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New approach for uncertain random multi-objective programming problems based on C_{ESD} criterion

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Abstract: To overcome the defects that the traditional approach for multi-objective programming under uncertain random environment (URMOP) neglects the randomness and uncertainty of the problem and the volatility of the results, a new approach is proposed based on expected value-standard deviation value criterion (C_{ESD} criterion). Firstly, the effective solution to the URMOP problem is defined; then, by applying sequence relationship between the uncertain random variables, the URMOP problem is transformed into a single-objective programming (SOP) under uncertain random environment (URSOP), which are transformed into a deterministic counterpart based on the C_{ESD} criterion. Then the validity of the new approach is proved that the optimal solution to the SOP problem is also efficient for the URMOP problem; finally, a numerical example and a case application are presented to show the effectiveness of the new approach.

Keywords: chance theory, independent-uncertain random multi-objective programming, expected value-standard derivation value criterion (C_{ESD} criterion).

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1. Introduction

The multi-objective programming (MOP) problem is a discipline developed from the 1970s, which is applied widely in management science, military science, operations research and so on. For example, in the construction of weapons and equipment systems, it is hoped for the largest combat effectiveness with the lowest cost; in the flight scheduling problem, it is expected for the highest mission safety with the shortest flight time. In these decision problems, the various objectives often contradict each other, which is difficult to find an optimal solution. Then, we may only weigh and compromise among the decision-making objectives, and select the most satisfactory plan [1–3].

Classical MOP mainly solves the problems in deterministic environment. In practice, we have to make decisions under indeterministic environment. How to solve MOP with multiple indeterministic factors has important significance. One of the common indeterministic phenomena is randomness, which can be solved by the probability theory with probability distributions obtained from enough samples. A random multi-objective programming (RMOP) problem is proposed [4–6].

Many events cannot accumulate enough and accurate data through experiments or other means. When dealing with such problems, experts in related fields are usually invited for the belief degree of each event [7]. However, since the expert's estimate of the event's belief degree is generally higher than the frequency of event in practice [8,9], if this type of problem is still solved by the probability theory, the conclusions are probably contrary to the facts [10]. Therefore, some scholars believed that the fuzzy theory [11] should be used to deal with the problem of expert belief, based on which, they studied the fuzzy multi-objective programming (FMOP) problem in [12–14]. Although FMOP has been widely used, many studies have shown that human uncertainty is not fuzzy [10], for which, applying the fuzzy theory to deal with uncertainty may lead to unrealistic situations. In order to overcome these defects, the uncertainty theory was established [7] by Liu in 2007 and refined [15] in 2010 to solve problems with experts' belief degree. At present, the uncertainty theory has grown into an important branch of mathematics dealing with belief degree. In 2009, Liu proposed the uncertain programing (UP) problem [16], which was applied in many areas. Then, the multi-objective programming under uncertain environment (UMOP) was proposed [17].

In reality, randomness and uncertainty often coexist in

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a complex system. In order to handle the problem, Liu proposed the chance theory, defined the uncertain random variable [18], and proposed the uncertain random programming [19]. Then, the multi-objective programming under uncertain random environment (URMOP) was proposed in 2014 [20], and the traditional solution approach was first proposed in this paper, which transformed URMOP into a counterpart under deterministic environment, which was then solved directly. The traditional approach solved the problem without considering uncertainty and randomness. Zheng et al. [21] presented another approach named linear weighted approach (LWA) under the expected value criterion (C_E criterion), based on which, the URMOP is transformed into a single-objective programming under uncertain random environment (URSOP), and then transformed into a counterpart under deterministic environment. Qi et al. [22] presented a new ideal point method (IPM) to solve the URMOP problem under the C_E criterion.

The equivalent model based on the C_E criterion reflects the average level that the objective function can reach under the influence of uncertain random factors. Uncertain random objective function, as a complex uncertain random variable, sometimes needs to investigate the degree of deviation from the expected value in practice, in particular, when the degree of deviation between the objective function and its expected value is very large, it is difficult to represent the objective function with the expected value only. It is necessary to describe the objective function together with the average level and the degree of deviation, and thus, the expected value-standard deviation value criterion (C_{ESD} criterion) is proposed, which can maintain the numerical characteristics of the first moment with the expected value, and reflect the deviation degree between the uncertain random objective function and its expected value.

The rest of this paper is organized in the following manner. In Section 2, some basic definitions are introduced and the basic framework of the new approach is proposed and the concepts such as Pareto efficient solution and C_{ESD} criterion are defined. In Section 3 and Section 4, several lemmas and theorems are proved to illustrate that the optimal solution of the single-objective programming (SOP) problem under the deterministic environment is efficient for the URMOP problem. A numerical example and a case application are presented to illustrate the feasibility of the new approach in Section 5. Finally, a brief summary and future research work are stated in Section 6.

