Harmonic Analysis in Frequency and Time Domain

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***Abstract*—This paper presents a review with a concise description and analysis of the fundamentals, characteristics, analytical details, merits, and drawbacks associated with existing methods in frequency and time domain for harmonic analysis in practical power networks. The description and analysis are centered on methods developed in the harmonic domain, hybrid frequency-time domain, and time domain, respectively. Validation of the reviewed methods for harmonic analysis, against one of the widely accepted digital simulators, such as EMTP, EMTDC, or MATLAB/SIMULINK, is reported in the cited individual contributions.**

***Index Terms*—Harmonic domain, Hartley transform, hybrid frequency-time domain, nonlinear elements, time varying.**

I. Introduction

A POWER system, operating under ideal conditions, is expected to be perfectly balanced, of a single frequency, and with sinusoidal current and voltage waveforms of constant amplitude. However, in practical power systems, this ideal operational mode is not encountered, since the network components, at a lesser or greater extent, have the undesirable effect of distorting the original sinusoidal waveforms. Nonlinear components and loads, such as power converters, flexible ac transmission systems (FACTS) devices, nonlinear saturation and hysteresis in components with magnetic cores, fluorescent lamps and arc furnaces, among others, represent the main contribution to this effect, known as *harmonic distortion*. Adverse quality of power effects due to harmonic distortion are, for instance, interference with communication, control and protection systems, reduction of the equipment's life span, and additional losses in the power system.

Diverse harmonic distortion issues, such as its causes, effect on quality of power, standards and mitigation techniques, are described in literature [1], [2].

Digital harmonic analysis relies on harmonic detection and prediction, respectively. The first processes in real-time data of the monitored harmonic content in the network, while the last relies on computer simulations to predict the harmonic distortion through implemented analytical models. The methods to be described in this paper belong to this second category.

Harmonic analysis has been carried-out using frequency, time and hybrid time-frequency domain methods. The conceptual and analytical details of these methods are concisely detailed in this contribution. A concise overview on simulation methods for harmonic analysis has been previously reported [3]. Their application was illustrated in a companion paper with examples and sample systems [4]. Further advances on methods for harmonic analysis in frequency and time domain are detailed in [5], [6] and more recently in [7], where in addition to the above, an alternate sub-division of methods for harmonic analysis is proposed.

This paper presents a concise yet detailed revision of theoretical fundamentals and principles of classical methods for harmonic analysis. A precise and simple classification of methods is given.

II. Methods for Harmonic Analysis

Different methods for harmonic analysis in frequency domain and time domain are currently detailed in the open literature. Hybrid frequency and time domain methods have been developed with the purpose of combining the individual advantages of the frequency and time domain methods. The fundamentals and theoretical principles of these methods are given next.

A. Frequency Domain

In general, available methods for harmonic analysis in the frequency domain are divided into direct method, iterative harmonic analysis and harmonic power flow methods, respectively.

1) Direct Method: The frequency response of the power system, as seen by the bus of interest, is obtained through injection of a one per-unit current or voltage at discrete frequency steps for the particular range of frequencies. The process is based on the solution of the network equation

$$[\mathbf{Y}]\mathbf{V} = \mathbf{I} \tag{1}$$

where $[\mathbf{Y}]$ is the network admittance matrix, $\mathbf{V}$ is the nodal voltage vector and $\mathbf{I}$ is vector of current injections, with only one nonzero entry.

The simplest current source method uses the sequence component framework by injecting ideal current sources into the power network [8]. In a later contribution, the solution is directly obtained in the phase domain for three-phase unbalanced systems [9]. Harmonic decoupled circuits are assumed in both methods.

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Under normal conditions, the classic ideal current source representation can be used quite accurately for most of the electronic loads. However, the ideal current injection may give erroneous results for arc furnaces and electronic converters under resonance conditions. Thus, to incorporate a dependence of the generated harmonic current with the system impedance may prove to be an important factor in planning studies.

