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Distributed fuzzy fault-tolerant consensus of leader-follower multi-agent systems with mismatched uncertainties

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Abstract: In this paper, the distributed fuzzy fault-tolerant tracking consensus problem of leader-follower multi-agent systems (MASs) is studied. The objective system includes actuator faults, mismatched parameter uncertainties, nonlinear functions, and exogenous disturbances under switching communication topologies. To solve this problem, a distributed fuzzy fault-tolerant controller is proposed for each follower by adaptive mechanisms to track the state of the leader. Furthermore, the fuzzy logic system is utilized to approximate the unknown nonlinear dynamics. An error estimator is introduced between the mismatched parameter matrix and the input matrix. Then, a selective adaptive law with relative state information is adopted and applied. When calculating the Lyapunov function's derivative, the coupling terms related to consensus error and mismatched parameter uncertainties can be eliminated. Finally, a numerical simulation is given to validate the effectiveness of the proposed protocol.

Keywords: distributed fuzzy fault-tolerant control (FTC), tracking consensus problem, leader-follower multi-agent system, mismatched parameter uncertainty.

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1. Introduction

In the past few decades, the dynamics of multi-agent systems (MASs) have been investigated in various science fields [1−3]. Among them, the study of the consensus in MASs can be traced back long ago. Based on a distributed model proposed by Reynolds in 1987, a simple model is primarily developed by Vicsek in [4] to study the emergence of self-ordered motion in the system.

Subsequently, Jadbabaie et al. [5] studied the consensus of Vicsek's model. In addition, Fax et al. [6] applied the graph and the matrix theory to solve the consensus problem via a coordinated position controller based on cooperative frameworks. Recently, the consensus problem has become a hot research topic because of its application in some special fields [7−11].

While investigating the consensus of MASs, people find out that the agents in MASs are usually under complex environments with various disturbances, such as channel noises, source noises, and sink noises. For these reasons, the stability of MASs with external disturbances has been studied in recent years [12−14].

In general, due to the existence and unavoidability of complex nonlinear functions in most real MASs [15], more and more consideration should be given to solve the nonlinear function. There are several different ways of tackling the nonlinear function, such as the neural network [16] and the fuzzy logic system (FLS) [17]. By the finite-time passivity, in [18], the finite-time synchronization of nonlinear MASs was investigated. In [19], the adaptive fuzzy containment control for nonlinear MASs with input delay was studied. Recently, a distributed consensus control method has been proposed by adaptive mechanisms in [20]. The mentioned method is fully distributed.

In addition, in MASs, the parameter uncertainties phenomena could not be ignored. The uncertainties parameters can be roughly categorized into matched parameter uncertainties and mismatched parameter uncertainties. In [21], the containment problem was investigated for a class of MASs in the presence of time-varying uncertainties. In [22], the consensus of fractional-order singular uncertain MASs is studied. The problem of fault diagnosis for uncertain MASs was considered in [23]. Note that the above works only consider the matched parameter uncertainties. Recently, there are several papers focused on the mismatched parameter uncertainties. In [24,25], the consensus of MASs with mismatched parameter uncer-

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tainties is considered.

In a real application, actuator faults are also considered as a significant problem. Actuator faults are normally caused by improper operation or component aging [26], and they are classified into four types: stuck, outage, bias, and loss of effectiveness (LOE), respectively. These failures have become a research topic due to the severe related degradation of global behavior caused by them [27–29]. For this shortcoming, fault-tolerant control (FTC) methods of various types, such as the hierarchical control scheme [30], the decentralized output sliding-mode controller [31], and the distributed learning control approach [13], are reported in previous kinds of literature. Besides, the distributed controller has drawn wide attention as a promising method to realize the consensus of MASs with four types of actuator faults. Besides, most works, which focus on the FTC, commonly assume the communication network topology is fixed.

However, this is mostly because while the communication network is switching, the Lyapunov function is invalid. Hence, the switching communication network (SCT) is more complex. In [32], a distributed FTC method is developed for SCT in MASs considering unknown uncertainties and external disturbance. This method removes the common assumption for the fixed communication topologies and compensates for the shortcoming of actuator faults, including the outage and stuck faults. Nevertheless, in real world, the nonlinear function satisfies the Lipschitz condition under some certain conditions.

In this paper, the tracking consensus of MASs with external disturbance, nonlinear function, and mismatched parameter uncertainties subject to actuator faults under switching communication topologies is studied. New ideas and innovative points of this article lie in the followings:

(i) Different from the existing papers dealing with actuator faults, such as [29–32], in this paper, the considered system with mismatched parameter uncertainties, nonlinear function, and actuator faults under switching communication topologies are more general. For the considered system, a new distributed fuzzy fault-tolerant controller is proposed via adaptive mechanisms.

(ii) The proposed controller's adaptive module is employed to estimate the weight vector norm in FLS rather than in traditional Lipschitz conditions.

(iii) For the case of matched uncertainties, the design of the distributed adaptive protocol is relatively simple. However, for the case of mismatched uncertainties under switching communication topologies, the existing controllers [21–25] cannot solve the issue in this work. Here, a norm error estimation is introduced between the mismatched parameter matrix and the input matrix. On this basis, a selective adaptive law with relative state information is adopted and applied. When calculating the derivative of the Lyapunov function, the coupling terms related to consensus error and mismatched parameter uncertainties can be eliminated.

This article is organized as follows. In Section 2, the preliminaries are introduced. In Section 3, the main results are shown. In Section 4, a simulation example based on the aircraft model is shown. Ultimately, conclusions are summarized in Section 5.

2. Preliminaries and problem statement

2.1 Graph theory

Consider a graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)})$ in the MAS, which is defined by several parameters, including a set of N nodes (a nonempty finite) $\mathcal{V} = \{\boldsymbol{v}_1, \cdots, \boldsymbol{v}_N\}$, a set of edges $\mathcal{E}_{\sigma(t)} = \mathcal{V} \times \mathcal{V}$, an associated weighted adjacency matrix $\mathcal{A}_{\sigma(t)} = [\boldsymbol{a}_{ij}(t)] \in \mathbf{R}^{N\times N}$, and a switching signal $\sigma(t) : [0, +\infty) \to \mathcal{P}$, where $\mathcal{P}$ is an indices set for total graphs. The $(\boldsymbol{v}_i, \boldsymbol{v}_j) \in \mathcal{E}_{\sigma(t)}$ is an edge which indicates the information exchange among agents i and j. $\boldsymbol{a}_{ij}(t)$ is a symbol of the weight of edge $(\boldsymbol{v}_i, \boldsymbol{v}_j)$, which satisfies $\boldsymbol{a}_{ij}(t) > 0$ if $(\boldsymbol{v}_i, \boldsymbol{v}_j) \in \mathcal{E}_{\sigma(t)}$ $(i \neq j)$, otherwise $\boldsymbol{a}_{ij}(t) = 0$. An uniformly bounded non-overlapping time interval $[t_d, t_{d+1})$ $(t_0 = 0, d = 1, 2, \cdots)$ exists with an infinite sequence, in which the interaction graph is time invariant in each time interval, meanwhile, a dwell time $\tau_d > 0$ exists which satisfies $t_{d+1} - t_d \geqslant \tau_d$. $\mathcal{N}_i(t) = \{j | (\boldsymbol{v}_j, \boldsymbol{v}_i) \in \mathcal{E}_{\sigma(t)}\}$ is the neighbor set of node i. In one graph, a sequence of connected edges indicates a path. An undirected graph will be obtained, if $\boldsymbol{a}_{ij}(t) = \boldsymbol{a}_{ji}(t)$. An undirected graph is considered to be connected when the condition that a path exists in each pair of nodes is met. The Laplacian matrix can be represented as $\mathcal{L}_{\sigma(t)} = [\boldsymbol{l}_{ij}(t)]$ with $\boldsymbol{l}_{ij}(t) = -\boldsymbol{a}_{ij}(t)$ $(i \neq j)$, $\boldsymbol{l}_{ij}(t) = \sum_{k=1,k\neq i}^{N} \boldsymbol{a}_{ik}(t) (i = j)$. Denote the leader adjacency matrix as $\boldsymbol{D}_{\sigma(t)} = \mathrm{diag}(\boldsymbol{d}_1(t), \cdots, \boldsymbol{d}_N(t))$ with $\boldsymbol{d}_i(t) > 0$ if and only if the node $\boldsymbol{v}_i$ can access the leader information.

