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A generalized geometric process based repairable system model with bivariate policy

MA Ning[1,*], YE Jimin[1], and WANG Junyuan[2]

1. College of Mathematics and Statistics, Xidian University, Xi'an 710071, China;
2. College of Sciences, China Jiliang University, Hangzhou 310018, China

Abstract: The maintenance model of simple repairable system is studied. We assume that there are two types of failure, namely type I failure (repairable failure) and type II failure (irrepairable failure). As long as the type I failure occurs, the system will be repaired immediately, which is failure repair (FR). Between the $(n-1)$th and the nth FR, the system is supposed to be preventively repaired (PR) as the consecutive working time of the system reaches $\lambda^{n-1}T$, where λ and T are specified values. Further, we assume that the system will go on working when the repair is finished and will be replaced at the occurrence of the Nth type I failure or the occurrence of the first type II failure, whichever occurs first. In practice, the system will degrade with the increasing number of repairs. That is, the consecutive working time of the system forms a decreasing generalized geometric process (GGP) whereas the successive repair time forms an increasing GGP. A simple bivariate policy (T,N) repairable model is introduced based on GGP. The alternative searching method is used to minimize the cost rate function $C(N,T)$, and the optimal $(T,N)^*$ is obtained. Finally, numerical cases are applied to demonstrate the reasonability of this model.

Keywords: renewal reward theorem, generalized geometric process (GGP), average cost rate, optimal policy, replacement.

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1. Introduction

Power systems and network systems are closely related to the modern life. Once these systems fail, people's lives and work will be paralyzed. In order to improve the system stability, reduce the probability of system failure and upgrade the operation efficiency, we try to find an optimal maintenance model and the corresponding optimal policy.

Take the production line of a food processing plant for example. Once the system fails, the entire plant will stop running. The downtime loss is very huge. Therefore, how to minimize the sudden failure rate of the production line is an urgent problem we have to consider. If an optimal model of the production line and its optimal strategy result in the lowest operation cost or a higher rate of return of a production circle which starts from the work to the replacement of the production line, this will have a great impact on the factory, reducing the production cost. In the increasingly fierce food processing market, cost reduction is an effective strategy to gain competitive advantages and compete against others. Therefore, reducing the cost while maintaining quality and quantity of the products are the important goals pursued by the factory.

Generally, the perfect maintenance model is frequently studied, in which a failed system after repair will be as good as new. However, this does not suit all the systems. Another minimum maintenance model is proposed as well, which means that the system can continue to work after being repaired but the system performance will degrade[1,2]. Moreover, for some systems, it is unrealistic to have a person observe their status for 24 hours. Then, Barlow et al. first came up with a model to detect the status of the system at some specific time points [3]. This model can effectively reduce the system's sudden failure rate, thus reducing the unit cost of the system operation. Nakagawa introduced a model in which the system is periodically checked to decide whether it needs to be replaced, and the optimal detection number that minimizes the system operating cost also is given [4]. Later on, Vaurio put forward a periodic inspection model with preventively repaired (PR) actions for normal operating systems and safety standby systems [5]. Cheng and Li also presented a periodic inspection model based on the geometric process (GP) for simple repairable systems [6]. In their model, the system is supposed to be repaired when its working time reaches T or encounters a failure, and it should be replaced when the number of failures reaches the fixed times N.

In practice, the accumulation of operation times,

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*Corresponding author.
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coupled with the increasing number of failure repair (FR) and the effect of environment, will cause loss to many systems. In other words, the consecutive working time of the system randomly decreases after FR while the repair time interval increases unexpectedly. The GP was proposed in [7,8] to model these monotonic processes. In Lam's study, the system will be replaced when the working time of the system reaches T or the failure number reaches N (only considering repairable failures). The GP is theoretically reasonable and it has also been verified in numerical examples. Lam et al. applied the GP to simulate a real data set. By comparing the GP with the homogeneous Poisson process and two non-homogeneous Poisson processes, it is shown that the result based on the GP is better than the others in the simulation of the data set [9]. The monotonic process modelled by the GP is uniformly decreasing, that is, the expectation of the consecutive working time of the system is uniformly reducing with the increasing number of failures. However, that is unreasonable, because there are many factors that cause the loss of the system to be different each time, such as the influence of the cumulative effect of the failures, the degree of each failure and the difference of each maintenance team, as well as the effect after the repair. Many researchers have been devoted to improving GP. The generalized GP (GGP) was proposed in [10,11]. In GGP, the geometric ratio of every repair is different, which is more reasonable.

Considering the maintenance and replacement of the system, many people have done research on two inspection models, including periodic inspection and random inspection. In [12], Wang considered two types of checks in the delay time setting. Nakagawa et al. summarized the strategies for periodic inspection and random inspection [13]. Chen et al. applied periodic and random inspection strategies to computer systems [14]. Cheng and Li studied the GP maintenance model proposed by Lam. They assumed that when the system failed, it can be detected by inspection [6]. Chen et al. introduced two kinds of failure competition, which are degradation failure and sudden failure. The preventive maintenance is carried out when the system performance level is degraded within a certain range, the degradation failure repair maintenance is carried out when the components are completely degraded, and the sudden failure repair maintenance is performed when the system performance level is in a certain range [15]. In those researches, they dealt with only one failure type. However, it is not enough to consider only one failure type in real life. Sheu et al. introduced a generalized replacement model, which attempts to deal with two failure types. And the system is replaced when the Nth type I failure or the first type II failure occurs, whichever occurs first. And the probability of occurrence of type II failure is related to the number of type I failure that has occurred since the last replacement [16,17].

