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System portfolio selection based on GRA method under hesitant fuzzy environment

LI Zhuoqian, DOU Yajie*, XIA Boyuan, YANG Kewei, and LI Mengjun

College of Systems Engineering, National University of Defense Technology, Changsha 410073, China

Abstract: The hesitant fuzzy set (HFS) is an important tool to deal with uncertain and vague information. In equipment system portfolio selection, the index attribute of the equipment system may not be expressed by precise data; it is usually described by qualitative information and expressed as multiple possible values. We propose a method of equipment system portfolio selection under hesitant fuzzy environment. The hesitant fuzzy element (HFE) is used to describe the index and attribute values of the equipment system. The hesitation degree of HFEs measures the uncertainty of the criterion data of the equipment system. The hesitant fuzzy grey relational analysis (GRA) method is used to evaluate the score of the equipment system, and the improved HFE distance measure is used to fully consider the influence of hesitation degree on the grey correlation degree. Based on the score and hesitation degree of the equipment system, two portfolio selection models of the equipment system and an equipment system portfolio selection case is given to illustrate the application process and effectiveness of the method.

Keywords: system portfolio selection, hesitant fuzzy set (HFS), grey relational analysis (GRA), score-hesitation tradeoff portfolio model.

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1. Introduction

The development plan of a weapon equipment system is important for improving the combat effectiveness of the army. Therefore, the senior decision maker needs to determine the amount of defense budget in the weapon equipment system demonstration stage, also the development of a weapon equipment system of systems must consider system portfolio decision-making and management. The portfolio selection theory [1] was proposed by Markowitz in 1952. In the military field, the earliest application of "portfolio selection" was proposed by Buede and Bresnick in 1992, applied to the investment project decision-making of the US Navy [2]. Since then, the portfolio selection theory has been widely used in practical issues such as weapon system planning [3], and defense acquisition project portfolio [4]. The portfolio selection process of weapon systems can be divided into two stages. The first is the evaluation, and the second is the portfolio selection.

In the process of evaluation, weapon system portfolio selection is located in the R&D stage. Because the equipment system is not put into service, acquisition of accurate quantitative data to describe the indices of the equipment system is difficult. The fuzzy set theory provides a method of solving the problem [5]. Typical forms of fuzzy sets are intuitionistic fuzzy sets [6,7], hesitant fuzzy sets (HFSs) [8], and linguistic fuzzy sets [9]. Compared with other fuzzy sets, the HFS is more suitable for equipment system evaluating, because the source of the data is multi-channel in the planning stage. These data are all possible and cannot be selected. Therefore, HFSs can completely retain all possible evaluation data.

Many researches about HFSs have been carried out to solve the decision making problems with uncertainties [10−15]. Grey relational analysis (GRA) is also a classical multi-attribute decision-making method. The grey system theory was proposed by Deng [16]. Some scholars [17−19] combined the GRA method with fuzzy information. However, there are few researches on the GRA under hesitant fuzzy environment. For the GRA method under hesitant fuzzy environment, the distance measure is of great importance. Xu and Xia proposed different forms of distance formulas based on the Hamming distance measure, the Euclidean distance measure, and the Hadolph distance measure [20,21]. A number of other extensions based on the above distance measures have been developed for HFSs [22−27], such as the extended HFS method based on the proposed similarity and entropy measures [28], and the hesitant fuzzy decision mak-

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*Corresponding author.
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ing method based on an improved signed distance [29].

In the process of portfolio, to deal with uncertain information, the portfolio problem is combined with the fuzzy theory. Tanaka and Guo [30] proposed a portfolio-selection model based on upper and lower exponential possibility distributions. Tsaur [31] proposed a fuzzy portfolio model focusing on different risk attitudes, risk aversion, risk neutrality, and risk seeking. Deng and Pan [32] compared multi-objective portfolio-selection models based on intuitionistic fuzzy optimization. Zhou and Xu [33] proposed an intuitionistic fuzzy portfolio-selection model to weigh returns and hesitancy. However, research on qualitative portfolio selection under hesitant fuzzy environment is still in an early stage. Zhou and Xu [34] proposed a portfolio-selection model for general investors and venture investors under hesitant fuzzy environment. Based on the prospect theory, Zhou et al. [35] developed a hesitant fuzzy portfolio-selection model considering the psychological behavior of experts.

For multi-attribute portfolio selection under hesitant fuzzy environment, the traditional portfolio-selection method is not available due to the uncertainty of information. This study uses GRA to aggregate the attribute values represented by hesitant fuzzy element (HFE), so that weapon system portfolio selection under uncertain information environment can be transformed into a portfolio-selection problem under deterministic information environment. The contributions of this study are as follows:

(i) On the basis of hesitant Hamming and Euclidean distance, with considering the hesitation degree, two new HFE distances are defined. These HFE distances satisfy the well-known basic axioms and the properties of trigonometric inequalities.

(ii) The GRA method based on the new HFE distance measure is proposed.

(iii) Two weapon system portfolio-selection models of the equipment systems are constructed, which trade off the score and hesitation under the hesitant fuzzy environment.

The remainder of this paper is organized as follows. Section 2 introduces definitions and concepts of HFS and HFE, and the basic process of the GRA method. In Section 3, two new hesitant distances are proposed, and the GRA method based on the new HFE distances is presented. Section 4 defines the score and hesitation of the weapon system and constructs portfolio-selection models that trade off the score and hesitation of the equipment system. Section 5 introduces the process of weapon system portfolio selection, based on GRA under hesitant fuzzy environment. In Section 6, a weapon system portfolio-selection example is presented, the results analysis and a comparative analysis are given. The conclusions of the study are presented in Section 7.

2. Preliminaries

In this section, definitions and concepts related to HFS and HFE, basic operations and other basic knowledge are introduced, and the GRA is defined.

2.1 HFS

Torra [8] developed the HFS, which was a kind of generalized fuzzy set where the membership degree of an element to a certain set could be described as several different values between 0 and 1. HFSs are effective in conditions with uncertainty or hesitancy in decision-making.

Definition 1 [8] Let X be a fixed set. An HFS on X is expressed in terms of a function such that when applied to X, it returns a subset of [0,1]. For simplicity, Xia and Xu [36] proposed the following mathematical expression for an HFS:

$$A = \{\langle x, h_A(x)\rangle \mid x \in X\} \tag{1}$$

where $h_A(x)$ is the set composed of several numbers in the interval [0,1], indicating the possibility that $x \in X$ belongs to set A. Xia and Xu [36] defined $h_A(x)$ as an HFE, which is the basic unit of an HFS.

Definition 2 [20] For any two HFEs of equal length $h_1 = \{\gamma_1^1, \gamma_1^2, \cdots, \gamma_1^{l_1}\}$ and $h_2 = \{\gamma_2^1, \gamma_2^2, \cdots, \gamma_2^{l_2}\}$, $l_1 = l_2 = l$, the elements in the two HFEs are arranged in an ascending order, where γ_1^λ and γ_2^λ $(\lambda = 1, 2, \cdots, l)$ are the λth largest values in h_1 and h_2, respectively.

Hesitant fuzzy Hamming distance:

$$d_H(h_1, h_2) = \frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda - \gamma_2^\lambda\right|. \tag{2}$$

Hesitant fuzzy Euclidean distance:

$$d_E(h_1, h_2) = \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_1^\lambda - \gamma_2^\lambda)^2}. \tag{3}$$

Definition 3 [37] For an HFE $h(x) = \{\gamma^1, \gamma^2, \cdots, \gamma^l\}$, l is the number of elements in $h(x)$. The hesitation degree of $h(x)$ is defined as follows:

$$\varphi_{h(x)} = \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}\left[\gamma^\lambda - \frac{1}{l}\sum_{\lambda=1}^{l}\gamma^\lambda\right]^2}. \tag{4}$$

Definition 4 In a multi-attribute decision making problem, the evaluation attribute can be divided into benefit and cost attributes. Due to different types of attributes, the dimensions or measures of attribute values are usually different. In order to ensure the compatibility of all attribute values, all attributes must be converted to the same compatible measure (or dimensionless index). Therefore, in order to eliminate the influence of different physical dimensions on the final decision-making results, the HFE normalization method proposed by Zhu et al. [38] is used for transforming. The transformation method is as follows:

$$h(x)' = \begin{cases} h^c(x), & \text{for the cost attribute} \\ h(x), & \text{for the benefit attribute} \end{cases} \tag{5}$$

where $h^c(x)$ is the complement set of $h(x)$, and $h^c(x) = \bigcup_{\gamma\in h}\{1-\gamma\}$.

