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A dynamic condition-based maintenance optimization model for mission-oriented system based on inverse Gaussian degradation process

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Abstract: An effective maintenance policy optimization model can reduce maintenance cost and system operation risk. For mission-oriented systems, the degradation process changes dynamically and is monotonous and irreversible. Meanwhile, the risk of early failure is high. Therefore, this paper proposes a dynamic condition-based maintenance (CBM) optimization model for mission-oriented system based on inverse Gaussian (IG) degradation process. Firstly, the IG process with random drift coefficient is used to describe the degradation process and the relevant probability distributions are obtained. Secondly, the dynamic preventive maintenance threshold (DPMT) function is used to control the early failure risk of the mission-oriented system, and the influence of imperfect preventive maintenance (PM) on the degradation amount and degradation rate is analysed comprehensively. Thirdly, according to the mission availability requirement, the probability formulas of different types of renewal policies are obtained, and the CBM optimization model is constructed. Finally, a numerical example is presented to verify the proposed model. The comparison with the fixed PM threshold model and the sensitivity analysis show the effectiveness and application value of the optimization model.

Keywords: inverse Gaussian (IG) process, imperfect preventive maintenance (PM), mission-oriented system, dynamic preventive maintenance threshold (DPMT), maintenance optimization.

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1. Introduction

A mission-oriented system is a system that, at different time during its lifetime, performs a mission with fixed duration [1]. Examples of such systems can be found in military systems such as the avionics portion of an airborne weapon system and in manufacturing equipment such as the manipulator arm working in production lines [2]. The performance of these systems degrades during mission execution. Different from the general system, the maintenance of the mission-oriented system cannot be carried out during the mission, and can only be carried out when the mission is completed or aborted. In addition, mission-oriented systems usually have to meet availability/reliability requirements during missions. Establishing a model of the system degradation process and formulating a reasonable maintenance policy are beneficial to ensuring the mission success. Zhao et al. [3,4] and Qiu et al. [5,6] respectively optimized the mission abort policies according to the degradation level and mission duration within the framework of different degradation processes to improve mission reliability and system survivability.

Practice shows that traditional maintenance modes (such as breakdown maintenance, periodic maintenance) tend to cause excessive or insufficient maintenance, and the maintenance efficiency is low. The appearance of condition-based maintenance (CBM) provides an opportunity to solve the above problems. It can formulate policies such as preventive maintenance (PM) or corrective maintenance (CM) based on the degradation state information and preset threshold of mission-oriented system with detection conditions. The advantages of CBM have attracted widespread attention from scholars, and it has been successfully applied to key areas such as industrial production [7–9] and national defense [10,11]. The content of CBM mainly includes three parts, that is, status data collection, remaining useful life (RUL) prediction and maintenance policy optimization.

At present, most research on CBM focuses on RUL prediction methods [12–16], but the research on maintenance policy optimization is still insufficient [17–19]. The degradation model based on CBM monitoring data is an important basis for maintenance policy optimization. Stochastic process takes into account the random disturb-

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ance of various factors in the system degradation process and is close to the actual operation of the system, so it is often used to model the degradation process [20].

Wiener process and Gamma process are the most concerned processes in the stochastic process model with continuous state, because they have good mathematical properties and clear physical explanations [20,21]. For example, Ma et al. [22] studied the reliability analysis and CBM optimization methods of dual-unit cooler, used multi-level Wiener process to characterize the degradation trend of the system, determined the reliability function of the system, and finally formulated a maintenance policy based on temperature monitoring information. Wang et al. [23] described equipment performance degradation characteristics through nonlinear Wiener process, introduced an uncertain failure threshold, predicted RUL of the equipment, and realized the optimal maintenance decision. However, a significant feature of Wiener process is that its degradation path is not necessarily monotonic, which loses its meaning in some degradation descriptions, because the degradation of many systems is irreversible. For this monotonic degradation, Gamma process is usually used for modeling. Phuc et al. [24] proposed a CBM policy based on Gamma degradation process. This model aimed to determine the best maintenance measure (perfect and imperfect) and inspection interval. Caballe et al. [25] proposed a CBM maintenance policy for system suffering from internal degradation and external sudden shocks. The internal degradation process is described by Gamma process.

Another stochastic process model which can describe the monotonic degradation process is the inverse Gaussian (IG) process proposed by Wasan [26]. The application of IG process in degradation was introduced by Wang et al. [27], and Ye et al. [21] further studied its physical meaning to support its application in engineering practice.ComparedwithGammaprocess,IGprocessismoreflexible in incorporating random effects and covariates, and can better explain the heterogeneity between different components and systems. For example, Chen et al. [20] proposed an IG process degradation model with random effects, and the objective function was to minimize the total operating cost including inspection costs, preventive or corrective replacement costs, and downtime costs to find the optimal inspection interval and maintenance policy. Wu et al. [9] proposed a dynamic CBM model based on IG process to solve the problem of equipment maintenance decision-making with dynamic degradation characteristics. The IG process incorporated random parameters. Finally, the multi-objective function was solved to obtain the optimal policy. Although the IG process has gradually attracted the attention of researchers, there are still few studies on the CBM model based on the IG process.

On the other hand, in engineering practice, the structure of the mission-oriented system is generally complicated, and the maintenance actions implemented on it are mostly imperfect, so the system can only be restored to a certain state between an as-good-as new state and an as-bad-as old state [28,29]. Therefore, some scholars have begun to consider the influence of imperfect maintenance in the study of CBM optimization. Zhang et al. [30] used stochastic improvement factor to reflect the impact of maintenance actions on the degradation rate of the system in the process of studying imperfect maintenance model using stochastic process. Liu et al. [31] used the delay time model to describe the degradation process, and used the service age regression factor to describe the impact of imperfect maintenance on the system life. Guo et al. [10] proposed a residual damage model to consider the influence of imperfect PM on the degradation amount. Although the above studies have achieved certain effects in maintenance policy optimization considering imperfect maintenance, they ignore the fact that imperfect maintenance affects both the degradation amount and degradation rate. Therefore, two types of effects must be taken into consideration in the modeling process [32]. In addition, the degradation process parameters in the degradation model proposed by Guo et al. [10] are fixed, so that the introduction of uncertainty is ignored. Its decision variable is also a fixed maintenance threshold, which is not conducive to reducing the risk of early failure of the system. Therefore, the dynamic adjustment of maintenance thresholds in different degradation stages is also a problem worthy of study.

In view of these practical needs, we develop a dynamic CBM optimization model for mission-oriented system based on IG degradation process. The main contributions of this paper are summarized as follows: Firstly, an IG process degradation model with random drift coefficients is established, and the analytical probability distributions of the correlation functions are derived in the sense of first hitting time, which solves the problem of describing the dynamic degradation process of such systems. Then, we propose a dynamic preventive maintenance threshold (DPMT) function to control the early failure risk of mission-oriented system. And we comprehensively consider the influence of imperfect PM on the degradation amount and degradation rate. Finally, according to the system evolution process and mission availability requirement, the probability formula for executing the related maintenance policy is obtained, and the optimal long-term expected cost ratio and the decision variables in DPMT are obtained by using the optimization algorithm. The experimental results show that the model proposed in this paper is more economical than the fixed PM threshold model. Mission requirement is a key factor that must be considered in the formulation of maintenance policies for such systems.

The remainder of this paper is organized as follows: Section 2 introduces the degradation model based on the IG process. Section 3 describes the system evolution process, proposes DPMT, and analyses the influence of imperfect PM. Section 4 establishes the CBM optimization model. Section 5 provides a numerical example to verify the effectiveness and application value of the proposed model. Section 6 draws the conclusion of this paper.

2. Degradation model based on IG process

In actual mission, the degradation of some mission-oriented systems is usually an irreversible damage accumulation. As a monotonous stochastic process, IG process can not only describe the characteristics of such systems, but also have more advantages in introducing random effects. Therefore, we build a model based on IG process.

2.1 IG process with random drift coefficient

Let $X(t)$ denote the degradation state of the system at time t, and assume that the degradation process $\{X(t), t \geqslant 0\}$ obeys IG distribution, i.e.,

$$X(t) \sim \mathrm{IG}(\beta\Lambda(t), \eta[\Lambda(t)]^2)$$

where $\beta, \eta > 0$ respectively represent the drift coefficient and diffusion coefficient of IG distribution, reflecting the rate and fluctuation of the degradation process. $\Lambda(t)$ represents the time scale function and monotonically increasing. According to the convention, $\Lambda(0) = 0$, and we use the linear form $\Lambda(t) = t$ to explain.

In addition to monotonicity, IG process also has the following properties:

Property 1 $X(t)$ has independent increments, that is, for $\forall t_2 > t_1 \geqslant t_2' > t_1'$, $X(t_2) - X(t_1)$ and $X(t_2') - X(t_1')$ are independent.

