Dynamic Self-Triggered Impulsive Synchronization of Complex Networks With Mismatched Parameters and Distributed Delay

Dong Ding, Ze Tang, Ju H. Park, *Senior Member, IEEE*, Yan Wang, and Zhicheng Ji

***Abstract*—Synchronization of complex networks with nonlinear couplings and distributed time-varying delays is investigated in this article. Since the mismatched parameters of individual systems, a kind of leader-following quasisynchronization issues is analyzed via impulsive control. To acquire appropriate impulsive intervals, the dynamic self-triggered impulsive controller is devoted to predicting the available instants of impulsive inputs. The proposed controller ensures the control effects while reducing the control costs. In addition, the updating laws of the dynamic parameter is settled in consideration of error bounds to adapt to the quasisynchronization. With the utilization of the Lyapunov stability theorem, comparison method, and the definition of average impulsive interval, sufficient conditions for realizing the synchronization within a specific bound are derived. Moreover, with the definition of average impulsive gain, the parameter variation scheme is extended from the fixed impulsive effects case to the time-varying impulsive effects case. Finally, three numerical examples are given to show the effectiveness and the superiority of proposed mathematical deduction.**

***Index Terms*—Dynamic self-triggered mechanism, distributed time-varying delay, extended parameter variation scheme, quasisynchronization.**

I. Introduction

IN LAST few decades, synchronization, as one of the most important collaborative phenomena, has attracted lots of scholars' interests. Since the rapid development in this field, complete synchronization has been well studied until now. In terms of delay types, proportional delay, a kind of unbounded delays, and its influence on synchronization were studied for the first time [1]; in field of convergence velocities, the unified stability analysis of finite time and fix time was well investigated with the utilization of the Filippov theorem [2]; in view of network topologies, cluster pinning synchronization of dynamical networks, which were composed of switched systems, was addressed via event-based strategy [3]. However, in our daily life, there occur very few situations to achieve complete synchronization. Therefore, other forms of synchronization including projective synchronization [4], quasisynchronization [15], [17], [27], [34], and secure synchronization [5] have attracted the attention of scholars. Among these synchronization phenomena, quasisynchronization has received extensive attention because of its value in industrial applications. In production activities, even for the same machine model, due to the differences in service experiences and maintenance methods, the mathematical description of their system models will be inconsistent. From the perspective of control theorem, the problem can be viewed as parameter mismatch [6], [7]. Consequently, these machines are difficult to achieve complete synchronization and can only maintain consistence within a certain error range, which motivates scholars to conduct research on quasisynchronization.

Although different types of controllers, such as feedback controllers [8] and intermittent controllers [21], [22], show the capabilities of facilitating synchronous behavior, the adjacency information are invariably needed. That is, the information exchanges in the communication channels are continuously active or piecewise continuously active [8], [9]. In other words, the above two methods occupy lots of communication resources during operation. In order to reduce the active instants of communication channels, an effective discrete control method called impulsive control is obviously more favorable [14], [15], [25]–[28].

Actually, impulsive control scheme has its advantages on control efficiency, and there still leaves the possibility of improvements in terms of cost saving. Consider the characteristic that the impulsive controller is only activated at a few of impulse instants, and many recent investigations have been done on designing the conditions for its activations. For instance, a kind of communication protocols was designed in [10] to force error vectors to converge to zero by the transient information exchange among different nodes in a complex network. Moreover, it was proved that bounded

Manuscript received 24 January 2022; accepted 17 April 2022. Date of publication 13 May 2022; date of current version 13 January 2023. This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFB1701903; in part by the National Natural Science Foundation of China under Grant 61803180 and Grant 61973138; in part by the Natural Science Foundation of Jiangsu Province under Grant BK20180599; in part by the China Postdoctoral Science Foundation Funded Project under Grant 2020M681484; and in part by the 111 Project under Grant B12018. The work of Dong Ding was supported by the China Scholarship Council (CSC). The work of Ju H. Park was supported by the National Research Foundation of Korea under Grant 2019R1A5A8080290. This article was recommended by Associate Editor C.-Y. Su. *(Corresponding authors: Ju H. Park; Zhicheng Ji.)*

Dong Ding, Ze Tang, Yan Wang, and Zhicheng Ji are with the Key Laboratory of Advanced Process Control for Light Industry, Ministry of Education and the School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China (e-mail: 6171905029@stu.jiangnan.edu.cn; zcjisci@gmail.com).

Ju H. Park is with the Department of Electrical Engineering, Yeungnam University, Gyeongsan 38541, Republic of Korea (e-mail: jessie@ynu.ac.kr).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCYB.2022.3168854.

Digital Object Identifier 10.1109/TCYB.2022.3168854

synchronization can be ensured by proposing an event-based quantized communication protocol in [11]. In [29], a decentralized event-triggered condition containing the current sampling vector and the last sampling vector is constructed to address the drive-response synchronization. Although the above mentioned impulsive schemes could guarantee the efficiency of control effects by designing suitable triggering conditions, unfortunately, most of the event-triggered mechanisms require real-time monitoring, which runs counter to the original intention of the impulsive controller (see [10], [11], [38], and references therein). Even though in [7] and [12], a discrete event-triggered mechanism was proposed, which reduced the number of monitoring times through periodic communication. Regrettably, the selection of the monitoring intervals still relies on experience rather than quantitative conditions.

In addition, the sensors and actuators are ordinarily equipped with limited resources in the actual production process. Therefore, proposing a kind of efficiency and energy-saving methods for both controlling and monitoring [42] is the priority work. Since it was proved in [13] that a complex network could achieve complete synchronization by utilizing appropriate coupling strength, then it is reasonable to rely on a suitable coupling strength of the complex network to keep the errors between each system and the target state within a certain range if possible. Naturally, it is intuitionally to design a controller that can effectively reduce the state differences between the control objects and the target in the early stage of system evolution, and as time goes on, the control inputs could be gradually decreased. Recent advances in event-based mechanisms, especially the self-triggered mechanisms [23], [24], [35], make the monitoring and controlling in low cost to be possible. For example, by setting the triggering condition with an exponential function, a self-triggered impulsive controller whose impulsive effects obeyed certain normal distribution was proposed in [23] to synchronize the multiagent systems. Even though some complete synchronization theorems were derived by such kinds of event-based mechanisms, how self-triggered impulsive controller contributes to the quasisynchronization remains poorly understood.

As we all knew that time delays caused by information transmission in the communication channels are usually regarded as an adverse factor on synchronization. To model the transmission delays, studies on time delays existed in the systems or the couplings, or even in both simultaneously have been well studied [14]–[19]. To proceed, scholars have considered that delays may remain in the procession from receiving the instructions to the execution of the controllers, including the intermittent controllers, the impulsive controllers, and so on [19], [20]. In terms of the diversity of the delay types, although the coupling time delay was considered in [14], the extent to which distributed delay has deep relevance to synchronization is still unclear. In addition, coupled delays and distributed delays should be considered as with different sizes rather than the same [37]. Furthermore, considering the spatial characteristics of complex networks, it is particularly critical to investigate the influence of distributed time delays on synchronization results, such as error bound and convergence velocity, in another word, delay-dependent criteria should be given.

Motivated by above-mentioned issues, this article intends to study the quasisynchronization for a class of complex networks with hybrid time-varying delays, including coupling delay and distributed delay. Noticeably, the extended parameter variation method is introduced to the case with time-varying impulsive effects. This article differs from some previous works mainly in the following three aspects.

1) This article studies a leader-following issue for a class of complex networks with nonlinear couplings, distributed time-varying delay, and mismatched parameters. Comparing with [14]–[17], the model analyzed in this article is more desirable.
2) By choosing an appropriate triggering function, the impulsive instants are predicted. Accordingly, the active time duration of monitors will be greatly reduced comparing with the event-triggered mechanism. Also, the dynamic self-triggered mechanism is proved to be more flexible than the static self-triggered mechanism [24] in reducing the number of triggering instants. It is worth mentioning that since the quasisynchronization is studied in this article rather than the complete synchronization as in [23], we will show how the dynamic self-triggering parameter is adopted with the improved adaptive updating laws.
3) Comparing with some previous literatures, which discussed the synchronizing and desynchronizing functions corresponding to different ranges of fixed impulsive effects [25]–[27], the time-varying impulsive effects are considered in this article. The extended parameter variation method, the definition of average impulsive gain, and the definition of average impulsive interval and comparison principle are jointly introduced and applied in order to derive some less conservative quasisynchronization results.

Notations: $\mathbb{R}^n$ represents the n-dimensional Euclidean space in this article. The notation T denotes the transpose of a matrix or a vector. $\mathfrak{N}$ represents the set of nonnegative integers. $\lambda(\mathcal{Q})$ represents the eigenvalues of matrix $\mathcal{Q}$. I_n is equivalent to the identity matrix of order n. Inequality $X < (\leq) 0$ is used to denote a real negative-definite (negative-semidefinite) matrix. $\|\gamma\| = \sqrt{\gamma^T\gamma}$ is used to present norm of vector γ. Notation $\mathcal{N}_i$ stands for the set of nodes, which are directly connected to the ith node. In following context, dimensions of matrices and vectors will be explained specifically.

II. Model Selection and Preliminaries

In this section, the mathematical model of complex networks will be given first. Then, some useful definitions, lemmas, and assumptions will be introduced. In addition, the Zeno behavior will be discussed.

A. Model Description

Consider the ith follower system whose dynamic property is described by

$$\dot{z}_i(t) = -\Upsilon_{A_i} z_i(t) + \Upsilon_{B_i} \tilde{h}(z_i(t)) + u_i(t) + b \sum_{j=1}^{N} c_{ij} \Gamma_1 \tilde{g}(z_j(t - \tau_1(t))) + d \sum_{j=1}^{N} \psi_{ij} \Gamma_2 \int_{t-\tau_2(t)}^{t} z_j(s) ds \quad (1)$$

where $z_i(t) = [z_i^1(t), z_i^2(t), \ldots, z_i^n(t)]^T \in \mathbb{R}^n$ denotes the state vector of the ith system for $i = 1, 2, \ldots, N$. Υ_{A_i} and $\Upsilon_{B_i} \in \mathbb{R}^{n\times n}$ are the connected weight matrices of the ith system. Nonlinear functions $\tilde{h}(\cdot)$ and $\tilde{g}(\cdot)$ represent some processions, which cannot be observed directly. Positive matrices $\Gamma_1 = \text{diag}\{\gamma_1^1, \gamma_1^2, \ldots, \gamma_1^n\}$ and $\Gamma_2 = \text{diag}\{\gamma_2^1, \gamma_2^2, \ldots, \gamma_2^n\} \in \mathbb{R}^{n\times n}$ are inner coupling matrices. Matrices $C = (c_{ij})_{N\times N}$ and $\Psi = (\psi_{ij})_{N\times N}$ are outer coupling matrices. It should be declared that c_{ij} and $\psi_{ij} > 0$ if there is a connection from the jth system to the ith system, otherwise, c_{ij} and $\psi_{ij} < 0$. In addition, C and Ψ are assumed to be diffusive, that is, $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$ and $\psi_{ii} = -\sum_{j=1, j\neq i}^{N} \psi_{ij}$. Positive scalars b and d are the coupling strengths of coupling matrices C and Ψ, correspondingly. $0 < \tau_1(t), \tau_2(t) \leq \tilde{\tau}$ are time-varying delays, which satisfy $0 < \dot{\tau}_1(t), \dot{\tau}_2(t) < 1$. The detailed design of controller $u_i(t)$ will be introduced in the following part.

