Journal of Systems Engineering and Electronics
Vol. 33, No. 3, June 2022, pp.665 – 673

A self-adaptive grey forecasting model and its application

TANG Xiaozhong[1,2] and XIE Naiming[1,*]

1. College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China;
2. Department of Industry and Finance, Huangshan Vocational and Technical College, Huangshan 245000, China

Abstract: GM(1,1) models have been widely used in various fields due to their high performance in time series prediction. However, some hypotheses of the existing GM(1,1) model family may reduce their prediction performance in some cases. To solve this problem, this paper proposes a self-adaptive GM(1,1) model, termed as SAGM(1,1) model, which aims to solve the defects of the existing GM (1,1) model family by deleting their modeling hypothesis. Moreover, a novel multi-parameter simultaneous optimization scheme based on firefly algorithm is proposed, the proposed multi-parameter optimization scheme adopts machine learning ideas, takes all adjustable parameters of SAGM(1,1) model as input variables, and trains it with firefly algorithm. And Sobol' sensitivity indices are applied to study global sensitivity of SAGM(1,1) model parameters, which provides an important reference for model parameter calibration. Finally, forecasting capability of SAGM(1,1) model is illustrated by Anhui electricity consumption dataset. Results show that prediction accuracy of SAGM(1,1) model is significantly better than other models, and it is shown that the proposed approach enhances the prediction performance of GM(1,1) model significantly.

Keywords: grey forecasting model, GM(1,1) model, firefly algorithm, Sobol' sensitivity indices, electricity consumption prediction.

DOI: 10.23919/JSEE.2022.000061

1. Introduction

Grey system theory was pioneered by Deng [1] focusing on solving uncertainty problems with a small amount of data. Unlike statistical models and machine learning models, grey system theory requires very little data to uncover the intrinsic development law of an uncertain system. Over the past several decades, grey system theory has been widely applied in various fields and achieved good results [2−4]. Grey system theory is composed of grey prediction, grey incidence analysis, grey control, and grey clustering, etc. [2−5], in which grey prediction is an important part and has been widely used in various fields, such as industry [6,7], agriculture [8], energy [9−11], and tourism [12−14].

GM(1,1) model, as a basic grey prediction model, has been successfully applied in energy, transportation, and other fields and achieved good results [15,16]. However, as a newly born theory, GM(1,1) model still has some defects that need to be solved. To improve the prediction accuracy of GM(1,1) model, scholars have carried out a lot of creative researches on the improvement of the GM (1,1) model in recent decades. The improved methods can be divided into several categories.

(i) Initial condition optimization. The initial value of the traditional grey model usually takes $x^{(0)}(1)$, which contradicts the new information priority principle. To settle this problem, scholars have conducted creative research on initial value optimization and achieved fruitful results, For example, Dang et al. took the last data item of $\boldsymbol{X}^{(1)}$ as the initial condition of the GM(1,1) model [17]. Wang et al. proposed a novel initial value calculation method by linear weighting the first and last item of $X^{(1)}$ [18]. Xiong et al. took the weighted average value of all data items of $X^{(1)}$ as the initial condition and constructed a new GM(1,1) model [19]. Although this method considers all known information in determining the initial value, the fixed weights affect the stability of the GM(1,1) model's predictive performance.To address this challenge, Ding proposed a new initial value construction method based on dynamic weighting coefficients [20]. Zhu et al. constructed a novel initial condition optimization method by using dynamic fractional-order weighted coefficients, these studies have greatly improved the predictive capability of GM(1,1) model [21].

(ii) Background value optimization. The background value of the traditional grey model is obtained by per-

Manuscript received February 15, 2021.
*Corresponding author.
This work was supported by the National Natural Science Foundation of China (72171116;71671090), the Fundamental Research Funds for the Central Universities (NP2020022), the Key Research Projects of Humanities and Social Sciences in Anhui Education Department (SK2021A1018), and Qinglan Project for Excellent Youth or Middle-aged Academic Leaders in Jiangsu Province, China.

forming an adjacent average generating operation on $X^{(1)}$. However, when the time series changes sharply, this background value calculation method will reduce the prediction accuracy of grey prediction model to some extent. To address this issue, a large number of background value optimization algorithms, such as linear value insert method [22,23], integration formula [24], and Simpson formula [25], have been proposed one after another over the past decades, which have significantly enhanced the predictive power of grey prediction model.

(iii) Accumulated generating operator optimization. Accumulated generating operation is an important modeling basis for grey system theory. Traditional GM(1,1) models are modeled based on first order cumulative generation series, which is contrary to the new information priority principle [26]. To address this challenge, Wu et al. creatively introduced the idea of fractional order into grey modeling, which significantly improved the prediction power of the grey prediction model [26]. Since then, many scholars have devoted themselves to the research of fractional order grey modeling and achieved fruitful results [27−29].

(iv) Hybrid grey models. GM(1,1) model can achieve accurate prediction when the original data sequence shows a stable increase trend. However, when the modeling sequence shows irregular fluctuation, the prediction performance of grey prediction model is poor. To address this challenge, scholars have introduced the idea of combinatorial modeling and constructed a series of hybrid grey prediction models, including Grey-Fuzzy models [30], Grey-Markov models [13−14,31], Grey-Fourier models [32,33], and other hybrid grey prediction models [34-36]. These optimization techniques have greatly improved the prediction power of the GM(1,1) model.