Definition 5 For two HFEs h_1 and h_2, l_1 is the number of elements in h_1 and l_2 is the number of elements in h_2. If $l_1 \neq l_2$, then extend the HFE with fewer elements until the two HFEs have the same number of elements. From the existing extension rules [39]:

(i) If the decision-maker is a risk averter and pessimistic about the outcome, the minimum value of the HFE with fewer elements is added to expand the HFE until the number of elements in the two HFEs is equal.

(ii) If the decision-maker is a risk seeker and optimistic about the outcome, the maximum value of the HFE with fewer elements is added to expand the HFE until the number of elements in the two HFEs is equal.

(iii) If the decision-maker is risk neutral, the average value of the HFE with fewer elements is added to expand the HFE until the number of elements in the two HFEs is equal.

2.2 GRA

GRA [16] judges whether the relationship between different sequences is close according to the sequence curve, and measures the similarity of the change trend for system factors according to the distance measure between corresponding points of the sequence. It has been widely applied to multi-attribute decision making. The calculation process is as follows:

(i) The reference sequence is $X_0 = (x_0(j), j = 1, 2, \cdots, m)$, and the comparison sequence is $X_i = (x_i(j), j = 1, 2, \cdots, m)$, where $i = 1, 2, \cdots, n$.

(ii) Dimensionless processing of the reference sequence and the comparison sequence.

(iii) The grey relational coefficient of X_i and X_0 about j is $\xi(x_0(j), x_i(j))$, defined as

$$\xi(x_0(j), x_i(j)) = \frac{\min_i \min_j |x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|}{|x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|} \tag{6}$$

where $i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$, and ρ is the resolution coefficient, $\rho \in [0, 1]$.

(iv) The grey relational degree between X_i and X_0 is defined as

$$r(x_0, x_i) = \frac{1}{m}\sum_{j=1}^{m} \xi(x_0(j), x_i(j)). \tag{7}$$

3. GRA under hesitant fuzzy environment

In this section, we combine the hesitant fuzzy theory with the grey relational theory. Two new HFE distance measures are proposed considering the hesitation degree. Based on these new distance measures, the HFS grey relational coefficient and the grey relational degree are defined.

3.1 New distance measures for HFEs

The original Hamming distance (3) and Euclidean distance (4) do not reflect the difference in the hesitation degree between the two HFEs. Two improved HFE distance measures are proposed that fully consider the hesitancy between HFEs.

For two HFEs of equal length $h_1 = \{\gamma_1^1, \gamma_1^2, \cdots, \gamma_1^{l_1}\}$ and $h_2 = \{\gamma_2^1, \gamma_2^2, \cdots, \gamma_2^{l_2}\}$, $l_1 = l_2 = l$, where γ_1^λ and γ_2^λ $(\lambda = 1, 2, \cdots, l)$ are the λth largest values in h_1 and h_2, respectively. φ_{h_1} and φ_{h_2} are the hesitation degrees of h_1 and h_2, respectively. The improved hesitant Hamming distance is defined as

$$d_{\mathrm{IH}}(h_1, h_2) = \frac{1}{l}\sum_{\lambda=1}^{l} \left|\gamma_1^\lambda - \gamma_2^\lambda\right| + \left|\varphi_{h_1} - \varphi_{h_2}\right|. \tag{8}$$

The improved hesitant Euclidean distance is defined as

$$d_{\mathrm{IE}}(h_1, h_2) = \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l} (\gamma_1^\lambda - \gamma_2^\lambda)^2 + (\varphi_{h_1} - \varphi_{h_2})^2}. \tag{9}$$

Let h_1, h_2, and h_3 be three HFEs, and $l_1 = l_2 = l_3 = l$. d_{IH} represents the improved hesitant Hamming distance between the two HFEs; d_{IE} represents the improved hesitant Euclidean distance between the two HFEs. d_{IH} and d_{IE} satisfy the following:

(i) $d_{\mathrm{IH}}(h_1, h_2) \geqslant 0$, $d_{\mathrm{IE}}(h_1, h_2) \geqslant 0$;

(ii) $d_{\mathrm{IH}}(h_1, h_2) = 0$ if and only if $h_1 = h_2$, $d_{\mathrm{IE}}(h_1, h_2) = 0$ if and only if $h_1 = h_2$;

(iii) $d_{\mathrm{IH}}(h_1, h_2) = d_{\mathrm{IH}}(h_2, h_1)$, $d_{\mathrm{IE}}(h_1, h_2) = d_{\mathrm{IE}}(h_2, h_1)$;

(iv) $d_{\mathrm{IH}}(h_1, h_2) + d_{\mathrm{IH}}(h_2, h_3) \geqslant d_{\mathrm{IH}}(h_1, h_3)$, $d_{\mathrm{IE}}(h_1, h_2) + d_{\mathrm{IE}}(h_2, h_3) \geqslant d_{\mathrm{IE}}(h_1, h_3)$.

Proof

(i) $d_{\mathrm{IH}}(h_1, h_2) \geqslant 0$, $d_{\mathrm{IE}}(h_1, h_2) \geqslant 0$

$$d_{\mathrm{IH}}(h_1, h_2) = \frac{1}{l}\sum_{\lambda=1}^{l} \left|\gamma_1^\lambda - \gamma_2^\lambda\right| + \left|\varphi_{h_1} - \varphi_{h_2}\right|$$

$\frac{1}{l}\sum_{\lambda=1}^{l} \left|\gamma_1^\lambda - \gamma_2^\lambda\right| \geqslant 0$, and $\left|\varphi_{h_1} - \varphi_{h_2}\right| \geqslant 0$, so $d_{\mathrm{IH}}(h_1, h_2) \geqslant 0$.

$$d_{\mathrm{IE}}(h_1,h_2)=\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_1^\lambda-\gamma_2^\lambda)^2+(\varphi_{h_1}-\varphi_{h_2})^2}$$

$$\frac{1}{l}\sum_{\lambda=1}^{l}\left(\gamma_1^\lambda-\gamma_2^\lambda\right)^2\geqslant 0,(\varphi_{h_1}-\varphi_{h_2})^2\geqslant 0,\ \text{so } d_{\mathrm{IE}}(h_1,h_2)\geqslant 0.$$

(ii) $d_{\mathrm{IH}}(h_1,h_2)=0$ if and only if $h_1=h_2$, $d_{\mathrm{IE}}(h_1,h_2)=0$ if and only if $h_1=h_2$

$d_{\mathrm{IH}}(h_1,h_2)=0$ means that $\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_2^\lambda\right|=0$ and $\left|\varphi_{h_1}-\varphi_{h_2}\right|=0$. That is, $\gamma_1^\lambda-\gamma_2^\lambda=0\ (\lambda=1,2,\cdots,l)$, thus $h_1=h_2$. Conversely, if $h_1=h_2$, then $\gamma_1^\lambda-\gamma_2^\lambda=0(\lambda=1,2,\cdots,l)$, thus $d_{\mathrm{IH}}(h_1,h_2)=0$. Similarly, $d_{\mathrm{IE}}(h_1,h_2)=0$ if and only if $h_1=h_2$.

