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Reconfigurability evaluation method for input-constrained control systems

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Abstract: This paper proposes a quantitative reconfigurability evaluation method for control systems with actuator saturation and additive faults from the perspective of system stability. Placing the saturated feedback law in the convex hull of a group of auxiliary linear controls, the sufficient reconfigurability conditions for the system under additive faults are derived using invariant sets. These conditions are then expressed as linear matrix inequalities (LMIs) and applied to quantify the degree of reconfigurability for the fault system. The largest fault magnitude for which the system can be stabilized, the largest initial state domain from which all the trajectories are convergent, and the minimum final state domain to which the trajectories will converge are investigated. The effectiveness of the proposed method is illustrated through an application example.

Keywords: control reconfigurability, additive fault, actuator saturation, invariant set, fault-tolerant control.

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1. Introduction

A control system should be designed to have both excellent performance in normal situations and high dependability in the case of fault conditions. Therefore, a fault-tolerant control (FTC) strategy is needed to maintain or gracefully degrade control objectives at the occurrence of faults [1,2]. Many efforts have been devoted to FTC [2,3] which can be divided into passive fault-tolerant control (PFTC) [4,5] and active fault-tolerant control (AFTC) [6,7]. In PFTC, a single controller is designed to be robust against a group of faults while AFTC reacts to faults actively by fault detection and diagnosis (FDD) and control reconfiguration (CR). Therefore, AFTC is more flexible and effective than PFTC and has attracted a lot of attention [8].

The prerequisite of developing an FTC strategy for a control system is that the system must have enough FTC capability. However, the existing studies mostly focus on how to develop advanced FTC methods without paying much attention to fundamental FTC capability analysis, which has seriously hindered the application of these advanced FTC methods in real systems. Given this, a system attribute called reconfigurability is defined to assess the system ability to recover its function by CR after the occurrence of a fault [9,10]. The reconfigurability can be used as a guideline for system design, including the structure and the algorithm, to improve the FTC capability.

The existing reconfigurability analysis methods can be divided into qualitative methods and quantitative methods. The qualitative methods, such as the functional structure-based method [9,11], determine only whether a system is reconfigurable when some fault occurs, which is a binary result. The quantitative methods [10−12], on the other hand, can further measure the degree of reconfigurability and provide more valuable information for designers to develop a system structure or an FTC scheme more efficiently. To quantitatively measure the reconfigurability of the control systems with both sensor and actuator faults, Wu [10] proposed the controllability- and observability-based reconfigurability evaluation methods, which was then extended to various complex systems and applied in the design of fault-tolerant power systems [13,14] and multirotor unmanned aerial vehicles (UAVs) [15,16]. Considering the actuator fault in particular, along with the energy limitation, Staroswiecki [12] proposed an energy-based reconfigurability evaluation method with the aid of controllability Gramian.

Although the existing studies have made great contributions to control reconfigurability research, they have some deficiencies. First, their reconfigurability evalua-

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tion indices are designed for control systems with multiplicative faults rather than additive faults which also occur frequently in practical engineering. Second, practical operating conditions, such as actuator saturation, are rarely considered. Indeed, actuator saturation occurs in many practical control systems, which can degrade the dynamic performance of the systems, leading to a loss of stability and considerably impacting system operation [17]. Hence, it is of great practical significance to consider these actual influence factors in the reconfigurability analysis. Third, the exiting methods relevant to actuator faults are mostly based on the controllability Gramian [18,19] and consequently are only appliable for controllable systems. Sometimes, especially when the control objectives have been degraded because of the actuator faults, we are more concerned about system stability. Therefore, the reconfigurability analysis methods should be studied from the perspective of system stability as well.

Given the above, a novel stability-based quantitative reconfigurability evaluation method is developed in this paper for control systems with actuator saturation and additive faults. The main contributions are as follows.

(i) The additive actuator faults are taken into account in reconfigurability research for control systems.

(ii) Actuator saturation is considered in reconfigurability research, and the stability-based sufficient reconfigurability conditions are derived by placing the saturated feedback law in the convex hull of a group of auxiliary linear controls.

(iii) The degree of reconfigurability is quantified for the fault control system based on the sufficient reconfigurability conditions.

In the reconfigurability quantization process, the following three problems are considered:

(i) Can the system be stabilized after some fault occurs?

(ii) What is the range of the initial state domain from which all the system trajectories will be convergent?

(iii) What is the range of the final state domain to which all the system trajectories starting from a certain initial state domain will converge?

This paper is organized as follows. In Section 2, the problems to be addressed are formulated. In Section 3, the system reconfigurability is qualitatively judged by deriving the sufficient conditions. In Section 4, the degree of reconfigurability is further quantified based on the sufficient conditions in the linear matrix inequality (LMI) form. Section 5 presents an application example. Finally, the conclusions are summarized in Section 6.

