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Lanchester equation for cognitive domain using hesitant fuzzy linguistic terms sets

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Abstract: Intelligent wars can take place not only in the physical domain and information domain but also in the cognitive domain. The cognitive domain will become the key domain to win in the future intelligent war. A Lanchester equation considering cognitive domain is proposed to fit the development tendency intelligent wars in this paper. One party is considered to obtain the exponential enhancement advantage on combat forces in combat if it can gain an advantage in the cognitive domain over the other party according to the systemic advantage function. The operational effectiveness of the cognitive domain in war is considered to consist of a series of indicators. Hesitant fuzzy sets and linguistic term sets are powerful tools when evaluating indicators, hence the indicators are scored by experts using hesitant fuzzy linguistic terms sets here. A unique hesitant fuzzy hybrid arithmetical averaging operator is used to aggregate the evaluation.

Keywords: Lanchester equation, cognitive domain, hesitant fuzzy set, hesitant fuzzy hybrid arithmetical averaging operator.

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1. Introduction

In military strategy research and combat modeling, mathematical models are used to analyze military conflicts and combat conditions, thus deepening the understanding of these operations. The Lanchester equation is an ordinary differential equation system that describes the continuous mutual consumption of time of two opposite forces in fierce confrontation. It was proposed by Lanchester, a British automotive engineer, in 1914. It is the first theory to systematically analyze the forces of two confronting parties during combat. In this paper, by extending the Lanchester equation, a study is carried out on the effect of three major domains for war confrontation on the development of war situation to discover and better describe the winning mechanism of intelligent wars.

The Lanchester equation is also called the "combat force attrition theory and combat dynamics". The state variables included in the equation represent the number of combatants (or weapon systems) active at any given time in the combat. Each equation expresses the change rate of a state variable as a function of other state variables. According to the different assumptions about the posture of operations or tactics situation, Lanchester equation-based models distinguish to some extent. The Lanchester equation mainly has two patterns of manifestation, the definitive pattern and the random pattern.

In recent years, extensive studies have been conducted on the Lanchester equation. MacKay et al.[1] and Johnson et al.[2] used the Lanchester equation to fit historical combat data. Keane used partial differential equations to capture the mutual influences of space and each component [3]. Some approximate solutions for the random type of Lanchester equation have been presented [4]. Kress et al. extended the model to a conflict consisting of three parties [5]. Scholars have proposed a series of improved or extended Lanchester equation models depending on actual war situations. For example, Cao et al. considered the effect of personnel's psychological factors present in modern wars[6]. By improving the classical Lanchester equation, the effect reflecting the personnel's psychological factors in wars was added in the equation, and thus a combat model reflecting personnel's psychological factors was proposed [6]. Deitchmen proposed an improved Lanchester equation regarding guerrilla warfare, which considered the different damaging results of both combat parties [7]. Tang et al. proposed an information warfare model that merges some factors, such as personnel psychology, combat cooperation, information advantage, and decision-making advantage [8]. In addition, Kress proposed a new Lanchester equation model with regard to

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unconventional warfare, such as rebellion and multilateral conflicts [9]. This indicates the strong advantages and wide use of Lanchester equation in dealing with the combat [10−13] or problems similar to combat [14,15].

Future wars can take place not only in the physical domain and information domain but also in the cognitive domain. The cognitive domain will become the key domain to win in the future intelligent war. The Center for Strategic and Budgetary Assessments (CSBA) published a report 'Taking back the seas: transforming the U.S. surface fleet for decision-centric warfare' to use the above viewpoint to design new operational concepts [16].

Through the above analysis, it can be found that the vast majority of studies related to Lancester equations mainly consider the effect of the physical domain and the information domain in the war until now. However, there are very few papers related to the effect of cognitive domain in the war, much less to Lanchester equation for the cognitive domain, because it is hard to figure out the role of the cognitive domain in the intelligent war.