Property 2 $X(t) - X(s)$ obeys IG distribution

$$\mathrm{IG}(\beta[\Lambda(t) - \Lambda(s)], \eta[\Lambda(t) - \Lambda(s)]^2), \ \forall t > s \geqslant 0.$$

When the system is performing a mission, it is often affected by factors such as the environment. Some equipment and components have a high early failure rate and great uncertainty. As the system operates, its degradation fluctuation gradually become stable [33,34]. Therefore, the immobilization of the parameters of IG process deviates from the actual situation, which is not conducive to describing the dynamic changes of the system degradation characteristics. Pan et al. [35] proposed an IG process model with random effect parameters to estimate RUL. Wu et al. [9] extended the dynamic change of the drift coefficient to be related to time $\mathrm{IG}(\beta[\Lambda(t) - \Lambda(s)], \eta[\Lambda(t) - \Lambda(s)]^2), \forall t > s \geqslant 0$. Based on this, we propose an IG process with random drift coefficients, which is used to derive the probability distributions of the mission-oriented system and build the CBM optimization model. Details are as follows:

$$\begin{cases} \dfrac{1}{\beta} = \mu_\beta + \dfrac{B(t)}{\sigma_\beta} \\ X(t) \sim \mathrm{IG}(\beta t, \eta t^2) \end{cases} \tag{1}$$

where $B(t)$ is a standard Brownian motion, μ_β and σ_β respectively represent the mean parameter and the variance parameter of $1/\beta$. $1/\beta$ in (1) can also be expressed as $1/\beta \sim \mathrm{N}(\mu_\beta, t/\sigma_\beta^2)$. It can be found that the variance $D(1/\beta) = t/\sigma_\beta^2$ of $1/\beta$ increases with time t. Therefore, the variance $D(\beta)$ of β decreases with the increase of time t. That is, the drift coefficient gradually stabilizes, which conforms to the description of the mission-oriented system above. The detailed proof of the above conclusion can be found in [9].

2.2 Correlation probability distribution

In many engineering applications, when the degradation first reaches a predetermined threshold level, the system is considered to be malfunctioning [36]. Therefore, the system life can be defined according to the first hitting time. The probability distributions of the first hitting time play a key role in deriving RUL and determining the optimal CBM policy. Let the PM threshold be ω_p and the failure threshold be ω_f, and the time when the system first reaches ω_p and ω_f from time zero is respectively defined as

$$T_p = \inf\{t | X(t) \geqslant \omega_p, t \geqslant 0\}, \tag{2}$$

$$T_f = \inf\{t | X(t) \geqslant \omega_f, t \geqslant 0\}. \tag{3}$$

According to the monotonicity of the IG process, when β is fixed, the conditional cumulative distribution function (CDF) of T_f is obtained as

$$\begin{aligned} F_{T_f|\beta}(t|\beta) &= P(T_f \leqslant t|\beta) = P(X(t) \geqslant \omega_f|\beta) = \\ &1 - F_{\mathrm{IG}}(\omega_f|\beta t, \eta t^2) = \Phi\left[\sqrt{\frac{\eta}{\omega_f}}\left(t - \frac{\omega_f}{\beta}\right)\right] - \\ &\exp\left(\frac{2\eta t}{\beta}\right)\Phi\left[-\sqrt{\frac{\eta}{\omega_f}}\left(t + \frac{\omega_f}{\beta}\right)\right] \end{aligned} \tag{4}$$

where $F_{\mathrm{IG}}(\cdot)$ represents the conditional CDF of IG distribution, and $\Phi(\cdot)$ represents the CDF of the standard normal distribution. By taking the derivative of t, the conditional probability density function (PDF) of T_f is obtained as

$$f_{T_f|\beta}(t|\beta) = 2\sqrt{\frac{\eta}{\omega_f}}\phi\left[\sqrt{\frac{\eta}{\omega_f}}\left(t-\frac{\omega_f}{\beta}\right)\right]- \frac{2\eta}{\beta}\exp\left(\frac{2\eta t}{\beta}\right)\Phi\left[-\sqrt{\frac{\eta}{\omega_f}}\left(t+\frac{\omega_f}{\beta}\right)\right] \tag{5}$$

where $\phi(\cdot)$ represents the PDF of the standard normal distribution.

The above $F_{T_f|\beta}(t|\beta)$ is the conditional CDF of T_f with respect to β. According to the total probability formula, the following formula can be obtained as

$$F_{T_f}(t) = \int_\Omega F_{T_f|\beta}(t|\beta)f(\beta)\mathrm{d}\beta = E_\beta\left[F_{T_f|\beta}(t|\beta)\right] \tag{6}$$

where $f(\beta)$, Ω, and $E_\beta[\cdot]$ represent the PDF, the parameter space, and the mathematical expectation of the random effect parameter β, respectively. According to Lemma 1 in [35], the integral of (6) can be calculated explicitly.

Lemma 1 If $Z\sim \mathrm{N}(\mu,\sigma^2)$ and $A,B,D\in\mathbf{R}$, the following results hold:

$$E_Z[\exp(AZ)\Phi(B+DZ)] = \exp\left(A\mu+\frac{A^2}{2}\sigma^2\right)\Phi\left(\frac{B+D\mu+AD\sigma^2}{\sqrt{1+D^2\sigma^2}}\right). \tag{7}$$

With $1/\beta\sim \mathrm{N}(\mu_\beta,t/\sigma_\beta^2)$, we substitute (4) into (6), and then use Lemma 1 to get the analytical expression of (6) as

$$F_{T_f}(t) = \Phi\left(\sqrt{\frac{\eta}{\omega_f}}\cdot\frac{\sigma_\beta t-\mu_\beta\sigma_\beta\omega_f}{\sqrt{\sigma_\beta^2+\eta\omega_f t}}\right)-\exp\left(2\eta\mu_\beta t+\frac{2\eta^2t^3}{\sigma_\beta^2}\right)\cdot \Phi\left(-\sqrt{\frac{\eta}{\omega_f}}\cdot\frac{(\sigma_\beta^2+2\eta\omega_f t)t+\mu_\beta\sigma_\beta^2\omega_f}{\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2\omega_f t}}\right). \tag{8}$$

Using the derivative of (8) with respect to t, the expression of $f_{T_f}(t)$ can be obtained as

$$f_{T_f}(t) = \sqrt{\frac{\eta}{\omega_f}}\cdot\frac{2\sigma_\beta^3+5\eta\omega_f\sigma_\beta t+3\eta^2\omega_f^2t^2/\sigma_\beta}{\left(\sqrt{\sigma_\beta^2+\eta\omega_f t}\right)^3}\cdot \phi\left(\sqrt{\frac{\eta}{\omega_f}}\cdot\frac{\sigma_\beta t-\mu_\beta\sigma_\beta\omega_f}{\sqrt{\sigma_\beta^2+\eta\omega_f t}}\right)-\left(2\eta\mu_\beta+\frac{6\eta^2t^2}{\sigma_\beta^2}\right)\cdot \Phi\left(-\sqrt{\frac{\eta}{\omega_f}}\cdot\frac{(\sigma_\beta^2+2\eta\omega_f t)t+\mu_\beta\sigma_\beta^2\omega_f}{\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2\omega_f t}}\right)\cdot \exp\left(2\eta\mu_\beta t+\frac{2\eta^2t^3}{\sigma_\beta^2}\right). \tag{9}$$

At the same time, according to (8), the CDF of the degradation process $\{X(t),t\geqslant 0\}$ can be further deduced in reverse as

$$F_{\mathrm{IG}}(x;t) = \Phi\left(\sqrt{\frac{\eta}{x}}\cdot\frac{\mu_\beta\sigma_\beta x-\sigma_\beta t}{\sqrt{\sigma_\beta^2+\eta xt}}\right)+\exp\left(2\eta\mu_\beta t+\frac{2\eta^2t^3}{\sigma_\beta^2}\right)\cdot \Phi\left(-\sqrt{\frac{\eta}{x}}\cdot\frac{(\sigma_\beta^2+2\eta xt)t+\mu_\beta\sigma_\beta^2x}{\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2xt}}\right). \tag{10}$$

By further taking the derivative of x, the PDF of the degradation process $\{X(t),t\geqslant 0\}$ can be obtained as

$$f_{\mathrm{IG}}(x;t) = \frac{1}{2}\sqrt{\frac{\eta}{x}}\left[\phi\left(\sqrt{\frac{\eta}{x}}\cdot\frac{\mu_\beta^i\sigma_\beta x-\sigma_\beta t}{\sqrt{\sigma_\beta^2+\eta xt}}\right)\cdot \frac{\mu_\beta^i\sigma_\beta^3+\sigma_\beta^3t/x+2\sigma_\beta\eta t^2}{\left(\sqrt{\sigma_\beta^2+\eta xt}\right)^3}+\exp\left(2\eta\mu_\beta^i t+\frac{2\eta^2t^3}{\sigma_\beta^2}\right)\cdot \phi\left(-\sqrt{\frac{\eta}{x}}\cdot\frac{(\sigma_\beta^2+2\eta xt)t+\mu_\beta^i\sigma_\beta^2x}{\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2xt}}\right)\cdot\frac{\sigma_\beta^6t/x-\mu_\beta^i\sigma_\beta^6}{\left(\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2xt}\right)^3}\right]. \tag{11}$$