Although the above studies improve prediction accuracy of the GM(1,1) model to some extent, these studies are all constructed on the hypothesis that some adjustable parameters are known. When original observations show great randomness, assumptions made by these studies may increase their prediction errors. To solve this problem, this paper proposes a novel self-adaptive GM(1,1) model (denoted as SAGM(1,1)) by removing the modeling assumptions of the existing GM(1,1) model family. Moreover, a novel multi-parameter simultaneous optimization scheme based on firefly optimization algorithm is designed in this paper, the new multi-parameter simultaneous optimization scheme adopts machine learning ideas, takes all the adjustable parameters of SAGM(1,1) model as input variables, and trains it with firefly algorithm. Besides, Sobol’ sensitivity indices are employed to study the global sensitivity of the proposed SAGM(1,1) model adjustable parameters, which provides an important reference for model parameter calibration.

The rest of this paper is arranged as follows: Basic theories of the GM(1,1) model are introduced in Section 2. A novel self-adaptive GM(1,1) model and its parameter optimization scheme are proposed in Section 3. In Section 4, forecasting capability of the SAGM(1,1) model is illustrated by Anhui electricity consumption dataset. Finally, Section 5 concludes this paper.

2. GM(1,1) model

GM(1,1) model, as a basic grey prediction model, has been successfully utilized in various fields and achieved good performances. The general modeling procedure of GM(1,1) model can be shown with Definitions 1−3.

Definition 1 [2] Suppose $\boldsymbol{X}^{(0)}=[x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(n)]$, $n\geqslant 4$ is an original time series, $\boldsymbol{X}^{(1)}=[x^{(1)}(1),\ x^{(1)}(2),\cdots,x^{(1)}(n)]$) is the first-order accumulating generation sequence of $\boldsymbol{X}^{(0)}$, where

$$x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i),\quad k=1,2,\cdots,n. \tag{1}$$

Definition 2 [2] Suppose $\boldsymbol{X}^{(0)}$ and $\boldsymbol{X}^{(1)}$ as shown in Definition 1, $\boldsymbol{Z}^{(1)}=[z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(n)]$ is the background value of $\boldsymbol{X}^{(1)}$,where

$$z^{(1)}(k)=0.5(x^{(1)}(k)+x^{(1)}(k-1)),\quad k=2,3,\cdots,n.$$

Differential equation:

$$x^{(0)}(k)+az^{(1)}(k)=b \tag{2}$$

is called GM(1,1) model.

Definition 3 [2] Equation

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+ax^{(1)}(t)=b \tag{3}$$

is called the whitening differential equation of (2). Parameters a and b are usually calculated by least square method, and the calculation formula is defined as

$$\left[\hat{a},\ \hat{b}\right]^{\mathrm{T}}=(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y} \tag{4}$$

where

$$\boldsymbol{B}=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix},\ \boldsymbol{Y}=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}.$$

Set initial value $x^{(1)}(1)=x^{(0)}(1)$, then time response function of (3) can be obtained.

$$\hat{x}^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}}\right)\mathrm{e}^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}} \tag{5}$$

The predicted values of $\boldsymbol{X}^{(0)}$ can be obtained by

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k). \tag{6}$$

3. Optimized GM(1,1) modeling approach based on firefly algorithm

In this section,we propose the SAGM(1,1) model. Then a new multi-parameter simultaneous estimation method is designed. The flow chart of SAGM(1,1) model is shown in Fig. 1.

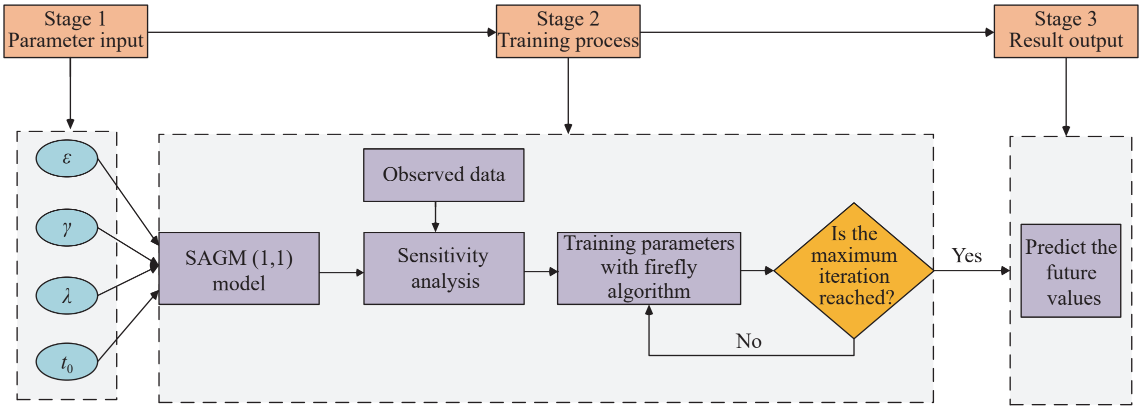

Fig. 1 Flow chart of the proposed SAGM(1,1) model

3.1 Optimization scheme of the GM(1,1) model adjustable parameters

In this subsection, optimization scheme of the GM(1,1) model adjustable parameters is designed.

