

# Joint optimization of inspection, maintenance, and spare ordering policy considering defective products loss

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**Abstract:** This paper proposes a joint inspection-based maintenance and spare ordering optimization policy that considers the problem of integrated inspection, preventive maintenance, spare ordering, and quality control for a four-state single-unit manufacturing system. When an inspection detects a minor defect, a second phase inspection is initiated and a regular order is placed. Product quality begins to deteriorate when the system undergoes a severe defect. To counter this, an advanced replacement of the minor defective system is carried out at the  $J$ th second phase inspection. If a severe defect is recognized prior to the  $J$ th inspection, or if system failure occurs, preventive or corrective replacement is executed. The timeliness of replacement depends on the availability of spare. We adopt two modes of ordering: a regular order and an emergency order. Meanwhile, a threshold level is introduced to determine whether an emergency order is preferred even when the regular order is already ordered but has not yet arrived. The optimal joint inspection-based maintenance and spare ordering policy is formulated by minimizing the expected cost per unit time. A simulation algorithm is proposed to obtain the optimal two-phase inspection interval, threshold level and advanced replacement interval. Results from several numerical examples demonstrate that, in terms of the expected cost per unit time, our proposed model is superior to some existing models.

**Keywords:** maintenance, two-phase inspection, spare ordering, three-stage failure process, delay-time model.

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## 1. Introduction

Preventive maintenance is extensively used to prevent sudden failures in many industrial environments, such as power systems, manufacturing systems, critical infrastructures, and military equipment [1]. Most maintenance policies assume that spare parts used for replacement are

always available, and they ignore the impact of system defects on product quality. However, this is not congruent with the real world. First, the delivery time of spare parts is not negligible. Second, a defective system can cause defective products, which can lead to economic losses. Therefore, the joint optimization of inspection, maintenance, spare ordering, and quality control is of great significance.

In most studies, maintenance policies are generally divided into two categories: corrective maintenance (CM) and preventive maintenance (PM). According to the age/condition information, PM can then be further divided into time-based maintenance and condition-based maintenance [2–6]. Moreover, these maintenance policies have been widely used in industry [7–9]. In terms of the joint policy of maintenance and spare ordering, most joint optimization models of time-based maintenance and spare ordering concentrate on the age-based replacement policy. An age-based replacement policy with a random lead time was first discussed by Nakagawa et al. [10]. Osaki et al. [11] then studied a joint age-based replacement and spare ordering policy that optimized the age replacement interval  $T$ . This policy assumes that only one spare unit is ordered at time 0, and it is delivered after a random lead time. Armstrong et al. [12] and Park et al. [13] extended the model of Osaki and Yamada [11] by relaxing the assumption of ordering time to simultaneously seek the optimal ordering time and age replacement interval,  $T$ . Additionally, Chien et al. [14–16] considered a system that is subject to shocks, extending the models in Nakagawa et al. [10] and Osaki et al. [11] by introducing minimal repair and repair cost.

Several papers have investigated joint condition-based maintenance and spare ordering optimization in which the systems are monitored continuously. A decision model for component replacement and spare parts inventory was developed by Elwany et al. [17] to enable the dynamic

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updating of replacement and inventory decisions by computing remaining life distributions using condition-based in-situ sensor data. Rausch et al. [18] proposed a joint production and spare part inventory control policy driven by condition-based maintenance for a critical unit, where the preventive maintenance threshold and the base-stock level of spare parts are the decision variables. For deterioration systems that need manual inspections, to the best of our knowledge, the joint policy of inspection-based preventive maintenance and spare ordering was originally proposed by Wang et al. [19]. They assumed that the single-unit system was inspected regularly, and only one kind of ordering mode was considered; moreover, the purpose of modeling was to optimize the thresholds related to spare ordering and preventive replacement. With the same assumptions, Wang et al. [20] and Zhang et al. [21] extended the joint policy to a deteriorating system with multi-identical items. Further, Farhad et al. [22] and Zhu et al. [23] relaxed the assumption of the ordering mode, considering two modes of spare ordering—a regular order and an emergency order—to optimize both the inspection interval and inventory policy for deterioration systems. However, the failure processes of the systems in these models do not use the delay-time concept. The delay-time concept was adopted by Wang [24,25] to model a joint inspection-based preventive maintenance and inventory strategy for multi-unit systems. The joint decision for a single-unit system with more than three discrete states was studied by Zhao et al. [26,27], these two researches adopted an irregular inspection policy, and the two modes of spare ordering were introduced. However, the joint optimization models using a three-stage failure process mentioned above did not consider the impact of system defects on product quality. This presents a challenge in actual manufacturing systems. If system defects are ignored, it could lead to defective products. Advanced replacement in a minor defective state could therefore be valuable. Therefore, we propose a joint policy of inspection-based preventive maintenance and spare ordering in which inspection is carried out within a two-phase policy, where advanced replacement of a minor defective system, defective products loss, and two modes of spare ordering (a regular order and an emergency order) are considered. This will be modeled for a single-unit system subject to a three-stage failure process, where the objective is to minimize the expected cost per unit time.

The remaining parts of this paper are as follows. Section 2 introduces the model notations and problem description. The joint optimization model is formulated in Section 3. Section 4 describes the proposed discrete event-driven simulation algorithm procedures and presents two special cases for comparison. Section 5 provides a case

study for the blast furnace of a steel mill, to validate the application of our proposed model. Finally, Section 6 concludes the paper and presents possible directions for future research.

## 2. Notations and problem description

### 2.1 Notations

Notations used in this paper are presented in Table 1.

Table 1 Notations

$X, Y, Z$	Random variable representing the duration of the normal, minor defective, and defective states
$x, y, z$	Respective durations of the normal, minor defective and severe defective states
$f_G(g)$	Probability density function (PDF) of $G$ ( $G = X, Y, Z$ ; $g = x, y, z$ )
$T$	The initial inspection interval
$J, \theta$	Threshold level as decision variables
$P$	Production rate
$L_r, L_e$	Lead times of the regular order and the emergency ( $L_r > L_e$ )
$C_{in}$	Unit inspection cost
$C_r, C_e$	Replacement cost by a regular ordered spare and an emergency ordered spare ( $C_e > C_r$ )
$C_f$	Economic loss caused by a failure
$C_h$	Holding cost per unit time
$C_w$	Shortage cost per unit time
$C_s$	Unit loss of a defective item

### 2.2 System statement

Consider a single-unit manufacturing system that undergoes a three-stage failure process, that is, the system has four states: normal, minor defective, severe defective, and failed. In the normal state, the system works properly and needs no intervention. In the minor defective state, the system is still operational but inspections may reveal minor defects that do not affect the quality of products. In the severe defective state, the system is still operational but inspections may reveal severe defects that negatively affect the quality of products. In the failed state, the system stops working immediately. To study the inspection, maintenance, and spare ordering policy of such a system, we follow some basic assumptions:

(i) The system is inspected with the initial fixed interval  $T$ , and inspections are perfect in that the normal and defective states can be recognized. In contrast, the failed state is self-announced.

(ii) If a minor defective state is first detected at  $T_k$ , the subsequent inspection interval is halved, that is, the second phase inspection is executed. Meanwhile, a regular order is placed and the spare part will be delivered after lead time  $L_r$ .

(iii) The quality of products begins to decline when the system enters a severe defective state. The proportion of

defective items at time  $t$  is assumed as  $\beta((t - x - y)/z)$ , where  $x+y \leq t \leq x+y+z$ .

(iv) If the system still undergoes the minor defective state at time  $T_{k,j}$ , where  $T_{k,j} = T_k + \frac{2j}{T}$  and  $j = J$ , replacement requires to be done immediately, known as an advanced replacement (AR).

(v) A preventive replacement (PR) is carried out when a severe defective state is found, and a corrective replacement (CR) is required to be carried out at the point of failure.

(vi) All of the replacement activities can bring the system to the “as good as new” state.