(iii) $d_{\mathrm{IH}}(h_1,h_2)=d_{\mathrm{IH}}(h_2,h_1)$, $d_{\mathrm{IE}}(h_1,h_2)=d_{\mathrm{IE}}(h_2,h_1)$

$$d_{\mathrm{IH}}(h_1,h_2)=\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_2^\lambda\right|+\left|\varphi_{h_1}-\varphi_{h_2}\right|=\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_2^\lambda-\gamma_1^\lambda\right|+\left|\varphi_{h_2}-\varphi_{h_1}\right|=d_{\mathrm{IH}}(h_2,h_1).$$

$$d_{\mathrm{IE}}(h_1,h_2)=\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_1^\lambda-\gamma_2^\lambda)^2+(\varphi_{h_1}-\varphi_{h_2})^2}=\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_2^\lambda-\gamma_1^\lambda)^2+(\varphi_{h_2}-\varphi_{h_1})^2}=d_{\mathrm{IE}}(h_2,h_1).$$

(iv) $d_{\mathrm{IH}}(h_1,h_2)+d_{\mathrm{IH}}(h_2,h_3)\geqslant d_{\mathrm{IH}}(h_1,h_3)$, $d_{\mathrm{IE}}(h_1,h_2)+d_{\mathrm{IE}}(h_2,h_3)\geqslant d_{\mathrm{IE}}(h_1,h_3)$

$$d_{\mathrm{IH}}(h_1,h_2)+d_{\mathrm{IH}}(h_2,h_3)\geqslant d_{\mathrm{IH}}(h_1,h_3)$$
$$\Leftrightarrow \frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_2^\lambda\right|+\left|\varphi_{h_1}-\varphi_{h_2}\right|+\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_2^\lambda-\gamma_3^\lambda\right|+\left|\varphi_{h_2}-\varphi_{h_3}\right|\geqslant\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_3^\lambda\right|+\left|\varphi_{h_1}-\varphi_{h_3}\right|$$

For any $\lambda\in(1,2,\cdots,l)$,

$$\left|\gamma_1^\lambda-\gamma_2^\lambda\right|+\left|\gamma_2^\lambda-\gamma_3^\lambda\right|\geqslant\left|\gamma_1^\lambda-\gamma_3^\lambda\right|$$
$$\Leftrightarrow(\left|\gamma_1^\lambda-\gamma_2^\lambda\right|+\left|\gamma_2^\lambda-\gamma_3^\lambda\right|)^2\geqslant\left|\gamma_1^\lambda-\gamma_3^\lambda\right|^2$$
$$\Leftrightarrow 2(\gamma_2^\lambda)^2-2\gamma_1^\lambda\gamma_2^\lambda+2\left|\gamma_1^\lambda-\gamma_2^\lambda\right|\left|\gamma_2^\lambda-\gamma_3^\lambda\right|-2\gamma_2^\lambda\gamma_3^\lambda\geqslant 2(\gamma_2^\lambda)^2-2\gamma_1^\lambda\gamma_2^\lambda+2(\gamma_1^\lambda-\gamma_2^\lambda)(\gamma_2^\lambda-\gamma_3^\lambda)-2\gamma_2^\lambda\gamma_3^\lambda=-2\gamma_1^\lambda\gamma_3^\lambda.$$

Therefore,

$$\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_2^\lambda\right|+\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_2^\lambda-\gamma_3^\lambda\right|\geqslant\frac{1}{l}\sum_{\lambda=1}^{l}\left|\gamma_1^\lambda-\gamma_3^\lambda\right|.$$

Similarly,

$$\left|\varphi_{h_1}-\varphi_{h_2}\right|+\left|\varphi_{h_2}-\varphi_{h_3}\right|\geqslant\left|\varphi_{h_1}-\varphi_{h_3}\right|.$$

Obviously,

$$d_{\mathrm{IH}}(h_1,h_2)+d_{\mathrm{IH}}(h_2,h_3)\geqslant d_{\mathrm{IH}}(h_1,h_3).$$

Similarly,

$$d_{\mathrm{IE}}(h_1,h_2)+d_{\mathrm{IE}}(h_2,h_3)\geqslant d_{\mathrm{IE}}(h_1,h_3)$$
$$\Leftrightarrow\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_1^\lambda-\gamma_2^\lambda)^2+(\varphi_{h_1}-\varphi_{h_2})^2}+\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_2^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_2}-\varphi_{h_3})^2}\geqslant\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}(\gamma_1^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_1}-\varphi_{h_3})^2}$$
$$\Leftrightarrow\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}[(\gamma_1^\lambda-\gamma_2^\lambda)^2+(\varphi_{h_1}-\varphi_{h_2})^2]}+\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}[(\gamma_2^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_2}-\varphi_{h_3})^2]}\geqslant\sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}[(\gamma_1^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_1}-\varphi_{h_3})^2]}.$$

According to the Cauchy inequality: $\sqrt{a^2+b^2}+\sqrt{c^2+d^2}\geqslant\sqrt{(a+c)^2+(b+d)^2}$. Let $a=\gamma_1^\lambda-\gamma_2^\lambda$, $b=\varphi_{h_1}-\varphi_{h_2}$, $c=\gamma_2^\lambda-\gamma_3^\lambda$, $d=\varphi_{h_2}-\varphi_{h_3}$. Thus, $\gamma_1^\lambda-\gamma_3^\lambda=a+c$, $\varphi_{h_1}-\varphi_{h_3}=b+d$.

For any $\lambda\in(1,2,\cdots,l)$, it is true that

$$\sqrt{(\gamma_1^\lambda-\gamma_2^\lambda)^2+(\varphi_{h_1}-\varphi_{h_2})^2}+\sqrt{(\gamma_2^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_2}-\varphi_{h_3})^2}\geqslant\sqrt{(\gamma_1^\lambda-\gamma_3^\lambda)^2+(\varphi_{h_1}-\varphi_{h_3})^2}.$$

Obviously,

$$d_{\mathrm{IE}}(h_1,h_2)+d_{\mathrm{IE}}(h_2,h_3)\geqslant d_{\mathrm{IE}}(h_1,h_3).$$ □

3.2 GRA between HFSs

For two HFSs on the fixed set $X=\{x_1,x_2,\cdots,x_m\}$, $A=\{\langle x_i,h_A(x_i)\rangle|x_i\in X, i=1,2,\cdots,m\}$ and $B_j=\left\{\left\langle x_i,h_{B_j}(x_i)\right\rangle|x_i\in X, i=1,2,\cdots,m, j=1,2,\cdots,n\right\}$, with $h_A(x_i)=\{\gamma_{Ai}^1,\gamma_{Ai}^2,\cdots,\gamma_{Ai}^{l_{Ai}}\}$, $h_{B_j}(x_i)=\{\gamma_{B_ji}^1,\gamma_{B_ji}^2,\cdots,\gamma_{B_ji}^{l_{B_ji}}\}$, $i=1,2,\cdots,m$, and $j=1,2,\cdots,n$. Then, the Grey relational coefficient between the HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$ is defined as

$$\xi(h_A(x_i),h_{B_j}(x_i))=\frac{\min\limits_j\min\limits_i\left\{d(h_A(x_i),h_{B_j}(x_i))\right\}+\rho\max\limits_j\max\limits_i\left\{d(h_A(x_i),h_{B_j}(x_i))\right\}}{d(h_A(x_i),h_{B_j}(x_i))+\rho\max\limits_j\max\limits_i\left\{d(h_A(x_i),h_{B_j}(x_i))\right\}}\quad(10)$$

where ρ is the resolution coefficient $\rho \in [0,1]$, and $d(h_A(x_i), h_{B_j}(x_i))$ is the distance between HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$, which can be $d_{\rm IH}$ or $d_{\rm IE}$.

