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Efficient unequal error protection for online fountain codes

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Abstract: In this paper, an efficient unequal error protection (UEP) scheme for online fountain codes is proposed. In the build-up phase, the traversing-selection strategy is proposed to select the most important symbols (MIS). Then, in the completion phase, the weighted-selection strategy is applied to provide low overhead. The performance of the proposed scheme is analyzed and compared with the existing UEP online fountain scheme. Simulation results show that in terms of MIS and the least important symbols (LIS), when the bit error ratio is 10^{-4}, the proposed scheme can achieve 85% and 31.58% overhead reduction, respectively.

Keywords: online fountain code, random graph, unequal error protection (UEP), rateless code.

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1. Introduction

Channel coding includes fixed rate codes and rateless codes. The most studied fixed rate codes are low density parity check (LDPC) codes [1,2] and polar codes [3,4]. Fountain codes, also called rateless codes, are a class of erasure correction codes with the property that a potentially infinite coded symbols can be generated from k source symbols. The receiver can decode the source symbols successfully when it receives $k(1+\varepsilon)$ output symbols, where ε is the overhead. The first practical realization of fountain codes is Luby transform (LT) codes [5], which were proposed by Luby in 2002. After that, researchers have studied the overhead performance [6] and application scenarios [7,8] of LT codes. The unequal error protection (UEP) [9] and intermediate performance [10,11] have also been studied. In 2006, Shokrollahi proposed Raptor codes [12]. Due to its excellent performance, Raptor codes have been greatly developed [13,14]. After that, other rateless codes were also proposed, such as spinal codes [15] and analog fountain codes [16].

Fountain codes are designed to transmit data without feedback. During the decoding process, the current decoding state changes constantly. Therefore, the degree generated from the degree distribution is not optimal, which results in high overhead and low decoding efficiency. Based on these, fountain codes with feedback have been studied in [17–20]. The scheme in [17] used the feedback to inform the transmitter the most useful information symbols. In [18], Hashemi et al. proposed a scheme that applies the nonuniform selection distribution based on the feedback to provide intermediate performance. Based on these work, online fountain codes were proposed by Cassuto et al. [21]. Due to low overhead and low complexity, more and more researchers have begun to study online fountain codes [22–27].

The UEP property is necessary for several applications where some part of data needs more protection. For example, in H.264 video transmission, the transmitted data includes I-frame data and P-frame data. The I-frame data is more important than P-frame data, so the I-frame data needs more protection than P-frame data. In addition, in virtual reality (VR) video transmission, the data in the field of view (FOV) requires lower bit error ratio (BER) than data in non-FOV. Therefore, the research on UEP is valuable. Fountain codes with UEP property were studied in [28] and [29]. These two strategies are important for fountain codes. However, for online fountain codes, due to its staged coding structure, the UEP scheme needs to be specially designed according to the stage. Online fountain codes with UEP were first studied in [30]. The weighted-selection strategy and expanding windows strategy are applied at the build-up phase and the completion phase, respectively. However, the overhead of the scheme is high and the BER performance is poor. An efficient UEP scheme is necessary to make online fountain codes better used in video transmission. Therefore, we propose a novel scheme to improve the UEP performance of online fountain codes.

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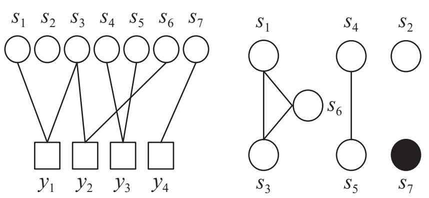
In our proposed method, in the build-up phase, a non-random selection strategy is applied in the most important symbols (MIS) region. This increases the proportion of MIS in the largest component at the end of the build-up phase. Therefore, it increases the proportion of MIS in all recovered symbols. At the same time, the average degree of the MIS outside the largest component increases. Thus, at the completion phase, the probability of becoming a neighbor of useful symbols increases. As a result, the probability of MIS being decoded increases, and the BER performance is better. At the completion phase, based on the analysis of the useful symbol's generation probability, the weighted-selection scheme is adopted to increase the useful symbol's generation probability when the coded symbols are generated in the least important symbols (LIS) region. Thus the overall overhead is reduced.

The rest of this paper is organized as follows. Section 2 introduces the online fountain codes with UEP. The proposed UEP scheme is presented in Section 3. Section 4 provides an analytical method for the proposed UEP scheme. The simulation results are given in Section 5. Section 6 concludes the paper.

