

Abstracts of Previous Tutorials in this Series

TUTORIAL I

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MULTIPLE HYPOTHESIS TRACKING FOR MULTIPLE TARGET TRACKING

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Multiple hypothesis tracking (MHT) is generally accepted as the preferred method for solving the data association problem in modern multiple target tracking (MTT) systems. This paper summarizes the motivations for MHT, the basic principles behind MHT and the alternative implementations in common use. It discusses the manner in which the multiple data association hypotheses formed by MHT can be combined with multiple filter models, such as used by the interacting multiple model (IMM) method. An overview of the studies that show the advantages of MHT over the conventional single hypothesis approach is given. Important current applications and areas of future research and development for MHT are discussed.

A STAP OVERVIEW

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This tutorial provides a brief overview of space-time adaptive processing (STAP) for radar applications. We discuss space-time signal diversity and various forms of the adaptive processor, including reduced-dimension and reduced-rank STAP approaches. Additionally, we describe the space-time properties of ground clutter and noise-jamming, as well as essential STAP performance metrics. We conclude this tutorial with an overview of some current STAP topics: space-based radar, bistatic STAP, knowledge-aided STAP, multi-channel synthetic aperture radar and non-sidelooking array configurations.

CLASS-SPECIFIC CLASSIFIER: AVOIDING THE CURSE OF DIMENSIONALITY

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This article describes a new probabilistic method called the “class-specific method” (CSM). CSM has the potential to avoid the “curse of dimensionality” which plagues most classifiers

which attempt to determine the decision boundaries in a high-dimensional feature space. In contrast, in CSM, it is possible to build classifiers without a common feature space. Separate low-dimensional feature sets may be defined for each class, while the decision functions are projected back to the common raw data space. CSM effectively extends the classical classification theory to handle multiple feature spaces. It is completely general, and requires no simplifying assumption such as Gaussianity or that data lies in linear subspaces.

“STATISTICS 101” FOR MULTISENSOR, MULTITARGET DATA FUSION

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This tutorial summarizes the motivations, concepts, techniques, and applications of finite-set statistics (FISST), a system-level, “top-down” direct generalization of ordinary single-sensor, single-target engineering statistics to the multisensor, multitarget realm. FISST provides powerful new conceptual and computational methods for dealing with multisensor, multitarget, and multi-evidence data fusion problems. The paper begins with a broad-brush overview of the basic concepts of FISST. It describes how conventional single-sensor, single-target formal Bayesian modeling is carefully extended to general data fusion problems. We examine a simple example: joint detection and tracking of a possibly non-existent maneuvering target in heavy clutter. The tutorial concludes with a commentary on certain criticisms of FISST.

TUTORIAL II

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A REVIEW OF DISTRIBUTED POWER SYSTEMS PART I: DC DISTRIBUTED POWER SYSTEM

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The present development state in dc distributed power systems (DPS) is comprehensively reviewed in this tutorial. Basic distributed structures and their characteristics are described. The system level design considerations are discussed. The profile of current technologies is drawn. Finally, the issues and challenges in this

research area are identified. These issues include not only improving efficiency, but also increased concerns regarding the cost and complexity of power supplying systems.

FUNDAMENTALS OF ENERGY-CONSTRAINED SENSOR NETWORK SYSTEMS

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This article is an overview of energy-constrained sensor networks, focusing on energy-conserving communications and signal processing strategies. We assume battery-driven nodes, employing robust communications, with little or no fixed infrastructure. Our discussion includes architectures, communications connectivity, capacity and scalability, mobility, network localization and synchronization, distributed signal processing, and cross-layer issues. Because energy is a precious system resource, all aspects of the network must be designed with energy savings in mind. In particular, transmissions and idle listening must be minimized, which implies the use of duty cycling to the maximum extent possible. When external assets are available, for tasks such as network synchronization and node geolocation, these can greatly relieve the energy burden and significantly enhance network lifetime.