2. Problem formulation

Consider the following linear time-invariant system with additive faults:

$$\begin{cases} \dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\mathrm{sat}(\boldsymbol{u}(t))+\boldsymbol{F}\boldsymbol{f}(t) \\ \boldsymbol{x}(t_0)=\boldsymbol{x}_0 \end{cases} \tag{1}$$

where $\boldsymbol{x}\in\mathbf{R}^n$, $\boldsymbol{u}\in\mathbf{R}^m$, and $\boldsymbol{f}\in\mathbf{R}^{m_a}$ are the system state, control input, and additive fault vectors, respectively, and m_a is the dimension of the fault vector, $\mathbf{R}$ denotes the Euclidean space; $\boldsymbol{x}_0$ is the state at the initial time instant t_0; $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{F}$ are constant matrices of corresponding dimensions. The mapping $\mathrm{sat}(\cdot):\mathbf{R}^m\rightarrow\mathbf{R}^m$ is the standard unity saturation function and the non-unity saturation can be absorbed into $\boldsymbol{B}$ and $\boldsymbol{u}$.

Remark 1 The existing reconfigurability evaluation methods are mostly proposed for control systems with multiplicative faults rather than additive faults which are also very common in practical engineering. To remedy this situation, this paper focuses on reconfigurability evaluation for the control systems with additive faults.

Due to the input saturation and the resource limitation of the system, only the additive faults with limited magnitude are concerned in this paper. Otherwise, the reconfiguration cost will exceed the system capacity, resulting in unreconfigurability. Given this, the faults discussed in this paper are assumed to be in the following set:

$$\mathcal{F}_\delta^\infty:=\left\{\boldsymbol{f}:\mathbf{R}_+\rightarrow\mathbf{R}^{m_a}:\boldsymbol{f}^{\mathrm{T}}(t)\boldsymbol{f}(t)\leqslant\delta,\forall t\right\} \tag{2}$$

where $\mathbf{R}_+$ denotes the set of positive real numbers. The subscript “T” indicates matrix transposition. $\delta 0$ is a constant that reflects the fault magnitude.

A state feedback controller is utilized to stabilize the system:

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t). \tag{3}$$

For a given feedback gain $\boldsymbol{K}\in\mathbf{R}^{n\times m}$, define the state domain where actuator saturation does not occur as

$$\mathcal{L}(\boldsymbol{K}):=\{\boldsymbol{x}\in\mathbf{R}^n:|\boldsymbol{k}_i\boldsymbol{x}|\leqslant 1, i=1,2,\cdots,m\} \tag{4}$$

where $\boldsymbol{k}_i$ is the ith row of $\boldsymbol{K}$, and $\mathbf{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices.

Let $\mathcal{M}$ be a set of $m\times m$ diagonal matrices whose elements are 0 or 1. The set $\mathcal{M}$ has 2^m elements in total and each element is denoted as $\boldsymbol{M}_i$ (i=1,2,$\cdots$,2^m). Let $\boldsymbol{M}_i^-=\boldsymbol{I}-\boldsymbol{M}_i$ and clearly, $\boldsymbol{M}_i^-\in\mathcal{M}$.

Before giving the main result of this paper, the following Lemma from [20,21] is introduced.

Lemma 1 Let $\boldsymbol{u},\boldsymbol{v}\in\mathbf{R}^{\mathbf{m}}$. Suppose that $|v_i|\leqslant 1$ for all $i\in[1,m]$, then

$$\mathrm{sat}(\boldsymbol{u})\in\mathrm{co}\{\boldsymbol{M}_i\boldsymbol{u}+\boldsymbol{M}_i^-\boldsymbol{v}:i\in[1,2^m]\} \tag{5}$$

where $\mathrm{sat}(\boldsymbol{u})=[\mathrm{sat}(u_1), \mathrm{sat}(u_2),\cdots,\mathrm{sat}(u_m)]^{\mathrm{T}}$, and

$\mathrm{sat}(u_i) = \mathrm{sgn}(u_i)\min\{1, |u_i|\}$. $\mathrm{co}\{\cdot\}$ denotes the convex hull of "$\cdot$".

Definition 1 (Reconfigurability) The fault system (1) is control reconfigurable with the initial state set $\boldsymbol{X}_0 \subset \mathbf{R}^n$ under a given fault $\boldsymbol{f} \in \mathcal{F}_\delta^\infty$ if there exists at least one controller that is stabilizing for system (1) with $\forall \boldsymbol{x}_0 \in \boldsymbol{X}_0$.

Definition 2 (Invariant set) A set in $\mathbf{R}^n$ is said to be invariant if all the trajectories starting from it will remain in it regardless of $\boldsymbol{f} \in \mathcal{F}_\delta^\infty$.

Let $\boldsymbol{V}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$, where $\boldsymbol{P} \in \mathbf{R}^{n\times n}$ is a positive definite symmetric matrix. The set

$$\boldsymbol{\Omega}(\boldsymbol{P},\rho) := \left\{\boldsymbol{x} \in \mathbf{R}^n : \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} \leqslant \rho\right\}$$

is an invariant set if and only if $\dot{\boldsymbol{V}}(\boldsymbol{x}) = 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}\sigma(\boldsymbol{u}))$ is negative definite for all $\boldsymbol{x} \in \boldsymbol{\Omega}(\boldsymbol{P},\rho)$.

3. Reconfigurability qualification

The reconfigurability is analyzed qualitatively in this section and the sufficient conditions of reconfigurability are given below.