Based on the grey correlation coefficient between HFEs, the grey correlation degree between A and B_j is defined as

$$r(A,B_j)=\frac{1}{n}\sum_{i=1}^{n}\xi(h_A(x_i),h_{B_j}(x_i)),\ i=1,2,\cdots,m; j=1,2,\cdots,n. \tag{11}$$

4. Score-hesitation trade-off system portfolio selection model

The classical Markowitz portfolio-selection method can be used to obtain the optimal investment ratio according to the return-risk tradeoff rule. The return and risk correspond to the average and variance of the quantitative data, respectively. Similarly, based on the definition of score and hesitation degree of alternatives, this section proposes a system portfolio model to trade off the score and hesitation degree under hesitant fuzzy environment.

Considering the score of the equipment system based on the hesitant fuzzy grey correlation analysis, the positive and negative reference sequences are determined. The correlation degree between the scheme and the positive reference sequence is r^{+}, and the correlation degree between the scheme and the negative reference sequence is r^{-}. The score of the equipment system can then be expressed as

$$v = r^{+}/(r^{+}+r^{-}). \tag{12}$$

The uncertain information of the equipment system can be measured by hesitation, described by the aggregate value of the hesitation degree of each indicator of the equipment system as

$$\varphi_j = \frac{1}{m}\sum_{i=1}^{m}\varphi_{h(i)},\ i=1,2,\cdots m; j=1,2,\cdots,n. \tag{13}$$

Based on the score and hesitation degree, the portfolio-selection rule under hesitant fuzzy environment can be expressed as Mod1:

$$\max S(P) = \sum_{i=1}^{n} w_i v_i$$
$$\min H(P) = \sum_{i=1}^{n} w_i \varphi_i$$
$$\text{s.t.} \begin{cases} l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{14}$$

where $S(P)$ represents the total score of portfolio P, v_i is the score of the equipment system a_i, and w_i is the investment ratio of a_i. $H(P)$ is the total hesitation of the portfolio and φ_i is the hesitation of a_i. In Mod1, the optimal portfolio is the Pareto optimal solution in the case of the maximum score and the minimum hesitation, also known as a non-dominated solution. Such solutions cannot accurately guide decision-makers to invest.

We can adjust the decision-maker preference to hesitation, and consider that the decision-maker can obtain the maximum value under bearable hesitation, rewriting Mod1 as Mod2. The hesitation threshold α can be determined by the hesitancy preference parameter $\theta \in [0,1]$. Different θ values represent different investor tolerances to the hesitation of the portfolio; greater values of θ indicate greater bearable hesitation. After determining the hesitation threshold, Mod1 is transformed into a single objective model, Mod2:

$$\max S(P) = \sum_{i=1}^{n} w_i v_i$$
$$\text{s.t.} \begin{cases} H(P) = \sum_{i=1}^{n} w_i \varphi_i \leqslant \alpha \\ l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{15}$$

where

$$\alpha = \theta H(P)_{\rm Max} + (1-\theta) H(P)_{\rm Min} \tag{16}$$

$$H(P)_{\rm Min} = \min H(P)$$
$$\text{s.t.} \begin{cases} l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{17}$$

$$H(P)_{\rm Max} = \max H(P)$$
$$\text{s.t.} \begin{cases} l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases}. \tag{18}$$

Mod2 provides decision-makers' different hesitation attitudes with acceptable hesitation to obtain the maximum value and the portfolio ratio. Furthermore, we can consider the minimum hesitation under an acceptable total score, such as in Mod3. The acceptable threshold score in Mod3 can be determined by the score preference parameter $\psi \in [0,1]$. Mod3 is defined as

$$\min H(P) = \sum_{i=1}^{n} w_i \varphi_i$$
$$\text{s.t.} \begin{cases} S(P) = \sum_{i=1}^{n} w_i v_i \geqslant \beta \\ l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{19}$$

where

$$\beta = \psi S(P)_{\rm Max} + (1-\psi) S(P)_{\rm Min} \tag{20}$$

$$S(P)_{\text{Min}} = \min S(P)$$
$$\text{s.t.} \begin{cases} l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \tag{21}$$

$$S(P)_{\text{Max}} = \max S(P)$$
$$\text{s.t.} \begin{cases} l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} . \tag{22}$$

For the decision-maker, under acceptable hesitation, the maximum score is represented by Mod2; the minimum hesitation obtained under an acceptable score is represented by Mod3. Both portfolio-selection models can be adjusted according to the preference information of the decision-maker, and the optimal portfolio proportion under constraint conditions can be obtained.

5. Portfolio-selection process under hesitant fuzzy environment

In this section, a portfolio selection process based on hesitant fuzzy GRA is provided based on the portfolio-selection models.

Assuming that the decision-maker must determine the optimal investment portfolio of equipment systems $\{S_1, S_2, \cdots, S_n\}$, the criteria $\{c_1, c_2, \cdots, c_m\}$ are set to evaluate each equipment system, and quantified in terms of HFEs. The equipment systems can be expressed by the HFE matrix $\boldsymbol{H} = [h_{ij}]_{n\times m} (i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$, which is constructed based on h_{ij}. The portfolio selection process of an equipment system under hesitant fuzzy environment is shown in Fig. 1.

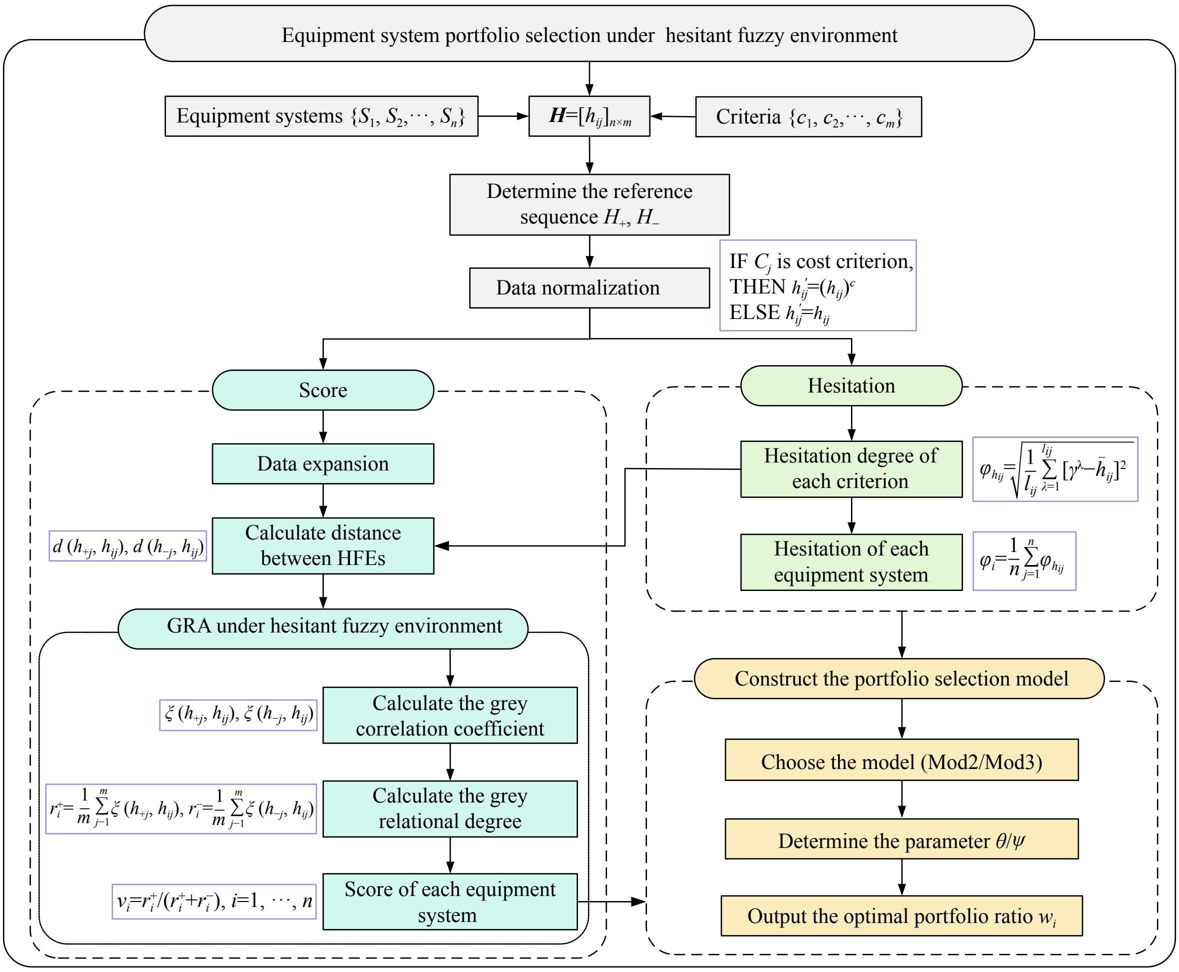

Fig. 1 Portfolio-selection process of equipment system under hesitant fuzzy environment

