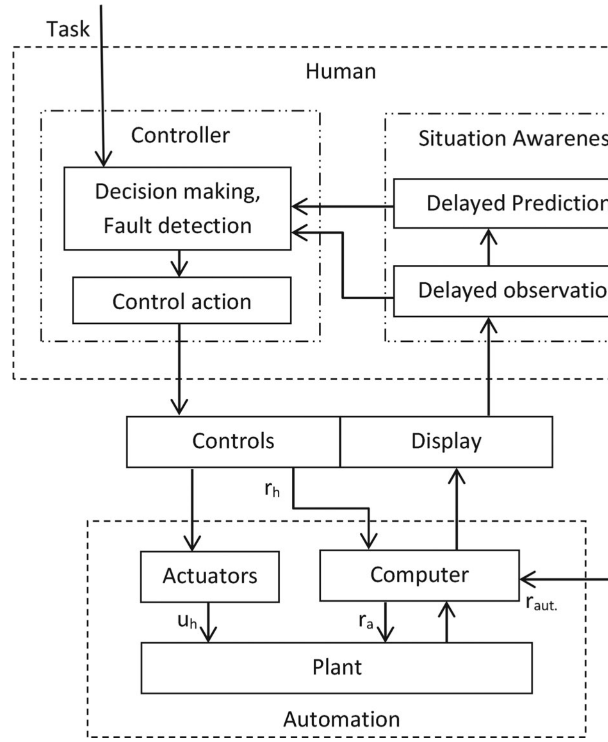


Fig. 1. Simple schematic model of a human controlling a system.

Under this assumption, information processing should occur in working memory [2].

Note that insufficient understanding about the situation is not solely due to lack of information in the user interface and the improper set of tasks, as it can also be the result of nondeterministic automation or a user who is unfamiliar with system dynamics, e.g., as in category II of pilot-induced oscillations [36].

In this paper, we particularly consider the case when the user is familiar with the deterministic dynamics of the system and our goal is to answer what information has to be presented on the display to achieve the desired task. We therefore model the user as a specific type of estimator who attempts to attain SA. This modeling provides us a tool for both the analysis and the synthesis of the user interface.

### A. Problem Definition

For analyzing human-automation interaction, Jamieson and Vicente [37] suggested a feedback loop model in which feeding back important signals to the user could help them identify and localize the source of failure in the elements of the system. Sheridan suggested a set of frameworks for systems with different levels of autonomy [38]. In [39], using a framework for supervisory control, Cummings showed that it is necessary for the user of some systems to act in collaboration with the plant, rather than exclusively acting as a supervisor.

Fig. 1 presents our schematic model of the human-automation interaction.

For this study, we consider a delay-free system whose evolution is modeled by the LTI model:

$$\dot{x}(t) = Ax(t) + Bu_h(t) + Fr_a \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector. The inputs in (1) can be categorized as the known input, which is the low-level human input $u_h(t) \in \mathbb{R}^{m_b}$ controlled by the user and the input, which is the time-invariant reference trajectory $r_a \in \mathbb{R}^{m_f}$ tracked by the automation. We consider the reference trajectories to be unknown, unless they are measured in the display. In (1), the matrices $A$, $B$, and $F$ have compatible dimensions. We also assume that no poles of the system have a zero real part.

*Assumption 2:* Matrix A has no eigenvalues on the imaginary axis.

Based on our specific application of user interface design, we assume that no combination of the states and the reference trajectories is available in the display. In addition, the output consists of two sets of measurements

$$\begin{aligned} y_1(t) &= Cx(t) \\ y_2(t) &= Dr_a \end{aligned} \tag{2}$$

where $y_1 \in \mathbb{R}^{p_x}$ is the set of measured combinations of states and $y_2 \in \mathbb{R}^{p_r}$ is the set of measured reference trajectories in the display.

In Model 1, we consider the user to be a function that maps the displayed information to the user input. This mapping involves different stages of information processing. In our framework, we similarly consider the user to first obtain SA, then decide on the required action, and finally act on the system.

To have a successful HAI, both the data-driven (bottom-up) information processing and the goal-directed (top-down) processing for SA are considered to be vital [40], [41]. The emphasis of the goal-driven processing is on paying direct attention to and then processing the most important information related to the goal. Awh *et al.* [42] and Fougnie [43] discuss how attention acts to filter out the unnecessary information at both the early stages of perception and the late stage of processing the information in working memory. Overall, one main role of attention during the postperceptual stage of processing information is to reduce or cancel out the distractions, while comprehending the target [44].

The adverse effect of irrelevant information on the ability of the user to understand and perform a desired task has been studied. For example, Pasolunghi *et al.* [45] concluded that "the problem-solving ability is related to the ability of reducing the memory accessibility of nontarget and irrelevant information." Carretti *et al.* [46] and [47] identified that the lack of capability of selecting information relevant to the task and suppressing irrelevant information will result in poor performance of the working memory and poor comprehension. For more challenging tasks, during the processing and manipulating of information in the executive working memory, the users will have higher focus on the actual goal and irrelevant task information will have less effect [43].

Thus, it is desirable for one to concentrate on the target and suppress irrelevant information. Therefore, it is not realistic to assume that the user reconstructs all observable and predictable states to only make use of the desired functional of the states of the system. We therefore have the following assumption.

*Assumption 3:* For the purpose of reconstructing the desired functional, the user does not estimate all observable states unless reconstructing all observable states is feasible and necessary for the estimation of the desired functional.

Based on Assumption 3 and since we consider the comprehension and prediction of the unmeasured information to be a challenging task that can occupy the executive working memory of the user, we consider the user to behave as a functional estimator in order to process the information. We believe that this model is a reasonable start to address perfect users, who perceive all information provided in the display and can fully concentrate on what they are asked to do.