PROBABILISTIC DATA ASSOCIATION TECHNIQUES FOR TARGET TRACKING WITH APPLICATIONS TO SONAR, RADAR AND EO SENSORS

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We present an overview of the probabilistic data association (PDA) technique and its application for different target tracking scenarios, in particular for low observable (LO) (low SNR) targets. A summary of the PDA technique is presented. The use of the PDA technique for tracking low observable targets with passive sonar measurements is presented. This “target motion analysis” is an application of the PDA technique, in conjunction with the maximum likelihood (ML) approach, for target motion parameter estimation via a batch procedure. The use of the PDA technique for tracking highly maneuvering targets combined radar resource management is described. This illustrates the application of the PDA technique for recursive state estimation using the interacting multiple model (IMM) estimator with probabilistic data association filter (PDAF) (IMMP-DAF). Then we present a flexible (expanding and contracting) sliding-window parameter estimator using the PDA approach for tracking the state of a maneuvering target using measurements from an electro-optical (EO) sensor. This, while still a batch procedure, has the flexibility of varying the batches de-

pending on the estimation results in order to make the estimation robust to target maneuvers as well as target appearance or disappearance.

NONLINEAR FILTERS: BEYOND THE KALMAN FILTER

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Nonlinear filters can provide estimation accuracy that is vastly superior to extended Kalman filters for some important practical applications. We compare several types of nonlinear filters, including: particle filters (PFs), unscented Kalman filters, extended Kalman filters, batch filters and exact recursive filters. The key practical issue in nonlinear filtering is computational complexity, which is often called “the curse of dimensionality.” It has been asserted that PFs avoid the curse of dimensionality, designed PFs with good proposal densities sometimes avoid the curse of dimensionality, but not otherwise. Future research in nonlinear filtering will exploit recent progress in quasi-Monte Carlo algorithms (rather than boring old Monte Carlo methods), as well as ideas borrowed from physics (e.g., dimensional interpolation) and new mesh-free adjoint methods for solving PDEs. This tutorial was written for normal engineers, who do not have nonlinear filters for breakfast.

MULTIBASELINE CROSS-TRACK SAR INTERFEROMETRY: A SIGNAL PROCESSING PERSPECTIVE

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Synthetic aperture radar interferometry (InSAR) is a powerful and increasingly expanding technique for measuring the topography of a surface, its changes over both short- and long-time scale, and other changes in the detailed characteristics of the surface. We provide a tutorial description of recent results of the research activity at the University of Pisa on multibaseline (MB) InSAR processing. The main focus is on the problem of retrieving both heights and radar reflectivities of natural layover areas by means of a cross-track InSAR (XTI-SAR) system with a uniform linear array (ULA). It is formulated as the problem of detecting and estimating a multi-component signal corrupted by multiplicative noise—the speckle in the radar imaging jargon—and by additive white Gaussian noise. Application to the InSAR problem of both nonparametric and parametric modern spectral estimation techniques is described. The problem of estimating the number of signal components in the presence of speckle is also addressed. Finally, a brief mention is given to recent research trends on robust methods for nonperfectly calibrated arrays, on processing for non-ULA configurations, and on MB SAR tomography, which is an extension of MB SAR interferometry for the full 3D mapping of semitransparent volume scattering layers. State of the art of other advanced multichannel interferometric techniques is also briefly recalled.

TUTORIAL III

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A REVIEW OF DISTRIBUTED POWER SYSTEMS PART II: HIGH FREQUENCY AC DISTRIBUTED POWER SYSTEMS

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The present development state in high frequency (HF) ac distributed power systems (DPS) is reviewed. First, background and motivations of developing HF AC-DPS are addressed. Two types of basic HF AC-DPSs based on sine wave and square/PWM (pulse-width modulated) wave bus are described, and the system level design considerations are discussed. Further, the issues and challenges in this research area are identified. These issues include high electromagnetic interference (EMI) level, difficulty to back up power, nonredundant system structure and limited post-regulation capability, etc. Finally, a viable HF AC-DPS is proposed, which is expected to yield effective EMI trade-off and system redundancy.

