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Reliability modelling based on dependent two-stage virtual age processes

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Abstract: This paper proposes reliability and maintenance models for systems suffering random shocks arriving according to a non-homogeneous Poisson process. The system degradation process include two stages: from the installation of a new system to an initial point of a defect (normal stage), and then from that point to failure (defective stage), following the delay time concept. By employing the virtual age method, the impact of external shocks on the system degradation process is characterized by random virtual age increment in the two stages, resulting in the corresponding two-stage virtual age process. When operating in the defective state, the system becomes more susceptible to fatigue and suffers from a greater aging rate. Replacement is carried out either on failure or on the detection of a defective state at periodic or opportunistic inspections. This paper evaluates system reliability performance and investigates the optimal opportunistic maintenance policy. A case study on a cooling system is given to verify the obtained results.

Keywords: reliability evaluation, delay-time model, virtual age process, opportunistic maintenance.

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1. Introduction

Common signals of defects can be observed in most industrial systems. In such cases, three system states are involved including normal, defective and failure states. Since Christer firstly proposed the delay time concept [1] to model such two-stage failure processes, numerous models have been established to evaluate system failure risks and the optimal maintenance policy over the past several decades [2–8]. The durations in the normal and defective stages are assumed to be independent in existing studies. However a variety of industrial systems are required to perform tasks under random environment. Considering a degrading system in a fixed baseline environment, if the same system operates in a more severe environment (regime), the system lifetime will be stochastically smaller than that under the baseline environment. To establish a hazard rate correspondence between the systems in two regimes, the baseline environment is regarded as a reference one [9].

Considering the effect of shocks on system failure behavior, the random durations of the two stages are dependent as a result of the same shock process [10,11]. Therefore, it is of both theoretical and practical interest to propose a dependent two-stage failure process by considering the external shock process shared by the two stages. The methods employed in existing literature to characterize the effect of external shocks on the system failure process are degradation-based and shot-noise process-based.

For systems with measurable deterioration states, degradation-based reliability models are studied the most with a variety of real-world applications [12–16]. In the case of system degradation, paths were not evident, the dependence between system internal degradation and eternal shocks was firstly studied via the shot-noise process in [17], where each shock resulted in random increase in the failure rate. Since then, numerous models have been established on the basis of the process to study the system reliability performance and maintenance optimization problems [18–25].

This paper proposes a reliability evaluation method utilizing the virtual age method firstly proposed by Kijima [26] to model imperfect repair whose effect can be expressed by a reduction of the system virtual age. The virtual age method has spurred tremendous increase in the literature of imperfect maintenance [27–30]. Finkelstein [9] proposed two different approaches to define the virtual age of degrading systems. The first one was based on the fact that systems aged faster in more severe environment and the system virtual age was greater than the elapsed time. The latter one was based on an observed level of individual ageing. Motivated by the first approach, this paper characterizes the effect of external

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shocks on the two-stage failure behavior by virtual age increment. To be specific, in a baseline environment the virtual age of a system is the elapsed time since it is put into operation. Upon the arrival of an external shock, the system virtual age has a random jump.

As a consequence of the same external shock process, the durations of the two stages are dependent. Furthermore, in our new model, we assume that the system suffers a greater degradation rate in the defective stage [31, 32]. As far as we know, the virtual age method has been mainly applied in the maintenance modelling and has been seldom studied in the evaluation of system reliability. We make contributions by considering a two-stage virtual age process with accelerating ageing effect in the defective stage. In this paper, closed-form reliability formulas considering the damage caused by random shocks are derived by using the two-stage virtual age process.

Designing preventive maintenance policies is of crucial importance to mitigate the system failure risk, particularly from the perspective of balancing the tradeoff between the cost of maintenance and system failures [33,34]. During system operation, preventive maintenance can be performed opportunistically via unexpected shutdown [35–40]. Although opportunistic maintenance actions of industrial systems are cheaper than periodic maintenance, its execution time is largely determined by the arrival time of unexpected shutdown. This may give rise to the problem that opportunistic maintenance may not be able to remove defects timely, or its execution is too frequent to incur excessive maintenance cost. Hence, scheduling opportunistic or periodic maintenance alone may be sub-optimal from the perspective of reducing maintenance costs. For this reason, this article incorporates both periodic and opportunistic maintenance to deal with the two-stage failure process, which aims to realize the better allocation of maintenance resources. The optimal opportunistic maintenance policy is studied based on the dependent two-stage virtual age process. We make the following scientific contributions:

(i) Establishing a two-stage degradation model in a shock environment;

(ii) Utilizing the two-stage virtual age process to model the influence of shocks on system degradation;

(iii) Deriving the system reliability function by using the two-stage virtual age process;

(iv) Studying the optimal opportunistic maintenance policy.

The rest of the paper is organized as follows. Section 2 presents the two-stage virtual age process and the opportunistic maintenance policy. Section 3 derives system reliability formulas considering virtual age increments. Section 4 evaluates the cost performance and investigates the optimal inspection interval. Case study to illustrate the theoretical results is given in Section 5. Conclusions are presented in Section 6.

