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Two-level consensus modeling with utility and cost constraints

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Abstract: There exist many two-level group consensus problems with different psychological behaviors of decision makers. To deal with these group consensus problems and reach a stable consensus, based on the principles and methods of grey system, utility theory and group consensus, we use grey utility function to describe and reflect decision makers' opinion preferences in different subgroups and different levels, and then we construct a two-level group consensus method with a moderator, and exploit it to solve the negotiation problems of the natural gas subsidy.

Keywords: two-level group consensus, group negotiation, utility function, natural gas subsidy.

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1. Introduction

In some group decision making (GDM) problems, the level of DMs may be different, and DMs (DMs) may spontaneously form interest subgroups. The formation of consensus direction is controlled by DMs with high level. There exist many two-level group decision making problems such as trade dispute among states, pollutant emission reduction negotiation [1], bilateral auctions [2], and China urban demolition negotiations [3], which take on the characteristics of two level. The decision-making environment is complex and uncertain, so it is difficult for DMs to accurately express information, and they may give opinions in the forms of interval number. DMs in different levels often express different psychological behaviors. To reach a stable consensus, we must fully consider their psychological behavior. To describe and deal with the two-level group consensus problems with a moderator, based on the utility theory and the minimum cost consensus method, we construct a two-level maximum utility consensus approach. On the one hand, we construct an interactive consensus model between the moderator and DMs. On the other hand, we construct an interactive consensus model between DMs from different interest subgroups. We hope to show the influence of the moderator's compensation strategy on the modification of DM's opinions, and the influence of the DM's modification strategy on cost consensus. Finally, we exploit a group negotiation problems of natural gas subsidy to test and verify the proposed model.

Existing research on GDM rarely considers the interaction mechanism between the moderator and DMs. As the main participants in GDM, DMs in different interest subgroups may seeking different benefits. In view of this, this paper will propose a two-level consensus modeling method with utility and cost constraints. The proposed models in this paper have the following contributions:

(i) This paper defines the nonlinear grey utility function, which can reflect individual preferences effectively and obtain the optimal consensus opinion accurately.

(ii) Consider the different levels of DMs, and meet the utility preference of the moderator and DMs to construct the two-level optimization model.

This paper is organized as follows: Section 2 discusses the related researches. Section 3 introduces some concepts of grey utility function and constructs two two-level GDM models. In Section 4, we use the proposed model to solve the problem of natural gas subsidy. Section 5 presents sensitivity analysis, and we put forward some management suggestions to the government. Section 6 concludes this paper and points out the shortcomings and possible research directions in future.

2. Literature review

GDM is to find a collective solution based on the preferences expressed by a group of DMs [4]. The consensus process is the core of GDM. The formation of consensus

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requires the synthesis of the opinions of DMs with different knowledge backgrounds, different values, and different positions [5–9].

The complexity of the real environment makes the GDM process have two difficulties: (i) Due to the different cultural and educational background of DM, they usually express their views in various ways. (ii) DMs want their own opinion value to be fully reflected [10], so it is difficult for them to communicate with each other spontaneously and reach a compromise consensus. Even if such a consensus result is obtained, a great price (time, cost, and other resources) is required.

From the perspective of resource consumption, the moderator tries to keep the total cost (such as time and expenses) of the final consensus as low as possible. DMs are often hesitant and unable to give precise opinions. For this kind of decision-making environment, the minimum adjustment cost consensus model in hesitant language [11,12] is proposed. In fact, group consensus may be a two-stage process and different decision-making individuals may give a language term set with different number of terms or even a preference matrix with completely different structures [13,14]. This requires assembling different preference representation structures into a collective preference vector through standardization [15]. Different random distributions can increase the availability of uncertain values [16], but when the opinions of DMs are random, it is difficult to determine the scope of opinions by knowing their probability distribution, fuzzy language is introduced to consensus model [17–21]. Also, The GDM problem needs a feedback mechanism to help multiple inconsistent experts to reach consensus in GDM by allowing to select different feedback parameters according to individual consensus degree [22]. The cost chance constraint makes the model more applicable to real-world decision making [23]. Based on the above discussion, we know that cost and compensation are dual in cost consensus. This is an economic behavior, and there must be a game among participants. Therefore, there are some research on cost consensus models based on Stackelberg game [24] and non-cooperative behaviors [25].

Only considering low cost is not conducive to reaching a stable consensus. Utility preference [1,3,26,27], consensus fairness [28,29], and other factors are also crucial to the consensus process and the selection process in GDM problems.

Consider that the existing consensus model either only considers the cost of decision-making (difficult to reflect the value of the DM in GDM), or only discusses the utility of the DM's preference (difficult to clarify resource consumption in GDM). How to maximize the efficiency of consensus decision-making at a limited cost is widely researched [26]. A GDM model with non-linear utility preference constraints and limited consensus cost constraints can be used to deal with the problem of pollutant emission reductions among government and manufacturing companies [1]. Using concave, convex, S-shaped, inverted S-shaped, inverted U-shaped and other types of multi-stage volatility utility functions and their combinations to represent the dynamic preferences and consensus level of negotiators, researchers can solve the problem of urban house demolition [3]. Compared with previous studies, this model considers both the negotiation cost and the DM's preference structure. This paper constructs a stochastic optimization group consensus model with minimum cost and maximum utility, which can reach a stable consensus [27].

Although these studies have systematically discussed the issue of minimum-cost consensus in GDM, but there are still some issues that have not been considered: (i) The existing models cannot reflect the interaction mechanism between the moderator and interest subgroups, interactions within interest subgroups well. (ii) Some scholars proposed a minimum cost consensus model with linear cost function and quadratic cost function, but these models do not consider different interest groups and different levels. In order to reach a consensus more effectively and quickly, it is worth to study the two-level consensus model.

Sometimes DMs use grey language to express their preferences, which requires a utility function applicable to grey language to reflect their opinion preferences. Consensus decision-making needs to fully consider the DMs' attitudes, and the nonlinear utility function can simulate the DMs' psychological preferences. Therefore, this study defines the grey nonlinear utility function to solve the consensus problem with grey information.