2. Preliminaries

In this section, we will introduce several related definitions and theorems as well as the basic framework of URMOP, which is helpful to prove and understand the following.

2.1 Chance theory

Definition 1[18] Chance space

Uncertainty space and probability space are represented by (Γ, L, M) and (Ω, A, Pr), respectively. The product of them is represented by $(\Gamma, L, M) \times (\Omega, A, \mathrm{Pr})$, which is called a chance space.

Definition 2[18] Uncertain random variable

An uncertain random variable is represented by a function ξ, which is mapped from $(\Gamma, L, M) \times (\Omega, A, \mathrm{Pr})$ to the set of real numbers.

Remark Any Borel set of real numbers is denoted by B, then, $\{\xi \in B\}$ is an event in $L \times A$. A random variable and an uncertain variable are denoted by η and τ, respectively, then, an uncertain random variable is represented by $\xi(\eta, \tau)$. When ξ does not change with η, it would degenerate to an uncertain variable. When ξ does not change with τ, it would degenerate to a random variable. As a result, a random variable and an uncertain variable are both special cases of uncertain random variables.

Definition 3[18] Chance distribution

An uncertain random variable is denoted by a function ξ. Then, the chance distribution $\Phi(x)$ for any $x \in \mathbf{R}$ is denoted by $\Phi(x) = Ch\{\xi \leqslant x\}$.

Definition 4[18] Expected value of uncertain random variable

An uncertain random variable is denoted by ξ, the expected value of which can be defined as follows:

$$\mathrm{E}[\xi] = \int_0^{+\infty} Ch\{\xi \geqslant x\} \mathrm{d}x - \int_{-\infty}^0 Ch\{\xi \leqslant x\} \mathrm{d}x \tag{1}$$

where at least one of the two integrals is finite.

Definition 5[18] Variance value of uncertain random variable

An uncertain random variable is denoted by ξ, the variance value of which is defined as follows:

$$V[\xi] = \mathrm{E}[(\xi - e)^2] = \mathrm{E}[\xi^2] - \mathrm{E}[\xi]^2 \tag{2}$$

where the uncertain random variable $(\xi - e)^2$ is nonnegative, that is, $(\xi - e)^2 \in [0, +\infty)$.

Definition 6 Standard deviation value of uncertain random variable

An uncertain random variable is denoted by ξ, the standard deviation value of which is defined as follows:

$$\sigma[\xi] = \sqrt{V[\xi]} = \sqrt{\mathrm{E}[(\xi - e)^2]}. \tag{3}$$

Theorem 1[18] Uncertain random variables on $(\Gamma, L, M) \times (\Omega, A, \mathrm{Pr})$ are denoted by $\xi_1, \xi_2, \cdots, \xi_n$ and a measurable function is denoted by f. Then, $\xi = f(\xi_1,$

$\xi_2, \cdots, \xi_n$) is also an uncertain random variable which is determined by $\xi(\eta,\tau) = f(\xi_1(\eta,\tau), \xi_2(\eta,\tau), \cdots, \xi_n(\eta,\tau))$ for all $(\eta,\tau) \in \Gamma \times \Omega$.

Theorem 2[19] Let $\eta_i (i = 1,2,\cdots,m)$ be random variables which are independent and denoted by Ψ_i, the probability distribution of η_i. Let $\tau_i (i = 1,2,\cdots,n)$ be uncertain variables which are independent and denoted by Υ_i, the regular uncertainty distribution of τ_i. Assume that $f(\eta_1,\eta_2,\cdots,\eta_m, \tau_1,\tau_2,\cdots,\tau_n)$ is a measurable function which strictly increases with respect to $\tau_1,\tau_2,\cdots,\tau_k$ and strictly decreases with respect to $\tau_{k+1},\tau_{k+2},\cdots,\tau_n$. Then the uncertain random variable

$$\xi = f(\eta_1,\eta_2,\cdots,\eta_m,\tau_1,\tau_2,\cdots,\tau_n) \tag{4}$$

has a chance distribution

$$\Phi(x) = \int_{\mathbf{R}^m} F(x; y_1, y_2, \cdots, y_m) \cdot \mathrm{d}\Psi_1(y_1)\mathrm{d}\Psi_2(y_2)\cdots\mathrm{d}\Psi_m(y_m) \tag{5}$$

where $F(x; y_1, y_2, \cdots, y_m)$ is the root α of

$$f(y_1,y_2,\cdots,y_m,\Upsilon_1^{-1}(\alpha),\cdots,\Upsilon_k^{-1}(\alpha), \Upsilon_{k+1}^{-1}(1-\alpha),\cdots,\Upsilon_n^{-1}(1-\alpha)) = x. \tag{6}$$