2. Model formulation

2.1 Two-stage virtual age process

The considered system is working in shock environment where shocks arrive stochastically according to a non-homogenous Poisson process (NHPP) $\{N(t), t \geqslant 0\}$ whose intensity function is $\mu(t)$. The system experiences the normal and defective stages. Let X and Y with reliability functions $R_X(t)$ and $R_Y(t)$ be the duration of the normal and defective stages respectively. In a baseline environment, the system virtual age is the elapsed time since it is put into operation and the corresponding ageing rate equals 1. In the presence of random shocks, the system virtual age $V(t)$ during the normal state is given as

$$V(t) = t + \sum_{i=1}^{N(0,t)} Z_i \tag{1}$$

where Z_i is the virtual age increment caused by the ith external shock and $N(0,t)$ denotes the number of external impacts in $(0,t)$.

Denote the defect arrival time by $\tilde{X}$ in the random environment. Upon the arrival of a defective state, the aging rate increases from 1 to $1+\beta$, where β denotes the constant ageing rate increment. Note that the normal and defective stages suffer the same external shock process, the random virtual age in the defective stage is related to the defect arrival time $\tilde{X}$ and is given as

$$V(t) = t + \sum_{i=1}^{N(0,t)} Z_i + \sum_{i=1}^{N(\tilde{X},t)} \beta(t - S_i) \tag{2}$$

where $S_i (i = 1, 2, \cdots)$ is the arrival time of the ith shock in the defective state and $N(\tilde{X},t)$ is the number of shocks in $(\tilde{X},t)$. Combining the two cases in (1) and (2), the system virtual age can be given as

$$V(t) = \begin{cases} t + \sum_{i=1}^{N(0,t)} Z_i, & t < \tilde{X} \\ t + \sum_{i=1}^{N(0,t)} Z_i + \sum_{i=1}^{N(\tilde{X},t)} \beta(t - S_i), & \tilde{X} \leqslant t < \tilde{X} + \tilde{Y} \end{cases}. \tag{3}$$

2.2 Maintenance strategy

To detect the defect timely, periodic inspections are typical maintenance actions. However, periodic inspections can only be performed when the system is shut down and occupies the production time, which may cause high downtime cost. On the other hand, unscheduled shutdown provides additional opportunities for maintenance such as inspections and replacements. A combination of periodic and opportunistic inspections helps to reduce the maintenance cost significantly.

In this paper, inspections provided for the system are block-based, i.e., periodic inspection is equally spaced with interval T according to the calendar time. As assumed in most literature, unexpected shutdown arrives according to a homogenous Poisson process (HPP) $\{\Lambda(t), t > 0\}$ with intensity λ. The system is replaced upon a failure or the detection of the defective state. System renewal may occur at a failure and inspection. Three possible renewal cases are illustrated in Fig. 1.

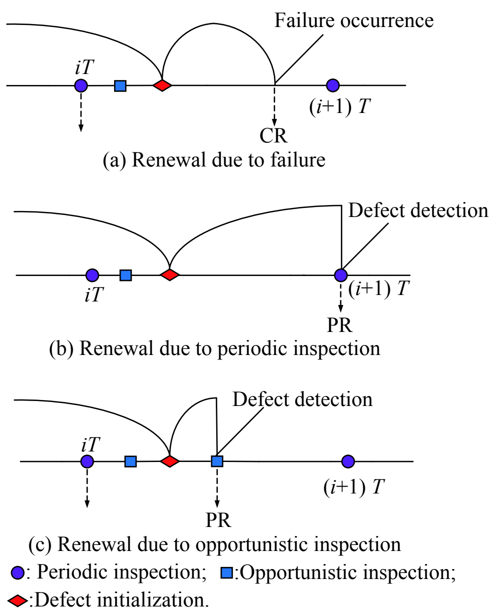

Fig. 1 Three renewal cases of the system

From Fig. 1 (a), we can see that the system fails before any opportunistic inspection and is correctively replaced (CR).

From Fig. 1 (b), we can see that the defective state is identified at periodic inspection and a preventive replacement (PR) is performed.

From Fig. 1 (c), we can see that the defective state is detected via an opportunistic inspection and a PR is carried out.

