Coarse-fine joint target parameter estimation method based on AN-RSC in OFDM passive radar

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Abstract: In this paper, we study the accuracy of delay-Doppler parameter estimation of targets in a passive radar using orthogonal frequency division multiplexing (OFDM) signal. A coarse-fine joint estimation method is proposed to achieve better estimation accuracy of target parameters without excessive computational burden. Firstly, the modulation symbol domain (MSD) method is used to roughly estimate the delay and Doppler of targets. Then, to obtain high-precision Doppler estimation, the atomic norm (AN) based on the multiple measurement vectors (MMV) model (MMV-AN) is used to manifest the signal sparsity in the continuous Doppler domain. At the same time, a reference signal compensation (RSC) method is presented to obtain highprecision delay estimation. Simulation results based on the OFDM signal show that the coarse-fine joint estimation method based on AN-RSC can obtain a more accurate estimation of target parameters compared with other algorithms. In addition, the proposed method also possesses computational advantages compared with the joint parameter estimation.

Keywords: passive radar, orthogonal frequency division multiplexing (OFDM) signal, atomic norm (AN), parameter estimation.

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1. Introduction

Passive radar, which utilizes the existing illuminators of opportunity to achieve target detection and tracking, has attracted increasing interest in recent years [1−3]. In addition, with the rapid popularization of digital broadcasting using orthogonal frequency division multiplexing (OFDM) modulation, such as digital audio broadcasting (DAB) [4], digital video broadcasting-terrestrial (DVB-T) [5], and digital terrestrial multimedia broadcasting (DTMB) [6], some novel estimations of target parameters are designed that make full use of the two-dimen-

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sional (2D) structure of the OFDM signal. In [7], a modulation symbol domain (MSD) method was proposed to estimate the delay and Doppler of the targets. Because the estimation is less susceptible to imperfect autocorrelation characteristics of baseband signals, this method provides a significantly higher peak to sidelobe ratio. However, the delay and Doppler resolution of the MSD are limited by signal bandwidth and accumulation time, respectively. To cope with this limitation, spatial spectrum estimation algorithms or sparse methods can be used in target parameter estimation, but they may incur higher computational complexity [8,9].

The multiple signal classification (MUSIC) algorithm, a typical subspace classification method, is widely used for target parameter estimation due to its good resolution and accuracy [10]. However, the estimation accuracy of MUSIC algorithm is susceptible to the signal to noise ratio (SNR), which leads to poor estimation performance in noisy environments. With the development of sparse signal representation and later the compressed sensing (CS) theory [11−13], sparse methods have been developed for improving the accuracy of parameter estimation [14]. When the CS algorithm has been considered in range Dopper (RD) map generation, the dictionary consists of the template signals with discrete delays and Doppler frequencies [15,16], and therefore the delays and frequencies of targets are assumed to be distributed on some fixed delay-frequency grids [17]. However, since the targets and clutter are often determined by parameters (delays and Doppler frequencies) in a continuous domain, the discretization tends to lead to model mismatches and poor recovery performance [18].

In order to address the grid mismatch problem caused by traditional sparse methods, a mathematical theory of continuous sparse recovery was proposed to achieve super-resolution $[19,20]$. Zheng et al. $[21]$ used the atomic norm (AN) constraint to formulate the sparsity of signals in the delay-Doppler plane, and the results have

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better performance compared with the existing high-resolution methods based on CS and 2D-MUSIC algorithms. However, if AN is directly applied to delay-Doppler parameter estimation with a long integration time and high sampling frequency, although the alternating direction method of multipliers (ADMM) [22] was used in [21] to effectively solve the semidefinite program (SDP), it still needs a lot of computation and may hinder the realtime application for it involves convex optimization. To further reduce the computational complexity of the algorithm, another approach is to utilize a series of onedimensional (1D) searches that alternate between optimizations on one variable while keeping the other variable fixed, which better achieves 2D joint estimation without the heavy computational burden [17,23].

Based on the methods above, we propose a new parameter estimation method for OFDM-based passive radar. A coarse-fine joint estimation method is presented, in which only a few batches are selected for parameter estimation to reduce calculations. Firstly, the method proposed in [7] is used to roughly estimate the delay and Doppler frequency of targets. Then the AN and reference signal compensation (RSC) methods are used in the Doppler domain and delay domain, respectively, and the generated Topeplitz matrix of AN no longer needs huge storage space. In the Doppler dimension, the AN based on the multiple measurement vectors (MMV) [24] model (MMV-AN) is used to improve the processing ability. In the delay dimension, the RSC is presented to overcome the loss of SNR caused by the block processing of the signal, which can easily achieve the accuracy and robustness of delay estimation.

The remainder of the paper is organized as follows. In Section 2, the passive radar signal model in the subcarrier domain is described. In Section 3, a new coarse-fine joint parameter estimation AN-RSC method for OFDM passive radar is proposed. In Section 4, the effectiveness of the proposed method is presented via numerical simulation and computational complexity analysis. Finally, conclusions are drawn in Section 5.