As mentioned above, when the severe defect appears, the system negatively affect the quality of products, thus we assume that the quality of products begins to decline. Based on Driessen et al. [28], we define  $\beta((t - x - y)/z)$  to express the proportion of defective items at time  $x+y \leq t \leq x+y+z$ . This can be explained since the proportion of defective items at time  $t$  depends on the system failure progress, and the failure progress can be depicted by the relative duration  $\frac{t-x-y}{z}$  in the severe defective state. The more the system degrades in the severe defective state, the higher the proportion of defective items produced, that is, the proportion of defective items  $\beta((t - x - y)/z)$  is increasing in  $(t - x - y)/z$ . Hence, following Bouslah et al. [29], we have

$$\beta\left(\frac{t-x-y}{z}\right) = \beta_0 + \eta \left(1 - \exp\left(-\lambda\left(\frac{t-x-y}{z}\right)^\gamma\right)\right) \quad (1)$$

where  $\beta_0$  is the proportion of defective items when the minor defective state arrives,  $\eta$  is the boundary considered in the quality deterioration, and  $\lambda$  and  $\gamma$  are positive constants. These parameters can be obtained from historical data adopting the least squares or the maximum likelihood methods [30].

### 2.3 A joint inspection, maintenance, and spare ordering policy

When the system needs to be replaced, whether the spare has been ordered should be firstly concerned. If it has not, an emergency order with a higher ordering cost and shorter lead time is placed, and we define  $S = \text{I}$  to express this scenario.  $S = \text{II}$  means the regular order has been previously ordered but has not yet arrived.  $S = \text{III}$  indicates that the regular order has been delivered, thus the replacement required can be conducted immediately.

Actually, it is possible that the emergency order is preferred even when the regular order is already ordered. Therefore, we define  $\Gamma$  as the time interval from the point in which a replacement is needed to the delivery time of the regular order’s spare, if  $\Gamma$  is not longer than a threshold  $\theta$  ( $L_e < \theta < L_r$ ), the system should retain stopping until delivery of the spare; otherwise, an emergency order should be made immediately.

Clearly, according to the system state and the state of the regular ordered spare, the inspection, the replacement, and reorder decisions can be determined. Fig. 1 gives the specific decision-making process.

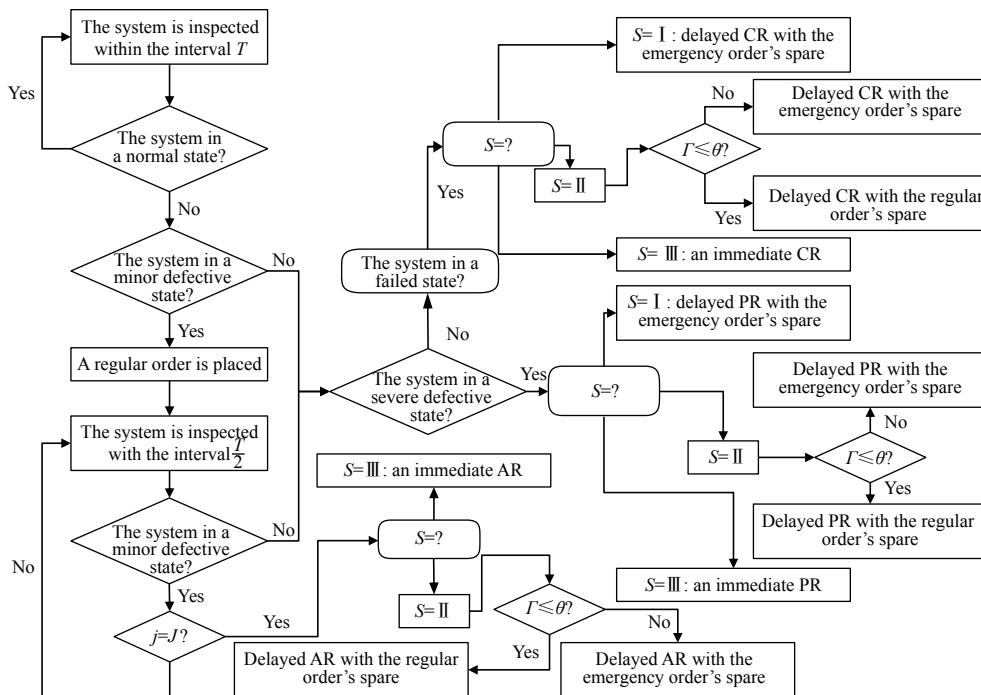


Fig. 1 Decision-making process flow chart

We denote our inspection, maintenance, and spare ordering policy by  $(T, J, \theta)$ , since the initial inspection interval  $T$ , the threshold level  $J$  and  $\theta$  are the decision variables that we are interested in. And our aim is to minimize the long-run expected total cost per unit time,  $EC(T, J, \theta)$ , by optimizing the joint policy.

### 3. Model formulation

As detailed in Section 2, there are three different renewal scenarios based on the state of the system at the renewal points: (i) an advanced renewal, when the system is found to be in a minor defective state; (ii) a preventive renewal, when the system is found to be in a severe defective state; and (iii) a corrective renewal, when the system fails. Furthermore, eight mutually exclusive possible scenarios are provided relying on the state of the spare from a regular order when replacement is required.

**Scenario 1** A renewal cycle is completed due to an AR is carried out under the condition of  $S = \text{II}$ .

Fig. 2 illustrates that an advanced replacement is required at time  $T_{k,J}$ , the regular order's spare has not arrived. As we mentioned previously, managers need to decide whether an emergency order should be placed or not. Clearly, if  $T_k + L_r - T_{k,J}$  is longer than  $\theta$ , an emergency order is placed immediately and the AR is delayed until the delivery time of the emergency order's spare (see case E<sub>1</sub> in Fig. 2). However, since  $J \geq 1$ , the condition  $\frac{2(L_r - \theta)}{T} > 1$  must be met. Therefore, the renewal cycle cost of such a case includes the inspection cost, the replacement cost by an emergency ordered spare and the shortage cost, it can be given as

$$C_1(T, J, \theta) = [(k + J)C_{in} + C_e + C_w L_e] \cdot I\left(J - \frac{L_r - \theta}{T/2}\right) \cdot V\left(\frac{L_r - \theta}{T/2}\right) \quad (2)$$

where  $I(m) = \begin{cases} 1, & m < 0 \\ 0, & \text{otherwise} \end{cases}$ ,  $V(a) = \begin{cases} 1, & a > 1 \\ 0, & \text{otherwise} \end{cases}$ , and  $k = 1, \dots, \infty$ .

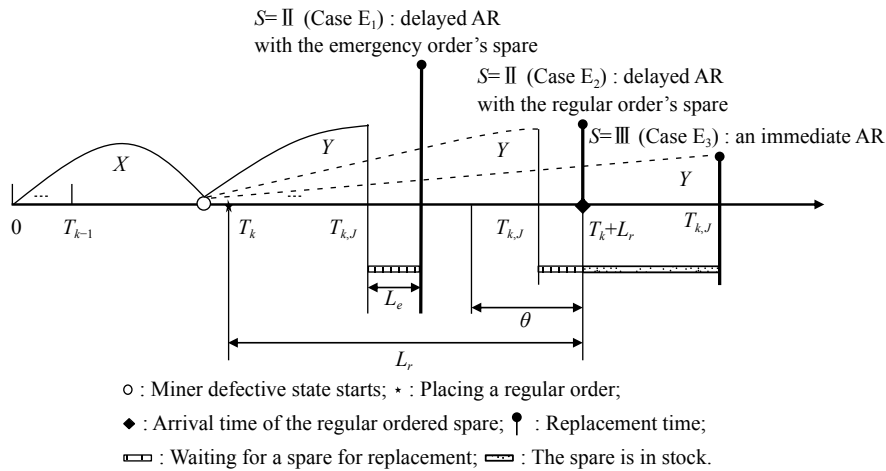


Fig. 2 Illustration of Scenario 1 and Scenario 2

The corresponding renewal cycle length is

$$L_1(T, J, \theta) = [T_{k,J} + L_e] \cdot I\left(J - \frac{L_r - \theta}{T/2}\right) \cdot V\left(\frac{L_r - \theta}{T/2}\right). \quad (3)$$

As can be seen from Case 2 in Fig. 2, the condition  $J \geq \frac{2(L_r - \theta)}{T}$  and  $J < \frac{2L_r}{T}$  are met, managers prefer to delay the AR until the arrival time of the regular order's spare. Analogously, there exists the condition  $\frac{2L_r}{T} > 1$  in this situation, since  $J \geq 1$ . The renewal cycle cost consists of the inspection cost, the replacement cost by a regular ordered spare and the shortage cost, and it can be obtained as

$$C_2(T, J, \theta) = [(k + J)C_{in} + C_r + C_w(T_k + L_r - T_{k,J})] \cdot I\left(J - \frac{L_r}{T/2}\right) \cdot I'\left(J - \frac{L_r - \theta}{T/2}\right) \cdot V\left(\frac{L_r}{T/2}\right) \quad (4)$$

where  $I'(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$  and  $k = 1, \dots, \infty$ .