We also consider the user to be highly trained and experienced with the dynamics of the automation. This specifically means that the user is capable of perceiving information on the inputs, outputs, and their derivatives. We therefore ignore the information-acquisition delay of the user. The user can then make use of the perceived information to reconstruct and predict important states of the system. Reconstruction and prediction of the desired states of the system are however considered to be delayed and we thus model the user as a delayed observer/predictor for the functional:

$$z_0(t+\tau) = Tx(t+\tau), \qquad 0 \leq \tau \tag{3}$$

where $\tau$ defines the prediction horizon. In (3), the task matrix $T \in \mathbb{R}^{l\times n}$ is comprised of $l$ linear combinations of the states. We formulate the task as a function $f : \mathbb{R}^l \to \mathbb{R}^s$ with $s$ subtasks [4], [13]:

$$\mathcal{F} = \{x \mid f(Tx) \geq 0\}. \tag{4}$$

In some cases, it is not possible to estimate the functional $z_0(t+\tau)$ directly and it is necessary for the user to also estimate the functional $Rx(t+\tau)$ such that $R \in \mathbb{R}^{\mathcal{X}\times n}$. We select the rows of $R$ to be linearly independent from the rows of $T$. Hence, we introduce the extended functional as

$$z(t+\tau) = \begin{bmatrix} T \\ R \end{bmatrix} x(t+\tau) \tag{5}$$

where $R$ is selected such that $L = [T^T, R^T]^T$ is of full row rank. For cases that $Tx(t+\tau)$ can be estimated directly, we have $R = \emptyset$.

We also make the following two assumptions:

*Assumption 4:* The user is provided with no additional information beyond what is on the display.

*Assumption 5:* The users have knowledge about the derivatives of their own inputs, up to $\lambda$ derivatives as well as have knowledge about the derivatives of the outputs, up to $\gamma$ derivatives.

Our purpose is to evaluate the correctness of the task and the available displayed information. For this purpose, we evaluate the existence of a stable delayed functional observer and a stable delayed functional predictor with access to the extended output and input and model the user as such an observer/predictor.

### B. Problem Formulation

Based on Assumption 5, we introduce the extended output vector and the extended input vector as

$$Y_{0:\gamma}(t) = [{y_1}^T(t), \dot{y}_1^T(t), \cdots, {y_1}^{(\gamma)^T}(t)]^T$$
$$U_{0:\lambda}(t) = [{u_h}^T(t), \dot{u}_h^T(t), \cdots, {u_h}^{(\lambda)^T}(t)]^T \quad (6)$$

hence, as we consider $\gamma \in \{0, 1\}$ and $\lambda \in \{0, 1\}$, only two cases for the extended output vector and two cases for the extended input vector may exist.

Analytically, the extended output vector can be written as

$$Y_{0:\gamma}(t) = O_\gamma x(t) + M_{1,0:\gamma} U_{0:\gamma}(t) + M_{2,\gamma} r_a \quad (7)$$

where for $\gamma \in \{0, 1\}$, the observability matrix $O_\gamma \in \mathbb{R}^{(\gamma+1)p_x \times n}$, a Toeplitz matrix $M_{1,0:\gamma} \in \mathbb{R}^{(\gamma+1)p_x \times (\gamma+1)p_x}$, and matrices $M_{2,\gamma} \in \mathbb{R}^{(\gamma+1)p_x \times p_r}$ and $U_{0:\gamma}(t) \in \mathbb{R}^{(\gamma+1)p_x}$ are defined as follows:

$$O_0 = C, \qquad O_1 = [C^T, A^T C^T]^T$$
$$M_{1,0:0} = 0, \qquad M_{1,0:1} = \begin{bmatrix} 0 & 0 \\ CB & 0 \end{bmatrix}$$
$$M_{2,0} = 0, \qquad M_{2,1} = \begin{bmatrix} 0 \\ CF \end{bmatrix}$$
$$U_{0:0}(t) = u_h, \quad U_{0:0}(t) = [{u_h}^T(t), \dot{u}_h^T(t)]^T. \quad (8)$$

In addition to the inputs, outputs, and their derivatives, we also give the user the ability to incorporate the measured trajectories in estimating the desired states. We therefore aim to model the user as an estimator of the form

$$\dot{\omega}(t) = N\omega(t) + J_1 Y_{0:\gamma}(t) + J_2 y_2(t) + HU_{0:\lambda}(t)$$
$$\hat{z}(t) = \omega(t - \tau_1) + EY_{0:\gamma}(t) \quad (9)$$

which produces delayed or nondelayed estimates of current or upcoming values of a desired functional of states. In (9), $\omega(t) \in \mathbb{R}^{l+\mathcal{X}}$ is the state of the estimator and $\tau_1$ is the estimation delay. It is desirable to determine a stable matrix $N$ and matrices $J_1$, $J_2$, $H$, and $E$ with compatible dimensions to make the estimation error asymptotically approach zero. From (9), it is clear that we only apply the delay term on the desired states, which need to be processed and estimated in the working memory—that is, the set of desired states, which are not directly available to the user.

Having the estimator (9) to estimate the functional (5) of the system (1), the prediction error is

$$\begin{aligned} e(t) &= \hat{z}(t) - z(t+\tau) \\ &= \omega(t-\tau_1) + EY_{0:\gamma}(t) - Lx(t+\tau) \\ &= \omega(t-\tau_1) + EO_\gamma x(t) + EM_{1,0:\gamma} U_{0:\gamma}(t) \\ &\quad + EM_{2,\gamma} r_a - Lx(t+\tau) \end{aligned} \quad (10)$$

with the error dynamics

$$\begin{aligned} \dot{e}(t) &= N\omega(t-\tau_1) + J_1 O_\gamma x(t-\tau_1) \\ &\quad + J_1 M_{1,0:\gamma} U_{0:\gamma}(t-\tau_1) + J_1 M_{2,\gamma} r_a \\ &\quad + J_2 D r_a + HU_{0:\lambda}(t-\tau_1) + EO_\gamma Ax(t) \\ &\quad + EO_\gamma B u_h(t) + EO_\gamma F r_a \\ &\quad + EM_{1,0:\gamma} U_{1:\gamma+1}(t) - LAx(t+\tau) \\ &\quad + LBu_h(t+\tau) - LFr_a. \end{aligned} \quad (11)$$

Note that in (11), by setting $\tau = 0$, we obtain the error dynamics for the observation and by setting $\tau > 0$ we obtain the error dynamics for the prediction.