Hesitant fuzzy sets (HFSs) [17,18] considered a case of several possible values when establishing membership degrees. Thus, HFSs can provide a highly comprehensive and detailed description of the hesitant information of decision makers. Linguistic term sets (LTSs) [19,20] are considerably useful and helpful for making decisions. Hesitant fuzzy LTSs provide a linguistic and computational basis to increase the richness of linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammars by using comparative terms [21−23]. Aggregation operators are useful tools when integrating evaluation information [24]. Thus it is very suitable for us to use it in evaluating attributes. With the expansion of application scope of HFSs, it is becoming more and more important to integrate and process the hesitant fuzzy information effectively. However, there are still few studies in determining the parameters of Lanchester equation using hesitant fuzzy LTSs.

In this paper, we first review several different forms of Lanchester equations. Then their limitations are analysed and a Lanchester equation for the cognitive domain is proposed by us to fit the coming intelligent wars. We think one party can obtain the exponential enhancement advantage on combat forces in combat if it can gain an advantage over the other party in the cognitive domain. In order to analyse the effectiveness of the cognitive domain, it is considered to consist of a series of indicators. The attributes are scored by experts using hesitant fuzzy LTSs here. At last, simulation results show that the party which has the cognitive asymmetric advantage will have a clear advantage. The results represent that gaining competitive advantages in the cognitive domain is very important for winning future wars. The cognitive domain will play an important role in the future intelligent warfare.

2. Preliminaries

2.1 The first linear law of Lanchester

The first linear law of Lanchester is applicable to combat environments such as cold weapon warfare in ancient times. Within such combat backgrounds, it is assumed that the combat forces are exposed to each other, and engagements comprise of one-to-one combats by each unit. The unit loss rates of both combatants are determined by the effectiveness of opponent combat. Let $B = B(t)$ and $R = R(t)$ be state variables that represent the number of living fighters (or equipment) of a blue and a red army at time t, respectively. At this time, the loss rates of both parties are constants. The combat loss equation [6] takes the following form:

$$\begin{cases} \dfrac{\mathrm{d}B}{\mathrm{d}t} = -\beta \\ \dfrac{\mathrm{d}R}{\mathrm{d}t} = -\alpha \end{cases}. \tag{1}$$

The state equation can be obtained through variable separation:

$$\beta(B_0 - B) = \alpha(R_0 - R). \tag{2}$$

2.2 The second linear law of Lanchester

The second linear law of Lanchester is applicable to non-line-of-sight firepower. In this circumstance, a party's firepower is determined not only by the size of its remaining forces, but also by the other's target density. As the firepower is non-line-of-sight, the possibility of obtaining the targets is to some extent determined by the number of such targets at a given region. For this reason, this type of model is called a surface firepower model. In this condition, the combat loss equation can be expressed as

$$\begin{cases} \dfrac{\mathrm{d}B}{\mathrm{d}t} = -\beta BR \\ \dfrac{\mathrm{d}R}{\mathrm{d}t} = -\alpha BR \end{cases}. \tag{3}$$

The state equation is the same as the first linear law, i.e., $\beta(B_0 - B) = \alpha(R_0 - R)$ obtained through variable separation.

2.3 The square-law of Lanchester

The Lanchester square-law is also called the classical Lanchester equation. In this model, both combat parties' tactics and command control are assumed to be at their best state. Also, it is assumed that: (i) the combat forces of both parties are exposed to each other; (ii) any party can concentrate superior firepower to attack the other party; (iii) the opportunity of each combat unit to shoot any opponent combat unit is roughly equal. The unit loss rates of both combat parties are determined by the opponent's combat force and are also proportional to the number of units on the other side.

$$\begin{cases} \frac{\mathrm{d}B}{\mathrm{d}t} = -\beta R \\ \frac{\mathrm{d}R}{\mathrm{d}t} = -\alpha B \end{cases}. \quad (4)$$

The state equation can be obtained through variable separation as follows:

$$\beta\left(B_0^2 - B^2\right) = \alpha\left(R_0^2 - R^2\right) \quad (5)$$

where B_0 and R_0 are the scales of forces of two sides at the initial state of warfare, respectively. Especially, we can obtain the equivalent conditions that end of warfare by way of mutual destruction is $\beta B_0^2 = \alpha R_0^2$. The side with a larger product of loss rate with initial force square is the winner. Meanwhile, we observe that the loss rate has a linear influence, but the influence of force is quadric. When the loss rate doubles, the effective loss also doubles. However, when the combat personnel quantity doubles, a quadratic effect will be generated. For this reason, the surface firepower model is called square-law, and it emphasizes a significant principle of concentrating firepower in military tactic.