3. System reliability analysis

Incorporating the periodic inspection complicates the system reliability analysis since the system can be preventively replaced at inspections. This section firstly studies the system reliability function without considering the effect of maintenance actions. Let $f_X(x)$ be the density function of X. Then $R_s(t)$ can be obtained by conditioning on the defect arrival time as

$$R_s(t) = P(\tilde{X} + \tilde{Y} > t) =$$
$$R_{\tilde{X}}(t) + \int_0^t P(\tilde{Y} > t - x | X = x) f_{\tilde{X}}(x)\mathrm{d}x =$$
$$R_{\tilde{X}}(t) + \int_0^t R_{\tilde{Y}}(t, x) f_{\tilde{X}}(x)\mathrm{d}x. \tag{4}$$

According to the virtual age process in the normal stage defined in (3), the reliability function of $\tilde{X}$ can be derived as

$$R_{\tilde{X}}(t) = P\left(X > t + \sum_{i=1}^{N(0,t)} Z_i\right) =$$
$$\sum_{n=0}^{\infty} P\left(X > t + \sum_{i=1}^{N(0,t)} Z_i | N(0,t) = n\right) P(N(0,t) = n) =$$
$$P(X > t) P(N(0,t) = 0) +$$
$$\sum_{n=1}^{\infty} P\left(X > t + \sum_{i=1}^{n} Z_i\right) P(N(0,t) = n). \tag{5}$$

The number of external impacts in (x, t), $N(x, x+t)$, follows Poisson distribution with parameter $\int_x^t \mu(x)\mathrm{d}x$, then the probability mass function of $N(0,t)$ can be given as

$$P(N(0,t) = n) = \frac{\left(\int_0^t \mu(x)\mathrm{d}x\right)^n \exp\left(-\int_0^t \mu(x)\mathrm{d}x\right)}{n!}. \tag{6}$$

Denote $f^{<n>}(z)$ as the n-fold convolution of Z. Utilizing (6), then $R_{\tilde{X}}(t)$ is given as

$$R_{\tilde{X}}(t) = R_X(t) \exp\left(-\int_0^t \mu(x)\mathrm{d}x\right) +$$
$$\sum_{n=1}^{\infty} \int_0^{\infty} R_X(t+z) f_Z^{(n)}(z)\mathrm{d}z \frac{\left(\int_0^t \mu(x)\mathrm{d}x\right)^n \exp\left(-\int_0^t \mu(x)\mathrm{d}x\right)}{n!}. \tag{7}$$

Then we derive $R_{\tilde{Y}}(t, x)$ in (4). $R_{\tilde{Y}}(t, x)$ can be obtained by conditioning on the number of arrived shocks as

$$R_{\tilde{Y}}(t, x) =$$
$$\sum_{n=0}^{\infty} P\left(Y > t - x + \sum_{i=1}^{N(x,t)} Z_i + \beta(t - S_i), N(x,t) = n | X = x\right). \tag{8}$$

Equation (8) can be further derived as

$$
\begin{aligned}
&P\left(Y>t-x+\sum_{i=1}^{N(x,t)}Z_i+\beta(t-S_i),N(x,t)=n|X=x\right)=\\
&\int_x^t\cdots\int_x^{s_2}\underbrace{P\left(Y>t-x+\sum_{i=1}^{N(x,t)}Z_i+\beta(t-S_i)|X=x,N(x,t)=n,S_1=s_1,\cdots,S_n=s_n\right)}_{P_1}\cdot\\
&\underbrace{f(s_1,\cdots,s_n|N(x,t)=n)}_{P_2}\underbrace{P(N(x,t)=n)}_{P_3}\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n.
\end{aligned}\tag{9}
$$

The first part in (9) denotes the probability that the duration of the defective stage is greater than $t-x$ given that n shocks arrive in (x,t). By (2), we have

$$
\begin{aligned}
&P\left(Y>t-x+\sum_{i=1}^{N(x,t)}Z_i+\beta(t-S_i)|X=x,N(x,t)=n,S_1=s_1,\cdots,S_n=s_n\right)=\\
&\int_0^\infty R_Y\left(t-x+\sum_{i=1}^{n}\beta(t-s_i)+z\right)f_Z^{(n)}(z)\mathrm{d}z.
\end{aligned}\tag{10}
$$

The second part denotes the joint probability density of n shock arrival times given that they occur in (x,t). As the intensity function of the arrival process of shocks is $\mu(t)$, then the joint probability density function in (9) can be given as

$$
f(s_1,s_2,\cdots,s_n|N(x,t)=n)=\frac{n!\prod_{i=1}^{n}\mu(s_i)}{\left(\int_x^t\mu(x)\mathrm{d}x\right)^n}.\tag{11}
$$

Note that $N(x,t)$ follows Poisson distribution with parameter $\int_x^t\mu(x)\mathrm{d}x$, then the third term in (9) can be given as

$$
P(N(x,t)=n)=\frac{\left(\int_x^t\eta(x)\mathrm{d}x\right)^n\exp\left(-\int_x^t\mu(x)\mathrm{d}x\right)}{n!}.\tag{12}
$$

By (10)−(12), the joint distribution in (9) is given as

$$
\begin{aligned}
&P\left(Y>t-x+\sum_{i=1}^{N(x,t)}Z_i+\beta(t-S_i),N(x,t)=n|X=x\right)=\\
&\int_x^t\cdots\int_x^{s_2}\int_0^\infty\exp\left(-\int_x^t\mu(x)\mathrm{d}x\right)R_Y\left(t-x+\sum_{i=1}^{n}\beta(t-s_i)+z\right)f_Z^{(n)}(z)\mathrm{d}z\prod_{i=1}^{n}\mu(s_i)\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n.
\end{aligned}\tag{13}
$$