The leader system is modeled as

$$\dot{s}(t) = -\Upsilon_{A_s} s(t) + \Upsilon_{B_s} \tilde{h}(s(t)) \quad (2)$$

where $s(t) = [s^1(t), s^2(t), \ldots, s^n(t)]^T \in \mathbb{R}^n$. Υ_{A_s} and $\Upsilon_{B_s} \in \mathbb{R}^{n\times n}$ are the connected weight matrices of the leader system. Moreover, to design a suitable controller, the following three steps are essential.

1) *Control Scheme:* Impulsive controller $u_i(t)$ and measurement error $\hat{e}_i(t)$ are adopted by

$$u_i(t) = \kappa \sum_{k=1}^{\infty} \hat{e}_i(t) \beta\left(t - t_k^i\right) \quad (3)$$

$$\hat{e}_i(t) = \sum_{j\in\mathcal{N}_i} \xi_{ij}\left(z_j(t) - z_i(t)\right) - w_i(z_i(t) - s(t)) \quad (4)$$

where $i = 1, 2, \ldots, N$, constants κ and w_i denote the impulsive effect and the control gains of the ith node. $\beta(t - t_k^i)$ stands for the Dirac impulsive function. $\Xi = (\xi_{ij})_{N\times N}$ is the distributed coupling matrix, which satisfies $\xi_{ij} > 0$ if there exist control signals sent from the jth to the ith system and $\xi_{ii} = -\sum_{j=1, j\neq i}^{N} \xi_{ij}$.

2) *Triggering Condition:* To determine the next triggering instant t_{k+1}^i, the following condition is proposed:

$$t_{k+1}^i = \inf\left\{t > t_k^i : \varphi_i(t) - \mu_i(t) > 0\right\} \quad (5)$$

where $\varphi_i(t) = \|\hat{e}_i(t_k)\|^2 (e^{\rho(t - t_k^i)} - 1) - \theta\|\hat{e}_i(t_k)\|^2 - e^{-l(t-t_0)}$ and ρ, θ, and l are positive constants. From (5), it can be observed that $\|\hat{e}_i(t_k)\|^2 (e^{\rho(t-t_k^i)} - 1)$ will reset to zero at every $t = t_k$, which leads to a negative $\varphi_i(t_k)$ at beginning of each impulsive interval $[t_k, t_{k+1})$. Consequently, the controller will not be triggered continuously. The setting of variable $\mu_i(t)$ will be discussed soon. $\{t_k^i\}$ satisfies $0 = t_0^i < t_1^i < \cdots < t_k^i < \ldots$ For later usage, all triggering instants are ordered as $\{t_k\} = \bigcup_{i=1}^{N}\{t_w^i\}$, $w \in \mathfrak{N}$. The new impulse sequence satisfies $t_0 < t_1 < \cdots < t_k < \ldots$, $k \in \mathfrak{N}$.

3) *Updating Laws:* The dynamic self-triggered function $\mu_i(t)$ will be updated by

$$\dot{\mu}_i(t) = -\epsilon_1 \mu_i(t) - \epsilon_2 \varphi_i(t) + \epsilon_3 e_b \quad (6)$$

where ϵ_1, ϵ_2, and ϵ_3 are positive constants and $e_b \in \{e_{b_1}, e_{b_2}, e_{b_3}\}$ denotes the error bound, which will be discussed next.

By jointly considering (1)–(6) and defining the error vector as $e_i(t) = z_i(t) - s(t)$, the following result could be derived:

$$\begin{cases} e_i(t) = -\Upsilon_{A_i} e_i(t) + \Upsilon_{B_i} h(e_i(t)), & t \neq t_k \\ \quad +b\sum_{j=1}^{N} c_{ij}\Gamma_1 g(e_j(t - \tau_1(t))) + \mathcal{Y}_i(s(t)) & \\ \quad +d\sum_{j=1}^{N} \psi_{ij}\Gamma_2 \int_{t-\tau_2(t)}^{t} e_j(s)ds & \\ \Delta e_i(t_k) = e_i\left(t_k^+\right) - e_i\left(t_k^-\right), & t = t_k \\ \quad = \kappa\left(\sum_{j=1}^{N} \xi_{ij} e_j\left(t_k^-\right) - w_i e_i\left(t_k^-\right)\right) & \end{cases} \quad (7)$$

where $k \in \mathfrak{N}$, $\mathcal{Y}_i(s(t)) = -(\Upsilon_{A_i} - \Upsilon_{A_s})s(t) + (\Upsilon_{B_i} - \Upsilon_{B_s})\tilde{h}(s(t))$, $h(e_i(t)) = \tilde{h}(z_i(t)) - \tilde{h}(s(t))$, and $g(e_j(t - \tau_1(t))) = \tilde{g}(z_j(t - \tau_1(t))) - \tilde{g}(s(t - \tau_1(t)))$.

Remark 1: In some previous literature, such as [25] and [28], the impulsive control protocols were applied by combining the continuous negative feedback control protocol to increase the control efficiency and speed up the synchronization procession. However, during the actual production process, it is difficult to let continuous actuators and discrete actuators receive the same type of signals. Therefore, as it can be seen from (3) that only the impulsive control protocol is considered in this article. As it is known to us that the impulsive intervals are key points in the impulsive controller. A question remains about the triggering strategy of impulses. Therefore, a type of event-based rather than time-based control strategy [36], [37] is considered in this article. Although both the event-triggered and the self-triggered mechanisms have shown their capabilities on providing assistance with discrete control signal inputs, that is, determining appropriate impulsive intervals and saving control costs [7], [10], [11], [24], the event-triggered mechanism requires continuous monitoring [38], which is contrary to the intention of saving control costs. Therefore, by designing a suitable self-triggering condition, this article establishes a kind of self-triggered impulsive controllers, which only occupies communication channels in very few instants. It is worth mentioning that the event-triggered mechanism will be applied if substituting $\|\hat{e}_i(t_k)\|$ with $\|\hat{e}_i(t)\|$ in (5).

B. Related Preliminaries

Definition 1 [30]: Non-Zeno behavior is declared for a kind of self-triggered impulsive controllers if there exists a scalar $\mathcal{S} > 0$ satisfying that

$$t_{k+1} - t_k \geq \mathcal{S}, \ k \in \mathfrak{N}$$

holds for any instants belonging to time sequence $\{t_k\}$.

Definition 2: Quasisynchronization between follower systems (1) and leader system (2) is said to be achieved if error state vector $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T$ converges exponentially below the bound of an existed set $\mathcal{B} = \{e(t) \in \mathbb{R}^{Nn} | \|e(t)\| \leq e_b\}$ with any initial value $e_i(0)$ for $t \to +\infty$.

Definition 3 [31]: In this article, the number of impulses available times will be estimated in two ways, therefore, two following definitions of average impulsive interval are given.

1) For the sequence $\zeta = \{t_0, t_1, \ldots\}$, with assumption that average impulsive interval is less than T_a, number of impulsive times N_ζ can be estimated by

$$\frac{T-t}{T_a} - N_0 \leq N_\zeta(T,t) \leq \frac{T-t}{T_a} + N_0 \ \forall T \geq t \geq 0 \tag{8}$$

where T_a and N_0 are positive scalars.

2) With no more assumption, the average impulsive interval T_a can be simply defined by

$$T_a = \lim_{t \to +\infty} \frac{t - t_0}{N_\zeta(t, t_0)}. \tag{9}$$

Definition 4 [32]: Consider time series $\{t_k\}$ that impulsive controller will be triggered at every instant t_k. $\kappa(t_1), \kappa(t_2), \ldots, \kappa(t_{N_\zeta})$ are the time-varying impulsive effects in respect to instants $t_1, t_2, \ldots, t_{N_\zeta}$. Then, define $N_\zeta(t, t_0)$ as the number of impulses from instant t_0 to t, it gives us

$$\bar{\kappa} = \lim_{t \to +\infty} \frac{|\kappa(t_1)| + |\kappa(t_2)| + \cdots + |\kappa(t_{N_\zeta})|}{N_\zeta(t, t_0)}, \ k \in \mathfrak{N}. \tag{10}$$

Lemma 1 [33]: Suppose $V(t)$ is continuous at $t \neq t_k$, $t \geq 0$, and $V(t)$ satisfies:

1) $V(t_k^-) = \lim_{t \to t_k^-} V(t)$ exists;
2) $V(t_k) = V(t_k^+) = \lim_{t \to t_k^+} V(t)$;
3) $V(t) \in PC([t_0 - \tilde{\tau}, t_0], \mathbb{R})$

if there exist scalars ε, χ, and r, for two functions $V_a(t)$ and $V_b(t)$ whose properties are same with $V(t)$, such that

$$\begin{cases} D^+V_a(t) \leq -\varepsilon V_a(t) + \chi\left(\sup_{t-\tau_1 \leq s \leq t} V_a(s)\right) \\ \qquad\qquad + d\int_{t-\tau_2(t)}^{t} V_a(s)ds, \ t \neq t_k, \ t \geq t_0 \\ V_a(t_k^+) \leq rV_a(t_k^-), \ \ t = t_k, \ t \in [t_0 - \tilde{\tau}, t_0] \end{cases} \tag{11}$$

$$\begin{cases} D^+V_b(t) > -\varepsilon V_b(t) + \chi\left(\sup_{t-\tau_1 \leq s \leq t} V_b(s)\right) \\ \qquad\qquad + d\int_{t-\tau_2(t)}^{t} V_b(s)ds, t \neq t_k, \ t \geq t_0 \\ V_b(t_k^+) = rV_b(t_k^-), \ \ t = t_k, \ t \in [t_0 - \tilde{\tau}, t_0] \end{cases} \tag{12}$$

$k \in \mathfrak{N}$, then, there holds $V_a(t) \leq V_b(t)$ for $t > 0$ if $V_a(t) \leq V_b(t)$ holds for $-\tilde{\tau} \leq t \leq 0$.

Assumption 1: For n-dimensional vectors $\mathcal{A}_1$ and $\mathcal{A}_2$, the nonlinear Lipschitz functions $\tilde{h}(\cdot)$ and $\tilde{g}(\cdot)$ satisfy the following inequalities:

$$\left\|\tilde{h}(\mathcal{A}_1) - \tilde{h}(\mathcal{A}_2)\right\| \leq \eta_1 \|\mathcal{A}_1 - \mathcal{A}_2\|$$
$$\|\tilde{g}(\mathcal{A}_1) - \tilde{g}(\mathcal{A}_2)\| \leq \eta_2 \|\mathcal{A}_1 - \mathcal{A}_2\|$$

where positive scalars η_1 and η_2 are the linearization coefficients.