2. Signal model and problem representation

The duration of each OFDM symbol is $T_s = T + T_G$, which is composed of an elementary symbol duration T and a guard interval duration T_G . Assume that there are *N*c orthogonal subcarriers in each OFDM symbol, and the *n*th subcarrier frequency is $f_n = n\Delta f$, where $\Delta f = 1/T$ is We study the passive radar system utilizing OFDM signals, which can be viewed as parallel streams of multiple single-carrier signals with orthogonal carrier waveforms, each modulated using different transmission data [7]. The transmitted data is divided into multiple OFDM symbols.

the subcarrier interval. The transmitted signal in the *m*th OFDM symbol can be modeled as

$$
x_m(t) = \sum_{n=0}^{N_c-1} s_m[n] \exp(j2\pi n\Delta ft) \cdot \xi(t - mT_s),
$$

$$
mT_s - T_G \leq t \leq mT_s + T
$$
 (1)

s^m[*n*] $(n = 0, 1, \dots, N_c - 1; m = 0, 1, \dots, N_{sym} - 1)$ is complex modulation data with the *n*th subcarrier of the *m*th OFDM symbol, and

$$
\xi(t) = \begin{cases} 1, & t \in [-T_G, T] \\ 0, & \text{otherwise} \end{cases} . \tag{2}
$$

and noise. Assume there are K paths, it can be modeled A typical passive radar includes two channels, a reference channel and a surveillance channel. In OFDM-based passive radar, on the surveillance side, suppose downconversion has been carried out, and only baseband signals are considered below. The received signals contain direct path interference, target echoes, multipath signals, as

$$
y(t) = \sum_{m=0}^{N_{sym}-1} \sum_{k=1}^{K} A_k \exp(j2\pi f_k t) \cdot
$$

$$
x_m(t - \tau_k) + \omega(t) \approx
$$

$$
\sum_{m=0}^{N_{sym}-1} \sum_{k=1}^{K} A_k \exp(j2\pi f_k m T_s) \cdot
$$

$$
x_m(t - \tau_k) + \omega(t)
$$
 (3)

where τ_k and f_k denote as the delay and Doppler shift of the *k*th path, respectively; A_k is an attenuation coefficient containing path loss, reflection and processing gains; $\omega(t)$ is additive noise. The approximation in (3) is derived from the constant phase rotation approximation within an OFDM block [21].

*m*th of (3) in the OFDM symbol, then the signal in the *n*th subcarrier can be expressed as Performe Fourier transform on the surveillance signal

$$
r_m[n] = \sum_{k=1}^{K} \alpha_k s_m[n] \exp(j2\pi f_k m T_s) \cdot \exp(-j2\pi f_n \tau_k) + v_m[n]
$$
 (4)

where α_k is the complex amplitude of the kth target, $\alpha_k = A_k T$, and $v_m[n]$ is a complex Gaussian variable with zero mean and variance δ^2 .

and the clean reference signal $x_{ref} = s_m[n]$ can be obtained In order to implement a processor for the estimation of delay and Doppler frequency, the transmitted information has to be removed from the surveillance channel firstly. In OFDM-based passive radar, the reference signal includes a direct path signal and multipath signals, by reconstruction [25]. Therefore, the transmitted signal

(QPSK) [7]. Let D be a new matrix, and its element $D_{n,m}$ is equal to the receiving symbol $r_m[n]$ divided by the reference symbol $x_m[n]$ [26], **D** is given as can be effectively removed from the surveillance information symbols by the complex division of elements for low-order modulation schemes such as binary phase shift keying (BPSK) and quadrature phase shift keying

$$
D = \left[\begin{array}{cccc} D_{0,0} & D_{0,1} & \cdots & D_{0,N_{\text{sym}}-1} \\ D_{1,0} & D_{1,1} & \cdots & D_{1,N_{\text{sym}}-1} \\ \vdots & \vdots & & \vdots \\ D_{N_{\text{c}}-1,0} & D_{N_{\text{c}}-1,1} & \cdots & D_{N_{\text{c}}-1,N_{\text{sym}}-1} \end{array}\right] \tag{5}
$$

where

$$
D_{n,m} = r_m[n]/s_m[n] = \sum_{k=1}^{K} \alpha_k \exp(j2\pi f_k mT) \cdot \exp(-j2\pi f_n \tau_k) + \varpi_m[n].
$$

The problem is to estimate the unknown parameters

reflections α_k , delays τ_k , and Doppler shifts f_k of targets in (5).

3. Coarse-fine joint estimation method based on AN-RSC

In this section, a coarse-fine joint estimation method based on AN-RSC is presented to achieve high-precision estimation of Doppler frequency and delay respectively with OFDM-based passive radar data, and the signal processing flow chart can be seen in Fig. 1. There are three main modules: rough estimation of Doppler and delay parameters, precise estimation of Doppler parameters, and precise estimation of delay parameters. First, the rough estimation of the parameters of the interesting targets is conducted by the MSD method. Then, combined with the rough estimation parameters, AN and RSC methods are used to realize the high-precision estimation of Doppler and delay, respectively. The specific operation of each module is as follows.