Theorem 3[18] Let ξ be an uncertain random variable and denoted by $\Phi(x)$, the chance distribution. Then we arrive at that

$$\mathrm{E}[\xi] = \int_0^{+\infty} (1-\Phi(x))\mathrm{d}x - \int_{-\infty}^0 \Phi(x)\mathrm{d}x. \tag{7}$$

Theorem 4[19] Let $\eta_i (i = 1,2,\cdots,m)$ be random variables which are independent and denoted by Ψ_i, the probability distribution of η_i, and let $\tau_1,\tau_2,\cdots,\tau_n$ be uncertain variables. Assume that f is a measurable function. Then we arrive at that the uncertain random variable $\xi = f(\eta_1,\eta_2,\cdots,\eta_m, \tau_1,\tau_2,\cdots,\tau_n)$ has an expected value

$$\mathrm{E}[\xi] = \int_{\mathbf{R}^m} \mathrm{E}[f(y_1,y_2,\cdots,y_m,\tau_1,\tau_2,\cdots,\tau_n)] \cdot \mathrm{d}\Psi_1(y_1)\mathrm{d}\Psi_2(y_2)\cdots\mathrm{d}\Psi_m(y_m) \tag{8}$$

where $\mathrm{E}[f(y_1,y_2,\cdots,y_m,\tau_1,\tau_2,\cdots,\tau_n)]$ is the expected value of the uncertain variable $f(y_1,y_2,\cdots,y_m,\tau_1,\tau_2,\cdots,\tau_n)$ for any real numbers $y_1,y_2,\cdots,y_m$.

2.2 Model of URMOP

If the objective function $f_i(\boldsymbol{x},\boldsymbol{\xi}_i) (i = 1,2,\cdots,p)$ contains different uncertain random variables, it is an independent multi-objective programming under uncertain random environment (I-URMOP) which is shown as below:

$$\begin{cases} \min\limits_{\boldsymbol{x}\in\mathbf{R}^n} F(\boldsymbol{x},\boldsymbol{\xi}) = \\ \quad F(f_1(\boldsymbol{x},\boldsymbol{\xi}_1), f_2(\boldsymbol{x},\boldsymbol{\xi}_2), \cdots, f_p(\boldsymbol{x},\boldsymbol{\xi}_p)) \\ \text{s. t.} \\ d_i(\boldsymbol{x},\boldsymbol{\xi}_i) \leqslant 0,\ i = 1,2,\cdots,m \end{cases} \tag{9}$$

where the symbols and assumptions used are as follows:

(i) $\boldsymbol{x}$ is a decision variable vector defined on $\mathbf{R}^n$; and $\boldsymbol{\xi}_j = \left(\xi_1^j, \cdots, \xi_n^j\right)$ is a continuous uncertain random vector which is defined on the chance space, whose components are independent and uncertainty distribution of which are known.

(ii) $f_j(\boldsymbol{x},\boldsymbol{\xi}_j)$ is a measurable function which is defined on $\mathbf{R}^n \times \mathbf{R}^n$. It can be seen from Theorem 1, that $f_j(\boldsymbol{x},\boldsymbol{\xi}_j)$ $(j = 1,\cdots,p)$ are also uncertain random variables. The objective function $F(\boldsymbol{x},\boldsymbol{\xi})$ is a convex vector function with respect to $\boldsymbol{x}$.

(iii) The feasible solution set satisfies the following conditions: nonempty, compactness, convexity.

Definition 7 C_{ESD} criterion

Two uncertain random variables are denoted by $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, the domain of which are $(\Gamma, L, M) \times (\Omega, A, \mathrm{Pr})$. Then, if and only if $\mathrm{E}[\boldsymbol{\xi}_1] \leqslant \mathrm{E}[\boldsymbol{\xi}_2]$ and $\sigma[\boldsymbol{\xi}_1] \leqslant \sigma[\boldsymbol{\xi}_2]$, we arrive at that $\boldsymbol{\xi}_1 <= \boldsymbol{\xi}_2$, where $\mathrm{E}[\cdot]$ and $\sigma[\cdot]$ stand for the expected value and the standard deviation value of these uncertain random variables, respectively.

Definition 8 C_{ESD}-Pareto efficient solution of URMOP

For the I-URMOP problem, the feasible solution is denoted by $\boldsymbol{x}^*$, the condition of which is there is no feasible solution $\bar{\boldsymbol{x}}$ satisfying the condition that $f_j(\bar{\boldsymbol{x}},\boldsymbol{\xi}_j) <= f_j(\boldsymbol{x}^*,\boldsymbol{\xi}_j)$ $(j = 1,2,\cdots,p)$, besides, there exists at least one indicator $j_0 \in [1,p]$ satisfying $f_{j_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{j_0}) < f_{j_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{j_0})$, where $\boldsymbol{x}^*$ is an efficient solution to the problem.