This is Part II of a two part review of Distributed Power Systems. *Part I: DC Distributed Power System* appeared in

Tutorial II, August 2005. See *Abstracts*, page 87, for more details.

BIOMETRIC SECURITY TECHNOLOGY

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This paper presents an overview of the main topics related to biometric security technology, with the central purpose to provide a primer on this subject.

Biometrics can offer greater security and convenience than traditional methods for people recognition. Even if we do not want to replace a classic method (password or handheld token) by a biometric one, for sure, we are potential users of these systems, which will even be mandatory for new passport models. For this reason, it is useful to be familiarized with the possibilities of biometric security technology.

OVERVIEW OF GENERALIZED MONOPULSE ESTIMATION

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Monopulse is an established technique for radar angle estimation. One can show that monopulse estimation is based on a general approximation derived from maximum likelihood (ML) estimation. This tutorial provides a derivation of this relation and presents extensions of this monopulse principle to multi-dimensional array and parameter estimation problems, in particular to

space-time adaptive processing (STAP) with reduced dimension, subarrays and generalized sidelobe canceller (GSLC) configurations. The performance of these monopulse applications can be predicted by exploiting the distribution of the monopulse ratio. It is demonstrated that this distribution is more realistic than the Cramér-Rao bound (CRB). Several examples of performance of monopulse estimators are given for thinned and fully filled planar arrays, adaptive beamforming with and without low sidelobes, GSLC, and STAP. Finally, conditions for estimates with low variance are discussed.

TUTORIAL IV

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A BEGINNER'S GUIDE TO INTERFEROMETRIC SAR CONCEPTS AND SIGNAL PROCESSING

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Interferometric synthetic aperture radar (IFSAR, also abbreviated as InSAR) employs pairs of high resolution SAR images to generate high quality terrain elevation maps using phase interferometry methods. IFSAR provides an all-weather, day/night capability to generate measurements of terrain elevation on a dense grid of sample points with accuracies of ones of meters. Both spaceborne and airborne IFSAR systems are in use.

In this paper we present a tutorial introduction to the concepts, techniques, and applications of IFSAR. After a brief introduction to digital elevation models (DEMs) and digital terrain elevation data (DTED), the fundamental IFSAR equation relating interferometric phase measurements to terrain elevation is derived from simple geometric considerations. The central section of the paper describes the major algorithmic steps required to form an IFSAR terrain map. Finally, variations of IFSAR for mapping terrain elevation or reflectivity changes are briefly described. A web site at users.ece.gatech.edu/~mrichard/AESS_IFSAR.htm provides access to color versions of many of the IFSAR images included in this paper.

LORAN DATA MODULATION: A PRIMER

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Loran has provided navigation service since 1958. Though not originally designed with data broadcast capabilities, Loran's versatility has enabled data to be broadcast with great benefits.

Research in the last two decades has resulted in a tremendous increase in the data capacity of Loran thereby increasing its utility. Currently, a modernized Loran is being evaluated for its capability to backup GPS and data modulation is an integral part of this Loran design. An overview and analysis of Loran modulation techniques is provided.

TEMPLATE-BASED TARGET IDENTIFICATION AND CONFUSION MATRICES

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One-dimensional high-range-resolution (HRR) and two-dimensional range-Doppler-imaging (RDI) radar represent possible sensor technologies where template-based techniques can be applied to perform combat identification (CID). The majority of the research reported in these areas consists of empirical studies. This article provides a theoretical basis for understanding some of the fundamental trade-offs associated with these CID techniques, such as the following.

- 1) What are the relative advantages of RDI over HRR radar or of finer versus coarser resolution in the HRR process?
- 2) What is the relative advantage of coherent over noncoherent processing?
- 3) How do target correlations, signal-to-noise ratio (SNR), and target scintillation affect the ability to identify targets?