3.1.1 Background value

Inappropriate background value will reduce prediction accuracy of GM(1,1) model [24,37]. The background value of the traditional GM(1,1) model is obtained by performing an adjacent average generating operation on $\boldsymbol{X}^{(1)}$. However, when time series change sharply, this background value calculation method will reduce the prediction accuracy of GM(1,1) model. To address this issue, we introduce a dynamic linear interpolation method to optimize the background value. This method not only improves the prediction accuracy of the traditional grey prediction model, but also satisfies the original intention of simplicity of grey system theory, whose calculation formula is given as

$$z^{(1)}(k)=\varepsilon x^{(1)}(k)+(1-\varepsilon)x^{(1)}(k-1),\quad 0\leqslant\varepsilon\leqslant 1. \tag{7}$$

3.1.2 Initial condition

The initial value of traditional grey forecasting model usually takes $x^{(0)}(1)$, which is contrary to the new information priority principle in grey system theory. To address this issue, we propose a new technique for determining initial value based on existing research results, and the new proposed initial value calculation method is defined as

$$x^{(1)}(t_0)=\beta_1 x^{(1)}(1)+\beta_2 x^{(1)}(2)+\cdots+\beta_m x^{(1)}(m) \tag{8}$$

where $\beta_k=\lambda^{m-k}\Big/\sum_{k=1}^{m}\lambda^{m-k}\ (0<\lambda<1), t_0$ is the time input parameter. It is easy to prove that the newly constructed initial value dynamic weighted coefficients satisfy the normalization condition, $\beta_1+\beta_2+\cdots+\beta_m=1$.

In addition, the newly proposed initial value also satisfy $\beta_1<\beta_2<\cdots<\beta_m$, when parameter λ belongs to (0,1). The following formula can clearly illustrate this fact:

$$\frac{\lambda^{m-1}}{\sum_{k=1}^{m}\lambda^{m-k}}<\frac{\lambda^{m-2}}{\sum_{k=1}^{m}\lambda^{m-k}}<\cdots<\frac{\lambda^{m-m}}{\sum_{k=1}^{m}\lambda^{m-k}}.$$

The time response function of (3) with the optimized initial condition can be derived.

$$\hat{x}^{(1)}(k)=\left(\beta_1 x^{(1)}(1)+\beta_2 x^{(1)}(2)+\cdots+\beta_m x^{(1)}(m)-\frac{\hat{b}}{\hat{a}}\right)\mathrm{e}^{-\hat{a}(k-t_0)}+\frac{\hat{b}}{\hat{a}}. \tag{9}$$

3.1.3 Accumulated generating operator

Accumulative generation operation is an important feature that distinguishes grey modeling from other modeling methods. Traditional first-order accumulative generation operation not only contradicts the principle of new information priorities but also fails to effectively explore nonlinear components embedded in uncertainty systems,which restricts the prediction accuracy of grey forecasting models to some extent [26]. In this paper, it is believed that the order of accumulation generating operator is an unknown parameter, which needs to be determined according to the characteristics of observed datasets. To further improve adaptability of GM(1,1) model, we introduce the idea of fractional order accumulative generation operation, which was first studied by Wu et al. [26].

3.2 The proposed SAGM(1,1) model

In this part, a new self-adaptive grey forecasting model is proposed, termed as SAGM(1,1) model, specific modeling steps are given in Definition 4.

Definition 4 Let $\boldsymbol{X}^{(0)}=[x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(m)]$ ($m\geqslant 4$) be a non-negative original time series, γ is the order of the accumulated generating operator, $\boldsymbol{X}^{(\gamma)}=[x^{(\gamma)}(1),x^{(\gamma)}(2),\cdots,x^{(\gamma)}(m)]$ is the γ-order accumulated generating sequence (γ-AGO) of $\boldsymbol{X}^{(0)}$, where

$$x^{(\gamma)}(k)=\sum_{i=1}^{k}\begin{pmatrix}k-i+\gamma-1\\ k-i\end{pmatrix}x^{(0)}(i),\quad k=1,2,\cdots,m.$$

Differential equation

$$x^{(\gamma)}(k)-x^{(\gamma)}(k-1)+a_\gamma z^{(\gamma)}(k)=b_\gamma \tag{10}$$

is called SAGM(1,1) model with γ order accumulated, and its whitening equation can be expressed as

$$\frac{\mathrm{d}x^{(\gamma)}(t)}{\mathrm{d}t}+a_\gamma x^{(\gamma)}(t)=b_\gamma \tag{11}$$

where

$$z^{(\gamma)}(k)=\varepsilon x^{(\gamma)}(k)+(1-\varepsilon)x^{(\gamma)}(k-1),\quad 0\leqslant\varepsilon\leqslant 1$$

is called the background value of grey derivative. Parameters a_γ and b_γ can be calculated by least-square method, its calculation formula can be given by

$$\left[\hat{a}_\gamma,\ \hat{b}_\gamma\right]^{\mathrm{T}}=\left(\boldsymbol{B}_\gamma{}^{\mathrm{T}}\boldsymbol{B}_\gamma\right)^{-1}\boldsymbol{B}_\gamma{}^{\mathrm{T}}\boldsymbol{Y}_\gamma \tag{12}$$

where

$$\boldsymbol{B}_\gamma=\begin{bmatrix}-z^{(\gamma)}(2) & 1\\ -z^{(\gamma)}(3) & 1\\ \vdots & \vdots\\ -z^{(\gamma)}(m) & 1\end{bmatrix},\ \boldsymbol{Y}_\gamma=\begin{bmatrix}x^{(\gamma)}(2)-x^{(\gamma)}(1)\\ x^{(\gamma)}(3)-x^{(\gamma)}(2)\\ \vdots\\ x^{(\gamma)}(m)-x^{(\gamma)}(m-1)\end{bmatrix}.$$

