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Threat sequencing of multiple UCAVs with incomplete information based on game theory

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Abstract: The threat sequencing of multiple unmanned combat air vehicles (UCAVs) is a multi-attribute decision-making (MADM) problem. In the threat sequencing process of multiple UCAVs, due to the strong confrontation and high dynamics of the air combat environment, the weight coefficients of the threat indicators are usually time-varying. Moreover, the air combat data is difficult to be obtained accurately. In this study, a threat sequencing method of multiple UCAVs is proposed based on game theory by considering the incomplete information. Firstly, a zero-sum game model of decision maker ($\mathcal{D}$) and nature ($\mathcal{N}$) with fuzzy payoffs is established to obtain the uncertain parameters which are the weight coefficient parameters of the threat indicators and the interval parameters of the threat matrix. Then, the established zero-sum game with fuzzy payoffs is transformed into a zero-sum game with crisp payoffs (matrix game) to solve. Moreover, a decision rule is addressed for the threat sequencing problem of multiple UCAVs based on the obtained uncertain parameters. Finally, numerical simulation results are presented to show the effectiveness of the proposed approach.

Keywords: threat sequencing, multiple unmanned combat air vehicles (UCAVs), multi-attribute decision-making (MADM), game theory, incomplete information.

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1. Introduction

With the rapid development of information technology and the gradual deepening of the world's military revolution, the unmanned combat air vehicle (UCAV) has gradually become an important entity in modern air combat [1,2]. Compared with manned aircraft, UCAV has many combat advantages, such as flexible use, rapid response, low cost, and zero casualties [3]. Since multiple UCAVs coordinated operations have greater offensive advantages, they often participate in combat as a swarm [4,5]. However, when the UCAVs are hired by the enemy, the large-scale scattered UCAV targets will bring great challenges to our attack and defense. We know that a reasonable threat sequencing of multiple UCAVs is a prerequisite for the attack-defense decision-making problem. Therefore, it is crucial to study an efficient threat sequencing approach for multiple UCAVs.

In fact, the threat sequencing problem of multiple UCAVs means to convert several threat attribute values into the comprehensive ones, so as to sequence the threat targets based on the comprehensive threats [6,7]. Essentially, the threat sequencing of multiple UCAVs is a multi-attribute decision-making (MADM) problem. However, due to the high complexity and strong dynamics of the air combat environment, it is difficult to obtain the complete information needed for threat sequencing. In the threat sequencing problem, the incomplete information is mainly manifested in two aspects: (i) the time-varying weight coefficients of different threat indicators; (ii) the inaccurate air combat data. The incompleteness of information brings great challenges to the threat sequencing problem, which is seldom considered in the existing literature [8]. Therefore, it is of far-reaching significance to develop an effective method for the threat sequencing problem of multiple UCAVs under incomplete information conditions [9,10].

In the existing literature, the methods to obtain the weights of an MADM problem can be classified into the subjective weighting methods, the objective weighting methods, and the combined weighting methods. The subjective weighting methods have a long history, but their subjective randomness is greater. Commonly used subjective weighting methods include the language measurement method [11], the Delphi method [12], and the analytic hierarchy process (AHP) [13]. The objective weighting methods have a stronger mathematical theoretical basis than the subjective weighting methods, and their calculation process is relatively complicated. However, as entirely data-based approaches, they do not take into

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account the perception of the decision maker. Commonly used objective weighting methods include the entropy method [14], the dispersion maximization method [15], the principal component analysis method [16], and the multiple targets programming method [17]. Compared with the previous two methods, the combined weighting method can both take into account the importance of the decision maker's perception and the numerical rules contained in decision data. Essentially, the combination weighting method is the fusion of multiple individual weights obtained by different subjective and objective weightings [18]. Although many methods have been proposed to determine the weights of an MADM problem with inaccurate data, their strong subjectivity and poor flexibility cannot be well applied to the actual threat sequencing of multiple UCAVs in the high dynamics and strong confrontation environment.

Game theory is an effective mathematical tool to study the strategic interaction of two or more decision makers. Since Von Neumann et al. [19] started the study of the game theory, it has been applied in the fields of economics, political science, international relations, engineering, computer science, military, biological evolution, etc [20–22]. In [23], a matrix game model was established to generate maneuvering decisions for low-flying aircraft during one-on-one air combat over hilly terrain. The maneuvering decisions were made by comparing scores of two aircrafts' orientation, range, velocity, and terrain clearance. In [24], a two-person cooperative game was presented to achieve user cooperation diversity for time division multiple access (TDMA) based commercial cooperative communication networks. It was proved that the game is indeed a two-person bargaining problem, which had a unique Nash equilibrium solution. In [25], a stochastic differential game approach was proposed to model the duopolistic competition with sticky prices, and its feedback Nash equilibrium was analytically derived. In [26], a zero-sum game approach was proposed, which was an innovative approach for MADM problems.

In this study, borrowing the idea from [26], we construct a virtual person Nature ($\mathcal{N}$) to determine the uncertain parameters (the weight coefficient parameters of the threat indicators and the interval parameters of the threat matrix), which acts as an "intelligent enemy" for decision maker ($\mathcal{D}$). This is based on the following consideration: in a system with a lack of prior knowledge, $\mathcal{D}$ has no knowledge of the uncertain parameters at all, so these parameters are considered to be determined by $\mathcal{N}$. Moreover, the threat sequencing process is modeled as a zero-sum game of $\mathcal{D}$ and $\mathcal{N}$, where $\mathcal{D}$ chooses a probability distribution over the threat targets to maximize its payoff, and $\mathcal{N}$ chooses an uncertain parameter to minimize $\mathcal{D}$'s payoff. Nash equilibrium, a widely adopted solution concept in the game theory, is used to represent the uncertain parameters and the decisions of $\mathcal{D}$.

Note that in [26], it was assumed that the real values of different threat indicators are determined by the same interval parameter. However, in air combat, these threat indicators are independent of each other. Moreover, the uncertainty of threat data comes from the measurement errors of the sensors, electronic countermeasures, electromagnetic interference, etc. As a consequence, the interval parameters of these threat indicators have no correlation. On this account, we assume that the real values of different threat indicators are determined by different interval parameters, which is more in line with the actual air combat. Then, the established zero-sum game is transformed into a matrix game to solve. Finally, the threat sequencing result is given based on the solved Nash equilibrium.

The main contributions of this study are stated as follows:

(i) A threat sequencing method of multiple UCAVs that considers incomplete information (the time-varying weight coefficients of different threat indicators, and the inaccurate air combat data) is proposed.

(ii) A zero-sum game model of $\mathcal{D}$ and $\mathcal{N}$ with fuzzy payoffs is addressed for the threat sequencing problem of multiple UCAVs with incomplete information, where $\mathcal{D}$ chooses a probability distribution over the threat targets to maximize its threat, and $\mathcal{N}$ chooses the incomplete information parameters to minimize $\mathcal{D}$'s threat.

(iii) The established zero-sum game with fuzzy payoffs is solved by transforming it into a zero-sum game with crisp payoffs (matrix game). Therefore, the threat sequencing result of multiple UCAVs is obtained under incomplete information conditions.