Fig. 1 Flow chart of the estimation of Doppler and delay

3.1 Rough estimation of target parameters

Construct a new sparse matrix X whose non-zero elements x_{τ,f_D} represent reflective targets or clutter, given as

$$
x_{\tau, f_D} = \begin{cases} \alpha_k, & \tau = \tau_k; f_D = f_k \\ 0, & \text{otherwise} \end{cases} . \tag{6}
$$

Then, (5) can be apporximated as

$$
\boldsymbol{D} = \boldsymbol{F}_{N_c} \boldsymbol{X} \boldsymbol{F}_{N_{sym}}^{\mathrm{H}} + \boldsymbol{\omega} \tag{7}
$$

where F_{N_c} and $F_{N_{sym}}$ denote Fourier transform matrices of different sizes $N_c \times N_c$ and $N_{sym} \times N_{sym}$, respectively; ω is the additive noise matrix; $(\cdot)^H$ represents the conjugate transformation of a matrix. To obtain high precision parameter estimation effectively, the method proposed in [7] is used to estimate the target parameters roughly. For Doppler parameter estimation, a classical spectrum estimation method is used, formally, the inverse discrete Fourier transform (IDFT) is performed for each column of the matrix *D* in (7), one has

$$
Y_{1} = F_{N_c}^{H} D = F_{N_c}^{H} F_{N_c} X F_{N_{sym}}^{H} + F_{N_c}^{H} \omega = X F_{N_{sym}}^{H} + F_{N_c}^{H} \omega.
$$
 (8)

every row of the matrix Y_1 in (8). The resulting matrix Y The discrete Fourier transform (DFT) is calculated for in (9) directly represents a 2D RD map:

$$
\boldsymbol{Y} = \boldsymbol{Y}_1 \boldsymbol{F}_{N_{\text{sym}}} = \boldsymbol{X} + \boldsymbol{F}_{N_{\text{e}}}^{\text{H}} \boldsymbol{\omega} \boldsymbol{F}_{N_{\text{sym}}} = \boldsymbol{X} + \boldsymbol{\tilde{\omega}}. \tag{9}
$$

interesting targets expressed as $G_{\tau,f} \in \mathbb{R}^{P \times 2}$, where P rep- $G_{\tau,f}(p, :)=\left(g_{\tau}^p, g_{f}^p\right)(p \in [1, P])$ denotes the delay bin g_{τ}^p and Doppler shift g_f^p of the pth target. From the RD map given in (9), we can preliminarily obtain the delay and Doppler frequency parameters of the resents the total number of the point targets of interest.

3.2 Doppler parameters estimation

In this subsection, the high-precision estimation of the

becomes at most N_c parallel univariate estimation process. In (8), the *i*th row of the matrix Y_1 be expressed as Doppler frequency of the interested point target is mainly carried out. The multivariable estimation problem in (7)

$$
y_i = \sum_{k=1}^{K} c_k \exp(j2\pi f_k m T_s) \in \mathbf{C}^{1 \times N_{sym}} \tag{10}
$$

where c_k and $exp(j2\pi f_k mT_s)$ denote the coefficient and atom corresponding to the kth scattering center respectively, and f_k is the normalized frequency. For (10), the following continuous dictionary set is established:

$$
\mathcal{A} = \{ a(f) : f \in [0, 1] \}. \tag{11}
$$

Note that the method in $[24]$, the MMV at a single step while the joint processing exploiting in a unified framework can provide a sufficient condition for exact recovery. To facilitate the practical operation, we adopt the method of partial extraction of data, which can be expressed as

$$
\boldsymbol{Y}_{1_sub} = \boldsymbol{\varPsi}_p \boldsymbol{Y}_1 \tag{12}
$$

where $\Psi_p \in \mathbf{R}^{P \times N_c}$ denote a partial identity matrix, which *is* determined by delay bin $G_{\tau,f}(:, 1)$ obtained by (9). Therefore, the MMV-AN for the signal $Y_{1 \text{ sub}} \in \mathbb{C}^{P \times N_{sym}}$ can be defined as

$$
\left\| \boldsymbol{Y}_{1_sub} \right\|_{\mathcal{A}} = \inf_{\boldsymbol{c}_p, f_k} \left\{ \sum_p \left\| \boldsymbol{c}_p \right\|_2 : \boldsymbol{Y}_{1_sub} = \sum_p \boldsymbol{c}_p \boldsymbol{a}(f_{k,p}) \right\}.
$$
 (13)

where $\inf\{\cdot\}$ is the infimum

We can recover \hat{Y}_{1} _{sub} using the MMV-AN minimization as follows:

$$
\hat{\mathbf{Y}}_{1_sub} = \arg \min_{\hat{\mathbf{Y}}_{1_sub}} \left\| \hat{\mathbf{Y}}_{1_sub} \right\|_{\mathcal{A}}
$$
\ns.t.
$$
\left\| \hat{\mathbf{Y}}_{1_sub} - \mathbf{Y}_{1_sub} \right\|_{\mathrm{F}}^2 \leq \varepsilon.
$$
\n(14)