Regarding the interaction between harmonic sources, measurements have shown changes in the current waveform. In most cases, the voltages were reasonably steady and only slightly distorted, implying only minor interaction with the system [10]. In this specific case, the harmonic source seems affected by the system impedance than by the harmonic interaction between sources. The occurrence of a parallel resonance can cause a significant reduction in the harmonic current generated at that frequency [11]. Two alternate ways can be used to represent the harmonic sources and thus to minimize this problem: 1) The nonideal current source is used, and the shunt impedance is estimated. This would particularly apply to arc furnaces as the internal impedance of an arc furnace cannot be assumed infinite. Experiments have suggested an inductance L in series with a resistance R, defined as [12], should be incorporated, i.e.,

$$L = (X_{\mathrm{LV}} + X_{\mathrm{TF}})/\omega \tag{2}$$

$$R = S_N \cos(\phi)/3I_N = L/\tan(\phi) \tag{3}$$

where X_{LV} is the reactance of the LV connection of the furnace, X_{TF} is the leakage inductance of the furnace transformer, L is the furnace short-circuit inductance calculated for the melting period, S_N is the apparent power and I_N is the apparent current. Alternately, if a capacitance is connected very close to the harmonic source, the system impedance and the injected currents can significantly change. Therefore, the produced harmonic currents should be specified together with the shunt impedance or capacitance directly connected to the supply point. 2) The voltage source or current re-injection method can be used. This would consist of correcting the estimated harmonic current injected due to changes in the system impedance. The following procedure is used:

a) Harmonic currents are specified by assuming an inductive supply system.
b) Harmonic voltages are calculated by multiplying the estimated currents by the system inductive reactance.
c) The new currents are calculated by dividing the voltages in (b) by the actual system impedance and are injected to perform the study.

A similar method has been incorporated into harmonic penetration programs where a voltage is applied instead of corrected currents [10], [13]. The inductive reactance is assumed equal to the leakage reactance of the transformer connecting the harmonic source. Injected harmonic currents are generally phase unbalanced; therefore, unbalances should be represented.

Voltage and Current Excitations: Power system nonlinearities manifest themselves as harmonic current sources. Harmonic voltage sources are sometimes used to represent the distortion background in the network, prior to the installation of the new nonlinear load. Moreover, some power-electronic apparatus based on GTO and IGBT switching act as voltage sources behind an impedance.

A system containing harmonic voltages at some busses and harmonic current injections at other busses can be solved by partitioning the admittance matrix and performing a partial inversion. This hybrid solution procedure allows the unknown bus harmonic voltages and unknown harmonic currents to be obtained. If $\mathbf{V}_k$ represents the known voltage sources and $\mathbf{I}_k$ the unknown variables, the remaining busses are represented as a harmonic current injection $\mathbf{I}_i$, which can be either zero or specified by a harmonic current source. The harmonic voltage vector $\mathbf{V}_i$ represents the unknown variables.

Partition of the matrix equation to separate the two types of nodes gives

$$\begin{bmatrix} \mathbf{Y}_{ii} & \mathbf{Y}_{ik} \\ \mathbf{Y}_{ki} & \mathbf{Y}_{kk} \end{bmatrix} \begin{bmatrix} \mathbf{V}_i \\ \mathbf{V}_k \end{bmatrix} = \begin{bmatrix} \mathbf{I}_i \\ \mathbf{I}_k \end{bmatrix}. \tag{4}$$

The unknown voltage vector $\boldsymbol{V}_i$ is obtained from

$$[\mathbf{Y}_{ii}]\mathbf{V}_i = \mathbf{I}_i - [\mathbf{Y}_{ik}]\mathbf{V}_k. \tag{5}$$

The harmonic currents injected by the harmonic voltage sources are calculated as

$$\mathbf{I}_k = [\mathbf{Y}_{ki}]\mathbf{V}_i + [\mathbf{Y}_{kk}]\mathbf{V}_k. \tag{6}$$

Some additional processing is required to obtain the reduced admittance matrix, which is not a result of the solution.

A sparsity-oriented technique for the solution of nonlinear networks with hybrid voltage and current excitations has been proposed for harmonic analysis [14].

2) Iterative Harmonic Analysis (IHA): The IHA is based on the following iterative process of the sequential substitutions Gauss type:

1) The harmonic producing device is modeled as a supply voltage-dependent current source, represented at each iteration by a fixed harmonic current source.
2) The problem is first solved using an estimated supply voltage to obtain the harmonic currents. In turn, harmonic currents are used to obtain the harmonic voltages.
3) These harmonic voltages allow the computation of more accurate harmonic currents. The iterative solution process stops once the changes in harmonic currents are sufficiently small [15]–[17].

Distorted and nondistorted conditions can be handled with this method. One of the main advantages of the IHA method is that the power network components can be modeled in a closed form, with time domain simulation.