The following sections of this study are organized as follows: In Section 2, the threat sequencing problem of multiple UCAVs is first described, then some basic concepts of interval numbers and zero-sum games are reviewed. In Section 3, a zero-sum game model with fuzzy payoffs is established for our threat sequencing problem, and a solution method is subsequently proposed for the established game. In Section 4, the effectiveness of the proposed method is verified by numerical simulations. Section 5 presents the conclusions.

2. Problem formulation and preliminaries

2.1 Problem description

Consider the following scenario: suppose that in the air combat, the Red has m UCAVs denoted as $\boldsymbol{R}_1, \boldsymbol{R}_2, \cdots, \boldsymbol{R}_m$; the Blue has n UCAVs denoted as $\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots, \boldsymbol{B}_n$. For the Red, the threats of the Blue UCAVs can be characterized by five threat indicators: angle threat, speed threat, height threat, distance threat, and air combat capability threat. These threat indicators can be calculated by the approaches in [27–30], and the detailed expressions are omitted here to simplify the related descriptions.

In air combat, the threat data is usually in the form of interval numbers due to the measurement errors of various sensors, electronic countermeasures, electromagnetic interference, etc. Therefore, the threat of UCAV $\boldsymbol{B}_i$ $(i=1,2,\cdots,n)$ can be characterized by a 5-dimensional vector: $\boldsymbol{B}_i=(\tilde{b}_{i1},\tilde{b}_{i2},\cdots,\tilde{b}_{i5})$, where $\tilde{b}_{ij}=[b^L_{ij},b^U_{ij}]$ is an interval number, representing the threat degree of $\boldsymbol{B}_i$'s jth threat indicator. In consequence, an interval threat matrix with $n\times 5$ dimension [26] is obtained as follows :

$$\tilde{\boldsymbol{B}} = \begin{array}{c} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \\ \vdots \\ \boldsymbol{B}_n \end{array} \begin{pmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \cdots & \tilde{b}_{15} \\ \tilde{b}_{21} & \tilde{b}_{22} & \cdots & \tilde{b}_{25} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \cdots & \tilde{b}_{n5} \end{pmatrix}. \tag{1}$$

Here, we denote $\boldsymbol{B}^L=(b^L_{ij})_{n\times 5}$, $\boldsymbol{B}^U=(b^U_{ij})_{n\times 5}$, which are called the threat lower bound matrix and the threat upper bound matrix, respectively.

We consider the threat sequencing problem of $\boldsymbol{B}_1,\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$ with time-varying weight coefficients of different threat indicators and interval threat data. The real threat value of the jth attribute of $\boldsymbol{B}_i$ is considered to be a value in interval $\tilde{b}_{ij}$. As discussed above, the real values of different threat indicators are determined by different interval parameters. If we denote the interval parameter of attribute j as λ_j $(j=1,2,\cdots,5)$, then the threat matrix has the following form:

$$\boldsymbol{B}^{\lambda}=(b^{\lambda_j}_{ij})_{n\times 5}= \begin{array}{c} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \\ \vdots \\ \boldsymbol{B}_n \end{array} \begin{pmatrix} \lambda_1 b^L_{11}+(1-\lambda_1)b^U_{11} & \lambda_2 b^L_{12}+(1-\lambda_2)b^U_{12} & \cdots & \lambda_5 b^L_{15}+(1-\lambda_5)b^U_{15} \\ \lambda_1 b^L_{21}+(1-\lambda_1)b^U_{21} & \lambda_2 b^L_{22}+(1-\lambda_2)b^U_{22} & \cdots & \lambda_5 b^L_{25}+(1-\lambda_5)b^U_{25} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1 b^L_{n1}+(1-\lambda_1)b^U_{n1} & \lambda_2 b^L_{n2}+(1-\lambda_2)b^U_{n2} & \cdots & \lambda_5 b^L_{n5}+(1-\lambda_5)b^U_{n5} \end{pmatrix} \tag{2}$$

where $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\cdots,\lambda_5)$, $\lambda_j\in[0,1]$ $(j=1,2,\cdots,5)$.

Our goal is to give the threat sequencing of $\boldsymbol{B}_1$, $\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$ by obtaining the time-varying weight coefficient parameter $\boldsymbol{\omega}$ and the interval parameter $\boldsymbol{\lambda}$ of the threat matrix.

To facilitate the following discussion, we first give the operation rules of interval numbers. If we denote $\tilde{c}=[c^L,c^U]$ and $\tilde{d}=[d^L,d^U]$ as two interval numbers, and g is a function, then the four arithmetic, the measure operator m, and the function operations of interval numbers [31,32] are given as follows:

$$\tilde{c}+\tilde{d}=\left[c^L+d^L,c^U+d^U\right], \tag{3}$$

$$\tilde{c}-\tilde{d}=\left[c^L-d^U,c^U-d^L\right], \tag{4}$$

$$\begin{aligned}\tilde{c}\cdot\tilde{d}=[&\min\{c^Ld^L,c^Ld^U,c^Ud^L,c^Ud^U\},\\ &\max\{c^Ld^L,c^Ld^U,c^Ud^L,c^Ud^U\}],\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\tilde{c}}{\tilde{d}}=\Bigg[&\min\left\{\frac{c^L}{d^L},\frac{c^L}{d^U},\frac{c^U}{d^L},\frac{c^U}{d^U}\right\},\\ &\max\left\{\frac{c^L}{d^L},\frac{c^L}{d^U},\frac{c^U}{d^L},\frac{c^U}{d^U}\right\}\Bigg],\end{aligned} \tag{6}$$

$$m(\tilde{c},\tilde{d})=\sqrt{(c^L-d^L)^2+(c^U-d^U)^2}, \tag{7}$$

$$g(\tilde{c})=\left[\min\{g(c^L),g(c^U)\},\max\{g(c^L),g(c^U)\}\right]. \tag{8}$$

2.2 Review of basic concepts and theories of two-person zero-sum games

In this part, the basic concepts of two-person zero-sum games are introduced, then the solution algorithm and some properties of zero-sum games are reviewed [33,34].

As a class of non-cooperative games, two-person zero-sum games (also called matrix games) are widely used in various decision-making scenarios [35]. Usually, a two-person zero-sum game of $\mathcal{D}$ and $\mathcal{N}$ [33] can be expressed by a $n\times q$ matrix:

$$\boldsymbol{B}=\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1q} \\ b_{21} & b_{22} & \cdots & b_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nq} \end{pmatrix} \tag{9}$$

where b_{ik} $(i=1,2,\cdots,n;\ k=1,2,\cdots,q)$ is $\mathcal{D}$'s payoff when $\mathcal{D}$ chooses its ith pure strategy and $\mathcal{N}$ choose its kth pure strategy, and $\mathcal{N}$'s payoff is $-b_{ik}$ under the same strategies choice.

In a two-person zero-sum game, the players usually choose their strategies not deterministically, but in the form of the probability distribution (called mixed strategy). Formally, a vector $\boldsymbol{\gamma}=(\gamma_1,\gamma_2,\cdots,\gamma_n)\in\mathbf{R}^n$ is called as a mixed strategy of $\mathcal{D}$, if it satisfies $\sum\limits_{i=1}^{n}\gamma_i=1$, $\gamma_i\geqslant 0$ $(i=1,2,\cdots,n)$. Similarly, a vector $\boldsymbol{\pi}=(\pi_1,\pi_2,\cdots,$

$\pi_q) \in \mathbf{R}^q$ is defined as a mixed strategy of $\mathcal{N}$, if it satisfies $\sum_{k=1}^{q} \pi_k = 1$, $\pi_k \geqslant 0$ $(k = 1,2,\cdots,q)$.