Suppose $x^{(\gamma)}(t_0)=\beta_1x^{(\gamma)}(1)+\beta_2x^{(\gamma)}(2)+\cdots+\beta_mx^{(\gamma)}(m)$ is the initial condition, t_0 is the time input parameter, then the solution of whitening differential (11) can be obtained.

$$\hat{x}^{(\gamma)}(k)=\left(\beta_1x^{(\gamma)}(1)+\beta_2x^{(\gamma)}(2)+\cdots+\beta_mx^{(\gamma)}(m)-\frac{\hat{b}_\gamma}{\hat{a}_\gamma}\right)\mathrm{e}^{-\hat{a}_\gamma(k-t_0)}+\frac{\hat{b}_\gamma}{\hat{a}_\gamma} \tag{13}$$

where

$$\beta_k=\frac{\lambda^{m-k}}{\sum\limits_{k=1}^{m}\lambda^{m-k}},\quad 0<\lambda<1;k=1,2,\cdots,m.$$

The simulated and predicted values $\hat{\boldsymbol{X}}^{(0)}$ can be obtained by performing an γth inverse accumulated generating operation (γ-IAGO) on $\hat{\boldsymbol{X}}^{(\gamma)}$.

$$\hat{x}^{(0)}(k)=\sum_{i=1}^{k}\begin{pmatrix}k-i-\gamma-1\\ k-i\end{pmatrix}\hat{x}^{(\gamma)}(i),\quad k=1,2\cdots,m \tag{14}$$

3.3 Parameter estimation of SAGM(1,1) model

In this subsection, a novel multi-parameter simultaneous optimization algorithm is proposed based on firefly algorithm, the new multi-parameter simultaneous optimization algorithm adopts machine learning ideas, takes all adjustable parameters of SAGM(1,1) model as input variables, and trains it with firefly algorithm. The parameter training mechanism can be expressed as

$$\min f(\varepsilon,\gamma,\lambda,t_0)=\frac{1}{m-1}\sum_{k=2}^{m}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|$$

$$\text{s.t.}\begin{cases}\hat{x}^{(\gamma)}(k)=\left(\sum\limits_{k=1}^{m}\beta_kx^{(\gamma)}(k)-\dfrac{\hat{b}_\gamma}{\hat{a}_\gamma}\right)\mathrm{e}^{-\hat{a}_\gamma(k-t_0)}+\dfrac{\hat{b}_\gamma}{\hat{a}_\gamma}\\ x^{(\gamma)}(t_0)=\beta_1x^{(\gamma)}(1)+\beta_2x^{(\gamma)}(2)+\cdots+\beta_mx^{(\gamma)}(m)=\sum\limits_{k=1}^{m}\beta_kx^{(\gamma)}(k)\\ \beta_k=\dfrac{\lambda^{m-k}}{\sum\limits_{k=1}^{m}\lambda^{m-k}}\\ \left[\hat{a}_\gamma,\ \hat{b}_\gamma\right]^{\mathrm{T}}=\left(\boldsymbol{B}_\gamma{}^{\mathrm{T}}\boldsymbol{B}_\gamma\right)^{-1}\boldsymbol{B}_\gamma{}^{\mathrm{T}}\boldsymbol{Y}_\gamma\\ x^{(\gamma)}(k)=\sum\limits_{i=1}^{k}\begin{pmatrix}k-i+\gamma-1\\ k-i\end{pmatrix}x^{(0)}(i)\\ \hat{x}^{(0)}(k)=\sum\limits_{i=1}^{k}\begin{pmatrix}k-i-\gamma-1\\ k-i\end{pmatrix}\hat{x}^{(\gamma)}(i)\\ z^{(\gamma)}(k)=\varepsilon x^{(\gamma)}(k)+(1-\varepsilon)x^{(\gamma)}(k-1)\\ 0<\lambda<1,\ 0\leqslant\varepsilon\leqslant 1,\ k=1,2,\cdots,m\end{cases} \tag{15}$$

It can be seen that (15) is essentially a nonlinear constrained optimization model, and it is difficult to directly seek the exact solution by ordinary methods. Based on this, a firefly algorithm is employed to search optimal values of $\varepsilon, \gamma, \lambda$, and t_0.

Firefly algorithm was pioneered by Yang in 2008 [38]. It is a bionic optimization algorithm based on swarm intelligence. Over the past decade, firefly algorithm has been widely used in various fields due to its high computational efficiencies, simple structure, and strong searchability [39,40]. The mechanism of firefly algorithm is provided below.

Assume that $\boldsymbol{X}_i = [x_{i1}, x_{i2}, \cdots, x_{iM}]$ $(i = 1,2,\cdots,N)$ represents the ith firefly in the population, where N and M represent population size and dimension size, respectively. The attractiveness between fireflies $\boldsymbol{X}_i$ and $\boldsymbol{X}_j\,(i \neq j)$ can be calculated by

$$\beta(r_{ij}) = \beta_0 \mathrm{e}^{-\gamma r_{ij}^2} \tag{16}$$

where β_0 is the attractiveness when $r_{ij} = 0$, γ denotes light absorption coefficient, and r_{ij} is the spatial distance between firefly $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$, whose calculation formula can be expressed as

$$r_{ij} = \|\boldsymbol{X}_i - \boldsymbol{X}_j\| = \sqrt{\sum_{m=1}^{M} (x_{im} - x_{jm})^2}. \tag{17}$$

Suppose firefly $\boldsymbol{X}_i$ is less bright than firefly $\boldsymbol{X}_j$, then the firefly $\boldsymbol{X}_i$ will move to the firefly $\boldsymbol{X}_j$, this movement can be defined as

$$x_{im}(t+1) = x_{im}(t) + \beta_0 \mathrm{e}^{-\gamma r_{ij}^2}(x_{jm}(t) - x_{im}(t)) + \alpha\left(\mathrm{rand} - \frac{1}{2}\right) \tag{18}$$

where parameter t is the number of iterations, x_{im} and x_{jm} are the mth dimensions of $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$, respectively, rand is a random number uniformly distributed in the range [0,1], and α is the step factor and $\alpha \in [0,1]$.