Since in general the delay and the value of prediction horizon are small, we can write

$$\begin{aligned} u_h(t) &= u_h(t-\tau_1) + \tau_1 \dot{u}_h(t-\tau_1) \\ u_h(t+\tau) &= u_h(t-\tau_1) + (\tau+\tau_1)\dot{u}_h(t-\tau_1) \\ &\quad + \tau\tau_1 \ddot{u}(t-\tau_1). \end{aligned} \quad (12)$$

The states of the continuous time system (1) evolve as

$$\begin{aligned} x(t+\tau) &= e^{A\tau} x(t) + c \\ x(t) &= e^{A\tau_1} x(t-\tau_1) + c_1 \end{aligned} \quad (13)$$

where

$$c = \int_t^{t+\tau} e^{A(t+\tau-T)} [B \;\; F] \begin{bmatrix} u_h(T) \\ r_a \end{bmatrix} dT$$
$$c_1 = \int_{t-\tau_1}^{t} e^{A(t-T)} [B \;\; F] \begin{bmatrix} u_h(T) \\ r_a \end{bmatrix} dT. \quad (14)$$

By introducing the variables

$$\begin{aligned} \delta_1 &\triangleq (e^{A\tau} - I)A^{-1}B \\ \delta_2 &\triangleq ((e^{A\tau} - I)A^{-1} - \tau I)A^{-1}B \\ \delta_3 &\triangleq (e^{A\tau} - I)A^{-1}F \\ \theta_1 &\triangleq (e^{A\tau_1} - I)A^{-1}B \\ \theta_2 &\triangleq ((e^{A\tau_1} - I)A^{-1} - \tau_1 I)A^{-1}B \\ \theta_3 &\triangleq (e^{A\tau_1} - I)A^{-1}F \\ \eta_1 &\triangleq e^{A(\tau+\tau_1)} \\ \eta_2 &\triangleq \delta_3 + e^{A\tau}\theta_3 \\ \eta_3 &\triangleq \delta_1 + e^{A\tau}\theta_1 \\ \eta_4 &\triangleq \delta_2 + \tau_1\delta_1 + e^{A\tau}\theta_2 \\ \eta_5 &\triangleq \tau_1\delta_2 \end{aligned} \quad (15)$$

and from (12) and (13), we can write

$$\begin{aligned} x(t) &= e^{A\tau_1} x(t-\tau_1) + \theta_1 u(t-\tau_1) + \theta_2 \dot{u}(t-\tau_1) + \theta_3 r_a \\ x(t+\tau) &= \eta_1 x(t-\tau_1) + \eta_2 u(t-\tau_1) + \eta_3 \dot{u}(t-\tau_1) \\ &\quad + \eta_4 \ddot{u}(t-\tau_1) + \eta_5 r_a. \end{aligned} \quad (16)$$

Hence, under the assumption that $\gamma$ and $\lambda$ are selected from $\{0, 1\}$, the error dynamics can be written as

$$\begin{aligned}\dot{e}(t) = &Ne(t) + (NL\eta_1 - LA\eta_1 \\ &+ [E\ J_1\ K\ J_2\ H]Q_{1,1})x(t-\tau_1) + (NL\eta_2 - LA\eta_2 \\ &+ [E\ J_1\ K\ J_2\ H]Q_{1,2} - LF)r_a + (NL\eta_3 - LA\eta_3 \\ &+ [E\ J_1\ K\ J_2\ H]Q_{1,3} - LB)u(t-\tau_1) + (NL\eta_4 \\ &- LA\eta_4 + [E\ J_1\ K\ J_2\ H]Q_{1,4} - LB(\tau+\tau_1)) \\ &\times \dot{u}(t-\tau_1) + (NL\eta_5 - LA\eta_5 + [E\ J_1\ K\ J_2\ H]Q_{1,5} \\ &- LB\tau\tau_1)\ddot{u}(t-\tau_1) \end{aligned} \quad (17)$$

where $K \triangleq J_1 - NE$ and $Q_{1,i}$ are defined in (18), as shown at the bottom of the page. for $i \in \{1, \dots, 5\}$

In (18), $M_{1,0} = 0_{p_x \times m_b}$ and $M_{1,1} = \begin{bmatrix} 0 \\ CB \end{bmatrix}$.

From (17), a $\tau_1$-delayed estimator exists and can be designed to estimate the desired functional $z(t+\tau)$ if and only if there exists a set $(E, J_1, N, J_2, H)$ , where $H \triangleq [H_a \quad H_b]$, with a stable $N$ to always satisfy

$$\begin{aligned} NL\eta_1 + [E\ J_1\ K\ J_2\ H]Q_{1,1} &= Q_{2,1} \\ NL\eta_2 + [E\ J_1\ K\ J_2\ H]Q_{1,2} &= Q_{2,2} \\ NL\eta_3 + [E\ J_1\ K\ J_2\ H]Q_{1,3} &= Q_{2,3} \\ NL\eta_4 + [E\ J_1\ K\ J_2\ H]Q_{1,4} &= Q_{2,4} \\ NL\eta_5 + [E\ J_1\ K\ J_2\ H]Q_{1,5} &= Q_{2,5} \end{aligned} \quad (19)$$

with $Q_{2,i}$ defined in (20), as shown at the bottom of the page. for $i \in \{1, \dots, 5\}$

In summary, using the above conditions, we can formulate the problem as follows. We seek to

1) evaluate the satisfaction of (19) for a desired task $T$, a given delay $\tau_1$, and a given amount of prediction horizon $\tau$ to determine whether it is possible for the user to attain SA regarding the desired task and thus make correct decisions toward its accomplishment.
2) obtain the model of the user by solving (19) for $N$ and $\begin{bmatrix} E & J_1 & K & J_2 & H \end{bmatrix}$ (which also satisfy $K = J_1 - NE$).
3) seek the triplet $(C, D, R)$ (if there exists any), with a minimum cardinality of (C,D), which satisfies conditions in (19) to determine the required information to be displayed. Note that, we define the cardinality of (C,D) as $rank(C) + rank(D)$.

## III. Methodology

In Section III-A, we derive the existence conditions of an extended delayed/nondelayed functional observer/predictor of form (9) for the system (1). We suggest a design method for such an estimator in Section III-B.

### A. Existence Conditions for an Extended Functional Estimator

For LTI systems under shared-control and assuming availability of the derivatives of the inputs and outputs, we obtain the necessary and sufficient conditions for the existence of a delayed/nondelayed functional observer and predictor.

Recall that we consider a full row rank functional $Lx(t+\tau)$, with $Lx(t+\tau) = \begin{bmatrix} T \\ R \end{bmatrix} x(t+\tau)$, whose components are $Tx(t+\tau)$ and $Rx(t+\tau)$. Therefore, reconstructing $Lx(t+\tau)$ is sufficient for the reconstruction of the desired task $Tx(t+\tau)$. Our goal is to investigate the existence of and then design an observer of form (9) to reconstruct the functional $Lx(t+\tau)$. Mathematically, this is equivalent to finding a solution for (19).