The expected payoff of $\mathcal{D}$ under the mixed strategies $\boldsymbol{\gamma}$ and $\boldsymbol{\pi}$ [33] is defined by

$$U'(\boldsymbol{\gamma},\pi) \triangleq \boldsymbol{\gamma}\boldsymbol{B}\boldsymbol{\pi}^{\mathrm{T}} = \sum_{i=1}^{n}\sum_{k=1}^{q}\gamma_i b_{ik}\pi_k, \tag{10}$$

and the expected payoff of $\mathcal{N}$ is defined as $-U'(\boldsymbol{\gamma},\boldsymbol{\pi})$. In the game theory, Nash equilibrium is a widely adopted solution concept. Formally, a strategy vector $[\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*]$ is called a Nash equilibrium of the matrix game in (9), if the following inequalities hold for each $\mathcal{D}$'s mixed strategy $\boldsymbol{\gamma}$, and each $\mathcal{N}$'s mixed strategy $\boldsymbol{\pi}$:

$$U'(\boldsymbol{\gamma},\boldsymbol{\pi}^*) \leqslant U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*) \leqslant U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}).$$

It is well known that the matrix game in (9) can be solved by solving a pair of dual linear programming, and this fact is given by the following lemma:

Lemma 1 [34] Assume that the elements of matrix $\boldsymbol{B}$ are positive in matrix game (9), if $\bar{\boldsymbol{\gamma}} = (\bar{\gamma}_1,\bar{\gamma}_2,\cdots,\bar{\gamma}_n)$ and $\bar{\boldsymbol{\pi}} = (\bar{\pi}_1,\bar{\pi}_2,\cdots,\bar{\pi}_q)$ are the optimal solutions of the dual linear programming (11) and (12), respectively, then $(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)$ is a Nash equilibrium of the matrix game in (9), where $v = \left(\sum_{i=1}^{n}\bar{\gamma}_i\right)^{-1}$, $\boldsymbol{\gamma}^* = v\bar{\boldsymbol{\gamma}}$, $\boldsymbol{\pi}^* = v\bar{\boldsymbol{\pi}}$.

$$\begin{aligned} &\min \sum_{i=1}^{n}\gamma_i \\ &\text{s.t.} \begin{cases} \sum_{i=1}^{n}\gamma_i b_{ik} \geqslant 1, & k = 1,2,\cdots,q \\ \gamma_i \geqslant 0, & i = 1,2,\cdots,n \end{cases} \end{aligned} \tag{11}$$

$$\begin{aligned} &\max \sum_{k=1}^{q}\pi_k \\ &\text{s.t.} \begin{cases} \sum_{k=1}^{q}\pi_k b_{ik} \leqslant 1, & i = 1,2,\cdots,n \\ \pi_k \geqslant 0, & k = 1,2,\cdots,q \end{cases} \end{aligned} \tag{12}$$

Remark 1 The above lemma requires that the elements of the payoff matrix $\boldsymbol{B}$ are positive. If there is a non-positive situation, we can add an enough large positive constant for each element of $\boldsymbol{B}$ to guarantee the elements of the newly obtained matrix $\hat{\boldsymbol{B}}$ positive. Then, the Nash equilibria of $\boldsymbol{B}$ and $\hat{\boldsymbol{B}}$ is exactly the same [36].

3. Threat sequencing of multiple UCAVs

In this section, a threat sequencing method is proposed for the threat sequencing problem of multiple UCAVs with incomplete information. First, a zero-sum game model of $\mathcal{D}$ and $\mathcal{N}$ with fuzzy payoffs is established to obtain the weight coefficient parameters of different indicators and the interval parameters of the threat matrix. Then, the established zero-sum game with fuzzy payoffs is transformed into a zero-game with crisp payoffs (the matrix game) to solve. Finally, a decision rule for the threat sequencing of multiple UCAVs is given.

3.1 Zero-sum game model with fuzzy payoffs

In the threat sequencing problem of multiple UCAVs with time-varying weight coefficient parameters of different threat indicators and interval parameters of the threat matrix, we introduce a virtual person: $\mathcal{N}$, which is regarded as an "intelligent enemy" of $\mathcal{D}$. The goal of $\mathcal{N}$ is to minimize $\mathcal{D}$'s threat by selecting the weight coefficient parameter $\boldsymbol{\omega}$ of different threat indicators, and the interval parameter $\boldsymbol{\lambda}$ of the threat matrix. Since $\mathcal{D}$ is uncertain about what parameters $\mathcal{N}$ will choose, one will choose a probability distribution over the UCAVs $\boldsymbol{B}_1,\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$ to maximize the threat.

On the basis of [26], we establish a zero-sum game $G = \langle \mathcal{D},\mathcal{N},\Gamma,\overline{\Omega\times\Lambda},\tilde{\boldsymbol{B}},U\rangle$ for the threat sequencing problem of multiple UCAVs, where

(i) $\mathcal{D}$ is the maximizing player. It chooses a probability distribution $\boldsymbol{\gamma}$ over the set of its threat targets $\boldsymbol{B}_1,\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$ to maximize its payoff.

(ii) $\mathcal{N}$ is the minimizing player. It chooses the weight coefficient parameter $\boldsymbol{\omega}$ of different threat indicators and interval parameter $\boldsymbol{\lambda}$ of the threat matrix to minimize $\mathcal{D}$'s payoff.

(iii) Γ is the strategy set of $\mathcal{D}$. It is the set of all probability distributions over its threat targets $\boldsymbol{B}_1,\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$. Γ can be written as follows:

$$\Gamma \triangleq \left\{\boldsymbol{\gamma} \in \mathbf{R}^n \middle| \sum_{i=1}^{n}\gamma_i = 1, 0 \leqslant \gamma_i \leqslant 1\right\} \tag{13}$$

(iv) $\overline{\Omega\times\Lambda}$ is the strategy set of $\mathcal{N}$. It is a set of $\Omega\times\Lambda$ with the restriction: if $\boldsymbol{\omega}_j = 0$, then $\lambda_j = 0$, where

$$\Omega = \left\{\boldsymbol{\omega} \in \mathbf{R}^5 \,|\boldsymbol{\omega}_j \geqslant 0, \sum_{j=1}^{n}\omega_j = 1\right\}$$

is the set of all weight coefficients of the five threat indicators, and

$$\Lambda = \{\boldsymbol{\lambda} = (\lambda_1,\lambda_2,\cdots,\lambda_5)\,|0 \leqslant \lambda_j \leqslant 1\}$$

is the set of all possible interval parameters of threat matrix $\tilde{\boldsymbol{B}}$.

(v) $\tilde{\boldsymbol{B}}$ is the interval payoff matrix of $\mathcal{D}$. The elements of $\tilde{\boldsymbol{B}}$ are all interval numbers, as expressed in (1).

