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# Firepower distribution method of anti-ship missile based on coupled path planning

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**Abstract:** Anti-ship missile coordinated attack mission planning is a complex multi-objective optimization problem with multiple combinations of platforms, strong decision-making constraints, and tightly coupled links. To avoid the coupling disorder between path planning and firepower distribution and improve the efficiency of coordinated attack mission planning, a firepower distribution model under the conditions of path planning is established from the perspective of decoupling optimization and the algorithm is implemented. First, we establish reference coordinate system of firepower distribution to clarify the reference direction of firepower distribution and divide the area of firepower distribution; then, we construct an index table of membership of firepower distribution to obtain alternative firepower distribution plans; finally, the fitness function of firepower distribution is established based on damage income, missile loss, ratio of efficiency and cost of firepower distribution, and the mean square deviation of the number of missiles used, and the alternatives are sorted to obtain the optimal firepower distribution plan. According to two simulation experiments, the method in this paper can effectively solve the many-to-many firepower distribution problem of coupled path planning. Under the premise of ensuring that no path crossing occurs, the optimal global solution can be obtained, and the operability and timeliness are good.

**Keywords:** anti-ship missile, path planning, firepower distribution, cooperation, weapon-target assignment (WTA), coupled.

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## 1. Introduction

Modern naval battles and coordinated anti-ship missile attacks have become the most important and only long-range means of attacking the sea. Due to the coupling relationship between path planning and firepower distribution, in addition to the high-intensity, multi-directional, multi-wave saturation attack effect, it is necessary to avoid the problems of path crossing and range waste [1,2], which makes the anti-ship missiles coordinated attack mission planning a complex multi-objective optimization problem with multiple combinations of platforms, strong decision-making constraints, and tightly coupled links.

Theoretically, for offline pre-planning problems such as cooperative anti-ship missile path planning, the exact calculation method and the intelligent optimization method can generally be used to solve the problem. However, considering the complexity of cooperative planning and the rapidity of time requirements, the intelligent optimization method becomes the current first choice for solving the cooperative offline path pre-planning problem.

Based on the existing problem model of cooperative path planning, scholars mainly refer to the application of intelligent optimization algorithms in other unmanned aerial vehicles to study the cooperative path planning method of anti-ship missiles based on intelligent optimization algorithms. Zandavi et al. [3] proposed a heuristic optimized missile path planning model based on 3 degree of freedom (3DOF) simulation to compare the performance of the genetic algorithm and the particle swarm algorithm, and the results showed that the genetic algorithm is closer to reality, but the particle swarm algorithm is more effective in solving the path planning problem due to its acceptable results and shorter computation time. Yang et al. [4] improved the genetic algorithm for multi-bomb cooperative path planning by first generating optimized paths that satisfy both the maximum number of waypoints constraint and the minimum path segment length constraint through population initialization, then using subpopulation categorization to distinguish similar paths, and finally fine-tuning the child paths to generate

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optimal and multiple suboptimal paths. Lei et al. [5] improved the Voronoi diagram to establish a multi-missile path model with the optimization objectives of minimizing the total cost of the path and minimizing the maximum time difference to reach the target, abstracted the collaborative path planning as a distributed constrained optimization problem, and used the asynchronous distributed constrained optimal algorithm and the dynamic planning optimal protocol algorithm to find the collaborative path. Qin et al. [6] proposed a path planning method for automatic parking systems based on direction map search and geometric curves. Liu et al. [7] proposed a collaborative path planning method for multi-autonomous underwater vehicle (AUV) under the influence of time-varying ocean currents based on the dynamic programming algorithm. Fan et al. [8] proposed a new manned/unmanned aerial vehicle (MAV/UAV) collaborative combat network model construction method based on a complex network. Hu et al. [9] proposed a distributed self-organizing method for UAV swarm search-attack mission planning. Yang et al. [10] solved the vehicle routing problem with rich cooperation. There is also the problem of ship scheduling [11]. Wang et al. [12] proposed a fast course planning method for anti-ship missiles based on geometric principle. Lin et al. [13] presented a collaborative complete coverage and path planning (CCPP) algorithms for single robot and multi-robot systems. Hu et al. [14] designed an improved Gaussian mixture model (GMM)/Gaussian mixture regression (GMR)-based path planning method for collaborative robots (cobots) to achieve adaptive obstacle avoidance in dynamic manufacturing.

In recent years, scholars have combined collaborative path planning and weapon-target assignmentas (WTA) into one problem, and then solved it by segments or decoupled optimization. Zeng et al. [15] decomposed the problem into two levels of multi-objective fire allocation and time-optimal collaborative path planning, and used three methods to solve the problem: non-dominated sorting genetic algorithm II (NSGA-II), multi-objective evolutionary algorithm based on decomposition (MOEA/D), and the $\varepsilon$-constrained MOEA/D algorithm (decomposition-based MOEA with the $\varepsilon$-constraint, DMOEA-$\varepsilon$C). Through comparison, it is found that DMOEA-$\varepsilon$C is the best choice for solving the multi-target firepower distribution problem, and combining it with the time-optimal cooperative path planning method can generate a missile flanking attack solution that is significantly better than the rule-based method. Jia et al. [16] proposed a method framework for multi-platform collaborative surprise path planning, decoupling the path planning process from the selection of optimization metrics and the establishment of constraints, and finding the path control points through particle swarm optimization algorithms. Jia et al. [17] also decoupled mission assignment and path planning for optimization, using genetic algorithms to find the mission assignment scheme and the loop flow artificial potential field method to find the path planning scheme by using a genetic algorithm to find the firepower distribution scheme and a circular flow artificial potential field method to find the path planning scheme to achieve the simultaneous arrival of multiple platforms. Wang et al. [18] constructed the task auction architecture and auction revenue function with the distributed contract network auction algorithm, combined with the simulated annealing algorithm to coordinate the task execution order, used the A* algorithm to complete the range estimation between two task points, and completed the collaborative path preliminary planning simultaneously in the firepower distribution stage to determine the best task execution order, so as to realize the tight coupling between firepower distribution and collaborative path planning. Zhen et al. [19,20] proposed an intelligent self-organizing algorithm (ISOA) and a UAV swarm intelligent collaborative task planning schema based on artificial potential field and ant colony algorithm. Li et al. [21] proposed a practical cooperative occupation decision-making methodology for use with multiple unmanned combat aerial vehicles (UCAVs). Sprenger et al. [22] proposed a methodology to solve a cooperative transportation planning problem and to assess its performance. Fernandez et al. [23] proposed a new vehicle routing problem that arises in an urban area where several carriers operate and some of their customers have demand of service for more than one carrier. Li et al. [24] proposed an intelligent agent-based model for the decision-making process during fleet operations by combining real-time optimization with artificial intelligence.

The above methods have real bottlenecks such as difficult coupling and decoupling processing in the vertical decision-making process and difficulty in ensuring global optimality in the design of the solution algorithm. The existing coupling processing and decoupling optimization are either through “partition-coordination” or “predict-adjustment.” In essence, they are both decomposition modeling and algebraic optimization, and only locally optimal solutions can be obtained.