C. Analysis of Non-Zeno Behavior

Non-Zeno behavior should be declared before stability discussion. According to (5), we have

$$\left\|\hat{e}_i(t_k)\right\|^2 \left(e^{\rho(t-t_k^i)} - 1\right) - \theta \left\|\hat{e}_i(t_k)\right\|^2 - e^{-l(t-t_0)} - \mu_i(t) > 0. \tag{13}$$

Since the target is to let systems be synchronized within a bound, variable $\|\hat{e}_i(t_k)\|^2$ will not converge to 0, it can be easily obtained that

$$t_{k+1}^i - t_k^i > \frac{1}{\rho} \ln\left(1 + \theta + \frac{\mu_i(t_k^i)}{\left\|\hat{e}_i(t_k^i)\right\|^2}\right) > 0 \tag{14}$$

which demonstrates that Zeno behavior is excluded with the designed control protocol.

Remark 2: Different from many works focused on the static self-triggered mechanism, dynamic self-triggered mechanism is studied by introducing a time-varying function $\mu_i(t)$. Since the quasisynchronization is considered rather than complete synchronization [1]–[3], the dynamic self-triggered function $\mu_i(t)$ is monopoly decreasing and may converge to zero, that is, the dynamic self-triggered mechanism will be invalid and it turns to be a static self-triggered mechanism as discussed in [35]. Therefore, the error bound e_b is introduced to establish a nonmonopoly updating law on the dynamic self-triggered scheme. It should be noticed that the updating laws in [23] could be thought as one of the special cases in our paper. It can be found that $\dot{\mu}_i(t)$ is closely related to $\varphi_i(t)$, which indicates that a larger value of measurement error would lead to a smaller $\mu_i(t)$. As a consequence, the impulsive controller would be easier to be triggered. On the contrary, the triggering intervals would get larger with an enhanced $\mu_i(t)$. This is the way how the dynamic self-triggering function $\mu_i(t)$ coordinates the relationship between the energy consumption and the triggering efficiency.

III. Quasisynchronization via Dynamic Self-Triggered Impulsive Control

In this section, more detailed conditions for achieving the quasisynchronization of a kind of complex networks with hybrid time-varying delays via designed control scheme will be given. First, fixed impulsive effects with different functions will be discussed, and then, time-varying impulsive effects will be analyzed. We will show how the parameter variation method is extended in this research.

A. Self-Triggered Scheme With Synchronizing and Desynchronizing Impulsive Effects

In this part, the quasisynchronization via dynamic self-triggered impulsive controller will be investigated. By introducing the comparison systems, the convergence rate and the error bound are specifically estimated.

Theorem 1: Consider the complex dynamical networks (1) satisfying Assumption 1. The average impulsive interval is less than T_a as defined in Definition 3-1). If there exist positive constants η_1, η_2, π, and α_i such that as the following.

Case I:

1) Matrix

$$\begin{bmatrix} -rI_{Nn} & \kappa(\Xi - W)\otimes I_n + I_{Nn} \\ * & -I_{Nn} \end{bmatrix} < 0. \tag{15}$$

2) $r \in (0, 1)$, that is, $\lambda(\Xi - W) \in (-[2/\kappa], 0)$.

3) Inequality

$$r^{N_0}\left(\frac{\ln r}{T_a} - \varepsilon\right) + \left(\chi + d\pi\tau_2^2\right) < 0 \tag{16}$$

then the controlled error network (7) globally and exponentially converges to the following compact set:

$$\mathcal{B}_1 = \left\{ e(t) \in \mathbb{R}^{Nn} \middle| \|e(t)\| \le e_{b1} = \sqrt{\frac{\sum_{i=1}^{N} \alpha_i q_i^2}{-r^{N_0}\left(\frac{\ln r}{T_a} - \varepsilon\right) - \left(\chi + d\pi\tau_2^2\right)}} \right\} \tag{17}$$

with convergence velocity $(\lambda/2)$, where $\varepsilon = -\lambda_{\max}\{-2\Upsilon_A + 2\eta_1\Upsilon_B + b(C\otimes\Gamma_1)(C\otimes\Gamma_1)^T + d(\Psi\Psi^T)\otimes(\Gamma_2\pi^{-1}\Gamma_2) + I_{Nn}\}$, $\chi = b\eta_2$, $\Upsilon_A = I_N \otimes \Upsilon_{A_i}$, $\Upsilon_B = I_N \otimes \Upsilon_{B_i}$, and λ is the unique solution of equation $\lambda + ([\ln r/T_a] - \varepsilon) + r^{-N_0}(\chi \exp(\lambda\tau_1) + d\pi\tau_2([\exp(\lambda\tau_2) - 1]/\lambda) = 0$.

Case II: If condition 1) in case I is satisfied and 2') $r \in (1, +\infty)$, that is, $\lambda(\Xi - W) \in (-\infty, -(2/\kappa)) \cup (0, +\infty)$; and 3') inequality

$$r^{-N_0}\left(\frac{\ln r}{T_a} - \varepsilon\right) + \left(\chi + d\pi\tau_2^2\right) < 0 \tag{18}$$

then the controlled error network (7) globally and exponentially converges to the following compact set:

$$\mathcal{B}_2 = \left\{ e(t) \in \mathbb{R}^{Nn} \middle| \|e(t)\| \le e_{b2} = \sqrt{\frac{\sum_{i=1}^{N} \alpha_i q_i^2}{-r^{-N_0}\left(\frac{\ln r}{T_a} - \varepsilon\right) - \left(\chi + d\pi\tau_2^2\right)}} \right\} \tag{19}$$

with convergence velocity $(\acute{\lambda}/2)$, where $\varepsilon = -\lambda_{\max}\{-2\Upsilon_A + 2\eta_1\Upsilon_B + b(C\otimes\Gamma_1)(C\otimes\Gamma_1)^T + d(\Psi\Psi^T)\otimes(\Gamma_2\pi^{-1}\Gamma_2) + I_{Nn}\}$, $\chi = b\eta_2$, $\Upsilon_A = I_N \otimes \Upsilon_{A_i}$, $\Upsilon_B = I_N \otimes \Upsilon_{B_i}$, and $\acute{\lambda}$ is the unique solution of equation $\acute{\lambda} + ([\ln r/T_a] - \varepsilon) + r^{N_0}(\chi \exp(\acute{\lambda}\tau_1) + d\pi\tau_2([\exp(\acute{\lambda}\tau_2) - 1]/\acute{\lambda})) = 0$.

Proof: Consider the following Lyapunov functional:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t),\ i = 1, 2, \ldots, N. \tag{20}$$

When $t \in [t_{k-1}, t_k)$, calculating the derivative of $V(t)$ gives

$$\begin{aligned}
\dot{V}(t) = &-\sum_{i=1}^{N} e_i^T(t)\Upsilon_{A_i}e_i(t) + \sum_{i=1}^{N} e_i^T(t)\Upsilon_{B_i}h(e_i(t)) \\
&+ b\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}e_i^T(t)\Gamma_1 g(e_j(t-\tau_1(t))) \\
&+ d\sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N}\psi_{ij}\Gamma_2\int_{t-\tau_2(t)}^{t} e_j(s)ds \\
&+ \sum_{i=1}^{N} e_i(t)\mathcal{Y}_i(s(t)) \\
\le &-e^T(t)\left(I_N \otimes \Upsilon_{A_i}\right)e(t) + \eta_1 e^T(t)\left(I_N \otimes \Upsilon_{B_i}\right)e(t) \\
&+ \frac{1}{2}be^T(t)(C\otimes\Gamma_1)(C\otimes\Gamma_1)^T e(t) \\
&+ \frac{1}{2}b\eta_2 e^T(t-\tau_1(t))e(t-\tau_1(t)) \\
&+ de^T(t)(\Psi\otimes\Gamma_2)\int_{t-\tau_2(t)}^{t} e(s)ds \\
&+ \frac{1}{2}e^T(t)e(t) + \frac{1}{2}\sum_{i=1}^{N}\mathcal{Y}_i^T(s(t))\mathcal{Y}_i(s(t))
\end{aligned} \tag{21}$$

where $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T$, $e(t-\tau_1(t)) = [e_1^T(t-\tau_1(t)), e_2^T(t-\tau_1(t)), \ldots, e_N^T(t-\tau_1(t))]^T$, and

$$\begin{aligned}
de^T(t)(\Psi\otimes\Gamma_2)\int_{t-\tau_2(t)}^{t} e(s)ds &\le \frac{1}{2}de^T(t) \\
&\times \left(\Psi\Psi^T \otimes \Gamma_2\pi^{-1}\Gamma_2\right)e(t) + \frac{1}{2}d\pi\tau_2\int_{t-\tau_2(t)}^{t} e^T(s)e(s)ds.
\end{aligned}$$

For the stable time T of the chaotic oscillator $s(t)$, define $\sup_{t>T}\|\mathcal{Y}_i(s(t))\|^2 = \alpha_i q_i^2$. In accordance with (20) and (21), it yields that

$$\begin{aligned}
\dot{V}(t) \le &\frac{1}{2}e^T(t)\Big(-2\Upsilon_A + 2\eta_1\Upsilon_B + b(C\otimes\Gamma_1)(C\otimes\Gamma_1)^T \\
&+ d\left(\Psi\Psi^T\right)\otimes\left(\Gamma_2\pi^{-1}\Gamma_2\right) + I_{Nn}\Big)e(t) \\
&+ \frac{1}{2}b\eta_2 e^T(t-\tau(t))e(t-\tau(t)) \\
&+ \frac{1}{2}d\tau_2\int_{t-\tau_2(t)}^{t} e^T(s)(I_N\otimes\pi)e(s)ds + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 \\
\le &\varepsilon V(t) + \chi V(t-\tau_1(t)) + \pi d\tau_2\int_{t-\tau_2(t)}^{t} V(s)ds \\
&+ \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2.
\end{aligned} \tag{22}$$

When $t = t_k$, we have

$$e\left(t_k^+\right) = \left(\kappa(\Xi - W)\otimes I_n + I_{Nn}\right)e\left(t_k^-\right). \tag{23}$$

Taking (23) into (20) gives following inequality:

$$\begin{aligned}
&(\kappa(\Xi - W)\otimes I_n + I_{Nn})^T(\kappa(\Xi - W)\otimes I_n + I_{Nn}) \\
&< rI_{Nn}.
\end{aligned} \tag{24}$$

Therefore, we have

$$V\left(t_k^+\right) \le rV\left(t_k^-\right). \tag{25}$$

To handle the comparison principle, the following comparison system, which is right-hand continuous, is intuitively constructed with considering the results in (22) and (25)

$$\begin{cases}
\dot{\varsigma}(t) = -\varepsilon\varsigma(t) + \chi\varsigma(t-\tau_1(t)) + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota \\
\qquad +d\pi\tau_2\int_{t-\tau_2(t)}^{t}\varsigma(s)ds,\ t \ne t_k,\ k \in \mathfrak{N} \\
\varsigma\left(t_k^+\right) = r\varsigma\left(t_k^-\right),\ t = t_k \\
\varsigma(0) = \sum_{i=1}^{N}\|f_i(0)\|^2,\ -\tilde{\tau} \le t \le 0.
\end{cases} \tag{26}$$

It can be further calculated that

$$\varsigma(t) = M(t,0)\varsigma(0) + \int_0^t M(t,s)\Bigg(\chi\varsigma(t-\tau_1(t)) + d\pi\tau_2 \times \int_{s-\tau_2(s)}^{s} \varsigma(x)dx + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)ds \quad (27)$$

where $M(t, s)$ is regarded as the Cauchy matrix of the following linear impulsive system:

$$\begin{cases} \dot{\varsigma}(t) = -\varepsilon\varsigma(t), \ t \neq t_k, \ k \in \mathfrak{N} \\ \varsigma(t_k^+) = r\varsigma(t_k^-), \ t = t_k. \end{cases} \quad (28)$$

Then, the quasisynchronization conditions with two different functions of impulsive effects will be discussed separately.

