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Stochastic stabilization of Markovian jump cloud control systems based on max-plus algebra

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Abstract: In this paper, stochastic stabilization is investigated by max-plus algebra for a Markovian jump cloud control system with a reference signal. For the Markovian jump cloud control system, there exists framework adjustment whose evolution is satisfied with a Markov chain. Using max-plus algebra, a max-plus stochastic system is used to describe the Markovian jump cloud control system. A causal feedback matrix is obtained by exponential stability analysis for a causal feedback controller of the Markovian jump cloud control system. A sufficient condition is given to ensure existence on the causal feedback matrix of the causal feedback controller. Based on the causal feedback controller, stochastic stabilization in probability is analyzed for the Markovian jump cloud control system with a reference signal. Simulation results are given to show effectiveness of the causal feedback controller for the Markovian jump cloud control system.

Keywords: Markovian jump cloud control system, causal feedback controller, max-plus algebra, max-product algebra, stochastic stabilization.

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1. Introduction

Since its birth, cloud computing has attracted much attention for significate advantages in data processing, computing power and communication security [1]. With expansion of application scenarios, cloud computing is applied into many complex applications which usually have time-varying characteristics and switching structures [2,3]. A time-varying data cloud computing system is used to reduce communication burden of individual agents in a low-power nonlinear multi-agent system [2]. In [3], a multi-order Markov chain framework is designed for anomaly detection uncertainties which are caused by structural change of a cloud server system. Nowadays, new-type systems on cloud computing are investigated such as cloud control systems [4], mobile cloud computing systems [5] and mCloud systems [6]. For a cloud control system, there exist numerous delays which are generated by information transmission and computing processes [7,8]. Based on cloud predictive control schemes, networked delays from information transmission are handled to achieve consensus control for a networked multiagent system with cloud computing [7]. In [8], time delays of computing processes are analyzed on predictive cloud control for a networked multiagent system with quantized signals and deniel of service (DoS) attacks. During a cloud control process, a processing framework of a cloud control system is usually changeable for the reason of that the processing framework is adjusted according to control expectation in real time [9]. Moreover, framework adjustment of a cloud control system is often dependent on states in the last sampling time solely such that it is reasonable to take the framework adjustment as a Markovian jump process [10]. Therefore, it is an interesting work to study stochastic control for a cloud control system with a Markovian jump process and time delays.

It is well known that discrete-event systems are suitable to be taken as cloud control systems which are used to model many engineered systems [11]. Nonlinearity can be solved by linear equations of max-plus algebra in a discrete-event system such that there exists a good idea to apply max-plus algebra in cloud control systems [12,13]. Based on max-plus algebra, a very large scale integration array processor is modeled as a max-plus linear system for dimension reduction and feedback stabilization [12]. In [13], time schedule of a multi-legged robot is proposed as max-plus linear equation sets which transform discrete states into continuous states. Moreover, there are some stochastic control strategies on max-plus algebra for

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discrete-event switching systems [14]. A model predictive control strategy is designed for a switching max-plus-linear system which consists of a switching process within different operation modes [15]. It is known from [16] that model predictive control with a nonlinear non-convex constraint is given to achieve linear programming for a switching max-plus-linear system. Note that operability is dependent on stability of Markovian jump systems in which stochastic stabilization should be considered for feedback controller design [17,18]. With a memoryless state feedback controller, sufficient conditions for exponential stabilization are proposed for a stochastic Cohen-Grossberg neural network with Markovian jump parameters [19]. In [20], remote stabilization was investigated for an erasure channel using multi-step Lyapunov bounds. To the best of our knowledge, there are few results on max-plus based stochastic control for Markovian jump cloud control systems, which motivates our current research work.

In this paper, a max-plus stochastic system with a Markov chain is taken as the Markovian jump cloud control system. A causal feedback controller is obtained by exponential stability of an autonomous max-product system obtained from the max-plus stochastic system. A sufficient condition is proposed for the causal feedback controller on a causal feedback matrix in the max-plus stochastic system. Main contributions of this paper are summarized as follows:

(i) Max-plus algebra is utilized for model establishment on a Markovian jump cloud control system with a reference signal.

(ii) A causal feedback matrix is introduced to a causal feedback controller for the Markovian jump cloud control system.

(iii) A sufficient condition of stochastic stabilization is used for exponential stability of the Markovian jump cloud control system.