Theorem 1 For the fault system (1) with initial state set $\boldsymbol{X}_0$, if there exist a feedback gain $\boldsymbol{K} \in \mathbf{R}^{n\times m}$, matrices $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} \in \mathbf{R}^{n\times n}$, $\boldsymbol{H} \in \mathbf{R}^{m\times n}$, and a positive number $\eta 0$, satisfying the following conditions:

$$\begin{cases} \boldsymbol{X}_0 \in \boldsymbol{\Omega}(\boldsymbol{P},\delta) & \text{(6a)} \\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]+ & \\ \quad \dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\eta\boldsymbol{P}<0,\ \ i=1,2,\cdots,2^m & \text{, (6b)} \\ \boldsymbol{\Omega}(\boldsymbol{P},\delta) \subset \mathcal{L}(\boldsymbol{H}) & \text{(6c)} \end{cases}$$

then system (1) is reconfigurable with $\boldsymbol{X}_0$ and all the trajectories starting from inside $\boldsymbol{X}_0$ will remain in it.

Proof Select the Lyapunov function.

$$\boldsymbol{V}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} \tag{7}$$

The derivative of $\boldsymbol{V}(\boldsymbol{x})$ along the trajectories of the system is

$$\dot{\boldsymbol{V}}(\boldsymbol{x}) = 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{x}} = 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}[\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\mathrm{sat}(\boldsymbol{u}(t))+\boldsymbol{F}\boldsymbol{f}(t)]. \tag{8}$$

Given that $\boldsymbol{\Omega}(\boldsymbol{P},\delta) \subset \mathcal{L}(\boldsymbol{H})$, according to Lemma 1, $\forall \boldsymbol{x} \in \boldsymbol{\Omega}(\boldsymbol{P},\delta)$,

$$\begin{aligned} \dot{\boldsymbol{V}}(\boldsymbol{x}) &= 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}[\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})\boldsymbol{x}+\boldsymbol{F}\boldsymbol{f}] = \\ & 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}[\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]\boldsymbol{x}+2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{F}\boldsymbol{f} \leqslant \\ & 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}[\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]\boldsymbol{x}+ \\ & \frac{1}{\eta}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}+\eta\boldsymbol{f}^{\mathrm{T}}\boldsymbol{f} \leqslant \\ & \boldsymbol{x}^{\mathrm{T}}\left[2\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}))+\frac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}\right]\boldsymbol{x}+\eta\delta, \\ & i = 1,2,\cdots,2^m. \end{aligned} \tag{9}$$

When (6) is satisfied, we have

$$\dot{\boldsymbol{V}}(\boldsymbol{x}) \leqslant -\eta\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}+\eta\delta. \tag{10}$$

Therefore, $\forall \boldsymbol{x} \in \boldsymbol{\Omega}(\boldsymbol{P},\delta)$, $\dot{\boldsymbol{V}}(\boldsymbol{x}) \leqslant 0$. Since $\boldsymbol{X}_0 \in \boldsymbol{\Omega}(\boldsymbol{P},\delta)$, $\dot{\boldsymbol{V}}(\boldsymbol{x}) \leqslant 0$ holds for $\forall \boldsymbol{x} \in \boldsymbol{X}_0$ and thereby all the trajectories starting from $\boldsymbol{X}_0$ will remain in it.

Theorem 1 is proved. □

Theorem 1 offers a sufficient criterion for control reconfigurability, but it only concerns the stability of the system without considering its convergence effect. In practice, the fault system is expected to have the ability to reduce the impact of the faults as much as possible, making the system trajectory from $\boldsymbol{X}_0$ converge to a smaller domain $\boldsymbol{X}_\infty$ ($\boldsymbol{X}_\infty \subset \boldsymbol{X}_0$).

Assume that the initial state domain of the system is $\boldsymbol{X}_0 = \boldsymbol{\Omega}(\boldsymbol{R}_0, 1)$ and the expected final state domain is $\boldsymbol{X}_\infty = \boldsymbol{\Omega}(\boldsymbol{R}_\infty, 1)$. The following theorem is given to develop the condition under which all the trajectories starting from $\boldsymbol{\Omega}(\boldsymbol{R}_0, 1)$ converge to $\boldsymbol{\Omega}(\boldsymbol{R}_\infty, 1)$.

Theorem 2 For the fault system (1), if there exist $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} \in \mathbf{R}^{n\times n}$, $\boldsymbol{H}_1$, $\boldsymbol{H}_2 \in \mathbf{R}^{m\times n}$ and positive numbers $\eta 0$, $0\varepsilon_\infty 1$, such that

$$\begin{cases} \boldsymbol{\Omega}(\boldsymbol{R}_0, 1) \subset \boldsymbol{\Omega}(\boldsymbol{P},\delta) & \text{(11a)} \\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta) \subset \boldsymbol{\Omega}(\boldsymbol{R}_\infty, 1) & \text{(11b)} \\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ & \\ \quad \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]+\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\eta\boldsymbol{P}<\boldsymbol{0}, & \\ \quad i = 1,2,\cdots,2^m & \text{, (11c)} \\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ & \\ \quad \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]+\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\dfrac{\eta}{\varepsilon_\infty}\boldsymbol{P}<\boldsymbol{0} & \text{(11d)} \\ \quad i = 1,2,\cdots,2^m & \\ \boldsymbol{\Omega}(\boldsymbol{P},\delta) \subset \mathcal{L}(\boldsymbol{H}_1) & \text{(11e)} \\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta) \subset \mathcal{L}(\boldsymbol{H}_2) & \text{(11f)} \end{cases}$$

then for all $\varepsilon \in [\varepsilon_\infty, 1]$, $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon\delta)$ is also invariant, which consequently implies that all the trajectories starting from $\boldsymbol{\Omega}(\boldsymbol{R}_0, 1)$ will converge to $\boldsymbol{\Omega}(\boldsymbol{R}_\infty, 1)$.