The life distribution function of the system is obtained by (8). Let $X_{0:k}=\{x_0,x_1,\cdots,x_k\}$ denote the degradation data obtained at time $0=t_0<t_1<\cdots<t_k$, where $x_k=X(t_k)$ denotes the degradation state of the system at time t_k. Based on the failure threshold ω_f, the RUL of the system at time t_k is defined as

$$R_k = \inf\{r_k|X(t_k+r_k)\geqslant\omega_f, r_k\geqslant 0\}. \tag{12}$$

According to the independent increments property and homogeneous Markov property, we have

$$R_k = \inf\{r_k|X(t_k+r_k)\geqslant\omega_f, r_k\geqslant 0\} = \inf\{r_k|X(t_k+r_k)-X(t_k)\geqslant\omega_f-x_k, r_k\geqslant 0\} = \inf\{r_k|X(r_k)\geqslant\omega_f-x_k, r_k\geqslant 0\}. \tag{13}$$

Using the conclusion of (8), we can obtain

$$F_{R_k}(r_k) = \Phi\left(\sqrt{\frac{\eta}{\omega_f-x_k}}\cdot\frac{\sigma_\beta r_k-\mu_\beta\sigma_\beta(\omega_f-x_k)}{\sqrt{\sigma_\beta^2+\eta(\omega_f-x_k)r_k}}\right)- \exp\left(2\eta\mu_\beta r_k+\frac{2\eta^2r_k^3}{\sigma_\beta^2}\right)\cdot\Phi\left(-\sqrt{\frac{\eta}{\omega_f-x_k}}\cdot \frac{(\sigma_\beta^2+2\eta(\omega_f-x_k)r_k)r_k+\mu_\beta\sigma_\beta^2(\omega_f-x_k)}{\sqrt{\sigma_\beta^4+\eta\sigma_\beta^2(\omega_f-x_k)r_k}}\right). \tag{14}$$

3. Dynamic CBM policy and the influence of imperfect PM

Most of the current maintenance policy optimization methods often start from the inherent degradation performance of the system, and rarely consider the impact of the mission on the maintenance policy. During the execution of the mission-oriented system, when the system degradation state reaches the relevant threshold, PM cannot be executed immediately. In addition, PM is usually imperfect, i.e., PM cannot restore the system as new. Based on the above analysis, we describe the system evolution process and discuss the impact of imperfect PM in this section.

3.1 Assumptions

(i) The duration of each mission is a constant greater than zero, denoted by τ. At the end of the mission, the system is inspected perfectly and a fixed cost C_I is incurred. The inspection time is ignored.

(ii) If $X(t) \geqslant \omega_f$ is satisfied during a mission, the mission fails and the CM is executed immediately to restore the system to the same state as the new system, resulting in fixed costs. A fixed cost C_C is incurred.

(iii) The PM action is imperfect. It cannot restore the system to its original state, which affects both the degradation amount and degradation rate.

(iv) If it is inspected that the system state satisfies $\omega_p \leqslant X(t) < \omega_f$ at the end of the mission, PM is executed immediately before the next mission. The unit time cost of PM is C_P, and the maintenance time cannot be ignored. In practice, as the degree of degradation increases, the expectation of each imperfect PM duration shows an increasing trend.

(v) If $X(t) < \omega_p$ is satisfied at the end of the mission, the system continues to operate without performing any operations.

(vi) When the system operating time reaches $U\tau$, a preventive replacement (PR) will be performed to restore the system to a new state and a fixed cost C_R is incurred. $U \in \mathbf{Z}^+$ denotes the maximum number of missions the system can perform.

(vii) Before the system fails or reaches $U\tau$, if the system availability constraint is violated, PR needs to be performed and the same cost C_R is incurred.

(viii) All renewals (including both CM and PR) consume the same amount of time.

(ix) The PM threshold ω_p is a binary function of the number of missions and the number of PM, rather than a fixed value, i.e., the system has different values of ω_p at different times.

3.2 System evolution process

Fig. 1 visualizes the mission process and renewals of the system. $\{W_j\}_{j\in\mathbf{N}^+}$ is a renewal sequence, W_j denotes the duration of the jth renewal cycle. $T_i^{(j)}$ denotes the ith PM interval time of the jth renewal cycle. $M_i^{(j)}$ denotes the duration of the ith PM action in the jth renewal cycle. ξ denotes the duration of each renewal. $t_{i,j}$ and $t_{i,j}^*$ respectively denote the start time and end time of the ith PM action in the jth renewal cycle. $t_{0,j}$ denotes the initial time of the first mission in the jth renewal cycle. t_j denotes the initial time of the jth renewal action.

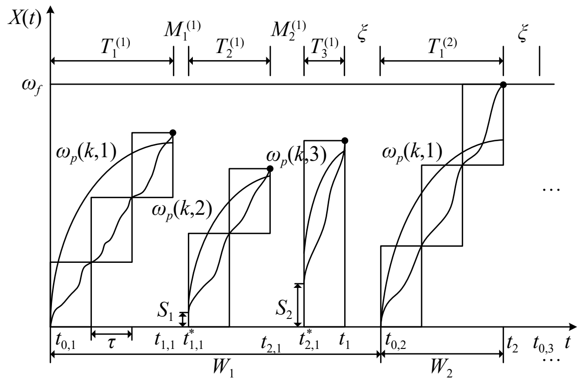


Fig. 1 The system evolution process

As shown in Fig. 1, we can see that after each imperfect PM, the degradation process of the system will start from a certain non-zero value (S_1, S_2) and gradually increase $(S_2 > S_1)$. This value will not return to zero until it is renewed. S_1 and S_2 are called residual damage, which will shorten the PM interval time.

3.3 Definition of DPMT

Most researches on maintenance policy assume that the PM threshold ω_p is fixed. However, many systems have greater uncertainty and higher failure rates during early operation. If ω_p adopts a higher level and is fixed, the early risk cost of the system to perform missions will increase [37]. Although the monotone control limit policy proposed in [20] and [38] can help reduce the risk of early failure, it lacks a unified mathematical form for description, which is not conducive to actual operation. Therefore, we propose a DPMT function $\omega_p(k,i)$ which is monotonous and non-decreasing with operation time. The expression is as follows:

$$\omega_p(k,i) = g(k,i;\theta) = c - \frac{1}{\int_0^k g_1(h_1;a\tau)\,\mathrm{d}h_1 + \int_0^i g_2(h_2;b)\,\mathrm{d}h_2} \tag{15}$$

where $\omega_p(k,i)$ denotes the DPMT after the kth mission in the ith PM cycle. $\theta = (a\tau, b, c)$ is the unknown parameter set, $g(k,i;\theta)$ is the general functional form of k and i. g_1, g_2 can be linear or non-linear function of k, i, respectively. In addition, $c < \omega_f$, this is because if c is set to ω_f, the system delays PM and increases the risk of failure, which loses the meaning of PM. In this paper, we take the simplest linear form as an example to illustrate. The analytical expression of $\omega_p(k,i)$ is as follows:

$$\omega_p(k,i) = c - \frac{1}{(a\tau k + bi)}, \quad a,b > \frac{1}{c} \tag{16}$$

where a, b, and c are variable parameters, $c < \omega_f$.

It can be seen from (16) that ω_p is relatively conservative in the initial stage of system operation, which achieves the purpose of controlling early failures. After multiple missions, ω_p gradually increases, it will ensure the effective mission time of the system and avoid excessive maintenance.

3.4 PM duration model

The length of PM duration usually depends on the initial system state, the PM threshold and the overall evolution of the system state. In fact, a severely degraded system will take longer to maintain than a slightly degraded one.

Let $M_{k,i}$ denote the duration of the ith PM. For any $i \neq j$, $M_{k,i}$ is independent of $M_{k,j}$. Based on the expectation model of $M_{k,i}$ in [39], we propose an improved model based on DPMT. Details are as follows:

$$\mathrm{E}[M_{k,i}] = \gamma\omega_p(k,i)\exp(i\lambda\omega_p(k,i)) \tag{17}$$

where $\gamma > 0$, $\lambda \geqslant 0$, $k = 0,1,\cdots$, $i = 1,2,\cdots$, and $M_{k,0} = 0$. Assume that the parameters γ and λ are independent of $\omega_p(k,i)$. It can be found that the expectation of $M_{k,i}$ increases with the increase of the PM threshold $\omega_p(k,i)$ and the number of PM, and the improvement result is more realistic.

3.5 Influence of imperfect PM

In engineering practice, the imperfect PM performed on the mission-oriented system will affect both the degradation amount and the degradation rate, which is specifically manifested in the change of residual damage and random drift coefficient.

(i) Residual damage

According to the DPMT function proposed in this paper and the residual damage model proposed in [10], the PDF of residual damage S_i is obtained as

$$f(s_i) = \begin{cases} \dfrac{\alpha^{i-1}\varepsilon}{(1-\exp(-\alpha^{i-1}\varepsilon))\omega_p(k,i)} \cdot \exp\left(-\dfrac{\alpha^{i-1}\varepsilon(\omega_p(k,i)-s_i)}{\omega_p(k,i)}\right), \\ s_i \in [0, \omega_p(k,i)] \\ 0, \quad \text{others} \end{cases} \tag{18}$$

where S_i is a simplified representation of $X(t^*_{i,j})$ and $S_0 = 0$. In addition, $\alpha > 1$, $\varepsilon > 0$.