Two-level GDM, which refers to the decision-making process that DMs are in a different level, maybe some are in an upper level and others are in a lower level. The upper-level DMs consider many complex factors, and the lower-level has multiple DMs, each factor considered by the lower-level DMs are relatively single and specific, and there is often only one goal that needs to be optimized. The GDM model that aims to reach a consensus in this decision environment is called two-level consensus model.

The opinions of DMs in different interest groups may differ greatly, and it is difficult to quickly reach a consensus. Therefore, this study constructs a two-level group consensus model. The lower-level consensus is the consensus within each interest group, and each interest group will give a consensus opinion, and the coordinator will promote the upper-level consensus based on this consensus opinion.

This study builds a model based on the negotiation between the government and the villagers on natural gas subsidies to further explain the rationality and economic significance of the proposed model. The main contributions are: (i) Construct a two-level group consensus model, which can more appropriately describe the level differences of DMs in a complex decision-making environment. (ii) Introduce the grey utility function into the consensus model and select the non-linear grey utility function to effectively reflect personal preferences. (iii) The maximum utility model with limited compensation costs can not only measure the utility level of the entire group, but also measure the consensus degree.

3. Two-level group consensus modeling with utility and cost constraints

There are many complex consensus problems in real life. DMs are at different levels, and it is impossible for them to negotiate. This kind of complex consensus problems need to be negotiated level by level. For example, in the Sino-US trade war, the global consensus needs to consider the consensus among members of various interest groups, the consensus within various countries, and the consensus among countries. For the stability of consensus results, the negotiation process must consider the utility preferences of DMs at all levels. At the same time, we should control the negotiation cost.

The negotiation of the natural gas subsidies is a typical two-level group consensus issue. In order to reach a global consensus on subsidies, a consensus must be reached among villagers in each village, that is, reaching a consensus at the lower-level. Furthermore, consider the consensus opinion within each village as the original opinion of the upper-level group consensus to reach the upper-level consensus. In order to solve the negotiation problem of natural gas subsidy, we define the grey utility function. We construct a two-level group consensus model with grey utility, and compares the difference between the two-level minimum cost group consensus model (T-MCCM) and the two-level maximum utility consensus model (T-MUCM). We use the above two models to calculate the optimal consensus opinions and achievable utility level of DMs at all levels, and the cost of the whole consensus process.

3.1 Definition of grey utility function

Utility refers to the specific value assigned by the DM to a specific result according to his own preferences [30]. In different problems and backgrounds, DMs often show different preference attitudes and satisfaction levels when giving opinions. Utility functions play a fundamental role in the research reflecting the preference structure of DMs because of its monotonicity and unevenness [31]. In this paper, this kind of utility preference is called the grey utility preference, and it is described quantitatively by the grey possibility function. The grey probability function describes the possibility of a grey number taking a certain value, or the possibility that a specific value of a grey number is the true value [32]. Generally speaking, the distribution of the grey probability function is greatly affected by the profit of the DM, because under the premise of the rational person's assumption, the DM always maximizes his own interests as the purpose of decision-making activities.

We can see that grey probability function is similar to utility function. Based on the definition of the grey probability function, combined with the utility theory, we can define the grey utility function. Then, the monotonicity, convexity-concavity, and the gradient of grey utility function for DM's grey opinion can reflect DM's psychological changes, which can describe the utility preference of the DM. Therefore, introducing grey utility function into the group consensus model can satisfy the utility preference of DMs and ensure the stability of the consensus to a certain extent.

Suppose that the judgment interval of DM is o_i. Any value belonging to the grey interval can reflect one utility value $U(x)$ of DM . Without loss of generality, we suppose that $U(x)$ is a single-variable continuous function that satisfies $0 \leqslant U(x) \leqslant 1$. $U'(x)$ denotes the unit change of the DM's utility level that will be defined as the marginal utility of the DM's opinion. It means DM's utility changes as a result of their unit change of opinions.

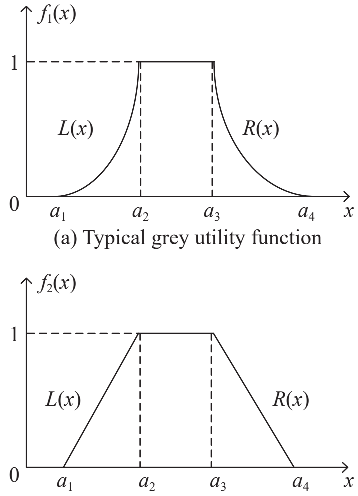
Definition 1 The typical grey utility function is determined by the starting point and the end point, including left-up and right-down continuous functions. Fig. 1(a) shows the shape of the typical grey utility function.

The function expression of Fig. 1(a) can be expressed as

$$f_1(x) = \begin{cases} L(x), & x \in [a_1, a_2) \\ 1, & x \in [a_2, a_3] \\ R(x), & x \in (a_3, a_4] \end{cases} \tag{1}$$

where $L(x)$, $R(x)$ stand for the left increasing function and the right decreasing function, respectively. $[a_2, a_3)$ is a peak area. a_1 and a_4 are the starting point and the ending point respectively. a_2 and a_3 are the turning points. In practical application, due to the limitation of programming and calculation, the $L(x)$ and $R(x)$ are often described as a straight line, as shown in Fig. 1(b). The function expression of Fig. 1(b) can be expressed as

$$f_2(x)=\begin{cases} L(x)=\dfrac{x-a_1}{a_2-a_1}, & x\in[a_1,a_2) \\ 1, & x\in[a_2,a_3] \\ R(x)=\dfrac{a_4-x}{a_4-a_3}, & x\in(a_3,a_4] \end{cases}. \tag{2}$$