Proof It can be simply proved from constraints (11a) and (11b) and the properties of the convex function that

$$\begin{aligned} & [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ \\ & \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H})]+\frac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\frac{\eta}{\varepsilon}\boldsymbol{P}<\boldsymbol{0}. \end{aligned} \tag{12}$$

Let $\boldsymbol{h}_{1,i}$ and $\boldsymbol{h}_{2,i}$ be the ith rows of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$. Then, it can be seen from [22] that conditions (11e) and (11f) are equivalent to

$$\begin{bmatrix} \dfrac{1}{\delta} & h_{1,i} \\ h_{1,i}^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} \geqslant \boldsymbol{0},\ \begin{bmatrix} \dfrac{1}{\varepsilon_\infty\delta} & h_{2,i} \\ h_{2,i}^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} \geqslant \boldsymbol{0},\quad i \in [1, m].$$

Because $\varepsilon \in [\varepsilon_\infty, 1]$, there exist $\lambda \in [0,1]$ satisfying

$$\frac{1}{\varepsilon} = \lambda + \frac{1-\lambda}{\varepsilon_\infty}. \tag{13}$$

Let $\boldsymbol{H} = \lambda \boldsymbol{H}_1 + (1-\lambda)\boldsymbol{H}_2$, then

$$\begin{bmatrix} \dfrac{1}{\varepsilon\delta} & \boldsymbol{h}_i \\ \boldsymbol{h}_i^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} =$$

$$\begin{bmatrix} \left(\lambda + \dfrac{1-\lambda}{\varepsilon_\infty}\right)\dfrac{1}{\delta} & \lambda h_{1,i} + (1-\lambda) h_{2,i} \\ \lambda h_{1,i}^{\mathrm{T}} + (1-\lambda) h_{2,i}^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} =$$

$$\lambda \begin{bmatrix} \dfrac{1}{\delta} & h_{1,i} \\ h_{1,i}^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} + (1-\lambda) \begin{bmatrix} \dfrac{1}{\varepsilon_\infty \delta} & h_{2,i} \\ h_{2,i}^{\mathrm{T}} & \boldsymbol{P} \end{bmatrix} \geqslant \mathbf{0} \tag{14}$$

which is equivalent to $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon\delta) \subset \mathcal{L}(\boldsymbol{H})$. Similar to Theorem 1, we can derive that $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon\delta)$ is an invariant set from (12) and (14). Because of the arbitrariness of ε over $[\varepsilon_\infty, 1]$, all the trajectories starting from $\boldsymbol{\Omega}(\boldsymbol{P},\delta)$ will converge to $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)$. Further, from (11a) and (11b), all the trajectories starting from $\boldsymbol{\Omega}(\boldsymbol{R}_0,1)$ will converge to $\boldsymbol{\Omega}(\boldsymbol{R}_\infty,1)$. Theorem 2 is proved. □

Theorem 2 provides a sufficient condition of system reconfigurability under certain convergence requirements.

4. Reconfigurability quantification

Based on the reconfigurability qualification given by Theorem 1 and Theorem 2, the reconfigurability is further quantified in this section. The reconfigurability of control systems can be reflected from three aspects: (i) the largest fault magnitude δ^* for which the system can be stabilized; (ii) the largest initial state domain $\boldsymbol{X}_0^*$ starting from which all the trajectories will converge to a given final state domain $\boldsymbol{X}_\infty$; (iii) the minimum final state domain $\boldsymbol{X}_\infty^*$ to which all the trajectories start from a given initial state domain $\boldsymbol{X}_0$ will converge. This paper quantitatively analyzes the reconfigurability of control systems from these three aspects.

4.1 The largest fault magnitude

The largest fault magnitude δ^* for which the system can be stabilized reflects the fault tolerance range of the control system. Given an initial state domain $\boldsymbol{X}_0 = \boldsymbol{\Omega}(\boldsymbol{R}_0,1)$, the following optimization problems can be used to estimate δ^*:

$$\sup_{\boldsymbol{P}>0,\eta>0,\boldsymbol{K},\boldsymbol{H}} \delta$$

$$\text{s.t.} \begin{cases} \boldsymbol{\Omega}(\boldsymbol{R}_0,1) \in \boldsymbol{\Omega}(\boldsymbol{P},\delta) & (15\text{a}) \\ [\boldsymbol{A} + \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K} + \boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}[\boldsymbol{A}+ \\ \quad \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K} + \boldsymbol{M}_i^-\boldsymbol{H})] + \dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P} + \eta\boldsymbol{P} < \mathbf{0}, \\ \quad i = 1,2,\cdots,2^m & (15\text{b}) \\ \boldsymbol{\Omega}(\boldsymbol{P},\delta) \subset \mathcal{L}(\boldsymbol{H}) & (15\text{c}) \end{cases}$$