If the duration in the defective state in a baseline environment follows exponential distribution with a failure rate η, then by substituting (11)−(13) into (10), we can obtain that

$$
\begin{aligned}
&P\left(Y>t-x+\sum_{i=1}^{N(x,t)}Z_i+\beta(t-S_i),N(x,t)=n|X=x\right)=\\
&\int_x^t\cdots\int_x^{s_2}\int_0^\infty\exp\left(-\int_x^t\mu(x)\mathrm{d}x\right)\exp\left(-\eta\left(t-x+\sum_{i=1}^{n}\beta(t-s_i)+z\right)\right)f_Z^{(n)}(z)\mathrm{d}z\prod_{i=1}^{n}\mu(s_i)\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n=\\
&\int_x^t\cdots\int_x^{s_2}\int_0^\infty\exp\left(-\int_x^t\mu(x)\mathrm{d}x-\eta(t-x)\right)\exp\left(-\eta\left(\sum_{i=1}^{n}\beta(t-s_i)+z\right)\right)f_Z^{(n)}(z)\mathrm{d}z\prod_{i=1}^{n}\mu(s_i)\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n=\\
&\int_x^t\cdots\int_x^{s_2}\int_0^\infty\exp\left(-\int_x^t\mu(x)\mathrm{d}x-\eta(t-x)\right)\exp(-\eta z)f_Z^{(n)}(z)\mathrm{d}z\prod_{i=1}^{n}\mu(s_i)\exp(-\eta(\beta(t-s_i)))\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n.
\end{aligned}\tag{14}
$$

By the property of integral, (14) can be further simplified as

$$P\left(Y > t-x+\sum_{i=1}^{N(x,t)} Z_i+\beta(t-S_i), N(x,t)=n|X=x\right)=\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)\right)\cdot$$
$$\int_x^t\cdots\int_x^t\int_0^\infty \exp(-z) f_Z^{(n)}(z)\mathrm{d}z I_{s_1<s_2<\cdots<s_n}\prod_{i=1}^n \mu(s_i)\exp(-\eta(\beta(t-s_i)))\mathrm{d}s_1\mathrm{d}s_2\cdots\mathrm{d}s_n] =$$
$$\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)\right)\cdot\frac{\int_0^\infty \exp(-\eta z) f_Z^{(n)}(z)\mathrm{d}z\left(\int_x^t \mu(s)\exp(-\eta(\beta(t-s)))\mathrm{d}s\right)^n}{n!}=$$
$$\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)\right)\cdot\frac{\left(\int_x^t\int_0^\infty \mu(s)\exp(-\eta(\beta(t-s)))\exp(-\eta z) f_Z(z)\mathrm{d}z\mathrm{d}s\right)^n}{n!}=$$
$$\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)\right)\cdot\frac{\left(\int_x^t \mu(s)\exp(-\eta(\beta(t-s)))M(\eta)\mathrm{d}s\right)^n}{n!}.$$

Summing all possible values of n, $R_{\tilde{Y}}(t,x)$ can be given as

$$R_{\tilde{Y}}(t,x)=\sum_{n=0}^\infty P\left(Y > t-x+\sum_{i=1}^{N(x,t)} Z_i+\beta(t-S_i), N(x,t)=n|X=x\right)=\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)\right)\cdot$$
$$\sum_{n=0}^\infty\frac{\left(\int_x^t \mu(s)\exp(-\eta(\beta(t-s)))M(\eta)\mathrm{d}s\right)^n}{n!}=\exp\left(-\int_x^t \mu(x)\mathrm{d}x-\eta(t-x)+\int_x^t \mu(s)\exp(-\eta(\beta(t-s)))M(\eta)\mathrm{d}s\right). \tag{15}$$

Remark Since a larger value of virtual age increment corresponds to a lower reliability performance, the upper and lower bounds of $R_s(t)$, $R_s^l(t)$ and $R_s^u(t)$, can be respectively obtained by setting ($Z_i=\infty$, $\beta=\infty$) and ($Z_i=0$, $\beta=0$), as shown below:

$$\begin{cases} R_s^u(t)=R_X(t)+\int_0^t R_Y(t-x)f_X(t)\mathrm{d}t \\ R_s^l(t)=R_X(t)\exp\left(-\int_0^t \mu(x)\mathrm{d}x\right)+\int_0^t R_Y(t-x+\beta t)f_X(x)\mathrm{d}x \end{cases}. \tag{16}$$

4. Optimal inspection policy

This section establishes the cost model under the proposed maintenance policy. Different renewal scenarios are analyzed and the cost performance is evaluated.

4.1 Probabilities of the three renewal scenarios

Considering the proposed maintenance policy, the system may be replaced upon a failure and inspection both periodically and opportunistically. The occurrence probabilities of these three renewal cases are given below.

Case 1 The system is renewed correctively.