However, the slow convergence characteristic and narrow stability margin of the IHA has limited its application to the solution of practical power systems. To ensure convergence, numerical diagonal dominance of the matrix of system parameters is required. However, this is not a satisfied condition by weak or poorly damped systems or near sharply tuned resonant frequencies [15], [16]. A double-iterative method has been proposed in [17] to improve the convergence characteristics of the IHA.

3) Harmonic Power Flow Method (HPF): The HPF method takes into account the voltage-dependent nature of power components. In general, the voltage and current harmonic equations are simultaneously solved using Newton-type algorithms [18]–[25].

A general frame of reference where nodes, phases, phase unbalance, linear, nonlinear and time-varying components, harmonics and harmonic cross-coupling effects are explicitly represented and iteratively solved in a unified fashion was formally introduced in [23] as Harmonic Domain (HD); its principles are found in an earlier contribution [19].

The solution is based on a linearization process around a particular operation point. This is a valid condition only in a close neighborhood of the operation point. The linearization process results in a Norton harmonic equivalent where the phase unbalance and harmonic cross-coupling effects are explicitly represented. The computational effort increases in direct proportion to the size of the system to be analyzed and to the number of harmonics explicitly represented.

The main steps of the HD solution process can be summarized as follows [23].

1) Input data, given by a load flow solution, to obtain terminal voltages, generator electromotive forces (emfs), and load admittances, at fundamental frequency.
2) Form the system harmonic admittance matrix (linear network).
3) Linearize each nonlinear component in the Harmonic Domain around an operation point $\mathbf{I}_{\mathrm{b}}$, $\mathbf{V}_b$ as

$$\Delta\mathbf{V} = \mathbf{V} - \mathbf{V}_b \tag{7}$$

$$\Delta\mathbf{I} = \mathbf{I} - \mathbf{I}_b. \tag{8}$$

For each nonlinear function, usually representing a nonlinear component, the linearized general equation has the form [23],

$$\Delta\mathbf{I} = [\mathbf{F}]\Delta\mathbf{V} \tag{9}$$

where, as shown in (10) at the bottom of the page, the substitution of (7) and (8) into (9) gives

$$\mathbf{I} = [\mathbf{F}]\mathbf{V} + \mathbf{I}_N \tag{11}$$

where

$$\mathbf{I}_N = \mathbf{I}_b - [\mathbf{F}]\mathbf{V}_b. \tag{12}$$

4) Equation (11) can be interpreted as a Norton Harmonic equivalent for the nonlinear component and represents the linearization process in the Harmonic Domain [23].
5) Combine linear and linearized networks, in the unified representation for the entire system, i.e.,

$$\Delta\mathbf{I} = [\mathbf{Y}_j]\Delta\mathbf{V} \tag{13}$$

where $\Delta\mathbf{I}$ is the vector of incremental currents having the contribution of nonlinear components, $\mathbf{\Delta V}$ is the vector of incremental voltages and $[\mathbf{Y}_j]$ is the admittance matrix of linear and nonlinear components. The later components are represented in each case by the computed Norton harmonic equivalent (11).
6) Solve the full linearized system to calculate harmonic voltages.
7) Check if convergence criterion is met; if done proceed with output of results otherwise re-start from step 3.

This is a numerically robust methodology having, in addition, good convergence properties [23].

Detailed models of power system components, such as the synchronous machine [26]–[28], the power transformer [21], [29]–[32], arc furnaces [33], fluorescent lamps [33], [34], Thyristor Commutated Reactors (TCRs) [35], the power converter [36]–[39], HVDC systems [40], [41], adjustable speed drives (ASDs) [42], static var systems (SVSs) [43], the unified power flow controller (UPFC) [44], the static compensator (STATCOM) [45], the Static Synchronous Series Compensator (SSSC) [46] and for the interaction between generation and transmission systems [47]–[50] have been developed in the HD.

4) Dynamic Harmonic Domain Method (DHD): In [25], the HD has been expanded to also represent the interaction between fundamental, harmonic and inter-harmonic frequency components. In other contributions [50]–[52] a Newton method is proposed based on the instantaneous power balance formulation for the representation of linear and nonlinear loads. While the DHD modeling of individual power system elements is relatively straightforward, the interfacing of several components [50] is still quite challenging, and is an ongoing topic of research.