Therefore, the mathematical expectation of S_i is

$$\mathrm{E}[S_i] = \int_0^{\omega_p(k,i)} s_i f(s_i)\mathrm{d}s_i = \frac{\omega_p(k,i)}{1-\exp(-\alpha^{i-1}\varepsilon)} \cdot \left(1 - \frac{1-\exp(-\alpha^{i-1}\varepsilon)}{\alpha^{i-1}\varepsilon}\right). \tag{19}$$

It can be proved that $\mathrm{E}[S_i]$ will increase monotonously with the number of PM cycles [10]. Therefore, the existence of residual damage leads to a limited number of missions in each renewal cycle.

(ii) Random drift coefficient

Since we adopt the random drift coefficient described in (1), it is considered that the influence of imperfect PM on the degradation rate is reflected in the parameter μ_β of the random drift coefficient β, i.e., an update factor of μ_β will be generated after each PM. Therefore, the update formula of μ_β is

$$\mu_\beta^i = v^i \mu_\beta^0 \tag{20}$$

where μ_β^i denotes the value of μ_β after the ith PM cycle, v denotes the update factor of μ_β, μ_β^0 denotes the initial value of μ_β. At the beginning of a new renewal cycle, μ_β^i returns to μ_β^0.

4. CBM optimization model and solution

The CBM policy for a mission-oriented system is usually affected by the requirements of mission availability. In order to make the optimization model more in line with the actual mission, it is necessary to define the mission availability, and then determine the probability of executing PM after mission inspection according to the system evolution process, and the probability of the renewal cycle ending with different types of policies. After that, the mathematical expectation of related cost and time can be obtained. Finally, we construct the CBM optimization model in combination with the constraint conditions such as mission availability.

4.1 Definition of mission availability

Let $A1[i]$ denotes the mission availability of the ith PM cycle in each renewal cycle, where the ith PM cycle is composed of the ith PM interval time and the $(i-1)$th PM duration. The expression of $A1[i]$ is as follows:

$$A1[i] = \frac{\mathrm{E}[T_i]}{\mathrm{E}[T_i] + \mathrm{E}[M_{k,i-1}]} = \frac{\mathrm{E}[\mathrm{E}[T_i|S_{i-1}]]}{\mathrm{E}[\mathrm{E}[T_i|S_{i-1}]] + \mathrm{E}[M_{k,i-1}]} \tag{21}$$

where

$$\mathrm{E}[\mathrm{E}[T_i|S_{i-1}]] = \int_0^{\omega_p(k,i)} \left(\int_0^{U\tau} t f_{T_p}(t|s_{i-1})\mathrm{d}t\right) \cdot f(s_{i-1})\mathrm{d}s_{i-1} = \int_0^{\omega_p(k,i)} \left(tF_{T_p}(t|s_{i-1})|_0^{U\tau} - \int_0^{U\tau} F_{T_p}(t|s_{i-1})\mathrm{d}t\right) f(s_{i-1})\mathrm{d}s_{i-1}. \tag{22}$$

According to (8) and (14), we can obtain

$$F_{T_p}(t|s_{i-1}) = P(T_p < t|s_{i-1}) = \Phi\left(\sqrt{\frac{\eta}{\omega_p - s_{i-1}}}\cdot\frac{\sigma_\beta t - \mu_\beta^i\sigma_\beta(\omega_p - s_{i-1})}{\sqrt{\sigma_\beta^2 + \eta(\omega_p - s_{i-1})t}}\right) - \exp\left(2\eta\mu_\beta^i t + \frac{2\eta^2 t^3}{\sigma_\beta^2}\right)\Phi\left(-\sqrt{\frac{\eta}{\omega_p - s_{i-1}}}\cdot \frac{(\sigma_\beta^2 + 2\eta(\omega_p - s_{i-1})t)t + \mu_\beta^i\sigma_\beta^2(\omega_p - s_{i-1})}{\sqrt{\sigma_\beta^4 + \eta\sigma_\beta^2(\omega_p - s_{i-1})t}}\right) \quad (23)$$

where ω_p is the simplified form of $\omega_p(k,i)$.

For the mission-oriented system, according to the assumption (vii), if the mission availability $A1[i]$ of the PM cycle in the lifetime $U\tau$ is reduced to the limit level ζ, PR is be performed to restore the system to a new state.

4.2 PM probability model

The evolution of $X(t)$ is a renewal process. If the system state satisfies $X((k-1)\tau) < \omega_p(k-1,i)$ and $\omega_p(k,i) \leqslant X(k\tau) < \omega_f$ in the ith PM cycle, PM will be performed after the kth mission. The probability of this event under the condition that the residual damage is S_{i-1} is

$$P_{\rm PM}(k,i) = E_{S_{i-1}}[P(X((k-1)\tau) + S_{i-1} < \omega_p(k-1,i) \cap \omega_p(k,i) \leqslant X(k\tau) + S_{i-1} < \omega_f)] = \int_0^{\omega_p(k,i)}((1 - F_{T_p}((k-1)\tau|s_{i-1})) \int_0^{\omega_p(k,i)-s_{i-1}}\left(F_{\omega_p(k,i)-x}(\tau;x|s_{i-1}) - F_{\omega_f-x}(\tau;x|s_{i-1})\right)\cdot f_{\rm IG}(x;(k-1)\tau)\,{\rm d}x)f(s_{i-1}){\rm d}s_{i-1}. \quad (24)$$

For the expressions of $F_{\omega_p(k,i)-x}(t;x|s_{i-1})$ and $F_{\omega_f-x}(t;x|s_{i-1})$, it only needs to replace $(\omega_p - s_{i-1})$ in (23) with $(\omega_p - x - s_{i-1})$ and $(\omega_f - x - s_{i-1})$ respectively, which will not be shown in detail here. The detailed proof of (24) can be referred to [10].

4.3 Three types of renewal cycles and related probability models

The end types of the system renewal cycle can be divided into three types: the first type ends with CM, the second type ends with the system operating time reaching $U\tau$, and the third type ends with availability constraint.

(i) Renewal cycle ending with CM

Assume that the system has i $(i = 1,2,\cdots)$ PM cycles before CM. $k_i \in \mathbf{N}^+$ represents the number of missions in the i th PM cycle. The system fails randomly in the $((k_{i+1}-1)\tau, k_{i+1}\tau]$ interval of the $(i+1)$th PM cycle. In addition, the renewal cycle ending with CM needs to satisfy $A1[i] > \zeta$ for each PM cycle, otherwise the system evolution process is invalid. Based on (24), the probability that the renewal cycle ends with a given combination k_1, $k_2,\cdots,k_{i+1}$ is

$$P_1(i) = P_{\rm CM}(k_{i+1}, i+1)\prod_i P_{\rm PM}(k_i,i)I(A1[i]) \quad (25)$$

where $P_{\rm CM}(k_{i+1},i+1)$ is the probability of failure in the $((k_{i+1}-1)\tau, k_{i+1}\tau]$ interval of the $(i+1)$th PM cycle, the detailed proof of which is similar to (24). $I(A1[i])$ is an indicator function. The expressions of them are respectively

$$P_{\rm CM}(k_{i+1},i+1) = E_{S_i}[P(X((k_{i+1}-1)\tau) + S_i < \omega_p(k_{i+1}-1,i+1) \cap X(k_{i+1}\tau) + S_i \geqslant \omega_f)] = \int_0^{\omega_p(k_{i+1},i+1)}\left(\left(1 - F_{T_p}((k_{i+1}-1)\tau|s_i)\right)\cdot \int_0^{\omega_p(k_{i+1},i+1)-s_i} F_{\omega_f-x}(\tau;x|s_i)f_{IG}(x;(k_{i+1}-1)\tau){\rm d}x\right)f(s_i){\rm d}s_i, \quad (26)$$

$$I(A1[i]) = \begin{cases} 1, & A1[i] > \zeta \\ 0, & \text{others} \end{cases}. \quad (27)$$

In particular, when $i = 0$, i.e., no PM action is performed before the system failure, $P_1(0) = P_{\rm CM}(k_1,1)$.

Therefore, the expected operating time $\mathrm{E}[T_{11}]$, expected downtime $\mathrm{E}[T_{12}]$, and expected cost $\mathrm{E}[C_1]$ for this type are respectively

$$\mathrm{E}[T_{11}] = \sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}(k_1\tau + k_2\tau + \cdots + k_{i+1}\tau)P_1(i)\right), \quad (28)$$

$$\mathrm{E}[T_{12}] = \sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}(\mathrm{E}[M_{k_1,1}] + \mathrm{E}[M_{k_2,1}] + \cdots + \mathrm{E}[M_{k_i,i}] + \xi)P_1(i)\right), \quad (29)$$

$$\mathrm{E}[C_1] = \sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}(C_P\mathrm{E}[M_{k_1,1}] + C_P\mathrm{E}[M_{k_2,1}] + \cdots + C_P\mathrm{E}[M_{k_i,i}] + C_Ik_1 + C_Ik_2 + \cdots + C_Ik_{i+1} - C_I + C_C)P_1(i)\right), \quad (30)$$

where $\mathrm{Sum}(z) = i - z + 1 + \sum_{i'=1}^{z-1} k_{i'}$.