(a) Typical grey utility function

(b) Commonly used grey utility function

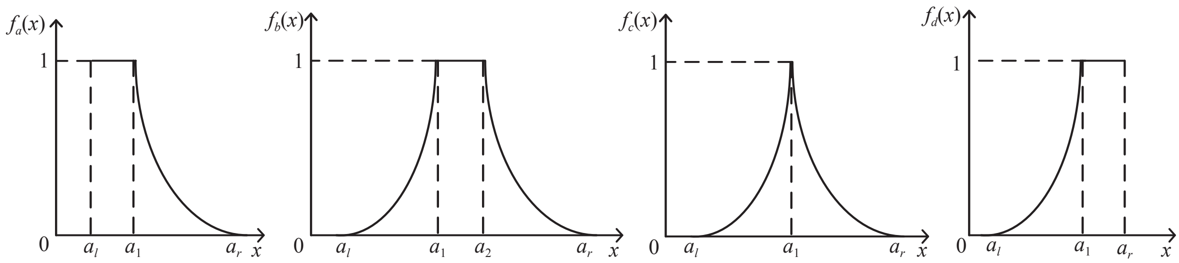
Fig. 1 Grey utility function

3.2 Other forms of grey utility function

People often can not give accurate opinions, and often contain some vague and grey information. Therefore, it is necessary to establish a utility function that can reflect grey information. Consider people's complex psychological behaviors, researchers divide the people's preference attitudes into three types: risk appetite, risk averse and risk neutral. Non-linear utility function can express the change of the DM's utility preference more flexibly and simulate the change process of decision psychology. And [1,33] pointed out that the parabolic and the S-shape are relatively simple and practical. Therefore, based on the existing utility function, combined with grey theory, we define the parabolic grey utility function and the S-shape grey utility function based on the sine function.

(i) Parabolic grey utility function

We can divide parabolic grey utility functions into left-biased parabolic grey utility functions, interval parabolic grey utility probability functions, triangular parabolic grey utility functions, and right-biased parabolic functions. Fig. 2 shows the shape of above grey utility functions.



(a) Left-biased parabolic grey utility function (b) Interval parabolic grey utility function (c) Triangular parabolic grey utility function (d) Right-biased parabolic grey utility function

Fig. 2 Parabolic grey utility function

The function expression of Fig. 2 can be given as follows:

$$f_a(x)=\begin{cases} 1, & x\in[a_l,a_1] \\ \left(\dfrac{a_r-x}{a_r-a_1}\right)^2, & x\in(a_1,a_r] \end{cases} \tag{3}$$

$$f_b(x)=\begin{cases} \left(\dfrac{x-a_l}{a_1-a_l}\right)^2, & x\in[a_l,a_1) \\ 1, & x\in[a_1,a_2] \\ \left(\dfrac{a_r-x}{a_r-a_2}\right)^2, & x\in(a_2,a_r] \end{cases} \tag{4}$$

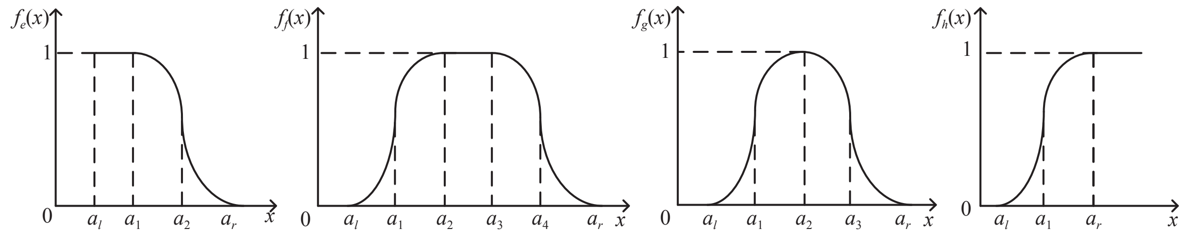
$$f_c(x)=\begin{cases} \left(\dfrac{x-a_l}{a_1-a_l}\right)^2, & x\in[a_l,a_1) \\ 1, & x=a_1 \\ \left(\dfrac{a_r-x}{a_r-a_1}\right)^2, & x\in(a_1,a_r] \end{cases} \tag{5}$$

$$f_d(x)=\begin{cases} \left(\dfrac{x-a_l}{a_1-a_l}\right)^2, & x\in[a_l,a_1] \\ 1, & x\in(a_1,a_r] \end{cases} \tag{6}$$

(ii) S-shape grey utility function based on sine function

We can divide the S-shape grey utility functions into the left-biased S-shape grey utility functions, the interval-type S-shape grey utility functions, and the right-biased S-shape grey utility functions. Fig. 3 shows the shape of the above grey utility functions. The function expression of Fig. 3 can be expressed as

$$f_e(x)=\begin{cases} 1, & x\in[a_l,a_1] \\ \dfrac{1}{2}-\dfrac{1}{2}\sin\dfrac{\pi}{a_r-a_1}(x-a_2), & x\in(a_1,a_r] \end{cases}, \tag{7}$$



(a) Left-biased S-shape grey utility function (b) Interval-type S-shape grey utility function (c) Triangular-type S-shape grey utility function (d) Right-biased S-shape grey utility function

Fig. 3 S-shape grey utility function based on sinusoidal function

$$f_f(x)=\begin{cases}\dfrac{1}{2}+\dfrac{1}{2}\sin\dfrac{\pi}{a_2-a_l}(x-a_1), & x\in[a_l,a_2)\\ 1, \quad x\in[a_2,a_3] & \\ \dfrac{1}{2}-\dfrac{1}{2}\sin\dfrac{\pi}{a_r-a_3}(x-a_4), & x\in(a_3,a_r]\end{cases}, \tag{8}$$

$$f_g(x)=\begin{cases}\dfrac{1}{2}+\dfrac{1}{2}\sin\dfrac{\pi}{a_2-a_l}(x-a_1), & x\in[a_l,a_2)\\ 1, \quad x=a_2 & \\ \dfrac{1}{2}-\dfrac{1}{2}\sin\dfrac{\pi}{a_r-a_2}(x-a_3), & x\in(a_2,a_r]\end{cases}, \tag{9}$$

$$f_h(x)=\begin{cases}\dfrac{1}{2}+\dfrac{1}{2}\sin\dfrac{\pi}{a_2-a_l}(x-a_1), & x\in[a_l,a_2]\\ 1, \quad x\in(a_2,a_r] & \end{cases}. \tag{10}$$

3.3 T-MUCM

Ben-Arie et al. [34] first introduced the concepts consensus level and consensus cost, and used it to build a minimum cost consensus model. The practical significance is that there is a moderator, and the moderator's opinion can be regarded as the group consensus opinion. In order to make the DMs change their original opinions, the moderator pays a certain fee to each DM. Of course, he hopes to reach a consensus with the smallest total cost.