(vi) U is the expected payoff function of $\mathcal{D}$. When $\mathcal{D}$ chooses a strategy $\boldsymbol{\gamma} \in \Gamma$, and $\mathcal{N}$ chooses a strategy

$(\boldsymbol{\omega},\boldsymbol{\lambda})\in\overline{\Omega\times\Lambda}$, the expected payoff $U(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})$ of $\mathcal{D}$ is defined as

$$U(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})=\boldsymbol{\gamma}\boldsymbol{B}^{\boldsymbol{\lambda}}\boldsymbol{\omega}^{\mathrm{T}}=\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i b_{ij}^{\lambda_j}\omega_j=$$
$$\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i[\lambda_j b_{ij}^{L}+(1-\lambda_j)b_{ij}^{U}]\omega_j \tag{14}$$

where $\boldsymbol{B}^{\boldsymbol{\lambda}}$ is the crisp payoff matrix determined by the parameter $\boldsymbol{\lambda}$ and the interval payoff matrix $\tilde{\boldsymbol{B}}$, as shown in (2). Since $\mathcal{N}$ is the "intelligent enemy" of $\mathcal{D}$, the expected payoff of $\mathcal{N}$ under strategy vector $(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})$ is defined as $-U(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})$.

Remark 2 The goal of $\mathcal{D}$ is to maximize its threat, and the goal of $\mathcal{N}$ is to minimize $\mathcal{D}$'s threat, so the "threat" is treated as the "payoff" in the game model.

Remark 3 The strategy set of $\mathcal{N}$ is not defined as $\Omega\times\Lambda$, but we impose a restriction on it: if $\omega_j=0$, then $\lambda_j=0$. This is for the convenience of proving the conclusion later. In fact, this restriction is reasonable, because if $\omega_j=0$, then $\mathcal{N}$ thinks that the index j has no value, so there is no need to materialize the data of index j. Therefore, when $\omega_j=0$, setting $\lambda_j=0$ has no effect on the threat sequencing problem.

Remark 4 In this paper, Nash equilibrium is used as the uncertainty parameters and the decision of $\mathcal{D}$. In fact, for the considered threat sequencing problem, the Nash equilibrium solution is optimal for both $\mathcal{D}$ and $\mathcal{N}$, which can be illustrated as follows: if $(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ is a Nash equilibrium of the game G, by definition, one can have $U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)\geqslant U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$. In other words, this approach can ensure that the payoff of $\mathcal{D}$ is not less than $U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$, regardless of whether the uncertainty parameter is $(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ or not.

The zero-sum game G established above is special, where the energy strategy of $\mathcal{N}$ consists of two parameters, $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}$. Therefore, it cannot be solved by the traditional zero-sum game solving algorithm. In the following subsection, G is transformed into a matrix game G' to solve.

3.2 Model transformation and solution

In this subsection, the matrix game G' is constructed based on G, and the relationship between the Nash equilibria of G and G' is established. Thus, G can be solved by solving G'.

In the following, we construct a matrix game $G'=\langle\mathcal{D},\mathcal{N},\Gamma,\Pi,\bar{\boldsymbol{B}},U'\rangle$ based on G, where

(i) $\mathcal{D}$ is the maximizing player, as defined in G.

(ii) $\mathcal{N}$ is the minimizing player, as defined in G.

(iii) Γ is the strategy set of $\mathcal{D}$. It is the same as $\mathcal{D}$'s strategy set defined in G, as shown in (13).

(iv) Π is the strategy set of $\mathcal{N}$. It is a set of probability distributions, given as follows:

$$\Pi\triangleq\left\{\boldsymbol{\pi}\in\mathbf{R}^{10}|0\leqslant\pi_k\leqslant 1,\sum_{k=1}^{10}\pi_k=1\right\}. \tag{15}$$

(v) $\bar{\boldsymbol{B}}$ is the payoff matrix of $\mathcal{D}$. It is an $n\times 10$ matrix composed of $\boldsymbol{B}^L$ and $\boldsymbol{B}^U$, which has the following form:

$$\bar{\boldsymbol{B}}\triangleq\begin{pmatrix} b_{11}^L & b_{12}^L & \cdots & b_{15}^L & b_{11}^U & b_{12}^U & \cdots & b_{15}^U\\ b_{21}^L & b_{22}^L & \cdots & b_{25}^L & b_{21}^U & b_{22}^U & \cdots & b_{25}^U\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ b_{n1}^L & b_{n2}^L & \cdots & b_{n5}^L & b_{n1}^U & b_{n2}^U & \cdots & b_{n5}^U \end{pmatrix}. \tag{16}$$

(vi) U' is the expected payoff function of $\mathcal{D}$. The payoff of $\mathcal{D}$ under the strategy vector $(\boldsymbol{\gamma},\boldsymbol{\pi})$ is given by

$$U'(\boldsymbol{\gamma},\boldsymbol{\pi})=\boldsymbol{\gamma}\bar{\boldsymbol{B}}\boldsymbol{\pi}^{\mathrm{T}}=$$
$$\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i b_{ij}^{L}\pi_j+\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i b_{ij}^{U}\pi_{j+5}, \tag{17}$$

and the payoff of $\mathcal{N}$ under $(\boldsymbol{\gamma},\boldsymbol{\pi})$ is defined as $-U'(\boldsymbol{\gamma},\boldsymbol{\pi})$.

Obviously, G' can be solved by the traditional zero-sum game solving algorithm. In the following, the relationship between the Nash equilibria of G and G' is established, so as to solve G by solving G'.

Firstly, we define a mapping f from $\overline{\Omega\times\Lambda}$ to Π as follows:

$$f(\boldsymbol{\omega},\boldsymbol{\lambda})\triangleq(\lambda_1\omega_1,\lambda_2\omega_2,\cdots,\lambda_5\omega_5,$$
$$(1-\lambda_1)\omega_1,(1-\lambda_2)\omega_2,\cdots,(1-\lambda_5)\omega_5) \tag{18}$$

where $(\boldsymbol{\omega},\boldsymbol{\lambda})\in\overline{\Omega\times\Lambda}$, $\boldsymbol{\omega}=(\omega_1,\omega_2,\cdots,\omega_5)$, $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\cdots,\lambda_5)$.

The payoffs of $\mathcal{D}$ in G and G' are equal under the action of mapping f, which is summarized as the following theorem:

Theorem 1 Consider the mapping f defined in (18), then we have that f is a one-to-one mapping from $\overline{\Omega\times\Lambda}$ to Π, and the following equations hold:

$$U(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})=U'(\boldsymbol{\gamma},f(\boldsymbol{\omega},\boldsymbol{\lambda})), \tag{19}$$

$$U'(\boldsymbol{\gamma},\boldsymbol{\pi})=U(\boldsymbol{\gamma},f^{-1}(\boldsymbol{\pi})). \tag{20}$$

Proof We first prove that f is a one-to-one mapping from $\overline{\Omega\times\Lambda}$ to Π, then we prove that (19) and (20) hold.

Step 1 We prove the first conclusion by showing that f is both injective and surjective.