Or, equivalently, the semidefinite program

$$
\{\hat{Y}_{1_{sub}}, T(u)\} =
$$
\n
$$
\arg \min_{u, \hat{Y}_{1_{sub}}} \frac{\tau}{2} [\text{tr}(T(u)) + \text{tr}(\Phi)]
$$
\n
$$
\text{s.t.} \begin{bmatrix} \boldsymbol{\Phi} & \hat{Y}_{1_{sub}} \\ (\hat{Y}_{1_{sub}})^{\text{H}} & T(u) \end{bmatrix} \geq 0,
$$
\n
$$
\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\text{H}}, \left\| \hat{Y}_{1_{sub}} - Y_{1_{sub}} \right\|_{\text{F}}^2 \leq \varepsilon \tag{15}
$$

where $\tau = 1/\sqrt{N_{\text{sym}}}$ and tr(·) is the trace of the input matrix; $\boldsymbol{\Phi} \in \mathbb{C}^{P \times P}$ is a Hermitian matrix; $\boldsymbol{u} \in \mathbb{C}^{N_{\text{sym}}}$ and $T(u) \in \mathbb{C}^{N_{\text{sym}} \times N_{\text{sym}}}$ is the Toeplitz matrix of a vector u ; the notation \geq indicates that the matrix is positive semi-definite. The problem of the semidefinite program in (15) can be solved by ready-made solvers such as self-dual-minimization (SeDuMi) $[27]$ and SDPT3 $[28]$. However, these solvers are usually slow when solving large-scale data, so a reasonably efficient algorithm based upon the ADMM is used in this paper. Express our problem in the appropriate ADMM form, then (15) can be rewritten as

$$
\{\hat{Y}_{1_sub}, T(u)\} = \arg\min_{u, \hat{Y}_{1_sub}} \frac{1}{2} ||\hat{Y}_{1_sub} - Y_{1_sub}||_F^2 + \frac{\tau}{2} [\text{tr}(T(u)) + \text{tr}(\Phi)]
$$

s.t.
$$
\begin{cases} U = \begin{bmatrix} \Phi & \hat{Y}_{1_sub} \\ (\hat{Y}_{1_sub})^H & T(u) \end{bmatrix} \ge 0 \\ \Phi = \Phi^H \end{cases}
$$
 (16)

By dualize the equality constraint by augmented Lagrangian, there are

$$
L(\hat{\mathbf{Y}}_{1_sub}, u, \boldsymbol{\Phi}, \boldsymbol{\Lambda}, U) =
$$
\n
$$
\frac{1}{2} \left\| \hat{\mathbf{Y}}_{1_sub} - \mathbf{Y}_{1_sub} \right\|_{\mathrm{F}}^2 + \frac{\tau}{2} \left[\text{tr}(\boldsymbol{T}(u)) + \text{tr}(\boldsymbol{\Phi}) \right] +
$$
\n
$$
\left\langle \boldsymbol{\Lambda}, U - \left[\begin{array}{cc} \boldsymbol{\Phi} & \hat{\boldsymbol{Y}}_{1_sub} \\ (\hat{\boldsymbol{Y}}_{1_sub})^{\mathrm{H}} & \boldsymbol{T}(u) \end{array} \right] \right\rangle +
$$
\n
$$
\frac{\rho}{2} \left\| U - \left[\begin{array}{cc} \boldsymbol{\Phi} & \hat{\boldsymbol{Y}}_{1_sub} \\ (\hat{\boldsymbol{Y}}_{1_sub})^{\mathrm{H}} & \boldsymbol{T}(u) \end{array} \right] \right\|_{\mathrm{F}}^2 \qquad (17)
$$

penalty term, and ρ is the penalty term coefficient. We set $\rho = 2\delta \sqrt{N_{\text{sym}} \log_2 N_{\text{sym}}}$ and δ is the standard variance in (8). Φ , $\Lambda \in \mathbb{C}^{(P+N_{\text{sym}})\times (P+N_{\text{sym}})}$, and $U \in \mathbb{C}^{(P+N_{\text{sym}})\times (P+N_{\text{sym}})}$ are where the fourth term on the right side of (17) is the of the noise obtained from base statistics without targets Hermitian matrices, and

$$
\begin{cases}\nA = \begin{bmatrix}\nA_{P \times P} & A_{P \times N_{\text{sym}}} \\
A_{N_{\text{sym}} \times P} & A_{N_{\text{sym}} \times N_{\text{sym}}}\n\end{bmatrix} \\
U = \begin{bmatrix}\nU_{P \times P} & U_{P \times N_{\text{sym}} \\
U_{N_{\text{sym}} \times P} & U_{N_{\text{sym}} \times N_{\text{sym}}}\n\end{bmatrix}.\n\end{cases} (18)
$$

The ADMM [29] contains the following update steps:

$$
(\hat{Y}_{1_{sub}}^{t+1}, \boldsymbol{u}^{t+1}, \boldsymbol{\Phi}^{t+1}) =
$$

arg min_{Y_{1sub},**u**,**u**, **u**, **v**, **A**^t, **U**^t), (19)}

$$
\hat{Y}_{1_sub}^{t+1} = \frac{1}{2\rho + 1} (Y_{1_sub} + \rho U'_{P \times N_{sym}} + 2(A'_{N_{sym} \times P})^{H} + \rho (U'_{N_{sym} \times P})^{H}),
$$
\n(20)

$$
\boldsymbol{\Phi}^{t+1} = \frac{1}{2} \boldsymbol{U}_{P\times P}^t + \frac{1}{2} (\boldsymbol{U}_{P\times P}^t)^H + \frac{1}{\rho} (\boldsymbol{U}_{P\times P}^t - \frac{\tau}{2} \boldsymbol{I}_{P\times P}), \qquad (21)
$$