Suppose there are m DMs, denoted as $E=\{e_1,e_2,\cdots,e_m\}$, where the DM i gives an original opinion $o_i\in\mathbf{R}$. o'_i represents the adjusted opinion of the DM, $\|o'_i-o_i\|$ expresses the deviation of the original opinion and the adjusted opinion. $\|\cdot\|$ is a certain distance measurement method. c_i represents the moderator's compensation for changes in the opinions of DMs. Ben-Arieh et al. [34] defined the linear consensus cost of opinion adjustment. Let o' represent the group consensus opinion, then the minimum cost consensus model is expressed as

$$\begin{aligned}&\min\sum_{i=1}^{m}c_i\|o'_i-o_i\|\\&\text{s.t.}\ \ \|o'_i-o'\|\leqslant\varepsilon,\quad i=1,2,\cdots,m\end{aligned} \tag{11}$$

where ε is the critical value, and the deviation between the adjusted opinion of the DM and the consensus opinion of the group does not exceed the critical value, which means that the DM can accept the consensus opinion. Subsequently, Ben-Arieh et al. [35] defined the quadratic cost function $f_i(o'_i)=c_i\|o'_i-o_i\|^2$. With the deepening of research, Gong proposed a minimum cost consensus model in which the decision opinion is an interval number:

$$\begin{aligned}&\min\sum_{i=1}^{m}c_i|o_i-o'|\\&\text{s.t.}\begin{cases}o_i\in[a_i,b_i], & i\in E\\ o'\in O\end{cases}\end{aligned} \tag{12}$$

where a_i,b_i are the upper and lower limits of the DM's interval opinion respectively.

Definition 2 Assume there are q equal-status sub-groups in a social network, and there are m DMs in each sub-group. D represents the set of all DMs, which can be expressed as

$$D=\left\{\begin{matrix}d_{11}\cdots d_{1j}\cdots d_{1m}\\ \vdots\quad\ \vdots\quad\ \vdots\\ d_{i1}\cdots d_{ij}\cdots d_{im}\\ \vdots\quad\ \vdots\quad\ \vdots\\ d_{q1}\cdots d_{qj}\cdots d_{qm}\end{matrix}\right\},$$
$$i=1,2,\cdots,q;j=1,2,\cdots,m.$$

If $\{o'*,o_1'*,o_2'*,\cdots,o_q'*\}$ can make the social network reach a global consensus, then both internal of all equal-status sub-groups and all sub-groups among q will reach consensus.

Definition 3 Every DM has a tolerable threshold for opinion adjustment. If the distance of opinion adjustment lower than this threshold, the DM will change his opinion. Otherwise, he will keep his own opinion at previous. The thresholds within and between sub-groups may be different, it can be expressed as ε_i and η_i respectively, where $\varepsilon,\eta\in[0,1]$.

The original opinions of the DMs i in each sub-group are $o_{ij}(i=1,2,\cdots,q;j=1,2,\cdots,m_q)$. The internal consensus opinion of the i interest subgroup are $o_i'(i=1,2,\cdots,q)$. $o'(i=1,2,\cdots,q)$ are consensus opinions of all sub-groups among q. $c_i(i=1,2,\cdots,q)$ rep resents the compensation provided by the moderator for opinions changes of sub-groups q. $c_i^j(i=1,2,\cdots,q;j=1,2,\cdots,m_q)$epresents the compensation provided by the moderator for the opinion changes of the DM d_{ij}.

In order to reach a global consensus on something, DMs must reach a consensus at the lower-level. Furthermore, view the consensus opinion within the lower-level as the original opinion of the upper-level group consensus to reach the upper-level consensus.

Based on this, the following two-level minimum cost consensus model can be constructed:

$$\begin{gathered}
\min\left\{c_1|o_1'-o'|+\cdots+c_j\left|o_j'-o'\right|+\cdots+c_q\left|o_q'-o'\right|\right\}\\
\min\left\{c_1^{\,1}|o_{11}-o_1'|+\cdots+c_j^{\,1}\left|o_{1j}-o_1'\right|+\cdots+c_{m_1}^{\,1}\left|o_{1m_1}-o_1'\right|\right\}\\
\vdots\\
\min\left\{c_1^{\,i}|o_{i1}-o_i'|+\cdots+c_j^{\,i}\left|o_{ij}-o_i'\right|+\cdots+c_{m_i}^{\,i}\left|o_{im_i}-o_i'\right|\right\}\\
\vdots\\
\min\left\{c_1^{\,q}\left|o_{q1}-o_q'\right|+\cdots+c_j^{\,q}\left|o_{qj}-o_q'\right|+\cdots+c_{m_q}^{\,q}\left|o_{qm_q}-o_q'\right|\right\}\\
\text{s.t.}\begin{cases}
|o_i'-o'|\leqslant\eta_i, & i=1,2,\cdots,q\\
\left|o_{ij}-o_i'\right|\leqslant\varepsilon_i, & i=1,2,\cdots,q;j=1,2,\cdots,m_i\\
o_{ij}\in[a_{ij},b_{ij}], & i=1,2,\cdots,q;j=1,2,\cdots,m_i\\
o_i',\quad o'\in O
\end{cases}
\end{gathered}\tag{13}$$

Among them, $\left|o_{ij}-o_i'\right|$ represents the deviation between original opinion o_{ij} of DM d_{ij} and the internal consensus opinion o_i' of the sub-interest group, ε_i is a critical value. The value of $\left|o_{ij}-o_i'\right|$ does not exceed the critical value ε_i, which means that the DM can accept the consensus opinion. $|o_i'-o'|$ represents the deviation between the consensus opinions o_i' of interest sub-groups and the global consensus opinion o', η_i is the critical value. The value of $|o_i'-o'|$ does not exceed the critical value η_i, indicating that the moderator can accept the consensus opinion.