(i) f is injective. If there is $f(\boldsymbol{\omega}^a,\boldsymbol{\lambda}^a)=f(\boldsymbol{\omega}^b,\boldsymbol{\lambda}^b)$, we prove that $(\boldsymbol{\omega}^a,\boldsymbol{\lambda}^a)=(\boldsymbol{\omega}^b,\boldsymbol{\lambda}^b)$. From the definition of f, we obtain

$$f(\boldsymbol{\omega}^a,\boldsymbol{\lambda}^a)=(\lambda_1^a\omega_1^a,\lambda_2^a\omega_2^a,\cdots,\lambda_5^a\omega_5^a,$$
$$(1-\lambda_1^a)\omega_1^a,(1-\lambda_2^a)\omega_2^a,\cdots,(1-\lambda_5^a)\omega_5^a),$$

$$f(\boldsymbol{\omega}^b,\boldsymbol{\lambda}^b)=(\lambda_1^b\omega_1^b,\lambda_2^b\omega_2^b,\cdots,\lambda_5^b\omega_5^b,\\(1-\lambda_1^b)\omega_1^b,(1-\lambda_2^b)\omega_2^b,\cdots,(1-\lambda_5^b)\omega_5^b).$$

Since $f(\boldsymbol{\omega}^a,\boldsymbol{\lambda}^a)=f(\boldsymbol{\omega}^b,\boldsymbol{\lambda}^b)$, we have

$$\lambda_j^a\omega_j^a=\lambda_j^b\omega_j^b, \tag{21}$$

$$(1-\lambda_j^a)\omega_j^a=(1-\lambda_j^b)\omega_j^b. \tag{22}$$

From (21) and (22), it holds that $\omega_j^a=\omega_j^b$ $(j=1,2,\cdots,5)$. If $\omega_j^a=\omega_j^b=0$, by the definition of $\overline{\Omega\times\Lambda}$, one can have $\lambda_j^a=\lambda_j^b=0$; if $\omega_j^a=\omega_j^b\neq 0$, according to (21), there is $\lambda_j^a=\lambda_j^b$. Therefore, it is proved that $(\boldsymbol{\omega}^a,\boldsymbol{\lambda}^a)=(\boldsymbol{\omega}^b,\boldsymbol{\lambda}^b)$.

(ii) f is surjective. For $\hat{\boldsymbol{\pi}}=(\hat{\pi}_1,\hat{\pi}_2,\cdots,\hat{\pi}_{10})\in\Pi$, we find $(\hat{\boldsymbol{\omega}},\hat{\boldsymbol{\lambda}})\in\overline{\Omega\times\Lambda}$, such that $f(\hat{\boldsymbol{\omega}},\hat{\boldsymbol{\lambda}})=\hat{\boldsymbol{\pi}}$. We construct $\hat{\boldsymbol{\omega}}=(\hat{\omega}_1,\hat{\omega}_2,\cdots,\hat{\omega}_5)$, and $\hat{\boldsymbol{\lambda}}=(\hat{\lambda}_1,\hat{\lambda}_2,\cdots,\hat{\lambda}_5)$ as follows:

$$\hat{\omega}_j\triangleq\hat{\pi}_j+\hat{\pi}_{j+5}, \tag{23}$$

$$\hat{\lambda}_j\triangleq\begin{cases}0, & \hat{\pi}_j+\hat{\pi}_{j+5}=0\\ \dfrac{\hat{\pi}_j}{\hat{\pi}_j+\hat{\pi}_{j+5}}, & \hat{\pi}_j+\hat{\pi}_{j+5}\neq 0\end{cases}, \tag{24}$$

and it is easy to know that $(\hat{\boldsymbol{\omega}},\hat{\boldsymbol{\lambda}})\in\overline{\Omega\times\Lambda}$. By the definition of f, we have

$$f(\hat{\boldsymbol{\omega}},\hat{\boldsymbol{\lambda}})=(\hat{\lambda}_1\hat{\omega}_1,\hat{\lambda}_2\hat{\omega}_2,\cdots,\hat{\lambda}_5\hat{\omega}_5,\\(1-\hat{\lambda}_1)\hat{\omega}_1,(1-\hat{\lambda}_2)\hat{\omega}_2,\cdots,(1-\hat{\lambda}_5)\hat{\omega}_5). \tag{25}$$

If $\hat{\omega}_j=\hat{\pi}_j+\hat{\pi}_{j+5}=0$, since $\hat{\pi}_j\geqslant 0$, $\hat{\pi}_{j+5}\geqslant 0$, we have $\hat{\pi}_j=\hat{\pi}_{j+5}=0$, thereby $\hat{\lambda}_j\hat{\omega}_j=0=\hat{\pi}_j$, $(1-\hat{\lambda}_j)\hat{\omega}_j=0=\hat{\pi}_{j+5}$; if $\hat{\omega}_j=\hat{\pi}_j+\hat{\pi}_{j+5}\neq 0$, $\hat{\lambda}_j\hat{\omega}_j=\hat{\pi}_j$, $(1-\hat{\lambda}_j)\hat{\omega}_j=\hat{\pi}_{j+5}$. As a result, we obtain $f(\hat{\boldsymbol{\omega}},\hat{\boldsymbol{\lambda}})=\hat{\boldsymbol{\pi}}$.

Step 2 According to (14), the payoff of $\mathcal{D}$ under the strategy vector $(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})$ is

$$U(\boldsymbol{\gamma},\boldsymbol{\omega},\boldsymbol{\lambda})=\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i\left[\lambda_j b_{ij}^L+(1-\lambda_j)b_{ij}^U\right]\omega_j. \tag{26}$$

By (17), the payoff of $\mathcal{D}$ under the strategy vector $[\boldsymbol{\gamma},f(\boldsymbol{\omega},\boldsymbol{\lambda})]$ is

$$U'(\boldsymbol{\gamma},f(\boldsymbol{\omega},\boldsymbol{\lambda}))=\boldsymbol{\gamma}\bar{\boldsymbol{B}}f(\boldsymbol{\omega},\boldsymbol{\lambda})^{\mathrm{T}}=\\\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i b_{ij}^L\lambda_j\omega_j+\sum_{i=1}^{n}\sum_{j=1}^{5}\gamma_i b_{ij}^U(1-\lambda_j)\omega_j. \tag{27}$$

It is obvious that (26) is equal to (27), thereby (19) is proved. In addition, one can have

$$U(\boldsymbol{\gamma},f^{-1}(\boldsymbol{\pi}))=U'(\boldsymbol{\gamma},f(f^{-1}(\boldsymbol{\pi})))=U'(\boldsymbol{\gamma},\boldsymbol{\pi}) \tag{28}$$

which proves that (20) holds. □

From the proof process of Theorem 1, we can give the inverse mapping $f^{-1}:\Pi\rightarrow\overline{\Omega\times\Lambda}$ of f as follows:

$$f^{-1}(\boldsymbol{\pi})\triangleq(\boldsymbol{\omega},\boldsymbol{\lambda}) \tag{29}$$

where

$$\omega_j=\pi_j+\pi_{j+5}, \tag{30}$$

$$\lambda_j=\begin{cases}0, & \pi_j+\pi_{j+5}=0\\ \dfrac{\pi_j}{\pi_j+\pi_{j+5}}, & \pi_j+\pi_{j+5}\neq 0\end{cases}. \tag{31}$$

The relationship of the Nash equilibria in G and G' is established by the following theorem:

Theorem 2 If $(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ is a Nash equilibrium of G, then $(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))$ is a Nash equilibrium of G'; conversely, if $(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)$ is a Nash equilibrium of G', then $(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))$ is a Nash equilibrium of G.