2. Problem statement and preliminaries

2.1 Max-plus algebra

For the real number set $\mathbf{R}$, an algebraic structure $(\mathbf{R}\cup\{-\infty\},\oplus,\otimes)$ is defined as max-plus algebra $\mathbf{R}_{\max}$ in which $\oplus$ and $\otimes$ are the main operations in max-plus algebra such that

$$a\oplus b=\max\{a,b\},\ a\otimes b=a+b$$

with $a,b\in\mathbf{R}_{\max}$. Therefore, it is obtained that c^{-1} is equal to $-c$ for $c\in\mathbf{R}_{\max}\cup\{+\infty\}$. Note that e and ε represent the identity element and zero element, respectively. In max-plus algebra, $\mathbf{R}_{\max}^{m\times n}$ represents the set of all the $m\times n$ matrices belonging to $\mathbf{R}_{\max}$. For two matrices $\boldsymbol{A}=(a_{ij})\in\mathbf{R}_{\max}^{m\times n}$, $\boldsymbol{B}=(b_{ij})\in\mathbf{R}_{\max}^{m\times n}$, the operation $\oplus$ is satisfied with

$$(\boldsymbol{A}\oplus\boldsymbol{B})_{ij}=a_{ij}\oplus b_{ij}\ .$$

With a matrix $\boldsymbol{C}=(c_{ij})\in\mathbf{R}_{\max}^{n\times o}$, the operation $\otimes$ is given as

$$(\boldsymbol{A}\otimes\boldsymbol{C})_{ij}=\bigoplus_{l=1}^{n}a_{il}\otimes c_{lj}\ .$$

Similarly, an algebraic structure $(\bar{\mathbf{R}}_{+},\oplus,\odot)$ is denoted as max-product algebra, where $\odot$ is the operation such that

$$a\odot b=a\cdot b,\ (\boldsymbol{A}\odot\boldsymbol{B})_{ij}=\bigoplus_{l=1}^{n}a_{il}\cdot b_{lj}\ .$$

For a matrix $\boldsymbol{D}=(d_{ij})\in\mathbf{R}_{\max}^{m\times m}$, $\boldsymbol{D}^{*}$ and $\boldsymbol{D}^{n}$ are denoted as

$$\begin{aligned}\boldsymbol{D}^{*}&=\boldsymbol{I}_{m}\oplus\boldsymbol{D}\oplus\boldsymbol{D}^{2}\oplus\cdots,\\ \boldsymbol{D}^{n}&=\underbrace{\boldsymbol{D}\otimes\boldsymbol{D}\otimes\cdots\otimes\boldsymbol{D}}_{n},\end{aligned}$$

where $\boldsymbol{I}_{m}$ represents the $m\times m$ identity matrix. In max-plus algebra, $\lambda(\boldsymbol{A})$ represents the maximum cycle mean. For a matrix $\breve{\boldsymbol{A}}=(\breve{a}_{ij})\in\mathbf{R}_{\max}^{m\times p}$, the precedence graph $\mathcal{G}(\breve{\boldsymbol{A}})$ of $\breve{\boldsymbol{A}}$ is a weighted directed graph with n nodes. In $\mathcal{G}(\breve{\boldsymbol{A}})$, there exists an $\breve{a}_{ij}$-weighted directed arc from node j to node i if $\breve{a}_{ij}$ is satisfied with $\breve{a}_{ij}\neq\varepsilon$. For max-plus algebra, a Petri net is a weighted directed graph which is described as a six-tuple such that

$$PN=(\mathcal{P},Q,F,W,M,M_{0})$$

where $\mathcal{P}=\{p_{1},p_{2},\cdots,p_{m}\}$ represents the location set, $Q=\{q_{1},q_{2},\cdots,q_{n}\}$ represents the transition set, F represents the set of directed arcs, W represents the weight function of directed arcs, M represents the status marking which is also called the Token, M_{0} represents the initial marking. Furthermore, the operation $\otimes$ is sometimes omitted for simplification.

2.2 Cloud control system

In a cloud control system, an instruction of a cloud parallel processor is often divided into three subprocesses, such as instruction fetch, instruction analysis and instruction execution [21]. Therefore, the cloud control system is obtained as shown in Fig. 1.

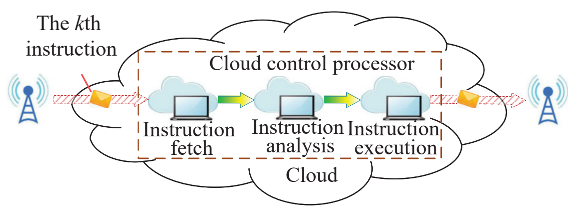

Fig. 1 Cloud control system

There exist multiple latches in the cloud control system to avoid instruction congestion [22]. During a cloud control process, the instruction processed by each subprocess is locked up by a latch until the next instruction is delivered to the subprocess. By a Petri net, the cloud control system is established as [23] in Fig. 2.