The corresponding renewal cycle length is

$$L_2(T, J, \theta) = [T_k + L_r] \cdot I\left(J - \frac{L_r}{T/2}\right) \cdot I'\left(J - \frac{L_r - \theta}{T/2}\right) \cdot V\left(\frac{L_r}{T/2}\right). \quad (5)$$

**Scenario 2** A renewal cycle is completed because an AR is carried out under the condition of  $S = \text{III}$ .

As illustrated in Fig. 2 (Case E<sub>3</sub>), the regular order's spare is available at  $T_{k,j}$ , thus, the AR can be performed immediately. Summating the inspection cost, the replacement cost by a regular ordered spare and the holding cost, the renewal cycle cost in this scenario is given as

$$C_3(T, J, \theta) = [(k + J)C_{in} + C_r + C_h(T_{k,j} - T_k - L_r)] \cdot I' \left( J - \frac{L_r}{T/2} \right) \quad (6)$$

where  $k = 1, \dots, \infty$ .

The corresponding renewal cycle length is

$$L_3(T, J, \theta) = T_{k,j} \cdot I' \left( J - \frac{L_r}{T/2} \right). \quad (7)$$

**Scenario 3** A renewal cycle is completed because a PR is carried out under the condition of  $S = I$ .

As can be seen from Fig. 3, a severe defective state is identified at time  $T_k$ , before which no minor defective state is detected. This indicates that the spare is not ordered, so an emergency order is placed at time  $T_k$  and the PR is performed at the arrival time of the emergency order's spare. This renewal cycle cost is the sum of the inspection cost, the replacement cost by an emergency ordered spare, the shortage cost and the loss of defective items. Consequently, we obtain the cost as follows:

$$C_4(T, J, \theta) = kC_{in} + C_e + C_wL_e + DC_1 \quad (8)$$

where  $k = 1, \dots, \infty$  and  $DC_1 = C_s P \int_{x+y}^{T_k} \beta \left( \frac{t-x-y}{z} \right) dt$ .

The corresponding renewal cycle length is

$$L_4(T, J, \theta) = T_k + L_e. \quad (9)$$

**Scenario 4** A renewal cycle is completed because a PR is carried out under the condition of  $S = II$ .

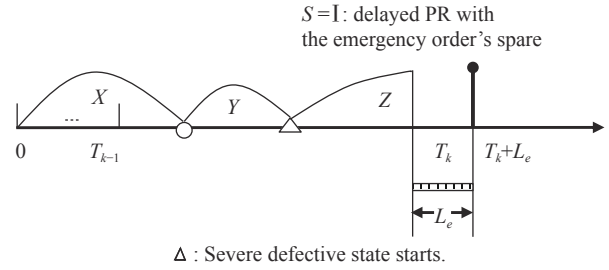


Fig. 3 Illustration of Scenario 3

A severe defective state is identified at time  $T_{k,j}$ , before which a minor defective state is first found at time  $T_k$ . However, the spare from a regular order has not arrived until  $T_{k,j}$ . As per assumption, when the time interval to  $T_k + L_r$  from  $T_{k,j}$  is longer than the threshold level  $\theta$ , that is, the condition  $j < \frac{2(L_r - \theta)}{T}$  is satisfied, managers prefer to place an emergency order at time  $T_{k,j}$  and the PR is delayed until  $T_{k,j} + L_e$  (see Case E<sub>1</sub> in Fig. 4). In such a case, the inspection cost, the replacement cost by an emergency ordered spare, the shortage cost and the loss of defective items constitute the renewal cycle cost

$$C_5(T, J, \theta) = [(k + j)C_{in} + C_e + C_wL_e + DC_2] \cdot I \left( j - \frac{L_r - \theta}{T/2} \right) \cdot V \left( \frac{L_r - \theta}{T/2} \right) \quad (10)$$

where  $k = 1, \dots, \infty, j = 1, \dots, J_{max}$ ,  $DC_2 = C_s P \int_{x+y}^{T_{k,j}} \beta \left( \frac{t-x-y}{z} \right) dt$ , and  $J_{max} = \begin{cases} J_{upp}, & J_{up} < J \\ J, & \text{otherwise} \end{cases}$ , here,  $J_{upp} = \begin{cases} J_{up} - 1, & U \left( \frac{L_r - \theta}{T/2} \right) = 1 \\ J_{up}, & \text{otherwise} \end{cases}$ , and  $J_{up} = \text{int} \left( \frac{L_r - \theta}{T/2} \right)$ , and we define  $\text{int}(u)$  returns to the integer part of a value  $u$ ,  $U(b) = \begin{cases} 1, & b = Z_+ \\ 0, & \text{otherwise} \end{cases}$ , and  $Z_+$  is a positive integer.

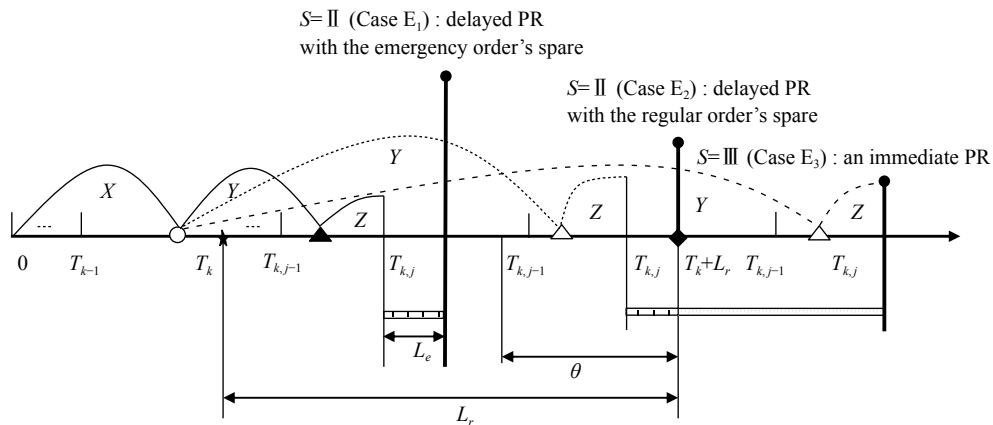


Fig. 4 Illustration of Scenario 4 and Scenario 5

The corresponding renewal cycle length is

$$L_5(T, J, \theta) = [T_{k,j} + L_e] \cdot I\left(j - \frac{L_r - \theta}{T/2}\right) \cdot V\left(\frac{L_r - \theta}{T/2}\right). \quad (11)$$