Proof If $(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ is a Nash equilibrium of G, it holds that

$$U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)\geqslant U(\boldsymbol{\gamma},\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*),$$

$$U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)\leqslant U(\boldsymbol{\gamma}^*,\boldsymbol{\omega},\boldsymbol{\lambda}).$$

Invoking Theorem 1, we obtain

$$U'(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))=U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)\geqslant\\U(\boldsymbol{\gamma},\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)=U'(\boldsymbol{\gamma},f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)),$$

$$U'(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))=U(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)\leqslant\\U(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}))=U'(\boldsymbol{\gamma}^*,f(f^{-1}(\boldsymbol{\pi})))=U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}).$$

In other words, one has

$$U'(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))\geqslant U'(\boldsymbol{\gamma},f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)),$$

$$U'(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))\leqslant U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}).$$

Thus, we proved that $(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*))$ is a Nash equilibrium of G'.

On the contrary, if $(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)$ is a Nash equilibrium of G', there is

$$U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)\geqslant U'(\boldsymbol{\gamma},\boldsymbol{\pi}^*),$$

$$U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)\leqslant U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}).$$

Applying Theorem 1, we have

$$U(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))=U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)\geqslant U'(\boldsymbol{\gamma},\boldsymbol{\pi}^*)=U(\boldsymbol{\gamma},f^{-1}(\boldsymbol{\pi}^*)),\\U(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))=U'(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)\leqslant U'(\boldsymbol{\gamma}^*,f(\boldsymbol{\omega},\boldsymbol{\lambda})=U(\boldsymbol{\gamma}^*,\boldsymbol{\omega},\boldsymbol{\lambda}).$$

Thus, one can have

$$U(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))\geqslant U(\boldsymbol{\gamma},f^{-1}(\boldsymbol{\pi}^*)),$$

$$U(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))\leqslant U(\boldsymbol{\gamma}^*,\boldsymbol{\omega},\boldsymbol{\lambda}).$$

Namely, we proved that $(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))$ is a Nash equilibrium of G. □

In the above discussion, the weight coefficient parameters $\boldsymbol{\omega}^*$ of different threat indicators and interval parameters $\boldsymbol{\lambda}^*$ of the threat matrix are calculated by solving the established game model. In the following, a decision rule is given for the threat sequencing problem of multiple UCAVs based on the calculated parameters.

3.3 Decision rule

In this subsection, a decision rule is presented for the threat sequencing of multiple UCAVs.

Given that $(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ is a Nash equilibrium of G, where $\boldsymbol{\gamma}^*=(\gamma_1^*,\gamma_2^*,\cdots,\gamma_n^*)$, $\boldsymbol{\omega}^*=(\omega_1^*,\omega_2^*,\cdots,\omega_5^*)$, $\boldsymbol{\lambda}^*=(\lambda_1^*,\lambda_2^*\cdots,\lambda_5^*)$, the threat degree $T(\boldsymbol{B}_i)$ of target $\boldsymbol{B}_i$ [26] can be defined as

$$T(\boldsymbol{B}_i)\triangleq\gamma_i^*\sum_{j=1}^{5}\omega_j^*b_{ij}^{\lambda_j^*}=$$

$$\gamma_i^*\sum_{j=1}^{5}\omega_j^*\left[\lambda_j^*b_{ij}^L+(1-\lambda_j^*)b_{ij}^U\right] \tag{32}$$

where $b_{ij}^{\lambda_j^*}$ is an element of $\boldsymbol{B}^{\lambda^*}$, as defined in (2).

Remark 5 The threat of $\boldsymbol{B}_i$ is defined as the expected threat of $\boldsymbol{B}_i$ when $\mathcal{D}$ and $\mathcal{N}$ choose their Nash equilibrium strategies, respectively.

Based on the threat degree equation, all threat degrees of the threat targets can be obtained. It is obvious that the higher the threat degree $T(\boldsymbol{B}_i)$, the greater the threat of $\boldsymbol{B}_i$. As a consequence, the threat sequencing result is given according to the threat degrees of these targets.

The flowchart of the proposed threat sequencing method is shown in Fig. 1, and the procedure of threat sequencing for air targets $\boldsymbol{B}_1,\boldsymbol{B}_1,\cdots,\boldsymbol{B}_n$ is given as follows:

Step 1 Based on the interval air combat data obtained by various sensors, calculate the interval threat matrix $\tilde{\boldsymbol{B}}$ in (1) according to the threat indicator calculation formulas in [27–29].

Step 2 Construct the matrix $\bar{\boldsymbol{B}}$ based on the threat matrix $\tilde{\boldsymbol{B}}$, as shown in (16).

Step 3 Calculate the Nash equilibrium $(\boldsymbol{\gamma}^*,\boldsymbol{\pi}^*)$ of G' by Lemma 1.

Step 4 According to Theorem 2, the Nash equilibrium $(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ of G is given as follows:

$$(\boldsymbol{\gamma}^*,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)=(\boldsymbol{\gamma}^*,f^{-1}(\boldsymbol{\pi}^*))$$

where f^{-1} is given in (29)–(31). Thus the weight coefficient parameter $\boldsymbol{\omega}^*$ and the interval parameter $\boldsymbol{\lambda}^*$ are obtained.

Step 5 Calculate the threat degrees $T(\boldsymbol{B}_i)$ of UCAV $\boldsymbol{B}_i$ $(i=1,2,\cdots,n)$ according to (32). In consequence, the threat sequencing result of $\boldsymbol{B}_1,\boldsymbol{B}_2,\cdots,\boldsymbol{B}_n$ is obtained according to the threat degrees of these targets.

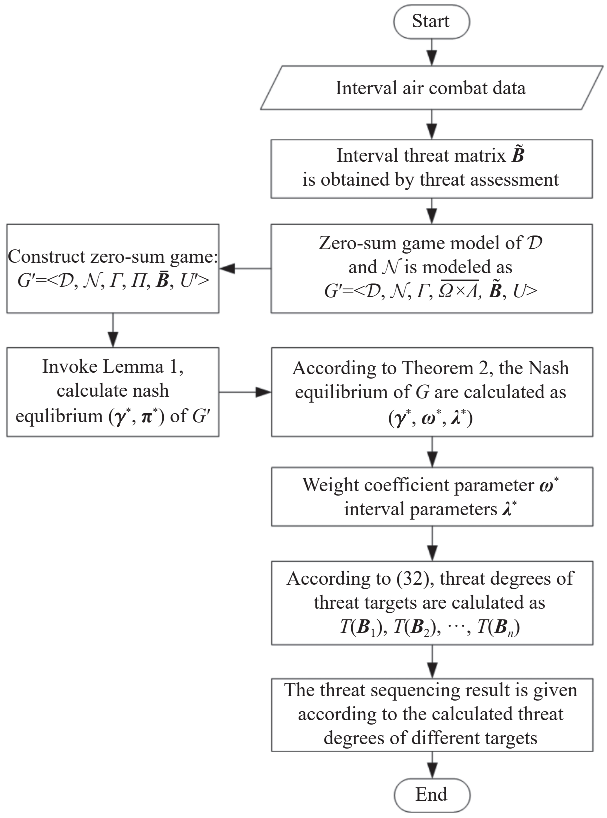

Fig. 1 Flowchart of threat sequencing of multiple UCAVs

4. Numerical simulations and comparison analyses

In this section, an example of one-to-four UCAVs air combat is given to illustrate the proposed method. Moreover, comparative analyses are conducted to show the effectiveness of the proposed method.