Constraints (15a) and (15b) are respectively equivalent to

$$\boldsymbol{R}_0 \geqslant \frac{\boldsymbol{P}}{\delta}\mathbf{0} \Leftrightarrow \begin{bmatrix} \dfrac{\boldsymbol{R}_0}{\delta} & \dfrac{\boldsymbol{I}_n}{\delta} \\ \dfrac{\boldsymbol{I}_n}{\delta} & \boldsymbol{P}^{-1} \end{bmatrix} \geqslant \mathbf{0} \tag{16}$$

and

$$\boldsymbol{P}^{-1}[\boldsymbol{A} + \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K} + \boldsymbol{M}_i^-\boldsymbol{H})]^{\mathrm{T}} + [\boldsymbol{A}+ \\ \boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K} + \boldsymbol{M}_i^-\boldsymbol{H})]\boldsymbol{P}^{-1} + \frac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}} + \eta\boldsymbol{P}^{-1} < \mathbf{0}. \tag{17}$$

From [22], 15(c) is equivalent to

$$\delta \leqslant \frac{1}{\boldsymbol{h}_j\boldsymbol{P}^{-1}\boldsymbol{h}_j^{\mathrm{T}}},\ j = 1,2,\cdots,m \tag{18}$$

where $\boldsymbol{h}_j$ is the jth row of $\boldsymbol{H}$. Furthermore, it can be transformed into

$$\begin{bmatrix} \dfrac{1}{\delta} & \boldsymbol{h}_j\boldsymbol{P}^{-1} \\ \boldsymbol{P}^{-1}\boldsymbol{h}_j^{\mathrm{T}} & \boldsymbol{P}^{-1} \end{bmatrix} \geqslant \mathbf{0}. \tag{19}$$

Let $\upsilon = \dfrac{1}{\delta}$, $\boldsymbol{Q} = \boldsymbol{P}^{-1}$, $\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{Q}$, and $\boldsymbol{Z} = \boldsymbol{K}\boldsymbol{Q}$. The optimization problem (15a)−(15c) can be rewritten as follows:

$$\inf_{\boldsymbol{Q}>0,\eta>0,\boldsymbol{Y},\boldsymbol{Z}} \upsilon$$

$$\text{s.t.} \begin{cases} \begin{bmatrix} \upsilon\boldsymbol{R}_0 & \upsilon\boldsymbol{I}_n \\ \upsilon\boldsymbol{I}_n & \boldsymbol{Q} \end{bmatrix} \geqslant \mathbf{0} & (20\text{a}) \\ (\boldsymbol{A}\boldsymbol{Q} + \boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z} + \boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y})^{\mathrm{T}} + \boldsymbol{A}\boldsymbol{Q}+ \\ \quad \boldsymbol{M}_i\boldsymbol{Z} + \boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y} + \dfrac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}} + \eta\boldsymbol{Q} < \mathbf{0}, \\ \quad i = 1,2,\cdots,2^m & (20\text{b}) \\ \begin{bmatrix} \upsilon & \boldsymbol{y}_j \\ \boldsymbol{y}_j^{\mathrm{T}} & \boldsymbol{Q} \end{bmatrix} \geqslant \mathbf{0},\quad j = 1,2,\cdots,m & (20\text{c}) \end{cases}$$

where $\boldsymbol{y}_j$ is the jth row of $\boldsymbol{Y}$.

If η is fixed, the optimization problem (20a)−(20c) is an LMI problem, which can be solved directly by using the LMI toolbox of Matlab. Varying η from 0 to ∞, the global optimal solution can be obtained, i.e., the largest fault magnitude δ^* for which the system is reconfigurable can be estimated as $\delta^* = \dfrac{1}{\upsilon^*}$ with the corresponding optimal control gain $\boldsymbol{K}^* = \boldsymbol{Z}^*\boldsymbol{Q}^{*-1}$, where $\boldsymbol{Q}^*$ and $\boldsymbol{Z}^*$ are the optimal solutions of $\boldsymbol{Q}$ and $\boldsymbol{Z}$.

4.2 The largest initial state set

Due to the nonlinear properties caused by the input constraints, it is difficult to calculate the largest initial state domain $\boldsymbol{X}_0^*$ accurately. Given this, the elliptic invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\delta)$ is used to estimate $\boldsymbol{X}_0^*$ based on Theorem 2. Accordingly, the problem of estimating the largest initial state domain of the system can be described as: given an expected final state domain $\boldsymbol{\Omega}(\boldsymbol{R}_\infty,1)$, design $\boldsymbol{K}$ so that the system has an invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\delta)\supset\alpha_0\boldsymbol{\Omega}(\boldsymbol{I}_n,1)$ with α_0 maximized, where α_0 is the inscribed radius of $\boldsymbol{\Omega}(\boldsymbol{P},\delta)$, and all the trajectory starting from it will converge into a smaller invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\boldsymbol{\Omega}(\boldsymbol{R}_\infty,1)$ $(0\varepsilon_\infty 1)$.