3.4 Sobol' sensitivity analysis

Sobol' sensitivity analysis is a global sensitivity analysis method, which was initiated by Sobol' in 1993 [41] and has been widely used in various fields [42,43]. The core algorithm of Sobol' indices is to decompose the total variance of the objective function into the sum of the variance of a single parameter and the variance of the interaction between multiple parameters, thereby obtaining the global sensitivity of the parameters.

Let $y = f(X)$ be an objective function, where $X = [x_1, x_2, \cdots, x_m]$, and $x_k\,(k = 1,2,\cdots,m)$ is the input variable of the objective function. The total variance of the objective function $y = f(X)$ can be expressed as

$$V(y) = \sum_{i=1}^{m} V_i + \sum_{1 \leqslant i \leqslant j \leqslant m} V_{ij} + \cdots + V_{1,2,\cdots,m} \tag{19}$$

where $V(y)$ stands for total variance, V_i is the partial variance of the input variable x_i, V_{ij} represents the interaction variance between variables x_i and x_j, and $V_{1,2,\cdots,m}$ is the variance of interaction among multiple input variables $x_i\,(i = 1,2,\cdots,m)$. Sobol' indices can be obtained by the following equations:

The first-order Sobol' index can be expressed as

$$S_i = \frac{V_i}{V(y)}. \tag{20}$$

The second-order Sobol' index can be expressed as

$$S_{ij} = \frac{V_{ij}}{V(y)}. \tag{21}$$

The total sensitivity index of the parameter x_i can be defined as

$$S_{Ti} = S_i + \sum_{i<j} S_{ij} + \cdots + S_{1,2,\cdots,m}. \tag{22}$$

4. Application

4.1 Experimental design

With the rapid development of Anhui's economy, electricity consumption in Anhui province has increased dramatically in recent years, especially since the 12th Five-Year Plan. Statistics show that the electricity consumption in Anhui province in 2019 was 230.068 billion kW·h, an increase of 2.13 times over 2010. An efficient power supply system is an important prerequisite for social and economic development. Therefore, an accurate power demand forecast is of great significance for the government to formulate correct energy policy and power planning. In this section, Anhui's annual electricity consumption dataset is applied to evaluate the prediction performance of the proposed SAGM(1,1) model. And the superiority of the SAGM(1,1) model is demonstrated by comparing to other competing models, including GM(1,1) model [2], the optimized GM(1,1) model (GM(1,1,$x^{(1)}(n)$)) [17], discrete grey forecasting model (DGM(1,1)) [44], grey system model with the fractional order accumulation (FGM(1,1)) [26], and the novel self-adaptive grey model (NSGM(1,1)) [20]. Moreover, three effective model evaluation indicators including mean absolute percent error (MAPE), root mean square error (RMSE), and mean absolute error (MAE) are employed to test the prediction performance of the proposed SAGM(1,1) model, which are

$$\mathrm{MAPE} = \frac{1}{N-1}\sum_{k=2}^{N}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%,$$

$$\mathrm{RMSE}=\sqrt{\frac{1}{N-1}\sum_{k=2}^{N}(x^{(0)}(k)-\hat{x}^{(0)}(k))^2},$$

$$\mathrm{MAE}=\frac{1}{N-1}\sum_{k=2}^{N}\left|x^{(0)}(k)-\hat{x}^{(0)}(k)\right|,$$

where $x^{(0)}(k)$ is the original observation at time k, $\hat{x}^{(0)}(k)$ is the predicted value at time k. According to the above indicators' calculation formulas, it can be concluded that the smaller the MAPE, RMSE, and MAE values are, the higher the prediction accuracy is.

The original dataset of Anhui's annual electricity consumption is collected from Anhui Statistical Yearbook [45−49]. The data from 2010 to 2018 are used as a training set for modeling, and the data of 2019 is used as a validation set to check the prediction power of the proposed SAGM(1,1) model. The experiment is implemented by programming in python3.7 running on Windows 10 with 64 bits 2.90 GHz Intel Core i5-10400 CPU and 16.00 GB of RAM.

4.2 Sensitivity analysis for SAGM(1,1) model adjustable parameters

To identify key parameters and improve the accuracy of model parameter calibration, Sobol' indices are used to calculate the global sensitivity of the SAGM(1,1) model adjustable parameters, the results are reported in Table 1. Table 1 shows that dynamic weighting parameter λ and time input parameter t_0 are the two important parameters, in which time input parameter t_0 is the most critical parameter, and its total sensitivity index is 0.66477. Moreover, there is a significant interaction between the parameters of the SAGM (1,1) model, as shown in Fig. 2. Therefore, multi-parameter simultaneous optimization algorithm is an important approach to improve the accuracy of the SAGM(1,1) model parameter calibration.