(ii) Renewal cycle ending with the system operating time reaching $U\tau$

Similar to the assumption in the first type, the number of missions executed by the system in the $(i+1)$th PM cycle reaches U and PR is performed. Meanwhile, in the previous i PM cycles, $A1[i] > \zeta$ is satisfied. Based on the given combination $k_1,k_2,\cdots,k_{i+1}$, the probability of this event is

$$P_2(i)=E_{S_i}\left[P\left(X\left(\left(U-1-\sum_{i'=1}^{i}k_{i'}\right)\tau\right)+S_i<\omega_p(k_{i+1}-1,i+1)\cap X\left(\left(U-\sum_{i'=1}^{i}k_{i'}\right)\tau\right)+S_i<\omega_f\right)\right]\cdot\prod_i P_{\rm PM}(k_i,i)I(A1[i])=\int_0^{\omega_p(k_{i+1},i+1)}((1-F_{T_p}((U-1-\sum_{i'=1}^{i}k_{i'})\tau|s_i))\int_0^{\omega_p(k_{i+1},i+1)-s_i}(1-F_{\omega_f-X}(\tau;x|s_i))\cdot f_{\rm IG}(x;(U-1-\sum_{i'=1}^{i}k_{i'})\tau){\rm d}x)f(s_i){\rm d}s_i\cdot\prod_i P_{\rm PM}(k_i,i)I(A1[i]). \tag{31}$$

In particular, when $i=0$, no PM action is performed when the number of missions executed by the system reaches U, and the probability of this event is

$$P_2(0)=\int_0^{\omega_{p(k_1,1)}}((1-FT_p((U-1)\tau))\cdot\int_0^{\omega_p(k_1,1)-s_0}(1-F\omega_f-x(\tau;x))f_{\rm IG}(x;(U-1)\tau){\rm d}x)f(s_0){\rm d}s_0. \tag{32}$$

Therefore, the expected operating time $\mathrm{E}[T_{21}]$, expected downtime $\mathrm{E}[T_{22}]$, and expected cost $\mathrm{E}[C_2]$ for this type are respectively

$$\mathrm{E}[T_{21}]=\sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}U\tau P(i)\right), \tag{33}$$

$$\mathrm{E}[T_{22}]=\sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}(\mathrm{E}[M_{k_1,1}]+\mathrm{E}[M_{k_2,1}]+\cdots+\mathrm{E}[M_{k_i,i}]+\xi)P_2(i)\right), \tag{34}$$

$$\mathrm{E}[C_2]=\sum_i\left(\sum_{k_1=1}^{U-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-\mathrm{Sum}(i+1)}(C_P\mathrm{E}[M_{k_1,1}]+C_P\mathrm{E}[M_{k_2,1}]+\cdots+C_P\mathrm{E}[M_{k_i,i}]+C_IU+C_R)P_2(i)\right). \tag{35}$$

(iii) Renewal cycle ending with availability constraint

With reference to the first two assumptions, in this scenario, the system performs PR due to availability constraint before reaching $U\tau$. In the previous i PM cycles, the system operating state satisfies $A1[i]>\zeta$, but at the end of the $(i+1)$th PM interval, $A1[i+1]\leqslant\zeta$. It should be noted that when $i=0$, since $A1=1$, the system will not perform PR due to availability constraint in this scenario. Therefore, based on the given combination $k_1,k_2,\cdots,k_{i+1}$, the probability of this event is

$$P_3(i)=P_{\rm PM}(k_{i+1},i+1)\bar{I}(A1[i+1])\cdot\prod_i P_{\rm PM}(k_i,i)I(A1[i]) \tag{36}$$

where $\bar{I}(A1[i+1])$ is an indicator function.

When $A1[i+1]\leqslant\zeta$, $\bar{I}(A1[i+1])=1$, otherwise $\bar{I}(A1[i+1])=0$.

Therefore, the expected operating time $\mathrm{E}[T_{31}]$, expected downtime $\mathrm{E}[T_{32}]$, and expected cost $\mathrm{E}[C_3]$ for this type are respectively

$$\mathrm{E}[T_{31}]=\sum_i\left(\sum_{k_1=1}^{U-1-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-1-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-1-\mathrm{Sum}(i+1)}(k_1\tau+k_2\tau+\cdots+k_{i+1}\tau)P_3(i)\right), \tag{37}$$

$$\mathrm{E}[T_{32}]=\sum_i\left(\sum_{k_1=1}^{U-1-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-1-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-1-\mathrm{Sum}(i+1)}(\mathrm{E}[M_{k_1,1}]+\mathrm{E}[M_{k_2,1}]+\cdots+\mathrm{E}[M_{k_i,i}]+\xi)P_3(i)\right), \tag{38}$$

$$\mathrm{E}[C_3]=\sum_i\left(\sum_{k_1=1}^{U-1-\mathrm{Sum}(1)}\sum_{k_2=1}^{U-1-\mathrm{Sum}(2)}\cdots\sum_{k_{i+1}=1}^{U-1-\mathrm{Sum}(i+1)}(C_P\mathrm{E}[M_{k_1,1}]+C_P\mathrm{E}[M_{k_2,1}]+\cdots+C_P\mathrm{E}[M_{k_i,i}]+C_Ik_1+C_Ik_2+\cdots+C_Ik_{i+1}+C_R)P_3(i)\right). \tag{39}$$

4.4 Optimization model and solution algorithm

For the mission-oriented system, we take the goal of minimizing the long-term expected cost ratio under the mission availability constraint, and the decision variables are a, b, and c in DPMT. According to the renewal reward theory, the CBM optimization model is expressed as

$$\min Z=\frac{\mathrm{E}[C]}{\mathrm{E}[T]}=\frac{\mathrm{E}[C_1]+\mathrm{E}[C_2]+\mathrm{E}[C_3]}{\mathrm{E}[T_1]+\mathrm{E}[T_2]+\mathrm{E}[T_3]}$$
$$\text{s.t.}\begin{cases}A1[i]\geqslant\zeta\\ \mathrm{E}[T_{m1}]\leqslant U\tau,\ m=1,2,3\\ a,b>\dfrac{1}{c}\\ 0<c<\omega_f\\ \omega_p(0,i)>s_{i-1}\end{cases} \tag{40}$$

where $\mathrm{E}[T_1]=\mathrm{E}[T_{11}]+\mathrm{E}[T_{12}]$, $\mathrm{E}[T_2]=\mathrm{E}[T_{21}]+\mathrm{E}[T_{22}]$, $\mathrm{E}[T_3]=\mathrm{E}[T_31]+\mathrm{E}[T_{32}]$.

Seeking the optimal solution of (40) is indeed a tough task. In order to obtain the optimal long-term expected cost ratio and maintenance policy, a solution algorithm is designed based on the characteristics of this model. The detailed steps are as follows:

Step 1 Initialize model parameters.

Step 2 Set the decision variables a, b, c, and search in the feasible region from the smaller value according to the constraint of (40), set i=0.

Step 3 Calculate $\mathrm{E}[T_{m1}]\_i$, $\mathrm{E}[T_{m2}]\_i$, and $\mathrm{E}[C_m]\_i$ ($m=1,2,3$), then calculate the objective function.

Step 4 Set i=i+1, and calculate $A1[i]$. Then calculate $\mathrm{E}[T_{m1}]\_i$, $\mathrm{E}[T_{m2}]\_i$, and $\mathrm{E}[C_m]\_i$ ($m=1,2,3$), based on the assumptions of three renewal cycles.

Step 5 Calculate $\mathrm{E}[T_{m1}]\_i$=$\mathrm{E}[T_{m1}]\_i$+ $\mathrm{E}[T_{m1}]\_i-1$, $\mathrm{E}[T_{m2}]\_i$=$\mathrm{E}[T_{m2}]\_i$+ $\mathrm{E}[T_{m2}]\_i-1$, and $\mathrm{E}[C_m]\_i$=$\mathrm{E}[C_m]\_i$+ $\mathrm{E}[C_m]\_i$, then calculate the objective function.

Step 6 Repeat Steps 4 and 5 until the objective function value no longer changes.

Step 7 Adjust the decision variables a, b, and c slightly within the search range, reset i=0, and repeat Steps 3 to 6.

Step 8 Determine the minimum objective function value in (40), and find the relevant decision variable values.

5. A numerical example

This section verifies the above optimization model through simulation experiments, and obtains the optimal solution of the objective function and related decision variables. Then, we compare the model proposed in this paper (referred to as Model 1) with the fixed PM threshold model (referred to as Model 2). Finally, the sensitivity analysis of the model parameters is carried out.

5.1 Parameter setting

It is assumed that the degradation process of the mission-oriented system follows IG process described by (11). The model parameters of IG process are μ_β^0=0.5, v=1.1, σ_β=100, and η=0.4. The residual damage model parameters are α=1.2 and ε=0.1. The PM duration model parameters are γ=0.1 and λ=0.05. The duration of each renewal action is ξ=2 h. The mission duration is τ=8 h, and the maximum mission number that the system can perform is U=7 h. The failure threshold is ω_f=10, and the mission availability constraint of a PM cycle is ζ=0.95. The relevant maintenance cost parameters are C_I=5 yuan, C_P=50 yuan, C_R=200 yuan, C_C=500 yuan.

5.2 Analysis and comparison of results

(i) Experimental results

Since the residual damage and PM duration increase with the increase of DPMT and the number of PM cycles, the system operation process is always renewed due to availability constraint. Therefore, the PM cycle number i can get an upper limit i^* through experiments to simplify the calculation process. The experimental results are shown in Fig. 2.