This problem can be formulated as

$$\sup_{\boldsymbol{P}>0,\eta>0,0<\varepsilon_\infty<1,\boldsymbol{K},\boldsymbol{H}_1,\boldsymbol{H}_2}\ \alpha_0$$

$$\text{s.t.}\begin{cases}\alpha_0\boldsymbol{\Omega}(\boldsymbol{I}_n,1)\subset\boldsymbol{\Omega}(\boldsymbol{P},\delta) & \text{(21a)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\boldsymbol{\Omega}(\boldsymbol{R}_\infty,1) & \text{(21b)}\\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_1)]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ & \\ \quad\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_1)]+\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\eta\boldsymbol{P}<\boldsymbol{0} & \text{(21c)}\\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_2)]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_2)]+ & \\ \quad\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\dfrac{\eta}{\varepsilon_\infty}\boldsymbol{P}<\boldsymbol{0},i=1,2,\cdots,2^m & \text{(21d)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\delta)\subset\mathcal{L}(\boldsymbol{H}_1) & \text{(21e)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\mathcal{L}(\boldsymbol{H}_2) & \text{(21f)}\end{cases}.$$

Let $\boldsymbol{Q}=\boldsymbol{P}^{-1}$, $\boldsymbol{Y}_1=\boldsymbol{H}_1\boldsymbol{Q}$, $\boldsymbol{Y}_2=\boldsymbol{H}_2\boldsymbol{Q}$, and $\boldsymbol{Z}=\boldsymbol{K}\boldsymbol{Q}$, then (21a)–(21f) can be transformed into the following forms:

$$\sup_{\boldsymbol{Q}>0,\eta>0,0<\varepsilon_\infty<1,\boldsymbol{Z},\boldsymbol{Y}_1,\boldsymbol{Y}_2}\ \alpha_0$$

$$\text{s.t.}\begin{cases}\begin{bmatrix}\delta\boldsymbol{I}_n & \alpha_0\boldsymbol{I}_n\\ \alpha_0\boldsymbol{I}_n & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0} & \text{(22a)}\\ \begin{bmatrix}\boldsymbol{I} & \varepsilon_\infty\delta\boldsymbol{R}_\infty\boldsymbol{Q}\\ \varepsilon_\infty\delta\boldsymbol{R}_\infty\boldsymbol{Q} & \boldsymbol{I}\end{bmatrix}\geqslant\boldsymbol{0} & \text{(22b)}\\ (\boldsymbol{A}\boldsymbol{Q}+\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_1)^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{Q}+ & \\ \quad\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_1+\dfrac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}+\eta\boldsymbol{Q}<\boldsymbol{0} & \text{(22c)}\\ (\boldsymbol{A}\boldsymbol{Q}+\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_2)^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{Q}+ & \\ \quad\mathrm{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_2+\dfrac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}+\dfrac{\eta}{\varepsilon_\infty}\boldsymbol{Q}<\boldsymbol{0}, & \\ \quad i=1,2,\cdots,2^m & \text{(22d)}\\ \begin{bmatrix}\dfrac{1}{\delta} & \boldsymbol{y}_{1j}\\ \boldsymbol{y}_{1j}^{\mathrm{T}} & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0} & \text{(22e)}\\ \begin{bmatrix}\dfrac{1}{\varepsilon_\infty\delta} & \boldsymbol{y}_{2j}\\ \boldsymbol{y}_{2j}^{\mathrm{T}} & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0},\ j=1,2,\cdots,m & \text{(22f)}\end{cases}$$

where $\boldsymbol{y}_{1j}$ and $\boldsymbol{y}_{2j}$ are the jth row of $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, respectively. The problem (22a)–(22f) is an LMI problem when η and ε_∞ are fixed. Varying η from 0 to ∞ and ε_∞ from 0 to 1, the global optimal solution can be obtained to estimate the largest initial state domain $\boldsymbol{X}_0^*\approx\boldsymbol{\Omega}(\boldsymbol{P},\delta)$.

4.3 Minimum final state domain

Similar to the above, the minimum final state domain $\boldsymbol{X}_\infty^*$ is estimated by the elliptic invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)$ based on Theorem 2. This problem can be formulated as: given an initial state domain $\boldsymbol{\Omega}(\boldsymbol{R}_0,1)$, design $\boldsymbol{K}$ so that the system has an invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\delta)\supseteq\boldsymbol{\Omega}(\boldsymbol{R}_0,1)$, and all the trajectory starting from it will converge into a smaller invariant set $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\alpha_\infty\boldsymbol{\Omega}(\boldsymbol{I}_n,1)$ with α_∞ minimized, where $0\varepsilon_\infty 1$ and α_∞ is the circumscribed radius of $\boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)$.