2.2 Problem formulation

Consider a team of nonlinear MAS links with $N+1$ agents. In this system, the dynamic of the leader is illustrated by

$$\dot{\boldsymbol{x}}_0 = (\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{x}_0 + \boldsymbol{B}\boldsymbol{u}_0 \tag{1}$$

where $\boldsymbol{x}_0 \in \mathbf{R}^n, \boldsymbol{u}_0 \in \mathbf{R}^m$, and $\Delta\boldsymbol{A}(t)$ indicate the state of leader, input, and mismatched parameter uncertainties, respectively. It is assumed here that $(\boldsymbol{A}, \boldsymbol{B})$ is stabilizable. For the mismatched parameter uncertainties, according to [24] and [25], it is assumed that $\Delta\boldsymbol{A}(t) = \boldsymbol{D}\boldsymbol{N}(t)$ where $\boldsymbol{D}$ is a known real constant matrix, and $\boldsymbol{N}(t)$ satisfies $\|\boldsymbol{N}(t)\| \leqslant \theta^*$ with the unknown constant parameter θ^*.

The ith follower is illustrated by

$$\dot{\boldsymbol{x}}_i = (\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{x}_i + \boldsymbol{B}(\boldsymbol{u}_i^F + \boldsymbol{f}_i(\boldsymbol{x}_i) + \boldsymbol{w}_i) \tag{2}$$

where $i = 1, \cdots, N$; $\boldsymbol{x}_i \in \mathbf{R}^n, \boldsymbol{u}_i^F \in \mathbf{R}^m$, $\boldsymbol{w}_i \in \mathbf{R}^m$, and $\boldsymbol{f}_i(\boldsymbol{x}_i) \in \mathbf{R}^m$ represent the state, control input, time-varying exogenous disturbance, and nonlinear function, respectively.

Remark 1 The system models (1) and (2) indicate the mismatched parameter uncertainties MASs. They are widely considered in early papers to achieve consensus of leader-follower MASs (see [24,25] for details).

Inspired by [25], for agent i, actuator faults in mode m ($m = 1, \cdots, M$) are described by

$$\boldsymbol{u}_{i,k}^{Fm} = \boldsymbol{\rho}_{i,k}^m \boldsymbol{u}_{i,k} + \boldsymbol{\psi}_{i,k}^m \tag{3}$$

where $k = 1, 2, \cdots, h$. $\boldsymbol{u}_{i,k}^{Fm}$ represents the actuator of agent i. $\boldsymbol{\rho}_{i,k}^m$ is an unknown time-varying efficiency factor. $\boldsymbol{\psi}_{i,k}^m$ denotes the stuck value. There is $0 \leqslant \underline{\rho}_{i,k}^m \leqslant \rho_{i,k}^m \leqslant \overline{\rho}_{i,k}^m \leqslant 1$. $\underline{\rho}_{i,k}^m$ and $\overline{\rho}_{i,k}^m$ are constants representing the lower bound and the upper bound of $\boldsymbol{\rho}_{i,k}^m$ respectively.

Remark 2 When $\underline{\boldsymbol{\rho}}_{i,k}^m$, $\overline{\rho}_{i,k}^m$ and $\overline{\psi}_{i,k}^m$ are different selected values, the fault model (3) describes different actuator faults. Table 1 illustrates the fault model.

Table 1 Actuator fault model

Fault mode	$\underline{\rho}_{i,k}^m$	$\overline{\rho}_{i,k}^m$	$\overline{\psi}_{i,k}^m$
Normal	1	1	0
LOE	>0	<1	0
Bias	1	1	≠0
Outage	0	0	0
Stuck	0	0	≠0

To simplify the presentation, (3) can be rewritten as follows:

$$\boldsymbol{u}_i^F = \boldsymbol{\rho}_i \boldsymbol{u}_i + \boldsymbol{\psi}_i \tag{4}$$

where

$$\boldsymbol{\psi}_i = [\psi_{i,1}, \cdots, \psi_{i,h}]^{\mathrm{T}},$$
$$\boldsymbol{\rho}_i = \mathrm{diag}(\rho_{i,1}, \cdots, \rho_{i,h}).$$

Remark 3 Before starting, the following lemmas and assumptions, more explanation for actuator faults will be given here. Abnormal operations or components aging, commonly appearing in the physical layer, are the main reasons of actuator faults. Thus, an appropriate controller will be proposed to compensate for actuator faults for they are urgent problems to be solved.

Assumption 1 The undirected graphs $\mathcal{G}_p (p \in \mathcal{P})$ are connected and fixed across each interval $[t_d, t_{d+1})$.

Remark 4 Assumption 1 is an essential condition in MASs under switching communication topologies.

Lemma 1 [15] If Assumption 1 holds, then $\mathcal{H}_p = \mathcal{L}_p + \mathcal{D}_p$ is positive definite.

Assumption 2 [32] The actuator bias fault is bounded, i.e., $\underline{\psi}_{i,k}^0 \leqslant \left\|\psi_{i,k}^m(t)\right\| \leqslant a\overline{\psi}_{i,k}^0$.

Remark 5 Assumption 2 indicates that the actuator bias fault is bounded. It is widely used in the robust FTC of MASs, and many practical systems satisfy this assumption (see [32] for more detail).

Assumption 3 $\mathrm{rank}[\boldsymbol{B\rho}_i] = \mathrm{rank}[\boldsymbol{B}]$.

Remark 6 Assumption 3 is a common assumption that can be used to solve the FTC problem of MASs.

Lemma 2 [26] If Assumption 3 holds, there exists $\boldsymbol{\mu}_i > 0$ such that $\boldsymbol{B\rho}_i\boldsymbol{B}^{\mathrm{T}} \geqslant \boldsymbol{\mu}_i\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}$.

Lemma 3 [19] Let the domain of continuous function $\boldsymbol{f}_i(x_i)$ be a compact set Ω.Then, for any $\boldsymbol{\pi}_i(t) > 0$, there exists a fuzzy logic system such that

$$\sup_{x_i \in \Omega} |\boldsymbol{\phi}_i^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{\vartheta}_i - f_i(\boldsymbol{x}_i)| \leqslant \boldsymbol{\pi}_i(t)$$

Using Lemma 3, it has

$$\boldsymbol{f}_i(\boldsymbol{x}_i) = \boldsymbol{\phi}_i^{\mathrm{T}}(\boldsymbol{x}_i)\boldsymbol{\vartheta}_i + \boldsymbol{\pi}_i(t)$$

where $\boldsymbol{\phi}_i(\boldsymbol{x}_i) = \mathrm{diag}(\boldsymbol{\phi}_{i1}^{\mathrm{T}}, \cdots, \boldsymbol{\phi}_{im}^{\mathrm{T}})$ indicates the fuzzy basis function with $\boldsymbol{\phi}_{iq} \in \mathbf{R}^l$. $\boldsymbol{\vartheta}_i = [\vartheta_{i1}, \cdots, \vartheta_{im}]^{\mathrm{T}}$ indicates the unknown parameter with $\boldsymbol{\pi}_{iq} \in \mathbf{R}^l (q = 1, \cdots, m)$. Moreover, there is an unknown constant $\boldsymbol{\pi}_0 > 0$ such that $\|\boldsymbol{\pi}_i(t)\| \leqslant \pi_0$.