Based on the above research, it can be found that regular inspection and preventive repair of the running system can effectively reduce the system's sudden failure rate and extend its life. Therefore, it is a meaningful measure to consider preventive repair and FR together. For more information on the application of preventive maintenance, please refer to [18–21]. Considering the effects of FR and cumulative operation of the system, the system is gradually deteriorating. Therefore, the continuous working time of the system decreases randomly and the continuous repair time increases randomly. Compared with GP and extended GP (EGP), GGP are more extensive. More applications of GP and EGP can be found in [22–29]. The GP describes a monotonic process of uniform deterioration, which is obviously more limited. The EGP combines the "perfect repair model" and the deterioration model after repair. Among them, the monotonic process is described by the GP. This model considering two situations of repair, which are "repair as new" and uniform deterioration. While the GGP can describe the monotonic process that its degree of deterioration has gradually increased unevenly. The deterioration process depends on the environment, conditions and effects of each FR, that is, it is reasonable that the degree of deterioration is increasing. With the increasing number of failures, the system becomes worse, and the degree of deterioration becomes larger. Therefore, the GGP can be broader and more reasonable to describe the monotonic process with different degrees of deterioration. In addition, considering two replacement modes, the maintenance model of the system is more abundant.

This paper introduces an optimal policy to solve this problem. We consider the model with two kinds of repair, which are PR and FR. When the system breaks down before the due testing time, it is assumed that there are two types of failure, namely type I and type II. As long as the type I failure occurs, the system will be FR. Between the $(n-1)$th and the nth FR, the system is supposed to be PR as the consecutive working time of the system reaches $\lambda^{n-1}T$, where λ and T are specified values. That is, the time interval of inspection is $\lambda^{n-1}T$, and λ is an indicator meaning that the detection interval is shortened, and T is a parameter. The system will go on working as soon as the repair is finished and will be replaced at the occurrence of the Nth type I failure or the first type II failure, whichever happens first. In practice, the system will degrade with the increasing number of repairs. That is, the consecutive working time of the system forms a decreasing process, while the time interval of repairs forms an increasing process. The GGP is used to illustrate these two processes, and a simple bivariate policy repairable model is introduced based on GGP. The alternative searching method is used to minimize the cost rate function $C(N,T)$, then the optimal $(T,N)^*$ is obtained. Finally, the numerical cases are applied to demonstrate the reasonability of this model.

Section 2 is devoted to the model establishment. Section 3 is devoted to the optimizing of the objective function, and an algorithm is presented to find the optimal solution $(T,N)^*$. Section 4 includes the numerical examples and the conclusions are given in Section 5.

2. Model analysis

2.1 Definitions

We give the definitions of stochastic order, GP and GGP.

Definition 1 Given two random variables X and Y, if for all t, there is

$$P(X \geqslant t) > P(Y \geqslant t),$$

then X is said to be stochastically larger than Y, which is denoted as $X >_{st} Y$ or $X <_{st} Y$ [30].

Definition 2 Given a random process $\{M_k, k = 1, 2, \cdots\}$, for all n, there is

$$M_k \leqslant_{st} (\geqslant_{st}) M_{k+1}.$$

Then, $\{M_k, k = 1, 2, \cdots\}$ is called the randomly increasing (decreasing) process.

Definition 3 A sequence of non-negative random variables $\{M_k, k \geqslant 1\}$ is said to be a GP with ratio $a > 0$, if

(i) the random variables are mutually independent;

(ii) for all integers $k \geqslant 1$, the cumulative distribution function (CDF) of M_k is $Q_k(t) = Q(a^{k-1}t)$ and $Q_1(t) = Q(t)$.

When $a > 1$, $\{M_k, k \geqslant 1\}$ is a decreasing GP; When $a < 1$, $\{M_k, k \geqslant 1\}$ is an increasing GP; When $a = 1$, $\{M_k, k \geqslant 1\}$ degenerates to a renewal process.

The ratio a is a positive constant, in the maintenance model, which means that the system uniformly deteriorates after FR. In fact, the loss rate of the system varies with the increasing number of FR and other factors.

Definition 4 A sequence of non-negative random variables $\{M_k, k \geqslant 1\}$ is called the GGP with ratios a_1, $a_2, \cdots$, if

(i) the random variables are mutually independent;

(ii) for all integers $k \geqslant 1$, $a_k > 0$;

(iii) for all integers $k \geqslant 1$, the CDF of M_k is $Q_k(t) = Q(A_k t)$, and $Q_1(t) = Q(t)$, $A_k = a_0 \cdot a_1 \cdot a_2 \cdots a_{k-1}$, $a_0 = 1$.

As defined above, for all integers $k \geqslant 1$, $a_k > 0$. When $a_k > 1$, $\{M_k, k \geqslant 1\}$ is a decreasing GGP, a typical assumption is that $1 = a_0 \leqslant a_1 \leqslant a_2 \leqslant \cdots$, that is, the loss rate of the system after each FR is increment; when $a_k < 1$, $\{M_k, k \geqslant 1\}$ is an increasing GGP, the assumption is that, $1 = a_0 \geqslant a_1 \geqslant a_2 \geqslant \cdots$, implies that the system deteriorates more and more rapidly with the increasing number of FR; specially, when $a_1 = a_2 = \cdots = a_{k-1} = a$, then the GGP degenerates into a GP.

2.2 Model assumptions

The maintenance model of the system is based on the following assumptions.

Assumption 1 A new system is installed at $t = 0$. The process for a system from being newly installed to being replaced is called a cycle. The process from the new installation to the completion of the first FR is called the first period of a cycle. The process from the completion of the $(n-1)$th FR to the completion of the nth FR is called the nth period in a cycle, and the process from the completion of the nth FR to the replacement is called the Nth period in a cycle .

Assumption 2 In the first period of a cycle, the inspection interval of the system after FR is T. Because the successive working time is a decrease process with the period index, we assume that the inspection interval of the nth period is $\lambda^{n-1}T$, where $0 < \lambda < 1$. In a period, after PR, the system is "good as it is before PR". In any period of a cycle, when the consecutive working time of the system reaches $\lambda^{n-1}T$, the worker will detect the system and perform PR. When the consecutive working time of the system is less than $\lambda^{n-1}T$, a failure occurs, the worker will carry out FR to the system, and the system enters the next period.

Assumption 3 The system is replaced by an identical new one at the occurrence of the Nth type I failure or the first type II failure, whichever occurs first. Type II failure may occur in any period of a cycle. As shown in Fig. 1 and Fig. 2.