2. Online fountain with UEP

In this section, we review the online fountain codes and UEP online fountain scheme. The uni-partite graph was introduced in [21] to represent the decoding state. As shown in Fig. 1, the circle nodes represent the variable nodes (source symbols), and the check nodes (coded symbols) are represented by the square nodes. In the bi-partite graph, if a coded symbol is the exclusive OR (XOR) of two source symbols, the coded symbol and the corresponding source symbol are connected by one edge. While in the uni-partite graph, only the source symbols are involved and represented by the circle nodes. Two nodes are connected with an edge if there is a coded symbol that is the XOR of the corresponding two input symbols. A component means a set that any two input nodes in this set are connected by an edge. If an input symbol is decoded by the decoder, it is colored black. Therefore, a component is decoded when one input symbol in this set is colored black.



(a) Bi-partite graph (b) Uni-partite graph

Fig. 1 A bi-partite graph and the corresponding uni-partite graph

The decoder receives the coded symbols and updates the uni-partite graph. Then the decoder feeds back the current decoding state to the encoder. The encoder gets the optimal degree based on the feedback. For the decoder, after XOR with the input symbols in the decoded component, the symbols of degree 1 and degree 2 are useful. Others are discarded. Obviously, a useful symbol belongs to the following two cases.

Case 1: A degree m received symbol is generated by the XOR of a single white symbol and $m-1$ black symbols.

Case 2: A degree m received symbol is generated by the XOR of two white symbols and $m-2$ black symbols.

The encoding process is divided into two phases, the build-up phase and the completion phase. A brief description is given as follows.

(i) Build-up phase

This phase consists of two steps. In the first step, the encoder generates degree 2 coded symbols and sends them to the receiver side. The objective is to build the largest component in the uni-partite to a size of $k\beta_0$ $(0<\beta_0<1)$, where k is the number of source symbols. In the second step, the encoder colors the largest component in black by sending degree 1 symbols.

In the UEP scheme, we assume that the source symbols are divided into two subsets with different importance, MIS and LIS. The number of MIS is denoted by $k\xi_1$, and the number of LIS is denoted by $k\xi_2$. Then we know $\xi_2 = 1-\xi_1$. The weighted-selection strategy is applied in the first step to select the source symbols in MIS and LIS. The two symbols can be selected within MIS and LIS separately or both in MIS and LIS with probabilities q_1, q_2, and q_3. While the second step in the UEP scheme is similar to online fountain codes.

(ii) Completion phase

In this phase, once the instantaneous decoding state changes, the decoder will feed back the current decoding state to the transmitter. The ratio of the number of current decoded symbols to the number of source symbols is denoted by β. The sum probability of Case 1 and Case 2 is denoted by $P_1(m,\beta)+P_2(m,\beta)$, where m is the degree of the coded symbols, and $P_1(m,\beta)$ and $P_2(m,\beta)$ are the probability of Case 1 and Case 2, respectively. The sum probability is a function of β and as the value of β increases, the sum probability becomes smaller. The optimal degree $\hat{m}$ satisfies

$$\hat{m} = \arg\max_{m} [P_1(m,\beta)+P_2(m,\beta)]. \tag{1}$$

$P_1(m,\beta)$ and $P_2(m,\beta)$ can be evaluated as follows:

$$P_1(m,\beta) = \binom{m}{1}\beta^{m-1}(1-\beta), \tag{2}$$

$$P_2(m,\beta) = \binom{m}{2}\beta^{m-2}(1-\beta)^2. \qquad (3)$$

The completion phase proceeds until successful decoding is achieved.

In the UEP scheme, the expanding-window strategy is applied at the completion phase. We set w_1 and w_2 for MIS and MIS+LIS, respectively. The encoder selects the window by the probability distribution $\theta(w) = \sum_{i=1}^{n} \theta_i w_i$ $(n = 2)$, where θ_1 and θ_2 represent the probability of selection at w_1 and w_2, satisfying $\theta_1 + \theta_2 = 1$. The decoder needs to feed back the current decoding state of MIS and LIS.

However, the full decoding overhead is high and the BER performance of MIS and LIS is poor. To solve these problems, we propose an efficient UEP scheme for online fountain codes described in detail in Section 3.