Because confusion matrices are often used to characterize the performance of CID systems, we provide analytical methods for calculating the entries in confusion matrices as a function of the issues cited above. These formulations provide analytical bases to guide system trade-off decisions. The organization of this paper is as follows. We begin with a short overview of HRR and RDI and then explore a number of ways to process the associated target templates that range from an ideal, theoretical approach to an approach that would be more feasible to implement within current-day radars. We first develop analytic template-based methodologies for constructing confusion matrix entries for nonscintillating targets for both coherent and noncoherent processing assumptions. The confusion matrix entries in these cases are conditional probabilities obtained from a simple rule: find the probability that among m (in general, correlated) random variables, each associated with a possible target, that any one is the largest. For the noncoherent case, the successful application of this rule requires the target template values to explicitly include the effects of thermal noise (noise-adjusted templates). We conclude by showing how to calculate theoretically optimum results (e.g., using maximum likelihood techniques) for noncoherent processing of targets that exhibit uncorrelated Swerling 1 scintillation in all resolution bins as a function of the SNR. This approach

allows us to include overall target intensity (e.g., total radar cross section (RCS)) as a further factor in the target decision process.

TUTORIAL V

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A TUTORIAL OVERVIEW OF ANOMALY DETECTION IN HYPERSPECTRAL IMAGES

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In this paper, a tutorial overview on anomaly detection for hyperspectral electro-optical systems is presented. This tutorial is focused on those techniques that aim to detect small man-made anomalies typically found in defense and surveillance applications. Since a variety of methods have been proposed for detecting such targets, this tutorial places emphasis on the techniques that are either mathematically more tractable or easier to interpret physically. These methods are not only more suitable for a tutorial publication, but also an essential to a study of anomaly detection. Previous surveys on this subject have focused mainly on anomaly detectors developed in a statistical framework and have been based on well-known background statistical models. However, the most recent research trends seem to move away from the statistical framework and to focus more on deterministic and geometric concepts. This work also takes into consideration these latest trends, providing a wide theoretical review without disregarding practical recommendations about algorithm implementation. The main open research topics are addressed as well, the foremost being algorithm optimization, which is required for embodying anomaly detectors in real-time systems.

ON BAYESIAN TRACKING AND DATA FUSION: A TUTORIAL INTRODUCTION WITH EXAMPLES

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This tutorial paper provides a short introduction to selected aspects of sensor data fusion by discussing characteristic examples. We consider three cases when fusion of sensor data is important: when emphasis is placed on data produced at different instants of time (i.e., target tracking), when data being collected from different sensor sources are important, and when we have data with background information on the sensor performance as well

as data with nonsensor context information. The feedback from data processing to the data acquisition process is illustrated by a sensor management example.

PARTICLE FILTER THEORY AND PRACTICE WITH POSITIONING APPLICATIONS

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The particle filter (PF) was introduced in 1993 as a numerical approximation to the nonlinear Bayesian filtering problem, and there is today a rather mature theory as well as a number of successful applications described in literature. This tutorial serves two purposes: to survey the part of the theory that is most important for applications and to survey a number of illustrative positioning applications from which conclusions relevant for the theory can be drawn.

The theory part first surveys the nonlinear filtering problem and then describes the general PF algorithm in relation to classical solutions based on the extended Kalman filter (EKF) and the point mass filter (PMF). Tuning options, design alternatives, and user guidelines are described, and potential computational bottlenecks are identified and remedies suggested. Finally, the marginalized (or Rao-Blackwellized) PF is overviewed as a general framework for applying the PF to complex systems.

The application part is more or less a stand-alone tutorial without equations that does not require any background knowledge in statistics or nonlinear filtering. It describes a number of related positioning applications where geographical information systems provide a nonlinear measurement and where it should be obvious that classical approaches based on Kalman filters (KFs) would have poor performance. All applications are based on real data and several of them come from real-time implementations. This part also provides complete code examples.