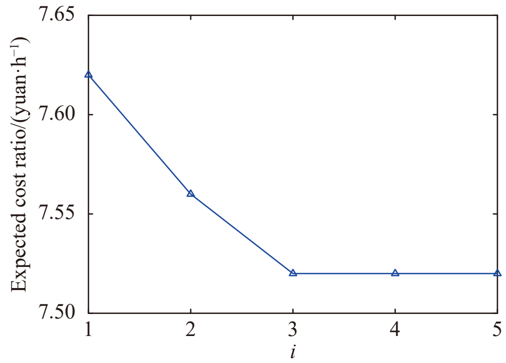


Fig. 2 Experimental results of upper limit of PM cycle

Observing Fig. 2 we can see that when $i \geqslant 3$, the long-term expected cost ratios corresponding to different PM cycle number i are consistent, therefore, the PM cycle upper limit is $i^*=3$. At the same time, according to the search results of the optimization algorithm for the decision variables a, b, c, when $a=3.46$, $b=0.17$, $c=6.2$, the long-term expected cost ratio reaches the minimum, and the minimum value is 7.52 yuan/h. So, the optimal maintenance policy of the mission-oriented system is obtained.

(ii) Comparison

In order to verify the role of DPMT in the system maintenance policy with dynamic degradation characteristics, we compare Model 1 and Model 2. The experimental parameter settings of Model 2 are the same as those of Model 1, and the experimental results are shown in Fig. 3.

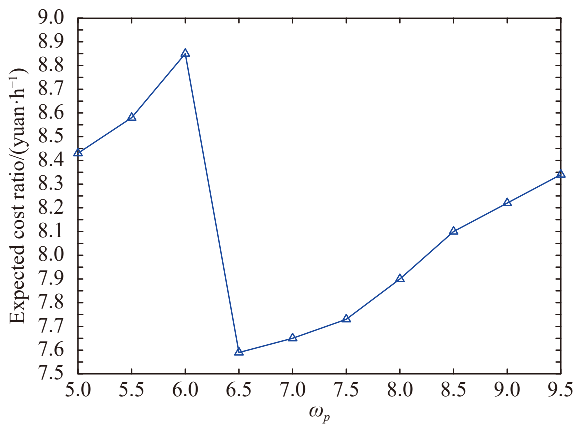


Fig. 3 Experimental results of Model 2

Observing Fig. 3, we can find that the optimal long-term expected cost ratio of Model 2 is 7.59 yuan/h. Obviously, the results of Model 1 proposed in this paper are better. The main reason is that Model 2 adopts a fixed PM threshold during the system renewal cycle, so that the high failure risk in the early stage is not controlled in time. During the last mission, the system state directly exceeds the failure threshold and the first renewal type occurs. The DPMT of the model we proposed reduces the risk of early failure with a smaller threshold in the early stage of system operation, and then increases the maintenance threshold to ensure the effective operation time of the system. This delays the occurrence of the first renewal type, and reduces maintenance cost.

5.3 Sensitivity analysis

In order to further verify the influence of model parameters on system maintenance cost and policy, in this paper, we divide the model parameters into three categories and use the control variable method to conduct sensitivity analysis from the maintenance cost, degradation characteristics and mission requirement.

First of all, we discuss the maintenance cost parameters C_I, C_P, C_R, and C_C. According to the sensitivity analysis method, we select ±20%, ±40%, ±60%, ±80%, and +100% respectively as the change range of maintenance cost parameters. The ranges of them satisfy $1 \leqslant C_I \leqslant 10$, $10 \leqslant C_P \leqslant 100$, $40 \leqslant C_R \leqslant 400$, and $100 \leqslant C_C \leqslant 1\,000$, and the other parameters are fixed. At the same time, in order to reflect the sensitivity of system maintenance cost and policy to the model parameters, the sensitivity coefficient is selected as the analysis index. The larger the sensitivity coefficient is, the more sensitive the optimization results are to this parameter. The expression of the sensitivity coefficient (SC) is

$$\mathrm{SC} = \frac{\Delta Z}{\Delta P} \times 100\% \tag{41}$$

where ΔZ denotes the optimization result change ratio, and ΔP denotes model parameter change ratio.

Fig. 4 shows the relevant results of the experimental process. What needs to be explained here is that from (16), it can be seen that among the decision variables a, b, and c, c has the greatest influence on $\omega_p(k,i)$. Therefore, this paper selects decision variable c as a representative to briefly explain the influence of model parameters on system maintenance policy.

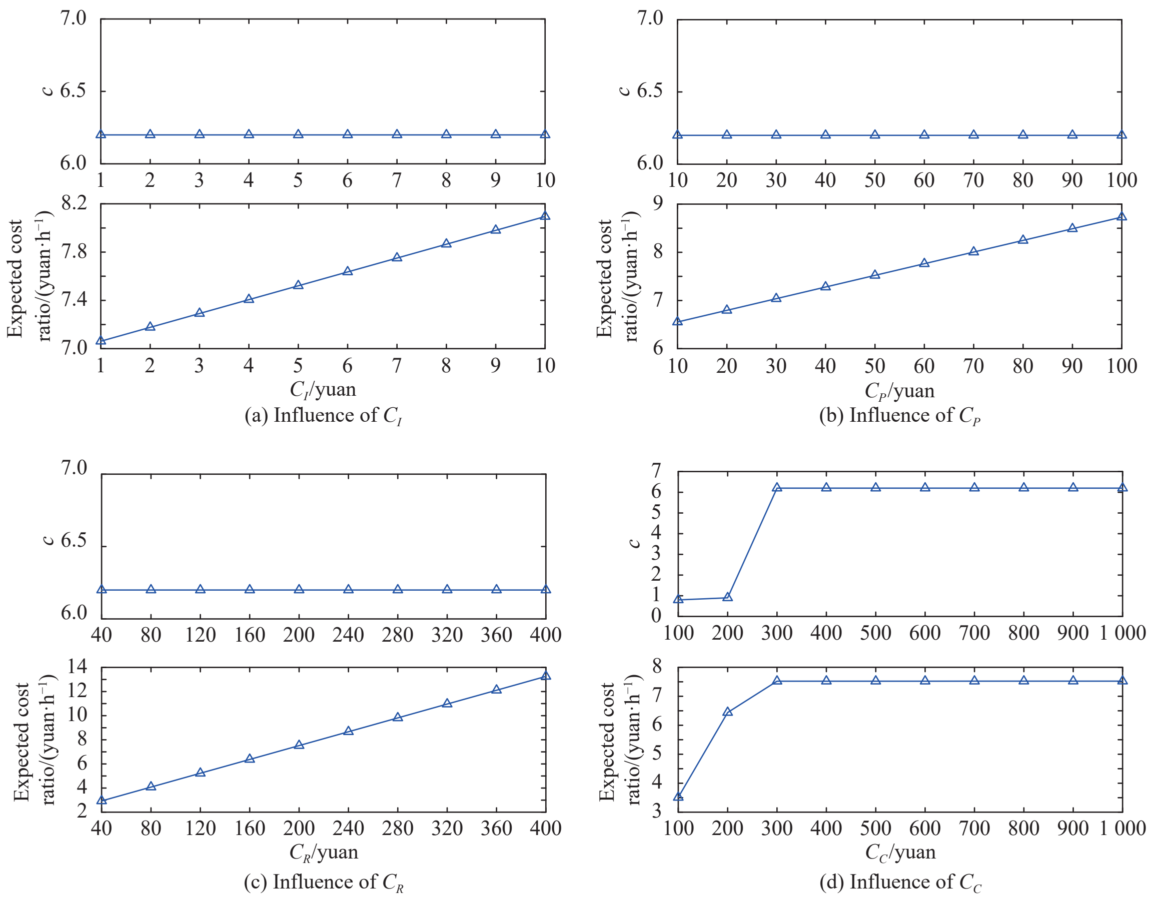


Fig. 4 Influence of C_I, C_P, C_R, and C_C on optimal expected cost ratio and policy

According to Fig. 4, it is easy to know that among the four maintenance cost parameters, only the change of C_C has an effect on the maintenance policy. When C_C=100 or 200, the optimal decision variable c changes suddenly and is equal to 0.8 or 0.9, because $C_C \leqslant C_R$ at this moment. The reduction of c will increase the mission availability, thereby preventing the system from performing the third renewal type that has a higher cost due to availability constraint. However, in practice, it is usually specified that $C_R < C_C$, which means that the four maintenance cost parameters have no effect on the maintenance policy within the range of changes in accordance with the actual meaning (−40%–+100%). Therefore, Table 1 only shows the sensitivity analysis results of the optimal expected cost ratio.