Observed from Case E<sub>2</sub> in Fig. 4, a delayed PR with a regular ordered spare is performed at  $T_k + L_r$ , this means the condition  $j \geq \frac{2(L_r - \theta)}{T}$ ,  $j < \frac{2L_r}{T}$  and  $\frac{2L_r}{T} > 1$  are satisfied. The renewal cycle cost can be obtained by summing the inspection cost, the replacement cost by a regular ordered spare, the shortage cost and the loss of defective items

$$C_6(T, J, \theta) = [(k + j)C_{in} + C_r + C_w(T_k + L_r - T_{k,j}) + DC_2] \cdot I'(\delta) \cdot V\left(\frac{L_r}{T/2}\right) \quad (12)$$

where  $k = 1, \dots, \infty, j = J_{min}, \dots, J_{max}, \delta = J - J_{min}$ ,

$$J_{min} = \begin{cases} J_{up}, & U\left(\frac{L_r - \theta}{T/2}\right) = 1 \\ J_{up} + 1, & \text{otherwise} \end{cases}, J_{max} = \begin{cases} J'_{up}, & J_u < J \\ J, & \text{otherwise} \end{cases}$$

$$J'_{up} = \begin{cases} J_u - 1, & U\left(\frac{L_r}{T/2}\right) = 1 \\ J_u, & \text{otherwise} \end{cases}, \text{ and } J_u = \text{int}\left(\frac{L_r}{T/2}\right).$$

The corresponding renewal cycle length is

$$L_6(T, J, \theta) = [T_k + L_r] \cdot I'(\delta) \cdot V\left(\frac{L_r}{T/2}\right). \quad (13)$$

**Scenario 5** A renewal cycle is completed because a PR is carried out under the condition of  $S = III$ .

As can be seen from Case E<sub>3</sub> in Fig. 4, the PR is carried out immediately since the regular order's spare is in stock at time  $T_{k,j}$ , thus, the condition  $j \geq \frac{2L_r}{T}$  is met. The inspection cost, the replacement cost by a regular ordered spare, the holding cost and the loss of defective items are incurred, thus, the renewal cycle cost can be given as

$$C_7(T, J, \theta) = [(k + j)C_{in} + C_r + C_h(T_{k,j} - T_k - L_r) + DC_2] \cdot I'\left(j - \frac{L_r}{T/2}\right) \cdot I'(\delta') \quad (14)$$

where  $k = 1, \dots, \infty, j = J'_{min}, \dots, J, \delta' = J - J'_{min}$ , and

$$J'_{min} = \begin{cases} J_u, & U\left(\frac{L_r}{T/2}\right) = 1 \\ J_u + 1, & \text{otherwise} \end{cases}.$$

The corresponding renewal cycle length is

$$L_7(T, J, \theta) = T_{k,j} \cdot I'\left(j - \frac{L_r}{T/2}\right) \cdot I'(\delta'). \quad (15)$$

**Scenario 6** A renewal cycle is completed because a CR is carried out under the condition of  $S = I$ .

As shown in Fig. 5, the minor defective, severe defec-

tive, and failed states start within the inspection interval  $(T_{k-1}, T_k)$ , which implies that no regular order is placed before failure time  $T_f$ . Thus, an emergency order is placed when failure occurs, and the system is replaced at  $T_f + l_e$ . Therefore, the renewal cycle cost not only includes the inspection cost, the replacement cost by an emergency ordered spare, the shortage cost, the loss of defective items, but also the economic loss caused by a failure, and it can be given as

$$C_8(T, J, \theta) = [(k - 1)C_{in} + C_e + C_w L_e + C_f + DC_3] \quad (16)$$

where  $k = 1, \dots, \infty, DC_3 = C_s P \int_{x+y}^{T_f} \beta\left(\frac{t-x-y}{z}\right) dt$ , and  $T_f = x + y + z$ .

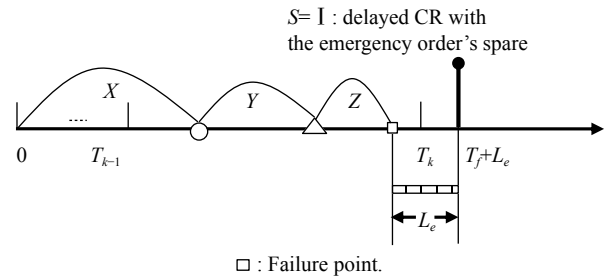


Fig. 5 Illustration of Scenario 6

The corresponding renewal cycle length is

$$L_8(T, J, \theta) = T_f + L_e. \quad (17)$$

**Scenario 7** A renewal cycle is completed because a CR is carried out under the condition of  $S = II$ .

The system fails after the regular order is placed before the spare arrives, as illustrated in Fig. 6. An emergency order is placed at the failure time  $T_f$  under the condition of  $T_k + L_r - T_f > \theta$ , and the CR has to be delayed until  $T_f + L_e$ . Moreover, the severe defective state must end in some halved inspection interval  $(T_{k,j-1}, T_{k,j})$  ( $j = 1, \dots, J_{up}$ ). However, when  $U\left(\frac{L_r - \theta}{T/2}\right) = 0$ , the failed state may start within the interval  $(T_{k,J_{up}}, T_k + L_r - \theta)$ . In particular,  $j < J$  must be satisfied. Thus, the renewal cycle cost for each event in this case can be calculated as

$$C_9(T, J, \theta) = [(k + j - 1)C_{in} + C_e + C_w L_e + C_f + DC_3] \cdot V\left(\frac{L_r - \theta}{T/2}\right) \quad (18)$$

where  $j = 1, \dots, J_s, J_s = \begin{cases} J_{up}, & J_{up} < J \\ J, & \text{otherwise} \end{cases}$ , and  $k = 1, \dots, \infty$ .

$$C_{10}(T, J, \theta) =$$

$$[(k + J_{up})C_{in} + C_e + C_w L_e + C_f + DC_3] \cdot U'\left(\frac{L_r - \theta}{T/2}\right) \cdot I(v) \quad (19)$$

where  $k = 1, \dots, \infty$ ,  $U'(c) = \begin{cases} 1, & c \neq Z_u \\ 0, & \text{otherwise} \end{cases}$ , and  $v = J_{up} - J$ .

The renewal cycle lengths for each event can be given respectively as

$$L_9(T, J, \theta) = [T_f + L_e] \cdot V\left(\frac{L_r - \theta}{T/2}\right), \quad (20)$$

$$L_{10}(T, J, \theta) = [T_f + L_e] \cdot U'\left(\frac{L_r - \theta}{T/2}\right) \cdot I(v). \quad (21)$$

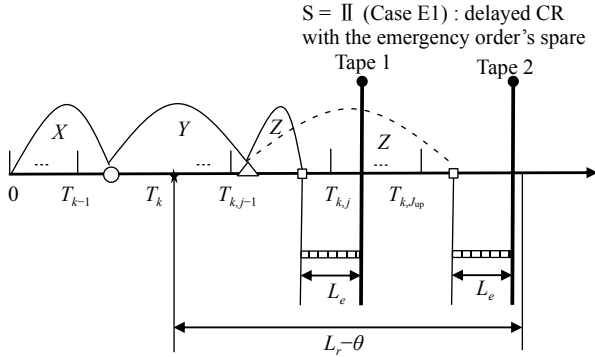


Fig. 6 Illustration of renewal Case E1 in Scenario 7

It is seen from Fig. 7, under the condition of  $T_k + L_r - T_f \leq \theta$ , waiting for the regular order's spare is a choice that managers prefer, and the delayed CR is performed until  $T_k + L_r$ .

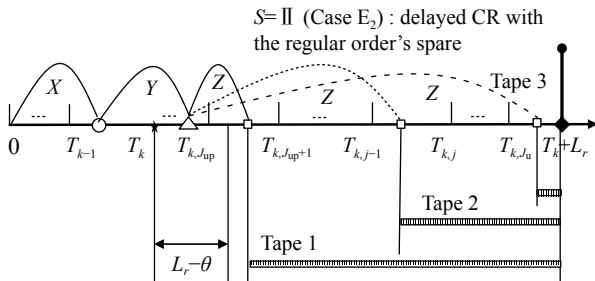


Fig. 7 Illustration of renewal Case E2 in Scenario 7

Three situations are considered depending on the interval in which a failure occurs, and the renewal cycle cost for each situation can be obtained respectively as

$$C_{11}(T, J, \theta) = [(k + J_{up})C_{in} + C_r + C_w(T_k + L_r - T_f) + C_f + DC_3] \cdot U'\left(\frac{L_r - \theta}{T/2}\right) \cdot I(v) \quad (22)$$

where  $k = 1, \dots, \infty$ .

$$C_{12}(T, J, \theta) = [(k + j - 1)C_{in} + C_r + C_w(T_k + L_r - T_f) + C_f + DC_3] \cdot I'(v) \quad (23)$$

where  $k = 1, \dots, \infty$ ,  $j = J_{low}, \dots, J_p$ ,  $q = J_u - J$ ,

$$J_{low} = \begin{cases} J_{up} + 1, & U\left(\frac{L_r - \theta}{T/2}\right) = 1 \\ J_{up} + 2, & \text{otherwise} \end{cases}, \quad J_p = \begin{cases} J_u, & q < 1 \\ J, & \text{otherwise} \end{cases},$$

and  $v' = J - J_{low}$ .

$$C_{13}(T, J, \theta) = [(k + J_u)C_{in} + C_r + C_w(T_k + L_r - T_f) + C_f + DC_3] \cdot U'\left(\frac{L_r}{T/2}\right) \cdot I(q) \quad (24)$$

where  $k = 1, \dots, \infty$ .