Step 1 Obtain the HFE matrix $\boldsymbol{H} = [h_{ij}]_{n\times m}$ and determine the positive and negative reference sequences. The criteria values of each equipment system and the positive and negative reference sequences are normalized according to Definition 4. All cost criteria data should be transformed into benefit criteria data.

Step 2 Calculate the hesitation. Equation (4) is used to calculate the hesitation degree for each criterion of the equipment system and reference sequences, and (13) is used to calculate the hesitation.

Step 3 According to the decision-maker's risk attitude, expand the criteria data of each equipment system and reference sequences using the rules in Definition 5.

Step 4 Use (8) or (9) to calculate the distance between the corresponding criteria data of each equipment system and the reference sequence.

Step 5 Based on the GRA between HFSs, calculate the score of each equipment system. The grey relational coefficient ξ_{ij}^{+}, ξ_{ij}^{-} $(i=1,2,\cdots,n; j=1,2,\cdots,m)$ of the criteria between each equipment system and the reference sequences is calculated by using (10), and using the distance measure from the result of Step 4. According to the grey correlation coefficient, the grey relational degrees r_i^+ and r_i^- $(i=1,2,\cdots,n; j=1,2,\cdots,m)$ are calculated by using (11). Equation (12) is used to calculate the score of each equipment system.

Step 6 Select portfolio model Mod2 or Mod3 according to the needs of the decision-maker. If Mod2 is chosen, the hesitation threshold α must be calculated according to the decision maker's hesitation preference parameter θ using (16)–(18). If Mod3 is chosen, the score threshold β must be calculated according to the decision maker's score preference parameter ψ using (20)–(22). The model is solved and the optimal equipment portfolio ratio w_i $(i=1,2,\cdots,n)$ is obtained.

6. Illustration example

This section provides an example of equipment system portfolio selection under hesitant fuzzy environment to fully demonstrate the application of the portfolio-selection model.

Decision-makers must invest resources in the eight equipment systems $\{S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_8\}$ to support the development of equipment. Four key criteria $\{c_1,c_2,c_3,c_4\}$ are set for equipment system evaluation; c_4 is a cost criterion, and the rest are benefit criteria. Considering the development needs of the equipment system of systems, the lower bounds of the investment proportion for each equipment system are set as $l_i=(0.05,0.05,0.1,0.05,0.1,0.05,0.15,0.05)$, and the upper bounds are set as $u_i=(0.2,0.2,0.3,0.1,0.2,0.1,0.45,0.1)$. Because the eight equipment systems are still in the R&D stage, accurate data and quantified information for each equipment system are unavailable. Thus, the conventional portfolio theory is unsuitable. Instead, the proposed portfolio models can be applied under hesitant fuzzy environment. The relevant departments provide these imprecise criteria described by the HFEs h_{ij} $(i=1,2,\cdots,8; j=1,2,3,4)$. The hesitant fuzzy matrix $\boldsymbol{H}=[h_{ij}]_{8\times4}$ is constructed based on h_{ij}, as presented in Table 1.

Table 1 The hesitant fuzzy matrix

System	Criteria			
	c_1	c_2	c_3	c_4
S_1	{0.45}	{0.35, 0.55, 0.60}	{0.70, 0.75, 0.80}	{0.60, 0.70}
S_2	{0.35, 0.65, 0.75}	{0.20, 0.35}	{0.55, 0.7}	{0.45, 0.50}
S_3	{0.75, 0.45}	{0.34, 0.45}	{0.62, 0.75}	{0.30, 0.45, 0.60}
S_4	{0.55, 0.70}	{0.30, 0.56}	{0.3, 0.70}	{0.40}
S_5	{0.35, 0.67}	{0.50, 0.60}	{0.73}	{0.42, 0.50}
S_6	{0.30, 0.46, 0.63}	{0.45}	{0.35, 0.90}	{0.35, 0.54}
S_7	{0.40, 0.45}	{0.22, 0.38, 0.50}	{0.70, 0.80, 0.85}	{0.70, 0.75}
S_8	{0.50, 0.70}	{0.30, 0.55}	{0.55, 0.67}	{0.55}

6.1 Equipment system portfolio-selection process and calculations

The following calculation example specifically illustrates the selection process of a weapon equipment system portfolio based on hesitant fuzzy GRA. In the following example, Rule (ii) in Definition 5 is used for data expansion, (10) is used for distance measure, and Mod2 is applied to solve the problem.

Step 1 The positive and negative reference sequences are set as the maximum and the minimum values of each criterion. The positive reference sequence is $H_+=\{\{0.75\},\{0.6\},\{0.9\},\{0.75\}\}$, and the negative reference sequence is $H_-=\{\{0.3\},\{0.2\},\{0.3\},\{0.25\}\}$. In practical application, the positive and negative reference sequences can be set according to the actual problem, and each element in the reference sequence is also an HFE. The criteria data of each equipment system and reference sequences are normalized according to Definition 4. Considering the equipment system S_1 and c_4 as an example, h_{14} is transformed into $h_{14}'=(h_{14})^c=\{1-0.6,1-0.7\}=\{0.4,0.3\}$.

Step 2 The hesitation degree is calculated by using (4). For example, the hesitation degree of each criterion of the equipment system S_1 is calculated as

$$\varphi_{11}=\sqrt{\sum_{\lambda=1}^{1}(\gamma^{\lambda}-\bar{h}_{11})^2}=\sqrt{(0.45-0.45)^2}=0,$$

$$\varphi_{12}=\sqrt{\frac{1}{3}\sum_{\lambda=1}^{3}(\gamma^{\lambda}-\bar{h}_{12})^2}=$$

$$\sqrt{\frac{1}{3}[(0.35-0.50)^2+(0.55-0.50)^2+(0.60-0.50)^2]}\approx 0.108,$$

$$\varphi_{13}=\sqrt{\frac{1}{3}\sum_{\lambda=1}^{3}(\gamma^{\lambda}-\bar{h}_{13})^2}=$$

$$\sqrt{\frac{1}{3}[(0.70-0.75)^2+(0.75-0.75)^2+(0.80-0.75)^2]}\approx 0.041,$$

$$\varphi_{14}=\sqrt{\frac{1}{2}\sum_{\lambda=1}^{2}(\gamma^{\lambda}-\bar{h}_{14})^2}=$$

$$\sqrt{\frac{1}{2}[(0.40-0.35)^2+(0.30-035)^2}=0.050.$$

The hesitation is calculated by using (13). For example, the hesitation of the equipment system S_1 is $\varphi_1=\frac{1}{4}\sum_{i=1}^{4}\varphi_{h_{1i}}=\frac{1}{4}(0+0.108+0.041+0.05)\approx 0.049\,7$.

Step 3 The data of each criterion of the equipment systems and reference sequences are extended according to Rule (ii) in Definition 5. For S_1, h_{11} is extended as $\{0.45,0.45,0.45\}$, and h_{14} is extended as $\{0.45,0.45,0.45\}$.