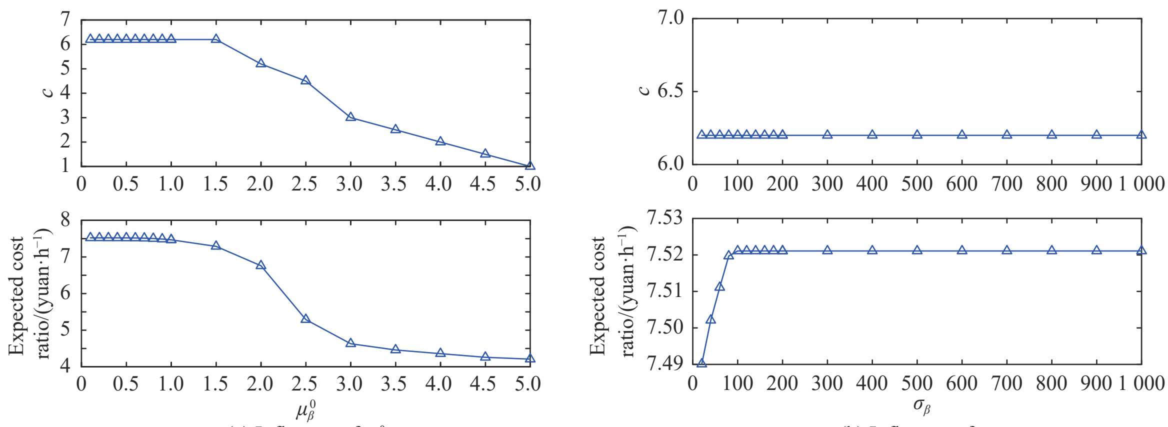
From Fig. 4 and Table 1, we can see that the optimal long-term expected cost ratio changes linearly with the increase of C_I, C_P, and C_R. C_C has no effect within the actual change range (−40%–+100%). This means that DPMT successfully prevents the occurrence of CM. In addition, Table 1 shows that the optimal long-term expected cost ratio is most sensitive to C_R.

Regarding the degradation characteristics and mission requirement parameters, due to their respective characteristics, sensitivity analysis is not limited to the range of parameter changes in the general sense.

Next, we discuss the degradation characteristic parameters μ_β^0 and α. The ranges of the two satisfy $0.1 \leqslant \mu_\beta^0 \leqslant 5$ and $20 \leqslant \sigma_\beta \leqslant 1\,000$ respectively, and the other parameters are fixed. Fig. 5 and Table 1 show the experimental results and sensitivity analysis results respectively.

Table 1 Sensitivity analysis results of optimal expected cost ratio on C_I, C_P, C_R, and C_C

Parameter		Initial value (0%)	−80%	−60%	−40%	−20%	+20%	+40%	+60%	+80%
Optimal expected cost ratio/(yuan·h^{-1})	C_I	7.52	7.06	7.18	7.29	7.41	7.64	7.75	7.87	7.98
	C_P		6.55	6.80	7.04	7.28	7.76	8.01	8.25	8.49
	C_R		2.93	4.08	5.23	6.37	8.67	9.82	10.96	12.11
	C_C		3.51	6.44	7.52	7.52	7.52	7.52	7.52	7.52
Difference from initial value/(yuan·h^{-1})	C_I	–	−0.46	−0.34	−0.23	−0.11	0.12	0.23	0.35	0.46
	C_P		−0.97	−0.72	−0.48	−0.24	0.24	0.49	0.73	0.97
	C_R		−4.59	−3.44	−2.29	−1.15	1.15	2.30	3.44	4.59
	C_C		−4.01	−1.08	0	0	0	0	0	0
SC /%	C_I	–	7.65	7.54	7.65	7.31	7.98	7.65	7.76	7.65
	C_P		16.12	15.96	15.96	15.96	15.96	16.29	16.18	16.12
	C_R		76.30	76.24	76.13	76.46	76.46	76.46	76.24	76.30
	C_C		66.66	23.94	0	0	0	0	0	0



(a) Influence of μ_β^0 (b) Influence of σ_β

Fig. 5 Influence of μ_β^0 and σ_β on optimal expected cost ratio and policy

We can find that when μ_β^0 increases from 0.1 to 5, the optimal long-term expected cost ratio decreases from 7.52 yuan·h^{-1} to 4.21 yuan·h^{-1}, and the corresponding optimal decision variable c decreases from 6.2 to 1. This is because as μ_β^0 increases, the system degradation rate decreases, and the reduction of the decision variable c in the optimal DPMT will prevent the system from increasing the failure risk and maintenance cost due to long-term inability to carry out PM actions. At the same time, it reduces the system renewal probability due to availability constraint, and further saves maintenance cost. For σ_β, when σ_β increases from 20 to 1 000, the optimal long-term expected cost ratio increases from 7.49 yuan·h^{-1} to 7.52 yuan·h^{-1}, while the corresponding optimal decision variable c remains unchanged at 6.2. This is because the increase in σ_β will cause a larger variance $D(\beta)$ of the random drift coefficient β, that is, the instability will increase, which will increase system maintenance cost. In summary, it is easy to see from Table 2 that μ_β^0 has more obvious influence on system maintenance cost and policy than σ_β.

Table 2 Sensitivity analysis results of optimal expected cost ratio and policy on μ_β^0 and σ_β

Parameter		Initial value (0%)	−80%	−60%	−40%	−20%	+20%	+40%	+60%	+80%	+100%	+200%	+300%	+400%	+500%	+600%	+700%	+800%	+900%
Optimal expected cost ratio/(yuan·h^{-1})	μ_β^0	–	7.52	7.52	7.52	7.52	7.52	7.51	7.50	7.49	7.47	7.29	6.76	5.29	4.63	4.46	4.36	4.26	4.21
	σ_β	7.52	7.49	7.50	7.51	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52	7.52
Difference from initial value/(yuan·h^{-1})	μ_β^0	–	0	0	0	0	0	−0.01	−0.02	−0.03	−0.05	−0.23	−0.76	−2.23	−2.89	−3.06	−3.16	−3.26	−3.31
	σ_β	–	−0.03	−0.02	−0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SC /%	μ_β^0	–	0	0	0	0	0	−0.33	−0.44	−0. 50	−0. 66	−1.53	−3.37	−7.41	−7.69	−6.78	−6.00	−5.42	−4.89
	σ_β	–	0.50	0. 44	0. 33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c	μ_β^0	–	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	5.2	4.5	3.0	2.5	2.0	1.5	1.0
	σ_β	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2
Difference from c	μ_β^0	–	0	0	0	0	0	0	0	0	0	0	−1.0	−1.7	−3.2	−3.7	−4.2	−4.7	−5.2
	σ_β	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SC/%	μ_β^0	–	0	0	0	0	0	0	0	0	0	0	−5.38	−6.86	−10.32	−9.95	−9.68	−9.48	−9.32
	σ_β	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Finally, we discuss the mission requirement parameters. The variation range of ζ satisfies $0.91 \leqslant \zeta \leqslant 0.99$, and the other parameters are fixed. Fig. 6 and Table 3 show the experimental results and sensitivity analysis results respectively.

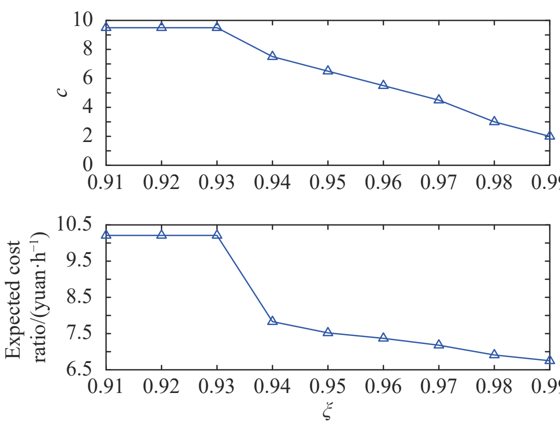


Fig. 6 Influence of ζ on optimal expected cost ratio and policy

As shown in Table 3, we can find that when ζ increases from 0.91 to 0.99, the optimal long-term expected cost ratio decreases from 10.21 yuan·h^{-1} to 6.75 yuan·h^{-1}, and the corresponding optimal decision variable c decreases from 9.5 to 2. And when ζ is in the interval of [0.91, 0.93], the expected cost ratio and decision variable c remain unchanged at 10.21 yuan·h^{-1} and 9.5, respectively. The main reason for this phenomenon is that the initial availability ζ is set slightly lower, which leads to a fixed number of occurrences of the third renewal type in the entire system evolution process or even no occurrence, so that the optimal solution does not change. After that, the optimal long-term expected cost ratio and decision variable c become smaller, because the increase in availability makes part of the first renewal type become the third type, and the PR cost C_R is lower than the CM cost C_C, so the maintenance cost will gradually decrease. At the same time, the higher the availability requirement, the lower the decision variable c in the optimal DPMT can enable the system evolution process to carry out as many low-cost PM actions as possible, instead of directly performing more expensive renewal due to availability constraint.

In summary, comparing the sensitivity coefficients in Tables 1 to 3, it is obvious that mission requirement has an important effect on system maintenance cost and policy, and is a key factor that cannot be ignored in maintenance optimization.