$$
\boldsymbol{u}^{t+1} = \frac{1}{\rho} \boldsymbol{\Gamma} \text{conj}(g(\boldsymbol{\Lambda}_{N_{\text{sym}} \times N_{\text{sym}}}) + \rho g(\boldsymbol{U}_{N_{\text{sym}} \times N_{\text{sym}}}) - \frac{\tau}{2} \boldsymbol{e}_1), \quad (22)
$$

$$
U^{t+1} = \arg\min_{\geq 0} L(\hat{Y}_{1_{\text{sub}}}^{t+1}, \boldsymbol{u}^{t+1}, \boldsymbol{\Phi}^{t+1}, \boldsymbol{\Lambda}^{t}, U^{t}), \qquad (23)
$$

$$
\Lambda^{t+1} = \Lambda^t + \rho \bigg(U^{t+1} - \bigg[\begin{array}{cc} \boldsymbol{\Phi}^{t+1} & \hat{Y}_{1 \text{ sub}}^{t+1} \\ (\hat{Y}_{1 \text{ sub}}^{t+1})^{\text{H}} & \boldsymbol{T}(\boldsymbol{u}) \end{array} \bigg] \bigg), \tag{24}
$$

where the superscript t represents the t th iteration; $conj(·)$ means taking conjugate for each element of the input matrix; $I_{p\times p}$ is a $p\times p$ indenfity matrix; $a = g(A)$ indicates that the matrix \boldsymbol{A} is converted into a vector \boldsymbol{a} , and $a_i = \text{sum}(A_{p,q} | q - p + 1 = i); e_1$ is the first column vector of the identity matrix; $\mathbf{\Gamma} = \text{diag}(1/N_{\text{sym}}, 1/\text{N}$ $N_{sym} - 1, \dots, 1$); \hat{Y}_{1_sub} can be obtained after iterative conto search the peak for each row of \hat{Y}_1 _{sub}. vergence. Finally, to obtain the Doppler parameter estimation of targets, the dual solution method in [21] is used

3.3 Delay parameters estimation

delay vector τ_{offset} is compensated on the reference signal, and the compensated signal in the *m*th OFDM symbol, the signal in the *n*th subcarrier can be expressed as After obtaining the accurate estimation of the Doppler frequency of targets, the RSC method is proposed to estimate their delay with high precision. Firstly, a known

$$
f_m^l[n] = s_m[n] \exp(j2\pi \tau_{\text{offset}}(l) f_n), \quad l = 1, 2, \cdots, L \tag{25}
$$

can be expressed as $\tau_{\text{offset}} = [(L-1)/L,(L-2)/L,\cdots,0]^T$, and L is equivalent to the sampling multiple. where the *l* is the delay vector index. The delay vector

Let $\boldsymbol{F}_{\text{offset}}$ be a three-dimensional (3D) matrix after RSC, shaded part in Fig. 2 is represented as $\mathbf{F}_{\text{offset}}(:, : , l)$ whose (n,m) th element is $f_m^l[n]$ in the *l*th layer matrix of $\mathbf{F}_{\text{offset}}$. and the structure is shown in Fig. 2. The matrix of the

Fig. 2 Structure of the matrix *F*offset

Let $\hat{\mathbf{D}} \in \mathbb{C}^{N_c \times N_{sym} \times L}$ be a new matrix with its entries the element-wise division of the received symbols $r_m[n]$ and compensated reference symbols $f_m^l[n]$, given as

$$
\hat{D}'_m[n] = \frac{r_m[n]}{f_m^l[n]}, \ l = 1, 2, \cdots, L. \tag{26}
$$

each layer of the 3D matrix $\hat{\boldsymbol{D}}$, one has Therefore, the influence of Doppler parameters should be eliminated firstly, performing DFT on the Doppler dimension. Formally, DFT is performed on each row in

$$
\hat{\bm{D}}_1(:,:,l)=\hat{\bm{D}}(:,:,l)\bm{F}_{N_{\text{sym}}},\ l=1,2,\cdots,L.
$$
 (27)

 $\hat{\boldsymbol{D}}_1$ (:, :, *l*) in (27), one has Then, performing IDFT on each column of the matrix

$$
\hat{\bm{D}}_2(:, :, l) = \bm{F}_{N_c}^{\rm H} \hat{\bm{D}}_1(:, :, l). \tag{28}
$$

The resulting matrix is $\hat{\mathbf{D}}_2 \in \mathbb{C}^{N_c \times N_{sym} \times L}$, to speed up pro-*L* = 10 and extend five delay bins forward and backward example, the rough estimated delay is g^p_τ of the target p, $[g_{\tau}^p - 5, g_{\tau}^p + 5]$ $(5 \le g_{\tau}^p \le N_c - 6)$. The signal to be estimated of the pth target can be expressed as cessing time and reduce the storage memory, we set respectively based on the rough estimated delay. For then the range of delay fine estimation is taken as

$$
\hat{\boldsymbol{D}}_2([g^p_\tau - 5, g^p_\tau + 5], g^p_f, :)\in \mathbb{C}^{11 \times 1 \times 10}.
$$
 (29)