T-MCCM aims to obtain the best consensus opinion with the lowest cost from the perspective of the moderator. However, this impairs personal interests and perceived utility of DMs. Therefore, it is necessary to introduce utility functions to reflect each DM's perceived utility of the consensus result. That is to say, the goal of T-MUCM not only considers cost constraint, but also achieving the maximum utility of each DM and moderator. Then, we can establish the following model:

$$\begin{gathered}
\max\lambda\\
\max\lambda_1\\
\vdots\\
\max\lambda_i\\
\vdots\\
\max\lambda_q\\
\text{s.t.}\begin{cases}
c_1|o_1'-o'|+\cdots+c_j\left|o_j'-o'\right|+\cdots+c_q\left|o_q'-o'\right|+\\
\quad c_1^{\,1}|o_{11}-o_1'|+\cdots+c_j^{\,1}\left|o_{1j}-o_1'\right|+\cdots+\\
\quad c_{m_1}^{\,1}\left|o_{1m_1}-o_1'\right|+c_1^{\,i}\left|o_{i1}-o_j'\right|+\cdots+c_j^{\,i}\left|o_{ij}-o_i'\right|+\cdots+\\
\quad c_{m_i}^{\,i}\left|o_{im_i}-o_i'\right|+\cdots c_1^{\,q}\left|o_{q1}-o_q'\right|+\cdots+\\
\quad c_j^{\,q}\left|o_{qj}-o_q'\right|+\cdots+c_{m_q}^{\,q}\left|o_{qm_q}-o_q'\right|\leqslant B\\
|o_i'-o'|\leqslant\eta_i,\quad i=1,2,\cdots,q\\
\left|o_{ij}-o_i'\right|\leqslant\varepsilon_i,\quad i=1,2,\cdots,q;j=1,2,\cdots,m_i\\
\lambda_i\leqslant f(o_{ij}),\quad i=1,2,\cdots,q;j=1,2,\cdots,m_i\\
\lambda\leqslant f(o_i'),\quad i=1,2,\cdots,q\\
o_{ij}\in[a_{ij},b_{ij}],\quad i=1,2;j=1,2,3,4\\
o_i',\quad o'\in O
\end{cases}.
\end{gathered}\tag{14}$$

Among them, the compensation that the moderator willing to provide is B, so the cost of the entire consensus process should be less than or equal to B. $\lambda_i\leqslant f(o_{ij})$ indicates the utility constraint of the DM d_{ij}, $\left|o_{ij}-o_i'\right|\leqslant\varepsilon_i$ indicates the utility constraint of the moderator. The utility level λ can be used to measure the global consensus level, and the utility level $\lambda_i(i=1,2,\cdots,q)$ can be used to measure the consensus level of the interest sub-group. The closer to 1, the higher the utility level λ and λ_i.

4. Case study

With the rapid development of the world economy, energy consumption continues to grow. In this situation, the demand for clean, high-calorific natural gas energy has increased substantially. The global demand for natural gas is growing at an annual rate of 2.4%, and this growth rate is expected to remain until 2030. China has always used coal and gasoline as main energy sources for production and life. The massive use of coal causes large-scale environmental pollution, and affects the sustainable development of environment, economy, and energy. Natural gas is the cleanest energy source. Substituting natural gas for coal and oil can effectively solve many problems. In 2016, the Ministry of Industry and Information Technology issued "Industrial Green Development Plan (2016−2020)", pointing out that resource and environmental issues are common challenges facing mankind. And promoting green growth and implementing Green New Deal are the common choices of major economies in the world. In the same period, the State Council of China issued relevant documents such as "Key Projects in the

12th Five-Year Plan for Air Pollution Prevention and Control in Key Areas", "Air Pollution Prevention Action Plan", and "Implementation of 'Coal to Gas' and 'Coal to Electricity' Centralized Use of Clean Energy". Then most cities successively implement the "Coal to Gas" and "Coal to Electricity" plans. The main measure is to provide cash subsidies to users.

Jiangsu Province issued the "Implementation Opinions on Promoting the Coordinated and Stable Development of Natural Gas", which provides appropriate subsidies to low-income villagers in rural areas. The subsidy amount can be determined through negotiation between the local government and the users, and the maximum subsidy of per household shall not exceed a certain amount.

The negotiation of the natural gas subsidies is a typical two-level group consensus issue. In order to reach a global consensus on subsidies, a consensus must be reached among villagers in each village, that is, reach a consensus at the lower-level. Furthermore, view the consensus opinion within each village as the original opinion of the upper-level group consensus to reach the upper-level consensus.

Based on this background, to study the consensus on natural gas subsidies in two villages in Jiangsu. To encourage villages to reach a consensus subsidies opinion, the government acts as a moderator to give one-time subsidies to villagers. The whole village is the upper-level DM and the villagers are the lower-level DMs. Assuming the two villages have equal status, there is no priority subsidy. And because of their own cognition, different villagers have different requirements for subsidies, which are given in the form of grey interval numbers.

Based on the above analysis, without loss of generality, it is assumed that there are two villages, each with four villagers. The symbols of the villagers are as follows:

$$D = \left\{d_{11}, d_{12}, d_{13}, d_{14}, d_{21}, d_{22}, d_{23}, d_{24}\right\}.$$

The original opinion of villager i in the first village j is $o_{ij}(i = 1,2; j = 1,2,3,4)$, o_{ij} are given in the form of a grey interval (all the number are in thousands of yuan).

$$\boldsymbol{O} = [\boldsymbol{o}_{11}\ \ \boldsymbol{o}_{12}\ \ \boldsymbol{o}_{13}\ \ \boldsymbol{o}_{14}\ \ \boldsymbol{o}_{21}\ \ \boldsymbol{o}_{22}\ \ \boldsymbol{o}_{23}\ \ \boldsymbol{o}_{24}] = \\ [\,[6,23]\ \ [3,16]\ \ [5,18]\ \ [4,19] \\ [5,25]\ \ [3,22]\ \ [7,24]\ \ [2,25]\,]$$

The internal consensus opinion of village i is $o_i'(i = 1,2)$, that is, the original opinion of the consensus process between two villages. The consensus opinion which government hopes to reach is overall consensus opinion o'. $c_i(i = 1,2)$ represents the government's compensation for changes in the opinions of villages $i(i = 1,2)$.