TUTORIAL VI

REVIEW OF RANGE-BASED POSITIONING ALGORITHMS

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Intel Mobile Communications GmbH

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PETER J. G. TEUNISSEN

Curtin University of Technology

GIOVANNI BELLUSCI

Xsens

This tutorial reviews algorithms which turn measured ranges into position solutions. From their basic mathematical principles, we relate and compare relevant aspects of these algorithms. Special attention is given to the direct (non-iterative) algorithms, which are frequently applied in indoor positioning. Most of them are shown to be essentially the same, as they can be related through applying

different weighting schemes. This tutorial is intended as a useful guide to help researchers and system designers evaluate and select appropriate range-based positioning algorithms for their applications at hand.

TUTORIAL VII

BASIC TRACKING USING NONLINEAR 3D MONOSTATIC AND BISTATIC MEASUREMENTS

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Monostatic and bistatic position and Doppler measurements used in radar and sonar systems are nonlinear transformations of a Cartesian state. These nonlinearities pose a challenge for many target tracking algorithms, causing the so-called contact lens problem, which describes the nonlinear appearance of the measurement probability density function in Cartesian coordinates. This tutorial considers methods for measurement filtering (tracking without considering data association) using a single-Gaussian approximation when monostatic and bistatic position and Doppler measurements are available. The connection between the cubature Kalman filter and numerous other filtering algorithms is shown, and the accuracy and consistency of different algorithms are compared through simulation. An effort is made to express the geometric relationships associated with multistatic tracking in a simple vectorial manner. This tutorial focuses on basic tracking, and the companion tutorials “Tracking Using 3D Monostatic and Bistatic Measurements in Refractive Environments” and “Basic Tracking Using Nonlinear Continuous-Time Dynamic Models” extend the results to more sophisticated physical models.

BASIC TRACKING USING NONLINEAR 3D MONOSTATIC AND BISTATIC MEASUREMENTS IN REFRACTIVE ENVIRONMENTS

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The deleterious effects of atmospheric refraction are often overlooked in work on target tracking. This tutorial shows how the effects of refraction can be directly incorporated into tracking algorithms, improving the performance of tracking algorithms that make use of monostatic and bistatic measurements. Additionally, a technique for converting measurements from the radar’s refraction-corrupted local coordinate system (typically bistatic $r - u - v$ coordinates) into Cartesian coordinates is presented. The refraction-compensation algorithms can be used with arbitrary refraction models for which ray tracing techniques for solving boundary value problems and initial value problems are available, though extensions to more complicated propagation scenarios are possible. The algorithms

are run on a simple exponential refraction model to demonstrate their effectiveness. This tutorial builds upon the tutorial entitled “Basic Tracking Using Nonlinear 3D Monostatic and Bistatic Measurements” and is complemented by the companion tutorial “Basic Tracking Using Nonlinear Continuous-Time Dynamic Models.”

COHERENT MIMO RADAR: THE PHASED ARRAY AND ORTHOGONAL WAVEFORMS

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Coherent multiple-input multiple-output (MIMO) radar is a natural extension of the phased array antenna that has been used by radar systems for decades. This tutorial unifies concepts from the literature and provides a framework for the analysis of an arbitrary suite of MIMO radar waveforms. A number of gain patterns are introduced, which quantify the antenna performance of a MIMO radar, and the impact of the waveform characteristics (e.g., range sidelobes) is discussed.

TUTORIAL VIII

BASIC TRACKING USING NONLINEAR CONTINUOUS-TIME DYNAMIC MODELS

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Physicists generally express the motion of objects in continuous time using differential equations, whereas the majority of target tracking algorithms use discrete-time models. This tutorial considers the use of general, nonlinear, continuous time motion models for use in target tracking algorithms that perform measurements at specific, discrete times. The basics of solving/simulating deterministic/stochastic differential equations is reviewed. The difference between most direct-discrete and continuous-discrete tracking algorithms is the prediction step. Consequently, a number of continuous-time state prediction techniques are presented, focusing on derivative-free techniques. Consistent with common filtering techniques, such as the cubature Kalman filter, Gaussian approximations are used for the propagated state. Three dynamic models are considered for evaluating the performance of the algorithms: a highly nonlinear spiraling motion mode, a multidimensional geometric Brownian model, which has multiplicative noise, and an integrated Ornstein-Uhlenbeck process. Track initiation is also discussed.