The system fails and no production waits occur in the defective state (see Fig. 1 (a)). The probability of such a renewal can be given as

$$P_a(i,T)=P(iT<X, X+Y<(i+1)T, \Lambda(X,X+Y)=0)=\int_{iT}^{(i+1)T} P(Y<(i+1)T-x, \Lambda(x,x+Y)=0|X=x)f_X(x)\mathrm{d}x=$$
$$\int_{iT}^{(i+1)T}\int_x^{(i+1)T} P(\Lambda(x,x+y)=0|X=x)\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x=\int_{iT}^{(i+1)T}\int_x^{(i+1)T}\exp(-\lambda y)\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x. \tag{17}$$

Here $F_Y(y,x)$ is obtained from (15) and given as

$$F_Y(y,x)=P(Y\leqslant y-x|X=x)=1-R_Y(y,x). \tag{18}$$

Based on (17), the probability density of a failure renewal at time $iT+u$ $(0<u<T)$ is calculated as

$$f_a(i,T,u)=\int_{iT}^{iT+u}\exp(-\lambda(iT+u-x))f_Y(iT+u+x,x)f_X(x)\mathrm{d}x. \tag{19}$$

Case 2 The system is renewed by a periodic inspection.

The defect is inspected by a periodic inspection and then removed by PR (see Fig. 1 (b)). The probability can be given as

$$P_b(i,T)=$$
$$P(iT<X<(i+1)T, X+Y>(i+1)T, \Lambda(X,(i+1)T)=0)=$$

$$\int_{iT}^{(i+1)T} P(Y > (i+1)T-x, \Lambda(x,(i+1)T) = 0|X = x) f_X(x)\mathrm{d}x = \int_{iT}^{(i+1)T}\int_{x}^{(i+1)T} P(\Lambda(x,(i+1)T) = 0|X = x)\mathrm{d}F_Y(y,x) f_X(x)\mathrm{d}x = \int_{iT}^{(i+1)T}\int_{(i+1)T}^{\infty} \exp(-\lambda((i+1)T-x))\,\mathrm{d}F_Y(y,x) f_X(x)\mathrm{d}x. \tag{20}$$

Case 3 The system is renewed opportunistically.

In this scenario, the defective state is found and removed by a preventive replacement at an opportunistic inspection (see Fig. 1 (c)).

The number of inspections in a renewal cycle N_I is the sum of periodic and opportunistic inspections respectively denoted by N_I^P and N_I^O. $E(N_I)$ can be given as

$$E(N_I) = E\left(N_I^P\right) + E\left(N_I^O\right) = \sum_{i=0}^{\infty} i(P_a(i,T) + P_b(i,T)) + (i+1)P_c(i,T) + E\left(N_I^O\right). \tag{21}$$

The expectation of the number of opportunistic inspections in a renewal cycle $E(N_I^O)$ is derived by conditioning on the three renewal scenarios, as shown below:

$$E(N_I^O) = \underbrace{\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T} E(\Lambda(0,x+y)|\Lambda(x,x+y)=0)\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x}_{a} + \underbrace{\int_{iT}^{(i+1)T}\int_{(i+1)T}^{\infty} E(\Lambda(0,x+iT)|\Lambda(x,iT)=0)\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x}_{b} + \underbrace{\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T} E(\Lambda(0,s)|\Lambda(x,s)=1)R_Y(s,x)\lambda\exp(-\lambda s)f_X(x)\mathrm{d}x\mathrm{d}s}_{c} + \underbrace{\sum_{n=1}^{\infty}\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T}\frac{E(\Lambda(0,s)|\Lambda(x,s)=1)\lambda^{n+1}s^n}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)R_Y(s,x)\exp(-\lambda s)f_X(x)\mathrm{d}x\mathrm{d}s}_{c}. \tag{22}$$

Equation (22) can be simplified as

$$E(N_I^O) = \int_{iT}^{(i+1)T}\int_{x}^{\infty}\lambda x\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x + \int_{iT}^{(i+1)T}\int_{x}^{(i+1)T}(\lambda s+1)R_Y(s,x)\gamma\exp(-\lambda s)f_X(x)\mathrm{d}x\mathrm{d}s + \sum_{n=1}^{\infty}\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T}\frac{(\lambda s+1)R_Y(s,x)\lambda^{n+1}s^n\exp(-\lambda s)}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)f_X(x)\mathrm{d}x\mathrm{d}s.$$

The expected number of inspections in a renewal cycle can be given as

$$E(N_I) = \sum_{i=0}^{\infty} i(P_a(i,T)+P_b(i,T)) + (i+1)P_c(i,T) + \sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_{x}^{\infty}\lambda x\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x + \sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T}(\lambda s+1)R_Y(s,x)\lambda\exp(-\lambda s)f_X(x)\mathrm{d}x\mathrm{d}s + \sum_{i=0}^{\infty}\sum_{n=1}^{\infty}\int_{iT}^{(i+1)T}\int_{x}^{(i+1)T}\frac{(\lambda s+1)R_Y(s,x)\lambda^{n+1}s^n\exp(-\lambda s)}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)f_X(x)\mathrm{d}x\mathrm{d}s.$$