According to the actual subsidy situation in Jiangsu Province that can be searched, we have appropriately simplified the subsidy value $c_1 = 2, c_2 = 3$.

$c_i^j\ (i = 1,2; j = 1,2,3,4)$ represents the government's compensation for changes in the opinions of villagers d_{ij} in the village $i(i = 1,2)$,

$$\boldsymbol{C} = \left[c_1^{\ 1}\ \ c_1^{\ 2}\ \ c_1^{\ 3}\ \ c_1^{\ 4}\ \ c_2^{\ 1}\ \ c_2^{\ 2}\ \ c_2^{\ 3}\ \ c_2^{\ 4}\right] = \\ [1\ \ 2\ \ 3\ \ 1\ \ 1\ \ 2\ \ 1\ \ 3].$$

$$\eta = 5, \varepsilon_1 = 10, \varepsilon_2 = 12.$$

Based on the idea of Ben-Arieh et al. [34], minimum cost consensus, a two-level minimum cost consensus model (T-MCCM) is constructed considering the hierarchical relationship between the government and villagers in real life.

$$\min\ 2|\boldsymbol{o}_1' - \boldsymbol{o}'| + 3|\boldsymbol{o}_2' - \boldsymbol{o}'|$$

$$\min\ 1|\boldsymbol{o}_{11} - \boldsymbol{o}_1'| + 2|\boldsymbol{o}_{12} - \boldsymbol{o}_1'| + 3|\boldsymbol{o}_{13} - \boldsymbol{o}_1'| + 1|\boldsymbol{o}_{14} - \boldsymbol{o}_1'|$$

$$\min\ 1|\boldsymbol{o}_{21} - \boldsymbol{o}_2'| + 2|\boldsymbol{o}_{22} - \boldsymbol{o}_2'| + 1|\boldsymbol{o}_{23} - \boldsymbol{o}_2'| + 3|\boldsymbol{o}_{24} - \boldsymbol{o}_2'|$$

$$\text{s.t.}\begin{cases} |\boldsymbol{o}_i' - \boldsymbol{o}'| \leqslant 15, \quad i = 1,2 \\ \left|\boldsymbol{o}_{1j} - \boldsymbol{o}_1'\right| \leqslant 10, \quad j = 1,2,3,4 \\ \left|\boldsymbol{o}_{2j} - \boldsymbol{o}_2'\right| \leqslant 12, \quad j = 1,2,3,4 \\ \boldsymbol{o}_{11} = [6,23], \boldsymbol{o}_{12} = [3,16], \boldsymbol{o}_{13} = [5,18], \boldsymbol{o}_{14} = [4,19] \\ \boldsymbol{o}_{21} = [5,25], \boldsymbol{o}_{22} = [3,22], \boldsymbol{o}_{23} = [7,24], \boldsymbol{o}_{24} = [2,25] \\ \boldsymbol{o}_i', \quad \boldsymbol{o}' \in \boldsymbol{O} \end{cases}$$

Solve the above multi-objective programming problem with a linear weighting method, and set priority weights for the three goals according to the consensus priority.

In the above two-level minimum-cost consensus model, the consensus of the two villages must be achieved firstly. Therefore, the priority weights given to the two villages are both 0.4. Then, the weight to achieve the goal of global consensus is 0.2. Running Matlab 2016b can get the optimal solution of the above model.

$$[\boldsymbol{o}_{11}\ \ \boldsymbol{o}_{12}\ \ \boldsymbol{o}_{13}\ \ \boldsymbol{o}_{14}\ \ \boldsymbol{o}_{21}\ \ \boldsymbol{o}_{22}\ \ \boldsymbol{o}_{23}\ \ \boldsymbol{o}_{24}] = \\ [6.24\ \ 5.58\ \ 5.43\ \ 5.39\ \ 5.29\ \ 5.41\ \ 7.24\ \ 5.41].$$

It can be further calculated that the government subsidy amount for village 1 to reach consensus is 1 150 yuan, the government subsidy amount for village 2 to reach consensus is 2 080 yuan, and the government's consensus cost to reach global consensus is 3 290 yuan.

Only controlling the cost of subsidies is difficult to reflect the value of villagers in group consensus process. Therefore, the utility preferences of villagers are considered below, and control the total cost of subsidies within the acceptable range of the government.

From the government's perspective, to achieve green development, it is necessary to reach a consensus with the villagers and realize the "coal to gas" plan. Therefore, the utility of the government can be described as the right-biased S-shaped utility function shown in Fig. 4. The government's grey utility function is a right-biased S-shape based on complex risk attitudes, as shown in Fig. 4.