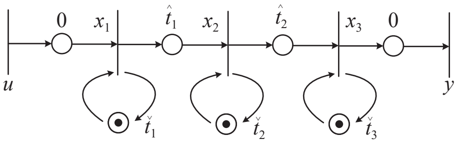

Fig. 2 Petri net of the cloud control system

According to Fig. 2, it is obtained for the kth instruction that the cloud control system is satisfied with

$$x_1(k) = \check{t}_1 x_1(k-1) \oplus u(k),$$

$$x_2(k) = \hat{t}_1 x_1(k) \oplus \check{t}_2 x_2(k-1),$$

$$x_3(k) = \hat{t}_2 x_2(k) \oplus \check{t}_3 x_3(k-1),$$

where $u(k)$ is the control input of the cloud parallel processor, $x_1(k)$, $x_2(k)$ and $x_3(k)$ are the beginning epochs of instruction fetch, instruction analysis and instruction execution, respectively, $\check{t}_1$, $\check{t}_2$ and $\check{t}_3$ represent the processing time units of instruction fetch, instruction analysis and instruction execution, respectively, $\hat{t}_1$ represents the transforming time unit between instruction fetch and instruction analysis, $\hat{t}_2$ represents the transforming time unit between instruction analysis and instruction execution. Therefore, the cloud control system is modeled as follows:

$$\boldsymbol{x}(k) = \hat{\boldsymbol{A}}\boldsymbol{x}(k) \oplus \check{\boldsymbol{A}}\boldsymbol{x}(k-1) \oplus \boldsymbol{B}u(k)$$

with

$$\hat{\boldsymbol{A}} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon \\ \hat{t}_1 & \varepsilon & \varepsilon \\ \varepsilon & \hat{t}_2 & \varepsilon \end{bmatrix},$$

$$\check{\boldsymbol{A}} = \begin{bmatrix} \check{t}_1 & \varepsilon & \varepsilon \\ \varepsilon & \check{t}_2 & \varepsilon \\ \varepsilon & \varepsilon & \check{t}_3 \end{bmatrix},$$

$$\boldsymbol{x}(k) = [x_1(k)\ \ x_2(k)\ \ x_3(k)]^{\mathrm{T}},$$

$$\boldsymbol{B} = [e\ \varepsilon\ \varepsilon]^{\mathrm{T}}.$$

According to [24], there exists instruction dependency of the cloud parallel processor in a cloud control process. During instruction dependency, the $(k+1)$th instruction has to enter the instruction analysis after the kth instruction leaves the instruction execution for the reason of that both the kth and $(k+1)$th instructions have the same source operand reference. Therefore, the cloud control system is transformed into the following form as Fig. 3.

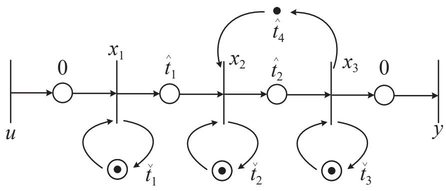

Fig. 3 Petri net with instruction dependency

In Fig. 3, the cloud control system is modeled as follows:

$$\boldsymbol{x}(k) = \hat{\boldsymbol{A}}\boldsymbol{x}(k) \oplus \check{\boldsymbol{A}}_1\boldsymbol{x}(k-1) \oplus \boldsymbol{B}u(k)$$

with

$$\check{\boldsymbol{A}}_1 = \begin{bmatrix} \check{t}_1 & \varepsilon & \varepsilon \\ \varepsilon & \check{t}_2 & \check{t}_4 \\ \varepsilon & \varepsilon & \check{t}_3 \end{bmatrix}$$

where $\check{t}_4$ represents the processing time units caused by instruction dependency. In practice, there also exist many other frameworks of the cloud control system, such as instruction lost, instruction failure and so on [24]. Note that a framework of an instruction in the cloud control system is dependent on the one of the previous adjacent instruction. It is reasonable to denote the framework adjustment into a Markov chain $y_k \in \{1, 2, \cdots, M\}$ with $M \in \mathbf{R}$. Moreover, denote a state transition matrix $\boldsymbol{C} \in \mathbf{R}^{M\times M}$ for the Markov chain y_k in which each evolution is satisfied with $c_{ij} = P(y_{k+1} = j | y_k = i)$. Based on the Markov chain y_k, a max-plus stochastic system is obtained for the Markovian jump cloud control system in the following form as

$$\boldsymbol{x}(k) = \hat{\boldsymbol{A}}(y_k)\boldsymbol{x}(k) \oplus \check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1) \oplus \boldsymbol{B}u(k) \tag{1}$$

where $\hat{\boldsymbol{A}}(y_k) \in \mathbf{R}_{\max}^{3\times3}$ and $\check{\boldsymbol{A}}(y_k) \in \mathbf{R}_{\max}^{3\times3}$ are the stochastic state matrices.

2.3 Objective statement

The following definitions and lemmas are proposed based on [25] and [26] to show main results in this paper.

Definition 1 A matrix is row G-astic if it has at least one nonzero in each row of the matrix.