Therefore, the renewal cycle lengths for each event of Case E2 in Scenario 7 can be given respectively as

$$L_{11}(T, J, \theta) = [T_k + L_r] \cdot U'\left(\frac{L_r - \theta}{T/2}\right) \cdot I(v), \quad (25)$$

$$L_{12}(T, J, \theta) = [T_k + L_r] \cdot I'(v'), \quad (26)$$

$$L_{13}(T, J, \theta) = [T_k + L_r] \cdot U'\left(\frac{L_r}{T/2}\right) \cdot I(q). \quad (27)$$

**Scenario 8** A renewal cycle is completed because a CR is carried out under the condition of  $S = III$ .

The system fails at time  $T_f$  after the delivery of the regular order's spare, that is,  $T_f \geq T_k + L_r$ . Consequently, an immediate CR is carried out at the time of failure, as shown in Fig. 8.

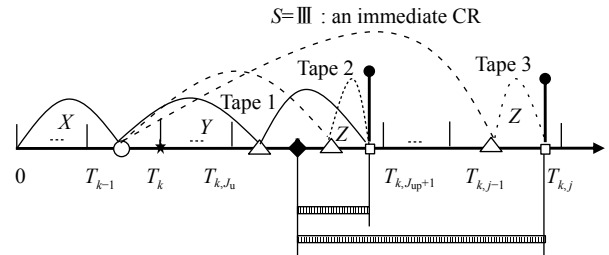


Fig. 8 Illustration of Scenario 8

The point of failure may fall into two possible intervals:  $(T_k + L_r, T_{k, J_{up}})$ , and  $(T_{k, j-1}, T_{k, j})$ ,  $j = J_{low}, \dots, J$ . Moreover, the time of the last inspection must be less than  $T_{k, j}$ , thus, the renewal cycle cost for each event in this scenario can be obtained respectively as

$$C_{14}(T, J, \theta) = [(k + J_u)C_{in} + C_r + C_h(T_f - T_k - L_r) + C_f + DC_3] \cdot U'\left(\frac{L_r - \theta}{T/2}\right) \cdot I(q) \quad (28)$$

where  $k = 1, \dots, \infty$ .

$$C_{15}(T, J, \theta) = [(k + j - 1)C_{in} + C_r + C_h(T_f - T_k - L_r) + C_f + DC_3] \cdot I'(q') \quad (29)$$

where  $k = 1, \dots, \infty, j = J'_{\text{low}}, \dots, J, q' = J - J'_{\text{low}}$ , and

$$J'_{\text{low}} = \begin{cases} J_u + 1, & U\left(\frac{L_r}{T/2}\right) = 1 \\ J_u + 2, & \text{otherwise} \end{cases}.$$

The renewal cycle length for each event can be given respectively as

$$L_{14}(T, J, \theta) = T_f \cdot U' \left( \frac{L_r - \theta}{T/2} \right) \cdot I(q), \quad (30)$$

$$L_{15}(T, J, \theta) = T_f \cdot I'(q'). \quad (31)$$

Based on the renewal cycle cost and length of eight different scenarios (15 different events) described above and adopting the renewal reward theory [31], the long-run expected cost per unit time can be obtained as

$$EC(T, J, \theta) = \lim_{t \rightarrow +\infty} \frac{C(T, J, \theta)}{t} = \frac{\sum_{e=1}^{15} C_e(T, J, \theta) \cdot N_e(t)}{\sum_{e=1}^{15} L_e(T, J, \theta) \cdot N_e(t)} \quad (32)$$

where  $C(T, J, \theta)$  is the total expected cost of the system in the period  $[0, t]$ , and  $N_e(t)$  represents the expected number of renewal event  $e$  during the same period  $[0, t]$ .

Obviously, our purpose is to design an optimal joint inspection, maintenance and spare ordering policy that minimizing the long-run expected cost per unit time. Therefore, the model can be summarized as the following non-linear, integer and stochastic optimization problem

$$\begin{aligned} & \min EC(T, J, \theta) \\ & \text{s.t.} \quad \begin{cases} T > 0 \\ J \geq 1 \\ L_e < \theta < L_r \\ T, J, \theta : \text{postive integers} \end{cases} \end{aligned} \quad (33)$$

## 4. Optimization methodology and some special cases

### 4.1 Optimization methodology

The jointly optimization model established above is extremely difficult to solve analytically since the complexity interactions between the state of system, inspection, maintenance, spare ordering and quality of items. For instance, the defective items loss is affected by the failure progress of the system, inspection interval and the production rate. The inspection interval not only influences the frequency of maintenance, but also the type of maintenance (AR, PR, or CR). The spare ordering decisions are also influenced by the inspection interval and maintenance activities. Furthermore, the specific maintenance activities rely on the states of the system, and the states of the system are random variables. Thus, we devise a dis-

crete event-driven simulation algorithm to determine the optimal inspection, maintenance, and spare ordering policy of our presented model, and it can effectively to imitate the stochastic and dynamic aspects of the system. It is noted that using a simulation algorithm to solve the non-linear, integer and stochastic problem has been widely applied in engineering practice.

Fig. 9 shows the simulation procedure for our model, and the main steps are as follows:

#### Step 1 Initialization

(i) Initialize the system and input the relevant parameter values.

(ii) Set the reasonable value range of decision variables.  $J_{\text{ub}}$  represent the maximum value of  $J$ .

(iii) Set the maximum iterative number to be  $N_{\text{max}}$  for each  $(T, J, \theta)$ .

(iv) At the beginning of each simulation, total cost  $C$  and total length  $L$  are all set to 0.

#### Step 2 Simulation process

(i) Rely on the distribution parameters of the three stages, and generate the corresponding random durations  $x, y,$  and  $z$ .

(ii) Use Box A to judge whether a PR or a CR needs to be carried out before a regular order is placed. If so, we turn to Box B; otherwise, the subsequent inspection interval is halved, and meanwhile, a regular order is placed and we denote  $T_{\text{ar}}$  and  $T_{\text{or}}$  to represent the arrival time of the regular order's spare and the time of regular order.

(iii) In Box B, if  $T_k < x + y + z$  is met, implying that a delayed PR is carried out under the condition of  $S = \text{I}$ , as described in Scenario 3; otherwise, a delayed CR is performed with the emergency order's spare, as depicted in Scenario 6.

(iv) Box C is used to judge whether a PR or a CR is required before an AR is needed at time  $T_k + \frac{jT}{2}$ . If so, simulation runs by Box D; otherwise, we turn to Box E.

(v) In Box D, whether a PR or a CR is needed should be further confirmed by comparing the inspection time  $T_k + \frac{jT}{2}$  with the failure time  $T_f = x + y + z$ . Both the PR and the CR are needed to further judge that whether regular order has been delivered. If the point  $T_k + \frac{jT}{2}$  or  $T_f$  is not less than  $T_{\text{ar}}$ , an immediate PR (as described in Scenario 5) or CR (as described in Scenario 8) is carried out; otherwise, we need to judge whether  $T_{\text{ar}} - T_k - \frac{jT}{2}$  or  $T_{\text{ar}} - T_f$  is longer than the threshold level  $\theta$ . Clearly, there exists two possible renewal events subject to a delayed PR/CR with the emergency order's spare (as depicted Case  $E_1$  in Scenario 4 or Scenario 7) or with the regular order's spare (as Case  $E_2$  depicted in Scenario 4 or Scenario 7).