5) Coupled $\mathbf{Y}$ *Matrix Method:* One of the recent progresses in harmonic analysis is the finding that common power-electronic devices such as thyristor-controlled reactors (TCRs) and

$$[\boldsymbol{F}] = \begin{pmatrix} c_o & \cdots & c_{n-1} & c_n & & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & & \\ c_{-(n-1)} & \ddots & c_o & \ddots & c_{n-1} & c_n & \\ c_{-n} & c_{-(n-1)} & \ddots & c_o & \ddots & c_{n-1} & c_n \\ & c_{-n} & c_{-(n-1)} & \ddots & c_o & \ddots & c_{n-1} \\ & & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & c_{-n} & c_{-(n-1)} & \cdots & c_0 \end{pmatrix} \tag{10}$$

bridge converters can be modeled using a coupled matrix equation [53]. For example, the model for dc drive has the following form,

$$\mathbf{I} = \mathbf{Y}^{+}\mathbf{V} + \mathbf{Y}^{-}\hat{\mathbf{V}} - \mathbf{Y}^{0}E_{\rm dc} \tag{14}$$

where $\mathbf{I}$ and $\mathbf{V}$ are the ac-side supply current and voltage harmonic vectors and $E_{\rm dc}$ is dc motor's internal voltage source. The linearity is due to the fact that the sequence admittance matrices, $\mathbf{Y}^{+}$, $\mathbf{Y}^{-}$, and $\mathbf{Y}^{0}$ are independent of $\mathbf{V}_{\rm ac}$. Because of this feature, an accurate, noniterative harmonic power flow method can be established, as explained below.

The method first computes the fundamental frequency power flow by treating the converters as constant power loads. From the results, the converter firing angles and dc voltage source are determined and the entries of Y matrices are then calculated. Term $\mathbf{Y}^{0}E_{\rm dc}$ becomes known as current sources.

The rest of the network is linear and can be modeled as a harmonically-decoupled admittance matrix as follows:

$$\begin{bmatrix} [\mathbf{I}]_5 \\ [\mathbf{I}]_7 \\ \cdots \end{bmatrix} = \begin{bmatrix} [\mathbf{Y}]_5 & 0 & \cdots \\ 0 & [\mathbf{Y}]_7 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} [\mathbf{V}]_5 \\ [\mathbf{V}]_7 \\ \cdots \end{bmatrix} \tag{15}$$

where $[\mathrm{I}]_h$ is the current injection vector at various buses, including the buses where the dc-drives are connected and where h is the harmonic order. Equations (14) and (15) are both linear equations. They can be solved together without iteration. The results are harmonic voltages at each bus $[\mathbf{V}]_h$ that have included the impact of harmonic currents from all harmonic sources. More details on this solution technique can be found in [53].

6) Multiphase Harmonic Analysis: In [22], the harmonic power flow problem is formulated in the multiphase domain, i.e., all three phases and other conductors such as neutrals of a distribution feeder are represented explicitly. Harmonic-producing loads and linear loads are also represented in three phases. This multiphase approach to harmonic analysis offers some unique advantages. This formulation is essential for assessing the impact of single-phase harmonic loads such as compact fluorescent lights and energy-saving home appliances on power distribution systems [54]. Although such loads produce insignificant harmonic currents individually, the collective effect of a large number of them can be substantial. They have become the main source of harmonic distortion in current residential distribution systems. The second advantage is the capability of assessing the impact of noncharacteristic harmonics. These harmonics are generated under unbalanced conditions and could cause problems since mitigation measures are normally not designed for them [55]. The third advantage is that a multiphase model can easily represent the harmonic phase-shift effects of various transformers on harmonics [56].

B. Time Domain

1) Conventional (Brute Force) Solution: In principle, the periodic steady-state solution of a power network can be obtained directly in the time domain by integration of the differential equations describing the system, once the transient response has died-out [57]. This brute force (BF) procedure [58] may require integration over considerable periods of time. It has been suggested only for cases where the periodic response can be obtained in a few cycles [16], such as sufficiently damped systems. The general description of nonlinear and time-varying elements is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{16}$$

where $\mathbf{xx}$ is the state vector of n elements.

The solution (16) is based on a conventional process such as Runge-Kutta (RK) or Trapezoidal Rule (TR). Although in the absence of numerical instability this process leads to the "exact" solution [58], for a broader scope of analysis it is inefficient and inadequate to obtain the periodic steady state of power systems.