Definition 2 An autonomous max-product system

$$\chi(k+1) = \boldsymbol{A}(\tilde{y}_k) \odot \chi(k),\ \chi(0) \in \mathbf{R}_+^n \tag{2}$$

is exponentially stable if there exist $a > 1$ and $L > 0$ such that

$$\|\chi(k)\| \leqslant L \cdot \|\chi(0)\| / a^k\ .$$

Definition 3 A max-product system

$$\begin{cases} \chi(k+1) = \boldsymbol{A}(\tilde{y}_k) \odot \chi(k) \oplus \boldsymbol{B}(\tilde{y}_k) \odot \varrho(k) \\ z(k) = \boldsymbol{C}(\tilde{y}_k) \odot \chi(k) \end{cases}$$

is bounded input bounded output in probability (BIBipO) stable if there exists a positive constant $\mathcal{M}_z$ for a positive constant σ and an initial condition $\chi(0)$ such that $P[\chi_i(k) \leqslant \mathcal{M}_z] > \sigma$ with $\sigma \in (0,1)$ in which $\chi_i(k)$ represents the entry at the ith row of $\chi(k)$. $\varrho(k)$ respresents the control input.

Lemma 1 [27] Consider the autonomous max-product system (2). If the autonomous max-product system (2) is mean norm exponentially stable, then a nonautonomous max-product system

$$\begin{cases} \chi(k+1) = \boldsymbol{A}(\tilde{y}_k)\odot\chi(k)\oplus\boldsymbol{B}(\tilde{y}_k)\odot\varrho(k) \\ z(k) = \boldsymbol{C}(\tilde{y}_k)\odot\chi(k) \end{cases}$$

is satisfied with $P(\|z(k)\| \leqslant M_z) > 1-\epsilon$ for a positive constant $\mathcal{M}_z > 0$ and an initial condition $\chi(0)$.

3. Main results

In this section, a causal feedback controller is investigated for the max-plus stochastic system (1) with a reference signal. According to [16], a reference signal is denoted as $\boldsymbol{r}(k) = kT + \boldsymbol{\xi}(k)$, where T is the positive constant, $\boldsymbol{\xi}(k)$ is the bounded vector. Moreover, the max-plus stochastic system (1) is rewritten as follows:

$$\begin{aligned} &\boldsymbol{x}(k) = \hat{\boldsymbol{A}}(y_k)\boldsymbol{x}(k)\oplus\check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1)\oplus\boldsymbol{B}u(k) = \\ &\left(\hat{\boldsymbol{A}}(y_k)\boldsymbol{x}(k)\oplus\check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1)\oplus\boldsymbol{B}u(k)\right)\otimes \\ &\hat{\boldsymbol{A}}(y_k)\oplus\check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1)\oplus\boldsymbol{B}u(k) = \\ &\left(\hat{\boldsymbol{A}}(y_k)\oplus\boldsymbol{I}\right)\check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1)\oplus \\ &\hat{\boldsymbol{A}}^2(y_k)\boldsymbol{x}(k)\oplus\left(\hat{\boldsymbol{A}}(y_k)\oplus\boldsymbol{I}\right)\boldsymbol{B}u(k) = \\ &\hat{\boldsymbol{A}}^*(y_k)\check{\boldsymbol{A}}(y_k)\boldsymbol{x}(k-1)\oplus\hat{\boldsymbol{A}}^*(y_k)\boldsymbol{B}u(k). \end{aligned} \tag{3}$$

Considering the max-plus stochastic system (3), a causal feedback controller is designed as

$$u(k) = \tilde{u}(k)\oplus v(k) \tag{4}$$

where $\boldsymbol{K}$ is the causal feedback matrix to be designed, $\tilde{u}(k) = \boldsymbol{K}\boldsymbol{x}(k-1)$ represents the regulating part of the causal feedback controller, $v(k) = kT$ represents the input part of the causal feedback controller. Based on the max-plus stochastic system (3), a stochastic system is calculated as

$$\boldsymbol{x}(k) = \hat{\boldsymbol{A}}^*(y_k)\left(\check{\boldsymbol{A}}(y_k)\oplus\boldsymbol{B}\boldsymbol{K}\right)\boldsymbol{x}(k-1)\oplus v(k)\ .$$

With the additional controller $v(k) = \varepsilon$, an autonomous stochastic system is obtained as

$$\boldsymbol{x}(k) = \hat{\boldsymbol{A}}^*(y_k)\left(\check{\boldsymbol{A}}(y_k)\oplus\boldsymbol{B}\boldsymbol{K}\right)\boldsymbol{x}(k-1)\ . \tag{5}$$

In [27], a max-product stochastic system is obtained from the autonomous stochastic system (5) as

$$\tilde{\boldsymbol{x}}(k) = \left(\tilde{\boldsymbol{A}}_0(y_k)\oplus\tilde{\boldsymbol{A}}_1(y_k)\right)\odot\tilde{\boldsymbol{x}}(k-1) \tag{6}$$

with

$$\begin{cases} \tilde{\boldsymbol{x}}(k) = \exp(\boldsymbol{x}(k))/\delta^k,\ \delta = \exp(T) \\ \tilde{\boldsymbol{A}}_0(y_k) = \exp(\hat{\boldsymbol{A}}^*(y_k)\check{\boldsymbol{A}}(y_k))/\delta \\ \tilde{\boldsymbol{A}}_1(y_k) = \exp(\hat{\boldsymbol{A}}^*(y_k)\boldsymbol{B}\boldsymbol{K})/\delta \end{cases}.$$

Lemma 2 Consider the max-product stochastic system (6) with $0<\alpha<1$ and $i = 1,2,\cdots,M$. If there exists a positive vector $\boldsymbol{h}(y_k)$ such that

$$\sum_{j=1}^{M} c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\odot\left(\tilde{\boldsymbol{A}}_0(i)\oplus\tilde{\boldsymbol{A}}_1(i)\right) \leqslant \alpha\cdot\boldsymbol{h}^{\mathrm{T}}(i), \tag{7}$$

then the max-product stochastic system (6) is exponentially stable.