Table 1 First order and total order sensitivity indices of the SAGM (1,1) model parameters

Parameter	First-order sensitivity	Total-order sensitivity
Background-value coefficient ε	−0.000 04	0.000 19
Accumulation order number γ	0.012 97	0.057 77
Dynamic weighting parameter λ	0.424 82	0.444 35
Time input parameter t_0	0.495 36	0.664 77

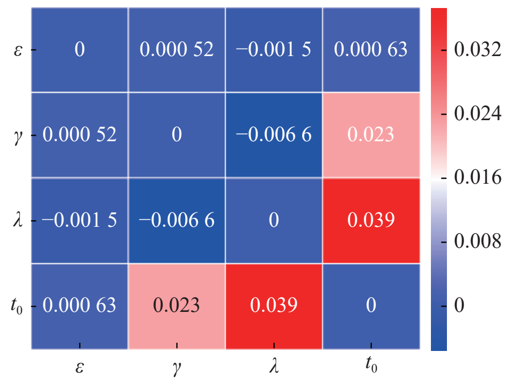

Fig. 2 Second-order interaction effect of the SAGM(1,1) model parameters

4.3 Results analysis

Based on the above analysis, firefly algorithm is employed to train the adjustable parameters of SAGM(1,1) model, and the related parameters setting of the firefly algorithm are shown in Table 2. The optimization trajectory of the SAGM(1,1) model is described in Fig. 3. It can be seen from Fig. 3 that the firefly algorithm has high efficiency to catch the optimal solution.

Table 2 Parameters setting for firefly algorithm

Parameter	Value
β_0	1
α	0.01
γ	1
Population size (maximum)	100
Number of iterations (maximum)	100

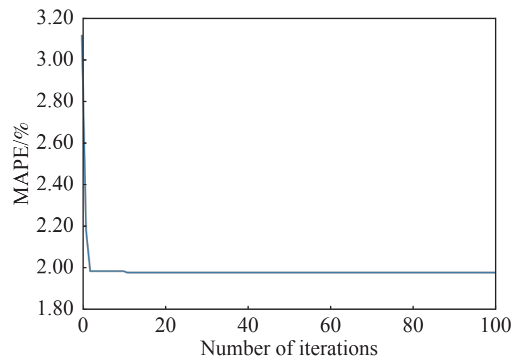

Fig. 3 Convergence curve for SAGM(1,1) model

The prediction results and errors of various grey models are listed in Table 3. It can be seen from Table 3 that compared with the competing models, SAGM(1,1) model has the smallest MAPE, RMSE, and MAE on both simulated period and predicted period. Therefore, the new proposed approach can significantly improve the prediction performance of GM(1,1) model.

Table 3 Comparison results of different models

Year	Actual data	GM(1,1)	DGM(1,1)	GM(1,1,$x^{(1)}(n)$)	NSGM(1,1) $\lambda = 0.149\,1$ $t_o = 10.055\,4$	FGM(1,1) $\gamma = 0.873\,5$	SAGM(1,1) $\epsilon = 0.589\,5$ $\gamma = 0.956\,5$ $\lambda = 0.269\,8$ $t_o = 7.599\,0$
2010	1 077.92	—	—	—	—	—	—
2011	1 221.19	1 258.263 6	1 259.136 2	1 258.594 0	1 246.088 4	1 213.424 5	1 248.598 0
2012	1 361.10	1 353.556 3	1 354.467 2	1 353.908 0	1 340.459 4	1 348.057 2	1 360.954 5
2013	1 528.07	1 456.065 8	1 457.015 9	1 456.440 1	1 441.977 4	1 470.820 2	1 472.185 8
2014	1 585.18	1 566.338 7	1 567.328 7	1 566.737 1	1 551.183 8	1 589.519 0	1 586.707 7
2015	1 639.79	1 684.963 0	1 685.993 4	1 685.387 0	1 668.660 8	1 707.817 9	1 706.481 9
2016	1 794.98	1 812.571 1	1 813.642 4	1 813.022 2	1 795.034 8	1 827.773 4	1 832.732 3
2017	1 921.48	1 949.843 4	1 950.955 9	1 950.323 4	1 930.979 5	1 950.716 4	1 966.396 9
2018	2 135.07	2 097.511 8	2 098.665 6	2 098.022 4	2 077.219 9	2 077.609 6	2 108.294 0
MAPE/%		2.057 6	2.058 4	2.058 1	2.037 7	1.975 3	1.971 3
RMSE		37.973 8	37.961 2	37.965 2	41.659 4	40.906 1	37.503 4
MAE		33.018 6	33.028 8	33.024 9	32.737 9	33.739 4	32.637 8
2019	2 300.68	2 256.363 7	2 257.558 7	2 256.906 8	2 234.535 5	2 209.214 7	2 259.195 3
MAPE/%		1.926 2	1.874 3	1.902 6	2.875	3.9756	1.803 1
RMSE		44.316 3	43.121 3	43.773 2	66.144 5	91.465 3	41.484 7
MAE		44.316 3	43.121 3	43.773 2	66.144 5	91.465 3	41.484 7

5. Conclusions

Since existing GM(1,1) model family all based on assumption that some adjustable parameters are known, the assumption made by these studies may reduce their prediction performance in some cases. To address this issue, this paper proposes a novel self-adaptive GM(1,1) optimization model (denoted as SAGM(1,1)) by removing modeling assumptions of existing GM(1,1) model family. The main conclusions of this paper can be summarized.