(vi) In Box E, the judgment of whether an AR is required should be made. If the system still undertakes a minor defective state at the inspection time  $T_k + \frac{JT}{2}$ , indicating that an AR needs to be carried out; otherwise, simulation runs by Box D. Analogously, when an AR is needed, whether regular order has been delivered should be confirmed. If the point  $T_{ar}$  is no longer than  $T_k + \frac{JT}{2}$ , an AR is performed immediately, as described in Scenario 2. Otherwise, a delayed AR with the emergency order's spare is carried out under the condition of  $T_{ar} - T_k - \frac{JT}{2} > \theta$  (as Case E<sub>1</sub> depicted in Scenario 1) or a

delayed AR with the regular order's spare is carried out under the condition of  $T_{ar} - T_k - \frac{JT}{2} \leq \theta$  (as Case E<sub>2</sub> depicted in Scenario 1).

**Step3** Simulation completed

Record the values of  $C$ ,  $L$ , and  $D$  after a renewal cycle is completed by running Box B, Box D, or Box E. If  $n < N_{max}$ , simulation continues to run; otherwise, we obtain the total cost  $C$  and total length  $L$  for each  $(T, J, \theta)$ . Further, the expected cost per unit time  $EC(T, J, \theta)$  is obtained by the total cost  $C$  divided by the total renewal length  $L$ .

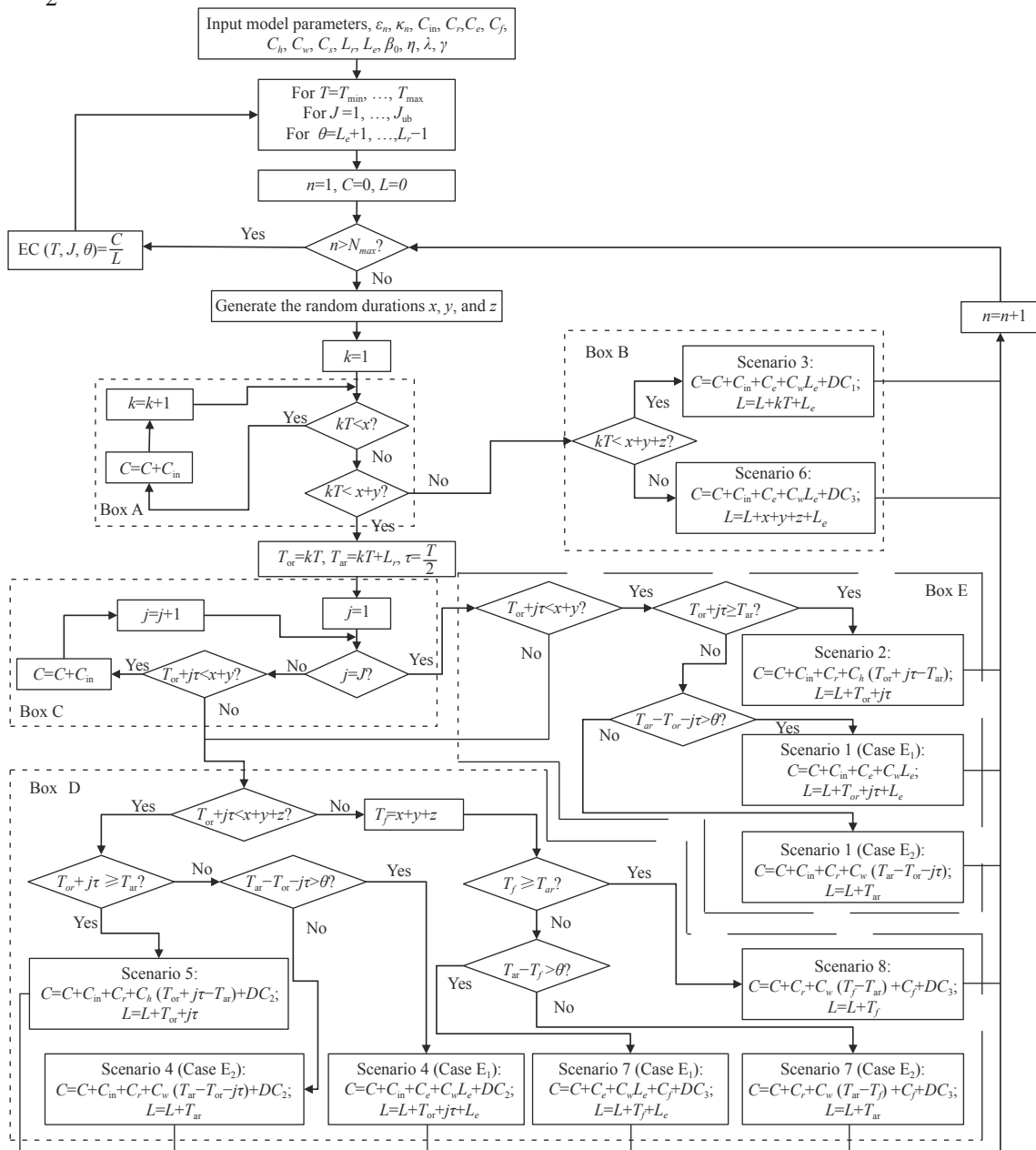


Fig. 9 Flow chart of the simulation algorithm

### 4.2 Some special cases

We introduce two further inspection-based maintenance and spare ordering policies (Models 2 and 3) as the special case of the policy presented in Section 3 (Model 1).

Model 2 does not allow an AR in the minor defective stage. Consequently, the system can only be renewed when the severe defective stage is detected or when failure occurs. Under this policy, Scenario 1 and Scenario 2 of Model 1 do not occur in Model 2. Therefore, (2) – (8) should be changed to 0. Based on this, we obtain the objective function of Model 2 and take  $T$  and  $\theta$  as the decision variables. Zhao et al. [26] adopted the same policy but did not take the defective items loss into consideration.

Model 3 uses the AR policy but does not allow an emergency order when the regular ordered spare has not arrived. Under this policy, Case  $E_1$  in Scenarios 1, 4, and 7 of Model 1 do not occur in Model 3. Hence, in Model 3  $T$  and  $J$  are the decision variables.

## 5. Numerical example

### 5.1 Initial parameter setting

To illustrate our model, we consider the refractory lining of the blast furnace in a steel mill. According to [26], the three stages failure distribution form of the refractory lining is more appropriate to be described by the two-parameter Weibull distribution. The probability density function of the two-parameter Weibull distribution can be represented by (34), in which  $\varepsilon_n$  and  $\kappa_n$  are the scale parameter and shape parameter, respectively. The values of these three sets of parameters are  $\varepsilon_1 = 0.019, \kappa_1 = 1.390; \varepsilon_2 = 0.031, \kappa_2 = 1.305; \text{ and } \varepsilon_3 = 0.088, \kappa_3 = 5.290$ . The cost and lead time parameters are given in Table 2. The chosen cost unit is 1 000 yuan and the chosen time unit is one day. Moreover, the failure distribution parameters, lead time parameters, and the majority of the cost parameters are adopted from literature [26]. Other parameters (see Table 3) can be obtained from historical information. Note that the day output of the system amounts to 2 200 t, that is,  $P = 2\,200$  t/d.

$$f_G(g, \varepsilon_n, \kappa_n) = \varepsilon_n \kappa_n (\varepsilon_n g)^{\kappa_n - 1} e^{-(\varepsilon_n g)^{\kappa_n}}, \quad \varepsilon_n > 0; \kappa_n > 0. \quad (34)$$

**Table 2 Cost and lead time parameters**

Parameter	$C_{in}$	$C_r$	$C_e$	$C_f$	$C_h$	$C_w$	$C_s$	$L_r$	$L_e$
Value	5	50	80	400	0.2	4	0.5	30	3