Case I (Synchronizing Impulsive Effects): In this case, impulsive effects are considered to be beneficial for synchronizing the coupled networks, that is, $0 < r < 1$. With the definition of average impulsive interval in Definition 3-1), it can be easily known

$$\begin{aligned} M(t,s) &= \exp(-\varepsilon(t-s))\prod_{s<t_k<t} r = \exp(-\varepsilon(t-s))r^{N_\zeta(t,s)} \\ &\leq \exp(-\varepsilon(t-s))r^{\frac{t-s}{T_a}-N_0} \\ &= r^{-N_0}\exp\Bigg(\bigg(\frac{\ln r}{T_a} - \varepsilon\bigg)(t-s)\Bigg). \end{aligned} \quad (29)$$

Substituting (29) into (27) derives

$$\begin{aligned} \varsigma(t) &= M(t,0)\varsigma(0) + \int_0^t M(t,s)\Bigg(\chi\varsigma(t-\tau_1(t)) \\ &\quad + d\pi\tau_2\int_{s-\tau_2(s)}^{s}\varsigma(x)dx + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)ds \\ &\leq r^{-N_0}\sum_{i=1}^{N}\|f_i(0)\|^2\exp\Bigg(\bigg(\frac{\ln r}{T_a} - \varepsilon\bigg)t\Bigg) \\ &\quad + \int_0^t r^{-N_0}\exp\Bigg(\bigg(\frac{\ln r}{T_a} - \varepsilon\bigg)(t-s)\Bigg) \\ &\quad \times \Bigg(\chi\varsigma(t-\tau_1(t)) + d\pi\tau_2\int_{s-\tau_2(s)}^{s}\varsigma(x)dx \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)ds. \end{aligned} \quad (30)$$

Define a function as $\vartheta(\lambda) = \lambda + (\ln r/T_a) - \varepsilon + r^{-N_0}\chi\exp(\lambda\tau_1) + r^{-N_0}d\pi\tau_2([\exp(\lambda\tau_2) - 1]/\lambda)$. One can easily obtain that $\vartheta(\lambda)$ is a continuous function for $\lambda \in \mathbb{R}$. By simple calculation, it can be derived that

$$\begin{aligned} \vartheta(0^+) &= \frac{\ln r}{T_a} - \varepsilon + r^{-N_0}\chi + \lim_{\lambda\to 0^+} r^{-N_0}d\pi\tau_2\frac{\exp(\lambda\tau_2)-1}{\lambda} \\ &= \frac{\ln r}{T_a} - \varepsilon + r^{-N_0}\chi + r^{-N_0}d\pi\tau_2^2 < 0. \end{aligned} \quad (31)$$

It can be noted that $\vartheta(0^+) < 0$ and $\vartheta(+\infty) > 0$. Calculate the derivative along parameter λ, and we have

$$\begin{aligned} \dot{\vartheta}(\lambda) &= 1 + \tau_1 r^{-N_0}\chi\exp(\lambda\tau_1) + d\pi\tau_2^2 r^{-N_0}\frac{\exp(\lambda\tau_2)}{\lambda} \\ &\quad - d\pi\tau_2 r^{-N_0}\frac{\exp(\lambda\tau_2)-1}{\lambda} \\ &= 1 + \tau_1 r^{-N_0}\chi\exp(\lambda\tau_1) + d\pi\tau_2 r^{-N_0} \\ &\quad \times \frac{\tau_2\lambda\exp(\lambda\tau_2) - \exp(\lambda\tau_2) + 1}{\lambda^2} > 0 \end{aligned} \quad (32)$$

which shows that $\vartheta(\lambda)$ is a monotonically increasing function, there exists at least one point $\lambda = \lambda_0$ letting $\vartheta(\lambda_0) = 0$.

Define $\omega = r^{-N_0}\sup_{-\tilde{\tau}\leq s\leq 0}\sum_{i=1}^{N}\|f_i(s)\|^2$. With $r \in (0, 1)$, $\lambda > 0$, for $t \in [-\tilde{\tau}, 0]$, it gives us

$$\begin{aligned} \varsigma(t) &\leq r^{-N_0}\sup_{-\tilde{\tau}\leq s\leq 0}\sum_{i=1}^{N}\|f_i(s)\|^2 \\ &< \omega\exp(-\lambda t) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{-r^{N_0}(\frac{\ln r}{T_a} - \varepsilon) - \chi - d\pi\tau_2^2}. \end{aligned} \quad (33)$$

It is hoped to extend the above inequality to $t > 0$. If it cannot be realized, then there at least exists a time instant $\hat{t}$ such that

$$\varsigma(\hat{t}) \geq \omega\exp(-\lambda\hat{t}) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{-r^{N_0}\Big(\frac{\ln r}{T_a} - \varepsilon\Big) - \chi - d\pi\tau_2^2}. \quad (34)$$

Denote $\sigma = \varepsilon - (\ln r/T_a)$, $\varrho = -r^{N_0}([\ln r/T_a] - \varepsilon) - \chi - d\pi\tau_2^2$. It implies from (30)–(34) that

$$\begin{aligned} \varsigma(\hat{t}) &\leq \omega\exp(-\sigma\hat{t}) + \int_0^{\hat{t}} r^{-N_0}\exp\Bigg(\bigg(\frac{\ln r}{T_a} - \varepsilon\bigg)(\hat{t}-s)\Bigg) \\ &\quad \times \Bigg(\chi\varsigma(s-\tau_1(s)) + d\pi\tau_2\int_{s-\tau_2(s)}^{s}\varsigma(x)dx \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)ds \\ &< \omega\exp(-\sigma\hat{t}) + \int_0^{\hat{t}} r^{-N_0}\exp(-\sigma(\hat{t}-s))\chi \\ &\quad \times \Bigg(\omega\exp(-\lambda(s-\tau_1(s))) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varrho}\Bigg)ds \\ &\quad + \int_0^{\hat{t}} r^{-N_0}\exp(-\sigma(\hat{t}-s))d\pi\tau_2 \\ &\quad \times \Bigg(\int_{s-\tau_2(s)}^{s}\Bigg(\omega\exp(-\lambda x) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varrho}\Bigg)dx\Bigg)ds \\ &\quad + \int_0^{\hat{t}} r^{-N_0}\exp(-\sigma(\hat{t}-s))\Bigg(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)ds \\ &\leq \omega\exp(-\sigma\hat{t}) + r^{-N_0}\chi\omega\exp(\tau_1\lambda)\exp(-\sigma\hat{t}) \\ &\quad \times \int_0^{\hat{t}}\exp((\sigma-\lambda)s)ds \\ &\quad + r^{-N_0}\chi\exp(-\sigma\hat{t})\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varrho}\int_0^{\hat{t}}\exp(\sigma s)ds \\ &\quad + \omega d\pi\tau_2 r^{-N_0}\frac{\exp(\lambda\tau_2)-1}{\lambda}\exp(-\sigma\hat{t})\int_0^{\hat{t}}\exp((\sigma-\lambda)s)ds \\ &\quad + d\pi\tau_2^2 r^{-N_0}\exp(-\sigma\hat{t})\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varrho}\int_0^{\hat{t}}\exp(\sigma s)ds \\ &\quad + r^{-N_0}\Bigg(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\Bigg)\exp(-\sigma\hat{t})\int_0^{\hat{t}}\exp(\sigma s)ds \end{aligned} \quad (35)$$

and inequality (35) can be rewritten into

$$\begin{aligned}\varsigma(\hat{t}) &\le \omega\exp(-\sigma\hat{t}) + \frac{\omega\exp(-\sigma\hat{t})}{\sigma-\lambda}r^{-N_0}\\ &\quad\times\left(\exp((\sigma-\lambda)\hat{t})-1\right)\left(\chi\exp(\tau_1\lambda)+d\pi\tau_2\frac{\exp(\tau_2\lambda)-1}{\lambda}\right)\\ &\quad + r^{-N_0}\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{\varrho}\left(\chi+d\pi\tau_2^2\right)\frac{1}{\sigma}\left(1-\exp(-\sigma\hat{t})\right)\\ &\quad + r^{-N_0}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota\right)\frac{1}{\sigma}\left(1-\exp(-\sigma\hat{t})\right). \qquad (36)\end{aligned}$$

According to mentioned solution $\vartheta(\lambda_0) = 0$ and condition (16), we have

$$\begin{aligned}\varsigma(\hat{t}) &\le \omega\exp(-\sigma\hat{t}) + \omega\left(\exp(-\lambda\hat{t})-\exp(-\sigma\hat{t})\right)\\ &\quad + \left(\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{\varrho} - r^{-N_0}\frac{1}{\sigma}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota\right)\right)\\ &\quad\times\left(1-\exp(-\sigma\hat{t})\right)\\ &\quad + r^{-N_0}\frac{1}{\sigma}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota\right)\left(1-\exp(-\sigma\hat{t})\right)\\ &= \omega\exp(-\lambda\hat{t}) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{\varrho}\left(1-\exp(-\sigma\hat{t})\right)\\ &\le \omega\exp(-\lambda\hat{t}) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{-r^{N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\left(\chi+d\pi\tau_2^2\right)} \qquad (37)\end{aligned}$$

which contradicts the assumption in (34). As a result, it indicates that the inequality (33) holds for $t \in [-\tilde{\tau}, +\infty]$. Therefore, one has

$$\begin{aligned}V(t) &= \frac{1}{2}e^T(t)e(t) \le \varsigma(t) \le \omega\exp(-\sigma\hat{t})\\ &\quad + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{-r^{N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\left(\chi+d\pi\tau_2^2\right)} \qquad (38)\end{aligned}$$

and it can be further rewritten into

$$\begin{aligned}\|e(t)\| &\le \sqrt{2r^{N_0}\sup_{-\tilde{\tau}\le s\le 0}\sum_{i=1}^{N}\|f_i(s)\|^2\exp\left(-\frac{\lambda}{2}t\right)}\\ &\quad + \sqrt{\frac{\sum_{i=1}^{N}\alpha_i q_i^2+2\iota}{-r^{N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\left(\chi+d\pi\tau_2^2\right)}}. \qquad (39)\end{aligned}$$

With $t \to +\infty$ and $\iota \to 0$, it can be further obtained that

$$\|e(t)\| \le e_{b1} = \sqrt{\frac{\sum_{i=1}^{N}\alpha_i q_i^2}{-r^{N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\left(\chi+d\pi\tau_2^2\right)}} \qquad (40)$$

which implies that the controlled error network (7) will eventually get stable via designed impulsive controller (3) and dynamic self-triggered mechanism (5) and (6) as $t \to +\infty$. Therefore, the proof of the leader-following synchronization issue with $0 < r < 1$ is finished. In addition, it should be stated that the exponential convergence rate is $(\lambda/2)$.