Then the compensated signal of the *p*th target can be expressed as

$$
\boldsymbol{d}_p = \text{reshape}((\hat{\boldsymbol{D}}_2([g_r^p - 5, g_r^p + 5], g_f^p, :)^T), [\cdot], 1) \in \mathbf{C}^{110 \times 1}
$$
\n(30)

where reshape $(\cdot, [\cdot], 1)$ means converting the matrix into found in vector d_p . a column vector. Finally, the accurate delay estimation is

which contains two adjacent delay targets ($g_{\tau}^{p_1} = 51.2$, $g_{\tau}^{p_2} = 52$). As shown in Fig. 3, the compensated signal To emphasize the effectiveness of RSC, Fig. 3 gives the delay dimension signal after compensation by RSC, obtained by RSC has the following two advantages: (i) the peak value is closer to the real value, and the amplitude of target estimation is increased accordingly; (ii) it has potential advantages in near-neighbor target recognition. These illustrate the necessity of the RSC method for target time delay estimation in passive radar.

Fig. 3 Comparison of uncompensated and RSC compensated signals in delay dimension

4. Simulation analysis

In this section, we evaluate the coarse-fine joint estimation method based on AN-RSC using metrics of root mean square error (RMSE) of target parameter and computational complexity.

4.1 Simulation setup

To verify the effectiveness of the proposed method, one illuminator transmits the 4 quadrature amplitude modulation (4QAM)-modulated OFDM signal in the simulations, which contains two parts, namely the frame header (420 symbols) and frame body (3 780 symbols). The baseband sampling rate (equal bandwidth sampling) is 7.56 MHz. 180 OFDM blocks for signal processing are used, namely approximately 0.1 s coherent processing interval. The three targets are assumed to be point scatterers in our simulations, and the values of the simulation parameters are listed in Table 1.

Table 1 Simulation parameters (input SNR: SNRⁱ)

| Parameter | Target 1 | Target 2 | Target 3 |
|------------------|-----------|----------|----------|
| Delay bin | 50.55 | 102.93 | 155.36 |
| Doppler shift/Hz | -106.23 | -59.36 | 105.45 |
| SNR/dB | -30 | -25 | -15 |

Fig. 4(a)−Fig. 4(c) present the Doppler shifts estimation for three simulation objectives in Table 1 by MSD, MMV-AN, optimized-MUSIC (OP-MUSIC) [29], extended OMP (EOMP) [14], and RSC methods. One can see that as the target input SNR increases, the estimation results of these algorithms are closer to the true Doppler position of the target. Compared with the MSD method, the Doppler estimation accuracy of these methods is improved, which proves the correctness of these algorithms. From Fig. 4(a)−Fig. 4(c), compared with the OP-MUSIC and EOMP algorithm, the estimated values of the MMV-AN and RSC are all closer to the real position of the target, which proves the performance of the algorithm in estimation accuracy.

Fig. 4 High precision Doppler estimation of three targets in Table 1 using different methods

Fig. 5 gives the simulation results for high accuracy estimation of delay by MSD, MMV-AN, OP-MUSIC, EOMP, and RSC methods. Due to the block processing of the signal, the signal SNR loss, and the input SNR of Target 1 is as low as −30 dB, which results in incorrect time delay estimation by the OP-MUSIC algorithm in Fig. $5(a)$. In Fig. $5(c)$, the value of delay estimation of the OP-MUSIC algorithm is close to the true value of delay of Target 3. These prove that the estimation accuracy of the OP-MUSIC algorithm is affected by the SNR of the signal. Compared with the OP-MUSIC algorithm, the EOMP algorithm still has better estimation results in a low SNR environment. During the simulation, it is found that the estimation accuracy of the EOMP algorithm increases with the increase of the grid density, but the targets are split into many scatterers once the coherence of the sensing matrix increases, which may get a false target position. From Fig. 5(a)−Fig. 5(c), compared with the AN, OP-MUSIC, and EOMP algorithms, the estimated value of the RSC method is closest to the real position of the target, and the amplitude of the target estimation is improved compared with the MSD method, which benefits target detection.

Fig. 5 High precision delay estimation of three targets in Table 1 using different methods

Table 2 lists the values of parameter estimation using different methods. As can be seen from the Table 2, these algorithms can make the high-precision estimation of the time delay and Doppler of the target. In the Doppler dimension, the methods of MMV-AN and RSC have better estimation accuracy than other algorithms. In the delay dimension, the proposed RSC method has better performance despite the low SNR compared to other algorithms.

Table 2 Parameter estimation of different methods

| Parameter | Method | Target 1 | Target 2 | Target 3 |
|------------------|-----------------|-----------|----------|----------|
| Doppler shift/Hz | True value | -106.23 | -59.36 | 105.45 |
| | MSD | -110 | -60 | 110 |
| | MMV-AN | -106.09 | -59.53 | 105.43 |
| | OP-MUSIC | -105.89 | -58.63 | 105.41 |
| | EOMP | -106.1 | -59.5 | 105.4 |
| | RSC | -106.07 | -59.52 | 105.44 |
| Delay bin | True value | 50.55 | 102.93 | 155.36 |
| | MSD | 51 | 103 | 155 |
| | MMV-AN | 51.7 | 102.6 | 155.3 |
| | OP-MUSIC | 368.4 | 101.7 | 155.4 |
| | EOMP | 51.7 | 102.6 | 155.3 |
| | RSC | 50.6 | 102.9 | 155.4 |