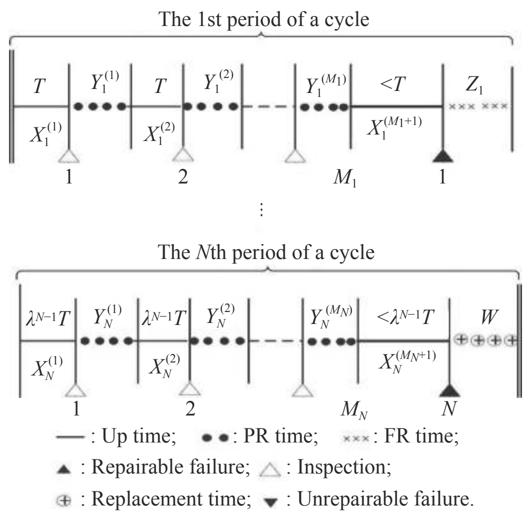

Fig. 1 System replaced when the number of type I failure reaches N

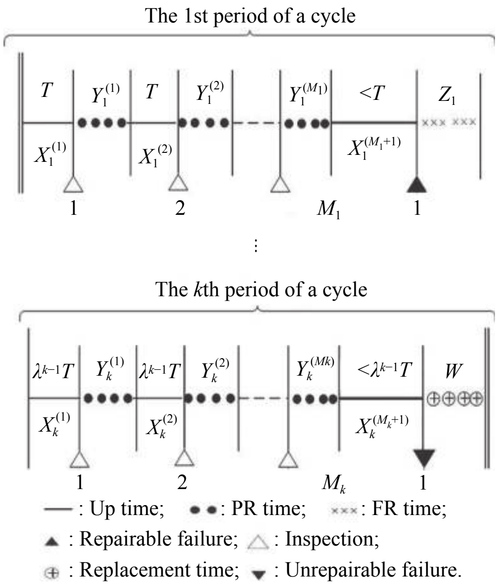

Fig. 2 System replaced at the occurrence of type II failure

Assumption 4 $X_n^{(i)}$ means the ith consecutive working time in the nth period of a cycle, $Y_n^{(j)}$ means the jth PR time in the nth period of a cycle, in a period, after the PR, the system is "good as it is before PR", that is, $\{X_n^{(i)}, i = 1,2,\cdots,M_n+1\}$ is a sequence of independent and identically distributed random variables; due to the same reason, $\left\{Y_n^{(j)}, j = 1,2,\cdots,M_n\right\}$ is also a sequence of independent and identically distributed random variables. M_n is the number of detection in the nth period.

Assumption 5 X_n, Y_n and Z_n denote the total working time, the total PR time and the FR time in the nth period of system, respectively. $X_n = \sum_{i=1}^{M_n+1} X_n^{(i)}$ and $Y_n = \sum_{j=1}^{M_n} Y_n^{(j)}$; $X_n (n = 1,2,\cdots)$, $Y_n (n = 1,2,\cdots)$, and $Z_n (n = 1,2,\cdots)$ all are mutually independent series. And X_n, Y_n, Z_n as well as the replacement time W_0 are also mutually independent.

Assumption 6 C_m is the cost of one inspection. r is the reward of the system working per unit time. C_p is the cost of PR per unit time. C_f is the cost of FR per unit time. d is the fixed cost of replacement. C_τ is the cost per unit time of replacement.

Assumption 7 The CDF of X_n, Y_n and Z_n can be formulated as below respectively:

$$F_n(t) = F(A_n t),$$

$$H_n(t) = H(B_n t),$$

$$G_n(t) = G(C_n t),$$

where $F_1(t) = F(t)$, $H_1(t) = H(t)$ and

$$A_n = \prod_{i=1}^{n-1} a_i,\ B_n = \prod_{i=1}^{n-1} b_i,\ C_n = \prod_{i=1}^{n-1} c_i,$$

where a_i denotes the geometric ratio of the ith FR about consecutive working time; b_i denotes the geometric ratio of the ith FR about consecutive PR time; c_i denotes the geometric ratio of the ith FR about the consecutive FR time. $a_i > 1, 0 < b_i, c_i < 1$ and $b_i < c_i$, because the impact of system degradation on PR is less than the impact on FR. $\mathrm{E}(X_1) = \lambda_1$, $\mathrm{E}(Y_1) = \mu_p$, $\mathrm{E}(Z_1) = \mu_f$.

2.3 Model analysis

The system has two replacement modes. In the first one, the system is replaced at occurrence of the Nth type I failure, and in the second one, the system is replaced at the occurrence of the first type II failure. Let L be the total number of failures (including type II failure) until the type II failure occurs. Let $\bar{P}_k = P(L < k)$ denote the probability that the first k occurred failures are type I failures, where $1 = \bar{P}_0 > \bar{P}_1 > \bar{P}_2 > \cdots$, further

$$P(L = k) = P(L > k-1) - P(L > k) =$$

$$\bar{P}_{k-1} - \bar{P}_k = \bar{P}_{k-1}\left(1 - \frac{\bar{P}_k}{\bar{P}_{k-1}}\right).$$

Therefore, when the kth failure occurs, it is classified into either a type I failure with probability $\eta_k = \bar{P}_k / \bar{P}_{k-1}$ or a type II failure with probability $\theta_k = 1 - \eta_k$. And $P(L > N)$ is the probability of the first replacement mode; $P(L = k), k \leqslant N$ is the probability of the second replacement mode.

The objective function of the maintenance model is the long-run average cost per unit of operating time $C(N,T)$, according to the updated reward theorem, the long-run average cost per unit of operating time is the average cost per unit time of a cycle. Hence, only consider one cycle. Thus

$$C(N,T) = \frac{\text{total expected costs in a renewal cycle}}{\text{expected operating time in a renewal cycle}}.$$

Let W be the operating time in one cycle, W_{I} be the operating time in the first replacement mode, and W_{II} be the operating time in the second replacement mode. Let R be the cost of the system in one cycle, R_{I} be the cost in the first replacement mode, and R_{II} be the cost in the second replacement mode. From Fig. 1 and Fig. 2, it is easy to obtain

$$W_{\mathrm{I}} = \sum_{n=1}^{N} X_n + \sum_{n=1}^{N}\sum_{j=1}^{M_n} Y_n^{(j)} + \sum_{n=1}^{N-1} Z_n + W_0,$$

$$W_{\mathrm{II}} = \sum_{n=1}^{k} X_n + \sum_{n=1}^{k}\sum_{j=1}^{M_n} Y_n^{(j)} + \sum_{n=1}^{k-1} Z_n + W_0.$$