Case II (Desynchronizing Impulsive Effects): In this case, the impulsive effects are considered to be negative in realizing synchronization, that is, $r > 1$. With the definition of average impulsive interval in Definition 3-1), the Cauchy matrix now could be estimated as

$$\begin{aligned}M(t,s) &= \exp(\varepsilon(t-s))r^{N_\zeta(t,s)}\\ &\le r^{N_0}\exp\left(\left(\frac{\ln r}{T_a}-\varepsilon\right)(t-s)\right). \qquad (41)\end{aligned}$$

With similar mathematical operations in (30)–(37), one obtains

$$\begin{aligned}V(t) < \varsigma(t) &< \omega\exp\left(-\acute{\lambda}t\right)\\ &\quad + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2+\iota}{-r^{-N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\chi-d\pi\tau_2^2}. \qquad (42)\end{aligned}$$

As $\iota \to 0$, and with $t \to +\infty$, it follows from (42) that:

$$\|e(t)\| \le e_{b2} = \sqrt{\frac{\sum_{i=1}^{N}\alpha_i q_i^2}{-r^{-N_0}\left(\frac{\ln r}{T_a}-\varepsilon\right)-\left(\chi+d\pi\tau_2^2\right)}} \qquad (43)$$

which implies that the controlled error network (7) will eventually get stable via designed impulsive controller (3) and dynamic self-triggered mechanism (5), (6) as $t \to +\infty$. Therefore, the proof of the leader-following synchronization issue with $r > 1$ is finished. In addition, it should be stated that the exponential convergence rate is $(\acute{\lambda}/2)$. Above all, the proof of synchronization with synchronizing and desynchronizing impulses is completed. ■

Remark 3: To decrease the control costs, many previous studies about pinning impulsive control mainly focused on reducing the quantity of controlled nodes at different control instants. For example, the nodes in the complex networks were rearranged with regard to $\|e_i(t)\|$ in [36], and only the first few of them were chosen to be forced with impulses at each t_k. Furthermore, a node selection method, which makes the proportion of the pinned nodes number obey $\eta_k \in (0, 1)$, was proposed in [33], and it helps to adjust the pinning number better. However, regardless of either the pinning impulsive method, there brings a problem that it is not efficient enough to control only a few nodes with the largest errors if the initial value distribution of the nodes in a complex network is relatively scattered, and it reflects that the error norm of each node is large. Besides, since the purpose is to reduce the number of awakened controllers at a certain moment, it is reasonable to design a separate triggering law for each node. Therefore, from this point of view, the self-triggered control protocol in this article can also be viewed as a kind of pinning impulsive mechanisms.

Remark 4: Until now, many efforts have been made to study the influence of time delays on synchronization [15]–[17]. For instance, it is concluded in [37] that the system delay has few influences on synchronization if the size of delay is not too large. Therefore, this article only considers the coupled delay and distributed delay. As a result, it can be achieved from (16) and (18) that Theorem 1 is delay dependent. Compared with [34], not only the influence of the distributed time delay is taken into account in the upper bound of the estimated error e_{b1} and e_{b2} but also the influence of both delays on the convergence rate is skillfully analyzed. It can be found that a larger distributed delay always leads to a larger upper bound of synchronization error.

B. Self-Triggered Scheme With Average Impulsive Gain

In the above section, the quasisynchronization conditions have been derived with various functions of impulsive effects while in this section, a more general result based on time-varying impulsive effects will be discussed. In addition, we will show how the parameter variation is extended with the definition of average impulsive gain.

Consider that the impulsive controller (3) has the time-varying impulsive effects $\kappa(t_k)$, then the corresponding controlled error network (7) could be converted into the following form:

$$\begin{cases} e_i(t) = -\Upsilon_{A_i}e_i(t) + \Upsilon_{B_i}h(e_i(t)), \quad t \neq t_k \\ \quad +b\sum_{j=1}^{N} c_{ij}\Gamma_1\tilde{g}\big(e_j(t-\tau_1(t))\big) + \mathcal{Y}_i(s(t)) \\ \quad +d\sum_{j=1}^{N} \psi_{ij}\Gamma_2\int_{t-\tau_2(t)}^{t} e_j(s)ds \\ \Delta e_i(t_k) = e_i\big(t_k^+\big) - e_i\big(t_k^-\big), \quad t = t_k \\ \quad = \kappa(t_k)\Big(\sum_{j=1}^{N}\xi_{ij}e_j\big(t_k^-\big) - w_ie_i\big(t_k^-\big)\Big) \end{cases} \tag{44}$$

where $k \in \mathfrak{N}$.

Theorem 2: Consider the complex dynamical networks (1) satisfying Assumption 1. Suppose that the number of impulses is denoted by N_ζ as defined in Definition 3-2). If there exist positive constants η_1, η_2, π, and α_i such that:

1) $\bar{r} = |[r(t_1) + r(t_2) + \cdots + r(t_{N_\zeta})]]/N_\zeta(t,s)|$;

2) inequality

$$-\varepsilon\bar{r}^{-N_\zeta} + \chi + d\pi\tau_2^2 < 0 \tag{45}$$

then the controlled error network (44) globally and exponentially converges to the following compact set:

$$\mathcal{B}_3 = \left\{ e(t) \in \mathbb{R}^{Nn} \middle| \|e(t)\| \leq e_{b3} = \sqrt{\frac{\sum_{i=1}^{N}\alpha_i q_i^2}{\varepsilon\bar{r}^{-N_\zeta} - \big(\chi + d\pi\tau_2^2\big)}} \right\} \tag{46}$$

with convergence velocity $(\bar{\lambda}/2)$, where $r(t_k) = \lambda_{\max}\{(\kappa(t_k)(\Xi - W)\otimes I_n + I_{Nn})^T(\kappa(t_k)(\Xi - W)\otimes I_n + I_{Nn})\}$, $\varepsilon = -\lambda_{\max}\{-2\Upsilon_A + 2\eta_1\Upsilon_B + b(C\otimes\Gamma_1)(C\otimes\Gamma_1)^T + d(\Psi\Psi^T)\otimes(\Gamma_2\pi^{-1}\Gamma_2) + I_{Nn}\}$, $\chi = b\eta_2$, $\Upsilon_A = I_N \otimes \Upsilon_{A_i}$, $\Upsilon_B = I_N \otimes \Upsilon_{B_i}$, and $\bar{\lambda}$ is the unique solution of equation $\bar{\lambda} - \varepsilon + \chi\exp(\bar{\lambda}\tau_1) + d\pi\tau_2([\exp(\bar{\lambda}\tau_2) - 1]/\bar{\lambda}) = 0$.

Proof: For $t \neq t_k$, inequality (22) in the proof of Theorem 1 obviously holds. When $t = t_k$, because of the impulses with limited energy, it gives that

$$(\kappa(t_k)(\Xi - W)\otimes I_n + I_{Nn})^T(\kappa(t_k)(\Xi - W)\otimes I_n + I_{Nn}) < r(t_k)I_{Nn}. \tag{47}$$

Therefore, we have

$$V\big(t_k^+\big) \leq r(t_k)V\big(t_k^-\big). \tag{48}$$

Due to the time-varying impulsive effects, the Cauchy matrix $M(t,s)$ could be calculated as

$$\begin{aligned} M(t,s) &= \exp(-\varepsilon(t-s))\prod_{s<t_k<t} r(t_k) \\ &= \exp(-\varepsilon(t-s))\left|\frac{r(t_1) + r(t_2) + \cdots + r(t_{N_\zeta})}{N_\zeta(t,s)}\right|^{N_\zeta(t,s)} \\ &\leq \bar{r}^{N_\zeta(t,s)}\exp(-\varepsilon(t-s)). \end{aligned} \tag{49}$$

With the definition of $\bar{\omega} = \bar{r}^{N_\zeta}\sup_{-\tilde{\tau}\leq s\leq 0}\sum_{i=1}^{N}\|f_i(s)\|^2$, it gives

$$\begin{aligned} \varsigma(t) &\leq \bar{r}^{N_\zeta}\sup_{-\tilde{\tau}\leq s\leq 0}\sum_{i=1}^{N}\|f_i(s)\|^2 \\ &< \bar{\omega}\exp\big(-\bar{\lambda}t\big) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varepsilon\bar{r}^{-N_\zeta} - \big(\chi + d\pi\tau_2^2\big)} \end{aligned} \tag{50}$$

for $-\tilde{\tau} < t < 0$. It is hoped to extend the above inequality to any $t > 0$. If it cannot be realized, then assume that there at least exists a time instant $\hat{t}$ satisfying $t > \hat{t}$ such that

$$\varsigma\big(\hat{t}\big) \geq \bar{\omega}\exp\big(-\bar{\lambda}\hat{t}\big) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varepsilon\bar{r}^{-N_\zeta} - \big(\chi + d\pi\tau_2^2\big)}. \tag{51}$$

Define $\bar{\sigma} = \varepsilon$, $\bar{\varrho} = \varepsilon\bar{r}^{-N_\zeta} - (\chi + d\pi\tau_2^2)$. By considering $-\varepsilon + \bar{r}^{N_\zeta}\chi + \bar{r}^{N_\zeta}d\pi\tau_2^2 < 0$ and $\bar{\lambda} - \varepsilon + \bar{r}^{N_\zeta}\chi\exp(\bar{\lambda}\tau_1) + \bar{r}^{N_\zeta}d\pi\tau_2([\exp(\bar{\lambda}\tau_2) - 1]/\bar{\lambda}) = 0$, it gives

$$\begin{aligned} \varsigma\big(\hat{t}\big) &\leq \bar{\omega}\exp\big(-\bar{\sigma}\hat{t}\big) + \frac{\bar{\omega}\exp\big(-\bar{\sigma}\hat{t}\big)}{\bar{\sigma} - \bar{\lambda}}\big(\exp\big((\bar{\sigma} - \bar{\lambda})\hat{t}\big) - 1\big) \\ &\quad \times \left(\bar{r}^{N_\zeta}\chi\exp(\tau_1\bar{\lambda}) + \bar{r}^{N_\zeta}d\pi\tau_2\frac{\exp(\tau_2\bar{\lambda}) - 1}{\bar{\lambda}}\right) \\ &\quad + \bar{r}^{N_\zeta}\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\bar{\varrho}}\big(\chi + d\pi\tau_2^2\big)\frac{1}{\bar{\sigma}}\big(1 - \exp\big(-\bar{\sigma}\hat{t}\big)\big) \\ &\quad + \bar{r}^{N_\zeta}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\right)\frac{1}{\bar{\sigma}}\big(1 - \exp\big(-\bar{\sigma}\hat{t}\big)\big) \end{aligned} \tag{52}$$

and it further yields

$$\begin{aligned} \varsigma\big(\hat{t}\big) &\leq \bar{\omega}\exp\big(-\bar{\sigma}\hat{t}\big) + \bar{\omega}\big(\exp\big(-\bar{\lambda}\hat{t}\big) - \exp\big(-\bar{\sigma}\hat{t}\big)\big) \\ &\quad + \left(\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\bar{\varrho}} - \bar{r}^{N_\zeta}\frac{1}{\bar{\sigma}}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\right)\right) \\ &\quad \times \big(1 - \exp\big(-\bar{\sigma}\hat{t}\big)\big) \\ &\quad + \bar{r}^{N_\zeta}\frac{1}{\bar{\sigma}}\left(\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota\right)\big(1 - \exp\big(-\bar{\sigma}\hat{t}\big)\big) \\ &\leq \bar{\omega}\exp\big(-\bar{\sigma}\hat{t}\big) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varepsilon\bar{r}^{-N_\zeta} - \big(\chi + d\pi\tau_2^2\big)} \end{aligned} \tag{53}$$

which contradicts to the assumption in (51). Thus, it proves that inequality (50) still holds for any $t > 0$. According to definition of $V(t)$, one could obtain

$$\begin{aligned} V(t) &= \frac{1}{2}e^T(t)e(t) \leq \varsigma(t) \\ &\leq \bar{\omega}\exp\big(-\bar{\sigma}\hat{t}\big) + \frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + \iota}{\varepsilon\bar{r}^{-N_\zeta} - \big(\chi + d\pi\tau_2^2\big)}. \end{aligned} \tag{54}$$