We proposed the region division model in [1], which has better performance for single platform single target. However, since path crossing may occur for multiple platforms and multiple targets, the present firepower distribution concept is introduced to ensure that path crossing does not occur during path planning. To address the above problems, this paper proposes a four-stage model of "reference of firepower distribution-division of area of firepower distribution-index table of membership-scheme preference" to optimize the firepower distribution method of coupled path planning. The coupled baseline system is established, and the decoupled affiliation degree guides the evolutionary direction to ensure that the global optimal solution is not lost due to the coupling from the source.

## 2. Origin and evolution of the concept of firepower distribution

In traditional concepts, the concept of firepower distribution is generally used in air defense operations, which refers to the matching decision-making process between ship-to-air missiles and artillery units and air attack targets. These are the problem of WTA in air defense operations [25–27]. However, with the rapid development of anti-ship missiles as a typical representative of anti-sea warfare equipment and technology, its operational use mode also changes accordingly. In anti-ship warfare, the timeliness of information, the time-sensitivity of targets, and the quick decision of the combat process are further enhanced and highlighted, which makes the status of these three in anti-ship warfare gradually improved and developed to the same height as their status in air defense warfare. This has created new combat requirements in anti-ship warfare. In response to combat requirements, corresponding decision-making process are required to support. Driven by new combat requirements, the concept of firepower distribution was gradually introduced into anti-ship operations [28], and it was enriched and developed, becoming a key decision-making link in anti-ship operations. The conceptual evolution of firepower distribution is shown in Fig. 1.

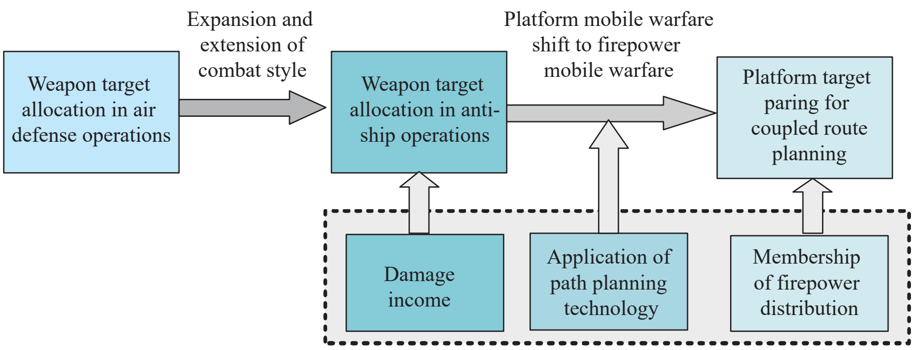


**Fig. 1 Conceptual evolution of firepower distribution**

The "anti-ship missile firepower distribution" according to the mission of the missile attack refers to the reasonable matching decision between the missile and the target. The corresponding decision-making instruction is set to the missile, and the target selection ability of the missile itself is used to select the intended attack target (firepower distribution target) to realize the commander's operational intentions and ensure operational coordination [29–33]. The essence is to reasonably distribute missile firepower to strike a predetermined target in accordance with the commander's combat intentions. The evaluation index is the probability of damage to the target by the missile. The technical basis and key lie in the ability of anti-ship missiles to select targets by firepower distribution. The realization form is to rely on missiles to autonomously complete the search, identification, selection, and attack of targets. This is not only the conceptual expansion and extension of its combat style (or aspects) from air defense to anti-ship for firepower distribution but also a leap-forward change in concepts and technology, which puts forward very high requirements for the intelligent combat level of anti-ship missiles. The whole process of anti-ship missile firepower distribution can be divided into three stages: (i) the commander makes the attack target decision; (ii) the missile user sets the attack decision instruction to the missile. (iii) the missile selects the predetermined attack target according to the decision instruction.

The theory of maritime firepower mobile warfare has led to the rapid development of naval weaponry and equipment technology as one of the important needs, and the anti-ship missile path planning technology has emerged. The application of path planning technology has greatly improved the combat effectiveness of anti-ship missiles. It has also made the traditional anti-ship decision-making links unable to meet the development requirements of maritime firepower mobile warfare. The firepower distribution is the most profoundly affected.

Before the path planning, the first key problem to be solved is to clarify the starting point and the target point of the anti-ship missile attack, and in the coordinated path planning is to clarify which launch platform attacks which target. This is the problem to be solved by firepower distribution. Therefore, the firepower distribution should be carried out before path planning, and reasonable firepower distribution is a prerequisite to ensure the smooth progress of path planning. If the firepower distribution is unreasonable, it will cause difficulty in path planning and even unsolvable situations. This puts forward new requirements for firepower distribution. The firepower distribution not only satisfies its own constraints but also be able to satisfy subsequent path planning requirements. The existing firepower distribution methods of anti-ship missiles are based on platform maneuvering warfare and direct anti-ship missiles, and do not consider firepower maneuvering factors such as path planning. This is far from the new requirements for firepower distribution. As a result, it is no longer possible to ensure that anti-ship missiles can fully exert their due combat effectiveness under the conditions of maritime firepower mobile warfare. Therefore, the concept and method of anti-ship missile firepower distribution must be improved and developed.

In view of the basic needs of coordinated path planning to study the issue of anti-ship missile firepower distribution, the first and foremost thing is to realize the conceptual transformation from platform maneuver warfare to firepower maneuver warfare. This paper proposes the method of regional division for collaborative path planning as a new anti-ship decision-making link under the conditions of maritime firepower maneuver warfare [34]. Regional division can not only avoid the occurrence of path crossing phenomenon but also be used as a bridge and tie for firepower distribution and path planning. Therefore, on the basis of satisfying its own constraints, firepower distribution should serve regional division and path planning. It can be defined as follows:

**Definition 1** Anti-ship missile firepower distribution is the process of coordinated path planning, according to the number of firepower units, the location of firepower units, the number of targets and the location of targets, each target is assigned to different firepower units according to certain rules and constraints, and each firepower unit carries out the attack separately so that the subsequent area division and path planning can be carried out smoothly, thereby ensuring the effective completion of the mission.

For Definition 1, there are two important points to explain:

(i) Definition 1 is to conceptually realize the transformation of firepower distribution from platform maneuver warfare to firepower maneuver warfare. The key is to create decision-making conditions for subsequent regional division and path planning. The regional division is based on a firepower unit (or launch platform). It focuses on the matching decision-making between the firepower unit and the target. However, it does not ignore the intelligent combat level of anti-ship missiles. The target selection capability of anti-ship missiles is still an important part of it.

(ii) The “rules” in Definition 1 belong to the category of firepower distribution methods, the basis for formulating rules is the evaluation index. It is obviously inappropriate to simply use the damage probability of anti-ship missiles as the evaluation index of firepower distribution for coordinated path planning. What evaluation index should be selected and how to formulate corresponding rules are all open-ended questions. For this reason, we put forward the concept of an index table of firepower distribution membership and use it as an evaluation index to study the firepower distribution method of anti-ship missiles under the condition of path planning.