*Lemma 1:* There exists a solution for (19) iff the following two conditions are simultaneously satisfied:

$$\begin{bmatrix} E & J_1 & K & J_2 & H \end{bmatrix} T_1 = T_2 \quad (21)$$

where $T_1 = Q_1 M_E$ and $T_2 = Q_2 M_E$ and $M_E$ is from (23)

$$N = Q_{2,i} H_i - \begin{bmatrix} E & J_1 & K & J_2 & H \end{bmatrix} Q_{1,i} H_i \quad \text{for } i \in \{1, \dots, 5\} \quad (22)$$

where $H_i$ is such that $L\eta_i H_i = I$, for $i \in \{1, \dots, 5\}$.

$$Q_1 \triangleq [\ Q_{1,1} \ |\ Q_{1,2} \ |\ Q_{1,3} \ |\ Q_{1,4} \ |\ Q_{1,5}\ ]$$

$$\triangleq \begin{bmatrix} O_\gamma A e^{A\tau_1} & O_\gamma(A\theta_3 + F) & O_\gamma(A\theta_1 + B) & O_\gamma(A\theta_2 + \tau_1 B) + M_{1,\gamma} & \tau_1 M_{1,\gamma} \\ O_\gamma(I - e^{A\tau_1}) & -O_\gamma\theta_3 & -O_\gamma\theta_1 & -O_\gamma\theta_2 - \tau_1 M_{1,\gamma} & 0 \\ O_\gamma e^{A\tau_1} & O_\gamma\theta_3 + M_{2,\gamma} & O_\gamma\theta_1 + M_{1,\gamma} & O_\gamma\theta_2 + \tau_1 M_{1,\gamma} & 0 \\ 0 & D & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & \lambda I & 0 \end{bmatrix} \quad (18)$$

$$Q_2 \triangleq [\ Q_{2,1} \ |\ Q_{2,2} \ |\ Q_{2,3} \ |\ Q_{2,4} \ |\ Q_{2,5}\ ]$$
$$\triangleq [\ LA\eta_1 \ |\ L(A\eta_2 + F) \ |\ L(A\eta_3 + B) \ |\ L(A\eta_4 + B(\tau + \tau_1)) \ |\ L(A\eta_5 + B\tau\tau_1)\ ]. \quad (20)$$

*Proof:* By selecting $E_i$s to satisfy $L\eta_i E_i = 0$ and $H_i$s defined earlier, we can define a full row-rank matrix

$$S_1 = \left[ M_H \mid M_E \right] \quad (23)$$

where

$$M_E = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 \\ 0 & 0 & E_3 & 0 & 0 \\ 0 & 0 & 0 & E_4 & 0 \\ 0 & 0 & 0 & 0 & E_5 \end{bmatrix}$$

$$M_H = \begin{bmatrix} H_1 & 0 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 & 0 \\ 0 & 0 & H_3 & 0 & 0 \\ 0 & 0 & 0 & H_4 & 0 \\ 0 & 0 & 0 & 0 & H_5 \end{bmatrix}. \quad (24)$$

Given that $S_1$ is of full row rank, postmultiplication of $S_1$ in (19) will not change the results. As a result of this postmultiplication, (21) and (22) are obtained to be an equivalent expression to (19).

In order for a stable solution for (19) to exist, a stable matrix $N$ and matrices $[E\ J_1\ K\ J_2\ H]$ have to exist to satisfy both (21) and (22).

Clearly, there exists a solution for (21) iff $\text{span}({T_2}^T) \subseteq \mathcal{R}({T_1}^T)$—that is,

$$rank \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = rank \left[ T_1 \right]. \quad (25)$$

*Proposition 1:* The condition (21) is satisfied iff

$$rank(LHS_1) = rank(RHS) \quad (26)$$

where

$$RHS \triangleq \begin{bmatrix} & & Q_1 & & \\ L\eta_1 & L\eta_2 & L\eta_3 & L\eta_4 & L\eta_5 \end{bmatrix} \quad (27)$$

and

$$LHS_1 \triangleq \begin{bmatrix} Q_2 \\ RHS \end{bmatrix}. \quad (28)$$

*Proof:* We can postmultiply $S_1$ from (23) in (27) and (28) to obtain

$$\begin{aligned} rank(RHS) &= rank(RHS \times S_1) \\ &= rank(L) + rank(T_1) \end{aligned} \quad (29)$$

and

$$\begin{aligned} rank(LHS_1) &= rank(LHS_1 \times S_1) \\ &= rank(L) + rank\left( \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right) \end{aligned} \quad (30)$$

respectively. From (29) and (30), we can show that $rank(RHS) = rank(LHS_1)$ iff $rank \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = rank \left[ T_1 \right]$. Thus, (26) is the necessary and sufficient condition for the existence of the solution to (21).

*Proposition 2:* The condition in (22) is satisfied, with a stable $N$, iff the following conditions are simultaneously satisfied.

1) For all $s \in \mathbb{C}$,

$$rank(LHS_{2,i}) = rank(RHS) \quad (31)$$

where $LHS_{2,i}$ is formulated in (32), as shown at the bottom of the page. In (32), $M_{E,i:j}$ is a block diagonal portion of $M_E$, defined in (24), which only contains $E_k$ on its diagonal, where $k \in \{1, \ldots, j\}$. In (31), $i \in \{1, \ldots, 5\}$

$$\begin{aligned}
LHS_{2,1} &\triangleq \begin{bmatrix} sL\eta_1 - LA\eta_1 & -Q_{2,2} & -Q_{2,3} & -Q_{2,4} & -Q_{2,5} \\ \hdashline & & Q_1 & & \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & M_{E,2:5} \end{bmatrix} \\
LHS_{2,2} &\triangleq \begin{bmatrix} -Q_{2,1} & sL\eta_2 - LA\eta_2 & -Q_{2,3} & -Q_{2,4} & -Q_{2,5} \\ \hdashline & & Q_1 & & \end{bmatrix} \begin{bmatrix} E_1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & M_{E,3:5} \end{bmatrix} \\
LHS_{2,3} &\triangleq \begin{bmatrix} -Q_{2,1} & -Q_{2,2} & sL\eta_3 - LA\eta_3 & -Q_{2,4} & -Q_{2,5} \\ \hdashline & & Q_1 & & \end{bmatrix} \begin{bmatrix} M_{E,1:2} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & M_{E,4:5} \end{bmatrix} \\
LHS_{2,4} &\triangleq \begin{bmatrix} -Q_{2,1} & -Q_{2,2} & -Q_{2,3} & sL\eta_4 - LA\eta_4 & -Q_{2,5} \\ \hdashline & & Q_1 & & \end{bmatrix} \begin{bmatrix} M_{E,1:3} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & E_5 \end{bmatrix} \\
LHS_{2,5} &\triangleq \begin{bmatrix} -Q_{2,1} & -Q_{2,2} & -Q_{2,3} & -Q_{2,4} & sL\eta_4 - LA\eta_4 \\ \hdashline & & Q_1 & & \end{bmatrix} \begin{bmatrix} M_{E,1:4} & 0 \\ 0 & I \end{bmatrix}.
\end{aligned} \quad (32)$$