According to the full probability formula, the total operating time of the system in a cycle is

$$W = W_{\mathrm{I}} I_{L>N} + \sum_{k=1}^{N} W_{\mathrm{II}} I_{L=k}.$$

The expectation of W is

$$\mathrm{E}(W) = \mathrm{E}(W_{\mathrm{I}})\bar{P}_N + \sum_{k=1}^{N} \mathrm{E}(W_{\mathrm{II}})\left(\bar{P}_{k-1} - \bar{P}_k\right) = \mathrm{E}\left[\sum_{n=1}^{N} X_n + \sum_{n=1}^{N}\sum_{j=1}^{M_n} Y_n^{(j)} + \sum_{n=1}^{N-1} Z_n + W_0\right]\bar{P}_N + \sum_{k=1}^{N} \mathrm{E}\left[\sum_{n=1}^{k} X_n + \sum_{n=1}^{k}\sum_{j=1}^{M_n} Y_n^{(j)} + \sum_{n=1}^{k-1} Z_n + W_0\right]\left(\bar{P}_{k-1} - \bar{P}_k\right). \quad (1)$$

The cost of system operating is

$$R_{\mathrm{I}} = C_p \sum_{n=1}^{N}\sum_{j=1}^{M_n} Y_n^{(j)} + C_f \sum_{n=1}^{N-1} Z_n + C_m \sum_{n=1}^{N} M_n + d + C_\tau W_0 - r\sum_{n=1}^{N} X_n,$$

$$R_{\mathrm{II}} = C_p \sum_{n=1}^{k}\sum_{j=1}^{M_n} Y_n^{(j)} + C_f \sum_{n=1}^{k-1} Z_n + C_m \sum_{n=1}^{k} M_n + d + C_\tau W_0 - r\sum_{n=1}^{k} X_n.$$

According to the full probability formula, the total cost of the system in a cycle is

$$R = R_{\mathrm{I}} I_{L>k} + \sum_{k=1}^{N} R_{\mathrm{II}} I_{L=k}.$$

The expectation of R is

$$\mathrm{E}(R) = \mathrm{E}(R_{\mathrm{I}})\bar{P}_N + \sum_{k=1}^{N} \mathrm{E}(R_{\mathrm{II}})\left(\bar{P}_{k-1} - \bar{P}_k\right) = \mathrm{E}\left(C_p \sum_{n=1}^{N}\sum_{j=1}^{M_n} Y_n^{(j)} + C_f \sum_{n=1}^{N-1} Z_n + C_m \sum_{n=1}^{N} M_n + d + C_\tau W_0 - r\sum_{n=1}^{N} X_n\right)\bar{P}_N + \sum_{k=1}^{N} \mathrm{E}\left(C_p \sum_{n=1}^{k}\sum_{j=1}^{M_n} Y_n^{(j)} + C_f \sum_{n=1}^{k-1} Z_n + C_m \sum_{n=1}^{k} M_n + d + C_\tau W_0 - r\sum_{n=1}^{k} X_n\right)\left(\bar{P}_{k-1} - \bar{P}_k\right). \quad (2)$$

Hence, the long-run average cost per unit of operating time is

$$C(N,T) = \frac{\mathrm{E}(R)}{\mathrm{E}(W)}. \quad (3)$$

Based on the definition of PR, in the nth period of a cycle, when the system continuously working time reaches $\lambda^{n-1}T$, detect the system. Let M_n be the number of detection that is the number of variables whose lifetime is greater than $\lambda^{n-1}T$ in series $\{X_n^{(i)}, i = 1,2,\cdots,M_n\}$, M_n obeys a geometric distribution

$$P(M_n = k) = P(X_n^{(1)} > \lambda^{n-1}T, X_n^{(2)} > \lambda^{n-1}T, \cdots, X_n^{(k)} > \lambda^{n-1}T, X_n^{(k+1)} \leqslant \lambda^{n-1}T) = P(X_n^{(1)} > \lambda^{n-1}T)P(X_n^{(2)} > \lambda^{n-1}T)\cdots P(X_n^{(k)} > \lambda^{n-1}T)P(X_n^{(k+1)} \leqslant \lambda^{n-1}T) = [1 - F_n(\lambda^{n-1}T)]^k F_n(\lambda^{n-1}T) = q_n^k p_n$$

where $p_n = F_n(\lambda^{n-1}T)$ and $q_n = 1 - F_n(\lambda^{n-1}T)$, the expectation of M_n is

$$\mathrm{E}(M_n) = \sum_{k=0}^{\infty} k q_n^k p^n = \frac{q_n}{p_n}. \quad (4)$$

Let X_n be the total working time in the nth period of a cycle,

$$X_n = M_n \lambda^{n-1} T + X_n^{M_n+1}.$$

Then, the expectation of X_n is

$$\mathrm{E}(X_n) = \lambda^{n-1} T \mathrm{E}(M_n) + \mathrm{E}(X_n^{M_n+1}). \quad (5)$$

Next given the CDF of $X_n^{M_n+1}$,

$$F_n(X_n^{M_n+1}) = P(X_n^{M_n+1} < t | X_n^{M_n+1} \leqslant \lambda^{n-1}T) = \frac{F_n(t)}{F_n(\lambda^{n-1}T)}.$$

The expectation of $X_n^{M_n+1}$ is

$$\mathrm{E}(X_n^{M_n+1}) = \int_0^{\lambda^{n-1}T} t \mathrm{d}\left(\frac{F_n(t)}{F_n(\lambda^{n-1}T)}\right). \quad (6)$$

According to the double expectation formula, the mean of $Y_n^{(j)}$ is

$$\mathrm{E}\left(\sum_{j=1}^{M_n} Y_n^{(j)}\right) = \mathrm{E}(M_n)\mathrm{E}(Y_n^{(j)}) = \frac{\mu_p}{B_n}\frac{q_n}{p_n}. \quad (7)$$