## 3. Firepower distribution model of coupled path planning

In order to adapt anti-ship operations to the development requirements of maritime firepower maneuver warfare, the concept of firepower distribution and its evolution is studied, the concept of anti-ship missile firepower distribution under the condition of path planning is proposed, and its definition is given in Section 2. This section solves the problem of how to distribute firepower for anti-ship missiles under the condition of path planning. That is the problem of firepower distribution method and modeling for cooperative path planning. This section divides the modeling of the firepower distribution of coordinated path planning into the following four steps to achieve: First, establish firepower distribution reference coordinate system and define the reference direction of firepower distribution. Second, carry out the area of firepower distribution in the reference coordinate system of firepower distribution, and obtain the area of firepower distribution under the jurisdiction of each platform. Third, calculate the membership degree of each target to the firepower distribution area of each platform, establish the firepower index table of membership of firepower distribution, and obtain all possible alternative firepower distribution plans. Finally, according to the fitness function of firepower distribution, sort the alternative firepower distribution schema, and obtain the optimal global solution of firepower distribution schema. Fig. 2. shows the modeling process of firepower distribution in cooperative path planning.

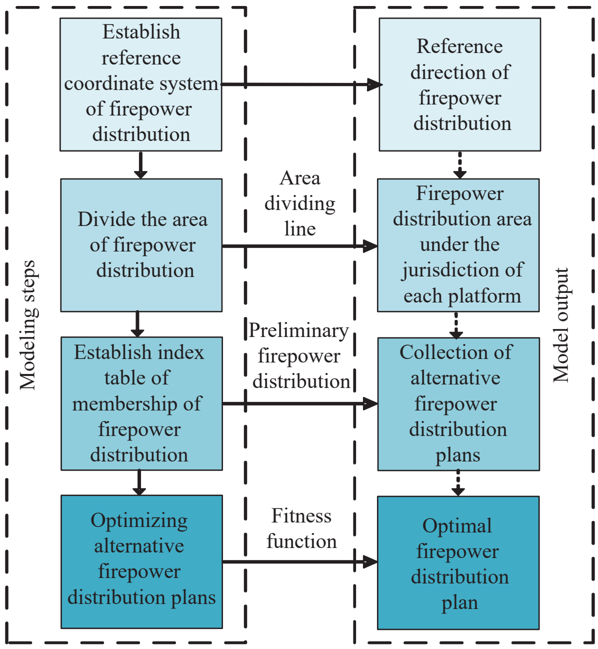


**Fig. 2 Conceptual evolution of firepower distributio**

### 3.1 Establishing a reference coordinate system of firepower distribution

When conducting a single-platform anti-ship missile attack, the reference direction of firepower distribution is specifically determined according to the attack situation of the platform on the target. Because of the compatibility of target firepower, the reference direction of firepower distribution of each platform is different from each other. However, the above-mentioned situation is obviously inappropriate when conducting firepower distribution modeling for cooperative path planning. When conducting coordinated path planning firepower distribution, not only the constraints of the single-platform anti-ship missile reference direction of firepower distribution must be considered, but more importantly, the mutual coordination constraints between multiple platforms must be considered. It is necessary to uniformly describe and determine the base direction of firepower distribution of anti-ship missiles on each platform. Make the following definition:

**Definition 2** The reference direction of firepower distribution is the reference direction used to uniformly measure the direction of anti-ship missile firepower distribution for each platform when conducting firepower distribution for cooperative path planning.

Establishing the reference direction of firepower distribution is the premise and basis for establishing the firepower distribution model. Different reference directions of firepower distribution will construct different firepower distribution situations (input of the model), and the reference direction of firepower distribution needs to be in a suitable coordinate system. Therefore, to establish a firepower distribution model, it is first necessary to establish its reference coordinate system. The following definitions are given:

**Definition 3** The baseline of firepower distribution is a ray pointing to the reference direction of the firepower with a certain point as the endpoint, and it is a specific expression of the reference direction of firepower distribution.

**Definition 4** The baseline angle of firepower distribution is the angle rotated from the due east (horizontal direction) on the chart to the baseline of the firepower distribution, with counter-clockwise as the positive, and used to measure the direction of the baseline of firepower distribution, denoted as $\alpha_s \in [0, 2\pi)$

**Definition 5** The normal line of firepower distribution is a ray that takes the endpoint of the baseline of firepower distribution as the endpoint and points to the direction of the baseline of firepower distribution rotated 90° counter-clockwise.

**Definition 6** The reference coordinate system of firepower distribution is a Cartesian coordinate system with the end of the baseline of firepower distribution as the origin and the baseline of firepower distribution and the normal line of firepower distribution as the two coordinate axes. The directions of baseline of firepower distribution and the directions of the firepower normal are the positive directions of the respective coordinate axes. The coordinate axes where the baseline of firepower distribution is called the baseline axis, denoted as $x_s$, and the coordinate distance on it called the baseline coordinate; the coordinate axis where the normal line of firepower distribution is called the normal axis, denoted as $y_s$, and the coordinate distance on it called the normal coordinates.

The reference coordinate system of firepower distribution is used to provide a unified description of the location of each platform, each target, and related area in the firepower distribution. From the above definition, it can be seen that the reference coordinate system of firepower distribution is determined by the baseline of firepower distribution, which is in turn determined by the baseline angle of firepower distribution. The reference coordinate system of firepower distribution is actually used to determine the baseline of firepower distribution and its baseline angle of firepower distribution. The baseline of firepower distribution is different, the location description provided is different, and the relevant area constructed is also different. In the firepower distribution modeling, the most important thing is that the baseline of firepower distribution needs to be selected and determined.

The following is a specific determination of the baseline of firepower distribution based on Definition 3 and Definition 4. As shown in Fig. 3, the dispersion center of the launch platform $O_w^c$ is taken as the endpoint of the baseline of firepower distribution, the angle is formed by the line between the endpoint and the dispersion center of target $O^c$, and the due east direction (horizontal direction) on the chart is taken as the baseline angle of firepower distribution $\alpha_s$.

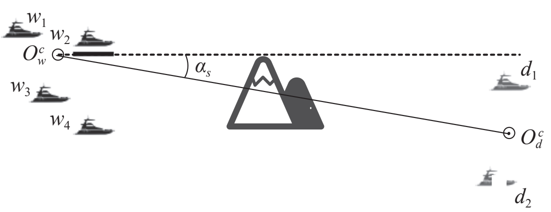


**Fig. 3 Determine the firepower baseline**

The positions of the dispersion center of the launch platform and the dispersion center of the target are the determining factors of the baseline of firepower distribution. Calculate them separately as follows:

Suppose the geographic coordinates of the dispersion center of launch platform $O_w^c$ are $(x_w^c, y_w^c)$.

$$O_w^c: \begin{cases} x_w^c = \dfrac{1}{N_w}\displaystyle\sum_{N_w}^{i=1} x_w^i \\ y_w^c = \dfrac{1}{N_w}\displaystyle\sum_{N_w}^{i=1} y_w^i \end{cases} \tag{1}$$

where $N_w$ is the number of launch platforms, and $(x_w^i, y_w^i)$ is the $i$th$(i = 1, 2, \cdots, N_w)$ geographic coordinates of the launch platform $w_i$.

Suppose the geographic coordinates of the dispersion center of target $O_d^c$ is $(x_d^c, y_d^c)$.

$$O_d^c: \begin{cases} x_d^c = \dfrac{1}{N_d}\displaystyle\sum_{N_d}^{i=1} x_d^i \\ y_d^c = \dfrac{1}{N_d}\displaystyle\sum_{N_d}^{i=1} y_d^i \end{cases} \tag{2}$$

where $N_d$ is the number of targets and $(x_d^j, y_d^j)$ is the $j$th$(j = 1, 2, \cdots, N_w)$ geographic coordinate of target $d_j$.