The probability density of a failure renewal at time $iT+u$ $(0<u<T)$ is calculated as

$$f_c(i,T,u) = \int_{iT}^{iT+u} R_Y(iT+u,x)\lambda\exp(-\lambda(iT+u))f_X(x)\mathrm{d}x + \sum_{n=1}^{\infty}\int_{iT}^{iT+u}\frac{R_Y(iT+u,x)\lambda^{n+1}(iT+u)^n\exp(-\lambda(iT+u))}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)f_X(x)\mathrm{d}x.$$

4.2 Average long-run cost rate

To assess the performance of the proposed maintenance policy, the average cost rate is evaluated as

$$w(T)=\frac{E(C)}{E(L)}. \tag{23}$$

where $E(C)$ and $E(L)$ are the expectations of the total cost and the length of a renewal cycle. $E(L)$ can be obtained based on (19), (20) and (22):

$$E(L)=\sum_{i=0}^{\infty}\int_0^T (iT+s)(f_a(i,T,s)+f_c(i,T,s))\mathrm{d}s+(i+1)TP_b(i,T). \tag{24}$$

The cost incurred in a renewal cycle includes the economic loss in defective state and maintenance cost. The expected economic loss $E(C_L)$ can then be given as

$$E(C_L)=c_l(E(L)-E(X)). \tag{25}$$

The maintenance cost is caused by periodic and opportunistic inspections and replacement. The expected replacement cost can be calculated as

$$E(C_R)=c_CE(N_C)+c_PE(N_P)=\sum_{i=0}^{\infty}c_CP_a(i,T)+c_P(P_b(i,T)+P_c(i,T)). \tag{26}$$

The expected number of inspections in a renewal cycle can be given as

$$E(N_I)=\sum_{i=0}^{\infty}i(P_a(i,T)+P_b(i,T))+(i+1)P_c(i,T)+\sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_x^{\infty}\lambda x\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x+$$
$$\sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_x^{(i+1)T}(\lambda s+1)R_Y(s,x)\lambda\exp(-\lambda s)f_X(x)\mathrm{d}x\mathrm{d}s+$$
$$\sum_{i=0}^{\infty}\sum_{n=1}^{\infty}\int_{iT}^{(i+1)T}\int_x^{(i+1)T}\frac{(\lambda s+1)R_Y(s,x)\lambda^{n+1}s^n\exp(-\gamma s)}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)f_X(x)\mathrm{d}x\mathrm{d}s. \tag{27}$$

Then the expected total cost incurred in a renewal cycle can be expressed as

$$E(C)=\sum_{i=0}^{\infty}c_CP_a(i,T)+c_P(P_b(i,T)+P_c(i,T))+$$
$$c_l\left(\sum_{i=0}^{\infty}\int_0^T(iT+s)(f_a(i,T,s)+f_c(i,T,s))\mathrm{d}s+(i+1)TP_b(i,T)-E(X)\right)+$$
$$c_I\left(\sum_{i=0}^{\infty}i(P_a(i,T)+P_b(i,T))+(i+1)P_c(i,T)+\sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_x^{\infty}\lambda x\exp(-\lambda y)\,\mathrm{d}F_Y(y,x)f_X(x)\mathrm{d}x+\right.$$
$$\sum_{i=0}^{\infty}\int_{iT}^{(i+1)T}\int_x^{(i+1)T}(\lambda s+1)R_Y(s,x)\lambda\exp(-\lambda s)f_X(x)\mathrm{d}s\mathrm{d}x+$$
$$\left.\sum_{i=0}^{\infty}\sum_{n=1}^{\infty}\int_{iT}^{(i+1)T}\int_x^{(i+1)T}\frac{(\lambda s+1)R_Y(s,x)\lambda^{n+1}s^n\exp(-\lambda s)}{n!}\left(1-\sum_{k=0}^{n-1}\frac{(\lambda x)^k\exp(-\lambda x)}{k!}\right)f_X(x)\mathrm{d}s\mathrm{d}x\right). \tag{28}$$

The expected cost rate can then be obtained by using (24) and (28). The analytical optimization of inspection interval is intractable due to the involvement of the complex failure process. The following modified artificial bee colony algorithm is developed for the maintenance cost optimization.

Step 1 Input the distribution parameters related to random variables Z,S,β as well as the cost parameters $c_C,c_P,c_l,\ c_I$.

Step 2 Initialize the maximal iteration number $\hat{N}$, the population size M; set the initial iteration number $N=1$.

Step 3 Generate the initial population T_m $(m=1,\cdots,M/2)$.

Step 4 Calculate the cost rate of the system $w(T_m)$; search the optimal inspection interval T^* and calculate its fitness under the current iterative number N.

Step 5 If the current iteration number $N<\hat{N}$, update population x^* using the Deb's rule, then set $N=N+1$ and go back to Step 4; otherwise go to Step 6.

Step 6 Output the minimum maintenance cost $w(T^*)$

and the corresponding solution T^*.