Algorithm 1 Procedure for Quasisynchronization via Dynamic Self-Triggered Impulsive Control Scheme

Require:

Initial states of each node $z_i(0)$, initial states of dynamical triggering parameters $\xi_i(t_0)$, initial impulsive effect $\kappa(t_1)$;

System parameters A_i and B_i, average impulsive gain $\bar{r}$, average impulsive interval T_a, specific error bound $\mathcal{B}_3$;

Updating parameters ϵ_1, ϵ_2, ϵ_3.

Ensure:

for t from t_0 to t_{end} **do**
  for i from 1 to N **do**
    Send adjacent node information to the i-th node.
    Update $\hat{e}_i(t)$ with Eq. (4).
    Update value of $\varphi_i(t)$ according to Eq. (5).
    Update $\mu_i(t)$ by Eq. (6).
    if $\varphi_i(t) - \mu_i(t) > 0$ **then**
      Update instant t as t^i_{k+1}.
      Update parameter $r(t_{k+1})$.
      Update the state $z_i(t)$ at impulsive instant t_{k+1}.
    else
      Let $z_i(t)$ remain unchanged.
    end if
  end for
end for

Analogously as deductions in (38)–(40), the following result could be carried out:

$$\|e(t)\| \leq \sqrt{2\bar{r}^{N_\zeta} \sup_{-\tilde{\tau}\leq s\leq 0} \sum_{i=1}^{N} \|f_i(s)\|^2 \exp\left(-\frac{\bar{\lambda}}{2}t\right)} + \sqrt{\frac{\frac{1}{2}\sum_{i=1}^{N}\alpha_i q_i^2 + 2\iota}{\varepsilon\bar{r}^{-N_\zeta} - \left(\chi + d\pi\tau_2^2\right)}}. \quad (55)$$

With $t \to +\infty$ and $\iota \to 0$, it follows from (55) that:

$$\|e(t)\| \leq e_{b3} = \sqrt{\frac{\sum_{i=1}^{N}\alpha_i q_i^2}{\varepsilon\bar{r}^{-N_\zeta} - \left(\chi + d\pi\tau_2^2\right)}} \quad (56)$$

which implies that the controlled error network (44) is eventually stable via the designed dynamic self-triggered impulsive controller as $t \to +\infty$. In addition, it should be stated that the exponential convergence rate is $(\bar{\lambda}/2)$. Therefore, the proof of the leader-following synchronization issue with time-varying impulsive effect is finished. ■

To concretely show the steps of the self-triggered mechanism for achieving quasisynchronization with the designed controller, Algorithm 1 is given.

Remark 5: As we mentioned before, the self-triggered mechanism saves energy consumption by reducing the number of monitoring, but correspondingly, it will weaken the control effects to a certain extent. Therefore, some works such as [42] limited the range of impulsive effects to ensure the synchronizing impulses. However, the results obtained are more conservative. Obviously, an impulse sequence containing hybrid impulses with different functions could describe the practical applications better. To be specific, the extensive of the parameter variation method, which is also the main reason for the different results of Theorems 1 and 2, is mainly manifested in (49), where different functions of time-varying impulsive effects are considered instead of only the general fixed impulsive effects [39]–[41], and then the ambiguous relationship between $V(t_k^+)$ and $V(t_k^-)$ is given. As a result, there is no need to discuss $r(t_k)$ with different ranges. In addition, the impulsive instants and the impulsive intervals could be adjusted by introducing the definition of average impulsive interval. As mentioned before, it is usually assumed that $(T - t/T_a) - N_0 \leq N_\zeta(t, s) \leq (T - t/T_a) + N_0$ as described in [26], [31], [33], and [41]. But this assumption on $N_\zeta(t, s)$ is removed in the proof procedure of Theorem 2, and therefore, a less conservative conclusion is obtained.

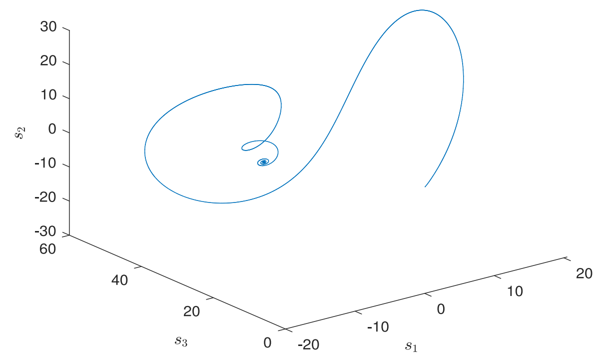

Fig. 1. Dynamic trajectory of system $s(t)$.

IV. Numerical Simulations

In this section, three numerical simulations will be displayed to prove the effectiveness of the derived theorems. Validity of controllers triggered by the dynamic self-triggered mechanism with both fixed and time-varying effects will be shown, respectively. In addition, estimated synchronization bound will be plotted.

Consider the following leader system $s(t)$:

$$\dot{s}(t) = -\Upsilon_{A_s} s(t) + \Upsilon_{B_s} \tilde{h}(s(t)) \quad (57)$$

where $\tilde{h}(s(t))$ is selected as a Lorenz system [43].

Choose matrices

$$\Upsilon_{A_s} = \begin{bmatrix} 1.5 & 0.06 & -0.3 \\ -1 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}, \quad \Upsilon_{B_s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The phrase trajectory of $s(t)$ is plotted in Fig. 1.

Example 1: In this example, synchronization via dynamic self-triggered impulsive control will be verified. To simplify the content, only synchronizing impulsive effects will be experienced.

Consider a complex network, which is composed of six nodes. For parameters of the complex network model, letting coupling strengths $b = 0.05$, $d = 0.1$, inner coupling matrices $\Gamma_1 = I_3$, $\Gamma_2 = 2I_3$, time-varying delays $\tau_1(t) = 0.01\cos(t)$, $\tau_2(t) = 0.05\max\{\cos(t), \sin(t)\}$.

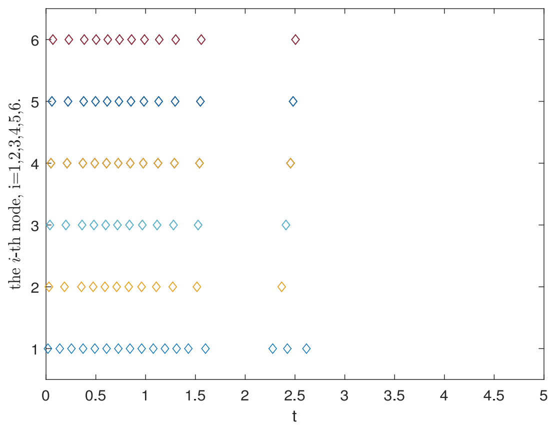

Fig. 2. Triggering instants of controllers equipped on the ith node with $\kappa = 0.5$.

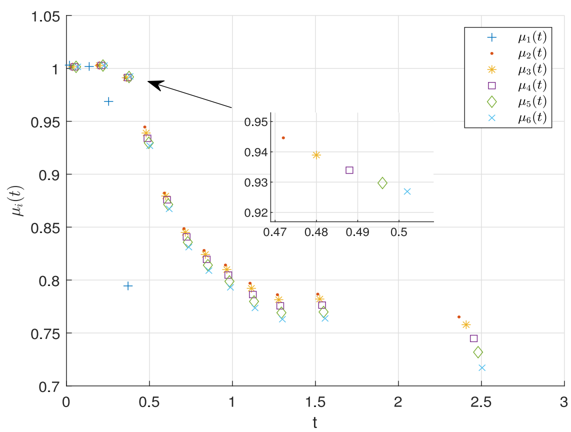

Fig. 3. Evolutionary points of $\mu_i(t)$ with $\kappa = 0.5$.

For the design of control protocol, choosing the impulsive effect $\kappa = 0.5$, $\omega_i = 0.35$, $N_0 = 3$, $r = 0.8$, $\pi = 10$, $\rho = 0.48$, $\theta = 0.75$, $l = 0.4$, $\epsilon_1 = 0.02$, $\epsilon_2 = 0.03$, and $\epsilon_3 = 1$, the initial value of dynamic parameter $\eta = [1, 1, 1, 1, 1, 1]^T$. It can be further computed that $\varepsilon = 0.6812$ and $r^{N_0}([\ln r/T_a] - \varepsilon) + (\chi + d\pi\tau_2^2) = -5.676 < 0$, which satisfies requirements in (16) and the estimated error bound is $e_{b1} = 1.409$.

Fig. 2 plots the triggering instants of each controller. Fig. 3 depicts the values of dynamic parameters $\mu_i(t)$ during the triggering instants t_k, $k \in \mathfrak{N}$. Fig. 4 shows the error curve with 76 triggers and it turns out that error vector $\|e(t)\|$ is less than error bound e_{b1}.

Example 2: In the above example, quasisynchronization is achieved by a kind of impulsive controllers with fixed impulsive effects, however, with time-varying impulsive effects, Theorem 1 will be no longer effective. Therefore, in this example, time-varying impulses will be introduced to verify Theorem 2.

Parameter $\kappa(t_k) = 0.4 + 0.1\sin(t_k)$ and the initial value of dynamic parameter $\eta = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1]^T$ are chosen, and other parameters are selected as the same in Example 1. It can be derived that $\bar{r} = 0.73$. The estimated error bound is $e_{b3} = 1.113$. It can be achieved that triggering instants of the ith controller are displayed in Fig. 5. Fig. 6 plots the value points of $\mu_i(t)$ during the triggering instants t_k, $k \in \mathfrak{N}$.