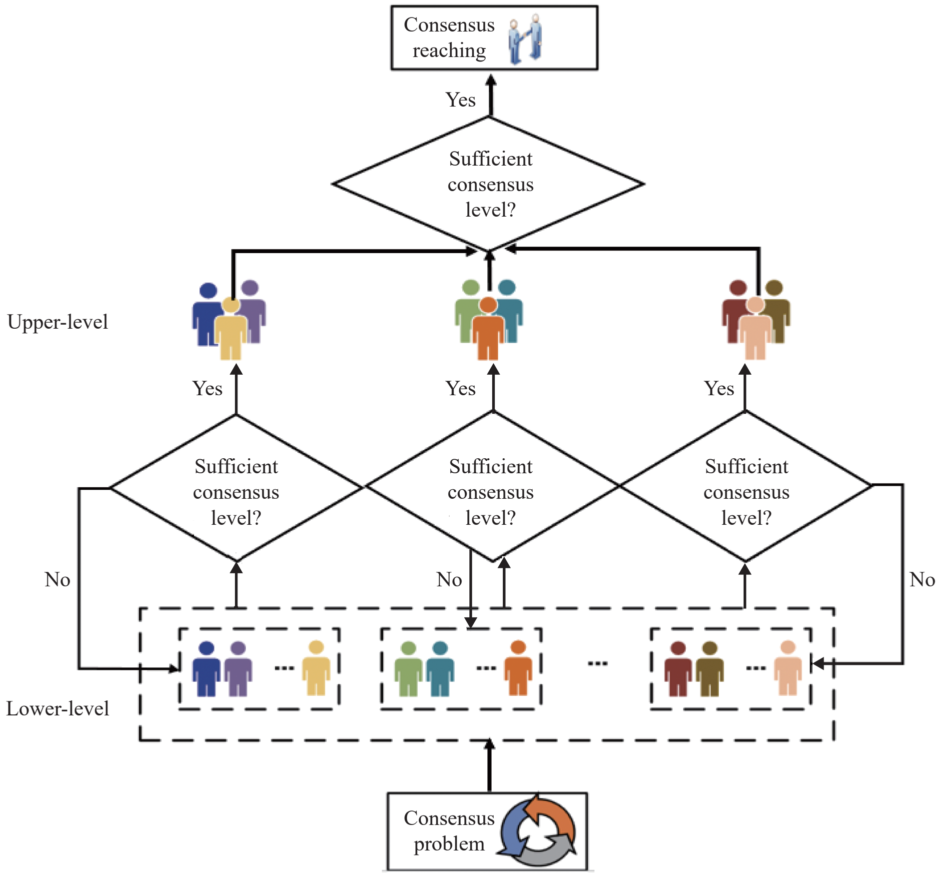


Fig. 4 Framework of two-level group consensus

In the lower subsidy interval $[a_l, a_1]$, the government coordinates the villagers' opinions through subsidies to the direction of global consensus. Government's performance will increase and the marginal utility will increase. The government is a risk-preference decision. To bear higher subsidy costs and diminish marginal utility, the government makes risk-averse decisions. In the interval $[a_1, a_2]$, in order to reach a global consensus, the government needs to bear higher subsidy costs, so the marginal utility decreases. Government makes risk-averse decisions.

The grey utility function of villagers is right-partial parabolic, as shown in Fig. 5. In the interval $[a_l, a_1]$, the utility of villagers increases with the increase of the amount of subsidies, and the marginal utility of the utility function increases. Then it reaches the highest point at a_1, and maintains the highest level of utility.

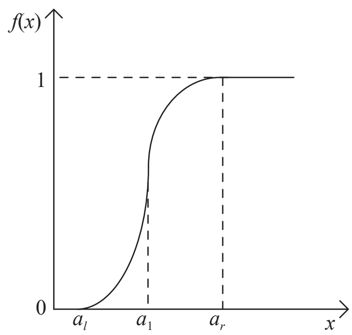


Fig. 5 Grey utility function of the government

Based on the above analysis, assuming that the government expects to pay 8 000 yuan to reach a consensus, and the inflection point of the government's grey utility function is $a_l = 2, a_1 = 16, a_r = 30$. The grey utility function of villagers are shown in Table1. Thus, this paper constructs T-MUCM as follows.

Table 1 Grey utility function reference point of villager

o_{ij}	$c_1{}^j$	a_l	a_1	o_{ij}	$c_2{}^j$	a_l	a_1
o_{11}	111	6	23	o_{11}	1	5	25
o_{12}	2	3	16	o_{12}	2	3	22
o_{13}	3	5	18	o_{13}	1	7	24
o_{14}	1	4	19	o_{14}	3	2	25

$$\begin{cases} \max\lambda \\ \max\lambda_1 \\ \max\lambda_2 \end{cases}$$

$$\text{s.t.}\begin{cases} 2\,|\boldsymbol{o}_1'-\boldsymbol{o}'|+3\,|\boldsymbol{o}_2'-\boldsymbol{o}'|+1\,|\boldsymbol{o}_{11}-\boldsymbol{o}_1'|+2\,|\boldsymbol{o}_{12}-\boldsymbol{o}_1'|+ \\ 3\,|\boldsymbol{o}_{13}-\boldsymbol{o}_1'|+1\,|\boldsymbol{o}_{14}-\boldsymbol{o}_1'|+|\boldsymbol{o}_{21}-\boldsymbol{o}_2'|+2\,|\boldsymbol{o}_{22}-\boldsymbol{o}_2'|+ \\ 1\,|\boldsymbol{o}_{23}-\boldsymbol{o}_2'|+3\,|\boldsymbol{o}_{24}-\boldsymbol{o}_2'|\leqslant 8 \\ |\boldsymbol{o}_i'-\boldsymbol{o}'|\leqslant 15,\quad i=1,2 \\ \left|\boldsymbol{o}_{1j}-\boldsymbol{o}_i'\right|\leqslant 10,\quad j=1,2,3,4 \\ \left|\boldsymbol{o}_{2j}-\boldsymbol{o}_i'\right|\leqslant 12,\quad j=1,2,3,4 \\ \lambda_1\leqslant(\dfrac{b_{1j}-\boldsymbol{o}_{1j}}{b_{1j}-a_{1j}}),\quad j=1,2,3,4 \\ \lambda_2\leqslant(\dfrac{b_{2j}-\boldsymbol{o}_{1j}}{b_{2j}-a_{1j}}),\quad j=1,2,3,4 \\ \lambda\leqslant\dfrac{1}{2}+\dfrac{1}{2}\sin\dfrac{\pi}{a_r-a_l}(\boldsymbol{o}'-a_1),\quad i=1,2 \\ \boldsymbol{o}_{11}=[6,23],\boldsymbol{o}_{12}=[3,16],\boldsymbol{o}_{13}=[5,18],\boldsymbol{o}_{14}=[4,19] \\ \boldsymbol{o}_{21}=[5,25],\boldsymbol{o}_{22}=[3,22],\boldsymbol{o}_{23}=[7,24],\boldsymbol{o}_{24}=[2,25] \\ \boldsymbol{o}_i',\quad \boldsymbol{o}'\in O \end{cases}$$

The government expects that subsidies consumed in the entire consensus process will not exceed 8 000 yuan. $|\boldsymbol{o}_i'-\boldsymbol{o}'|\leqslant 15$ represents that the opinion deviation which village 1 and village 2 can tolerate is 15, when they negotiate with each other. $\left|\boldsymbol{o}_{1j}-\boldsymbol{o}_i'\right|\leqslant 10$ means the opinions deviation which villagers in village 1 can tolerate is 10. $\left|\boldsymbol{o}_{2j}-\boldsymbol{o}_i'\right|\leqslant 12$ means the opinions deviation which villagers in village 2 can tolerate is 12. $\lambda_1\leqslant(b_{1j}-o_{1j})/(b_{1j}-a_{1j})$ and $\lambda_2\leqslant(b_{2j}-o_{1j})/(b_{2j}-a_{1j})$ represents that the utility constraint function of villagers is right-biased parabolic. $\lambda\leqslant 1/2+1/2\sin\pi/(a_r-a_l)(o'-a_1)$ means that utility constraint function of the government is the right-biased S-shaped utility function.