The above calculation method of the dispersion center is suitable for the situation where the distribution of each platform (target) is relatively uniform. If some platforms (targets) are far away from most other platforms (targets), the situation is shown in Fig. 4.

As shown in Fig. 4., by calculating according to (2), the dispersion center of the launch platform is $O_w^{c'}$, and the corresponding baseline angle of firepower distribution is $\alpha_s'$. If the firepower distribution is according to this reference direction, most of the launch platforms (as in Fig. 4, $w_1, w_2, w_3, w_4$) obviously cannot allocate a reasonable area when the subsequent area division is carried out. In order to solve this problem, when calculating the dispersion center, the situation is similar to the platform in Fig. 4. $w_5$ needs to be eliminated. For this reason, this paper first pre-processes the location data and then calculates it according to the calculation formula of the dispersion center.

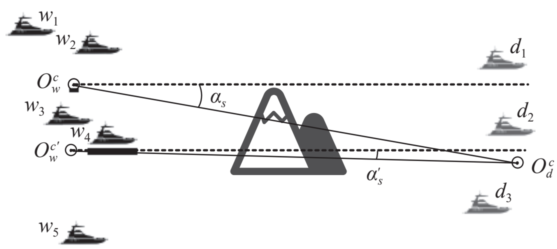


**Fig. 4 Scattering center position under extremely uneven distribution**

The steps of data pre-process are as follows:

**Step 1** According to the calculation formulas (1) and (2) of the dispersion center, calculate the position of the initial dispersion center $O_w^{c'}$ to obtain the coordinates $(x_w^{c'}, y_w^{c'})$ and calculate the position of the initial dispersion center $O_d^{c'}$ to obtain the coordinates $(x_d^{c'}, y_d^{c'})$.

**Step 2** Calculate the distance $D_w^i/D_d^j$ between each launch platform $w_i$ (target $d_j$) and the initial dispersion center $O_w^{c'}/O_d^{c'}$:

$$\begin{cases} D_w^i = \sqrt{(x_w^i - x_w^{c'})^2 + (y_w^i - y_w^{c'})^2} \\ D_d^j = \sqrt{(x_d^j - x_d^{c'})^2 + (y_d^j - y_d^{c'})^2} \end{cases}. \tag{3}$$

**Step 3** Determine whether $D_w^i/D_d^j$ is greater than a certain critical value $\varepsilon_w(\varepsilon_d)$. If it is, then the corresponding platform (target) is considered to be an interference platform (target), and remove it; otherwise, this platform (target) participates in the calculation of the dispersion center. Go to Step 4.

**Step 4** According to the calculation formulas (1) and (2) of the dispersion center, calculate the position of the dispersion center after removing the interference platform (target), and obtain the new coordinate of the dispersion center position $(x_w^c, y_w^c)$ and $(x_d^c, y_d^c)$.

After pre-processing the data, the final dispersion center of the launch platform $O_w^c$ and the target $O_d^c$ can be obtained. The baseline of firepower distribution can be determined. The baseline of firepower distribution is the ray that points to the target dispersion center $O_d^c$ with the dispersion center of the launch platform $O_w^c$ as the endpoint. According to Definition 4, the calculation formula of the baseline angle of firepower distribution $\alpha_s$ can be obtained:

$$\alpha_s = \begin{cases} \arctan\dfrac{y_d^c}{y_w^c}, & x_w^c < x_d^c; y_w^c \leqslant y_d^c \\ \dfrac{\pi}{2}, & x_w^c = x_d^c; y_w^c < y_d^c \\ \pi + \arctan\dfrac{y_d^c}{y_w^c}, & x_w^c > x_d^c \\ \dfrac{3\pi}{2}, & x_w^c = x_d^c; y_w^c > y_d^c \\ 2\pi + \arctan\dfrac{y_d^c}{y_w^c}, & x_w^c < x_d^c; y_w^c > y_d^c \end{cases}. \quad (4)$$

The reference coordinate system of firepower distribution is established. The geographic coordinates of the launch platform and the target are converted into the corresponding baseline coordinates and normal coordinates.

$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \begin{pmatrix} \cos\alpha_s & \sin\alpha_s \\ -\sin\alpha_s & \sin\alpha_s \end{pmatrix} \begin{pmatrix} x - x_w^c \\ y - y_w^c \end{pmatrix} \quad (5)$$

where $(x, y)$ is the geographic coordinates of the launch platform (target), $\alpha_s$ is the baseline angle of firepower distribution, $(x_s, y_s)$ is the reference coordinate of firepower distribution of the launch platform (target), $x_s$ is the distance of baseline coordinate, and $y_s$ is the distance of normal coordinate.

## 3.2 Division of area of firepower distribution

After the reference coordinate system of firepower distribution is established, the area of firepower distribution is divided into the reference coordinate system of firepower distribution to obtain the area of firepower distribution under the jurisdiction of each platform. Since the research goal of firepower distribution in this article is to achieve task coordination between launch platforms, the focus of the research is to solve the problem of path crossing. The following three assumptions may be made for the division of areas of firepower distribution:

(i) The target platform has entered the maximum effective range of the launching platform when the firepower distribution is started.

(ii) The performances of the anti-ship missiles of all launch platforms are roughly the same.

(iii) Each target platform poses the same threat to each launch platform.

As shown in Fig. 5, $w_i(i \leqslant N_w)$ is a launching platform, the size of $i$ is increased in the negative direction of the normal axis. Firepower distribution reference system coordinates are $(x_{\rm sw}^i, y_{\rm sw}^i)$, where $x_{\rm sw}^i$ is the distance of baseline coordinate, and $y_{\rm sw}^i$ is the distance of normal coordinate. $d_j(j \leqslant N_d)$ is the target platform, where the size of $j$ is increased sequentially along the negative direction of the normal axis. The reference coordinate of firepower distribution are $(x_{\rm sd}^j, y_{\rm sd}^j)$, where $x_{\rm sw}^i$ is the distance of the baseline coordinate, and $y_{\rm sw}^i$ is the distance of the normal coordinate.

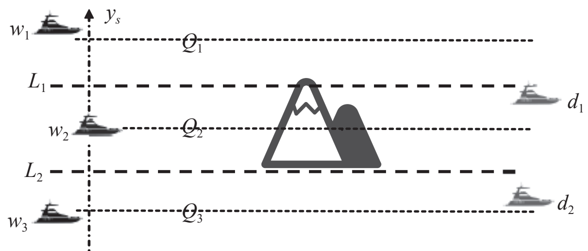

**Fig. 5** Scattering center position under extremely uneven distribution

The following definitions are given:

**Definition 7** A regional dividing line is a set of parallel lines parallel to the baseline axis. They divide the entire combat sea area into several areas, denoted as $L_m(m \leqslant N_w - 1)$, and the size of $m$ is increased in order along the negative direction of the normal axis. The position is the weighted average of the normal coordinates of two adjacent launch platforms.

$$L_m = \lambda \cdot y_{sw}^m + (1-\lambda) \cdot y_{sw}^{m+1}, \quad m \leqslant N_w - 1 \quad (6)$$

where $\lambda \in [0, 1]$ is the division weight coefficient, and the value is related to the performance of the anti-ship missiles of two adjacent launch platforms and the threat of each target to each launch platform.