Remark 7 [30] The basis function $\boldsymbol{\phi}_i(\cdot)$ in Lemma 3 satisfies $\boldsymbol{\phi}_i(\cdot)\boldsymbol{\phi}_i^{\mathrm{T}}(\cdot) \leqslant l\boldsymbol{I}$.

Define consensus error as $\boldsymbol{\delta}_i = \boldsymbol{x}_i - \boldsymbol{x}_0$, then

$$\dot{\boldsymbol{\delta}}_i = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{\delta}_i + \boldsymbol{B}[\boldsymbol{\rho}_i\boldsymbol{u}_i + \bar{\boldsymbol{\Delta}}_i + \boldsymbol{\phi}_i^{\mathrm{T}}\boldsymbol{\vartheta}_i + \boldsymbol{\psi}_i]$$

where $\bar{\boldsymbol{\Delta}}_i = \boldsymbol{\pi}_0 - \boldsymbol{u}_0 + \boldsymbol{w}_i$.

The relative information from neighbors $z_i^{\sigma(t)}$ is written by

$$z_i^{\sigma(t)} = \sum_{j=1}^{N} \boldsymbol{a}_{ij}(\boldsymbol{x}_i - \boldsymbol{x}_j) + \boldsymbol{d}_i(\boldsymbol{x}_i - \boldsymbol{x}_0) \tag{5}$$

where $\boldsymbol{x}_i - \boldsymbol{x}_j$ denotes the relative information form agent j to agent i.

Let $\boldsymbol{\delta} = [\boldsymbol{\delta}_1^{\mathrm{T}}, \cdots, \boldsymbol{\delta}_N^{\mathrm{T}}]^{\mathrm{T}}$, then

$$\boldsymbol{\delta} = (\boldsymbol{I} \otimes (\boldsymbol{A} + \Delta\boldsymbol{A}))\boldsymbol{\delta} + (\boldsymbol{I} \otimes \boldsymbol{B})(\boldsymbol{\rho u} + \bar{\boldsymbol{\Delta}} + \boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{\vartheta} + \boldsymbol{\psi}) \tag{6}$$

where

$$\boldsymbol{\rho} = \mathrm{diag}(\boldsymbol{\rho}_1, \cdots, \boldsymbol{\rho}_N),$$
$$\boldsymbol{u} = [\boldsymbol{u}_1^{\mathrm{T}}, \cdots, \boldsymbol{u}_N^{\mathrm{T}}]^{\mathrm{T}},$$
$$\boldsymbol{\psi} = [\boldsymbol{\psi}_1^{\mathrm{T}}, \cdots, \boldsymbol{\psi}_N^{\mathrm{T}}]^{\mathrm{T}},$$
$$\bar{\boldsymbol{\Delta}} = [\bar{\boldsymbol{\Delta}}_1^{\mathrm{T}}, \cdots, \bar{\boldsymbol{\Delta}}_N^{\mathrm{T}}]^{\mathrm{T}},$$
$$\boldsymbol{\phi} = \mathrm{diag}(\boldsymbol{\phi}_1^{\mathrm{T}}, \cdots, \boldsymbol{\phi}_m^{\mathrm{T}}),$$
$$\boldsymbol{\vartheta} = [\boldsymbol{\vartheta}_1^{\mathrm{T}}, \cdots, \boldsymbol{\vartheta}_m^{\mathrm{T}}]^{\mathrm{T}}.$$

The aim of this paper is to find a controller such that all followers asymptotically converge to the state of leader.

3. Main results

3.1 Distributed fuzzy fault-tolerant consensus protocol design

Before starting this subsection, we introduce the following notations for agent i:

$$\begin{cases} \boldsymbol{l}_{i,1} = c_0/\boldsymbol{\mu}_i \\ \boldsymbol{l}_{i,2} = (\theta^*)^2/\boldsymbol{\mu}_i \\ \boldsymbol{l}_{i,3} = \boldsymbol{\vartheta}_i/\boldsymbol{\mu}_i \\ \boldsymbol{l}_{i,4} = \bar{\boldsymbol{\varDelta}}_i/\boldsymbol{\mu}_i \\ \boldsymbol{l}_{i,5} = \boldsymbol{\varLambda}_i^{-1}(\boldsymbol{\psi}_i + \boldsymbol{y}_i) \end{cases} \tag{7}$$

where $c_0 > 1/(2\lambda_0)$ with

$$\lambda_0 = \min\{\lambda_{\min}(\mathcal{H}_p) : p \in \mathcal{P}\},$$
$$\boldsymbol{\varLambda}_i = \mathrm{diag}\{\boldsymbol{\varLambda}_{i,1}, \cdots, \boldsymbol{\varLambda}_{i,h}\},$$

and $\boldsymbol{y}_i = [y_{i,1}, \cdots, y_{i,h}]^{\mathrm{T}}$ are defined by

$$\boldsymbol{\varLambda}_{i,k} = \begin{cases} \rho_{i,k}, & \rho_{i,k} \neq 0, \\ 1, & \text{otherwise}, \end{cases}$$
$$y_{i,k} = \begin{cases} \sum_{s \in \varSigma_i} \bar{\psi}_k^{(s)} \psi_{i,s}, & k \neq \varSigma_i, \\ 0, & \text{otherwise}, \end{cases}$$

where scalars $\bar{\psi}_k^{(s)}$ for all $s \in \varSigma_i$ with

$$\varSigma_i = \{\bar{r}_\ell | \rho_{i,\bar{r}_\ell} = 0, \ell = 1, \cdots, \bar{\ell}, 0 \leqslant \bar{\ell} \leqslant h - \bar{r}_0\}$$

where $\bar{r}_0 = \mathrm{rank}(\boldsymbol{B})$.

An appropriate time-varying distributed controller is designed for agent i by

$$\boldsymbol{u}_i = \boldsymbol{u}_{i,1} + \boldsymbol{u}_{i,2} + \boldsymbol{u}_{i,3} + \boldsymbol{u}_{i,4} + \boldsymbol{u}_{i,5} \tag{8}$$

where

$$\boldsymbol{u}_{i,1} = -\hat{\boldsymbol{l}}_{i,1} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{z}_i^{\sigma(t)}, \tag{9}$$

$$\boldsymbol{u}_{i,2} = -\frac{1}{2}\eta(1 + \boldsymbol{v}_i)\hat{\boldsymbol{l}}_{i,2} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{z}_i^{\sigma(t)}, \tag{10}$$

$$\boldsymbol{u}_{i,3} = -\frac{\boldsymbol{B}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{z}_i^{\sigma(t)} \hat{\boldsymbol{l}}_{i,3}^2 \boldsymbol{l}^4}{\| \boldsymbol{z}_i^{\sigma(t)\mathrm{T}} \boldsymbol{P} \boldsymbol{B} \boldsymbol{\phi}_i^{\mathrm{T}} \| \hat{\boldsymbol{l}}_{i,3} + \boldsymbol{\chi}_i}, \tag{11}$$

$$\boldsymbol{u}_{i,4} = -\frac{\boldsymbol{B}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{z}_i^{\sigma(t)} \hat{\boldsymbol{l}}_{i,4}^2}{\| \boldsymbol{z}_i^{\sigma(t)\mathrm{T}} \boldsymbol{P} \boldsymbol{B} \| \hat{\boldsymbol{l}}_{i,4} + \boldsymbol{\chi}_i}, \tag{12}$$

$$\boldsymbol{u}_{i,5} = -(\hat{\underline{\boldsymbol{l}}}_{i,5} + (\hat{\bar{\boldsymbol{l}}}_{i,5} - \hat{\underline{\boldsymbol{l}}}_{i,5})\boldsymbol{\varUpsilon}_i), \tag{13}$$

$\boldsymbol{P} > 0$ with $\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} - 2c\lambda_0 \boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \frac{1}{\beta}\boldsymbol{I} + \boldsymbol{Q} < 0$, the uniform continuous function is written as $\boldsymbol{\chi}_i(t)$ with $\lim_{t\to+\infty} \int_{t_0}^{t} \boldsymbol{\chi}_i(s)\mathrm{d}s \leqslant \bar{\boldsymbol{\chi}}_i < +\infty$.