NAVIGATION USING INERTIAL SENSORS

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This tutorial provides an introduction to navigation using inertial sensors, explaining the underlying principles. Topics covered include accelerometer and gyroscope technology and their characteristics, strapdown inertial navigation, attitude determination, integration and alignment, zero updates, motion constraints, pedestrian dead reckoning using step detection, and fault detection.

TUTORIAL IX

DETECTION OVER SENSOR NETWORKS: A TUTORIAL

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The applications of sensor networks (SNs) are increasing since they facilitate real-time remote information monitoring and processing. Detection of an event is one of the main tasks of SNs in many applications. Sensors may transmit either raw or processed data to a fusion center (FC), where a final decision is taken. The problems and challenges that exist in detection over SNs and the previously proposed methods to deal with them are reviewed and described in this tutorial.

A SURVEY OF CORRELATED WAVEFORM DESIGN FOR MULTIFUNCTION SOFTWARE RADAR

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To change the transmit beam pattern, single antenna radar requires a change of antenna while multiple antenna array radar, such as phased-array, require amplitude/phase tapers, which are hardware components. Therefore, there is a genuine demand that the parameters of future radar systems be controlled through a software without changing any hardware or using amplitude/phase tapers. It is well known that the parameters of an antenna array radar can be controlled by transmitting suitable correlated waveforms. This approach provides more degree-of-freedom and if constant-envelope (CE) or low peak-to-average power ratio (PAPR) correlated waveforms are used, it allows us to control the parameters of the radar without changing any hardware. Therefore, this approach can be considered as a step towards a software radar. The aim of this article is to provide a survey of recent techniques used to design CE and low PAPR correlated waveforms and discuss the benefits and drawbacks of each.

GEOMETRY OF COMPLEX DATA

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Geometric algebra has been called a “unified language for mathematics and physics.” Sometimes known as Clifford algebra, it is based on the notion of an invertible product of vectors that captures the geometric relationship between two vectors, i.e., their relative magnitudes and the angle between them. This seemingly simple concept leads to a rich system of algebra and calculus that encompasses the diverse areas of complex numbers, quaternions, vectors, tensors, spinors, and differential forms. This tutorial provides a basic introduction to geometric algebra and presents formulations of known electrical engineering and signal processing concepts to illustrate some inherent advantages of geometric algebra for formulating and solving problems involving vectors. Being introductory, the goal of the tutorial is to introduce this emerging area that, although old as a mathematics discipline, has only recently started to garner significant attention in engineering communities. Geometric algebra should give another potentially powerful tool for pursuing research in any area that uses vectors.

RECURSIVE BAYESIAN FILTERING IN CIRCULAR STATE SPACES

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To facilitate recursive state estimation in the circular domain based on circular statistics, we introduce a general framework for estimation of a circular state based on different circular distributions. Specifically, we consider the wrapped normal (WN) distribution and the von Mises distribution. We propose an estimation method for circular systems with nonlinear system and measurement functions. This is achieved by relying on efficient deterministic sampling techniques. Furthermore, we show how the calculations can be simplified in a variety of important special cases, such as systems with additive noise, as well as identity system or measurement functions, which are illustrated using an example from

aeronautics. We introduce several novel key components, particularly a distribution-free prediction algorithm, a new and superior formula for the multiplication of WN densities, and the ability to deal with nonadditive system noise. All proposed methods are thoroughly evaluated and compared with several state-of-the-art approaches. ◆

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