When the performance of the anti-ship missiles of each platform is the same, and the threat level of each target to each launch platform is the same, the value of $\lambda$ is 0.5, and then (6) can be transformed into

$$L_m = \frac{y_{sw}^m + y_{sw}^{m+1}}{2}, \quad m \leqslant N_w - 1. \quad (7)$$

**Definition 8** The area of firepower distribution is a number of areas divided into the entire combat sea area by the area dividing line, denoted as $Q_k(k \leqslant N_w)$. The size of $k$ is increased along the negative direction of the normal axis, $Q_k$ corresponds to $w_k$, where $w_k$ is called area of firepower distribution. On both sides of the regional dividing line are two different areas of firepower distribution, and the boundary of the area of firepower distribution $Q_k$ is two adjacent regional dividing lines, the boundaries of the area of firepower distribution are respectively $L_{k-1}$ and $L_k$. The upper boundary $Q_1$ is set as $L_1 + \varepsilon_1$, and the lower boundary $Q_{N_k}$ is set as $L_{N_k-1} - \varepsilon_{N_k}$. The $w_k$ firepower distribution area $Q_k$ can be expressed as

$$Q_k \in \begin{cases} (L_k, L_1 + \varepsilon_1), & k = 1 \\ (L_k, L_{k-1}), & 1 < k < N_w \\ (L_{N_k-1} - \varepsilon_{N_k}, L_{k-1}), & k = N_w \end{cases} \quad (8)$$

where $\varepsilon_1$ and $\varepsilon_{N_w}$ are fixed values, and the values are determined according to the boundary of the combat sea area. In principle, the width of the upper and lower firepower distribution areas should be basically consistent with the width of the existing firepower distribution areas.

$$\begin{cases} L_0 = L_1 + \varepsilon_1 \\ L_{N_{w-1}} = L_{N_w-1} - \varepsilon_{N_w} \end{cases} \tag{9}$$

Formula (8) can be simplified as

$$Q_k \in (L_k, L_{k-1}), \quad 1 \leqslant k \leqslant N_w. \tag{10}$$

The regional dividing line $L_m (m \leqslant N_w - 1)$ can belong to either the area of firepower distribution $Q_m$ or the area of firepower distribution $Q_{m+1}$. Considering the simplicity and uniformity of the expression, (8) and (10) are both open intervals.

In a single area of firepower distribution, there are generally no more than two target platforms. If the number of target platforms exceeds two, the target platform in this area of firepower distribution can be regarded as a dense target group and treated as a target platform, and the center of the dispersion is the target position; if several adjacent platforms in the launch platform are relatively close, they are regarded as a dense platform group, and the dispersion center is taken as the platform position.

In order to meet the anti-ship missile's flight constraints and firepower distribution constraints and to avoid the anti-ship missile's path crossing, the following criterion can be made:

**Criterion 1** In the firepower distribution of coupled path planning, it is stipulated that each target is only related to its area of firepower distribution and adjacent areas of firepower distribution and has nothing to do with other areas of firepower distribution far away. If the target $d_j$ is in the area of firepower distribution $Q_k$, it can only be assigned to platform $w_k$ and its adjacent $w_{k-1}$ and platform $w_{k+1}$ (if it exists).

As shown in Fig. 5, target $d_1$ is in the area of firepower distribution $Q_2$; it can only be assigned to $w_2$, $w_1$ or $w_3$ the platform because if assigned to platform $w_4$, its path $w_2$ and $w_3$ will inevitably cross the path of the platform, attacking the target $d_2$.

### 3.3 Establishing the index table of membership of firepower distribution

After the area of firepower distribution is divided, the area of firepower distribution under the jurisdiction of each platform is clarified, and the basic basis for firepower distribution is constructed. However, not all targets are assigned to the platform corresponding to the area of firepower distribu-tion. Then, the next step is to provide a more direct basis for firepower distribution.

**Definition 9** The index table of membership of firepower distribution is the probability of assigning a target $d_j$ to the platform $w_i$, denoted as $\kappa_j^i$.

The concept of the index table of membership of firepower distribution provides a direct basis for firepower distribution. For example, if the target's $d_j$ index table of membership of firepower distribution to the platform $w_1$ is 0.8, and the firepower distribution $w_2$ index table of membership of firepower distribution to the platform is 0.2. The platform $w_1$ will attack $d_j$ with a greater probability. Under the premise of meeting other constraints, the target $d_j$ should be distributed to the platform $w_1$.

According to Criterion 1, if the target $d_j$ is in the area of firepower distribution $Q_k$, its index table of membership of firepower distribution is only for the platform $w_k$ and its adjacent $w_{k-1}$ and the platform $w_{k+1}$ (if it exists). As shown in Fig. 5, if the target $d_1$ is in the area of firepower distribution $Q_2$, the index table of membership of firepower distribution $d_1$ is only for the $w_2$, $w_1$, and $w_3$ platforms. If the target $d_j$ is in the area of firepower distribution $Q_k$, its index table of membership of firepower distribution $\kappa_j^i$ can be determined by the following formula:

$$\kappa_j^i = \begin{cases} 1 - \exp\left(-\dfrac{\left|y_{sd}^j - L_k\right|}{\left|L_k - y_{sw}^{k-1}\right|}\right), & i = k-1 \\ 1 - -\dfrac{\left|y_{sd}^j - \dfrac{L_k + L_{k-1}}{2}\right|}{\left|L_k - L_{k-1}\right|}, & i = k \\ 1 - \exp\left(-\dfrac{\left|y_{sd}^j - L_{k-1}\right|}{\left|L_k - y_{sw}^{k+1}\right|}\right), & i = k+1 \end{cases} . \tag{11}$$

It can be seen from (11) that:

(i) The target $d_j$ index table of membership of firepower distribution $\kappa_j^k$ of the platform $w_k$ ranges from 0.5 to 1. When the target $d_j$ is in the center of the area of firepower distribution $Q_k$, the target $d_j$ index table of membership of firepower distribution $\kappa_j^k$ to the platform $w_k$ is the largest, with a value of 1; when the target $d_j$ is at the lower boundary $L_k$ or upper boundary $L_{k-1}$ of the area of firepower distribution $Q_k$, the target $d_j$ index table of membership of firepower distribution $\kappa_j^k$ to the platform $w_k$ is set to 0.

(ii) When the target moves from the lower boundary to the upper boundary, the target's $d_j$ index table of membership of firepower distribution $\kappa_j^{k-1}$ to the platform $w_{k-1}$ gets larger and larger, while the index table of membership of firepower distribution $\kappa_j^{k+1}$ to the platform $w_{k+1}$ gets smaller and smaller.

(iii) Equation (11) satisfies a certain principle of non-intersection of missile paths and also satisfies Criterion 1.

After calculating the index table of membership of firepower distribution of all targets to their respective launch platforms, an index table of an anti-ship missile of membership of firepower distribution can be established. Table 1 shows the index table of membership of firepower distribution.