4.1 An example of threat sequencing

Suppose that for our UCAV $\boldsymbol{R}_0$, there are four enemy UCAVs, denoted as $\boldsymbol{B}_1,\boldsymbol{B}_2,\boldsymbol{B}_3,\boldsymbol{B}_4$, and their parameter values are presented in Table 1.

Table 1 Parameters of $\boldsymbol{B}_i$ and $\boldsymbol{R}_0$

Symbol	Description	$\boldsymbol{B}_1$	$\boldsymbol{B}_2$	$\boldsymbol{B}_3$	$\boldsymbol{B}_4$	$\boldsymbol{R}_0$
p_{ix}/km	X coordinate of position	[−14,−12]	[21,23]	[−17,−15]	[17,19]	0
p_{iy}/km	Y coordinate of position	[21,24]	[−17,−14]	[16,19]	[−15,−12]	0
p_{iz}/km	Z coordinate of position	[12,14]	[21,23]	[−4,−3]	[22,24]	0
v_{ix}/(km·h^{-1})	X coordinate of speed	[35,37]	[−56,−52]	[83,87]	[53,58]	[91,97]
v_{iy}/(km·h^{-1})	Y coordinate of speed	[56,59]	[93 98]	[73,78]	[84,87]	[61,64]

Continued

Symbol	Description	$\boldsymbol{B}_1$	$\boldsymbol{B}_2$	$\boldsymbol{B}_3$	$\boldsymbol{B}_4$	$\boldsymbol{R}_0$
$v_{iz}/(\mathrm{km\cdot h^{-1}})$	Z coordinate of speed	[53,56]	[71,73]	[84,88]	[94,97]	[37,40]
m_i/km	Missile attack distance	[30,33]	[31,34]	[29,31]	[26,28]	[42,44]
r_i/km	Radar detection distance	[81,87]	[82,89]	[90,94]	[84,87]	[91,98]
ε_i^m	Maneuverability performance	[1.3,1.5]	[1.5,1.6]	[2.4,2.6]	[1.4,1.6]	–
ε_i^f	Firepower performance	[0.4,0.5]	[0.7,0.9]	[0.6,0.7]	[0.5,0.6]	–
ε_i^d	Detection performance	[0.8,0.9]	[0.4,0.5]	[0.6,0.8]	[0.7,0.9]	–
ε_i^h	Handling performance	[0.6,0.7]	[0.5,0,6]	[0.3,0.4]	[0.4,0.5]	–
ε_i^s	Survival performance	[0.4,0.5]	[0.7,0.8]	[0.4,0.6]	[0.5,0.6]	–
ε_i^v	Voyage coefficient	[0.3,0.5]	[0.6,0.7]	[0.5,0.6]	[0.4,0.5]	–
ε_i^e	Electronic countermeasure performance	[0.3,0.4]	[0.5,0.6]	[0.7,0.9]	[0.4,0.5]	–

The threat sequencing process of $\boldsymbol{B}_1,\boldsymbol{B}_2,\boldsymbol{B}_3,\boldsymbol{B}_4$ is given as follows:

Step 1 The interval threat matrix $\tilde{\boldsymbol{B}}$ can be obtained by the threat indicators calculation formulas in [27–30], and interval number calculation rules in (3)–(8), which is given as

$$\tilde{\boldsymbol{B}}=\begin{pmatrix} [0.3,0.4] & [0.2,0.3] & [0.1,0.1] & [0.1,0.1] & [0.1,0.2] \\ [0.4,0.5] & [0.5,0.7] & [0.2,0.6] & [0.1,0.1] & [0.2,0.5] \\ [0.5,0.6] & [0.6,0.8] & [0.1,0.1] & [0.8,0.9] & [0.1,0.5] \\ [0.3,0.4] & [0.6,0.7] & [0.2,0.4] & [0.1,0.1] & [0.1,0.2] \end{pmatrix} \tag{33}$$

Step 2 According to (16), which is also shown in Fig. 2, construct matrix $\bar{\boldsymbol{B}}$ as follows:

$$\bar{\boldsymbol{B}}=\left(\begin{array}{ccccc:ccccc} 0.3 & 0.2 & 0.1 & 0.1 & 0.1 & 0.4 & 0.3 & 0.1 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.2 & 0.1 & 0.2 & 0.5 & 0.7 & 0.6 & 0.1 & 0.5 \\ 0.5 & 0.6 & 0.1 & 0.8 & 0.1 & 0.6 & 0.8 & 0.1 & 0.9 & 0.5 \\ 0.3 & 0.6 & 0.2 & 0.1 & 0.1 & 0.4 & 0.7 & 0.4 & 0.1 & 0.2 \end{array}\right)$$

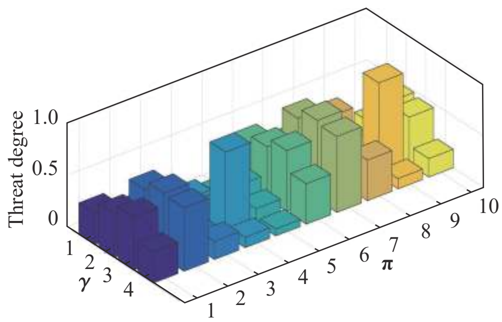

Fig. 2 Threat matrix $\bar{\boldsymbol{B}}$ of D in G'

Step 3 Obviously, the elements b_{ij} of matrix $\bar{\boldsymbol{B}}$ are all positive. According to Lemma 1, we first solve the following dual linear programming:

$$\min\sum_{i=1}^{4}x_i$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}b_{ij}x_i\geqslant 1\\ x_i\geqslant 0\end{cases}, \tag{34}$$

$$\max\sum_{j=1}^{10}y_i$$
$$\text{s.t.}\begin{cases}\sum_{j=1}^{10}b_{ij}y_i\leqslant 1\\ y_i\geqslant 0\end{cases}. \tag{35}$$

We can get that the following $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$ are the optimal solutions of (34) and (35), respectively:

$$\bar{\boldsymbol{x}}=(0,3.261\,2,0.548\,4,2.351\,7),$$

$$\bar{\boldsymbol{y}}=(0,0,5.237\,3,0.525\,9,0.398\,2,0,0,0,0,0).$$

Then, we have

$$\boldsymbol{\gamma}^*=v\bar{\boldsymbol{x}}=(0,0.529\,3,0.089,0.381\,7), \tag{36}$$

$$\boldsymbol{\pi}^*=v\bar{\boldsymbol{y}}=(0,0,0.85,0.085\,3,0.064\,6,0,0,0,0,0), \tag{37}$$

to form a Nash equilibrium of G', where $v=\left(\sum_{i=1}^{4}\bar{x}_i\right)^{-1}=0.162\,3$.