5. A case study on cooling system

5.1 Background

In this section, numerical illustrations are presented to verify the proposed two-stage degradation model. The cooling system is commonly used to adjust the temperature of the overheated engine within an appropriate range. Since safety-critical systems require a large amount of energy to complete the various tasks, the reliability modelling is crucial to cooling systems. The temperature of the engine will rise sharply due to frictions or other reasons, which results in irreversible damage. The cooling system can disperse redundant heat attached to the cylinder and it works by controlling the outflow of the coolant, which is widely used in industrial systems. Reliability analysis and maintenance optimization of the cooling systems play important roles in the operations management of the nuclear plant. Cooling systems are subject to corrosion due to the impact of wind, thunder and humidity. In addition to internal deterioration, external shocks can accelerate the failure process of cooling systems. According to the ageing rate, the failure process of cooling systems can often be divided into normal and defective stages. To this end, we model the degradation process of cooling systems by a two-stage accelerated virtual age process.

In this numerical study, the time to defective stage of cooling systems without external shocks follows Weibull distributions. That is, $\lambda_X(t)=1.3t^{0.3}$ and $\lambda_Y(t)=1.5t^{0.5}$. The failure rate increments of $\lambda_X(t)$ and $\lambda_Y(t)$ caused by external shocks are respectively $\delta=0.3$ and $\gamma=0.5$. External shocks are modelled by an NHPP whose intensity function is $\eta(t)=\rho t(\rho=0.1)$. In the following section, reliability evaluation and maintenance policy optimization are investigated for the considered cooling systems.

5.2 Reliability prediction

Using (15) and (16), one can obtain the system reliability function and its upper and lower bounds. We can observe in Fig. 2 that the system reliability changes slowly in $[0,0.5]$, but decreases sharply after the time exceeds 0.5. A possible source for this variation is a smaller arrival rate of external shocks at the early stage of operation. When the time is larger than 0.5, the cooling system suffers more frequent shocks, resulting in the fast decrease of system reliability. Fig. 3 indicates that (16) provides relatively precise upper and lower bounds of system reliability. In the following, we test the variation of the reliability function with respect to several degradation parameters.

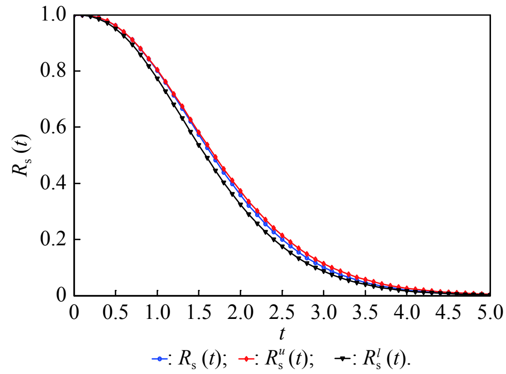

Fig. 2 System reliability function with corresponding lower and upper bounds

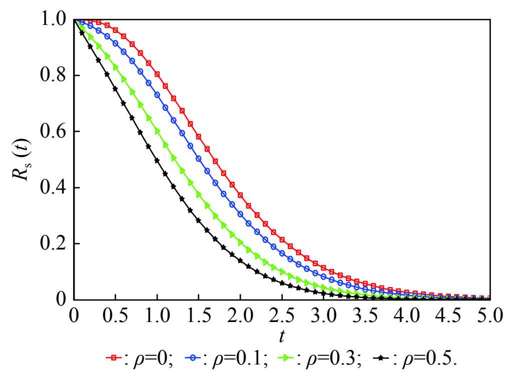

Fig. 3 Variation of $R(t)$ on ρ

In Fig. 3, the variation of the system reliability as a consequence of the arrival rate of shocks is illustrated. When external shocks arrive more frequently, the cooling system suffers more severe damage, resulting in lower system reliability. When $\rho=0$, the system is operated in a baseline environment and achieves the highest reliability. We can observe that the influence of ρ on system failure risk increases at the early stage and decreases when the operation time is large. The external damage is slight at the beginning of operation, thus corresponds to a small impact on system reliability. As the ageing of the system, external impacts are more frequent and a higher damage is caused, lowering system reliability more obviously.

Fig. 4 depicts the influence of δ on system reliability. System reliability is a decreasing function of δ because a larger value of δ corresponds to a higher external damage. Comparison of Fig. 4 and Fig. 5 reveals that γ has a smaller impact on system reliability due to the following reasons: (i) Parameter δ is related to the two stages and γ can only affect the defective stage; (ii) The ageing rate in the defective stage is accelerated.