3. Proposed scheme with efficient UEP

In this section, we introduce the efficient UEP scheme for online fountain codes. The proposed scheme is divided into two parts to achieve UEP property. In the build-up phase, the encoder gets a random number q and decides to select two symbols from MIS, LIS, or MIS+LIS based on the probabilities q_1, q_2, and q_3. If the encoder selects symbols from LIS and MIS+LIS, the selection strategy is random. The traversing-selection strategy is applied when the selected symbols are from MIS. We set M_1 for the symbols in MIS that have not been selected once. The symbols in MIS that have been encoded is stored in M_2. The number of symbols in M_i is represented by $|M_i| (i = 1, 2)$. In the encoding process, the encoder selects two symbols from M_1 and moves these two symbols to M_2. If $|M_1| = 1$, the output symbol is encoded by two input symbols, one is selected from M_1, and the other is selected from M_2. Then the encoder moves the symbol in M_1 to M_2. If M_1 is empty, the encoder selects two symbols from M_2. The encoding process continues until the encoder receives the acknowledge (ACK) from the decoder. We initialize ACK = 0, the M_1 to all the source symbols, and the M_2 to the empty. The detail description is shown in Algorithm 1.

Algorithm 1 Encoding of proposed scheme in the first phase

while ACK = 0 do
  Generate a random number q ($q \in [0,1]$);
  if $q < q_1$ then
    if $|M_1| \geqslant 2$ then
      Select symbols s_i and s_j randomly, and s_i, $s_j \in M_1$
      Store s_i and s_j in M_2 and delete them from M_1;
    else if $|M_1| = 1$
      Select symbols s_i and s_j, and $s_i \in M_1$, $s_j \in M_2$;
      Store s_i in M_2 and delete it from M_1;
    else if $|M_1| = 0$
      Select symbols s_i and s_j, and s_i, $s_j \in M_2$;
    end if
  else if $q_1 \leqslant q < q_1 + q_2$
    Select s_i and s_j randomly from LIS;
  else
    Select s_i and s_j from MIS and LIS, respectively;
  end if
  Generate the output symbol y by the XOR of s_i and s_j, $y \leftarrow s_i \oplus s_j$
  Send y to the receiver;
  if receive ACK from decoder then
    ACK ← 1;
  else
    ACK ← 0;
  end if
end while

As discussed in [22], the average degree of input symbols is less than 1 at the end of the build-up phase. Thus we apply this strategy to provide useful symbols with probability 1 at the start of the build-up phase. If we apply this strategy in LIS, more LIS symbols will be included in the largest component. Then the BER performance of MIS will be poor. Thus we select the symbols in LIS randomly.

At the completion phase, we apply the weighted-selection strategy. We know that as the value of β increases, the sum probability becomes smaller. Thus the probability that a symbol belongs to the useful symbol becomes smaller. At the completion phase, we assume the recovery ratio of MIS, LIS, and MIS+LIS is β_1, β_2, and β_3, respectively. $\beta_1 > \beta_2$ and

$$\beta_2 = \frac{\beta_3 - \xi_1\beta_1}{\xi_2} \qquad (4)$$

where $\beta_3 - \beta_2 > 0$. If a symbol is assigned to LIS, the probability of it belonging to the useful symbol is bigger when the encoder selects the degree based on β_2 instead of β_3. Thus we apply the weighted-selection strategy at the completion phase. Notice that if all the symbols in MIS are recovered, the coded symbols are generated from LIS.

4. Performance analysis

In this section, we analyze the performance of the proposed scheme by the analysis method introduced in [22]. We first analyze the performance in lossless channels. Then in a similar way, we analyze the performance of the proposed scheme in lossy channels.

4.1 Performance analysis in lossless channels

In the build-up phase, we calculate the average degree value of MIS symbols and LIS symbols, denoted by c_1 and c_2 respectively. In addition, the ratios of MIS symbols β_{01} and LIS symbols β_{02} to the largest component can also be calculated at the end of the build-up phase. Then we can calculate the expected number of output symbols N_{build}. At the completion phase, through two corollaries, we can calculate the expected number of output symbols required to decode MIS and LIS symbols, respectively. Then we can analyze the overhead performance of the proposed scheme in lossless channels.

At the end of the build-up phase, the decoding graph $\mathcal{G} = G(k, c/k)$ is a random graph based on k vertices, where c is the average degree of an input symbol. Then we get a lemma as follows based on the random graph.