3. Lanchester equation for asymmetric operation in cognitive domain

Future wars can occur simultaneously in three domains: the physical domain, the information domain, and the cognitive domain [25]. The physical domain is the traditional war domain, which consists of combat platforms, military installations, etc. It is the material base for the occurrence of wars. The information domain refers to the space of information generation, transmission, and sharing. It has been the key point in modern wars. The cognitive domain is the space reflecting people's learning and use of knowledge regarding equipment, combat beliefs, and combat ability. It is the apogee of strategies for future intelligent wars.

For the coming intelligent wars, cognitive asymmetric advantages gained in the cognitive domain can have an important influence on seizing the initiative of war and winning future wars. The ability to wage war in the cognitive domain has become the essential combat capability of modern militaries. The issue of reflecting the effects of the cognitive advantages of combat systems in the established Lanchester equation model remains unresolved. Here, the following aspects are considered:

(i) The problem of selecting the form of Lanchester equation. Depending on whether information support is present or not, the measured combat results obtained using different forms of the Lanchester equation show significant differences. In the presence of information support, the Lanchester square-law is usually used, while the second linear law is commonly used in the absence of information support. The first linear law of Lanchester is often specific to combat among individual soldiers in ancient times.

Presently, for the coming intelligent wars, both combat parties can realize real-time state information perception and interaction in complicated battlefield environments. On this basis, combat task orders can be formed dynamically according to fused state information without human factors for various war systems, automatically satisfying various combat task requirements. This basically forms a type of complete external information support and is similar to direct pointing and shooting. Direct pointing and shooting refer to cases where, at a certain time and space, the attacker weapon systems can shoot the target effectively due to a strong information perception of the battlefield and the exposure of the attacked party. Therefore, the first linear law of Lanchester, which describes combat among single battle units, and the second linear law, which describes indirect pointing and shooting under blindness, are both inapplicable. Moreover, under special balanced conditions, cognitive asymmetry is approximately equivalent to Lanchester square-law that both combat parties' tactics and command and control are at their best state.

(ii) The Lanchester equation describes the problem of asymmetric warfare. From the aspect of asymmetric effects, any combat unit of a combat system with cognitive advantages holds significant advantages compared to the combat ability of the enemy. This indicates that the combat forces' effective numerical value (including personnel and equipment) has been exponentially enhanced. By contrast, any combat unit of combat systems at cognitive disadvantages has been weakened to a large extent compared to the combat ability of the enemy, indicating the weakening of the combat forces' effective numerical value. For example, other factors being equal, the intelligence levels of various equipment of the red party are all higher than those of the blue party. According to the system advantage function, the overall equipment combat effectiveness will form a continuous product effect. This is even truer for the functions of the national strategy, warfare level, psychological area attack and defense, etc., in the war. It should be noted that, in contrast to the information and physical domains, the capacity coefficient of the cognitive domain exerts influence on the damage capabilities of the military force, as it serves as a measure of comparison of the capability coefficients of both combat parties. It means that, if both parties have a strong cognitive domain presence, they cannot gain an advantage over the other party, so they cannot obtain the exponential enhancement advantage on combat forces in combat.