Step 4 The distance between each equipment system and the corresponding criterion of the reference sequences is calculated. The distance of each criterion between S_1 and H_+ is calculated as

$$d_{\mathrm{IH}}(h_{11},h_{+1})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{11}}-\varphi_{h_{+1}}\right|=$$

$$\frac{1}{3}(|0.45-0.75|\times 3)+0=0.3,$$

$$d_{\mathrm{IH}}(h_{12},h_{+2})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{12}}-\varphi_{h_{+2}}\right|=$$

$$\frac{1}{3}(|0.35-0.6|+|0.55-0.6|+|0.6-0.6|)+0.108=0.208,$$

$$d_{\mathrm{IH}}(h_{13},h_{+3})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{13}}-\varphi_{h_{+3}}\right|=$$

$$\frac{1}{3}(|0.7-0.9|+|0.75-0.9|+|0.8-0.9|)+0.041=0.191,$$

$$d_{\mathrm{IH}}(h_{14},h_{+4})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{14}}-\varphi_{h_{+4}}\right|=$$

$$\frac{1}{3}(|0.4-0.7|+|0.3-0.7|+|0.4-0.7|)+0.05=0.383.$$

The distance between each criterion of the equipment system S_1 and H_- is calculated as

$$d_{\mathrm{IH}}(h_{11},h_{-1})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{11}}-\varphi_{h_{-1}}\right|=$$

$$\frac{1}{3}(|0.45-0.30|\times 3)+0=0.15,$$

$$d_{\mathrm{IH}}(h_{12},h_{-2})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{12}}-\varphi_{h_{-2}}\right|=0.408,$$

$$d_{\mathrm{IH}}(h_{13},h_{-3})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{13}}-\varphi_{h_{-3}}\right|=$$

$$\frac{1}{3}(|0.7-0.3|+|0.75-0.3|+|0.8-0.3|)+0.041=0.491,$$

$$d_{\mathrm{IH}}(h_{14},h_{-4})=\frac{1}{3}\sum_{\lambda=1}^{3}\left|\gamma_1^{\lambda}-\gamma_2^{\lambda}\right|+\left|\varphi_{h_{14}}-\varphi_{h_{-4}}\right|=$$

$$\frac{1}{3}(|0.4-0.25|+|0.3-0.25|+|0.4-0.25|)+0.05=0.167.$$

Step 5 Calculate the grey relational coefficient for each criterion between the equipment systems and the reference sequences using (10). For S_1, according to Step 4, $\min_j\min_i\{d(h_{ij},h_{+j})\}=0.083\,3$, and $\max_j\max_i\{d(h_{ij},h_{+j})\}=0.533\,3$. The grey relational coefficients of each criterion between the equipment system S_1 and the positive reference sequence are

$$\xi(h_{11},h_{+1})=$$

$$\frac{\min_j\min_i\{d_{\mathrm{IH}}(h_{ij},h_{+j})\}+0.5\max_j\max_i\{d_{\mathrm{IH}}(h_{ij},h_{+j})\}}{d_{\mathrm{IH}}(h_{11},h_{+1})+0.5\max_j\max_i\{d_{\mathrm{IH}}(h_{ij},h_{+j})\}}=$$

$$\frac{0.083\,3+0.5\times 0.533\,3}{0.3+0.5\times 0.533\,3}\approx 0.618,$$

$$\xi(h_{12},h_{+2})=\frac{0.083\,3+0.5\times 0.533\,3}{0.208+0.5\times 0.533\,3}\approx 0.737,$$

$$\xi(h_{13},h_{+3})=\frac{0.083\,3+0.5\times 0.533\,3}{0.191+0.5\times 0.533\,3}\approx 0.765,$$

$$\xi(h_{14},h_{+4})=\frac{0.083\,3+0.5\times 0.533\,3}{0.383+0.5\times 0.533\,3}\approx 0.538.$$

Similarly,

$$\min_j\min_i\{d(h_{ij},h_{-j})\}=0.058\,3,$$

$$\max_j\max_i\{d(h_{ij},h_{-j})\}=0.691\,7.$$

The grey relational coefficients of each criterion between the equipment system S_1 and the negative reference sequence are

$$\xi(h_{11},h_{-1}) = \frac{\min_j \min_i \{d_{\rm IH}(h_{ij},h_{-j})\} + 0.5\max_j \max_i \{d_{\rm IH}(h_{ij},h_{-j})\}}{d_{\rm IH}(h_{11},h_{-1}) + 0.5\max_j \max_i \{d_{\rm IH}(h_{ij},h_{-j})\}} = \frac{0.058\,3+0.5\times 0.691\,7}{0.15+0.5\times 0.691\,7} \approx 0.815,$$

$$\xi(h_{12},h_{-2}) = \frac{0.058\,3+0.5\times 0.691\,7}{0.408+0.5\times 0.691\,7} \approx 0.536,$$

$$\xi(h_{13},h_{-3}) = \frac{0.058\,3+0.5\times 0.691\,7}{0.491+0.5\times 0.691\,7} \approx 0.483,$$

$$\xi(h_{14},h_{-4}) = \frac{0.058\,3+0.5\times 0.691\,7}{0.167+0.5\times 0.691\,7} \approx 0.789.$$

According to the grey correlation coefficient, the grey relational degrees r_i^+ and r_i^- between the equipment systems and the reference sequences are calculated by using (11). The grey relational degree between the equipment system S_1 and the positive reference sequence is

$$r_1^+ = \frac{1}{4}\sum_{i=1}^{4}\xi(h_{1i},h_{+i}) = \frac{1}{4}(0.618+0.737+0.765+0.538) \approx 0.664\,6.$$

The grey relational degree between the equipment system S_1 and the negative reference sequence is

$$r_1^- = \frac{1}{4}\sum_{i=1}^{4}\xi(h_{1i},h_{-i}) = \frac{1}{4}(0.815+0.536+0.483+0.789) \approx 0.655\,7.$$

The score v_i of each equipment system is calculated by using (12). The score of the equipment system S_1 is $v_1 = r_1^+/(r_1^+ + r_1^-) = 0.664\,6/(0.664\,6+0.655\,7) = 0.503\,4$.

The grey correlation degree, score, and hesitation of the eight equipment systems are shown in Table 2.

Table 2 Grey correlation degree, score and hesitation of the eight equipment systems

System	r_i^+	r_i^-	v_i	φ_i
S_1	0.664 6	0.655 7	0.503 4	0.049 7
S_2	0.620 2	0.606 0	0.505 8	0.086 2
S_3	0.670 4	0.391	0.554 3	0.098 1
S_4	0.713 3	0.535 5	0.571 2	0.101 3
S_5	0.786 1	0.540 7	0.592 5	0.062 5
S_6	0.642 1	0.553 8	0.536 9	0.126 2
S_7	0.606 0	0.724 8	0.455 4	0.056 8
S_8	0.663 7	0.589 1	0.529 8	0.071 3

Step 6 Mod2 is applied to obtain the maximum score of the equipment system portfolio under acceptable hesitation. According to (16)–(18), $H(P)_{\rm Max} = 0.093\,9$ and $H(P)_{\rm Min} = 0.068\,2$, and $\alpha = \theta\times 0.093\,9 + (1-\theta)\times 0.068\,2$, Mod2 is expressed as

$$\max S(P) = \sum_{i=1}^{8} w_i v_i$$
$$\text{s.t.}\begin{cases} H(P) = \sum_{i=1}^{8} w_i\varphi_i \leqslant \alpha \\ l_i \leqslant w_i \leqslant u_i \\ \sum_{i=1}^{8} w_i = 1 \end{cases}. \quad (23)$$

The final optimal portfolio ratio of each equipment system with ten different hesitant preference parameters θ is shown in Table 3.