Lemma 1 [21] In a random graph $\mathcal{G} = G(k, c/k)$, given the ratio of largest component, the relationship between c and β_0 satisfies

$$\beta_0 + \mathrm{e}^{-c\beta_0} = 1 \tag{5}$$

where for each specified $\beta_0 < 1$, there is a unique parameter $c > 1$ that achieves it with high probability.

At the end of the build-up phase, we assume that the ratio of the largest component in MIS and LIS is β_{01} and β_{02}. We can know

$$\xi_1\beta_{01} + \xi_2\beta_{02} = \beta_0. \tag{6}$$

Because the symbols in LIS are uniformly selected at random, we can get

$$\beta_{02} + \mathrm{e}^{-c_2\beta_{02}} = 1. \tag{7}$$

For the symbols in MIS, after each of them is selected once, the encoder uniformly selects the symbols in MIS at random. We regard the two input symbols that have been connected as one node. Therefore, we get a random graph $\mathcal{G}' = G(k\xi_1/2, 2c_3/k\xi_1)$ with $k\xi_1/2$ vertices, where c_3 is the average degree of the nodes in MIS. Because the ratio of largest component in MIS is β_{01}, the ratio of the largest component in the random graph is also β_{01}. From Lemma 1, we can get

$$\beta_{01} + \mathrm{e}^{-c_3\beta_{01}} = 1. \tag{8}$$

Consider the relationship between c_1, c_2, and c_3. Firstly, based on the selection probabilities q_1, q_2, and q_3, we get the relationship between c_1 and c_2:

$$\frac{c_1}{c_2} = \frac{q_1\xi_2}{q_2\xi_1}. \tag{9}$$

Then the relationship between c_1 and c_3 satisfies

$$1 + \frac{c_s}{2} = c_1. \tag{10}$$

Combining (6)−(10), we get the following lemma.

Lemma 2 Given the selection probabilities q_1, q_2, and q_3, the ratio of the largest component β_0, we can get β_{01}, β_{02}, c_1, c_2, and c_3 as follows:

$$\begin{cases} \xi_1\beta_{01} + \xi_2\beta_{02} = \beta_0 \\ \beta_{02} + \mathrm{e}^{-c_2\beta_{02}} = 1 \\ \beta_{01} + \mathrm{e}^{-c_3\beta_{01}} = 1 \\ 1 + \dfrac{c_3}{2} = c_1 \\ \dfrac{c_1}{c_2} = \dfrac{q_1\xi_2}{q_2\xi_1} \end{cases}. \tag{11}$$

We can calculate the expected number of output symbols at the end of the build-up phase as

$$N_{\text{build}} = \frac{1}{2}k(\xi_1 c_1 + \xi_2 c_2) + \frac{1}{\beta_0}. \tag{12}$$

Then we analyze the performance of MIS and LIS at the completion phase, respectively. We assume that the selection probabilities of MIS and LIS are θ_1' and θ_2' ($\theta_1' + \theta_2' = 1$). We introduce the following lemma.

Lemma 3 [22] Denote by P the probability that an encoded symbol is a Case 1 or Case 2 symbol at the completion phase. The number of recovery symbols is represented by n. The degree value of the encoded symbol is represented by d, and $\hat{d}$ is the optimal degree value when n is determined. Then P can be calculated as follows:

$$P(n) = P_1\left(\hat{d}, \beta_0 + \frac{n}{k}\right) + P_2\left(\hat{d}, \beta_0 + \frac{n}{k}\right) \tag{13}$$

where

$$\hat{d} = \arg\max_d \left[P_1\left(d, \beta_0 + \frac{n}{k}\right) + P_2\left(d, \beta_0 + \frac{n}{k}\right)\right],$$

$$P_1(d, n) = \binom{d}{1}\left(\beta_0 + \frac{n}{k}\right)^{d-1}\left[1 - \left(\beta_0 + \frac{n}{k}\right)\right],$$

$$P_2(d, n) = \binom{d}{2}\left(\beta_0 + \frac{n}{k}\right)^{d-2}\left[1 - \left(\beta_0 + \frac{n}{k}\right)\right]^2.$$

Based on Lemma 3, we denote by P_m the probability that an encoded symbol in MIS is a useful symbol at the completion phase. Then $P_m(n) = P_1\left(\hat{d}, \beta_{01} + \dfrac{n}{k\xi_1}\right) + P_2\left(\hat{d}, \beta_{01} + \dfrac{n}{k\xi_1}\right)$.