**Table 1 Index table of membership of firepower distribution**

| Platform | Target | | | | |
|---|---|---|---|---|---|
| | $d_1$ | $d_2$ | $d_3$ | $\cdots$ | $d_{N_d}$ |
| $w_1$ | 0.4 | 0.8 | 0 | $\cdots$ | 0 |
| $w_2$ | 0.6 | 0.3 | 0 | $\cdots$ | 0 |
| $w_3$ | 0.4 | 0.7 | 0.2 | $\cdots$ | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $w_{N_d}$ | 0.4 | 0.8 | 0 | $\cdots$ | 0.5 |

After the index table of membership of firepower distribution is obtained, the preliminary firepower distribution can be performed according to the value of the index table of membership of firepower distribution. Two of the higher values of the index table of membership of firepower distribution of each target are selected as alternative firepower distribution schema, and then the alternative firepower distribution schema of each target are combined to obtain a complete set of alternative firepower distribution schema.

### 3.4 Optimizing alternative firepower distribution schema

After obtaining the set of alternative firepower distribution schema, the optimal firepower distribution schema is selected from the set of alternative firepower distribution schema. Then, we must first clarify the evaluation criteria and establish a suitable fitness function of firepower distribution.

**Definition 10** Damage income is the damage income of the anti-ship missile carried on the launching platform to the target, denoted as

$$b_d = \left(\sum_{j=1}^{N_d} n_j - 2n_c\right) \cdot \alpha_b \tag{12}$$

where $n_j$ is the number of anti-ship missiles that target $j$ is attacked. $n_c$ is the number of path crossing. Obviously, for anyone path crossing, the corresponding two anti-ship missiles fail at the same time. $\alpha_b$ is the benefit factor.

**Definition 11** Missile loss is the number of anti-ship missiles consumed in an attack, denoted as

$$c_m = \beta_c \cdot \sum_{j=1}^{N_d} n_j \tag{13}$$

where $\beta_c$ is the loss factor.

**Definition 12** The ratio of efficiency and cost of firepower distribution is the ratio between the damage income of the anti-ship missile carried on the launching platform to the target $b_d$ and the loss of the anti-ship missile in an attack $c_m$, denoted as

$$R_{bc} = \frac{b_d}{c_m} = \frac{\left(\sum_{j=1}^{N_d} n_j - 2n_c\right) \cdot \alpha_b}{\beta_c \cdot \sum_{j=1}^{N_d} n_j}. \tag{14}$$

Equation (14) is a typical calculation form of the ratio of efficiency-cost. However, for the military, the main consideration is to damage the target, while the consideration of loss is secondary. Therefore, in order to highlight the requirements for damage, the damage income is expressed in the form of an exponential function, and the ratio of efficiency and cost of firepower distribution can be calculated by the following formula:

$$R_{bc} = \frac{\exp\left[\left(\sum_{j=1}^{N_d} n_j - 2n_c\right) \cdot \alpha_b\right]}{\beta_c \cdot \sum_{j=1}^{N_d} n_j}. \tag{15}$$

When performing alternative firepower distribution optimization, in addition to considering the ratio of efficiency and cost of firepower distribution, it should also consider making the target as fully damaged as possible (assuming that the more missiles the target is attacked, the greater the probability of damage). Each target platform poses the same threat to each launch platform. When the number of launched anti-ship missiles is the same, the launched anti-ship missiles should be equally distributed to each target platform as much as possible. This is also to satisfy the requirement that each target platform is damaged as much as possible:

**Definition 13** The mean square deviation of the number of missiles used is the mean square deviation of the number of anti-ship missiles distribution by each target during an attack, denoted as

$$\sigma_n = \sqrt{\frac{\sum_{j=1}^{N_d}(n_j-\overline{n})^2}{N_d}}. \quad (16)$$

The average value of the number of anti-ship missiles distribution to each target, that is $\overline{n}$, mean of $n_j$ to $N_d$:

$$\overline{n} = \frac{\sum_{j=1}^{N_d} n_j}{N_d}. \quad (17)$$

Considering that the path does not crossing and all targets are damaged as much as possible, the fitness function of firepower distribution can be established.

$$f(n_j,n_c) = \gamma\cdot\frac{\exp\left[\left(\sum_{j=1}^{N_d} n_j - 2n_c\right)\cdot\alpha_b\right]}{\beta_c\cdot\sum_{j=1}^{N_d} n_j} - (1-\gamma)\cdot\sqrt{\frac{\sum_{j=1}^{N_d}(n_j-\overline{n})^2}{N_d}},\quad 1\leqslant j\leqslant N_d \quad (18)$$

where $\gamma$ is the weight coefficient.

After the fitness function of firepower distribution is established, the fitness function value of firepower distribution of each alternative firepower distribution plan can be calculated based on it, and the firepower distribution plan is optimized according to the size of the fitness function value of firepower distribution. The greater the value of the fitness function of firepower distribution, the greater the probability that the corresponding firepower distribution plan will be selected.

After the firepower distribution is completed, the optimal firepower distribution plan can be verified. The specific method is: first carry out path planning, and check again whether there is path crossing; if there is, reselect the suboptimal firepower distribution in the set of alternative firepower distribution plans, and then carry out path planning until the path planning of all anti-ship missiles is completed and there is no path crossing.

### 3.5 Algorithm steps

Design its firepower distribution algorithm according to the firepower distribution model of coupled path planning:

**Step 1** Set the size of the critical value $\varepsilon_w$ and $\varepsilon_d$.

**Step 2** Calculate the position coordinates of the dispersion center $O_w^c$ of the launch platform $(x_w^c, y_w^c)$, and the position coordinates $(x_d^c, y_d^c)$ of the target's dispersion center $O_d^c$ according to (1) and (2).

**Step 3** Calculate the distance $D_w^i(D_d^j)$ between each launch platform $w_i$ (target $d_j$) and the dispersion center $O_w^c(O_d^c)$ according to (3), and judge whether $D_w^i(D_d^j)$ is greater than the critical value $\varepsilon_w(\varepsilon_d)$. If it is, remove this platform (target) and turn Step 2; otherwise, go to Step 4.

**Step 4** Calculate the baseline angle $\alpha_s$ according to (4), and convert the geographic coordinates of the launch platform and the target platform into the corresponding baseline coordinates and normal coordinates according to (5);

**Step 5** Order $i=1, j=1, k=1, m=1$.

**Step 6** Determine whether $m\leqslant N_w-1$. If it is true, go to Step 7; otherwise, go to Step 8.

**Step 7** Calculate the position of the area dividing line $L_m$ according to (7), and go to Step 6.

**Step 8** Determine whether $k\leqslant N_w$ is true. If yes, go to Step 9; otherwise, go to Step 10.

**Step 9** Determine the scope of the area of firepower distribution $Q_k$ according to (10), and then go to Step 8.

**Step 10** Determine whether $j\leqslant N_d$. If it is true, go to Step 11; otherwise $j=1$, go to Step 12.

**Step 11** Calculate which area of firepower distribution it belongs to according to the target's $d_j$ firepower reference system coordinates $(x_{sd}^j, y_{sd}^j)$, go to Step 10.

**Step 12** Determine whether $j\leqslant N_d$. If it is true, go to Step 13; otherwise, go to Step 14.

**Step 13** Calculate the index table of firepower distribution membership $\kappa_j^i$ of the target $d_j$ according to (11), $j++$ and go to Step 12.

**Step 14** Establish the index table of membership of firepower distribution, and carry out preliminary firepower distribution based on the principle of first taking the larger one and obtain an alternative $N_p$ firepower distribution plan.