In (9), $\hat{\boldsymbol{l}}_{i,1}$ is the estimate of $\boldsymbol{l}_{i,1}$ and it is generated by

$$\dot{\hat{\boldsymbol{l}}}_{i,1} = -r_{i,1}\boldsymbol{\chi}_i \hat{\boldsymbol{l}}_{i,1} + r_{i,1} \| \boldsymbol{z}_i^{\sigma(t)\mathrm{T}} \boldsymbol{P}\boldsymbol{B} \|^2$$

where $r_{i,1}$ is a positive constant.

In (10), $\eta > \dfrac{\beta}{\lambda_0}, \boldsymbol{v}_i = \dfrac{\bar{\boldsymbol{v}}_i \bar{\boldsymbol{\eta}}_i \mathrm{sign}(\bar{\boldsymbol{\eta}}_i)}{\|\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^{\sigma(t)}\|^2 + \tau}$ with $\bar{\boldsymbol{\eta}}_i = \|\boldsymbol{D}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^{\sigma(t)}\|^2 - \|\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^{\sigma(t)}\|^2$, $\bar{\boldsymbol{v}}_i > 0$, and $\tau > 0$, $\hat{\boldsymbol{l}}_{i,2}$ is the estimation of $\boldsymbol{l}_{i,2}$ which is adjusted by

$$\dot{\hat{\boldsymbol{l}}}_{i,2} = -r_{i,2}\boldsymbol{\chi}_i \hat{\boldsymbol{l}}_{i,2} + \frac{r_{i,2}\eta(\hat{\boldsymbol{l}}_{i,2} \| \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^{\sigma(t)}\|^2)}{(\hat{\boldsymbol{l}}_{i,2} - \boldsymbol{\varsigma}_i) + \boldsymbol{\zeta}_i} + \frac{r_{i,2}\eta((\boldsymbol{\zeta}_i - \boldsymbol{\varsigma}_i) \| \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^{\sigma(t)}\|^2)}{(\hat{\boldsymbol{l}}_{i,2} - \boldsymbol{\varsigma}_i) + \boldsymbol{\zeta}_i}$$

where $r_{i,2}$ is a positive constant, $\boldsymbol{\varsigma}_i$ belongs to any neighbor of $\hat{\boldsymbol{l}}_{i,2}$, $\boldsymbol{\zeta}_i$ is determined by

$$\zeta_i = \begin{cases} \boldsymbol{0}, & \hat{\boldsymbol{l}}_{i,2} \neq \varsigma_i \\ \zeta_{i0}, & \hat{\boldsymbol{l}}_{i,2} = \varsigma_i \end{cases}$$

with $\boldsymbol{\zeta}_{i0}$ being a positive scalar.

Remark 8 The norm error estimator $\bar{\boldsymbol{\eta}}_i$ is introduced between the mismatched parameter matrix and the input matrix. On this basis, a selective adaptive law with relative state information is adopted and applied. When calculating the derivative of the Lyapunov function, the coupling terms related to consensus error and mismatched parameter uncertainties can be eliminated.

In (11), $\hat{\boldsymbol{l}}_{i,3}$ is the estimation of $\boldsymbol{l}_{i,3}$ and it is adjusted by

$$\dot{\hat{\boldsymbol{l}}}_{i,3} = -r_{i,3}\boldsymbol{\chi}_i \hat{\boldsymbol{l}}_{i,3} + r_{i,3} \| \boldsymbol{z}_i^{\sigma(t)\mathrm{T}} \boldsymbol{P}\boldsymbol{B}\boldsymbol{\phi}_i^{\mathrm{T}} \|$$

where $r_{i,3}$ is a positive constant.

In (12), $\hat{\boldsymbol{l}}_{i,4}$ is the estimation of $\boldsymbol{l}_{i,4}$, which is adjusted by

$$\dot{\hat{\boldsymbol{l}}}_{i,4} = -r_{i,4}\boldsymbol{\chi}_i \hat{\boldsymbol{l}}_{i,4} + r_{i,4} \| \boldsymbol{z}_i^{\sigma(t)\mathrm{T}} \boldsymbol{P}\boldsymbol{B} \|$$

where $r_{i,4}$ is a positive constant.

In (13), $\boldsymbol{\varUpsilon}_i = \mathrm{diag}(\boldsymbol{\varUpsilon}_{i,k}(\boldsymbol{\varXi}_{k,1})$, where $\boldsymbol{\varUpsilon}_{i,k}(\boldsymbol{\varXi}_{i,k}) = \boldsymbol{1}$, if $\boldsymbol{\varXi}_{i,k} \geqslant \boldsymbol{0}$, then $\boldsymbol{\varUpsilon}_{i,k}(\boldsymbol{\varXi}_{i,k}) = \boldsymbol{0}$; if $\boldsymbol{\varXi}_{i,k} < \boldsymbol{0}$ with $\boldsymbol{\varXi}_{i,k} = \boldsymbol{z}_i^{\sigma(t)\mathrm{T}}[\boldsymbol{K}_0^{\mathrm{T}}]_k$, $\hat{\bar{l}}_{i,5}$ and $\hat{\underline{l}}_{i,5}$ are adjusted by

$$\frac{\mathrm{d}\hat{\underline{l}}_{i,k_5}}{\mathrm{d}t} = \mathrm{Proj}_1\{\boldsymbol{\varXi}_{i,k}\} \triangleq \begin{cases} 0, & \hat{\underline{l}}_{i,k_5} > \bar{\psi}_{i,k}^0 \,;\, \boldsymbol{\varXi}_{i,k} \geqslant \boldsymbol{0} \text{ or } \hat{\underline{l}}_{i,k_5} < \underline{q}_{ik}\underline{\psi}_{i,k}^0 \,; \boldsymbol{\varXi}_{i,k}^{\mathrm{T}} \leqslant \boldsymbol{0} \\ r_{i,5}^k \boldsymbol{\varXi}_{i,k}, & \text{otherwise} \end{cases},$$

$$\frac{\mathrm{d}\hat{\bar{l}}_{i,k_5}}{\mathrm{d}t} = \mathrm{Proj}_2\{\boldsymbol{\varXi}_{i,k}\} \triangleq \begin{cases} \boldsymbol{0}, & (\hat{\bar{l}}_{i,k_5} > \bar{q}_{i,k}\bar{\psi}_{i,k}^0 \,;\, \boldsymbol{\varXi}_{i,k} \geqslant \boldsymbol{0}) \text{ or } \hat{\bar{l}}_{i,k_5} < \underline{\psi}_{i,k}^0 ; \boldsymbol{\varXi}_{i,k} \leqslant \boldsymbol{0} \\ r_{i,5}^k \boldsymbol{\varXi}_{i,k}, & \text{otherwise} \end{cases},$$

where $\hat{\bar{\boldsymbol{l}}}_{i,5}$ and $\hat{\underline{\boldsymbol{l}}}_{i,5}$ are the estimation of the upper and lower bounds of the unknown function $l_{i,5}$, respectively. $r_{i,5}^k$ is a positive constant, the scalars $\bar{q}_{i,k}$ and $\underline{q}_{i,k}$ are weighting parameters.