Proof Select a Lyapunov function $V(y_k) = \boldsymbol{h}^{\mathrm{T}}(y_k)\odot\tilde{\boldsymbol{x}}(k)$. Based on the Markov chain y_k, expectation on the Lyapunov function $V(y_{k+1})$ is obtained as

$$\mathrm{E}[V(y_{k+1})] = \sum_{j=1}^{M} c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\odot\tilde{\boldsymbol{x}}(k) = \sum_{j=1}^{M} c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\odot\left(\tilde{\boldsymbol{A}}_0(i)\oplus\tilde{\boldsymbol{A}}_1(i)\right)\odot\tilde{\boldsymbol{x}}(k-1).$$

Note that the Lyapunov function $V(y_k) = \boldsymbol{h}^{\mathrm{T}}(y_k)\odot\tilde{\boldsymbol{x}}(k)$ is satisfied with $\beta_1\|\boldsymbol{x}_k\| \leqslant \mathrm{E}[V(y_{k+1})] \leqslant \beta_2\|\boldsymbol{x}_k\|$ in which β_1 and β_2 are the positive constants. One has that

$$\begin{gathered} \mathrm{E}[\mathrm{E}[V(y_{k+1})]] \leqslant \mathrm{E}[\beta_2\|x_k\|] \Longleftrightarrow \\ \mathrm{E}[V(y_{k+1})] \leqslant \mathrm{E}[\beta_2\|x_k\|]. \end{gathered}$$

From the inequity (7), there exists $\mathrm{E}[V(y_{k+1})] \leqslant \alpha\cdot \mathrm{E}[V(y_k)]$. By the same way, it is obtained that $\mathrm{E}[V(y_{k+1})] \leqslant \alpha^k\cdot\mathrm{E}[V(y_1)]$ which is equal to

$$\mathrm{E}[\boldsymbol{h}^{\mathrm{T}}(k+1)\odot\boldsymbol{x}(k+1)] \leqslant \alpha^k\cdot\mathrm{E}[\boldsymbol{h}^{\mathrm{T}}(1)\odot\boldsymbol{x}(1)].$$

Let $\tilde{\beta}_1$ and $\tilde{\beta}_2$ represent the maximum entry and minimum entry, respectively. One has that

$$\mathrm{E}[\tilde{\beta}_1\cdot\|\boldsymbol{x}(k+1)\|] \leqslant \alpha^k\cdot\mathrm{E}[\tilde{\beta}_2\cdot\|\boldsymbol{x}(1)\|].$$

Therefore, the max-product stochastic system (6) is exponentially stable. This completes the proof. □

Theorem 1 If there exists at least an stochastic feedback matrix $\boldsymbol{K}$ which is satisfied with

$$\boldsymbol{K} \leqslant \boldsymbol{H}_2\hat{\boldsymbol{A}}^*(i)\boldsymbol{B}\,\diagdown\!\!\!\circ\left((\boldsymbol{H}_1\ominus\boldsymbol{H}_2)\,\hat{\boldsymbol{A}}^*(i)\check{\boldsymbol{A}}(i)\right)$$

where

$$\boldsymbol{H}_1 = \ln(\alpha\cdot\boldsymbol{h}^{\mathrm{T}}(i)),\ \boldsymbol{H}_2 = \ln\left(\sum_{j=1}^{M} c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\right),$$

$\diagdown\!\!\!\circ$ and $\ominus$ are defined as left division and subtraction respectively, based on the residuation theory, then the max-product stochastic system (6) is exponentially stable with the causal feedback controller (4).