This problem can be formulated as

$$\inf_{\boldsymbol{P}>0,\eta>0,0<\varepsilon_\infty<1,\boldsymbol{K},\boldsymbol{H}_1,\boldsymbol{H}_2}\ \alpha_\infty$$

$$\text{s.t.}\begin{cases}\boldsymbol{\Omega}(\boldsymbol{R}_0,1)\subset\boldsymbol{\Omega}(\boldsymbol{P},\delta) & \text{(23a)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\alpha_\infty\boldsymbol{\Omega}(\boldsymbol{I}_n,1) & \text{(23b)}\\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_1)]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ & \\ \quad\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_1)]+\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\eta\boldsymbol{P}<\boldsymbol{0} & \text{(23c)}\\ [\boldsymbol{A}+\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_2)]^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}[\boldsymbol{A}+ & \\ \quad\boldsymbol{B}(\boldsymbol{M}_i\boldsymbol{K}+\boldsymbol{M}_i^-\boldsymbol{H}_2)]+\dfrac{1}{\eta}\boldsymbol{P}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}+\dfrac{\eta}{\varepsilon_\infty}\boldsymbol{P}<\boldsymbol{0}, & \\ \quad i=1,2,\cdots,2^m & \text{(23d)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\delta)\subset\mathcal{L}(\boldsymbol{H}_1) & \text{(23e)}\\ \boldsymbol{\Omega}(\boldsymbol{P},\varepsilon_\infty\delta)\subset\mathcal{L}(\boldsymbol{H}_2),\ j=1,2,\cdots,m & \text{(23f)}\end{cases}.$$

Let $\boldsymbol{Q}=\boldsymbol{P}^{-1}$, $\boldsymbol{Y}_1=\boldsymbol{H}_1\boldsymbol{Q}$, $\boldsymbol{Y}_2=\boldsymbol{H}_2\boldsymbol{Q}$, and $\boldsymbol{Z}=\boldsymbol{K}\boldsymbol{Q}$. Then, (23a)–(23f) can be transformed into

$$\inf_{\boldsymbol{Q}>0,\eta>0,0<\varepsilon_\infty<1,\boldsymbol{Z},\boldsymbol{Y}_1,\boldsymbol{Y}_2}\ \alpha_\infty$$

$$\text{s.t.}\begin{cases}\begin{bmatrix}\delta\boldsymbol{R}_0 & \boldsymbol{I}_n\\ \boldsymbol{I}_n & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0} & \text{(24a)}\\ \begin{bmatrix}\alpha_\infty\boldsymbol{I}_n & \varepsilon_\infty\delta\boldsymbol{Q}\\ \varepsilon_\infty\delta\boldsymbol{Q} & \alpha_\infty\boldsymbol{I}_n\end{bmatrix}\geqslant\boldsymbol{0} & \text{(24b)}\\ (\boldsymbol{A}\boldsymbol{Q}+\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_1)^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{Q}+ & \\ \quad\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_1+\dfrac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}+\eta\boldsymbol{Q}<\boldsymbol{0} & \text{(24c)}\\ (\boldsymbol{A}\boldsymbol{Q}+\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_2)^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{Q}+ & \\ \quad\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{Z}+\boldsymbol{B}\boldsymbol{M}_i^-\boldsymbol{Y}_2+\dfrac{1}{\eta}\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}+\dfrac{\eta}{\varepsilon_\infty}\boldsymbol{Q}<\boldsymbol{0}, & \\ \quad i=1,2,\cdots,2^m & \text{(24d)}\\ \begin{bmatrix}\dfrac{1}{\delta} & \boldsymbol{y}_{1j}\\ \boldsymbol{y}_{1j}^{\mathrm{T}} & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0} & \text{(24e)}\\ \begin{bmatrix}\dfrac{1}{\varepsilon_\infty\delta} & \boldsymbol{y}_{2j}\\ \boldsymbol{y}_{2j}^{\mathrm{T}} & \boldsymbol{Q}\end{bmatrix}\geqslant\boldsymbol{0},\quad j=1,2,\cdots,m & \text{(24f)}\end{cases}.$$

Similar to (22a)–(22f), varying η from 0 to ∞ and ε_∞ from 0 to 1, the global optimal solution to the problem (24a)–(24f) can be obtained by the LMI toolbox.

5. Numerical simulation

In this section, a simulation example is presented to demonstrate the effectiveness of the proposed methods. The parameters of system (1) are

$$A=\begin{bmatrix}0.6 & -0.8\\ 0.8 & 0.6\end{bmatrix},\ B=\begin{bmatrix}2\\ 4\end{bmatrix},\ F=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}.$$

From (20), the maximum additive fault magnitude is a function of η. Given the initial state domain $X_0=\boldsymbol{\Omega}(I_n,1)$, the curve of the maximum additive fault magnitude with different values of η is shown in Fig. 1. The global optimal solution over $\eta\in(0,\infty)$ is $\delta^*=12.710\,6$ with $P^*=\begin{bmatrix}76.851\,6 & -69.972\,1\\ -69.972\,1 & 103.761\,4\end{bmatrix}$ and $K^*=\begin{bmatrix}23\,367 & -50\,943\end{bmatrix}$ when η=0.50.