3.2 Stability analysis

In this subsection, first, for convenience, we denote the notations:

$$\tilde{\boldsymbol{l}}_{i,k}=\hat{\boldsymbol{l}}_{i,k}-\boldsymbol{l}_{i,k},\ k=1,2,3,4,$$
$$\tilde{\underline{\boldsymbol{l}}}_{i,k_5}=\hat{\underline{\boldsymbol{l}}}_{i,k_5}-\underline{\boldsymbol{l}}_{i,k_5},\tilde{\bar{\boldsymbol{l}}}^{i,k_5}=\hat{\bar{\boldsymbol{l}}}_{i,k_5}-\bar{\boldsymbol{l}}_{i,k_5},$$
$$\boldsymbol{\Phi}=\sum_{i=1}^{N}\frac{1}{r_{i,1}}\boldsymbol{\mu}_i\tilde{\boldsymbol{l}}_{i,1}^2+\sum_{i=1}^{N}\frac{1}{r_{i,2}}\boldsymbol{\mu}_i\tilde{\boldsymbol{l}}_{i,2}^2+\sum_{i=1}^{N}\frac{1}{r_{i,3}}\boldsymbol{\mu}_i\tilde{\boldsymbol{l}}_{i,3}^2+$$
$$\sum_{i=1}^{N}\frac{1}{r_{i,4}}\boldsymbol{\mu}_i\tilde{\boldsymbol{l}}_{i,4}^2+\sum_{i=1}^{N}\sum_{k=1}^{m}\frac{\boldsymbol{\rho}_{i,k}}{r_{i,5}^k}(\underline{\varrho}_{i,k}(\tilde{\bar{\boldsymbol{l}}}_{i,k_5})^2+(1-\varrho_{i,k})(\tilde{\underline{\boldsymbol{l}}}_{i,k_5})^2),$$
$$\underline{\boldsymbol{\rho}}_{i,k}=\boldsymbol{\varUpsilon}_{i,k}(\boldsymbol{\varXi}_{i,k}),$$

then, a theorem is proposed to characterize the sufficient condition for the control objective.

Theorem 1 Consider systems (1) and (2) under Assumptions 1−3. Then the controller (8) ensures that the tracking consensus problem is solvable if the topology dwell time satisfies $\tau_0>\dfrac{\ln k}{b_0}$, where $k>\dfrac{\bar{a}_{\widehat{p}}}{\underline{a}_{\hat{p}}}$, for all $\hat{p},\widehat{p}\in\mathcal{P}$, and $b_0=\min_{p\in\mathcal{P}}(b_p)$.

Proof Now, a Lyapunov function is considered by

$$V_{\sigma(t)}=\boldsymbol{\delta}^{\mathrm{T}}(\mathcal{H}_{\sigma(t)}\otimes\boldsymbol{P})\boldsymbol{\delta}+\boldsymbol{\Phi}. \tag{14}$$

Based on the control objective, the proof process has been divided into Step 1 and Step 2.

Step 1 For $t\in[t_d,t_{d+1})$, let $\sigma(t)=p,p\in\mathcal{P}$. Herein, from (14), we have

$$\dot{V}_p\leqslant\boldsymbol{\delta}^{\mathrm{T}}(\mathcal{H}_p\otimes(\boldsymbol{PA}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P})-2c_0(\mathcal{H}_p^2\otimes\boldsymbol{PBB}^{\mathrm{T}}\boldsymbol{P}))\boldsymbol{\delta}+$$
$$\boldsymbol{\Theta}_1+\boldsymbol{\Theta}_2+\boldsymbol{\Theta}_3+\boldsymbol{\Theta}_4+\boldsymbol{\Theta}_5+\dot{\boldsymbol{\Phi}} \tag{15}$$

where

$$\boldsymbol{\Theta}_1=2c_0\boldsymbol{\delta}^{\mathrm{T}}(\mathcal{H}_p^2\otimes\boldsymbol{PBB}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{\delta}-2\sum_{i=1}^{N}\boldsymbol{\mu}_i\hat{\boldsymbol{l}}_{i,1}\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PBB}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^p$$
$$\boldsymbol{\Theta}_2=2\sum_{i=1}^{N}\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PD}\|\cdot\|\boldsymbol{N}\|\cdot\|\boldsymbol{\delta}_i\|-2\sum_{i=1}^{N}\eta(1+\boldsymbol{v}_i)\hat{\boldsymbol{l}}_{i,2}\times$$
$$\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\rho}_i\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^p,$$
$$\boldsymbol{\Theta}_3=-2\sum_{i=1}^{N}\frac{\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\rho}_i\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^p\hat{\boldsymbol{l}}_{i,3}^2\boldsymbol{l}^4}{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\phi}_i^{\mathrm{T}}\|\hat{\boldsymbol{l}}_{i,3}+\boldsymbol{\chi}_i}+2\sum_{i=1}^{N}\boldsymbol{\mu}_i\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\phi}_i^{\mathrm{T}}\|\boldsymbol{l}_{i,3},$$
$$\boldsymbol{\Theta}_4=-2\sum_{i=1}^{N}\frac{\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\rho}_i\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z}_i^p\hat{\boldsymbol{l}}_{i,4}^2}{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB}\|\hat{\boldsymbol{l}}_{i,4}+\boldsymbol{\chi}_i}+2\sum_{i=1}^{N}\boldsymbol{\mu}_i\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB}\|\boldsymbol{l}_{i,4},$$
$$\boldsymbol{\Theta}_5=-2\sum_{i=1}^{N}\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\rho}_i[\hat{\underline{\boldsymbol{l}}}_{i,5}+(\hat{\bar{\boldsymbol{l}}}_{i,5}-\hat{\underline{\boldsymbol{l}}}_{i,5})\boldsymbol{\varUpsilon}_i]+2\sum_{i=1}^{N}\boldsymbol{\mu}_i\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB}\|\boldsymbol{l}_{i,5}.$$

Let $\mathcal{I}$ be defined as follows:

$$\mathcal{I}=\{i_1,\cdots,i_{\bar{q}}|\rho_{\bar{q},r^{(\bar{q})}}=0,r^{(\bar{q})}\in\varSigma_{\bar{q}},0\leqslant\bar{q}\leqslant N\}.$$

For all $i\in\mathcal{I}$, there exist constants $\bar{v}_s^{(k)}$, $k\in\varSigma_i$, $s\notin\varSigma_i$ such that $[\boldsymbol{B}]_k=\sum_{s\notin\varSigma_i}\bar{v}_s^{(k)}[\boldsymbol{B}]_s$. From (7), it follows that

$$2\sum_{i=1}^{N}\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\psi}_i\leqslant2\sum_{i=1}^{N}\boldsymbol{z}_i^{p\mathrm{T}}\sum_{k=1}^{h}[\boldsymbol{PB}]_k\boldsymbol{\rho}_{i,k}\underline{l}_{i,k5}+$$
$$2\sum_{i=1}^{N}\boldsymbol{z}_i^{p\mathrm{T}}\sum_{k=1}^{h}[\boldsymbol{PB}]_k\boldsymbol{\rho}_{i,k}(\bar{l}_{i,k5}-\underline{l}_{i,k5})\boldsymbol{\varUpsilon}_{i,k}. \tag{16}$$