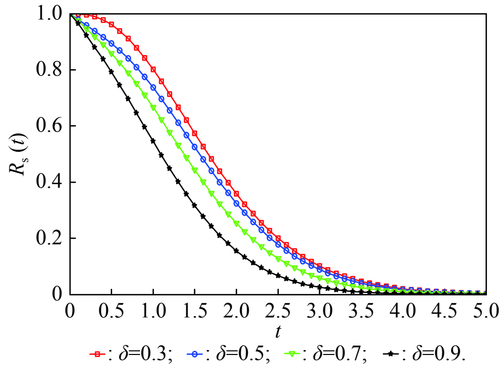

Fig. 4 Sensitivity analysis of $R(t)$ on δ

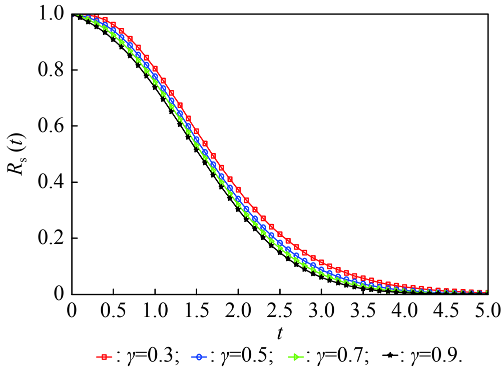

Fig. 5 Sensitivity analysis of $R(t)$ on γ

5.3 Optimal inspection strategy

The cost parameters are given as $c_s = 2\,000$, $c_C = 2\,500$, $c_P = 1\,000$ and $c_I = 300$. The intensity of the arrival process of production waits is $\lambda = 0.5$. Fig. 6 shows the average long-run cost rate when the inspection varies. It can be seen in Fig. 6 that it is optimal to inspect the system every 1.2 time units with the minimal cost rate of 1 794.

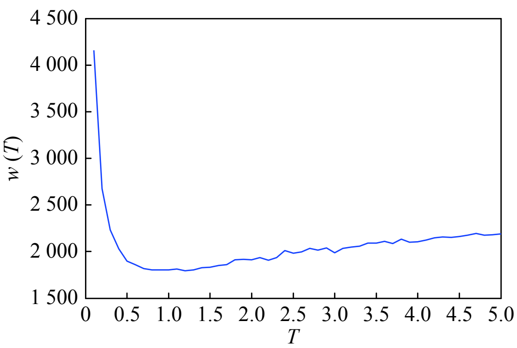

Fig. 6 Cost performance evaluation

Then, the variation of the cost rate regarding λ is studied. When $\lambda = 0$, opportunistic inspection is not considered, leading to a higher penalty cost in the defective stage. The optimal inspection interval should be identified to balance the tradeoff between the cost of the inspection and that incurred by the defective state. It can be observed in Fig. 7 that the optimal inspection frequency decreases when the production waits arrive more frequently as a consequence of the increased opportunistic inspections.

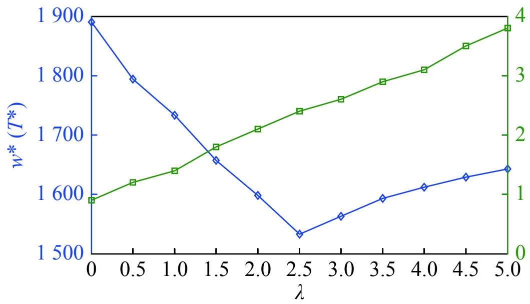

Fig. 7 Sensitivity analysis of the optimal policy on λ

To demonstrate the applicability of the considered maintenance strategy, sensitivity analysis on the cost of corrective replacement c_C is conducted as shown in Fig. 8. We can observe that when c_C increases from 1 500 to 5 000 with step size 500, the minimal cost rate increases significantly, indicating that the cost rate is sensitive to c_C. In contrast, the optimal inspection interval decreases since more timely detection of failures is required when the cost of failures increases.

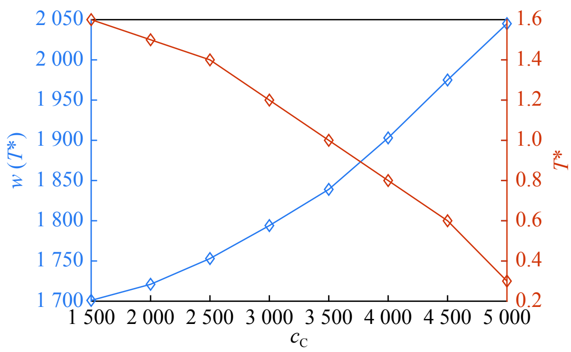

Fig. 8 Sensitivity analysis of the optimal solution on c_C

6. Conclusions and discussions

This paper proposes a two-stage degradation process using the virtual age process, where the failure rate increment caused by external shocks is considered. The system failure process includes normal and defective stages and its ageing rate accelerates in the defective stage. Several reliability indices are derived by using the nonhomogeneous Poisson process and maintenance policies are designed to calculate the minimal cost rate under different renewal scenarios. Finally, numerical illustrations are presented to verify the proposed model.

A number of extensions of the current study are worth investigating. First, it is assumed that time based maintenance is utilized, and one extension of the study is to consider condition-based maintenance [41−44]. Second, it would be beneficial to apply the proposed reliability prediction method and the maintenance policy for mission critical systems, which gives rise to a joint optimization problem where the policies of stopping of a mission and the system maintenance should be identified simultaneously [45−51].

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