(i) The proposed SAGM(1,1) model can effectively overcome the defects of the existing GM(1,1) model family, and significantly improve the prediction performance of the existing GM(1,1) model family in small sample time series prediction.

(ii) A new multi-parameter simultaneous optimization scheme is developed for SAGM(1,1) model. The new parameter optimization scheme adopts machine learning ideas, takes all adjustable parameters of SAGM(1,1) model as input variables, and trains it with firefly algorithm. The new scheme has greatly improved estimation accuracy of model parameters.

(iii) The global sensitivity of SAGM(1,1) model adjustable parameters are investigated by Sobol' method. Empirical results demonstrate that there is a strong interaction effect between the parameters of SAGM (1,1) model.

Although the proposed SAGM(1,1) model provides an effective solution for small sample time series prediction, there is still room for improvement. For example, SAGM (1,1) model is a univariable grey forecasting model and does not consider the influence of external factors. Therefore, studying the self-adaptive multivariable grey forecasting model is a valuable research direction. Moreover, the proposed SAGM(1,1) model can be further investigated by being integrated with the rolling mechanism or other circumstances.

References

[1] DENG J L. Control problems of grey systems. Systems and Control Letters, 1982, 1(5): 288–294.

[2] LIU S F, YANG Y J, FORREST J. Grey data analysis: methods, models and applications. Berlin: Springer, 2016.

[3] XIE N M, WANG R Z. A historic review of grey forecasting models. Journal of Grey System, 2017, 29(4): 1–29.

[4] TANG X Z, XIE N M. Research on the evaluation of tourism development potential of tea intangible cultural heritage based on grey clustering. Grey Systems: Theory and Applica-

tion, 2019, 9(3): 295–304.

[5] XIE N M, LIU S F. Interval grey number sequence prediction by using non-homogenous exponential discrete grey forecasting model. Journal of Systems Engineering and Electronics, 2015, 26(1): 96–102.

[6] HSU L C. Using improved grey forecasting models to forecast the output of opto-electronics industry. Expert Systems with Applications, 2011, 38(11): 13879–13885.

[7] TAN G J. The grey forecasting method of life test data of electronic equipments. Systems Engineering and Electronics, 2001, 23(9): 102–104. (in Chinese)

[8] OU S L. Forecasting agricultural output with an improved grey forecasting model based on the genetic algorithm. Computers and Electronics in Agriculture, 2012, 85: 33–39.

[9] XIE N M, PEARMAN A D. Forecasting energy consumption in China following instigation of an energy-saving policy. Natural Hazards, 2014, 74(2): 639–659.

[10] XIE N M, YUAN C Q, YANG Y J. Forecasting China's energy demand and self-sufficiency rate by grey forecasting model and Markov model. International Journal of Electrical Power and Energy Systems, 2015, 66: 1–8.

[11] WANG Y H, LU J. Improvement and application of GM(1, 1) model based on multivariable dynamic optimization. Journal of Systems Engineering and Electronics, 2020, 31(3): 593–601.

[12] TANG X Z, XIE N M, HU A Q. Forecasting annual foreign tourist arrivals to China by incorporating firefly algorithm into fractional non-homogenous discrete grey model. Kybernetes, 2021. DOI:10.1108/K-11-2020-0787.

[13] SUN X, SUN W S, WANG J Z, et al. Using a Grey-Markov model optimized by Cuckoo search algorithm to forecast the annual foreign tourist arrivals to China. Tourism Management, 2016, 52: 369–379.

[14] HU Y C, JIANG P, LEE P. Forecasting tourism demand by incorporating neural networks into Grey-Markov models. The Journal of the Operational Research Society, 2019, 70(1): 12–20.

[15] ZHOU W, HE J M. Generalized GM(1, 1) model and its application in forecasting of fuel production. Applied Mathematical Modelling, 2013, 37(9): 6234–6243.

[16] XIAO X P, DUAN H M, WEN J H. A novel car-following inertia gray model and its application in forecasting short-term traffic flow. Applied Mathematical Modelling, 2020, 87: 546–570.

[17] DANG Y G, LIU S F, CHEN K J. The GM models that $x(n)$ be taken as initial value. Kybernetes, 2004, 33(2): 247–254.

[18] WANG Y H, DANG Y G, LI Y Q, et al. An approach to increase prediction precision of GM(1, 1) model based on optimization of the initial condition. Expert Systems with Applications, 2010, 37(8): 5640–5644.

[19] XIONG P P, DANG Y G, YAO T X, et al. Optimal modeling and forecasting of the energy consumption and production in China. Energy, 2014, 77: 623–634.

[20] DING S. A novel self-adapting intelligent grey model for forecasting China's natural-gas demand. Energy, 2018, 162: 393–407.

[21] ZHU X Y, DANG Y G, DING S. Using a self-adaptive grey fractional weighted model to forecast Jiangsu's electricity consumption in China. Energy, 2020, 190: 116417.

[22] ZENG B, LI C. Improved multi-variable grey forecasting model with a dynamic background-value coefficient and its application. Computers & Industrial Engineering, 2018, 118: 278–290.

[23] ZENG X Y, YAN S L, HE F L, et al. Multi-variable grey model based on dynamic background algorithm for forecasting the interval sequence. Applied Mathematical Modeling, 2020, 80: 99–114.

[24] LIN Y H, LEE P C, CHANG T P. Adaptive and high-precision grey forecasting model. Expert Systems with Applications, 2009, 36(6): 9658–9662.