Substituting (16) into (15) yields

$$\dot{V}_p\leqslant\boldsymbol{\delta}^{\mathrm{T}}(\mathcal{H}_p\otimes(\boldsymbol{PA}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P})-$$
$$2c_0(\mathcal{H}_p^2\otimes\boldsymbol{PBB}^{\mathrm{T}}\boldsymbol{P})+\frac{1}{\eta}\boldsymbol{I}_N)\boldsymbol{\delta}+$$
$$2\sum_{i=1}^{N}\boldsymbol{\mu}_i\boldsymbol{\chi}_i\left(\frac{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB}\|\hat{\boldsymbol{l}}_{i,4}}{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB}\|\hat{\boldsymbol{l}}_{i,4}+\boldsymbol{\chi}_i}+\frac{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\phi}_i^{\mathrm{T}}\|\hat{\boldsymbol{l}}_{i,3}}{\|\boldsymbol{z}_i^{p\mathrm{T}}\boldsymbol{PB\phi}_i^{\mathrm{T}}\|\hat{\boldsymbol{l}}_{i,3}+\boldsymbol{\chi}_i}\right)-$$
$$\sum_{i=1}^{N}\boldsymbol{\mu}_i\boldsymbol{\chi}_i(\tilde{\boldsymbol{l}}_{i,2}^2+\tilde{\boldsymbol{l}}_{i,2}\boldsymbol{l}_{i,2}+\tilde{\boldsymbol{l}}_{i,3}^2+\tilde{\boldsymbol{l}}_{i,3}\boldsymbol{l}_{i,3}+\tilde{\boldsymbol{l}}_{i,4}^2+\tilde{\boldsymbol{l}}_{i,4}\boldsymbol{l}_{i,4}). \tag{17}$$

From (17), we have

$$\dot{V}_p\leqslant-\lambda_{\min}(\boldsymbol{Q})\sum_{i=1}^{N}\|\boldsymbol{\delta}_i\|^2+\sum_{i=1}^{N}\boldsymbol{\chi}_i\boldsymbol{\kappa}_i \tag{18}$$

where $\boldsymbol{\kappa}_i=\mu_i\left(\dfrac{\boldsymbol{l}_{i,2}^2}{4}+\dfrac{\boldsymbol{l}_{i,3}^2}{4}+\dfrac{\boldsymbol{l}_{i,4}^2}{4}\right)$.Then from (18), one has that $V_p(t)$ is nonincreasing. Thus, it means that all signals in (6) are bounded on every interval $[t_d,t_{d+1}]$.

Step 2 For $t\in[t_d,t_{d+1})$, we define $\bar{\boldsymbol{\Phi}}=\max_{p\in\mathcal{P}}\{\boldsymbol{\Phi}_p\}$. From the definition of $\mathcal{H}_p$, there exist $\bar{a}_p>0$ and $\underline{a}_p>0$ such that

$$\underline{a}_p\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P})\boldsymbol{\delta}\leqslant\boldsymbol{\delta}^{\mathrm{T}}(\mathcal{H}_p\otimes\boldsymbol{P})\boldsymbol{\delta}\leqslant\bar{a}_p\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P})\boldsymbol{\delta}.$$

By choosing the parameters $b_p<\dfrac{\underline{\lambda}_p}{\bar{a}_p}$, when $\|\boldsymbol{\delta}\|\geqslant\sqrt{\dfrac{2b_p\bar{\boldsymbol{\Phi}}}{-2b_p\bar{a}_p+\underline{\lambda}_p}}$, we have

$$\dot{V}_p(\tilde{\boldsymbol{\delta}})\leqslant-\underline{\lambda}_p\|\boldsymbol{\delta}\|^2\leqslant-2b_pV_p(\tilde{\boldsymbol{\delta}})$$

which implies

$$\dot{V}_p(\tilde{\boldsymbol{\delta}})\leqslant-2b_0V_p(\tilde{\boldsymbol{\delta}}),\ \forall p\in\mathcal{P} \tag{19}$$

when $\|\boldsymbol{\delta}\|\geqslant\bar{\delta}_1$, with $b_0=\min_{p\in\mathcal{P}}(b_p)$, $\delta_1=\max_{p\in\mathcal{P}}\sqrt{\dfrac{2b_p\bar{\Phi}}{-2b_p\bar{a}_p+\underline{\lambda}_p}}$.

Besides, for all $\hat{p},\widehat{p}\in\mathcal{P}$, when $\|\boldsymbol{\delta}\|\geqslant\bar{\delta}_2=\max_{\hat{p},\widehat{p}\in\mathcal{P}}\left(\sqrt{\dfrac{2\bar{\boldsymbol{\Phi}}}{k\underline{a}_{\hat{p}}-\bar{a}_{\hat{p}}}}\right)$ with $k>\dfrac{\bar{a}_{\widehat{p}}}{\underline{a}_{\hat{p}}}$, we have

$$\dot{V}_{\hat{p}}(\tilde{\boldsymbol{\delta}})\leqslant kV_{\widehat{p}}(\tilde{\boldsymbol{\delta}}). \tag{20}$$

For $T>0$, there exists an $h\geqslant0$ such that $t_h<T\leqslant t_{hz+1}$, then, from (19) and (20), when $\|\boldsymbol{\delta}\|\geqslant\bar{\delta}=\max(\delta_1,\delta_2)$, one

has

$$\dot V_{\sigma(T^-)}(T^-) \leqslant k^h \mathrm{e}^{-2r_0 T} V_{\sigma(t_0)}(t_0). \tag{21}$$

Since $\tau_0 > \dfrac{\ln k}{2b_0}$,when $T \in (t_h, t_{h+1})$, we have

$$\dot V_{\sigma(T^-)}(T^-) \leqslant \mathrm{e}^{-\frac{(h-1)\tau_0}{h\tau_1}\omega T} V_{\sigma(t_0)}(t_0). \tag{22}$$

where $\bar b = 2b_0 - \dfrac{\ln k}{\tau_0}$. And for $T = t_{h+1}$, we get

$$\dot V_{\sigma(T^-)}(T^-) \leqslant \mathrm{e}^{-\frac{\tau_0}{\tau_1}\bar b T} V_{\sigma(t_0)}(t_0). \tag{23}$$

Since $\tilde{\boldsymbol\delta}(t)$ is continuous at t_k with (22) and (23), we derive that $\boldsymbol\delta(t)$ will enter in the set $\Omega_0 = \{\boldsymbol\delta(t) | \|\boldsymbol\delta(t)\| \leqslant \bar{\boldsymbol\delta}\}$, that means $\lim_{t\to\infty}|\delta(t)\| \leqslant \bar{\boldsymbol\delta}$. The proof is finished. □

Remark 9 For the matched parameter uncertainties, according to the reference [21], $\Delta \boldsymbol A(t) = \boldsymbol B \boldsymbol N(t)$, where $\boldsymbol B$ is a known real constant matrix (1), $\boldsymbol N(t)$ satisfies $\boldsymbol N(t)^{\mathrm T}\boldsymbol N(t) \leqslant \boldsymbol I$. The $\boldsymbol u_{i,2}$ is designed as follows:

$$\boldsymbol u_{i,2} = -\tfrac{1}{2}\eta \hat{\boldsymbol l}_{i,2}\boldsymbol B^{\mathrm T}\boldsymbol P \boldsymbol z_i^{\sigma(t)},$$
$$\dot{\hat{\boldsymbol l}}_{i,2} = -r_{i,2}\chi_i \hat{\boldsymbol l}_{i,2} + r_{i,2}\eta \|\boldsymbol B^{\mathrm T}\boldsymbol P \boldsymbol z_i^{\sigma(t)}\|^2.$$

Remark 10 For the case $\Delta \boldsymbol A(t) = 0$, the $\boldsymbol u_{i,2}$ with $\dfrac{1}{2}\eta(1+v_i)\hat{\boldsymbol l}_{i,2}(t)\boldsymbol B^{\mathrm T}\boldsymbol P \boldsymbol z_i^{\sigma(t)}$ is removed from the controller (8).