Proof Note that transformation from the autonomous stochastic system (5) to the max-product stochastic system (6) is invertible [27]. Therefore, it is reasonable to transform the inequality (7) to the following inequality as

$$\delta\cdot\sum_{j=1}^{M}c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\odot\exp(\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right))\leqslant \delta\cdot\alpha\cdot\boldsymbol{h}^{\mathrm{T}}(i). \quad (8)$$

Taking natural logarithms on both sides of the inequality (8), it is obtained that

$$\ln\left(\delta\cdot\sum_{j=1}^{M}c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\leqslant \ln\left(\delta\cdot\alpha\cdot\boldsymbol{h}^{\mathrm{T}}(i)\right). \quad (9)$$

By shifting items and residuation theory, one has that

$$\boldsymbol{K}\leqslant\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\boldsymbol{B}\,\circ\!\!\!\!\setminus\left(\boldsymbol{H}_1\ominus\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i)\right) \quad (10)$$

This completes the proof. □

Based on Definition 1, it is known that the matrix $\hat{\boldsymbol{A}}^{*}(y_k)\boldsymbol{B}$ is row G-astic in the autonomous stochastic system (5). There exists the causal feedback matrix $\boldsymbol{K}$ which is satisfied with the inequity (10) if and only if rows of the matrix $\boldsymbol{H}_1\ominus\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i)$ are null for a null row in the matrix $\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\boldsymbol{B}$. Note that the causal feedback matrix $\boldsymbol{K}$ is a non-null row vector in the causal feedback controller (4). To ensure the inequity (10) valid, rows of $\boldsymbol{H}_1\ominus\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i)$ are null when the corresponding rows are null in matrix $\boldsymbol{H}_2\hat{\boldsymbol{A}}^{*}(i)\boldsymbol{B}$. Considering that it is difficult to obtain the causal feedback matrix $\boldsymbol{K}$ only by residuation theory, a theorem is proposed to assist existence analysis of $\boldsymbol{K}$ in the following.

Theorem 2 There exists an appropriate causal feedback matrix $\boldsymbol{K}$ with $\hat{h}=\boldsymbol{h}(1)\oplus\boldsymbol{h}(2)\cdots\oplus\boldsymbol{h}(M)$ if there is a positive constant τ such that

$$\tau\cdot\boldsymbol{h}(n)\geqslant\hat{\boldsymbol{h}},\ \lambda(\ln(\tau/\alpha)\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i))\leqslant e$$

holds for $i=1,2,\cdots,M$.

Proof Considering the inequity (9), one has that

$$\ln\left(\delta\cdot\sum_{j=1}^{M}c_{ij}\cdot\boldsymbol{h}^{\mathrm{T}}(j)\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\leqslant \ln\left(\delta\cdot\hat{\boldsymbol{h}}^{\mathrm{T}}\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right).$$

It is obtained that the inequity (9) holds when

$$\ln\left(\delta\cdot\hat{\boldsymbol{h}}^{\mathrm{T}}\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\leqslant \ln\alpha\ln\left(\delta\cdot\boldsymbol{h}^{\mathrm{T}}(i)\right).$$

One has that

$$\ln\left(\delta\cdot\hat{\boldsymbol{h}}^{\mathrm{T}}\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\ln\tau\leqslant \ln\alpha\ln\left(\delta\cdot\boldsymbol{h}^{\mathrm{T}}\right).$$

By further retraction, it is obtained that

$$\ln\left(\hat{\boldsymbol{h}}^{\mathrm{T}}\right)\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\ln\tau\leqslant\ln\alpha\ln\left(\boldsymbol{h}^{\mathrm{T}}\right). \quad (11)$$

According to [28], the inequity (11) has the same solution on $\ln(\hat{\boldsymbol{h}}^{\mathrm{T}})$ as

$$\ln\left(\hat{\boldsymbol{h}}^{\mathrm{T}}\right)\left(\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\rho\right)^{*}=\ln\left(\boldsymbol{h}^{\mathrm{T}}\right). \quad (12)$$

Note that existence of $\ln(\hat{\boldsymbol{h}}^{\mathrm{T}})$ is equivalent to a two-sided eigenproblem in [29]. According to Perron-Frobenius theory, spectral radius of $\rho\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)$ is equal to its maximal eigenvalue which is also the maximum cycle mean. With the row G-astic matrix $\hat{\boldsymbol{A}}^{*}(i)\boldsymbol{B}$, it is obtained that there exists at least one stochastic feedback $\boldsymbol{K}$ such that $\lambda((\hat{\boldsymbol{A}}^{*}(i)\left(\check{\boldsymbol{A}}(i)\oplus\boldsymbol{BK}\right)\rho)^{*})=e$. This completes the proof. □

Theorem 3 If there exist positive constant

$$T>\bigoplus_{n=1}^{M}\lambda(\hat{\boldsymbol{A}}^{*}(n)\left(\check{\boldsymbol{A}}(n)\oplus\boldsymbol{BK}\right))$$

and the causal feedback matrix $\boldsymbol{K}$ such that the max-product stochastic system (6) is mean norm exponentially stable, then the autonomous stochastic system (5) is stable in probability with a positive constant M_x such that

$$P[x_i(k)-kT\leqslant M_x]>1-\epsilon$$

where $0<\epsilon<1$, $x_i(k)$ represents the entry in the ith row of $x(k)$.