Practical power networks can be solved in the time domain with a state space matrix equation representation based on nonautonomous ordinary differential equations of the form

$$\dot{\mathbf{x}} = [\mathbf{A}]\mathbf{x} + [\mathbf{B}]\mathbf{u} \tag{17}$$

where $[\mathbf{A}]$ is the state matrix of size $n \times n$, $[\mathbf{B}]$ is the control or input matrix of size $n \times r$ and $\mathbf{u}$ is the input vector of dimension r.

Widely accepted digital simulators for electromagnetic transient analysis, such as EMTP and EMTDC can be used for steady-state analysis. However, the solution process can be potentially inefficient, as detailed before. Here, a discrete time domain solution for any integration step length Δt is adopted, where the basic elements of the power network, e.g., R, L and C are represented with Norton equivalents depending on Δt. Other power network elements are formed with the adequate combination of R, L and C parameters, which are in turn combined together for a unified solution of the complete network in the time domain, i.e.,

$$[\mathbf{G}]\mathbf{v}(t) = \mathbf{i}(t) + \mathbf{I}_H \tag{18}$$

where $[\mathbf{G}]$ is the conductance matrix, $\mathbf{v}(t)$ the unknown voltages at time t, $\mathbf{i}(t)$ the vector of nodal current sources and $\mathbf{I}_H$ the vector of past history current sources. The solution of (18) is obtained in EMTP and EMTDC using the TR method.

2) Fast Periodic Steady-State Solution: A method based on Newton iterations has been used to obtain the periodic steady-state solution of power networks without the computation of the complete transient [59]. In a later contribution [60], convergence speed-up in time domain of state variables to the Limit Cycle [58], [61] was achieved based on Newton methods in the time domain. The state variables $\mathbf{x}^{\infty}$ at the limit cycle are estimated with these methods as [60]

$$\mathbf{x}^{\infty} = \mathbf{x}^{i} + (\mathbf{I} - \mathbf{\Phi})^{-1}(\mathbf{x}^{i+1} - \mathbf{x}^{i}) \tag{19}$$

where

$$\mathbf{\Phi} = \partial\mathbf{x}(t+T)/\partial\mathbf{x}(t). \tag{20}$$

In (19), $\mathbf{x}^{\infty}$, $\mathbf{x}^{i}$ and $\mathbf{x}^{i+1}$ are the vectors of state variables at the limit cycle, beginning and end of the base cycle, respectively. On the other hand, $\mathbf{I}$ and $\mathbf{\Phi}$ are the unit and identification (state transition) matrices, respectively. Additionally, $\mathbf{x}^{i+1}$ and $\mathbf{x}^{i}$ are $\mathbf{x}(t+T)$ and $\mathbf{x}(t)$, respectively. The Newton method (19) is suitable for computing unstable and stable limit cycles. $\mathbf{x}^{i+1}$ is obtained through integration from $\mathbf{x}^i$ over one period. Once $\mathbf{x}^\infty$ is computed using (19), $\mathbf{x}^i$ is equated to $\mathbf{x}^\infty$ and the iteration (19) is repeated until two consecutive state vectors meet a convergence criterion error. Further details are given in [60].

Different ways to compute the state transition matrix have been proposed [60], [62]–[64]. In [60] the numerical differentiation (ND), direct approach (DA) and matrix exponential (ME) methods were proposed. Results were reported for the application of the ND and DA methods in nonlinear single-phase power systems. Taking advantage of the half-cycle waveform symmetry in practical power systems, the efficiency of the ND method has been improved in the order of at least 100%, with the enhanced numerical differentiation method (END) [62]. In a more recent contribution [63], a discrete exponential expansion (DEE) method was proposed, which allows a one-step identification of the $\mathbf{\Phi}$. In [64], the state transition matrix is calculated by transiently simulating over one period, a set of sensitivity circuits whose topologies are the same as that of the original system to be solved.