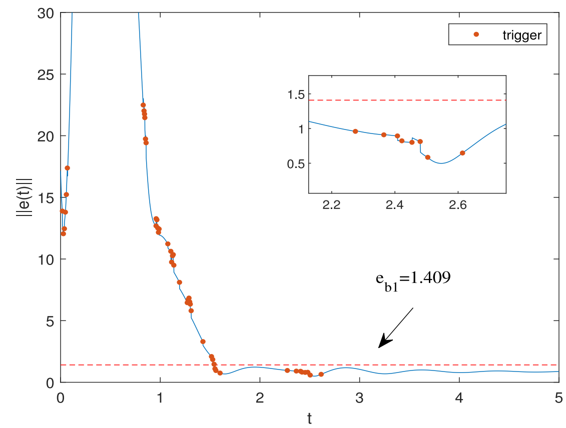

Fig. 4. Evolutionary curve of $\|e(t)\|$ and error bound e_{b1} with $\kappa = 0.5$.

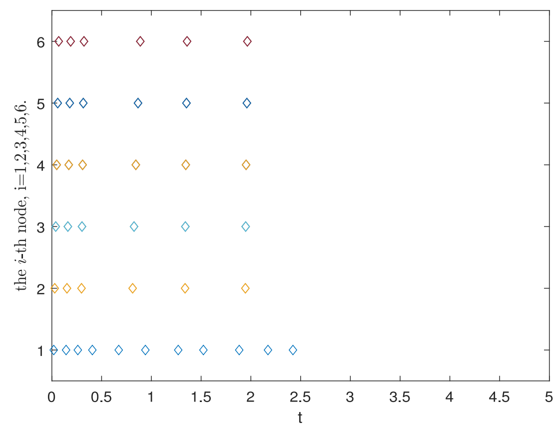

Fig. 5. Triggering instants of controllers equipped on the ith node with $\kappa(t_k) = 0.4 + 0.1\sin(t_k)$.

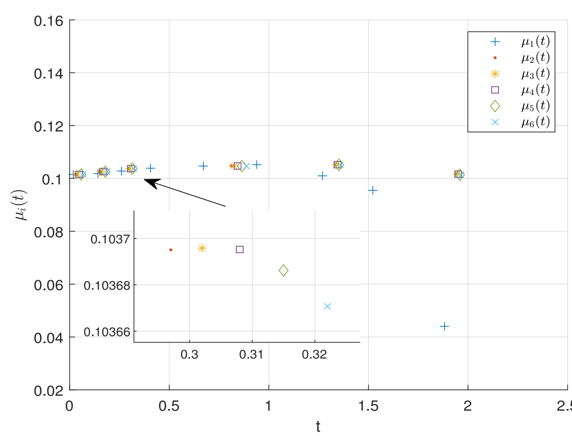

Fig. 6. Evolutionary points of $\mu_i(t)$ with $\kappa(t_k) = 0.4 + 0.1\sin(t_k)$.

It portrays in Fig. 7 that $\|e(t)\|$ is less than error bound e_{b3} with 41 times impulses.

Example 3: The above examples have shown the effectiveness of Theorems 1 and 2. In this example, a comparative experiment will be demonstrated to show the advantage of the given control protocol. The related parameters are chosen as the same with those in Theorem 2. The triggering conditions for different triggered mechanism is shown in Table I. Given the initial value $z_i(0)$, the evolution curves of each triggered

TABLE I
ANALYSIS OF DIFFERENT TRIGGERED MECHANISMS

Triggered mechanism	Triggering condition $(f_i(t) > 0)$	Monitoring	Controlling
Dynamic self-triggered mechanism	$\|\hat{e}_i(t_k)\|^2(e^{\rho(t-t_k^i)}-1)-\theta\|\hat{e}_i(t_k)\|^2-e^{-l(t-t_0)}-\mu_i(t)$	62 times	62 times
Static self-triggered mechanism [44]	$\|\hat{e}_i(t_k)\|^2(e^{\rho(t-t_k^i)}-1)-\theta\|\hat{e}_i(t_k)\|^2-e^{-l(t-t_0)}$	112 times	112 times
Event-triggered mechanism [23]	$\|\hat{e}_i(t_k)-e_i(t)\|^2-\frac{1}{2}\theta\|\hat{e}_i(t_k)\|^2-\frac{l}{(t-t_0)^2}$	continuous	26 times

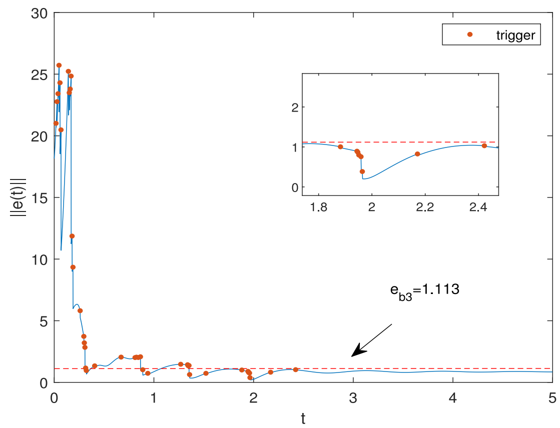

Fig. 7. Evolution curve of $\|e(t)\|$ and error bound e_{b3} with $\kappa(t_k) = 0.4 + 0.1\sin(t_k)$.

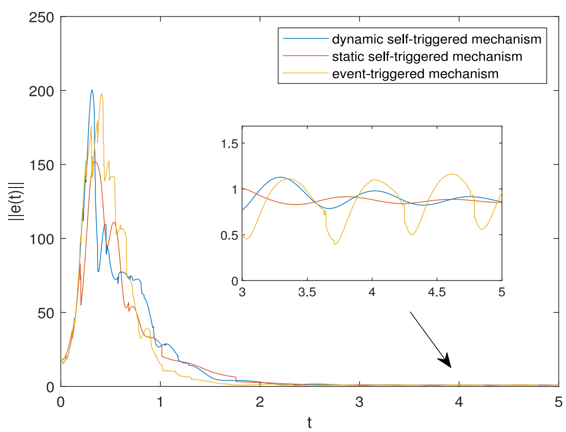

Fig. 8. Evolution curve of $\|e(t)\|$ with different triggered mechanisms.

mechanism are plotted in Fig. 8. It can be known that even if the number of monitoring times of the self-triggered mechanism are far less than the event-triggered mechanism [23], the convergence speed of synchronization does not drop significantly. In addition, it can also be observed that the improved dynamic self-triggered mechanism can further reduce the number of triggering instants compared to the static self-triggered mechanism [35], which shows the superiority of the control method proposed in this article.

Remark 6: One concern about the self-triggered mechanism is that whether the wake-up timing of the control signal at the next instant is timely and effective without continuous monitoring. Actually, in the early stage of the evolution of complex networks, the impulsive controller will be frequently triggered due to the increasing of the monitoring error $\hat{e}_i(t)$. Moreover, in the later stage of synchronization, with the help of coupling characteristics of complex networks, the number of impulsive triggering instants will be decreased and each node can still be synchronized to the target state $s(t)$ within a specific bound. As can be found in Figs. 4 and 7, the error evolution curves are rapidly reduced due to dense control signals in the early stage, and remains stable in the later stage due to the influence of couplings.

V. CONCLUSION

This article has been devoted to investigating synchronization of the complex network with hybrid time-varying delays via the self-triggered impulsive control. In consideration of mismatched parameters, the updating laws of dynamic self-triggered mechanism have been designed to cope with quasisynchronization issue. First, Zeno behavior has been excluded through mathematical analysis to verify the rationality of control scheme and updating laws. Second, with the Lyapunov stability theorem, extended parameter variation method, definition of average impulsive gain, and definition of average impulsive interval, sufficient synchronization conditions in the matter of both fixed and time-varying impulsive effects have been given. In addition, regarding different functions of impulsive effects, the convergence rate and precise synchronization bound have been elaborately derived. Finally, three numerical simulations have been conducted to illustrate the feasibility of proposed control scheme. In our future work, we plan to extend the parameter variation method to make it applicable to cases where deception attack signals are included. In addition, a kind of periodic self-triggered mechanisms may be considered to further coordinate the relationships between the number of triggering times and energy consumption.

REFERENCES

[1] Z. Tang, J. H. Park, Y. Wang, and W. X. Zheng, “Synchronization on Lur’e cluster networks with proportional delay: Impulsive effects method,” *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 51, no. 7, pp. 4555–4565, Jul. 2021, doi: 10.1109/tsmc.2019.2943933.

[2] G. Ji, C. Hu, J. Yu, and H. Jiang, “Finite-time and fixed-time synchronization of discontinuous complex networks: A unified control framework design,” *J. Franklin Inst.*, vol. 355, no. 11, pp. 4665–4685, 2018.

[3] L. Liu, W. Zhou, X. Li, and Y. Sun, “Dynamic event-triggered approach for cluster synchronization of complex dynamical networks with switching via pinning control,” *Neurocomputing*, vol. 340, pp. 32–41, May 2019.

[4] R. Guo, W. Lv, and Z. Zhang, “Quasi-projective synchronization of stochastic complex-valued neural networks with time-varying delay and mismatched parameters,” *Neurocomputing*, vol. 415, pp. 184–192, Nov. 2020.

[5] D. Liu and D. Ye, “Pinning-observer-based secure synchronization control for complex dynamical networks subject to DoS attacks,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 12, pp. 5394–5404, Dec. 2020.