Lemma 4 [22] Denote by N_{comp} the expected number of symbols at the completion phase. We can get

$$N_{\text{comp}}(n) = \left[1 - \frac{1}{2}(1 - \beta_0)c\right]\sum_{i=1}^{n}\frac{1}{P(i)}. \tag{14}$$

Based on Lemma 3 and Lemma 4, we can calculate the expected number of output symbols to recover the MIS and LIS symbols at the completion phase through the following corollaries.

Corollary 1 For MIS, with given k, β_{01}, c_1, and ξ_1, denote by $N_{m,\text{comp}}$ the expected number of output symbols to recover the MIS at the completion phase, we can get

$$N_{m,\text{comp}}(s) = \left[1 - \frac{1}{2}c_1(1-\beta_{01}) - \frac{1}{2}\beta_{01}\right] \sum_{i=1}^{s-k\xi_1\beta_{01}} \frac{1}{P_m(i)} \quad (15)$$

where s is the recovery symbols in MIS and $k\xi_1\beta_{01} < s \leqslant k\xi_1$.

Proof We first calculate the average degree of MIS symbols which is not included in the largest component at the end of the build-up phase. Denote it by d_m, we can calculate d_m as follows:

$$d_m = \frac{1}{2}(1-\beta_{01})c_3 + 1 = c_1(1-\beta_{01}) + \beta_{01}. \quad (16)$$

As analyzed in [22], the decoder receives a useful symbol, then on average one source symbol is recovered at the completion phase. Thus for MIS, the useful symbols that the decoder needs are denoted by $N_{m,\text{Ca12}}$. We can get

$$N_{m,\text{Ca12}}(n) = n - \frac{1}{2}nd_m = \left[1 - \frac{1}{2}c_1(1-\beta_{01}) - \frac{1}{2}\beta_{01}\right]n, \quad 0 < n \leqslant k\xi_1(1-\beta_{01}) \quad (17)$$

where n is the number of recovery symbols at the completion phase.

From Lemma 4, N_{comp} can be calculated as follows:

$$N_{m,\text{comp}}(n) = \left[1 - \frac{1}{2}c_1(1-\beta_{01}) - \frac{1}{2}\beta_{01}\right] \sum_{i=1}^{n} \frac{1}{P_m(i)}. \quad (18)$$

Then we can get

$$N_{m,\text{comp}}(s) = \left[1 - \frac{1}{2}c_1(1-\beta_{01}) - \frac{1}{2}\beta_{01}\right] \sum_{i=1}^{s-k\xi_1\beta_{01}} \frac{1}{P_m(i)}.$$

□

Then we can get the expected number of output symbols that the decoder receives to recover the MIS symbols at the completion phase. Denote it by $N_{tM,\text{comp}}$, we can get

$$N_{tM,\text{comp}}(s) = \frac{N_{m,\text{comp}}(s)}{\theta_1'}. \quad (19)$$

Therefore, the expected number that the decoder receives to recover the MIS symbols can be calculated as follows:

$$N_{tm}(s) = N_{\text{build}} + N_{tM,\text{comp}}(s). \quad (20)$$

Let $s = k\xi_1$, we can get the overhead of MIS.

For LIS, we calculate the average degree of LIS symbols, which is not included in the largest component at the end of the build-up phase. Denote it by d_l, then

$$d_l = c_2(1-\beta_{02}). \quad (21)$$

Similar to the analysis of MIS, Corollary 2 is introduced as follows.

Corollary 2 For LIS, with given k, β_{02}, c_2 and ξ_2, denote by $N_{l,\text{comp}}$ the expected number of output symbols to recover the LIS at the completion phase, we can get

$$N_{l,\text{comp}}(s') = \left[1 - \frac{1}{2}c_2(1-\beta_{02})\right] \sum_{i=1}^{s'-k\xi_2\beta_{02}} \frac{1}{P_l(i)} \quad (22)$$

where $P_l(n) = P_1\left(\hat{d}, \beta_{02} + n/(k\xi_2)\right) + P_2\left(\hat{d}, \beta_{02} + n/(k\xi_2)\right)$, s' is the recovery symbols in LIS and $k\xi_2\beta_{02} < s' \leqslant k\xi_2$.