4. Numerical example

In this section, an example based on a reduced-order aircraft model is presented. Consider leader-follower MASs (1) and (2) under the switching communication topologies with four followers labeled as 1, 2, 3, 4 and one leader labeled as 0. The communication switching topologies of the system is shown in Fig. 1. The topology switching signal of the example is given in Fig. 2. The matrices of system $\boldsymbol A, \boldsymbol B, \boldsymbol N(t)$, and $\boldsymbol D$ are selected as follows:

$$\boldsymbol A = \begin{bmatrix} 0.059 & 0.0496 & -0.868 \\ -5.513 & -0.939 & 0.665 \\ 0.068 & 0.026 & -0.104 \end{bmatrix},$$

$$\boldsymbol B^{\mathrm T} = \begin{bmatrix} 0.006 & 1.878 & -0.109 \\ 0.006 & 1.328 & -0.096 \\ 0.004 & 0.029 & -0.084 \\ 0 & 0.675 & 0.007 \\ 0.09 & 0.217 & -2.974 \end{bmatrix},$$

$$\boldsymbol D^{\mathrm T} = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0 & -0.05 & 0 \\ 0 & 0.01 & 0.01 \\ 0.05 & 0 & 0 \\ 0 & 0.05 & 0.1 \end{bmatrix},$$

$$\boldsymbol N(t) = \begin{bmatrix} 0.5\sin t & 0 & \cos t \\ 0 & -0.5\sin t & 0 \\ \sin t & 0 & -0.5\sin t \\ -\cos t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The nonlinear function is described as $\boldsymbol g_i = [-0.13\boldsymbol x_{i2}\boldsymbol x_{i3}, 0, 0, 0, 0]^{\mathrm T}$ $(i = 1,2,3,4)$.

The disturbance is $\boldsymbol w_i = [1 \ \ \sin t]^{\mathrm T}$. The input is

$$\boldsymbol r_j = -\alpha \boldsymbol B^{\mathrm T}\boldsymbol P \boldsymbol x_k + \boldsymbol B^{\mathrm T}\boldsymbol K_m \boldsymbol s_i$$

where $m = 3,4$, $\boldsymbol s_i = [1, \sin t]^{\mathrm T}$,

$$\boldsymbol K_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol K_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1.4 & 0 & 0 & 0 \end{bmatrix}.$$

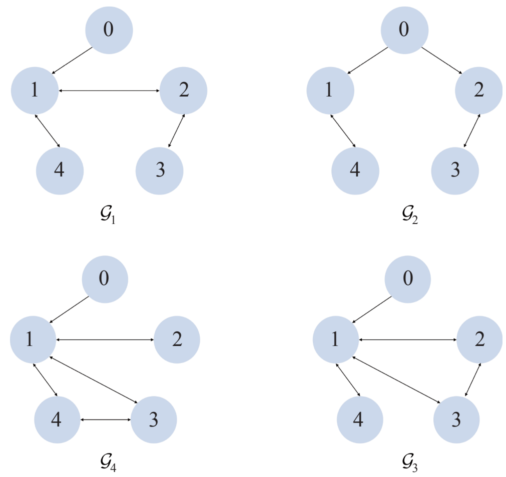

Fig. 1 Network topologies with four followers

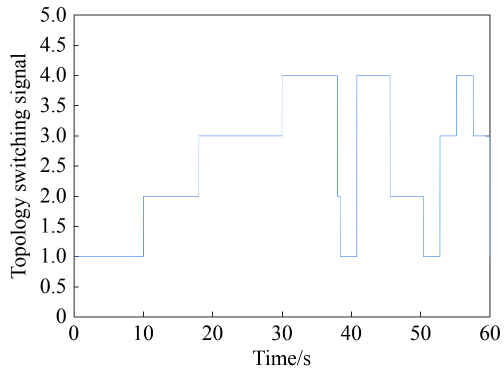

Fig. 2 Switching signal of communication graphs

The initial values and control parameters of this example are shown below:

$$\boldsymbol x_i(0) = [0.1, -1, 1.1]^{\mathrm T},$$
$$\hat l_{i,h}(0) = 0.1, \quad h = 0,2,3,4,$$
$$\sigma_i(t) = 30\exp(-0.5t),$$
$$\phi_i(\boldsymbol x_i) = \exp(-(\boldsymbol x_i - \hat{\boldsymbol l}_{i,1})^2/\sigma_i^2).$$

Moreover, in this example, for agent 2, the first actuator ha outage fault. For agent 3, the second actuator has bias fault, which is described by $\psi_{3,2} = 5 + 0.1\sin t$, $t \geqslant 30$ s. For agent 1, the third actuator has stuck fault that $\psi_{1,3} = 4 + 0.2\sin t$, $t \geqslant 48$ s. For agent 4, the second actuator has LOE faults.

The system in [33] fails to consider switching communication topologies, actuator faults, and mismatched parameter uncertainties. The state trajectories by using method (8) in this paper are shown in Fig. 3, Fig. 5, and Fig. 7. The state trajectories by using the method in [33] are shown in Fig. 4, Fig. 6, and Fig. 8. From these figures, it is easy to see that in the presence of the above actuator faults and mismatched parameter uncertainties under the switching communication topologies, the agents converge to zero by using the developed controller (8), while they are divergent by using the controller in [33]. The estimations of adaptive parameters are given in Fig. 9–Fig.13. Furthermore, these figures are provided to demonstrate the validity and applicability of the proposed control scheme.

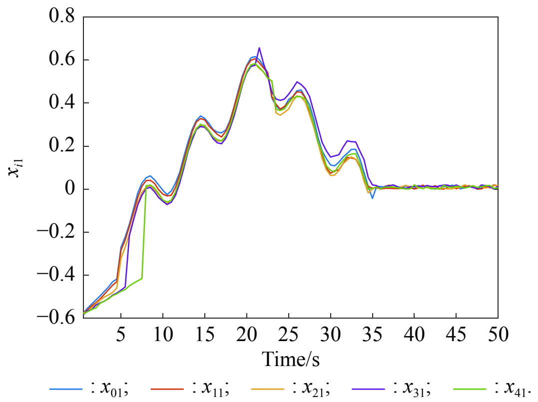

Fig. 3 Response curves of $x_{i1}(t)(i = 0, \cdots, 4)$ using controller (8)

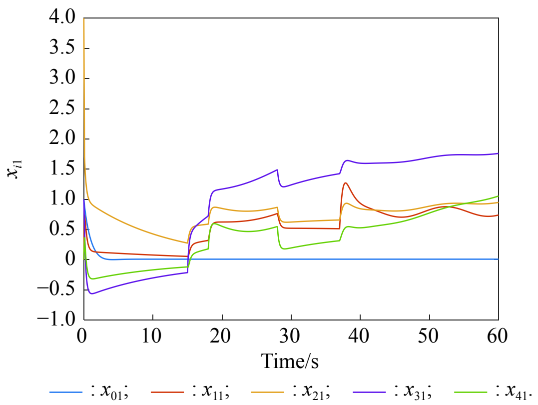

Fig. 4 Response curves of $x_{i1}(t)(i = 0, \cdots, 4)$ using controller in [33]