Modern combat systems are composed of multiple relatively independent systems with some functions, and each such system can be regarded as a node. For the emergence that the system effectiveness (capability) index remains unchanged while the whole combat effectiveness is mutated, the essence is the interaction among the nodes of systems. The change of the comprehensive effectiveness of the whole system is used to analyze and evaluate the occurrence. The system advantage function is the concentrated reflection of various subsystems' comprehensive effectiveness. Suppose that the capacity coefficients for the n parts (or the effectiveness indexes) of a certain combat system Y are represented as $y_1, y_2, \cdots, y_n$, and b is the weight coefficient. Then the systemic advantage function $F(y)$ for evaluating the overall system effectiveness can be established as follows:

$$\begin{gathered} F(y) = \varphi(y_1, y_2, \cdots, y_n) = \\ b_1 y_1 + b_2 y_2 + \cdots b_n y_n + b_{12} y_1 y_2 + \cdots b_{1n} y_1 y_n + \\ b_{21} y_2 y_1 + \cdots + b_{2n} y_2 y_n + \cdots + b_{n1} y_n y_1 + \cdots b_{12\cdots n} \prod_{i=1}^{n} y_i. \end{gathered} \tag{6}$$

The comprehensive effectiveness of the system is $\sum_{i=1}^{n} b_i y_i$ when each part functions independently. When the parts cooperate in pairs, the additionally obtained comprehensive effectiveness of the system is $\sum_{i,j=1, i\neq j}^{n} b_{ij} y_i y_j$.

When all the parts cooperate with each other, the additionally obtained comprehensive effectiveness of the system becomes $b_{12\cdots n} \prod_{i=1}^{n} y_i$.

In the intelligent warfare form, the parts in the combat system interact and cooperate with each other, and share information resources sufficiently, thereby realizing intelligent control and decision making in a distributed manner. A significant advantage can thus be gained in confrontation, which is equivalent to the exponential growth of the forces.

We assume that the cognitive domain capacity coefficients of the red and the blue party are $C_R \in [0,1]$ and $C_B \in [0,1]$, respectively. On the basis of the Lanchester square-law, the Lanchester equation of cognitive asymmetry is established as follows:

$$\begin{cases} \dfrac{\mathrm{d}R}{\mathrm{d}t} = -\beta I_B [B(t)]^{C_B/C_R} \\ \dfrac{\mathrm{d}B}{\mathrm{d}t} = -\alpha I_R [R(t)]^{C_R/C_B} \end{cases}. \tag{7}$$

Let $\lambda = C_{A_2}/C_2$ be the cognitive domain capacity coefficient ratio for the two parties, the equation can also be written as follows:

$$\begin{cases} \dfrac{\mathrm{d}R}{\mathrm{d}t} = -\beta I_B [B(t)]^{1/\lambda} \\ \dfrac{\mathrm{d}B}{\mathrm{d}t} = -\alpha I_R [R(t)]^{\lambda} \end{cases}. \tag{8}$$

In this paper, the cognitive domain is considered to mainly consist of the intelligent domain and the psychological domain. One army's cognitive domain capacity coefficient $C \in [0,1]$ can reflect its personnel's knowledge, beliefs' and abilities, as well as the intelligence level of weaponry, the levels of psychological warfare and public opinion warfare, etc. As shown in Fig. 1, it consists of two parts, i.e., the intelligence domain capacity coefficient and the psychological domain capacity coefficient. The intelligence domain capacity coefficient consists of x_1 and x_2, x_1 is the national strategy and warfare level coefficient, x_2 is the equipment intelligence level coefficient, while the psychological domain capacity coefficient consists of x_3 and x_4 , x_3 is the psychological attack and defense capacity coefficient, and x_4 is the soldier volitional quality coefficient.

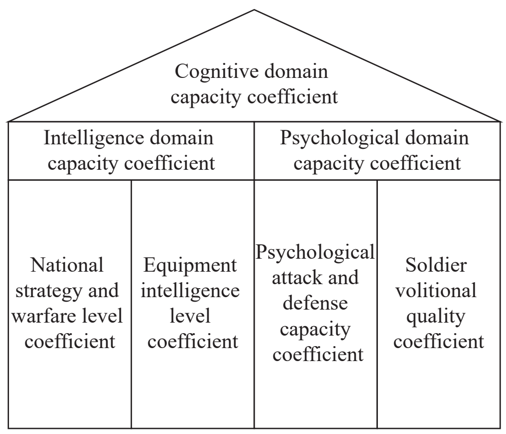

Fig. 1 Cognitive domain capacity coefficient composition

These coefficients cannot be quantitatively analyzed directly. Thus, in this paper, it is considered that in practical applications, the four attributes can be scored by experts using hesitant fuzzy LTSs, and then the hesitant fuzzy integration operator can be adopted to perform comprehensive integration and obtain the final cognitive domain capacity coefficient. All of the indices involved are benefit-type indexes.