when $\tau \neq 0$ and $\tau_1 \neq 0$, $i = 1$ when $\tau = 0$ and $\tau_1 = 0$, and $i \in \{1, \ldots, 4\}$ when $\tau = 0$ and $\tau_1 \neq 0$.

2) When $\tau_1 \neq 0$ and/or $\tau \neq 0$, there exists a $Z$ for which $(\Lambda_1 - Z\Gamma_1)$ has negative eigenvalues with acceptable magnitude

$$(\Lambda_1 - \Lambda_i) - Z(\Gamma_1 - \Gamma_i) = 0 \tag{33}$$

for $i \in \{2, \ldots, 5\}$ when $\tau_1 \neq 0$ and for $i \in \{2, \cdots, 4\}$ when $\tau = 0$.
In (33), $\Gamma_i$ and $\Lambda_i$ are

$$\begin{aligned}\Lambda_i &= Q_{2,i}H_i - T_2T_1^+Q_{1,i}H_i\\ \Gamma_i &= (I - T_1{T_1}^+)Q_{1,i}H_i.\end{aligned}$$

*Proof:* The solution to (21) has the form of

$$\begin{bmatrix} E & J_1 & K & J_2 & H\end{bmatrix} = T_2{T_1}^+ + Z(I - T_1{T_1}^+) \tag{34}$$

for an arbitrary matrix $Z$ with compatible dimension.

1) Proof of (31).
From (22) and (34), $N$ can be written as $N = \Lambda_i - Z\Gamma_i$ where

$$\begin{aligned}\Lambda_i &= Q_{2,i}H_i - T_2{T_1}^+Q_{1,i}H_i\\ \Gamma_i &= (I - T_1{T_1}^+)Q_{1,i}H_i.\end{aligned} \tag{35}$$

The eigenvalues of $N$ can be placed at any desired values iff $rank\begin{bmatrix} sI - \Lambda_i\\ \Gamma_i\end{bmatrix} = l + \mathcal{X}\ \forall s \in \mathbb{C}$.
We first introduce some required matrices to complete the proof. Choose full-row rank matrices

$$S_{a,1} = \begin{bmatrix} H_1 & E_1 & 0\\ \hline 0 & 0 & I\end{bmatrix}$$

$$S_{a,i} = \begin{bmatrix} 0 & I & 0 & 0\\ \hline H_i & 0 & E_i & 0\\ \hline 0 & 0 & 0 & I\end{bmatrix} \quad \text{for } i \in \{2,3,4\}$$

$$S_{a,5} = \begin{bmatrix} 0 & 0 & I\\ \hline H_1 & E_1 & 0\end{bmatrix} \tag{36}$$

a full column rank matrix

$$S_b = \begin{bmatrix} I & T_2{T_1}^+\\ 0 & (I - T_1{T_1}^+)\\ 0 & T_1{T_1}^+\end{bmatrix} \tag{37}$$

and a full row rank matrix

$$S_{c,i} = \begin{bmatrix} I & 0\\ {T_1}^+Q_{1,i}H_i & I\end{bmatrix} \quad \text{for } i \in \{1, \ldots, 5\}. \tag{38}$$

For each feasible value of $i$, by first postmultiplying $S_{a,i}$ in $LHS_{2,i}$ from (32) and then premultiplying $S_b$ and postmultiplying $S_{c,i}$ in the resulted matrix, the rank does not change and we have

$$rank(LHS_{2,i}) = rank\begin{bmatrix} sI - \Lambda_i\\ \Gamma_i\end{bmatrix} + rank[T_1]. \tag{39}$$

Comparing (39) and (29) and having $rank(L) = l + \mathcal{X}$, (31) is satisfied iff $rank\begin{bmatrix} sI - \Lambda_i\\ \Gamma_i\end{bmatrix} = l + \mathcal{X}\ \forall s \in \mathbb{C}$.

2) Proof of (33).
In proof of (31) we showed that, for each feasible value of $i$, the eigenvalues of matrix $N$ can be selected to have a desired values depending on a matrix $Z$ if $rank(LHS_{2,i}) = rank(RHS)$.
After satisfaction of (31), according to $N = \Lambda_i - Z\Gamma_i$, required is a common pair $(N, Z)$ with an stable $N$, which can satisfy the above equation for all feasible values of $i$. Thus, for any feasible pair of $(i, j)$, it is necessary to have $\Lambda_i - Z\Gamma_i = \Lambda_j - Z\Gamma_j$, which is the proof to (33).

From Lemma 1 and Propositions 1 and 2, we can introduce Theorem 1 on the existence of a stable delayed functional estimator to estimate the current and upcoming desired functional of the states of a system of interest.