**Step 15** Set the value size of $\alpha_b,\beta_c,\gamma$ and order $p=1$.

**Step 16** Determine whether $p\leqslant N_p$. If it is true, go to Step 17; otherwise, go to Step 18.

**Step 17** Calculate the fitness function value of firepower distribution of the first alternative firepower distribution plan paccording to (18), $p++$ and go to Step 16.

**Step 18** Check whether there is a path crossing. If yes, go to Step 15; otherwise, the optimal firepower distribution plan is the one with the largest fitness function value of firepower distribution, and the firepower distribution ends.

Fig. 6 shows the algorithm flow designed according to the above steps.

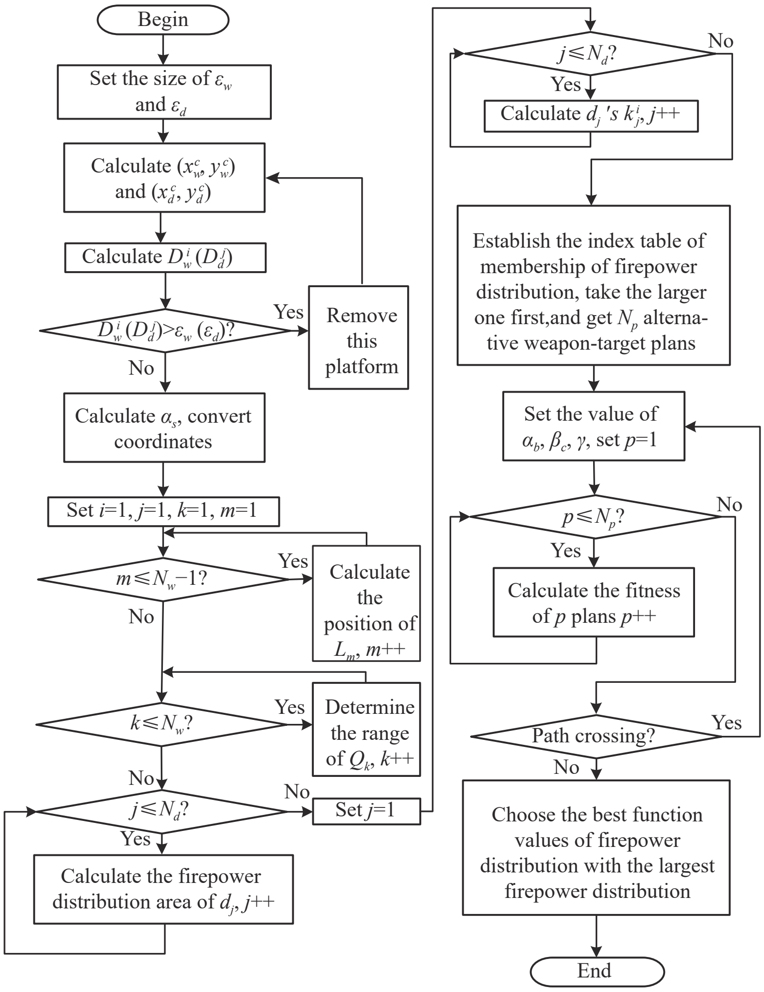


**Fig. 6 Conceptual evolution of firepower distribution**

## 4. Simulation experiment

**Simulation 1** Take the combat situation in Fig. 5 as an example, the red platforms $w_1$, $w_2$, and $w_3$ implement a multi-directional simultaneous attack on blue ships $d_1$ and $d_2$ anti-ship missiles in a certain sea area.

According to Subsection 3.1, the distance of baseline coordinate and distance of normal coordinate of the three platforms on the red side and the two ships on the blue side are shown in Table 2.

**Table 2 Coordinate data of the firepower reference system of the red and blue platforms (Simulation 1)**

| Firepower sub-base system | Red party | | | Blue party | |
|---|---|---|---|---|---|
| | $w_1$ | $w_2$ | $w_3$ | $d_1$ | $d_2$ |
| Baseline coordinates | 5.91 | −13.02 | 7.11 | 0.47 | −0.47 |
| Normal coordinate | 4.19 | −0.32 | −3.87 | 1.86 | −1.86 |

According to Subsection 3.2, the location of the area dividing line can be determined from Table 2.

$$L_1 = \frac{4.19 + (-0.32)}{2} = 1.935$$

$$L_2 = \frac{(-0.32) + (-3.87)}{2} = -2.095$$

It may be assumed $\varepsilon_1 = \varepsilon_{N_w} = 3$ that the specific scope of the area of firepower distribution $Q_1$, $Q_2$ and $Q_3$ can be determined:

$$Q_1 : (1.935, 4.935),$$
$$Q_2 : (-2.095, 1.935),$$
$$Q_3 : (-5.095, -2.095).$$

According to the distance of normal coordinate of each target and the interval of each area of firepower distribution, it can be determined that the target $d_1$ is located in $Q_2$ the middle and the target $d_2$ is located in $Q_2$ the middle. The table of area of firepower distribution shown in Table 3.

**Table 3 Table of area of firepower distribution (Simulation 1)**

| Blue party | $d_1$ | $d_2$ |
|---|---|---|
| Area of firepower distribution | $Q_2$ | $Q_2$ |

According to Subsection 3.3, calculate the membership of firepower distribution of each target to each platform respectively, and establish the index table of membership of firepower distribution, as shown in Table 4.

**Table 4 Index table of membership of firepower distribution (Simulation 1)**

| Red party | Blue party | |
|---|---|---|
| | $d_1$ | $d_2$ |
| $w_1$ | 0.4670 | 0.0367 |
| $w_2$ | 0.5186 | 0.5583 |
| $w_3$ | 0.0144 | 0.5197 |

A set of alternative firepower distribution schema can be obtained from Table 4, as shown in Table 5.

According to Subsection 3.4 and (8), for the above nine alternative firepower distribution plans, let $\alpha_b = \beta_c = 1$, $\gamma = 0.8$, calculate their fitness function values of firepower distribution fitness function values according to the fitness function of firepower distribution and summarize them in Table 6.

**Table 5 Collection of alternative firepower distribution plans (Simulation 1)**

| Serial number | Specific plan | $\sum_{j=1}^{N_d} n_j$ | $n_c$ |
|---|---|---|---|
| 1 | $w_1, w_2$ attack $d_1$<br>$w_2, w_3$ attack $d_2$ | 4 | 0 |
| 2 | $w_1, w_2$ attack $d_1$<br>$w_2$ attack $d_2$ | 3 | 0 |
| 3 | $w_1, w_2$ attack $d_1$<br>$w_3$ attack $d_2$ | 3 | 0 |
| 4 | $w_1$ attack $d_1$<br>$w_2, w_3$ attack $d_2$ | 3 | 0 |
| 5 | $w_1$ attack $d_1$<br>$w_2$ attack $d_2$ | 2 | 0 |
| 6 | $w_1$ attack $d_1$<br>$w_3$ attack $d_2$ | 2 | 0 |
| 7 | $w_2$ attack $d_1$<br>$w_2, w_3$ attack $d_2$ | 3 | 0 |
| 8 | $w_2$ attack $d_1$<br>$w_2$ attack $d_2$ | 2 | 0 |
| 9 | $w_2$ attack $d_1$<br>$w_3$ attack $d_2$ | 2 | 0 |

**Table 6 Fitness function value of firepower distribution (Simulation 1)**

| Program number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Adaptability | 10.7862 | 5.2147 | 5.2147 | 5.2147 | 2.8890 | 2.8890 | 5.2147 | 2.8890 | 2.8890 |

It can be concluded from Table 6 that Program 1 is the preferred firepower distribution plan. The result has the same fitness function value of firepower distribution beacuse that the fitness function of firepower distribution is only related to path crossing and the number of attacks on the target.