It is difficult to directly analyze these capacity coefficients quantitatively. Thus, in this paper, it is considered that in practical applications, the four attributes can be scored first by experts using hesitant fuzzy LTSs (or marked after further subdivision of the coefficients). Then the hesitant fuzzy integration operator can be utilized to perform comprehensive integration and obtain the ultimate cognitive domain capacity coefficients. All of the indexes involved are benefit-type indexes.

HFSs consider the case that we have a set of possible values when establishing the membership degree [17,26,27]. Thus HFSs can have a more comprehensive and detailed description of the decision-maker's hesitant information.

Definition 1 Let h_j $(j = 1,2,\cdots,n)$ be hesitant fuzzy elements, and the hesitant fuzzy hybrid arithmetical averaging (HFHAA) operator [17,28,29] is defined as

$$\text{HFHAA}(h_1,h_2,\cdots,h_n) = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}(1-\gamma_j)^{\frac{\omega_j\omega_{\sigma(j)}}{\sum_{j=1}^{n}\omega_j\omega_{\sigma(j)}}}\right\}. \tag{9}$$

Every index in respect of each army is evaluated by linguistic hesitant fuzzy numbers, whose membership degrees and non-membership degrees are from the LTSs [30,31] as follows:

$$S = \{s_{10} = \text{ extremely good}, s_9 = \text{ very good}, s_8 = \text{ fairly good}, s_7 = \text{ good}, s_6 = \text{ slightly good}, s_5 = \text{ fair}, s_4 = \text{ slightly poor}, s_3 = \text{ poor}, s_2 = \text{ fairly poor}, s_1 = \text{ very poor}, s_0 = \text{ extremely poor}\}.$$

We assume that experts' evaluation values for the red and the blue party are shown in Table 1.

Table 1 Experts' evaluation values for the red and the blue party

Parameter	x_1	x_2	x_3	x_4
B	$\{s_5,s_4\}$	$\{s_5\}$	$\{s_5,s_4,s_3\}$	$\{s_6,s_5\}$
R	$\{s_6\}$	$\{s_6,s_5\}$	$\{s_7,s_6\}$	$\{s_8,s_6\}$

It is assumed that the weight vector of the four capacity coefficient attributes provided by the experts is $\boldsymbol{\omega} = [0.29,0.28,0.21,0.22]^{\mathrm{T}}$, and the ordered position vector of the attributes provided by the experts is $\boldsymbol{\omega} = [0.5, 0.2,0.15,0.15]^{\mathrm{T}}$, meaning that the better attributes among the cognitive domain capacity coefficients are assigned with a larger weight.

The HFHAA operator is used to obtain the hesitant fuzzy elements h_B and h_R corresponding to army B and army R, respectively. Taking the case of army B as an example, h_B is calculated as

$$h_B = \text{HFHAA}(h_{B1},h_{B2},h_{B3},h_{B4}) = \text{HFHAA}(\{0.5,0.4\},\{0.5\},\{0.5,0.4,0.3\},\{0.6,0.5\}).$$

Due to $s(h_{B1})$ =0.45, $s(h_{B2})$ =0.5, $s(h_{B3})$ =0.4, $s(h_{B4})$ = 0.55, thus $h_{B4}>h_{B2}>h_{B1}>h_{B3}$, $\sigma(B1) = 4$, $\sigma(B2) = 2$, $\sigma(B3) = 1$, and $\sigma(B4) = 3$. Therefore,

$$\frac{\omega_1\omega_{\sigma(B1)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Bj)}} = 0.183\,2,$$

$$\frac{\omega_2\omega_{\sigma(B2)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Bj)}} = 0.235\,8,$$

$$\frac{\omega_3\omega_{\sigma(B3)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Bj)}} = 0.442\,1,$$