*Theorem 1:* For a system of the form (1), with $\gamma \in \{0, 1\}$ available derivatives of the outputs and $\lambda \in \{0, 1\}$ available derivatives of the inputs, there exists an estimator of the form (9) to make

1) nondelayed observations, with $\tau_1 = 0$ and $\tau = 0$,
2) delayed observations, with $\tau_1 \neq 0$ and $\tau = 0$,
3) nondelayed predictions, with $\tau_1 = 0$ and $\tau \neq 0$,
4) delayed predictions, with $\tau_1 \neq 0$ and $\tau \neq 0$,

iff, a functional $Rx$ exists to extend the desired task functional; as is defined in (5), to satisfy the condition (26) in Proposition 1 and conditions (31) and (33) in Proposition 2 as well as to satisfy the following condition:

1) When $\tau_1 \neq 0$, there exists a pair $(N, Z)$, with $Z$ satisfying (33) and $N$ being a stable matrix, to hold

$$N(T_{a,1} + ZT_{b,1}) = (T_{a,2} - T_{a,3}) + Z(T_{b,2} - T_{b,3}) \tag{40}$$

where

$$\begin{aligned}\begin{bmatrix} T_{a,1} & T_{a,2} & T_{a,3} & T_{a,4}\end{bmatrix} &\triangleq T_2{T_1}^+\\ \begin{bmatrix} T_{b,1} & T_{b,2} & T_{b,3} & T_{b,4}\end{bmatrix} &\triangleq (I - T_1{T_1}^+)\end{aligned} \tag{41}$$

and have compatible dimensions ($T_{a,4}$ and $T_{b,4}$ have $(p_r + (\lambda + 1)B)$ columns, and $T_{a,i}$ and $T_{b,i}$ have equal number of columns for $i \in \{1, 2, 3\}$).
In (41), $T_1 = Q_1M_E$ and $T_2 = Q_2M_E$ where $Q_1$ and $Q_2$ are obtained from (18) and (20). In (41), ${T_1}^+$ is the pseudoinverse of $T_1$.

*Proof:* The necessary and sufficient condition for the existence of a delayed/nondelayed estimator of form (9) to reconstruct the functional $Tx(t + \tau)$ of the system (1) is the existence of a matrix $R$ and a stable matrix $N$ to satisfy (19). This problem can be considered as two subproblems: 1) existence of a stable solution for (19), and 2) satisfaction of the condition $J_1 = K + NE$.

In the proofs of Propositions 1 and 2, we have already showed that the satisfaction of (26) is necessary and sufficient for the existence of solution to (21) and that satisfaction of (31) and (33) are necessary and sufficient for the existence of the solution to (22).

In addition to the existence of the solution for (21) and (22) as well as the stability of matrix $N$, it is required to choose $N$ to satisfy $J_1 \triangleq K + NE$.

When $\tau_1 = 0$, from (15), $\theta_i = 0$ which will result in $Q_{1,2} = 0$. Thus, $J_1$ can be selected arbitrarily and it will be straightforward to obtain $N$ to satisfy $J_1 \triangleq K + NE$.

On the other hand, when $\tau_1 \neq 0$, $J_1$ cannot be selected arbitrarily. Therefore, we need to select the pair $(N, Z)$ such that $N$ is stable and $J_1 = K + NE$. Recall that, from (34), the selection of $Z$ will affect the values of $[E\ J_1\ K\ J_2\ H]$. Equation (40) is obtained by plugging in (34) to $J_1 = K + NE$.

### B. Design Procedure for the Functional Estimator (9)

Assuming the existence of the estimator (9) to reconstruct the functional $Tx(t+\tau)$ of the system (1), we can determine the related estimator matrices $N$, $J_1$, $J_2$, $H$, and $E$ for delayed/nondelayed observation/prediction as follows:

1) To design a nondelayed observer (i.e., for $\tau = 0$ and $\tau_1 = 0$),
   a) Find a matrix $R$ to satisfy (26) and (31), then form the matrix $L$.
   b) Choose $H_1$ and $E_1$ such that $LH_1 = I$ and $LE_1 = 0$, respectively.
   c) Calculate $\Lambda_1$ and $\Gamma_1$ from (35).
   d) Choose $Z$ to have a stable $N = \Lambda_1 - Z\Gamma_1$ and obtain $J_2$, $E$, $K$, and $H$ from (34).
   e) Based on $N$, $E$, and $K$, calculate $J_1$ from $J_1 = K + NE$.
2) To design a delayed observer (i.e., for $\tau = 0$ and $\tau_1 \neq 0$),
   a) Find matrices $R$, $N$, and $Z$ to simultaneously satisfy (26), (31), (33), and (40), then form the matrix $L$.
   b) Based on $Z$, obtain $J_1$, $J_2$, $E$, $K$, and $H$ from (34).
3) To design a nondelayed predictor (i.e., for $\tau \neq 0$ and $\tau_1 = 0$),
   a) Find matrices $R$, $N$, and $Z$ to simultaneously satisfy (26), (31), and (33), then form the matrix $L$.
   b) Determine $J_2$, $E$, $K$, and $H$ from (34).
   c) Based on $N$, $E$, and $K$, calculate $J_1$ from $J_1 = K + NE$.
4) To design a delayed predictor (i.e., for $\tau \neq 0$ and $\tau_1 \neq 0$),
   a) Find matrices $R$, $N$, and $Z$ to simultaneously satisfy (26), (31), (33), and (40), then form the matrix $L$.
   b) Based on $Z$, obtain $J_1$, $J_2$, $E$, $K$, and $H$ from (34).

### C. Example

In this example, we validate our method for designing a delayed/nondelayed observer/predictor to estimate a desired functional. We assume $\gamma = 1$ (i.e., having access to the first derivative of the outputs) and $\lambda = 0$ (i.e., no derivative of the low-level input). Our goal is to evaluate the existence of a delayed-observer and a delayed-predictor to reconstruct $Tx(t+\tau)$, where $T = \begin{bmatrix} 0\ 1\ 1 \end{bmatrix}^T$ and then design such an estimator.

Consider a system of the form (1) with the following system matrices:

$$A = \begin{bmatrix} 0 & 1 & -10 \\ -2 & -3 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$
$$F = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \tag{42}$$

and $C = I_{3\times 3}$—that is, all states are measured. Besides, the reference trajectory is assumed to be available in the user interface $D = 1$. Given the system dynamics in (42), with all states being measured, the system is observable and predictable. However, based on our earlier discussion, the standard observability and predictability of the system is not enough for a human operator to accomplish the desired task. Under the conditions of Theorem 1, we can evaluate whether the human can attain SA about the task $Tx$.

For the system (42), the conditions in Theorem 1 for the existence of the delayed and nondelayed estimator, where $\tau \in \{0, 0.2\}$ and $\tau_1 \in \{0, 0.3\}$ are satisfied. Therefore, based on the technique suggested in Section III-B, we can design delayed/nondelayed observer/predictor for this system to reconstruct $Tx(t+\tau)$.