In order to evaluate the performance of the proposed method and others, the off-grid values of delay and Doppler are uniformly distributed within [−0.5,0.5], and the input SNR varies from −35 dB to 0 dB with a step of 5 dB. For each SNR value, 2000 Monte Carlo (MC) simulations are performed with independently generated measurements. The RMSEs of the delay-Doppler are utilized to evaluate the performance. The delay RMSE and Doppler RMSE are defined as

$$
\text{delay_RMSE}_{i}^{q} = \sqrt{\frac{1}{M_{\text{c}}}\sum_{m=1}^{M_{\text{c}}}\left(t_{i}^{q}-\hat{t}_{i,m}^{q}\right)^{2}},\qquad(31)
$$

Doppler_RMSE_i^q =
$$
\sqrt{\frac{1}{M_c} \sum_{m=1}^{M_c} (f_i^q - \hat{f}_{i,m}^q)^2}
$$
, (32)

where $\hat{\mathbf{t}}_{i,m}^q$ and $\hat{\mathbf{f}}_{i,m}^q$ denote the delay and Doppler estimates of the qth target at scan i in the *m*th MC run; t_i^q and f_i^q denote the true values of the delay and Doppler estimates of the q th target at scan i . In this paper, we randomly generate a target so that $q = 1$. $M_c = 2000$ represents the number of MC simulations.

The RMSEs using different estimation methods under different SNR conditions are shown in Fig. 6 . Fig. $6(a)$ and Fig. 6(b) show the delay RMSE and Doppler RMSE of the different SNR conditions, respectively. As shown in Fig. 6(a), the performance of the OP-MUSIC algorithm is easily affected by SNR. When the input SNR of the signal is greater than −30 dB, the estimation accuracy of the MMV-AN, OP-MUSIC, EOMP, and RSC methods is higher than that of the MSD method, which can

be used for fine estimation. Because the estimation accuracy of the signal is mainly affected by SNR and grid density, when the grid is sparse, the estimation accuracy is mainly affected by grid density. When the grid density is large enough, SNR is the main factor of accuracy. Therefore, the estimation accuracy of MSD remains almost unchanged. Compared with the OP-MUSIC, EOMP algorithms, the Doppler RMSEs of MMV-AN and RSC methods are lower, and they are close to the Cramer-Rao bounds (CRBs). Compared with the RSC method, when $SNR < -15$ dB, the accuracy of MMV-AN is equivalent to that of RSC. When the SNR is large enough, the estimation accuracy of MMV-AN is higher, and the accuracy of RSC tends to the grid density boundary. As shown in Fig. 6 (b), compared with other methods, the RSC method has the highest accuracy and stable robustness in delay estimation. The delay estimation accuracy of MMV-AN and OP-MUSIC algorithms is easily affected by signal noise, while the delay RMSEs of EOMP, MSD, and RSC methods remain basically unchanged. Due to the coherence of the sensing matrix, the estimation accuracy of the EOMP algorithm is limited. The accuracy of MSD and RSC methods is mainly related to the grid density, and the MSD is a special form of RSC (i.e., no signal compensation, compensation step $L = 0$). By increasing the step size, the estimation accuracy of the RSC method is greatly improved. In addition, in the simulations, the search accuracy of all algorithms is set to 0.1 delay unit, which results in the accuracy of these algorithms being limited by the grid density and not close to the CRB. It should be noted that increasing the search accuracy of the algorithm can improve the estimation accuracy, which is closer to CRB, but it also greatly increases the computational complexity of the algorithm.

Fig. 6 2000 MC simulation results for each SNR

4.2 Computational complexity

The computational complexities of the method for obtaining parameter estimates come from the following four stages: signals conversion from the time domain to the symbol domain, OFDM channel response estimation, target Doppler dimension estimation, and target delay dimension estimation. The first two stages of these methods are exactly the same, except for the third and fourth stages. Therefore, only the complexity analysis of parameter estimation is considered below.

via the ADMM, which requires $O((N_cN_{sym})^3)$; the other is targets by dual solution, which requires $O(N_f N_\tau N_c N_{sym})$, where N_f and N_τ are the number of Doppler and delay ters requires $O(N_c N_{sym}(\log_2(N_{sym}) + \log_2(N_c)))$. Then, the computational complexity as follows: $O((N_{sym} + P)^3)$ + $O(PN_fN_{sym})$, $O(N_fN_{sym})+O(PG_fN_fN_{sym})$, $O(PN_{sym}^2)$ + $O(2PN_{sym}^3) + O(PN_f(2N_{sym}(N_{sym}-K_f)+N_{sym}))$, and $O(2L_fN_{sym}^3)$ $N_{sym}(1 + \log_2(N_{sym}) + 0.5\log_2(N_c)))$, where G_f is the number of iterations, K_f represents the number of frequency sources, and L_f is equivalent to the sampling multiple The computational complexity of 2D-AN [21,30] is mainly reflected in two aspects: one is to solve the SDP to jointly estimate the delay and Doppler parameters of grids respectively. In this paper, the computational complexity of the coarse-fine joint estimation method is mainly reflected in three aspects: the coarse estimation of target parameters using (8) and (9), estimation of target Doppler parameters, and estimation of target delay parameters. Firstly, the coarse estimation of target parame-MMV-AN, EOMP, OP-MUSIC, and RSC methods are used to estimate the Doppler parameters respectively, and in Doppler domain. Finally, the parameter estimation of the delay dimension with MMV-AN, EOMP, OP-MUSIC, and RSC methods, respectively. It is not difficult to find that the complexity of the delay dimension of MMV-AN, EOMP, OP-MUSIC, and RSC methods are basically the same as the complexity expression of the Doppler dimension, except that the value of dimension and grid is different. The computational complexity of the