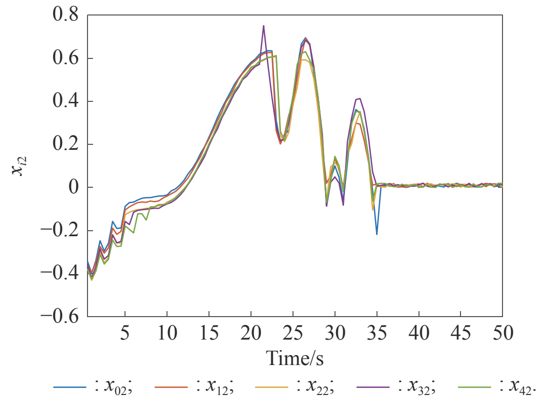

Fig. 5 Response curves of $x_{i2}(t)(i = 0, \cdots, 4)$ using controller (8)

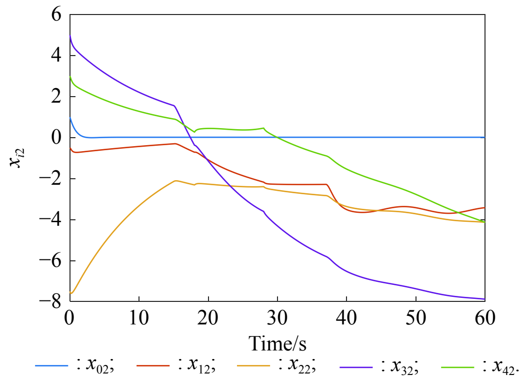

Fig. 6 Response curves of $x_{i2}(t)(i = 0, \cdots, 4)$ using controller in [33]

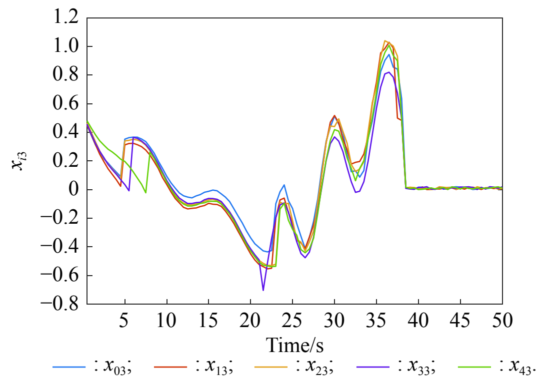

Fig. 7 Response curves of $x_{i3}(t)(i = 0, \cdots, 4)$ using controller (8)

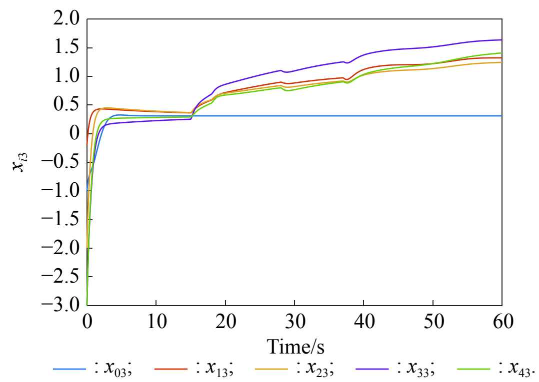

Fig. 8 Response curves of $x_{i3}(t)(i = 0, \cdots, 4)$ using controller in [33]

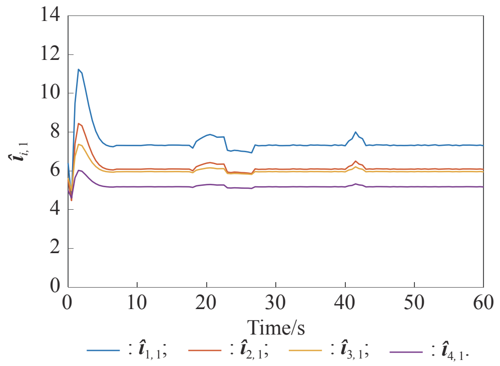

Fig. 9 Response curves of $\hat{l}_{i,1}(t)$ $(i = 1,\cdots,4)$

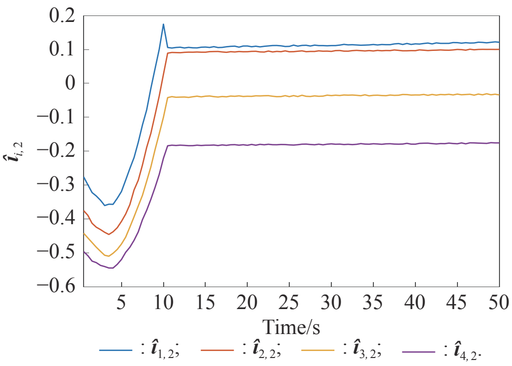

Fig. 10 Response curves of $\hat{l}_{i,2}(t)$ $(i = 1,\cdots,4)$

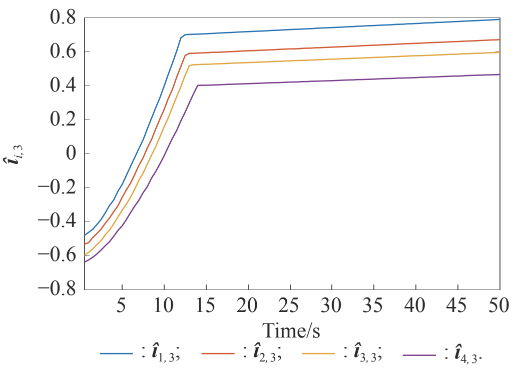

Fig. 11 Response curves of $\hat{l}_{i,3}(t)$ $(i = 1,\cdots,4)$

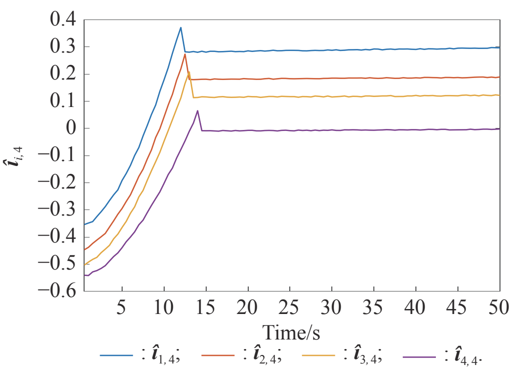

Fig. 12 Response curves of $\hat{l}_{i,4}(t)$ $(i = 1,\cdots,4)$

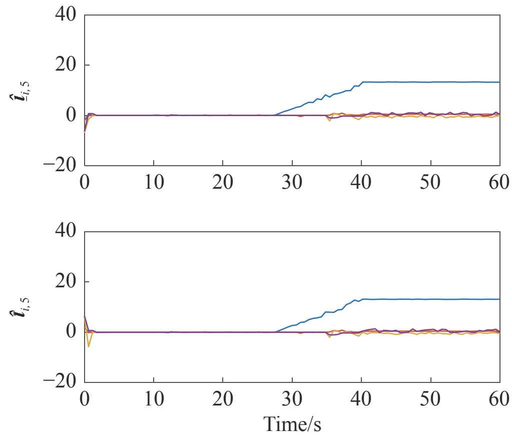

Fig. 13 Response curves of $\hat{\hat{l}}_{i,5}$ and $\hat{\underline{l}}_{i,5}$ $(i = 1,\cdots,4)$

5. Conclusions

In this paper, the tracking consensus problem for a class of nonlinear leader-follower MAS with external disturbance, mismatched parameter uncertainties, and actuator faults under switching communication topologies is studied. A new distributed fuzzy FTC is designed under the case of switching communication topologies and actuator faults. The effectiveness of the developed approach is shown by a simulation example.

It is worth pointing out that this paper only considers the distributed FTC problem under the undirected network topology case. Therefore, the extension of the current result to a more general switching directed topology is a challenging task, and this problem will be further investigated in the future work. Besides, it is challenging to achieve the consensus of MASs by the event-triggered scheme. Even though the convergence does not affect the established result in theorem, it is still an interesting future work for us.

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