The mean of Z_n and W_0 is

$$\mathrm{E}(Z_n) = \frac{\mu_f}{C_n} \quad \mathrm{E}(W_0) = \mu_\tau. \quad (8)$$

According to (3)−(8), we obtain

$$C(N,T)=\frac{\mathrm{E}(R)}{\mathrm{E}(W)}=\frac{C_p\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+C_f\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+C_m\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}+d+C_\tau\mu_\tau-r\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}}{\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}+\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau}. \quad (9)$$

3. Optimization

3.1 Replacement policy

Lam researched two kinds of univariate replacement policies [7,8]. One is policy T, where the system is replaced when its operating time reaches T. The other is policy N, where the system is replaced when its failure number reaches N. Zhang introduced a bivariate policy (N,T), in which the system is replaced at the operating age T or at the time of the Nth type II failure occurs, whenever occurs first [30]. It is showed that the policy (N,T) is better than both policy T and policy N. This paper aims to find the optimal $(T,N)^*$ by minimizing $C(N,T)$.

First, for any fixed $T_0>0$, we can find N_1 that makes $C(N,T_0)$ the smallest, N is a countable discrete variable, N_1 satisfies the following formula:

$$C(N+1,T)>C(N,T)\ \&\ C(N,T)<C(N-1,T)\Leftrightarrow$$
$$L(N,T)\geqslant d+C_\tau\mu_\tau \& L(N-1,T)<d+C_\tau\mu_\tau \quad (10)$$

where

$$L(N,T)=\frac{C_pL_1+C_fL_2+C_mL_3-rL_4}{\lambda(N+1,T)\bar{P}_N+\frac{\mu_p}{B_{N+1}}\frac{q_{N+1}}{p_{N+1}}\bar{P}_N+\frac{\mu_f}{C_N}\bar{P}_N}$$

where

$$L_1=\frac{\mu_p}{B_{N+1}}\frac{q_{N+1}}{p_{N+1}}\bar{P}_N\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau\right)-\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}\left(\lambda(N+1,T)\bar{P}_N+\frac{\mu_f}{C_N}\bar{P}_N\right)$$

$$L_2=\frac{\mu_f}{C_N}\bar{P}_N\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}+\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\mu_\tau\right)-\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k\left(\lambda(N+1,T)\bar{P}_N+\frac{\mu_p}{B_{N+1}}\frac{q_{N+1}}{p_{N+1}}\bar{P}_N\right)$$

$$L_3=\frac{q_{N+1}}{p_{N+1}}\bar{P}_N\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}+\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau\right)-\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k\left(\lambda(N+1,T)\bar{P}_N+\frac{\mu_p}{B_{N+1}}\frac{q_{N+1}}{p_{N+1}}\bar{P}_N+\frac{\mu_f}{C_N}\bar{P}_N\right)$$

$$L_4=\lambda(N+1,T)\bar{P}_N\left(\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau\right)-\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}\left(\frac{\mu_p}{B_{N+1}}\frac{q_{N+1}}{p_{N+1}}\bar{P}_N+\frac{\mu_f}{C_N}\bar{P}_N\right).$$

N_1 found in the above formula satisfies

$$C(N_1,T_0)=\min_N C(N,T_0).$$

Second, let $N=N_1$, we can find T_1 that makes $C(N_1,T)$ the smallest. T is a continuous random variable, and $C(N_1,T)$ is differentiated with respect to T, then T satisfies the following formula:

$$\partial C(N,T)/\partial T=0\Leftrightarrow K(N,T)=d+C_\tau\mu_\tau \quad (11)$$

where

$$K(N,T)=\frac{\left[C_p\left(\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}\right)'+C_m\left(\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}\right)'-r\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}\right)'\right]\cdot\left[\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}\right)+\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau\right]}{\left(\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}\right)'+\left(\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}\right)'-\left(C_p\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+C_f\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+C_m\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}-r\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}\right)}$$

where $(\cdot)'$ denotes the operator $\partial(\cdot)'/\partial T$.

Differentiating $C(N_1,T)$ with respect to T twice, we obtain

$$\frac{\partial^2 C(N,T)}{\partial T^2}=\frac{\begin{array}{c}(C_pP''+C_mM''-r\Lambda'')(\Lambda+P+F+\mu_\tau)^2-(C_pP+C_mM+C_fF-r\Lambda+d+\mu_\tau C_\tau)(P''+\Lambda'')(\Lambda+P+F+\mu_\tau)-\\2(C_pP'+C_mM'-r\Lambda')(P'+\Lambda')(\Lambda+P+F+\mu_\tau)+2(C_pP+C_mM+C_fF-r\Lambda+d+\mu_\tau C_\tau)(P'+\Lambda')^2\end{array}}{(\Lambda+P+F+\mu_\tau)^3} \quad (12)$$

where

$$P=\sum_{k=1}^{N}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1},$$

$$F=\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k,$$

$$M=\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1},$$

$$\Lambda=\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}.$$

$(\cdot)'$ and $(\cdot)''$ denote the operator $\partial(\cdot)/\partial T$ and $\partial^2(\cdot)/\partial T^2$ respectively.

When $\partial^2 C(N,T)/\partial T\geqslant 0$, it can prove that the existence of T_1 makes $C(N_1,T_1)=\min_T C(N_1,T)$. If $T_0=T_1$, then (N_1,T_1) is the optimal solution to the model. Let $(N,\ T)^*=(N_1,T_1)$.

Otherwise, let $T=T_1$ and repeat the above steps to find an N_2 that satisfies (10). Then, let $N=N_2$ and find a T_2 that satisfies (11), if $T_2=T_1$, then, (N_2,T_2) is the optimal solution to the model. Let $(N,\ T)^*=(N_2,T_2)$.

Otherwise, continue to repeat the above steps. Until the alternately updated sequence (N_k,T_k) stops or the amount of update reaches the set threshold. The alternative search algorithm for solving the optimal strategy is summarized as Algorithm 1. In practice, Algorithm 1 can always find the optimal solution to the model, $(N,\ T)^*=(N_k,T_k)$. In summary, $(N,\ T)^*$ is the optimal solution to $C(N,T)$, that is, the optimal replacement strategy [29].

$$C((N,\ T)^*)=\min\left[\min_N[C(N,T^*)],\min_T[C(N^*,T)]\right] \quad (13)$$

Algorithm 1

Input $C_m,C_p,C_f,C_\tau,r,d,\lambda,\lambda_1,\mu_p,\mu_f,\mu_\tau,q,k,\alpha$.