Proof With $\tilde{\boldsymbol{x}}(k)=\exp(\boldsymbol{x}(k))/\exp(kT)$, the max-plus stochastic system (3) is equivalent to the following nonautonomous system as

$$\tilde{\boldsymbol{x}}(k+1)=\left(\tilde{A}_0(y_k)\oplus\tilde{A}_1(y_k)\right)\odot\tilde{\boldsymbol{x}}(k)\oplus\boldsymbol{B}. \quad (13)$$

Based on Lemma 1, it is obtained that the nonautonomous system (13) is BIBipO stable. That is, $P(\|\boldsymbol{C}(\tilde{y}_k)\odot\boldsymbol{x}(k)\|\leqslant M_x)>1-\epsilon$ holds for any $\boldsymbol{C}(\tilde{y}_k)$. Letting $\boldsymbol{C}(\tilde{y}_k)=[e\ \varepsilon\ \varepsilon]$, one has that $P(\|\tilde{x}_1(k)\|\leqslant M_{x,1})>1-\epsilon_1$, where $M_{x,1}$ and ϵ_1 are the positive constants. By the same method, it is known that $P(\|\tilde{x}_2(k)\|\leqslant M_{x,2})>1-\epsilon_2$ and $P(\|\tilde{x}_3(k)\|\leqslant M_{x,3})>1-\epsilon_3$ with $\boldsymbol{C}(\tilde{y}_k)=[\varepsilon\ e\ \varepsilon]$ and $\boldsymbol{C}(\tilde{y}_k)=[\varepsilon\ \varepsilon\ e]$, respectively. Note that $M_{x,2}$, $M_{x,3}$, ϵ_2 and ϵ_3 are the positive constants. Therefore, there exists $P[x_i(k)-kT\leqslant M_x]>1-\epsilon$ with $M_x=M_{x,1}\oplus M_{x,2}\oplus M_{x,3}$ and $\epsilon=\epsilon_1\oplus\epsilon_2\oplus\epsilon_3$. This completes the proof. □

Remark 1 In the autonomous stochastic system (5), the maximum cycle mean $\lambda(\rho\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i))$ is sometimes larger than e. According to [30], the maximum cycle mean $\lambda(\rho\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i))$ is reduced by changing state units. For example, 1 s is replaced by 1/60 min, which reduces numerical values. By this way, it is reasonable to make $\lambda(\rho\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i))$ with arbitrarily ρ, $\hat{\boldsymbol{A}}(y_k)$ and $\check{\boldsymbol{A}}(y_k)$ satisfy the condition $\lambda(\rho\hat{\boldsymbol{A}}^{*}(i)\check{\boldsymbol{A}}(i))\leqslant e$.

Remark 2 According to [31], state variables have corresponding exponential upper bounds in max-plus stochastic systems with a stochastic distribution. Therefore, it is feasible to further reduce the differences $x_i(k)-kT$ between the state variables and reference signals in the max-plus stochastic system (3) with the specific stochastic distribution. Moreover, network attacks sometimes happen such that the time delays fluctuate in the cloud control system [32]. That is, there has to be at least one additional possible realization for the Markov chain y_k of the max-plus stochastic system (3) with the network attacks. In this case, it is interesting to improve the proposed method for the Markovian jump cloud control system (1) under network attacks.

4. Simulation results

In this section, two frameworks of the Markovian jump cloud control system (1) are selected as structures whose Petri nets are shown in Fig. 2 and Fig. 3. A max-plus stochastic system (3) for numerical simulation is chosen as a Markovian jump cloud control system (1) with

$$\hat{\boldsymbol{A}}(1)=\begin{bmatrix} \varepsilon & \varepsilon & \varepsilon \\ 1 & \varepsilon & \varepsilon \\ \varepsilon & 2 & \varepsilon \end{bmatrix},\ \hat{\boldsymbol{A}}(2)=\begin{bmatrix} \varepsilon & \varepsilon & \varepsilon \\ 3 & \varepsilon & \varepsilon \\ \varepsilon & 3 & \varepsilon \end{bmatrix},$$

$$\check{\boldsymbol{A}}(1)=\begin{bmatrix} 2 & \varepsilon & \varepsilon \\ \varepsilon & 5 & \varepsilon \\ \varepsilon & \varepsilon & 4 \end{bmatrix},\ \check{\boldsymbol{A}}(2)=\begin{bmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 3 & 6 \\ \varepsilon & \varepsilon & 7 \end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix} e & \varepsilon & \varepsilon \end{bmatrix}^{\mathrm{T}},$$

and the initial state $\boldsymbol{x}(0)=[0\ 3\ 4]^{\mathrm{T}}$. The Markov chain for the Markovian jump cloud control system (1) is obtained as $y_k=\{1,2\}$ in which the transition probability matrix $\boldsymbol{C}$ and the observation probability matrix $\boldsymbol{\Phi}$ are satisfied with

$$\boldsymbol{C}=\begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix},\ \boldsymbol{\Phi}=\begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}.$$

One of the possible realizations for the Markov chain y_k is shown in Fig. 4.