$$\frac{\omega_4\omega_{\sigma(B4)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Bj)}} = 0.138\,9,$$

$$h_R = \text{HFHAA}(h_{R1},h_{R2},h_{R3},h_{R4}) = \text{HFHAA}(\{0.5,0.4\},\{0.5\},\{0.5,0.4,0.3\},\{0.6,0.5\}) = \bigcup_{\gamma_{B1}\in h_{B1},\gamma_{B2}\in h_{B2}\gamma_{B3}\in h_{B3},\gamma_{B4}\in h_{B4}}\left\{1-\prod_{j=1}^{4}(1-\gamma_{Bj})^{\frac{\omega_j\omega_{\sigma(Bj)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Bj)}}}\right\} = \bigcup_{\gamma_{B1}\in h_{B1},\gamma_{B2}\in h_{B2},\gamma_{B3}\in h_{B3},\gamma_{B4}\in h_{B4}}\left\{1-\prod_{j=1}^{4}(1-\gamma_{B1})^{0.1832}\cdot(1-\gamma_{B2})^{0.2358}(1-\gamma_{B3})^{0.4421}(1-\gamma_{B4})^{0.1389}\right\} =$$

$$\{0.515\,3,0.5,0.474\,6,0.458\,0,0.437\,5,0.419\,8,0.498\,8,0.483\,0,0.456\,7,0.439\,6,0.418\,4,0.400\,1\},$$

$$s(h_B) = 0.458\,5,$$

$$h_R = \text{HFHAA}(h_{R1},h_{R2},h_{R3},h_{R4}) = \text{HFHAA}(\{0.6\},\{0.6,0.5\},\{0.7,0.6\},\{0.8,0.6\}).$$

Due to $s(h_{R1})$ =0.6, $s(h_{R2})$ =0.55, $s(h_{R3})$ =0.65, $s(h_{R4})$ = 0.7, thus $h_{R4}>h_{R3}>h_{R1}>h_{R2}$, $\sigma(R1) = 4$, $\sigma(R2) = 3$, $\sigma(R3) = 1$, and $\sigma(R4) = 2$. Therefore,

$$\frac{\omega_1\omega_{\sigma(R1)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Rj)}} = 0.185\,5,$$

$$\frac{\omega_2\omega_{\sigma(R2)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Rj)}}=0.179\ 1,$$

$$\frac{\omega_3\omega_{\sigma(R3)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Rj)}}=0.447\ 8,$$

$$\frac{\omega_4\omega_{\sigma(R4)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Rj)}}=0.187\ 6,$$

$$h_R=\mathrm{HFHAA}(h_{R1},h_{R2},h_{R3},h_{R4})=$$
$$\mathrm{HFHAA}(\{0.6\},\{0.6,0.5\},\{0.7,0.6\},\{0.8,0.6\})=$$
$$\bigcup_{\gamma_{R1}\in h_{R1},\gamma_{R2}\in h_{R2},\gamma_{R3}\in h_{R3},\gamma_{R4}\in h_{R4}}\left\{1-\prod_{j=1}^{4}(1-\gamma_{Rj})^{\frac{\omega_j\omega_{\sigma(Rj)}}{\sum_{j=1}^{4}\omega_j\omega_{\sigma(Rj)}}}\right\}.$$

It is assumed that country B and country R are in the state of confrontation, and their loss rates of forces are $\alpha=0.84$ and $\beta=0.6$, respectively. The initial troops for the blue and the red party are $B_0=100$ and $R_0=50$, respectively. Military experts evaluate the capacity coefficients in the cognitive domain for both parties.

Therefore, $\lambda=0.636\ 9/0.458\ 5$, and the simulation step length is set as 0.001. Matlab is used to numerically simulate (8), obtaining the evolution of combat forces as a function of the simulation step length when the two parties are in a confrontation equilibrium cognitively. The result is illustrated in Fig. 2.