All these methods approximate the state transition matrix expressed by (20) through finite-difference derivatives as,

$$\mathbf{\Phi} \approx \Delta\mathbf{x}(t+T)/\Delta\mathbf{x}(t). \tag{21}$$

A concise description of the ND, DA, END, DEE, and sensitivity circuit methods is given next.

a) Numerical Differentiation Method (ND) [60]: In this method, $\mathbf{\Phi}$ is computed in a column-by-column process. The kth column of $\mathbf{\Phi}$ is Φ_k for $k = 1, 2, \ldots, n$. This column can be computed by perturbing the kth state, i.e., let $\mathbf{x}(t) \rightarrow \mathbf{x}(t) + \Delta\mathbf{x}_k(t)$ and compute $\mathbf{x}(t+T) + \Delta\mathbf{x}_k(t+T)$ by numerical integration of (16) over one period with the initial condition $\mathbf{x}(t) \rightarrow \mathbf{x}(t) + \Delta\mathbf{x}_k(t)$. Notice that if $\Delta\mathbf{x}_k(t)$ is equal to $\varepsilon\mathrm{U}_k$, being ε a small real number, e.g., 10^{-6}, and U_k the kth column of a identity matrix of dimension n, for $k = 1, 2, \ldots, n$, then, considering (21),

$$\Delta\mathbf{x}_k(t+T) = \mathbf{\Phi}\varepsilon\mathrm{U}_k \tag{22}$$

and consequently

$$\Delta\mathbf{x}_k(t+T) = \varepsilon\Phi_k. \tag{23}$$

Therefore

$$\Phi_k = \Delta\mathbf{x}(t+T)/\varepsilon. \tag{24}$$

Each column of $\mathbf{\Phi}$ can be computed with (24). All n states of the system (16) must be perturbed separately in order to compute the n columns of the sensitivity matrix. Note that $n+1$ cycles must be computed before we can apply (19).

b) Direct Approach Method (DA) [60]: The difference between the DA and the ND methods is the way the vector $\Delta\mathbf{x}_k(t+T)$ is computed as well as the selection of the initial condition $\Delta\mathbf{x}_k(t)$. In the DA method, $\Delta\mathbf{x}_k(t+T)$ is obtained by direct integration of $\Delta\dot{\mathbf{x}} = \mathbf{J}(t)\Delta\mathbf{x}$ over one period T, using $\Delta\mathbf{x}_k(t)$ as initial condition. $\mathbf{J}(t)$ is the Jacobian matrix of $\mathbf{f}(\mathbf{x}, t)$. Notice that if $\Delta\mathbf{x}_k(t)$ is U_k, from (21)

$$\Delta\mathbf{x}_k(t+T) = \mathbf{\Phi}\mathrm{U}_k \tag{25}$$

or

$$\Delta\mathbf{x}_k(t+T) = \Phi_k. \tag{26}$$

This DA method also requires $n+1$ cycles to compute the n colums of $\mathbf{\Phi}$. $\mathbf{J}(t)$ is time-varying and is evaluated using $\mathbf{x}(t)$ from t to $t+T$.

c) Enhanced Numerical Differentiation Method (END) [62]: The END method exploits the half-wave symmetry of the input signal such as voltage and current sources. This methodology consists on the evaluation of (21) by the approximation of $\mathbf{x}(t+T)$ through the extrapolation of $\mathbf{x}(t+(T/2))$. Using this method, the integration of (16) for the computation of $\mathbf{x}(t+T)$ is not required to be carried-out over one full cycle of period T, but only over a half period, which can double the efficiency of the ND method.

Let $x_i(t)$ be the ith state variable of $\mathbf{x}(t)$. For practical power systems, in the neighborhood of the limit cycle, $x_i(t)$ satisfies the following condition if it contains dc component:

$$x_i(t+T) \approx x_i(t+T/2). \tag{27}$$

On the other hand, in the neighborhood of the limit cycle $x_i(t)$ satisfies the following condition if it does not contain dc component:

$$x_i(t+T) \approx -x_i(t+T/2). \tag{28}$$

In order to properly use the conditions (27) and (28), $x_i(t)$ with dc component is identified from those without dc component. If $x_i(t)$ has dc component, then it satisfies the following condition:

$$\frac{dx_i(t)}{dt}\frac{dx_i(t+T/2)}{dt} > 0. \tag{29}$$

In summary, if $x_i(t)$ satisfies (29), (27) is used to approximate $x_i(t+T)$ from $x_i(t+T/2)$, otherwise (28) is used. Notice that $x_i(t+T)$ and $x_i(t)$ are the ith elements of $\mathbf{x}^{i+1}$ and $\mathbf{x}^i$, respectively. Therefore, in order to compute $\mathbf{x}(t+T)$, integrate (16) over one half cycle with the initial condition $\mathbf{x}(t)$ to obtain $\mathbf{x}(t+T/2)$, and then use (27) and (28) to compute $\mathbf{x}(t+T)$ from $\mathbf{x}(t+T/2)$ In the same way, compute $\mathbf{x}(t+T) + \Delta\mathbf{x}_k(t+T)$ from $\mathbf{x}(t) \rightarrow \mathbf{x}(t) + \Delta\mathbf{x}_k(t)$ and calculate $\mathbf{\Phi}_k$ with (24) for $k = 1, 2, 3, \ldots n$. In this END method, $\Delta\mathbf{x}_k(t)$ is chosen as in the ND method described above.