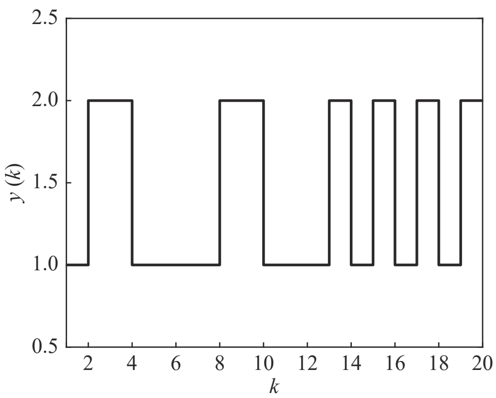

Fig. 4 Possible realization of Markov chain y_k

Based on Theorems 2 and 3, there exist $\boldsymbol{K}=[8\ \varepsilon\ \varepsilon]^{\mathrm{T}}$ and $T=10$ in the causal feedback controller $u(k)$. Therefore, the difference $x_i(k)-kT$ on the Markov chain y_k is presented in Fig. 5.

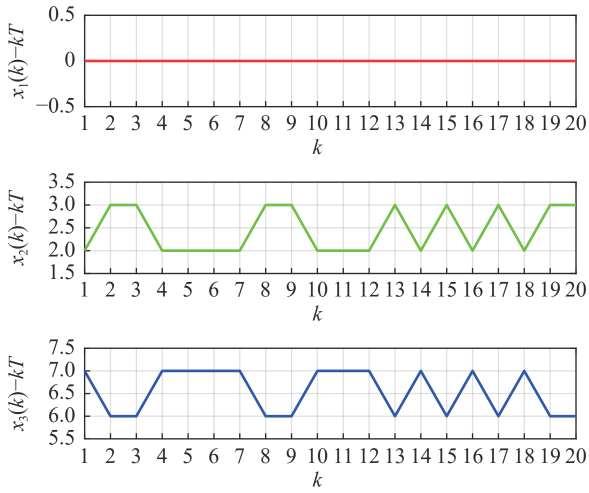

Fig. 5 Differences $x_i(k)-kT$ in the sample 1 of the Markov chain y_k

It is shown from Fig. 5 that the Markovian jump cloud control system (1) is stable in probability with the proposed causal feedback controller (4). Moreover, tracking between the state $\boldsymbol{x}(k)$ and the reference signal $r(k)$ are given in Fig. 6. Based on the causal feedback controller (4), the input $u(k)$ is shown in Fig. 7.

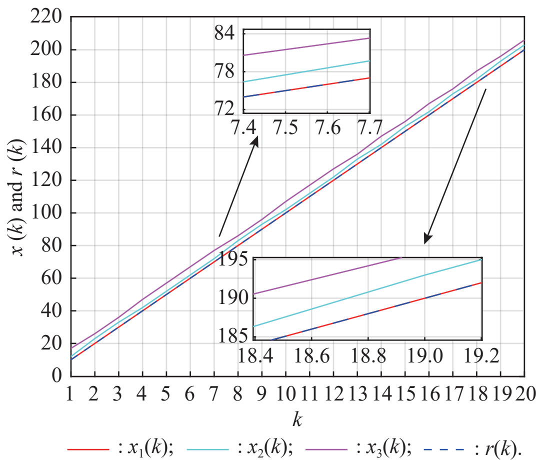

Fig. 6 Tracking between $\boldsymbol{x}(k)$ and $\boldsymbol{r}(k)$

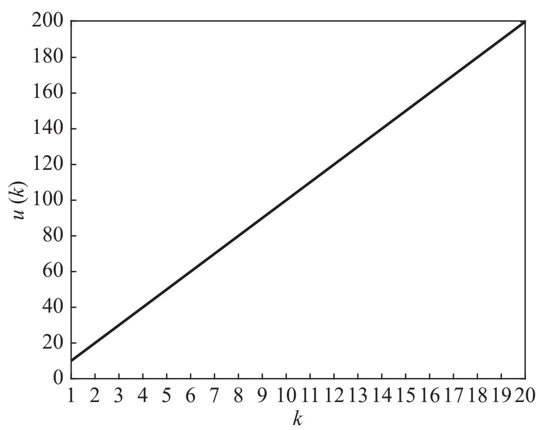

Fig. 7 Input $u(k)$

5. Conclusions

In this paper, a max-plus stochastic system with a Markov chain is established as the Markovian jump cloud control system. Based on residuation theory and exponential stability, a causal feedback matrix has been designed for a causal feedback controller to track a reference signal. For the causal feedback controller, a sufficient condition has been proposed for existence of the causal feedback matrix. By BIBipO stability analysis, stochastic stabilization has been shown for the Markovian jump cloud control system with the reference signal. Simulation results have shown effectiveness of the causal feedback controller for the Markovian jump cloud control system.

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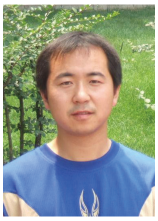

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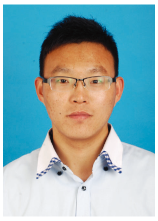

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