Step 1 Let $a_k=1+0.05(k-1)$, $b_k=1-0.05(k-1)$, $c_k=1-0.07(k-1)$.

Step 2 Let $k=1$.

Step 3 Let $T=T_0$, put it into (10), obtain N_k.

Step 4 Let $N=N_k$, put it into (11), obtain T_k.

Step 5 If $T_k=T_{k-1}$, then go to Step 6; otherwise, let $k=k+1$, and go to Step 3.

Step 6 $(N^*,\ T^*)=(N_k,\ T_k)$.

Output (N^*,T^*) and $C(N^*,T^*)$.

Stop

3.2 Special cases

Case 1 If $\bar{P}_k=1\ (k=0,1,2,\cdots)$, that is, the probability of the occurrence of type II failure is zero, hence, the system will be replaced only at the time of the Nth type I failure, then we obtain the following result:

$$C(N,T)=\frac{C_p\sum_{n=1}^{N}\frac{\mu_p}{B_n}\frac{q_n}{p_n}+C_f\sum_{n=1}^{N-1}\frac{\mu_f}{C_n}+C_m\sum_{n=1}^{N}\frac{q_n}{p_n}+(d+C_\tau\mu_\tau)-r\sum_{n=1}^{N}\lambda(n,T)}{\sum_{n=1}^{N}\lambda(n,T)+\sum_{n=1}^{N}\frac{\mu_p}{B_n}\frac{q_n}{p_n}+\sum_{n=1}^{N-1}\frac{\mu_f}{C_n}+\mu_\tau}.$$

Case 2 If the time of PR, FR and replacement are negligible, the fixed cost separately are C_0,C_1, and d, then the model degenerates into

$$C(N,T)=\frac{C_0\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}+C_1\sum_{k=1}^{N-1}\bar{P}_k+C_m\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}+d-r\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}}{\sum_{k=1}^{N}\lambda(k,T)\bar{P}_{k-1}}$$

Case 3 If $N\to\infty$, then $\bar{P}_N\to 0$, that is, the system is replaced only at the time of the occurrence of the first type II failure, then the model degenerates into

$$C(\infty,T)=\lim_{N\to\infty}C(N,T)=\frac{C_p\sum_{k=1}^{\infty}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+C_f\sum_{k=1}^{\infty}\frac{\mu_f}{C_k}\bar{P}_k+C_m\sum_{k=1}^{\infty}\frac{q_k}{p_k}\bar{P}_{k-1}+d+C_\tau\mu_\tau-r\sum_{k=1}^{\infty}\lambda(k,T)\bar{P}_{k-1}}{\sum_{k=1}^{\infty}\lambda(k,T)\bar{P}_{k-1}+\sum_{k=1}^{\infty}\frac{\mu_p}{B_k}\frac{q_k}{p_k}\bar{P}_{k-1}+\sum_{k=1}^{\infty}\frac{\mu_f}{C_k}\bar{p}_k+\mu_\tau}.$$

Case 4 If $T\to\infty$, then there is no inspection and PR. Only when the system fails, perform FR for the system. And $\lim_{T\to\infty}F_n(\lambda^{n-1}T)=1$, $\mathrm{E}(X_n)=\frac{\lambda_1}{A_n}$, Then, the model is degenerated into

$$C(N,\infty)=\lim_{T\to\infty}C(N,T)=$$

$$\frac{C_f\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+d+C_\tau\mu_\tau-r\sum_{k=1}^{N}\frac{\lambda_1}{A_k}\bar{P}_{k-1}}{\sum_{k=1}^{N}\frac{\lambda_1}{A_k}\bar{P}_{k-1}+\sum_{k=1}^{N-1}\frac{\mu_f}{C_k}\bar{P}_k+\mu_\tau}.$$

4. Numerical example

Assume that the CDF of operating time $X_n^{(i)}$ for any i is exponential, that is,

$$F_n(t)=F(A_nt)=1-\mathrm{e}^{-\frac{A_n}{\lambda_1}t},\ t\geqslant 0;\lambda_1>0;\ a_0=1;$$
$$a_k>1,\ k=1,2,\cdots,n-1;\ n=1,2,\cdots.$$

Then,

$$p_k=F_k(\lambda^{k-1}T)=1-\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T},$$
$$q_k=\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T},$$

$$\lambda(k,T)=\frac{1}{F_n(\lambda^{k-1}T)}\int_0^{\lambda^{k-1}T}\bar{F}_k(t)\,\mathrm{d}t=$$
$$\frac{\int_0^{\lambda^{k-1}T}\mathrm{e}^{-\frac{A_k}{\lambda_1}t}\mathrm{d}t}{1-\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T}}=\frac{\lambda_1}{A_k}.$$

Then (9) is given by

$$C(N,T)=\frac{C_p\mu_p\gamma+C_f\mu_f\beta+C_m\varphi+d+\mu_\tau C_\tau-r\phi}{\phi+\mu_p\gamma+\mu_f\beta+\mu_\tau}\tag{14}$$

where

$$\gamma=\sum_{k=1}^{N}\frac{1}{B_k}\frac{q_k}{p_k}=\sum_{k=1}^{N}\frac{1}{B_k}\frac{\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T}}{1-\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T}},$$

$$\beta=\sum_{k=1}^{N-1}\frac{\bar{P}_k}{C_k},$$

$$\varphi=\sum_{k=1}^{N}\frac{q_k}{p_k}\bar{P}_{k-1}=\sum_{k=1}^{N}\bar{P}_{k-1}\frac{\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T}}{1-\mathrm{e}^{-\frac{A_k}{\lambda_1}\lambda^{k-1}T}},$$

$$\phi=\sum_{k=1}^{N}\bar{P}_{k-1}\lambda(k,T)=\sum_{k=1}^{N}\bar{P}_{k-1}\frac{\lambda_1}{A_k}.$$

Now, let $a_n=1+0.05(n-1),b_n=1-0.05(n-1)$, $c_n=1-0.07(n-1)\lambda$=0.93, λ_1= 300, μ_p= 2, $\mu_f=4$, $\mu_\tau=30$, $C_m=80$, $r=400$, $C_p=100$, $C_f=200$, $d=3\,000$, $C_\tau=150$, specially, $\bar{P}_k=q^{k^\alpha}$, shown in Table 1.