**Simulation 2** Take the combat situation in Fig. 4 as an example, the red platforms $w_1, w_2$, $w_3, w_4$, and $w_5$ implement a multi-directional simultaneous attack on blue ships $d_1$, $d_2$, and $d_3$ anti-ship missiles in a certain sea area.

According to Subsection 3.1, the distance of baseline coordinate and distance of normal coordinate of the five platforms on the red side and the three ships on the blue side are shown in Table 7.

**Table 7 Coordinate data of the firepower reference system of the red and blue platforms (Simulation 2)**

| Firepower sub-base system | Red party | | | | | Blue party | | |
|---|---|---|---|---|---|---|---|---|
| | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $d_1$ | $d_2$ | $d_3$ |
| Baseline coordinate | −12.18 | −12.29 | 7.01 | 5.29 | 27.23 | 291.03 | −390.34 | 99.31 |
| Normal coordinate | 499.87 | 109.65 | 40.25 | −149.91 | −299.26 | 276.43 | 3.78 | −280.21 |

According to Subsection 3.2, the location of the area dividing line can be determined from Table 7.

$$L_1 = \frac{499.87 + 109.65}{2} = 304.76$$

$$L_2 = \frac{109.65 + 40.25}{2} = 74.95$$

$$L_3 = \frac{40.25 + (-149.91)}{2} = -54.83$$

$$L_4 = \frac{(-149.91) + (-299.26))}{2} = -224.59$$

It may be assumed $\varepsilon_1 = \varepsilon_{N_w} = 100$ that the specific scope of the area of firepower distribution $Q_1$, $Q_2$, $Q_3$, $Q_4$, and $Q_5$ can be determined:

$$Q_1 : (304.76, 404.76),$$
$$Q_2 : (74.95, 304.76),$$
$$Q_3 : (-54.83, 74.95),$$
$$Q_4 : (-224.59, -54.83),$$
$$Q_5 : (-324.59, -224.59).$$

According to the distance of normal coordinate of each target and the interval of each area of firepower distribution, it can be determined that the target $d_1$ is located in $Q_2$ the middle, the target $d_2$ is located in $Q_3$ the middle and the target $d_3$ is located in $Q_5$ the middle. The table of area of firepower distribution shown in Table 8.

**Table 8 Table of area of firepower distribution (Simulation 2)**

| Blue party | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|
| Area of firepower distribution | $Q_2$ | $Q_3$ | $Q_5$ |

According to Subsection 3.3, calculate the membership of firepower distribution of each target to each platform respectively, and establish the index table of membership of firepower distribution, as shown in Table 9.

**Table 9 Index table of membership of firepower distribution (Simulation 2)**

| Red party | Blue party | | |
|---|---|---|---|
| | $d_1$ | $d_2$ | $d_3$ |
| $w_1$ | 0.4157 | 0 | 0 |
| $w_2$ | 0.6234 | 0.2997 | 0 |
| $w_3$ | 0.1016 | 0.9516 | 0 |
| $w_4$ | 0 | 0.2713 | 0.2244 |
| $w_5$ | 0 | 0 | 0.9438 |

A set of alternative firepower distribution schema can be obtained from Table 9, as shown in Table 10.

**Table 10 Collection of alternative firepower distribution plans (Simulation 2)**

| Serial number | Specific plan | $\sum_{j=1}^{N_d} n_j$ | $n_c$ |
|---|---|---|---|
| 1 | $w_1, w_2, w_3$ attack $d_1$<br>$w_2, w_3, w_4$ attack $d_2$<br>$w_5$ attacks $d_3$ | 7 | 1 |
| 2 | $w_1, w_2$ attack $d_1$<br>$w_2, w_3, w_4$ attack $d_2$<br>$w_5$ attacks $d_3$ | 6 | 0 |
| 3 | $w_2$ attack $d_1$<br>$w_2, w_3, w_4$ attack $d_2$<br>$w_5$ attack $d_3$ | 5 | 0 |
| 4 | $w_1, w_2$ attack $d_1$<br>$w_2, w_3$ attack $d_2$<br>$w_5$ attack $d_3$ | 5 | 0 |
| 5 | $w_2$ attack $d_1$<br>$w_2, w_3$ attack $d_2$<br>$w_5$ attack $d_3$ | 4 | 0 |
| 6 | $w_1, w_2$ attack $d_1$<br>$w_3, w_4$ attack $d_2$<br>$w_5$ attack $d_3$ | 5 | 0 |
| 7 | $w_2$ attack $d_1$<br>$w_3, w_4$ attack $d_2$<br>$w_5$ attack $d_3$ | 4 | 0 |
| 8 | $w_1, w_2$ attack $d_1$<br>$w_3$ attack $d_2$<br>$w_5$ attack $d_3$ | 4 | 0 |
| 9 | $w_2$ attack $d_1$<br>$w_3$ attack $d_2$<br>$w_5$ attack $d_3$ | 3 | 0 |

According to Subsection 3.4 and (8), for the above nine alternative firepower distribution plans, let $\alpha_b = \beta_c = 1$, $\gamma = 0.8$, calculate their fitness function values of firepower distribution fitness function values according to the fitness function of firepower distribution and summarize them in Table 11.

**Table 11 Fitness function value of firepower distribution (Simulation 2)**

| Program number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Adaptability | 16.8672 | 53.6272 | 23.5575 | 23.6518 | 10.8253 | 23.6518 | 10.8253 | 10.8253 | 5.3561 |

It can be concluded from Table 11 that Program 2 is the preferred firepower distribution plan. From Table 10, it can be seen that the fitness function of firepower distribution penalizing the path crossing situation is relatively strict. If there is path crossing, the fitness function value of firepower distribution fitness function value is very small, which also verifies the rationality of the allocation fitness function, and judging from the set of alternative firepower distribution schema, there are not many cases of actual path crossing.

Comparing the two simulations, it can be concluded that the present firepower distribution method can reduce the the number of selectable firepower distribution schemes and reduce the calculation time. From Table 6 and Table 11 the optimal firepower distribution scheme is not affected by the platform outlier points and the path crossing.

## 5. Conclusions

This paper establishes a firepower distribution model under the condition of path planning from the perspective of decoupling optimization and implements the algorithm.

(i) The concept of anti-ship missile firepower distribution under the condition of path planning is proposed, and its formal definition is given.

(ii) The coupling disorder of path planning and firepower distribution is avoided by establishing a reference coordinate system of firepower distribution and dividing the area of firepower distribution.

(iii) According to the performance of the anti-ship missile and the coordination constraints, the membership criterion of firepower distribution is constructed, and the index table of membership of fire distribution is established to obtain all possible alternative firepower distribution schema.

(iv) The fitness function of firepower distribution is established based on the damage income, missile loss, the ratio of efficiency and cost of firepower distribution, and the mean square deviation of the number of missiles is used, and the optimal global solution of the firepower distribution scheme is obtained.

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