d) Discrete Exponential Expansion Method [63]: In the DEE method, the Jacobian of the Poincaré map $\mathbf{\Phi}$ is obtained by following a step-by-step identification procedure based on a recursive formulation. Each computation of $\mathbf{\Phi}$ in the DEE method requires a numerical integration over a single period of time T. The DEE method approximates the transition matrix as

$$\mathbf{\Phi} \approx \prod_{i=0}^{N-1} e^{\mathbf{J}_{N-i}\Delta t_{N-i}} \tag{30}$$

where the Jacobian matrix $\mathbf{J}_i$ is given by

$$\mathbf{J}_i = \partial\mathbf{f}(\mathbf{x}, t)/\partial\mathbf{x}|_{t=(t_i+t_{i-1})/2, \quad \mathbf{x}=(\mathbf{x}(t_i)+\mathbf{x}(t_{i-1}))/2}. \tag{31}$$

In (30) and (31), Δt_i is defined as $t_i - t_{i-1}$, N is the number of intervals in one period T, and t_i represents the ith element of the time vector from t to $t+T$.

In the ND, DA, END, and DEE methods, $\mathbf{x}^i$ represents the starting point of the Newton method and this can be computed by direct integration of (16) over some cycles using an appropriate initial condition.

e) Sensitivity Circuits [64]: The concept of sensitivity circuits is introduced in [64], which is briefly described here:

All the circuits to be solved can be described in terms of KCL, KVL, and branch constitutive equations (BCE). Taking the partial derivatives of these equations with respect to the initial conditions, $\mathbf{x}(0)$, results in the "sensitivity circuits". The Jacobian matrix can be directly obtained simulating these circuits over one period. The circuit topologies of the sensitivity circuits are identical to the original circuits, except that

1) all the independent sources in the sensitivity circuits are set to zero.
2) the initial conditions of the dynamic elements, such as L and C, with respect to its own sensitivity is set to 1, and all the other initial conditions are 0.

Therefore, such a method can be easily incorporated into an EMTP-type program.

The Newton methods described above have been applied to obtain the periodic steady-state solution of power systems containing synchronous machines [65], thyristor commutated switched capacitors (TCSCs) [66], TCRs [67], static var systems (SVS) [68], parallel ac/dc converters [69], unified power flow controller (UPFC) [70], dynamic voltage restorer (DVR) [71], STATCOM [72], asynchronous wind generator [73], and adjustable speed drives (ASD) [74], among others.

Remarks:

1) The ND method is based on a simple procedure for the identification of $\mathbf{\Phi}$. It is very suitable when the explicit determination of partial derivatives can not be easily obtained, such as in case of commutated devices operation.
2) The DA method has a straight forward application when the above restriction does not exist.
3) Taking advantage of the half wave symmetry of input signals, the END method can improve the efficiency of the original ND method in nearly 100%.
4) The DEE method identifies in a single step$\mathbf{\Phi}$, thus avoiding the sequential perturbation of state variables to identify, in turn, column-by-column $\mathbf{\Phi}$.
5) The efficiency of the Newton methods can substantially improve and the computational effort decrease with update variants of $\mathbf{\Phi}$ during the iterative process [60].
6) The sensitivity circuit approach can be improved by using Quasi-Newton's method, so that only a total of $\mathrm{n}+1$ one-period simulation is required throughout the iteration process [75].
7) A fast Fourier transform (FFT) or a discrete Fourier transform (DFT) is applied to obtain the harmonic content of the periodic steady-state solution.

The efficient computation in the time of the periodic steady state of electric networks can be applied for the initialization of software for the analysis of electromagnetic transients [76], [77].

C. Hybrid Methods

In a hybrid methodology [60], the power components are represented in their natural frames of reference, i.e., the frequency domain for the case of linear components, such as transmission lines with distributed, frequency dependent parameters, and the time domain for the case of nonlinear and time-varying components, e.g., loads.