Table 3 Optimal investment ratios with different θ

θ	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
0	0.2	0.05	0.05	0.05	0.25	0.05	0.199 8	0.150 2
1/9	0.2	0.05	0.05	0.072 9	0.25	0.05	0.05	0.277 1
2/9	0.2	0.05	0.05	0.168	0.25	0.05	0.05	0.182
3/9	0.2	0.05	0.05	0.263 2	0.25	0.05	0.05	0.086 8
4/9	0.118 7	0.05	0.05	0.3	0.25	0.05	0.05	0.131 3
5/9	0.05	0.05	0.101 2	0.3	0.25	0.05	0.05	0.148 8
6/9	0.05	0.05	0.2	0.3	0.25	0.05	0.05	0.05
7/9	0.05	0.05	0.2	0.3	0.25	0.05	0.05	0.05
8/9	0.05	0.05	0.2	0.3	0.25	0.05	0.05	0.05
1	0.05	0.05	0.2	0.3	0.25	0.05	0.05	0.05

6.2 Discussion and analysis of results

Different hesitation preference parameters affect the score and investment ratio of the equipment systems in Mod2. To better understand the influence of θ on the results, a comparison is made and the results are analyzed and discussed.

(i) The parameter θ affects the final score of the equipment system portfolio, as shown in Fig. 2. When $\theta \leqslant 0.657\,6$, the hesitation and the score of the equipment system portfolio increase with an increase in θ. When $\theta = 0.657\,6$, the maximum score of the equipment system portfolio is 0.556 9, and the hesitation is 0.085 1. When $\theta > 0.657\,6$, the hesitation and score do not change with θ. When $\theta = 0$, the maximum score is 0.527 7, and the hesitation is 0.068 2. It is observed that higher decision-maker hesitation correlates with a higher score.

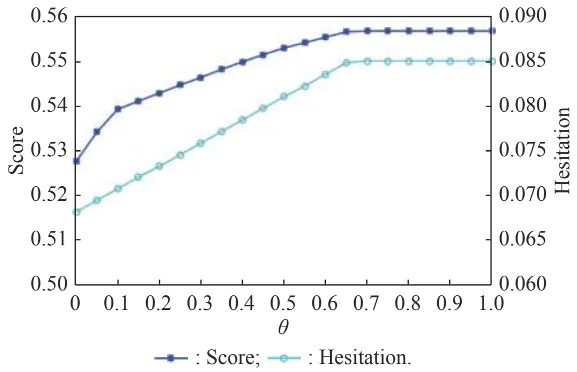

Fig. 2 Hesitation and score with different θ

(ii) The value of θ also affects the investment ratio of each equipment system. As shown in Fig. 3, for a smaller θ, the investment ratio of S_1 and S_8 is relatively decreased, and the investment ratio of S_3 and S_4 is relatively increased. However, regardless of how the acceptable hesitation changes, the investment ratio of S_5 is 0.25, which is in the preset upper bound. The investment ratio of S_2 and S_6 is 0.05, which is at the lower bound. When the acceptable hesitation of the decision-makers is relatively small, they invest more resources in the development of S_1, S_5, and S_8. When the acceptable hesitation is higher, they prefer to invest in equipment systems S_3, S_4 and S_5.

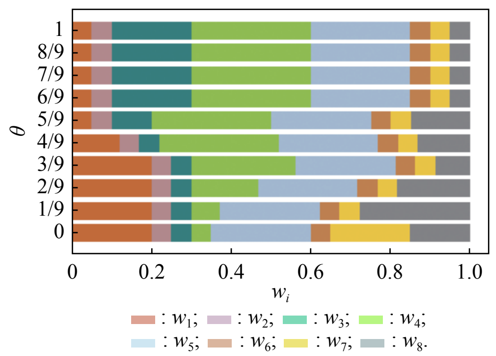

Fig. 3 Investment ratio of each equipment system with different θ

(iii) Combined with the score and hesitation of each equipment system, the result of the investment ratio is analyzed. The order of score for the eight equipment systems is $v_5 > v_4 > v_3 > v_6 > v_8 > v_2 > v_1 > v_7$. The order of hesitation for the eight equipment systems is $\varphi_6 > \varphi_4 > \varphi_3 > \varphi_2 > \varphi_8 > \varphi_5 > \varphi_7 > \varphi_1$ as shown in Fig. 4 The highest equipment system score is 0.592 5, for S_5; the hesitation is only 0.062 5, thus S_5 is worth investing for any hesitation preference. Although the score of S_4 is 0.571 2, and its hesitation is 0.101 3, the investment ratio of S_4 is relatively low when θ is relatively small, such as in S_3. If $v_i > v_j$ and $\varphi_i < \varphi_j$, the equipment system S_i is absolutely superior to the equipment system S_j. Equipment systems S_2, S_6, and S_7 are not absolutely superior to any equipment system. Except for S_7, which produces a large investment ratio when $\theta = 0$, the investment ratios of S_2, S_6, and S_7 are in the preset investment lower bound with any θ.

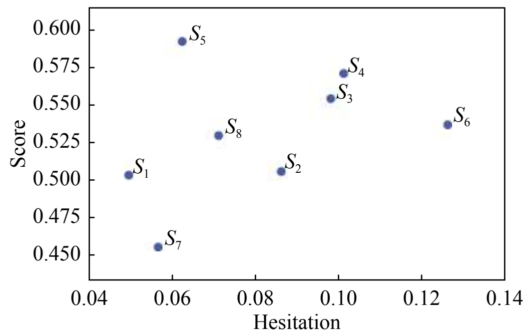

Fig. 4 Relationship between hesitation and score

The hesitation preference parameter θ can reflect an investor's hesitance preference, and its value affects the score and investment ratio of the equipment portfolio. However, its impact on the investment ratio depends on the tradeoff between the score and hesitation, because Mod2 aims to maximize the score of the equipment system portfolio with tolerable hesitation.

6.3 Comparative analysis

The equipment system portfolio-selection method proposed in this paper is mainly divided into two parts, firstly, the weapon/equipment system evaluation, then the system portfolio selection based on the evaluation results. The evaluation is essentially a multi-attribute decision making process under hesitant fuzzy environment. The following will carry out the comparisons based on the evaluation and portfolio selection process to illustrate the effectiveness, applicability, feasibility and efficiency of our method.

(i) Comparison and analysis of evaluation process

Our method is compared with the other three multi-attribute decision making methods which meet the problem in our paper under hesitant fuzzy environment. Applying the data in Table 2, the results of the evaluation are shown in Table 5. TOPSIS refers to the technique for order preference by similarity to an ideal solution. Based on the results in Table 4, we will illustrate the advantages of the evaluation process proposed in this paper from effectivity and applicability.

Table 4 Results from different evaluation methods

Method	System sorting
The method of this article	$v_5 \succ v_4 \succ v_3 \succ v_6 \succ v_8 \succ v_2 \succ v_1 \succ v_7$
The traditional GRA	$v_5 \succ v_4 \succ v_3 = v_8 \succ v_1 \succ v_6 \succ v_2 \succ v_7$
TOPSIS	$v_5 \succ v_3 \succ v_4 \succ v_1 \succ v_2 = v_6 \succ v_8 \succ v_7$
Average $\bar{h}(x)$	$v_7 \succ v_1 \succ v_5 \succ v_8 \succ v_3 \succ v_6 \succ v_4 \succ v_8$

The traditional GRA cannot distinguish S_3 from S_8, also the TOPSIS cannot distinguish S_2 from S_6. However, the method proposed in this article can clearly distinguish the advantages and disadvantages of the eight equipment systems. The reason is that the distance measures of the HFEs in the existing methods cannot distinguish the distance of HFEs in some cases. For example, $h_1 = \{0.3, 0.5\}$, $h_2 = \{0.2, 0.3\}$ and $h_3 = \{0.2, 0.7\}$. The distance between them measured by the traditional methods and the $d_{\rm IH}$, $d_{\rm IE}$ proposed in this article are shown in Table 5. The traditional Hamming distance and Euclidean distance cannot judge whether h_2 is closer to h_1 or h_3, which makes decision and evaluation more difficult. The distance measured by two distance measures proposed in this study shows that $d(h_1, h_3)$ is greater than $d(h_1, h_2)$. Therefore, it can be seen that the evaluation method proposed in this paper is more reasonable and more effective.