Step 4 According to Theorem 2, the weight coefficient parameter $\boldsymbol{\omega}^*$ and interval parameter $\boldsymbol{\lambda}^*$ can be calculated by the formula $(\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)=f^{-1}(\boldsymbol{\pi}^*)$, where the definition of f^{-1} is given in (29)−(31), thus we obtain

$$\boldsymbol{\omega}^*=(0,0,0.85,0.085\,3,0.064\,6), \tag{38}$$

$$\boldsymbol{\lambda}^*=(0,0,1,1,1). \tag{39}$$

Step 5 According to (32), the threat degrees $T(\boldsymbol{B}_i)$ $(i=1,2,\cdots,n)$ are calculated as

$$\begin{cases} T(\boldsymbol{B}_1)=0 \\ T(\boldsymbol{B}_2)=0.085\,9 \\ T(\boldsymbol{B}_3)=0.014\,4 \\ T(\boldsymbol{B}_4)=0.061\,9 \end{cases}.$$

Therefore, the threat sequencing result is given as follows:

$$\boldsymbol{B}_2 \succ \boldsymbol{B}_4 \succ \boldsymbol{B}_3 \succ \boldsymbol{B}_1$$

where $\succ$ means "is preferred to".

4.2 Results analysis and comparison

For the threat sequencing problem of multiple UCAVs, the weight coefficient parameter of different threat indicators are usually changeable with time-varying, and the threat matrix is in the form of intervals. To calculate the weight coefficient parameter $\boldsymbol{\omega}$ and interval parameter $\boldsymbol{\lambda}$, a zero-sum game model of $\mathcal{D}$ and $\mathcal{N}$ with fuzzy payoffs is proposed in this study. In the proposed game, $\mathcal{N}$ is regarded as a spoiler, who minimizes $\mathcal{D}$'s threat by choosing the weight coefficient parameter $\boldsymbol{\omega}$ and interval parameter $\boldsymbol{\lambda}$. In fact, in the traditional method, when the system is completely unknown, and there is no prior knowledge, the parameters $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}$ cannot be obtained by a convincing theory. However, in the proposed method, the parameters $\boldsymbol{\omega}^*$ and $\boldsymbol{\lambda}^*$ are obtained via the game theory and is therefore reasonable.

In the above numerical example, for the calculated value of $\boldsymbol{\omega}^*$ and $\boldsymbol{\lambda}^*$ in (38) and (39), we give 100 random values of $\boldsymbol{\gamma}^l$ $(l=1,2,\cdots,100)$. Under these parameters, the threat degrees $U(\boldsymbol{\gamma}^l,\boldsymbol{\omega}^*,\boldsymbol{\lambda}^*)$ $(l=1,2,\cdots,100)$ are presented in Fig. 3, which shows that the $\mathcal{D}$'s choice of $\boldsymbol{\gamma}^*$ in (36) is the target with the greatest threat value. In fact, the meaning of threat sequencing is to obtain the threat degrees of the targets. It is reasonable to select the largest threat target. Our method is to perform threat sequencing on the basis of selecting the largest threat target, which shows the effectiveness of our method.

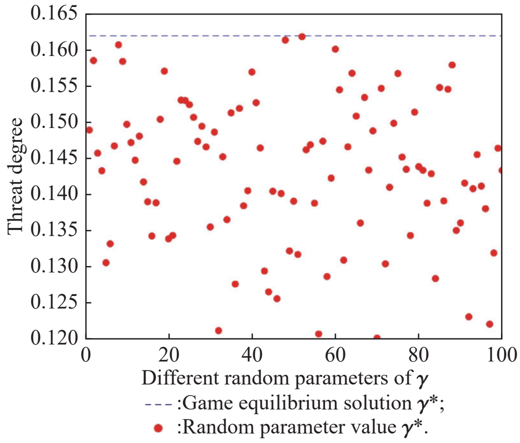

Fig. 3 Threat degrees of different γ^l under ω^* and λ^*

In multi-attribute decision-making problems, the technique for order preference by similarity to an ideal solution (TOPSIS) method is often used to select the best solution from alternatives with multiple attributes. A threat sequencing method based on the TOPSIS method was given in [37]. As a comparison of our method, in the following, the TOPSIS method is applied to conduct threat sequencing for $\boldsymbol{B}_1,\boldsymbol{B}_2,\boldsymbol{B}_3,\boldsymbol{B}_4$, and the calculation steps are as follows:

Step 1 The fuzzy decision matrix is shown in (33).

Step 2 Determine the positive negative ideal solution and the negative ideal solution as

$$I^+=\{[1,1],[1,1],[1,1],[1,1],[1,1]\},$$

$$I^-=\{[0,0],[0,0],[0,0],[0,0],[0,0]\},$$

respectively.

Step 3 Calculate the relative closeness coefficient of threat target $\boldsymbol{B}_i$ as

$$CC_i^*=\frac{D_i^-}{D_i^++D_i^-}, \tag{40}$$

where

$$D_i^+=\sqrt{\sum_{j=1}^{5}m(\tilde{b}_{ij},[1,1])}\ ,$$

$$D_i^-=\sqrt{\sum_{j=1}^{5}m(\tilde{b}_{ij},[0,0])}\ .$$

The calculation result is given as

$$\begin{cases} CC_1^*=0.201\,0 \\ CC_2^*=0.393\,4 \\ CC_3^*=0.499\,3 \\ CC_4^*=0.339\,2 \end{cases}.$$

Step 4 Rank the threat target $\boldsymbol{B}_1,\boldsymbol{B}_2,\boldsymbol{B}_3,\boldsymbol{B}_4$ according to the relative closeness coefficients. Therefore, the threat sequencing result based on the TOPSIS method is $\boldsymbol{B}_3 \succ \boldsymbol{B}_2 \succ \boldsymbol{B}_4 \succ \boldsymbol{B}_1$.

The result shows that our threat sequencing result is $\boldsymbol{B}_2 \succ \boldsymbol{B}_4 \succ \boldsymbol{B}_3 \succ \boldsymbol{B}_1$, while the result of the TOPSIS method is $\boldsymbol{B}_3 \succ \boldsymbol{B}_2 \succ \boldsymbol{B}_4 \succ \boldsymbol{B}_1$. This is because the TOPSIS method does not distinguish the importance of different threat indicators, and treats the weight coefficient of each indicator as the same. Hence, $\boldsymbol{B}_3$ has the greatest threat among the four targets. In our method, $\mathcal{N}$ is endowed with wisdom. Under its motivation to obtain the most payoffs, the weight coefficients of the threat indicators are $\omega^* = (0,0,0.85,0.085\,3,0.064\,6)$ and the uncertain parameters are $\lambda^* = (0,0,1,1,1)$. Note that in the threat sequencing problem of multiple UCAVs, the time-varying weight coefficients and interval parameters are difficult to effectively deal with by the traditional methods. It should be pointed out that the theory that we have established gives an effective theoretical support for this kind of problem, thereby ensuring the validity and rationality of our method.

5. Conclusions

In this study, for the threat sequencing problem of multiple UCAVs, a zero-sum game model of $\mathcal{D}$ and $\mathcal{N}$ with fuzzy payoffs has been proposed to obtain the weight coefficient parameters of the threat indicators and the interval parameters of the threat matrix. Moreover, a novel technique has been proposed to solve the established zero-sum game with fuzzy payoffs, whereby the final threat sequencing result has been provided. Finally, the effectiveness of the proposed method has been verified by numerical simulations. Future work will aim to extend the proposed method to more similar scenarios for MADM problems with incomplete information.

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