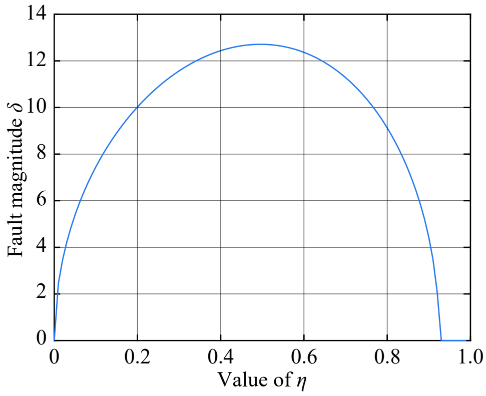

Fig. 1 Additive fault magnitude δ with different η

Given the expected final state domain $X_\infty=\boldsymbol{\Omega}(4I_n,1)$ and the additive fault magnitude $\delta=1$, the largest initial state domain X_0^* starting from which all the trajectories will converge to X_∞ is estimated by solving the optimization problem (22) to obtain the elliptic invariant set $\boldsymbol{\Omega}(P_0^*,1)$. The inscribed radius α_0 of the invariant set with different ε_∞ and η is shown as Fig. 2. The optimal solution $\alpha_0^*=1.678\,3$, with $P_0^*=\begin{bmatrix}0.171\,9 & -0.140\,6\\ -0.140\,6 & 0.247\,2\end{bmatrix}$ and $K_0^*=\begin{bmatrix}11\,077 & -35\,866\end{bmatrix}$, is obtained when $\varepsilon_\infty=0.016$ and $\eta=0.003\,6$. Fig. 3 presents the estimated largest initial state domain and some trajectories. It can be seen that the trajectories starting from $\boldsymbol{\Omega}(P_0^*,1)\supset X_0^*$ will converge to $\boldsymbol{\Omega}(P_0^*,0.016\delta)\subset X_\infty$.

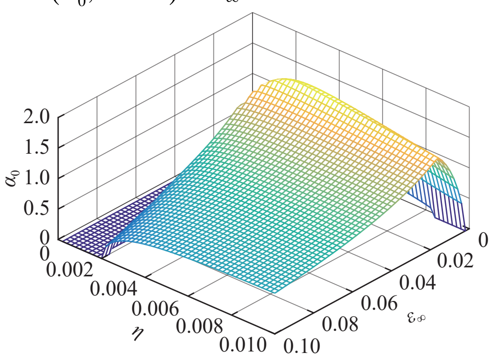

Fig. 2 Inscribed radius α_0 of the initial state domain with different ε_∞ and η

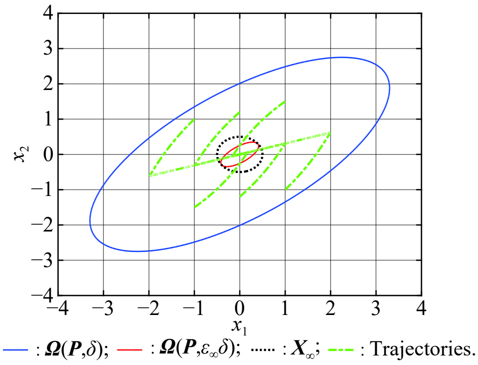

Fig. 3 Estimated largest initial state domain

Given the initial state domain $X_0=\boldsymbol{\Omega}(0.25I_n,1)$ and the additive fault magnitude $\delta=1$, the minimum final state domain X_∞^* to which all the trajectories from X_0 will converge is estimated by solving the optimization problem (24) to obtain the elliptic domain $\boldsymbol{\Omega}(P_\infty^*,1)$. The optimal solution $\alpha_\infty^*=0.956\,2$, with $P_\infty^*=\begin{bmatrix}0.116\,8 & -0.092\,7\\ -0.092\,7 & 0.185\,5\end{bmatrix}$ and $K_\infty^*=\begin{bmatrix}836.1 & -3\,392.9\end{bmatrix}$, is obtained when $\varepsilon_\infty=0.05$ and $\eta=0.005\,8$. The estimated minimum final state domain and some trajectories are shown in Fig. 4. It can be seen that the trajectories starting from $\boldsymbol{\Omega}(P_\infty^*,1)\supset X_0$ will converge to $\boldsymbol{\Omega}(P_\infty^*,0.05\delta)\subset X_\infty^*$.

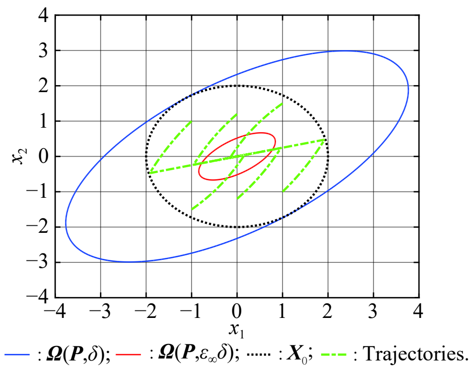

Fig. 4 Estimated minimum final state domain

6. Conclusions

This paper proposes a quantitative reconfigurability evaluation method for control systems with actuator saturation and additive faults from the perspective of stability. This method provides system designers with more information about the reconfigurability level of the control system, which can be used as a guideline for system design, including the structure and FTC algorithm, from the perspective of improving the reconfiguration capability of the system in the presence of faults.

In future work, other practical factors, such as time delays, finite horizons, and nonlinearities could be further considered. Moreover, the reconfigurability evaluation idea can be extended to other fault conditions, such as sensor faults and structural faults.

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