1) Nondelayed observer ($\tau = 0$ and $\tau_1 = 0$):
   Using the design procedure suggested in Section III-B, we can design a nondelayed observer as is illustrated in

$$\begin{bmatrix} N \\ J_2 \\ H \end{bmatrix} = \begin{bmatrix} -4.2204 \\ 0.0546 \\ 1.2624 \end{bmatrix}, \quad J_1 = \begin{bmatrix} -0.8084 \\ -0.2323 \\ -4.5347 \\ -0.1637 \\ -0.9632 \\ 0.3564 \end{bmatrix}^T$$
$$E = \begin{bmatrix} 0.2515 \\ 0.2605 \\ 1.0778 \\ 0.1342 \\ 0.3544 \\ -0.0000 \end{bmatrix}^T, \quad K = \begin{bmatrix} 0.2530 \\ 0.8672 \\ 0.0139 \\ 0.4028 \\ 0.5326 \\ 0.3564 \end{bmatrix}^T. \tag{43}$$

Fig. 2 shows the effectiveness of using the designed observer in tracking the desired functional of the states of system (42), while the observer has no delay.

We now use the designed observer (same matrices as above) to predict our desired functional, while the actual observer is delayed ($\tau_1 = 0.3$ and $\tau = 0.2$). The results are available in Fig. 3.

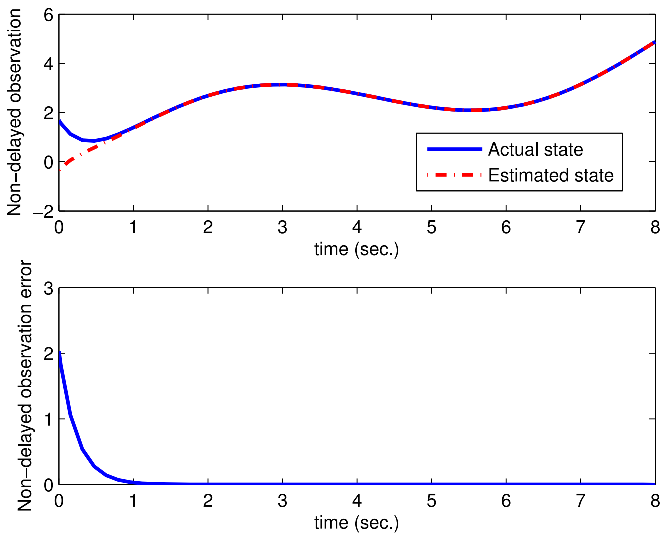


Fig. 2. Nondelayed observation of the desired functional $Tx$ of the states of the system (42).

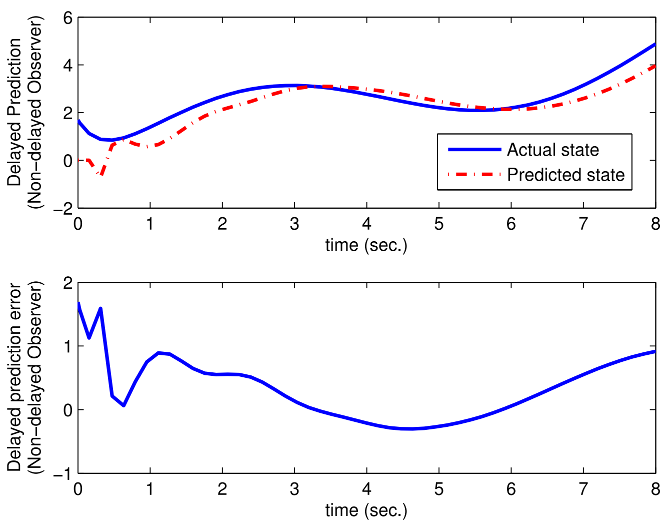


Fig. 3. Using a nondelayed observer matrices for predicting $Tx$, while the actual observer has also internal delay.

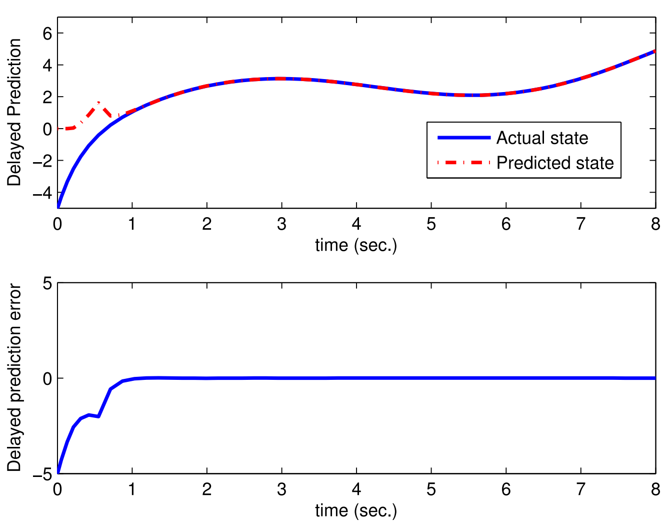


Fig. 4. Delayed prediction of the desired functional $Tx$ of the states of the system (42), with $\tau = 0.2$ and $\tau_1 = 0.3$.

From Fig. 3, it can be seen that the nondelayed observer matrices are not effective for precisely predicting a desired functional, while the structure of the actual observer is delayed too. Hence, a new estimator have to be designed to provide us with our desired results.

2) Delayed-predictor ($\tau = 0.2$ and $\tau_1 = 0.3$):
From the algorithm provided in Section III-B, we can design a delayed predictor for the reconstruction of the functional $Tx$. The predictor structure is provided as follows:

$$\begin{bmatrix} N \\ J_2 \\ H \end{bmatrix} = \begin{bmatrix} -8.4719 \\ 0.1800 \\ 1.8711 \end{bmatrix}, \quad J_1 = \begin{bmatrix} 0.6541 \\ 0.2090 \\ 0.3986 \\ -0.1792 \\ -0.4676 \\ -1.1273 \end{bmatrix}^T$$

$$E = \begin{bmatrix} -0.2100 \\ 0.1348 \\ 0.0114 \\ -0.0730 \\ 0.1040 \\ 0.2905 \end{bmatrix}^T, \quad K = \begin{bmatrix} -1.1250 \\ 1.3510 \\ 0.4956 \\ -0.7978 \\ 0.4136 \\ 1.3335 \end{bmatrix}^T. \tag{44}$$

Fig. 4 illustrates the simulation results of such a predictor. From this figure, it is possible to design a delayed estimator for system (42) to predict the desired functional $Tx$, which confirms the results of Theorem 1.
Since the prediction starts when $t \geq \tau$, it will result in the discontinuity observed in Fig. 4.