The proof of Corallary 2 is similar to that of Corollary 1.

Then the expected number the decoder receives to recover the LIS symbols can be calculated as follows:

$$N_{\text{ta}}(s') = N_{\text{build}} + \frac{N_{l,\text{comp}}(s')}{\theta_2'}. \quad (23)$$

Based on Corollary 1 and Corollary 2, we can calculate the performance of MIS and LIS in lossless channels. Notice that if the symbols in MIS are full recovered, the encoded symbols are assigned to LIS.

4.2 Performance analysis in lossy channels

In this subsection, we analyze the performance of the proposed UEP scheme in lossy channels. We focus on the first step of the selection in MIS since this step is not random. At the end of the first step, there are $1/2k\xi_1(1-\varepsilon)$ size-2 nodes and $k\xi_1\varepsilon$ size-1 nodes. The size-1 nodes have little effect on the largest component, so we ignore them. Denote by β_{03} the ratio of size-2 nodes included in the largest component, we can get $\beta_{03} = \beta_{01}/(1-\varepsilon)$ and $\beta_{03} + \mathrm{e}^{-c_3\beta_{03}} = 1$.

Denote by $N_{m,b}$ and $N_{l,b}$ the expected number of symbols in MIS and LIS required to build the largest component, respectively. We can get $N_{m,b}/q_1 = N_{l,b}/q_2$. Obviously, $N_{l,b} = k\xi_2 c_2/(2(1-\varepsilon))$, $N_{m,b}$ can be calculated in [22] as follows:

$$N_{m,b} = \frac{1}{2}k\xi_1 + \frac{k\xi_1 c_3}{4(1-\varepsilon)}. \quad (24)$$

We simplify $N_{m,b}/q_1 = N_{l,b}/q_2$. Then we get β_{01}, β_{02}, c_1, c_2 and c_3 as below:

$$\begin{cases} \xi_1\beta_{01} + \xi_2\beta_{02} = \beta_0 \\ \beta_{02} + \mathrm{e}^{-c_2\beta_{02}} = 1 \\ \beta_{03} + \mathrm{e}^{-c_3\beta_{03}} = 1 \\ \dfrac{c_1}{c_2} = \dfrac{q_1\xi_2}{q_2\xi_1} \\ \beta_{03} = \dfrac{\beta_{01}}{1-\varepsilon} \\ 2\xi_1 q_2(1-\varepsilon) + \xi_1 c_3 q_2 = 2\xi_2 c_2 q_1 \end{cases} . \quad (25)$$

Then we can calculate the expected number of coded symbols to end the build-up phase as follows:

$$N_{\text{build},\varepsilon} = \frac{k\xi_1 c_1 + k\xi_2 c_2}{2(1-\varepsilon)} + \frac{1}{\beta_0(1-\varepsilon)}. \tag{26}$$

At the completion phase, we first calculate the average degree of input symbols in MIS that is not included in the largest component [22] as below:

$$d_{m,\varepsilon} = \left[\frac{1}{2}(1-\beta_{03})c_3 + 1\right]\frac{(1-\varepsilon)(1-\beta_{03})}{(1-\varepsilon)(1-\beta_{03})+\varepsilon} + \frac{1}{2}c_3\frac{\varepsilon}{(1-\varepsilon)(1-\beta_{03})+\varepsilon}. \tag{27}$$

The average degree of input symbols in LIS that is not included in the largest component is $d_{l,\varepsilon} = c_2(1-\beta_{02})$. Because the selection is random at the completion phase, the analysis in lossless channels can be applied to lossy channels. Based on Lemma 4, we denote the expected number of output symbols by $N_{m,\text{comp},\varepsilon}$ and $N_{l,\text{comp},\varepsilon}$ to recover the MIS and LIS at the completion phase, then

$$N_{m,\text{comp},\varepsilon} = \frac{1-\frac{1}{2}d_{m,\varepsilon}}{1-\varepsilon}\sum_{i=1}^{s-k\xi_1\beta_{01}}\frac{1}{P_m(i)}, \tag{28}$$

$$N_{l,\text{comp},\varepsilon} = \frac{1-\frac{1}{2}d_{l,\varepsilon}}{1-\varepsilon}\sum_{i=1}^{s'-k\xi_2\beta_{02}}\frac{1}{P_l(i)}. \tag{29}$$

Similar to the analysis in lossless channels, we denote the expected number that the decoder receives by $N_{tM,\varepsilon}$ and $N_{ta,\varepsilon}$ to recover the MIS and LIS symbols, then

$$N_{tM,\varepsilon}(s) = N_{\text{build},\varepsilon} + \frac{N_{m,\text{comp},\varepsilon}(s)}{\theta_1'}, \tag{30}$$

$$N_{ta,\varepsilon}(s') = N_{\text{build},\varepsilon} + \frac{N_{l,\text{comp},\varepsilon}(s')}{\theta_2'}. \tag{31}$$

Thus we can calculate the performance of MIS and LIS in lossy channels.