 RSC method requires $O(L_rN_cN_{sym}log_2(N_c))$ + $O(2L_rN_cN_{sym}$. $(log_2(N_cN_{sym})+log_2(N_{sym})))$, which is related to the value of L and coarse estimation. The overall computational complexities of the different methods of solving the third and fourth stages are concluded in Table 3.

| Parameters to be estimated | Method | Computational cost | |
|----------------------------|-----------------|--|--|
| Delay and Doppler | $2D-AN$ | $O((N_cN_{\rm sym})^3 + N_fN_{\tau}N_cN_{\rm sym})$ | |
| | MSD | $O(N_c N_{\text{sym}} (\log_2(N_{\text{sym}}) + \log_2(N_c)))$ | |
| Delay | RSC | $O(2L_{\tau}N_{\rm c}N_{\rm sym}(\log_2(N_{\rm c}N_{\rm sym})+\log_2(N_{\rm sym})+0.5\log_2(N_{\rm c})))$ | |
| | MMV-AN | $O((N_c + P)^3 + PN_{\tau}N_c)$ | |
| | EOMP | $O(N_\tau N_c + PG_\tau N_\tau N_c)$ | |
| | OP-MUSIC | $O(PN_c^2 + 2PN_c^3 + PN_{\tau}(2N_c(N_c - K_{\tau}) + N_c))$ | |
| Doppler | RSC | $O(2L_fN_cN_{\rm sym}(1+\log_2(N_{\rm sym})+0.5\log_2(N_{\rm c})))$ | |
| | MMV-AN | $O((N_{\text{sym}} + P)^3 + PN_f N_{\text{sym}})$ | |
| | EOMP | $O(N_fN_{\text{sym}}+PG_fN_fN_{\text{sym}})$ | |
| | OP-MUSIC | $O(PN_{\text{sym}}^2 + 2PN_{\text{sym}}^3 + PN_f(2N_{\text{sym}}(N_{\text{sym}} - K_f) + N_{\text{sym}}))$ | |

Table 3 Comparison of the computational complexity of different methods

is given via a study case. Assume that $N_c = 3780$ and $N_{sym} = 180$ and that the parameter configuration for the AN-RSC is $P = 1$, $G = 3$, $N_f = 1.8 \times 10^5$, $L_f = 1000$, and L_{τ} = 10. The computational complexity of the coarse-fine joint method based on AN-RSC is 4.9799×10^8 . For algorithm is 3.1550×10^{17} , and its parameter configuration $N_{\tau} = 4\,200$. An intuitive comparison for computational complexity comparison, the computational complexity of the 2D-AN

MMV-AN is 3.8330×10^7 . The complexity of OP-MUSIC is 1.164×10^{10} with its parameter configuration $K_f = 1$. Due to the coherence of the sensing matrix, parameter configuration for the EOMP is $N_f = 1.8 \times 10^4$, and $G_f = G_\tau = 3$. So, the complexity of EOMP is $1.296\,0 \times 10^7$, and that of RSC is $1.964\,2 \times 10^{10}$. It can be AN, EOMP, OP-MUSIC and RSC are 5.4069×10^{10} , $6.350\,4 \times 10^7$, $1.740\,3 \times 10^{11}$, and $4.464\,8 \times 10^8$. It can be It can be concluded that the complexity of the coarsefine joint estimation method is lower than that of 2D-AN. In the Doppler estimation dimension, the complexity of the grid size of the EOMP algorithm is limited, and the seen that the complexity of RSC is the largest. Therefore, compared with the RSC method, the MMV-AN algorithm is more suitable for Doppler estimation. In the delay estimation dimension, the complexity of MMVseen that the algorithm complexity of EOMP is the smallest, OP-MUSIC is the largest, and RSC is close to EOMP. Thus, the coarse-fine joint estimation method based on AN-RSC is attractive in computational load for real-time implementation.

5. Conclusions

In this paper, the delay and Doppler of targets in OFDMbased passive radar are investigated. In order to reduce the complexity of joint estimation parameters, we have proposed a coarse-fine joint estimation method, in which the MMV-AN is used to perform sparse reconstruction of Doppler dimension, and then, the RSC method is used to perform sparse reconstruction of range dimension. Compared to previous OP-MUSIC and EOMP algorithms, we can achieve high-precision parameter estimation of the target scenario and do not suffer from the heavy computational burden. The effectiveness of the proposed method is verified by simulated data. In future work, the influence of adjacent targets on their parameter estimation will be taken into consideration. The high resolution of adjacent targets using the OFDM-based passive radar system is also an interesting topic.

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