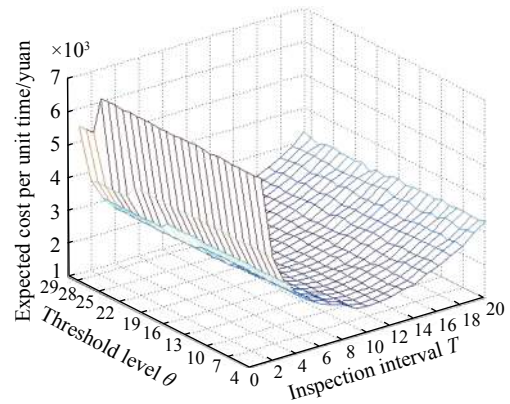
**Table 3 Other model parameters**

Parameter	$\beta_0$	$\eta$	$\lambda$	$\gamma$
Value	0.004	0.08	10	4

### 5.2 Result analysis and comparison

We calculate the expected cost per unit time of Models 1–3 based on the simulation algorithm presented in Section 4.1. It is noted that when using the simulation algorithm to conduct the numerical experiment of Model 2, simulation is not run to Box E since an advanced replacement does not allowed in Model 2. When conduct the numerical experiment of Model 3, simulation does not turn to Scenario 1 (Case  $E_1$ ), Scenario 4 (Case  $E_1$ ), and Scenario 7 (Case  $E_1$ ). We set the maximum simulation number  $N_{max}$  to be 1 000, and simulate 5 000 renewals for each decision variable, which are averaged to obtain the expected cost per unit time.

The optimal results of Model 1 is  $(T^*, J^*, \theta^*) = (10, 6, 16)$  with  $EC(T^*, J^*, \theta^*) = 1.7317$  (the expected cost per day is 1 731.7 yuan). This implies that the optimal policy of Model 1 is to perform an inspection every 10 days at the earlier stage, execute an advanced replacement at the 6th second phase inspection, and set the threshold level  $\theta$  as 16 days. Fig. 10 shows how the  $EC(T, J, \theta)$  changes with  $T$  and  $\theta$  when  $J = 6$ , and Fig. 11 illustrates the change trend of  $EC(T, J, \theta)$  along with  $T$  and  $J$  when  $\theta = 16$ . It can be seen from Fig. 10 and Fig. 11 that when the values of  $J$  and  $\theta$  are fixed, with the increase of  $T$ ,  $EC(T, J, \theta)$  first decreases and then goes up. Our interpretation is that the smaller inspection interval will lead to more frequent inspection actions, which further brings about higher inspection costs. However, if we check the system with a longer interval, an AR or a PR may be missed, thus, leading to a CR and resulting in a higher economic loss.



**Fig. 10  $EC(T, J, \theta)$  with regard to  $T$  and  $\theta$  when  $J = 6$**

In order to verify the effectiveness of our proposed inspection-based maintenance and spare ordering policy (Model 1), we conduct experiments to analyze the influence of  $C_{in}, L_e$ , and  $\gamma$  on the optimal solution. This is because that (i) the inspection interval is largely affected by  $C_{in}$ ; (ii) the lead time  $L_e$  has a great influence on the threshold level  $\theta$ . Besides,  $L_e$  is negatively related to  $C_e$  in engineering practice, hence, according to the values of  $L_e, C_e, L_r$ , and  $C_r$  given in Table 2, we develop a linear func-

tion  $C_e(L_e) = 10\left(\frac{1}{3} - \frac{L_e}{9}\right) + 80$ ; (iii)  $\gamma$  directly influences the proportion of defective items in the severe defective stage (see Fig. 12), further affects the economic loss because a PR or a CR is completed, thus, the point of an AR needed is largely impacted by  $\gamma$ .

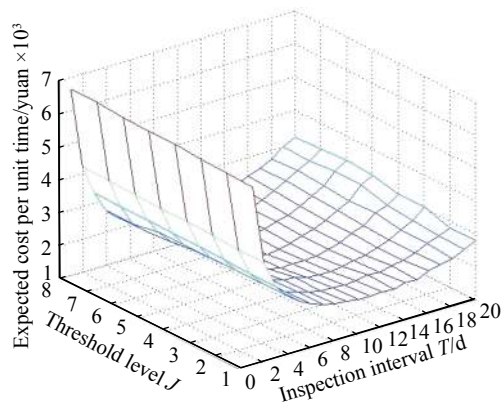


Fig. 11  $EC(T, J, \theta)$  with regard to  $T$  and  $J$  when  $\theta = 16$

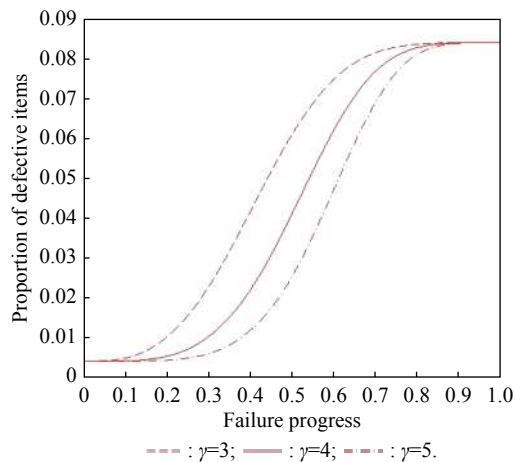


Fig. 12 Proportion of defective items for various values of  $\gamma$

Table 4 gives seven sets values of  $C_{in}$ ,  $L_e$ , and  $\gamma$ , and the optimization results under seven sets values of parameters for Models 1, 2, and 3 are given by Table 4.

Table 4 Seven sets values of  $C_{in}$ ,  $L_e$ , and  $\gamma$

Case	$C_{in}$	$L_e$	$\gamma$
Case 1	5	3	4
Case 2	10	3	4
Case 3	15	3	4
Case 4	5	9	4
Case 5	5	18	4
Case 6	5	3	3
Case 7	5	3	5

From Table 5, we observe that the optimal solution  $T^*$  increases,  $J^*$  and  $\frac{J^*T^*}{2}$  decreases when  $C_{in}$  is costly. This illustrates that less frequent inspections may miss a severe defective state, thus, an advanced replacement should be executed as early as possible to prevent the expensively defective items loss and failure. Besides, we can find that a longer lead time  $L_e$  leads to an increase in threshold  $\theta^*$ . This means waiting for the regular order's spare is more attractive than placing an emergency order since the difference between them becomes slighter as  $L_e$  increases. One interesting phenomenon is that the optimal inspection interval  $T^*$  increases when  $\gamma$  increases, but  $\frac{J^*T^*}{2}$  first increases and then decreases with the increasing of  $\gamma$ . Our interpretation of this is that a larger  $\gamma$  induces a decrease in the defective items loss, thus, an advanced replacement is less popular than a preventive maintenance ( $\frac{J^*T^*}{2}$  increases). However, with the continuous increase of  $T$ , less frequent inspections are performed and more defectives degenerate to failure, therefore, an advanced replacement is recommended to prevent the costly failure ( $\frac{J^*T^*}{2}$  decreases). Furthermore, it is reasonable that the optimal expected cost per unit time is positively related to  $C_{in}$  and  $L_e$  and negatively related to  $\gamma$ .