5. Simulation results

In this section, we verify the proposed analysis by comparing the simulation results. In addition, we show that the proposed UEP scheme performs better than the scheme in [30]. We define the overhead as the value of $\frac{N_{\text{ta}}-k}{k}$. And we assume $\xi_1 = \xi_2 = 0.5$ and $\beta_0 = 0.55$. In our proposed scheme, we set $q_1{:}q_2{:}q_3 = 0.582{:}0.388{:}0.03$ and $\theta_1' = 0.6$, $\theta_2' = 0.4$.

Fig. 2 shows the analysis and simulation performance of the proposed UEP scheme. In the simulation, we set $k = 10\,000$. The results are in the lossless channels and the lossy channels. In the lossy channels, we set $\varepsilon = 0.2$. We can see the proposed analysis performance matches well with the simulation results in lossless and lossy channels. It proves our analysis is accurate for MIS and LIS symbols.

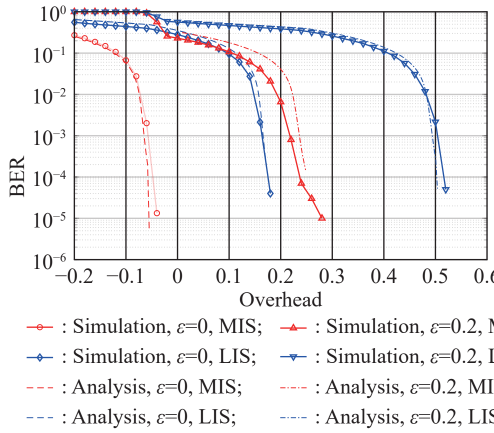


Fig. 2 Analysis and simulation performance of the proposed UEP scheme

Fig. 3 shows the BER performance of the scheme in [30] and the proposed scheme. We set $k = 1\,000$. When $\varepsilon = 0$, at BER of 10^{-4}, the overhead of the proposed scheme in MIS and LIS are 0.015 and 0.195. Comparing with the scheme in [30], in terms of MIS and LIS, the proposed scheme can achieve 85% and 31.58% overhead reduction, respectively. When the simulation is in the lossy channels, comparing with the schemes in [30], the overhead of the proposed scheme is lower when the BER drops to 10^{-4}. Therefore, the proposed scheme performs better in MIS and LIS.

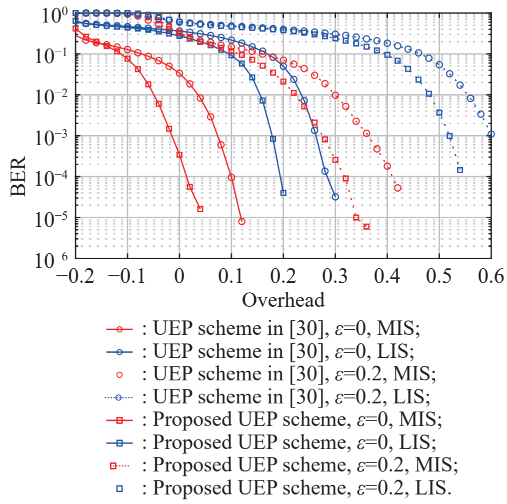


Fig. 3 Simulation performance of the proposed scheme and the scheme in [30]

6. Conclusions

An efficient UEP online fountain scheme is proposed in this paper. In the build-up phase, the selection strategy in

MIS is non-random, aiming to increase the number of MIS symbols in the largest component. It also improves the average degree of MIS symbols which are not included in the largest component. In the completion phase, the weighted-selection strategy is applied to provide lower overhead. We also analyze the performance of the proposed scheme in lossless and lossy channels. Both the analysis and simulation results show that the proposed scheme has better performance than conventional performance in MIS and LIS.

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