The voltages $\mathbf{V}$ at the load nodes where the nonlinear components are connected are iteratively obtained. The iterative solution for the entire system has the form [60]

$$[\mathbf{Y}_h]\Delta\mathbf{V} = \Delta\mathbf{I}. \tag{32}$$

The iterative computation of (32) is based on the following process [60]:

1) Obtain the steady-state solution given by a power flow method, at fundamental frequency.
2) Calculate a current mismatch $\Delta\mathbf{I}_h$ for all harmonics, followed by a voltage update $\Delta\mathbf{V}_h$. Starting from estimated $\mathbf{V}$ values. For the linear part, it obtains $\mathbf{I}_h = [\mathbf{Y}_h]\mathbf{V}_h$ for all harmonics, which defines $\mathbf{I}_L$; and for each nonlinear or time-varying load it performs a time domain simulation to obtain the periodic steady-state solution $\mathbf{i}(t)$ with $\mathbf{v}(t)$ as input. This is then transformed back to $\mathbf{I}_N$ in the harmonic domain. The mismatch vector $\Delta\mathbf{I}_h$ (containing all load buses) is then used with an appropriate iteration matrix $[\tilde{\mathbf{Y}}_h]$, equal or close to $\mathbf{Y}_h$, to obtain the update increment $\Delta\mathbf{V}_h$ as,
$$[\tilde{\mathbf{Y}}]\Delta\mathbf{V}_h = \Delta\mathbf{I}_h. \tag{33}$$
3) The time domain simulation is accelerated by noting that the dynamics of cycles in the neighborhood of a limit cycle is almost linear, so that the intercepts with a Poincare plane can be used to extrapolate to the limit cycle by a Newton Method based on a ND, END, DA, ME, or DEE procedure, respectively.
4) In convergence $\Delta\mathbf{I} = \mathbf{I_L} + \mathbf{I_N}$ tends to zero. However, before convergence, $\mathbf{V}$ is not yet accurately known so that a mismatch $\Delta\mathbf{I} = \mathbf{I_L} + \mathbf{I_N}$ will result. As seen from the bus, the system has the harmonic admittance $\mathrm{Y_h}$ plus the admittance of the nonlinear part; obtained with the procedure detailed in [60]. The solution of (33) will allow the determination of the voltage correction $\Delta\mathbf{V}_h$. The iterative process is re-started from step 1 if convergence is not yet achieved.

The methodology above described has been applied to the solution of single-phase power systems [60] and of a three-phase power network with a commutated device [78].

Another hybrid time and frequency domain formulation is detailed in [79]. Here, each nonlinear part or load is solved in the time domain using a shooting method [59] and the obtained frequency scan is combined and iteratively solved with the linear network represented in the frequency domain by a Norton equivalent at the interface load buses.

D. Hartley Transform

For the calculation of nonsinusoidal voltages and currents, especially in the presence of power-electronic apparatus, the Hartley transform (HT) has proven to be a computationally efficient tool [80]–[84]. The Hartley is a real-valued transform unlike the Fourier transform (FT) which is a complex-valued transform. Therefore, convolution operations calculated using HT show significant speedup while maintaining the same accuracy as the (FT).

III. Conclusion

A description has been given on the fundamentals of the methods for harmonic analysis in power systems, developed in the frames of reference of frequency, time, and hybrid time-frequency domain, respectively.

In general, harmonic power flow methods are numerically robust and have good convergence properties. However, their application to obtain the nonsinusoidal periodic solution of the power system may require the iterative process of a matrix equation problem of very high dimensions. The principles of a noniterative $\mathbf{Y}$ matrix method have been detailed.

Conventional brute force methodologies in the time domain for the computation of the periodic steady state in the power system are, in general, an inefficient alternative which, in addition, may not be sufficiently reliable, in particular, for the solution of poorly damped systems. The potential of Newton methods for the fast convergence of state variables to the Limit Cycle has been illustrated. Their application yields efficient time domain periodic steady-state solutions. In addition, the concepts of the sensitivity circuits approach have been described.

The principles of a hybrid time and frequency domain methodology have been detailed. They have been applied to the solution of single-phase power systems, and to a lower scale three-phase power network. Attention has been given to methods based on the Hartley transform for the calculation of nonsinusoidal voltages and currents, in the presence of power-electronic apparatus.

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