[6] R. Kumar and S. Das, "Weak, modified and function projective synchronization of Cohen–Grossberg neural networks with mixed time-varying delays and parameter mismatch via matrix measure approach," *Neural Comput. Appl.*, vol. 32, no. 11, pp. 7321–7332, 2020.
[7] S. Wang, Y. Cao, Z. Guo, Z. Yan, S. Wen, and T. Huang, "Periodic event-triggered synchronization of multiple memristive neural networks with switching topologies and parameter mismatch," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 427–437, Jan. 2021.
[8] X. Qiu, W. Lin, and Y. Zheng, "Function projective synchronization of complex networks with distributed delays via hybrid feedback control," *IEEE Access*, vol. 8, pp. 99110–99114, 2020.
[9] C. Xu, D. Tong, Q. Chen, W. Zhou, and Y. Xu, "Exponential synchronization of chaotic systems with stochastic noise via periodically intermittent control," *Int. J. Robust Nonlinear Control*, vol. 30, no. 7, pp. 2611–2624, 2020.
[10] A. Fan, J. Li, and C. Wei, "Distributed prescribed performance pinning synchronization for complex dynamical networks with event-triggered communication protocols," *J. Franklin Inst.*, vol. 357, no. 3, pp. 1790–1812, 2020.
[11] Z.-G. Wu, Y. Xu, Y.-J. Pan, P. Shi, and Q. Wang, "Event-triggered pinning control for consensus of multiagent systems with quantized information," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1929–1938, Nov. 2018.
[12] S. Wang, Y. Cao, S. Wen, Z. Guo, T. Huang, and Y. Chen, "Projective synchroniztion of neural networks via continuous/periodic event-based sampling algorithms," *IEEE Trans. Netw. Sci. Eng.*, vol. 7, no. 4, pp. 2746–2754, Oct.–Dec. 2020, doi: 10.1109/TNSE.2020.2985409.
[13] X. Liu and T. Chen, "Synchronization analysis for nonlinearly-coupled complex networks with an asymmetrical coupling matrix," *Phys. A Stat. Mech. Appl.*, vol. 387, nos. 16–17, pp. 4429–4439, 2008.
[14] X. Hai, G. Ren, Y. Yu, C. Xu, and Y. Zeng, "Pinning synchronization of fractional and impulsive complex networks via event-triggered strategy," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 82, Mar. 2020, Art. no. 105017.
[15] W. Sun *et al.*, "Quasi-synchronization of heterogeneous dynamical networks via event-triggered impulsive controls," *IEEE Trans. Cybern.*, vol. 52, no. 1, pp. 228–239, Jan. 2022, doi: 10.1109/tcyb.2020.2975234.
[16] X. Yang, Y. Liu, J. Cao, and L. Rutkowski, "Synchronization of coupled time-delay neural networks with mode-dependent average dwell time switching," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 12, pp. 5483–5496, Dec. 2020.
[17] L. Liu, H. Sun, L. Ma, J. Zhang, and Y. Bo, "Quasi-consensus control for a class of time-varying stochastic nonlinear time-delay multiagent systems subject to deception attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 11, pp. 6863–6873, Nov. 2021, doi: 10.1109/tsmc.2020.2964826.
[18] R. Vadivel, R. Suresh, P. Hammachukiattikul, B. Unyong, and N. Gunasekaran, "Event-triggered L_2-L_∞ filtering for network-based neutral systems with time-varying delays via T-S fuzzy approach," *IEEE Access*, vol. 9, pp. 145133–145147, 2021.
[19] Z. Guo, S. Gong, and T. Huang, "Finite-time synchronization of inertial memristive neural networks with time delay via delay-dependent control," *Neurocomputing*, vol. 293, pp. 100–107, Jun. 2018.
[20] J. Shi and Z. Zeng, "Global exponential stabilization and lag synchronization control of inertial neural networks with time delays," *Neural Netw.*, vol. 126, pp. 11–20, Jun. 2020.
[21] R. Tang, H. Su, Y. Zou, and X. Yang, "Finite-time synchronization of Markovian coupled neural networks with delays via intermittent quantized control: Linear programming approach," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Apr. 8, 2021, doi: 10.1109/TNNLS.2021.3069926.
[22] L. Cheng, Y. Yang, L. Li, and X. Sui, "Finite-time hybrid projective synchronization of the drive-response complex networks with distributed-delay via adaptive intermittent control," *Phys. A Stat. Mech. Appl.*, vol. 500, pp. 273–286, Jun. 2018.
[23] X. Tan, J. Cao, L. Rutkowski, and G. Lu, "Distributed dynamic self-triggered impulsive control for consensus networks: The case of impulse gain with normal distribution," *IEEE Trans. Cybern.*, vol. 51, no. 2, pp. 624–634, Feb. 2021.
[24] J. Almeida, C. Silvestre, and A. Pascoal, "Synchronization of multiagent systems using event-triggered and self-triggered broadcasts," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4741–4746, Sep. 2017.
[25] X. Yang, X. Li, J. Lu, and Z. Cheng, "Synchronization of time-delayed complex networks with switching topology via hybrid actuator fault and impulsive effects control," *IEEE Trans. Cybern.*, vol. 50, no. 9, pp. 4043–4052, Sep. 2020.
[26] Z. Tang, D. Xuan, J. H. Park, Y. Wang, and J. Feng, "Impulsive effects based distributed synchronization of heterogeneous coupled neural networks," *IEEE Trans. Netw. Sci. Eng.*, vol. 8, no. 1, pp. 498–510, Jan.–Mar. 2021.
[27] D. Ye and Y. Shao, "Quasi-synchronization of heterogeneous nonlinear multi-agent systems subject to DOS attacks with impulsive effects," *Neurocomputing*, vol. 366, pp. 131–139, Nov. 2019.
[28] Y. Li, J. Lou, Z. Wang, and F. E. Alsaadi, "Synchronization of dynamical networks with nonlinearly coupling function under hybrid pinning impulsive controllers," *J. Franklin Inst.*, vol. 355, no. 14, pp. 6520–6530, 2018.
[29] R. Vadivel, M. S. Ali, and Y. H. Joo, "Drive-response synchronization of uncertain Markov jump generalized neural networks with interval time varying delays via decentralized event-triggered communication scheme," *J. Franklin Inst.*, vol. 357, no. 11, pp. 6824–6857, 2020.
[30] Y. Fan, L. Liu, S. Member, G. Feng, and Y. Wang, "Self-triggered consensus for multi-agent systems with zeno-free triggers," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2779–2784, Oct. 2015.
[31] J. Lu, D. W. C. Ho, and J. Cao, "A unified synchronization criterion for impulsive dynamical networks," *Automatica*, vol. 46, no. 7, pp. 1215–1221, 2010.
[32] N. Wang, X. Li, J. Lu, and F. E. Alsaadi, "Unified synchronization criteria in an array of coupled neural networks with hybrid impulses," *Neural Netw.*, vol. 101, pp. 25–32, May 2018.
[33] W. He, F. Qian, and J. Cao, "Pinning-controlled synchronization of delayed neural networks with distributed-delay coupling via impulsive control," *Neural Netw.*, vol. 85, pp. 1–9, Jan. 2017.
[34] Z. Tang, J. H. Park, and J. Feng, "Impulsive effects on quasi-synchronization of neural networks with parameter mismatches and time-varying delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 908–919, Apr. 2018.
[35] M. Xing, F. Deng, and X. Zhao, "Synchronization of stochastic complex dynamical networks under self-triggered control," *Int. J. Robust Nonlinear Control*, vol. 27, no. 16, pp. 2861–2878, 2017.
[36] Z. Wu, D. Liu, and Q. Ye, "Pinning impulsive synchronization of complex-variable dynamical network," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 20, no. 1, pp. 273–280, 2015.
[37] X. Wang, X. Liu, K. She, and S. Zhong, "Pinning impulsive synchronization of complex dynamical networks with various time-varying delay sizes," *Nonlinear Anal. Hybrid Syst.*, vol. 26, pp. 307–318, Nov. 2017.
[38] H. Dai, W. Chen, J. Jia, J. Liu, and Z. Zhang, "Exponential synchronization of complex dynamical networks with time-varying inner coupling via event-triggered communication," *Neurocomputing*, vol. 245, pp. 124–132, Jul. 2017.
[39] Z. Xu, X. Li, and P. Duan, "Synchronization of complex networks with time-varying delay of unknown bound via delayed impulsive control," *Neural Netw.*, vol. 125, pp. 224–232, May 2020.
[40] N. Yu and W. Zhu, "Event-triggered impulsive chaotic synchronization of fractional-order differential systems," *Appl. Math. Comput.*, vol. 388, Jan. 2021, Art. no. 125554.
[41] L. Li and G. Mu, "Synchronization of coupled complex-valued impulsive neural networks with time delays," *Neural Process. Lett.*, vol. 50, no. 3, pp. 2515–2527, 2019.
[42] G. Wen, W. Yu, X. Yu, and J. Lü, "Complex cyber-physical networks: From cybersecurity to security control," *J. Syst. Sci. Complex.*, vol. 30, no. 1, pp. 46–67, 2017.
[43] T. Chen, X. Liu, and W. Lu, "Pinning complex networks by a single controller," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 54, no. 6, pp. 1317–1326, Jun. 2007.
[44] X. Tan, J. Cao, and X. Li, "Consensus of leader-following multiagent systems: A distributed event-triggered impulsive control strategy," *IEEE Trans. Cybern.*, vol. 49, no. 3, pp. 792–801, Mar. 2019.

Dong Ding received the B.S. degree in vehicle engineering from Shanghai Dianji University, Shanghai, China, in 2016. He is currently pursuing the Ph.D. degree with Jiangnan University, Wuxi, China.

He used to be a Visiting Scholar with Yeungnam University, Gyeongsan, Republic of Korea. His current research interests include complex dynamic networks, synchronization and control, and nonlinear systems.

Ze Tang received the Ph.D. degree in electrical engineering from Yeungnam University, Gyeongsan, South Korea, in 2017.

He joined Jiangnan University, Wuxi, China, in 2017 as an Associate Professor. In 2016, he was a Visiting Fellow with the School of Computing, Engineering and Mathematics, Western Sydney University, Penrith, NSW, Australia. His current research interests include complex networks, synchronization control, and nonlinear systems.

Ju H. Park (Senior Member, IEEE) received the Ph.D. degree in electronics and electrical engineering from the Pohang University of Science and Technology (POSTECH), Pohang, Republic of Korea, in 1997.

From May 1997 to February 2000, he was a Research Associate with the Engineering Research Center-Automation Research Center, POSTECH. He joined Yeungnam University, Gyeongsan, Republic of Korea, in March 2000, where he is currently the Chuma Chair Professor. He is a coauthor of the monographs *Recent Advances in Control and Filtering of Dynamic Systems With Constrained Signals* (New York, NY, USA: Springer-Nature, 2018) and *Dynamic Systems With Time Delays: Stability and Control* (New York, NY, USA: Springer-Nature, 2019) and is an Editor of an edited volume *Recent Advances in Control Problems of Dynamical Systems and Networks* (New York, NY, USA: Springer-Nature, 2020). His research interests include robust control and filtering, neural/complex networks, fuzzy systems, multiagent systems, and chaotic systems. He has published a number of articles in these areas.

Prof. Park has been a recipient of the Highly Cited Researchers Award by Clarivate Analytics (formerly, Thomson Reuters) since 2015 and listed in three fields, Engineering, Computer Sciences, and Mathematics, in 2019, 2020, and 2021. He also serves as an Editor for the *International Journal of Control, Automation and Systems*. He is also a Subject Editor/Advisory Editor/Associate Editor/Editorial Board Member of several international journals, including *IET Control Theory & Applications*, *Applied Mathematics and Computation*, *Journal of The Franklin Institute*, *Nonlinear Dynamics*, *Engineering Reports*, *Cogent Engineering*, the IEEE TRANSACTION ON FUZZY SYSTEMS, the IEEE TRANSACTION ON NEURAL NETWORKS AND LEARNING SYSTEMS, and the IEEE TRANSACTION ON CYBERNETICS. He is a Fellow of the Korean Academy of Science and Technology.

Yan Wang was born in Yancheng, Jiangsu, China, in 1978. She received the Ph.D. degree in control theory and engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2006.

She was a Visiting Scholar with the Department of Electrical and Computer Engineering, Luis Anna State University, Baton Rouge, LA, USA, from September 2013 to September 2014. She is a Professor and a Doctoral Supervisor with the Internet of Things Engineering School, Jiangnan University, Wuxi, China, the Yangtze River Young Scholar of the Ministry of Education, and the Outstanding Youth Fund Winner of Jiangsu Province.

Zhicheng Ji was born in Ningbo, Zhejiang, China, in 1959. He received the Ph.D. degree in power electronics and transmission from the China University of Mining and Technology, Xuzhou, Jiangsu, China, in 2004.

He was a Visiting Scholar with the University of Toronto, Toronto, ON, Canada, in 2003. He used to be the Vice Chancellor of Jiangnan University, Wuxi, China, where he is currently a Professor and a Doctoral Supervisor with the Internet of Things Engineering School. His current research interests include complex systems, artificial intelligence, and workshop scheduling.