Table 3 Sensitivity analysis results of optimal expected cost ratio and policy on ζ

Parameter	Initial value (0%)	−4%	−3%	−2%	−1%	+1%	+2%	+3%	+4%
Optimal expected cost ratio/(yuan·h^{-1})	7.52	10.21	10.21	10.21	7.83	7.37	7.18	6.91	6.75
Difference from initial value/(yuan·h^{-1})	–	2.69	2.69	2.69	0.31	−0.15	−0.34	−0.61	−0.77
SC /%	–	−849.57	−1 132.76	−1 699.14	−391.62	−189.49	−214.76	−256.87	−243.18
c	6.2	9.5	9.5	9.5	7.5	5.5	4.5	3.0	2.0
Difference from c	–	3.3	3.3	3.3	1.3	−0.7	−1.7	−3.2	−4.2
SC /%	–	−1 264.11	−1 685.48	−2 528.23	−1 991.94	−1 072.58	−1 302.42	−1 634.41	−1 608.87

6. Conclusions

Considering that in practical applications, the degradation process of some mission-oriented systems is monotonous and irreversible, the risk of failure in the early operation stage of the system is high, and the influence of imperfect PM is considered singly, this paper proposes a dynamic CBM optimization model for mission-oriented system that obeys IG degradation process. This model first adopts IG process with random drift coefficient to solve the problem of describing the dynamic degradation process of the system. Then we use the DPMT function to control the early failure risk of the mission-oriented system, and comprehensively consider the influence of imperfect PM on the degradation amount and degradation rate. Finally, in the process of constructing the CBM optimization model, the mission availability requirement is integrated to make the model more realistic. The experimental results show that the model proposed in this paper can effectively reduce the maintenance cost of the mission-oriented system compared with the fixed PM threshold model. At the same time, sensitivity analysis proves that mission requirements are more important than maintenance cost and degradation characteristics for the formulation of maintenance policies for such systems.

References

[1] WINOKUR H S, GOLDSTEIN L J. Analysis of mission-oriented systems. IEEE Trans. on Reliability, 1969, 18(4): 144–148.

[2] LIU B, XIE M, XU Z G, et al. An imperfect maintenance policy for mission-oriented systems subject to degradation and external shocks. Computers & Industrial Engineering, 2016, 102: 21–32.

[3] ZHAO X, SUN J L, QIU Q A, et al. Optimal inspection and mission abort policies for systems subject to degradation. European Journal of Operational Research, 2021, 292(2): 610–621.

[4] ZHAO X, FAN Y, QIU Q A, et al. Multi-criteria mission abort policy for systems subject to two-stage degradation process. European Journal of Operational Research, 2021, 295(1): 233–245.

[5] QIU Q A, CUI L R. Gamma process based optimal mission abort policy. Reliability Engineering & System Safety, 2019, 190: 106496.

[6] QIU Q A, KOU M, CHEN K, et al. Optimal stopping problems for mission oriented systems considering time redundancy. Reliability Engineering & System Safety, 2021, 205: 107226.

[7] CHENG G Q, ZHOU B H, LI L. Integrated production, quality control and condition-based maintenance for imperfect production systems. Reliability Engineering & System Safety, 2018, 175: 251–264.

[8] ABDELHAKIM K, CLAVER D, EL-HOUSSAINE A, et al. Integrated production quality and condition-based maintenance optimisation for a stochastically deteriorating manufacturing system. International Journal of Production Research, 2019, 57(8): 2480–2497.

[9] WU Z Y, GUO B, AXITA, et al. A dynamic condition-based maintenance model using inverse Gaussian process. IEEE Access, 2020, 8: 104–117.

[10] GUO C M, WANG W B, GUO B, et al. A maintenance optimization model for mission-oriented systems based on Wiener degradation. Reliability Engineering & System Safety, 2013, 111: 183–194.

[11] LI J F, CHEN Y X, XIANG H C, et al. Remaining useful life prediction for aircraft engine based on LSTM-DBN. Systems Engineering and Electronics, 2020, 42(7): 1637–1644. (in Chinese)

[12] WANG Z Z, CHEN Y X, CAI Z Y, et al. Methods for predicting the remaining useful life of equipment in consideration of the random failure threshold. Journal of Systems Engineering and Electronics, 2020, 31(2): 415–431.

[13] CAI Z Y, WANG Z Z, CHEN Y X, et al. Remaining useful lifetime prediction for equipment based on nonlinear implicit degradation modeling. Journal of Systems Engineering and Electronics, 2020, 31(1): 194–205.

[14] WANG Z Z, CHEN Y X, CAI Z Y, et al. Remaining useful lifetime prediction based on nonlinear degradation processes with random failure threshold. Journal of National University of Defense Technology, 2020, 42(2): 177–185. (in Chinese)

[15] CAI Z Y, WANG Z Z, ZHANG X F, et al. Online prediction method of remaining useful life for implicit nonlinear degradation equipment. Systems Engineering and Electronics, 2020, 42(6): 1410–1416. (in Chinese)

[16] WANG Z Z, CHEN Y X, CAI Z Y, et al. Remaining useful lifetime online prediction based on accelerated degradation modeling with the proportion relationship. Systems Engineering and Electronics, 2021, 43(2): 584–592. (in Chinese)

[17] SHIN J H, JUN H B. On condition based maintenance policy. Journal of Computational Design & Engineering, 2015, 2(2):

119–127.

[18] ALASWAD S, XIANG Y S. A review on condition-based maintenance optimization models for stochastically deteriorating system. Reliability Engineering & System Safety, 2017, 157: 54–63.

[19] CHEN Y X, WANG Z Z, CAI Z Y. Optimal maintenance decision based on remaining useful lifetime prediction for the equipment subject to imperfect maintenance. IEEE Access, 2020, 8: 6704–6716.

[20] CHEN N, YE Z S, XIANG Y S, et al. Condition-based maintenance using the inverse Gaussian degradation model. European Journal of Operational Research, 2015, 243(1): 190–199.

[21] YE Z S, CHEN N. The inverse Gaussian process as a degradation model. Technometrics, 2014, 56(3): 302–311.

[22] MA X Y, LIU B, YANG L, et al. Reliability analysis and condition-based maintenance optimization for a warm standby cooling system. Reliability Engineering & System Safety, 2020, 193: 106588.

[23] WANG Z Z, CHEN Y X, CAI Z Y, et al. Optimal replacement strategy considering equipment remaining useful lifetime prediction information under the influence of uncertain failure threshold. Journal of National University of Defense Technology, 2021, 43(1): 145–154. (in Chinese)

[24] PHUC D, VOISIN A, LEVRAT E, et al. A proactive condition-based maintenance strategy with both perfect and imperfect maintenance actions. Reliability Engineering & System Safety, 2015, 133: 22–32.

[25] CABALLE N C, CASTRO I T, PEREZ C J, et al. A condition-based maintenance of a dependent degradation-threshold-shock model in a system with multiple degradation processes. Reliability Engineering & System Safety, 2015, 134: 98–109.

[26] WASAN M T. On an inverse Gaussian process. Annals of Mathematical Statistics, 1967, 38(2): 638–645.

[27] WANG X, XU D H. An inverse Gaussian process model for degradation data. Technometrics, 2010, 52(2): 188–197.

[28] DONG W J, LIU S F, BAE S J, et al. A multi-stage imperfect maintenance strategy for multi-state systems with variable user demands. Computers & Industrial Engineering, 2020, 145: 106508.

[29] ZHOU Y, KOU G, XIAO H, et al. Sequential imperfect preventive maintenance model with failure intensity reduction with an application to urban buses. Reliability Engineering & System Safety, 2020, 198: 106871.

[30] ZHANG M M, GAUDOIN O, XIE M. Degradation-based maintenance decision using stochastic filtering for systems under imperfect maintenance. European Journal of Operational Research, 2015, 245(2): 531–541.

[31] LIU G H, CHEN S K, JIN H, et al. Optimum imperfect inspection and maintenance scheduling model considering delay time theory. Journal of Zhejiang University (Engineering Science), 2020, 54(7): 1298–1307. (in Chinese)

[32] PEI H, HU C H, SI X S, et al. Remaining life prediction information-based maintenance decision model for equipment under imperfect maintenance. Acta Automatica Sinica, 2018, 44(4): 719–729. (in Chinese)

[33] CAI Z Y, CHEN Y X, LI S L, et al. Residual lifetime prediction method with random degradation and information fusion. Journal of Shanghai Jiao Tong University, 2016, 50(11): 1778–1783. (in Chinese)

[34] CAI Z Y, CHEN Y X, GUO J S, et al. Remaining lifetime prediction for device with measurement error and random effect. Systems Engineering and Electronics, 2019, 41(7): 1658–1664. (in Chinese)

[35] PAN D H, LIU J B, CAO J D. Remaining useful life estimation using an inverse Gaussian degradation model. Neurocomputing, 2016, 185: 64–72.

[36] SI X S, WANG W B, HU C H, et al. Estimating remaining useful life with three-source variability in degradation modeling. IEEE Trans. on Reliability, 2014, 63(1): 167–190.

[37] WANG Z Z, CHEN Y X, CAI Z Y, et al. Real-time prediction of remaining useful lifetime for equipment with random failure threshold. Systems Engineering and Electronics, 2019, 41(5): 1162–1168. (in Chinese)

[38] ELWANY A H, GEBRAEEL N Z, MAILLART L M. Structured replacement policies for components with complex degradation processes and dedicated sensors. Operations Research, 2011, 59(3): 684–695.

[39] LIAO H T, ELSAYED E A, CHAN L Y. Maintenance of continuously monitored degrading systems. European Journal of Operational Research, 2006, 175(2): 821–835.

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