[25] MA X, WU W Q, ZHANG Y Y. Improved GM(1, 1) model based on Simpson formula and its applications. Journal of Grey System, 2019, 31(4): 33–46.

[26] WU L F, LIU S F, YAO L G, et al. Grey system model with the fractional order accumulation. Communications in Nonlinear Science and Numerical Simulation, 2013, 18(7): 1775–1785.

[27] WU L F, LIU S F, CUI W, et al. Non-homogenous discrete grey model with fractional-order accumulation. Neural Computing and Applications, 2014, 25(5): 1215–1221.

[28] MAO S H, GAO M Y, XIAO X P, et al. A novel fractional grey system model and its application. Applied Mathematical Modelling, 2016, 40: 5063–5076.

[29] MA X, XIE M, WU W Q, et al. The novel fractional discrete multivariate grey system model and its applications. Applied Mathematical Modelling, 2019, 70: 402–424.

[30] TSAUR R. Forecasting analysis by using fuzzy grey regression model for solving limited time series data. Soft Computing, 2008, 12(11): 1105–1113.

[31] JIA Z, ZHOU Z F, ZHANG H J, et al. Forecast of coal consumption in Gansu province based on Grey-Markov chain model. Energy, 2020, 199: 117444.

[32] HSU Y T, LIU M C, YEH J, et al. Forecasting the turning time of stock market based on Markov-Fourier grey model. Expert Systems with Applications, 2009, 36(4): 8597–8603.

[33] HU Y C. Forecasting tourism demand using fractional grey prediction models with Fourier series. Annals of Operations Research, 2021, 300(2): 467–491.

[34] FAN C, JIAO Y J, GUO Y F, et al. Forecasting the Chinese grain yield using hybrid ARIMA and error-corrected grey model base on the wavelet transform. Journal of Grey System, 2019, 31(4): 97–110.

[35] YAO T, WANG Z H. Crude oil price prediction based on LSTM network and GM (1, 1) model. Grey Systems: Theory and Application, 2020, 11(1): 80–94.

[36] MOONCHAI S, CHUTSAGULPROM N. Short-term forecasting of renewable energy consumption: augmentation of a modified grey model with a Kalman filter. Applied Soft Computing, 2020, 87: 105994.

[37] LI K W, LIU L, ZHAI J N, et al. The improved grey model based on particle swarm optimization algorithm for time series prediction. Engineering Applications of Artificial Intelligence, 2016, 55: 285–291.

[38] YANG X S. Nature-inspired metaheuristic algorithms. Cambridge: Luniver Press, 2008.

[39] XIAO L Y, SHAO W, LIANG T L, et al. A combined model based on multiple seasonal patterns and modified firefly algorithm for electrical load forecasting. Applied Energy, 2016, 167: 135–153.

[40] IBRAHIM I A, KHATIB T. A novel hybrid model for hourly global solar radiation prediction using random forests technique and firefly algorithm. Energy Conversion and Management, 2017, 138: 413–425.

[41] SOBOL'I M. Sensitivity analysis for non-linear mathematical models. Mathematical Modelling and Computational Experiment, 1993, 1(4): 407–414.

[42] ZHANG X Y, TRAME M N, LESKO L J, et al. Sobol sensitivity analysis: a tool to guide the development and evaluation of systems pharmacology models. CPT Pharmacometrics & Systems Pharmacology, 2015, 4: 69–79.

[43] NARIMAN N A, HUSSAIN R R, MOHAMMAD I I, et al. Global sensitivity analysis of certain and uncertain factors for a circular tunnel under seismic action. Frontiers of Structural and Civil Engineering, 2019, 13: 1289−1300.

[44] XIE N M, LIU S F. Discrete grey forecasting model and its optimization. Applied Mathematical Modelling, 2009, 33(2): 1173–1186.

[45] Anhui Statistical Bureau, NBS Survey Office in Anhui. Anhui Statistical Yearbook-2013. Beijing: China Statistics Press,2013. (in Chinese)

[46] Anhui Statistical Bureau, NBS Survey Office in Anhui. Anhui Statistical Yearbook-2015. Beijing: China Statistics Press,2015. (in Chinese)

[47] Anhui Statistical Bureau, NBS Survey Office in Anhui. Anhui Statistical Yearbook-2017. Beijing: China Statistics Press,2017. (in Chinese)

[48] Anhui Statistical Bureau, NBS Survey Office in Anhui. Anhui Statistical Yearbook-2019. Beijing: China Statistics Press,2019. (in Chinese)

[49] Anhui Statistical Bureau, NBS Survey Office in Anhui. Anhui Statistical Yearbook-2020. Beijing: China Statistics Press,2020. (in Chinese)

Biographies

TANG Xiaozhong was born in 1985. He received his B.S. degree from Huangshan University in 2008, and M.S. degree from Shenyang University of Technology in 2012. Now, he is an associate professor at Huang shan Vocational and Technical College. He has been pursuing his Ph.D. degree in Nanjing University of Aeronautics and Astronautics since 2018. His research interests include grey system theory and prediction modeling algorithm.

E-mail: tangxz1985@163.com

XIE Naiming was born in 1981. He received his B.S., M.S., and Ph.D. degrees in grey systems theory from Nanjing University of Aeronautics and Astronautics (NUAA). Now he is a professor in the College of Economics and Management of NUAA and an associate editor of Grey Systems: Theory and Application. His research interests include Grey Systems theory, production scheduling, and control.

E-mail: xienaiming@nuaa.edu.cn