Table 1 Optimal value (×10^4) of $C(N,T)$ on different q and α

q	$\alpha=0.5$			$\alpha=1$		
	N^*	T^*	$C(N^*,T^*)$	N^*	T^*	$C(N^*,T^*)$
q=0.1	6	22	−1.5700	1	2	−0.5119
q=0.2	7	26	−0.8441	3	18	−1.2321
q=0.3	2	7	−0.9706	2	7	−0.9706
q=0.4	3	10	−7.8686	4	21	−6.0050
q=0.5	5	19	−5.9212	4	19	−1.6152
q=0.6	5	18	−3.4665	14	20	−1.2975
q=0.7	7	31	−5.3722	6	30	−2.4538
q=0.8	**4**	**11**	**−9.0739**	5	19	−20.7206
q=0.9	8	37	−54.4648	3	7	−57.3538
q=1	4	10	−7.8596	4	10	−7.8596

We can see the influence of different q and α on $C(N,T)$. $\alpha=1$ means that the probability of the occurrence of type I does not depend on the number of failures since the last replacement. When $q=1$, then $\bar{P}_k=1$, that is, Case 1 in the last section.

Set $q=0.8$ and $\alpha=0.5$, shown as Fig. 3 and Table 2, we can find when $(N,T)^*=(4,11)$. The optimal value $C(N^*,T^*)$ is -9.0739, which means that the system should be replaced when the time interval of inspection in the first period of a cycle is 11 and the times of FR reaches 4. Then the optimal expected net cost rate is -9.0739×10^4 per unit time, that is the net profit is 9.0739×10^4 per unit time. Fig. 4 and Fig. 5 are the cases that T and N take different values. And the situations in Figs. 3−5 where there is a sharp decrease in the value of $C(N,T)$ may be because of parameter values of the example or the properties of this model.

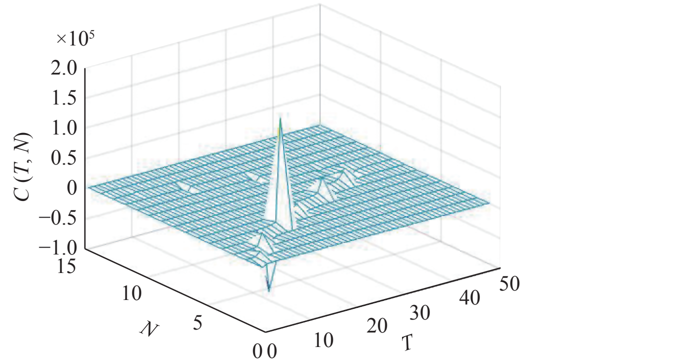

Fig. 3 Plot of $C(N,T)$, $N=[0,5,15]$, $T=[0,10,50]$

Table 2 Some results of $C(N,T)$ with $q=0.8$, $\alpha=0.5$

N	T								
	5	7	9	11	13	20	30	40	50
1	−0.0633	−0.0525	−0.0477	−0.0450	−0.0432	−0.0402	−0.0384	−0.0376	−0.0371
2	−0.2951	−0.1053	−0.0750	−0.0626	−0.0559	−0.0457	−0.0409	−0.0387	−0.0375
3	0.1188	2.3626	−0.2624	−0.1137	−0.0856	−0.0559	−0.0451	−0.0408	−0.0385
4	0.0488	0.0878	0.2070	**−9.0739**	−0.2520	0.0970	−0.0524	−0.0441	−0.0401
5	0.0305	0.0427	0.0618	0.0962	0.1759	−0.1781	−0.0682	−0.0501	−0.0426
6	0.0223	0.0277	0.0346	0.0440	0.0573	0.2652	−0.1193	−0.0621	−0.0466
7	0.0178	0.0205	0.0236	0.0275	0.0322	0.0630	2.9528	−0.0935	−0.0528
8	0.0151	0.0164	0.0180	0.0197	0.0217	0.0326	0.0834	−0.2612	−0.0576
9	0.0133	0.0139	0.0146	0.0153	0.0162	0.0205	0.0366	0.4738	−0.0355
10	0.0119	0.0120	0.0121	0.0122	0.0123	0.0130	0.0163	−0.0014	0.0078
11	0.0104	0.0099	0.0092	0.0083	0.0072	−0.0025	0.0449	0.0239	0.0205
12	0.0072	0.0041	−0.0015	−0.0142	−0.0732	0.0353	0.0246	0.0222	0.0211
13	−0.0524	0.0554	0.0337	0.0283	0.0259	0.0227	0.0213	0.0208	0.0205
14	0.0241	0.0225	0.0217	0.0213	0.0211	0.0206	0.0203	0.0202	0.0201
15	0.0205	0.0203	0.0202	0.0202	0.0201	0.0201	0.0200	0.0200	0.0200

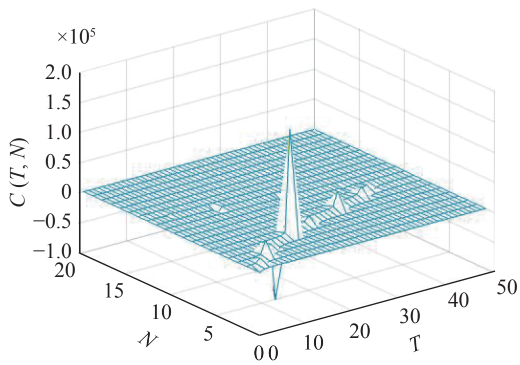

Fig. 4 Plot of $C(N,T)$, $N=[0,5,20]$, $T=[0,10,50]$

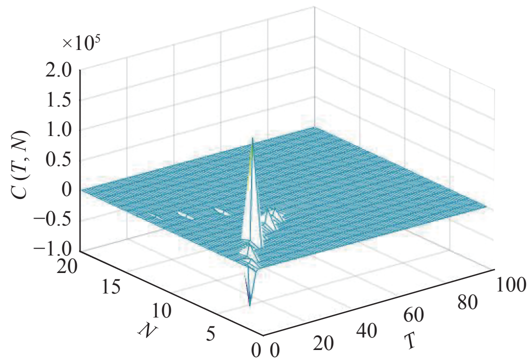

Fig. 5 Plot of $C(N,T)$, $N=[0,5,20]$, $T=[0,20,100]$

When $q = 1$, this model is degenerated into the model of special case 1. Shown as Fig. 6 and Table 3, we can find when $(N,T)^* = (4,10)$, the optimal value $C(N^*,T^*)$ is −7.859 6. Other special cases have similar results.