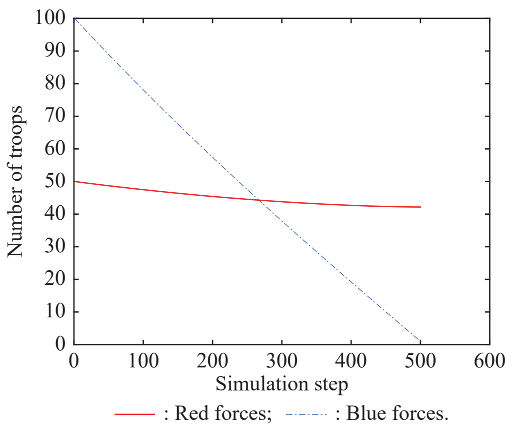

Fig. 2 Variation of the red and the blue forces

Next, the influence of the cognitive domain capacity coefficients on the outcome of war is analyzed. First, the case when the cognitive domain capacity coefficients of the two parties are equal is considered. The simulation step length is set as 0.001, and the input parameters are configured as follows: $\alpha=1$, $\beta=1$; $I_B=1$, $I_R=1$; $B_0=100$, $R_0=50$, $\lambda=1$. Matlab is used to numerically simulate (8), obtaining the evolution of combat forces as a function of the simulation step length when the red and the blue are in a confrontation equilibrium cognitively. The result is illustrated in Fig. 3 (the solid and dash lines respectively show the combat force coefficient for the red party and the blue party). As can be seen from Fig. 3, the blue wins the war at a small cost shortly after the start of the war.

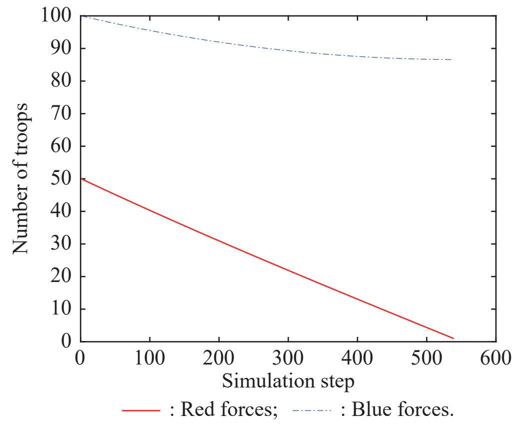

Fig. 3 Variation of the red and the blue forces when λ=1

Next, another scenario is considered when the cognitive domain capacity coefficient ratio for the two parties is $\lambda=1.2$. The rest of the parameters are configured as above. Matlab is used to numerically simulate (8), obtaining the variation of combat forces for the two parties versus the combat time, as illustrated in Fig. 4. In the end, the blue party still manages to win the war, though after a much longer combat time and with a lot of casualties.

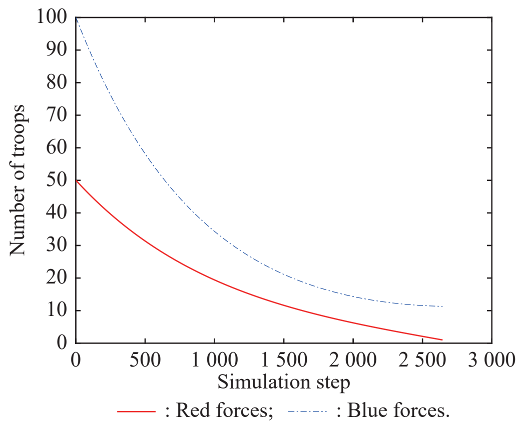

Fig. 4 Variation of the red and the blue forces when λ=1.2

When the cognitive domain capacity coefficient ratio for the two parties is $\lambda=1.25$ and the other parameters are fixed, Matlab is again used to numerically simulate (8). The variation of combat forces for the two parties with the combat time is thus obtained and shown in Fig. 5. Even though the ratio of initial combat troops for the two parties is 1:2, the red party has the cognitive asymmetric advantage and wins the war ultimately after a period of combat time.