Solve the above multi-objective programming problem with a linear weighting method. Set priority weights for the three goals according to the consensus priority. In the above two-level minimum cost consensus model, it is assumed that the weight to maximize the internal utility of village 1 is $1/3$, the weight to maximize the internal utility of village 2 is $1/3$, and the weight to maximize the global utility is $1/3$. Running Matlab 2016b can get the optimal solution of the above model.

$$(o',o_1',o_2')=(6.15,6.14,5.99),$$

$$(\lambda,\lambda_1,\lambda_1)=(0.01,0.85,0.84),$$

$$[o_{11}\ o_{12}\ o_{13}\ o_{14}\ o_{21}\ o_{22}\ o_{23}\ o_{24}]=$$
$$[6.92\ \ 6.60\ \ 6.21\ \ 6.14\ \ 6.00\ \ 5.45\ \ 7.86\ \ 5.34].$$

In this scenario, the subsidy required by the government to reach global consensus is 7 320 yuan. At this subsidy cost, village 1 can reach its highest utility level of 0.85 and village 2 can reach its highest utility level of 0.84. However, the higher subsidy decreases the government's utility level, and it only reaches 0.01. Constrained by the total subsidy cost of 8 000, all villagers' willingness to subsidize are close to the lower limit of their subsidy interval. From the perspective of economic benefits, except for the villagers o_{24}, the other seven villagers all obtain a larger subsidy at a higher utility level. However, the government needs to pay a greater subsidy cost, and can only obtain a smaller utility level at 0.01.

5. Sensitivity analysis

In order to further explore the practical economic meaning of T-MUCM, and analyze the impact of the government's expected subsidy on the utility of each subgroup, we analyze the sensitivity of subsidy cost budget B. Based on the sensitivity analysis results, we provide countermeasures and suggestions for the government to set reasonable subsidies. Sensitivity analysis is shown in Fig. 6.

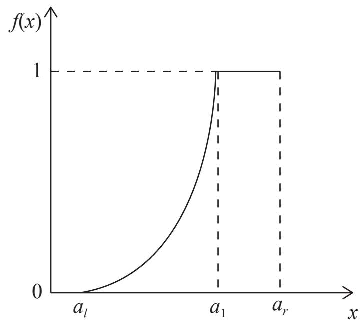


Fig. 6 Grey utility function of the villager

As it can be seen from Fig. 7, the global consensus utility level is "cyclic". When the government subsidy amount is between 5 and 6, the global consensus utility level exists a maximum of 0.011 9; When the government subsidy amount is between 7 and 8, over time, there is a maximum value of 0.010 4 in the global consensus utility level; When the government subsidy amount is between 9 and 10, there is a maximum value of 0.011 7 in the global consensus utility level. We can know that the government can determine the range of subsidy amount in advance according to the economic level of each village, and then maximize the consensus effect within the appropriate subsidy range.

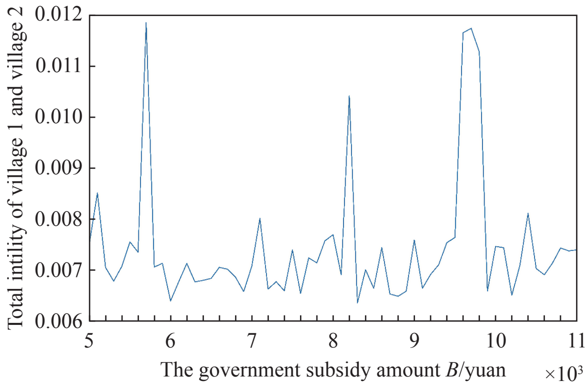


Fig. 7 Sensitivity analysis of *B*

6. Conclusions

There exists lots of two-level group consensus problems such as the allocation of pollution reduction quotas and natural gas subsidies in real life. Due to the complexity of the environment, it is difficult for DMs to express their opinions with a specific value. Based on this, we construct a hierarchical consensus model, where the government is the moderator, villagers are the lower-level DMs, and the whole villages are upper-level DMs. All decision-makers express their opinions in grey interval numbers.

Firstly, minimizing the subsidy as the objective function, we calculate the consumed cost in consensus process and the optimal consensus opinion of each interest subgroup. Further, we construct a maximum utility consensus model, and introduces a grey utility function to characterize the opinion preferences of DMs at all levels. We use the right-biased S-shape to describe the government's opinion preference, and uses the right-biased parabolic curve to describe the villagers' opinion preference. Under the constraint of government's expected subsidy budget to calculate utility level (consensus level) and optimal consensus opinion of each village and villager. The T-MUCM not only considers the cost of consensus, but also considers the opinions preferences of DMs at all levels, which greatly meets the psychological expectations of DMs. The utility goal is a method to measure the consensus level, which can make the consensus level be in a high stable state.

However, we only analyze two forms of utility functions, and assume that the DMs in each sub-group have the same opinion preference. In fact, different DMs have different opinion preferences. Therefore, the future research direction is to obtain real utility functions that can truly reflect psychological preferences of DMs, and use the real utility functions to reach the consensus. In addition, due to the limitation of programming complexity, we only consider a two-level hierarchical consensus model and only select four DMs at each level. In the future, we will try to obtain the real utility function of the DMs through machine learning and study how to construct a better applicability multi-level group consensus model.

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