Table 5 Hesitant fuzzy distances obtained by different distance measures

HFEs distance measure	Distance
The Hamming distance	$d_H(h_1,h_2) = d_H(h_1,h_3) = 0.15$
The Euclidean distance	$d_E(h_1,h_2) = d_E(h_1,h_3) = 0.158\,1$
The new Hamming distance proposed in this article	$d_{\rm IH}(h_1,h_2) = 0.2 < d_{\rm IH}(h_1,h_3) = 0.35$
The new Euclidean distance proposed in this article	$d_{\rm IE}(h_1,h_2) = 0.165\,8 < d_{\rm IE}(h_1,h_3) = 0.255$

On the other hand, the average is not suitable for multi-attribute decision making with different attribute types, because the average does not consider the compatibility and additivity of attributes. As shown in Table 4, the result obtained by the average is quite different from those obtained by the other three methods. In the first three methods, S_7 is the worst, however, evaluated by average, S_7 is the best, which is abnormal. The average never considers whether different attributes can be added. For example, there is no additivity between the fighter's detection coverage and strike accuracy.

Through the above comparison, it can be seen that the evaluation method proposed in this study has a better performance, with better discrimination, and is generally applicable.

(ii) Comparison and analysis of portfolio selection process

There are very few studies on portfolio selection under hesitant fuzzy environment. Therefore, the portfolio selection method in this study is compared with the hesitant fuzzy portfolio selection model (HFPSM) proposed by Zhou and Xu [34] to show the superiority of our method. Also for the data in Table 1, the method proposed by Zhou and Xu [34] is confronted with large computational quantity. Firstly, 138 operations are required to get the evaluation value $\bar{h}_i$ of each equipment system, for example,

$$\bar{h}_1 = \underbrace{\{0.935\,7, 0.955\,5, \cdots, 0.965\,35, 0.969\,2\}}_{l_1=24}.$$

According to the HFPSM, the objective function is shown as (24), if treat $\prod_{i=1}^{8} \gamma_i^{w_i} \left(\gamma_i \in \bar{h}_i\right)$ as a unit, then $s\left(\oplus_{i=1}^{8} w_i \bar{h}_i\right)$ is the sum of about 2.3×10^9 units, it is difficult to obtain the optimal solution, calculation time is in hours. The method proposed by Zhou and Xu [34] is only applicable when the amount of data is small. As the number of alternatives and criteria increases, the number of operations will increase in geometric progression, and the results cannot be obtained in a short time. Relatively, the objective function and the method proposed in this study are simpler. As shown in (25), the amount of calculation does not change with the increase of the amount of data, and it can be solved with Cplex in about 13.4 s.

$$\max s(\oplus_{i=1}^{8} w_i \bar{h}_i) =$$

$$1-\frac{1}{2\,293\,235\,712}\underbrace{\left\{\begin{array}{c} 0.935\,7^{w_1}\cdot 0.894\,7^{w_2}\cdot 0.981\,2^{w_3}\cdot 0.911\,8^{w_4}\cdot 0.963\,1^{w_5}\cdot 0.912\,4^{w_6}\cdot 0.901\,7^{w_7}\cdot 0.913\,4^{w_8}+ \\ 0.935\,7^{w_1}\cdot 0.894\,7^{w_2}\cdot 0.981\,2^{w_3}\cdot 0.911\,8^{w_4}\cdot 0.963\,1^{w_5}\cdot 0.912\,4^{w_6}\cdot 0.901\,7^{w_7}\cdot 0.948\,0^{w_8}+ \\ \vdots \\ 0.969\,2^{w_1}\cdot 0.961\,0^{w_2}\cdot 0.969\,8^{w_3}\cdot 0.984\,2^{w_4}\cdot 0.982\,2^{w_5}\cdot 0.989\,0^{w_6}\cdot 0.969\,1^{w_7}\cdot 0.975\,5^{w_8} \end{array}\right\}}_{2\,293\,235\,712\text{ units}} \tag{24}$$

$$\max S(P) =$$

$$\sum_{i=1}^{8} w_i v_i = 0.503\,4w_1 + 0.505\,8w_2 + 0.554\,3w_3 + 0.571\,2w_4 + 0.592\,5w_5 + 0.536\,9w_6 + 0.455\,4w_7 + 0.529\,8w_8 \tag{25}$$

7. Conclusions

The traditional weapon/equipment system portfolio-selection method requires a large amount of accurate index attribute data. However, the investment portfolio selection of an equipment system occurs in the R&D stage; the equipment has not been put into service, thus it is impossible to obtain accurate and quantitative index attribute data. Data that can be obtained has great uncertainty. The hesitant fuzzy theory provides a method to solve this type of problem in equipment portfolio selection.

The advantages of this study is to effectively describe the uncertain attribute information of the equipment system criteria by HFEs. The new HFE distance measures proposed in this paper consider the difference in uncertainty between HFEs. By using the score and hesitation of the equipment systems, two equipment portfolio-selection models are defined, Mod2 focuses on the decision-maker's hesitancy preference, and Mod3 focuses on the score preference. The process of equipment system portfolio-selection based on hesitant fuzzy GRA is presented. An example of equipment system portfolio selection is provided to illustrate the effectiveness of the method, and the results are analyzed.

There are some limitations in this study. In the data extension stage, different data-extending methods may affect the investment ratio, in the portfolio selection stage, the risk preference of the decision maker is not considered for guiding the portfolio selection. The investment selection of equipment systems under hesitant fuzzy environment is still early in its development. Much work remains to be done, and we will continue to study portfolio selection in this field.

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Biographies

LI Zhuoqian was born in 1997. She is a postgraduate student at National University of Defense Technology. Her research interests are weapon alternative decision and effectiveness evaluation, and portfolio decision analysis.
E-mail: zhuoqianli_nudt@163.com

DOU Yajie was born in 1987. He received his M.E. and Ph.D. degrees in management science and engineering from National University of Defense Technology. He is currently an associate professor in National University of Defense Technology. His research interests are weapon alternative decision and effectiveness evaluation, system-of-systems architecting and engineering management, and portfolio decision analysis.
E-mail: yajiedou_nudt@163.com

XIA Boyuan was born in 1994. He received his B.E. degree in management science and engineering from National University and Defense Technology, Changsha, China, in 2017. He is currently a Ph.D. candidate in the College of Systems Engineering, National University of Defense Technology. His research interests include system of systems engineering, complex systems, and systems evaluation and optimization.
E-mail: xiaboyuan11@nudt.edu.cn

YANG Kewei was born in 1977. He received his B.E. degree in systems engineering and Ph.D. degree in management science and engineering from the National University of Defense Technology, Changsha, Hunan, China, in 1999 and 2004, respectively. He is currently a professor and the Director of the Department of Management Science and Engineering, College of Systems Engineering, National University of Defense Technology. He was a visiting scholar with the Department of Computer Science at the University of York in the United Kingdom. His current research interests include intelligent agent simulation, defense acquisition, and system-of-systems requirement modeling.
E-mail: kayyang27@nudt.edu.cn

LI Mengjun was born in 1964. He received his B.E. degree in the School of Economics and Finance from Xi'an Jiaotong University, Xi'an, China, in 1992, and Ph.D. degree in management science and engineering from the National University of Defense Technology, Changsha, China, in 2005. He was a visiting scholar at the University of Birmingham in the United Kingdom, in 2013. He is currently a professor of the Department of Management Science and Engineering, College of Systems Engineering, National University of Defense Technology. His research interest is defense acquisition.
E-mail: mjli11260744@sina.com