Form Table 4 we can see that the mean of X_n is randomly decreasing with the number of PR. While the means of Y_n and Z_n are randomly increasing with the number of PR. And it is clear that $\mathrm{E}(X_n)$ is less than $\mathrm{E}(Z_n)$, when the number of PR is greater than or equal to 9. Hence, there is no need to repair the system when the number of PR is greater than 9.

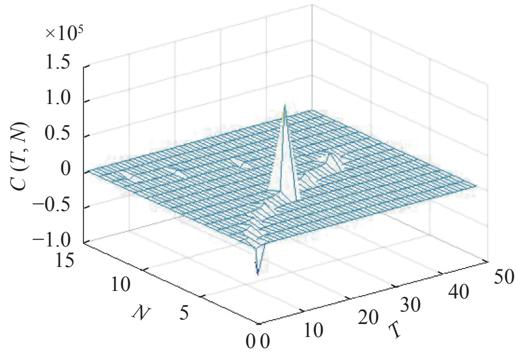

Fig. 6 Plot of $C(N,T)$ with $q = 1$, $N = [0,5,15]$, $T = [0,10,50]$

Table 3 Some results of $C(N,T)$ with $q = 1$

N	T							
	5	9	10	11	21	31	41	50
1	−0.063 3	−0.047 7	−0.046 2	−0.045 0	−0.039 9	−0.038 3	−0.037 5	−0.037 1
2	−0.199 8	−0.070 6	−0.064 6	−0.060 3	−0.045 1	−0.041 1	−0.039 2	−0.038 2
3	0.171 7	−0.151 6	−0.117 9	−0.099 1	−0.053 4	−0.045 0	−0.041 5	−0.039 7
4	0.058 1	0.434 4	**−7.859 6**	−0.446 3	−0.070 4	−0.051 5	−0.044 8	−0.041 6
5	0.034 3	0.077 7	0.100 1	0.134 7	−0.122 8	−0.064 4	−0.050 4	−0.044 6
6	0.024 1	0.039 8	0.045 5	0.052 3	12.805 2	−0.099 5	−0.061 2	−0.049 4
7	0.018 7	0.025 8	0.028 1	0.030 6	0.090 9	−0.409 9	−0.086 4	−0.056 8
8	0.015 4	0.018 9	0.019 9	0.021 0	0.040 0	0.129 1	−0.170 4	−0.063 0
9	0.013 3	0.014 8	0.015 3	0.015 8	0.023 3	0.047 8	−0.359 7	−0.039 0
10	0.011 7	0.011 9	0.012 0	0.012 1	0.013 3	0.020 3	0.002 6	0.007 3
11	0.010 0	0.008 4	0.007 9	0.007 3	−0.018 9	0.032 6	0.022 6	0.020 3
12	0.006 0	−0.007 2	−0.016 5	−0.036 6	0.030 0	0.023 7	0.021 8	0.021 0
13	−0.466 4	0.030 7	0.028 4	0.026 9	0.022 2	0.021 1	0.020 7	0.020 4
14	0.023 7	0.021 6	0.021 4	0.021 2	0.020 5	0.020 3	0.020 2	0.020 1
15	0.020 5	0.020 2	0.020 2	0.020 2	0.020 1	0.020 0	0.020 0	0.020 0

Table 4 Random change of the means of X_n, Y_n and Z_n

Expectation	n							
	1	2	3	4	6	8	9	10
$\mathrm{E}(X_n)$	300.000 0	285.714 3	259.740 3	225.861 1	150.547 1	85.797 2	**61.283 7**	42.264 6
$\mathrm{E}(Y_n)$	2.000 0	2.150 3	2.339 2	2.752 0	4.586 6	10.080 5	16.800 6	30.547 0
$\mathrm{E}(Z_n)$	4.000 0	4.301 1	5.001 3	6.330 7	13.527 1	45.730 7	**103.933 3**	280.909 0

5. Conclusions

This paper studies the maintenance model of a simple repairable system with two failure modes with PR, and models the consecutive working time and the consecutive repair time of the system based on the GGP, aiming at the minimum expected cost per unit time of the long-term operation of the system. We can find the optimal value by the alternative searching method. Then numerical examples are given. For the production line of a food processing plant we have mentioned, this model can effectively improve its stability, raise economic efficiency, as well as decrease the costs. It can be widely used in cold standby systems, power systems, network systems, etc.

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Biographies

MA Ning was born in 1994. She received her B.S. degree of statistics from the College of Mathematical Science in Shanxi University in 2017, M.S. degree of statistics from the College of Mathematics and Statistics, Xidian University in 2020. Her research interests include applied stochastic processes, maintenance theory and reliability analysis.
E-mail: chumnxi@163.com

YE Jimin was born in 1967. He received his Ph.D. degree in signal and information processing from Xidian University in 2005. Currently, he is a professor and doctoral supervisor in the College of Mathematics and Statistics, Xidian University. His research interests are risk theory, stochastic operation research and applied probability.
E-mail: jmye@mail.xidian.edu.cn

WANG Junyuan was born in 1987. He received his B.S. degree of mathematics and applied mathematics in 2011, Ph.D. degree of probability theory and mathematical statistics in 2020. He is a lecturer in China Jiliang University. His research interests include applied stochastic processes, maintenance theory and reliability analysis.
E-mail: junyuanyc@163.com