## IV. Application

Attaining SA is indispensable for an anesthetist to maintain the safety of the anesthetized patient [5], [48], [49]. According to [48], after perceiving the available displayed information and the information from the environment, the anesthetist has to integrate all the available data for the identification of the current and the future desired patient states. The estimation of the current states of the system is important for goal accomplishment and for fault detection. As is mentioned in [48], task prediction is also extremely important for the anesthetist to be proactive rather than just being reactive.

In order to model a patient under anesthesia, understanding the relationship between the dose of the drug and its pharmacological effect is necessary. This model consists of two submodels, the pharmacokinetic (PK) and the pharmacodynamic (PD) models. The PK model demonstrates the effect of the administered drug on the drug plasma concentration, and the PD model models the relationship between the drug concentration in the effect site and the observed effect of the drug.

We consider a simplified version of the PKPD model described in [50] and [51] to model the effect of drug

TABLE I
PKPD COEFFICIENT FOR A 21 YEARS OLD PATIENT WITH WEIGHT 100 kg

| | | | | | |
|---|---|---|---|---|---|
| $k_{10}$ | 0.0524 | $k_{12}$ | 0.2359 | $k_{13}$ | 0.0162 |
| $k_{21}$ | 0.0892 | $k_{31}$ | 0.0022 | $k_d$ | 0.1335 |
| $V_1$ | 0.3593 | $EC_{50}$ | 3.2 | $\gamma_h$ | 4.7 |

administration on the depth of hypnosis. The PK model in [51] is the well-known three-compartment model developed in [52] to evaluate the effect of propofol on the drug concentration in different compartments. Pharmacokinetically, a compartment is considered to be a group of tissues, which have similar kinetic characteristics. A three-compartment model has three states, i.e., concentrations in 1) the blood and highly perfused tissues (e.g., brain and liver), 2) the muscles and viscera, and 3) fat and bones. The PD model presented in [51] consists of three states, two of which are associated with the dynamics of the monitor. We, however, only consider the effect site concentration of the drug and linearize the Hill equation to obtain the depth of hypnosis based on this state of the system.

Considering no transport delay, the PKPD model for evaluating the depth of anesthesia is as (1) with

$$A = \begin{bmatrix} & A_{pk} & & 0 \\ k_d & 0 & 0 & -k_d \end{bmatrix}$$

$$B = \begin{bmatrix} B_{pk} \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0_{4\times 1} \end{bmatrix} \tag{45}$$

where

$$A_{pk} = \begin{bmatrix} -(k_{10}+k_{12}+k_{13}) & k_{12} & k_{13} \\ k_{21} & -k_{21} & 0 \\ k_{31} & 0 & -k_{31} \end{bmatrix}$$

$$B_{pk} = \begin{bmatrix} V_1^{-1} \\ 0 \\ 0 \end{bmatrix}. \tag{46}$$

In (45) and (46), $k_{ij}$ and $k_d$ are rate constants and $V_1$ is the volume of the plasma compartment.

The desired task which is controlling the depth of anesthesia can be defined as $Tx = \begin{bmatrix} 0 & 0 & 0 & \gamma_h(4EC_{50})^{-1} \end{bmatrix} x$, where $EC_{50}$ is the 50% effect concentration and $\gamma_h$ is the cooperativity coefficient. The values that we are using are presented in Table I.

From Theorem 1, no delayed predictor for system (45) can reconstruct the desired functional $Tx(t+\tau)$ when the measurement in the display is restricted to the depth of hypnosis $C = \begin{bmatrix} 0\ 0\ 0\ \gamma_h(4EC_{50})^{-1} \end{bmatrix}$.

We assume that, during the entire operation, the operating condition, i.e., PKPD model and coefficients, remains constant as well as the desired task of the anesthetist which is just to keep track of the DOA. Hence, it is possible to simply determine the displayed information in advance rather than the real-time determination of the information. We assume that unless a specific linear combination of the states has a physical meaning to the user, no linear combinations of states and no linear combinations of the trajectories are presented in the interface (i.e., $C$ and $D$ are often diagonal with the elements on the diagonal being zero or one). Therefore, for such a case, it is possible to determine the required displayed information by manipulating the matrices $C$ and $D$ and verifying the satisfaction of the conditions in Theorem 1, heuristically.

Based on the above discussion, it can be seen that for an estimator of the form (9) with $\tau_1 = 0.3$ estimation delay, it is necessary and sufficient to measure the blood-plasma drug concentration in addition to the depth of hypnosis in order to make correct and precise observations and predictions of the desired functional—that is, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_h(4EC_{50})^{-1} \end{bmatrix}$.

Hence, from the above analysis, we can conclude that it is not possible for anesthetists to precisely predict the effect of the drug administration on the depth of hypnosis unless they are provided with both the depth of hypnosis and the plasma concentration through the display. Unfortunately, plasma concentration measurement is beyond current state of technology and thus cannot be provided. In practice, open-loop population-based PKPD models are used to estimate and display both $C_p$ and $C_e$. However, due to significant interpatient variability, these estimates come with such large uncertainties that they are not likely to substitute for real measurements, hence hindering the ability of the anesthetist to accurately predict the depth of hypnosis. As future work, we are planning to study the effect of those uncertainties on the ability to predict the depth of hypnosis with a bounded error.

## V. DISCUSSION

In this paper, we focused on mathematical modeling of the process of attaining SA for the user. We considered the user to be a functional observer and a functional predictor whose estimations of the states of the system are delayed with possible knowledge about the derivatives of their own inputs and of the outputs. For a system that is controlled by the user and that tracks a desired reference trajectory with the aid of a computer, we presented a technique to evaluate whether it is possible to reconstruct and predict (with delay) a desired set of states given a set of displayed measurements. In addition to obtaining the existence conditions for the extended delayed functional observer/predictor with the availability of higher order derivatives of the inputs and the outputs, we presented a procedure to design such an estimator.

The proposed framework may provide a useful tool to the designers and system analysts for investigating the safety of the system during early stages of design. As this is one of the first attempts to model the process of attaining SA mathematically, it could serve as a foundation for more complicated models. Future directions should consider: 1) modifying our suggested human model to provide estimates within an acceptable interval, as opposed to precise estimates, 2) determining a rigorous technique for obtaining the required display information, and 3) considering the process under uncertainties (e.g., with parameter uncertainties and/or noise).

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