Table 5 Optimal results of Models 1, 2, and 3 under different values of  $C_{in}$ ,  $L_e$ , and  $\gamma$

Case	Optimal result of Model 1				Optimal result of Model 2			Optimal result of Model 3		
	$T^*$	$J^*$	$\theta^*$	$EC(T^*, J^*, \theta^*)$	$T^*$	$\theta^*$	$EC(T^*, \theta^*)$	$T^*$	$J^*$	$EC(T^*, J^*)$
Case 1	10	6	16	1.731 7	10	10	1.774 7	10	7	1.780 3
Case 2	12	5	23	2.227 5	11	16	2.335 3	12	5	2.273 7
Case 3	13	3	28	2.650 9	12	18	2.813 4	13	2	2.668 2
Case 4	9	7	19	1.744 6	10	18	1.778 8	10	8	1.807 8
Case 5	10	6	27	1.781 6	10	23	1.821 2	10	6	1.813 1
Case 6	10	4	16	1.845 0	9	17	1.910 8	9	6	1.880 2
Case 7	11	5	18	1.684 6	10	18	1.710 7	11	6	1.725 7

We can also observe from Table 5 that allowing an advanced replacement to be carried out during the minor de-

fective stage leads to a better result, since there always exists  $EC(T^*, J^*, \theta^*) < EC(T^*, \theta^*)$ . Additionally, the opti-

mal expected cost per unit time of Model 3 is longer than that of Model 1, indicating that considering an emergency order when the regular order is already ordered but has not yet arrived is superior to that not considering. Hence, we can conclude that the inspection-based maintenance and spare ordering policy of Model 1 is more capable of minimizing the expected cost per unit time of the system than that of Model 2 and Model 3.

## 6. Conclusions

Most existing research on inspection and preventive maintenance considers neither spare ordering nor that the system's defective state can reduce the quality of products. In our paper, defective products loss is considered, and we propose a joint policy of inspection-based maintenance and spare ordering for a four-state system. Specifically, the system uses a two-phase inspection schedule where the original inspection interval is halved when a minor defective stage is identified at an inspection. If the system is still in a minor defective stage at the  $J$ th second phase inspection, an advanced replacement is carried out. We assume that defective items are produced only during a severe defective stage. Thus, once a severe defective stage is detected, or when system failure occurs, preventive or corrective replacement is required. Furthermore, two modes of ordering (a regular order and an emergency order) are considered. When the minor defective stage is found, a regular spare order is placed. If no regular order has been previously placed, an emergency order is placed instead. Meanwhile, a threshold level is introduced to determine whether an emergency order is preferred even when the regular order is already ordered but has not yet arrived. We establish the optimization model, and a numerical example demonstrates that (i) including an advanced replacement policy is better than having no advanced replacement; (ii) allowing an emergency order when the regular order has not arrived is superior to that not allowing.

There are some interesting directions for future research. For example, we could relax our assumption that one spare unit is ordered and stored. The imperfect repair of severe defective systems and the monitoring of product quality could also be considered. Additionally, the single-unit system could be extended to a multi-unit system.

## References

- [1] JARDINE A K S, LIN D M, BANJEVIC D. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical Systems and Signal Processing*, 2006, 20(7): 1483–1510.
- [2] ALASWAD S, XIANG Y S. A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering & System Safety*, 2017, 157: 54–63.
- [3] DE JONGE B D, SCARF P A. A review on maintenance optimization. *European Journal of Operational Research*, 2020, 285(3): 805–824.
- [4] GHOFRANI F, HE Q, GOVERDE R M P, et al. Recent applications of big data analytics in railway transportation systems: a survey. *Transportation Research Part C: Emerging Technologies*, 2018, 90: 226–246.
- [5] ZOU G, BANISOLEIMAN K, GONZALEZ A, et al. Probabilistic investigations into the value of information: a comparison of condition-based and time-based maintenance strategies. *Ocean Engineering*, 2019, 188: 106181.
- [6] CARVALHO T P, SOARES F A, VITA R, et al. A systematic literature review of machine learning methods applied to predictive maintenance. *Computers & Industrial Engineering*, 2019, 137: 106024.
- [7] CHOLETTE M E, YU H Y, BORGHESEANI P, et al. Degradation modeling and condition-based maintenance of boiler heat exchangers using Gamma processes. *Reliability Engineering & System Safety*, 2019, 183: 184–196.
- [8] WEI G Z, ZHAO X J, HE S G, et al. Reliability modeling with condition-based maintenance for binary-state deteriorating systems considering zoned shock effects. *Computers & Industrial Engineering*, 2019, 130: 282–297.
- [9] YANG L, YE Z S, LEE C G, et al. A two-phase preventive maintenance policy considering imperfect repair and postponed replacement. *European Journal of Operational Research*, 2019, 274(3): 966–977.
- [10] NAKAGAWA T, OSAKI S. Optimum replacement policies with delay. *Journal of Applied Probability*, 1974, 11(1): 102–110.
- [11] OSAKI S, YAMADA S. Age replacement with lead time. *IEEE Trans. on Reliability*, 1976, 25(5): 344–345.
- [12] ARMSTRONG M J, ATKINS D R. A note on joint optimization of maintenance and inventory. *IIE Transactions*, 1998, 30(2): 143–149.
- [13] PARK Y T, SUN J. Optimum ordering policy for preventive age replacement. *Journal of Systems Science and Systems Engineering*, 2009, 18(3): 283–291.
- [14] CHIEN Y H, CHANG C C, SHEU S H. Optimal age-replacement model with age-dependent type of failure and random lead time based on a cumulative repair-cost limit policy. *Annals of Operations Research*, 2010, 181(1): 723–744.
- [15] SHEU S H, CHIEN Y H. Optimal age-replacement policy of a system subject to shocks with random lead-time. *European Journal of Operational Research*, 2004, 159(1): 132–144.
- [16] SHEU S H, CHANG C C, CHIEN Y H. Optimal age-replacement time with minimal repair based on cumulative repair-cost limit for a system subject to shocks. *Annals of Operations Research*, 2011, 186(1): 317–329.
- [17] ELWANY A H, GEBRAEEL N Z. Sensor-driven prognostic models for equipment replacement and spare parts inventory. *IIE Transactions*, 2008, 40(7): 629–639.
- [18] RAUSCH M, LIAO H L. Joint production and spare part inventory control strategy driven by condition based maintenance. *IEEE Trans. on Reliability*, 2010, 59(3): 507–516.
- [19] WANG L, CHU J, MAO W J. A condition-based order replacement policy for a single-unit system. *Applied Mathematical Modelling*, 2008, 32(11): 2274–2289.
- [20] WANG L, CHU J, MAO W J. A condition-based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure. *European Journal of Operational Research*, 2009, 194(1): 184–205.
- [21] ZHANG X H, ZENG J C. Joint optimization of condition-based opportunistic maintenance and spare parts provision-

ing policy in multiunit systems. *European Journal of Operational Research*, 2017, 262(2): 479–498.

- [22] FARHAD Z H, SCARF P, SYNTETOS A. Joint optimisation of inspection maintenance and spare parts provisioning: a comparative study of inventory policies using simulation and survey data. *Reliability Engineering & System Safety*, 2017, 168: 306–316.
- [23] ZHU S, VAN JAARSVELD W, DEKKER R. Spare parts inventory control based on maintenance planning. *Reliability Engineering & System Safety*, 2020, 193: 106600.
- [24] WANG W B. A joint spare part and maintenance inspection optimisation model using the Delay-Time concept. *Reliability Engineering & System Safety*, 2011, 96(11): 1535–1541.
- [25] WANG W B. A stochastic model for joint spare parts inventory and planned maintenance optimisation. *European Journal of Operational Research*, 2012, 216(1): 127–139.
- [26] ZHAO F, LIU X J, PENG R, et al. Joint optimization of inspection and spare ordering policy with multi-level defect information. *Computers & Industrial Engineering*, 2020, 139: 106205.
- [27] ZHAO F, XIE F F, SHI C H, et al. A joint inspection-based preventive maintenance and spare ordering optimization policy using a three-stage failure process. *Complexity*, 2017, 2017: 8319485.
- [28] DRIESSEN J P C, PENG H, VAN HOUTUM G J. Maintenance optimization under non-constant probabilities of imperfect inspections. *Reliability Engineering & System Safety*, 2017, 165: 115–123.
- [29] BOUSLAH B, GHARBI, A, PELLERIN R. Integrated production, sampling quality control and maintenance of deteriorating production systems with AOQL constraint. *Omega*, 2016, 61(4): 110–126.
- [30] HOSSAIN A, ZIMMER W. Comparison of estimation methods for Weibull parameters: complete and censored samples. *Journal of Statistical Computation and Simulation*, 2003, 73(2): 145–53.
- [31] ROSS S M. *Introduction to probability models*. New York: Elsevier, 2014.

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