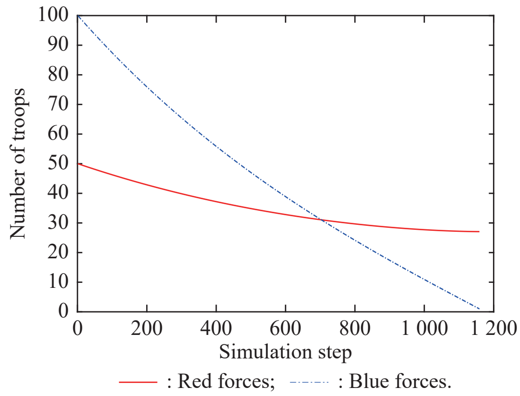

Fig. 5 Variation of the red and the blue forces when λ=1.25

Finally, the case of $\lambda = 1.5$ is considered, and the resulting variation of combat forces for the two parties with the combat time is obtained by numerically simulating (8) with Matlab, as shown in Fig. 6. Even with an initial troops ratio of 1:2, by taking a significant advantage of cognitive asymmetry, the red party is able to win the war with a minor cost and a short combat time. This fully demonstrates the exponential strengthening effect of the advantage in cognitive asymmetry.

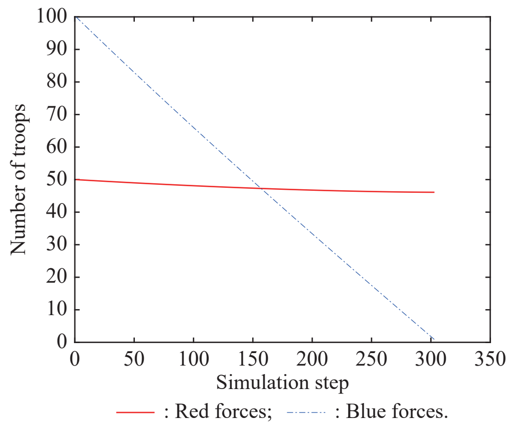

Fig. 6 Variation of the red and the blue forces when λ=1.5

From the above analysis, it can be found that the cognitive domain will become the key domain to win in the future intelligent war. We think there are some key points of winning a war in the future intelligent warfare.

Firstly, knowing beforehand and launching a pre-emptive strike will achieve higher levels by relying on intelligent equipments. Time is the only combat element that cannot be reversed. A pre-emptive attack, therefore, invariably plays a vital role in winning wars. Intelligent warfare can become more intense since intelligent equipment can make decisions autonomously and cloud networks can vie to gain time advantages. Secondly, unmanned clusters and community operations will become typical new methods of warfare to keep on top of the numbers. Being numerically superior to enemies is a concrete expression of applying "instant advantage" to combat forces. Being numerically superior to enemies in intelligent warfare implies having superior numbers of forces by implementing unmanned clusters or community operations. Thirdly, crossing domain, improving efficiency, and full integration will rise to a new level by relying on cognition domain to expand spatial advantages. Artificial intelligence will not only give rise to new operational space but also expand joint warfare in breadth and depth. Last but not least, controlling the enemies instead of destroying them will become a new way to win a war by relying on the cognition domain to gain psychological advantages. Thus, the purpose of "subduing enemies without a fight" can be achieved.

4. Conclusions

In this paper, we propose a Lanchester equation for the cognitive domain to fit the development tendency of intelligent wars. According to the systemic advantage function, we think one party will obtain the exponential enhancement advantage on combat forces in combat if they can gain an advantage in cognitive over the other party.

In order to analyze the effectiveness of the cognitive domain compared to other domains in war, the operational effectiveness of the cognitive domain in war is considered to consist of a series of indicators. HFSs and LTSs are powerful tools when evaluating attributes. Thus the indicators are scored by experts using hesitant fuzzy LTSs in the paper. Simulation results show that the party which has the cognitive asymmetrical advantage will have a clear advantage in the war. This fully demonstrates that winning future wars needs to gain competitive advantages in the cognitive domain. Several key points of winning a war in the future intelligent warfare are proposed at the end of the paper. The research is helpful to study the wining mechanism of intelligent warfare.

However, there is still much research to be done in the future. The indicator system of the cognitive domain needs to be refined. The influences that are imposed on the consequences to Lanchester equations by the changing cognitive domain capacity coefficient ratios for the two parties in multiple time periods of war also need to be studied.

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