Once we obtain the C_{ESD}-Pareto efficient solution, as needed, we can improve any one of the objective functions in a compromise way only by sacrificing one or more other objective functions.

The I-URMOP problem can be transformed into a URSOP problem in the following:

$$\begin{cases} \min\limits_{\boldsymbol{x}\in\mathbf{R}^n} U[F(\boldsymbol{x},\boldsymbol{\xi})] = \\ \quad U(f_1(\boldsymbol{x},\boldsymbol{\xi}_1), f_2(\boldsymbol{x},\boldsymbol{\xi}_2), \cdots, f_p(\boldsymbol{x},\boldsymbol{\xi}_p)) \\ \text{s. t.} \\ Ch\{d_i(\boldsymbol{x},\boldsymbol{\xi}_i) \leqslant 0\} \geqslant \alpha_i,\ i = 1,2,\cdots,q \end{cases} \tag{10}$$

where U represents a measurable function. Therefore, according to Theorem 1, we can arrive at that $U[F(\boldsymbol{x},\boldsymbol{\xi})]$ is an uncertain random variable and a convex vector function.

Thus, we can get a deterministic SOP problem under the C_{ESD} criterion, which is shown in the following:

$$\begin{cases} \min\limits_{\boldsymbol{x}\in\mathbf{R}^n} \mathrm{E}\{U[f(\boldsymbol{x},\boldsymbol{\xi})]\} + \sigma\{U[f(\boldsymbol{x},\boldsymbol{\xi})]\} = \\ \quad \mathrm{E}[F(f_1(\boldsymbol{x},\boldsymbol{\xi}_1), f_2(\boldsymbol{x},\boldsymbol{\xi}_2), \cdots, f_p(\boldsymbol{x},\boldsymbol{\xi}_p))] + \\ \quad \sigma[F(f_1(\boldsymbol{x},\boldsymbol{\xi}_1), f_2(\boldsymbol{x},\boldsymbol{\xi}_2), \cdots, f_p(\boldsymbol{x},\boldsymbol{\xi}_p))] \\ \text{s. t.} \\ Ch\{d_i(\boldsymbol{x},\boldsymbol{\xi}_i) \leqslant 0\} \geqslant \alpha_i,\ i = 1,2,\cdots,q \end{cases} \tag{11}$$

3. Proof of related lemmas

Several related lemmas are presented in the following, which play important roles in proving the validity of the new approach for the I-URMOP problem under the C_{ESD} criterion.

Lemma 1 The independent random variable is denoted by $\eta_i(i=1,2,\cdots,m)$ with probability distribution $\varPsi_i(i=1,2,\cdots,m)$ and probability density function $\psi_i(i=1,2,\cdots,m)$, respectively, and the uncertain variable is denoted by $\tau_i(i=1,2,\cdots,m)$ with regular uncertain distribution $\varUpsilon_i$ $(i=1,2,\cdots,m)$, respectively. Assume $\xi_i=g_i(\eta_i,\tau_i)$ is a measurable function which strictly increases with respect to $\tau_1,\tau_2,\cdots,\tau_k$ and strictly decreases with respect to $\tau_{k+1},\tau_{k+2},\cdots,\tau_n$. If $\xi_1<=\xi_2$, then for $\lambda\in\mathbf{R}^+$, we have $\lambda\xi_1<=\lambda\xi_2$.

Proof Since $\xi_1<=\xi_2$, according to the C_{ESD} criterion, we get

$$\mathrm{E}[g_1(\eta_1,\tau_1)]\leqslant\mathrm{E}[g_2(\eta_2,\tau_2)], \tag{12}$$

$$\sigma[g_1(\eta_1,\tau_1)]\leqslant\sigma[g_2(\eta_2,\tau_2)]. \tag{13}$$

Furthermore, we get

$$\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\varUpsilon_1^{-1}(\alpha))\mathrm{d}\alpha\mathrm{d}\varPsi_1(y)\leqslant \int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\varUpsilon_2^{-1}(\alpha))\mathrm{d}\alpha\mathrm{d}\varPsi_2(y). \tag{14}$$

We obtain

$$\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\varUpsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y\leqslant \int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\varUpsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \tag{15}$$

For $\lambda\in\mathbf{R}^+$, we get

$$\lambda\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\varUpsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y\leqslant \lambda\int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\varUpsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \tag{16}$$

Furthermore, we get

$$\int_{\mathbf{R}^m}\int_0^1 \lambda g_1(y_1,\varUpsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y\leqslant \int_{\mathbf{R}^m}\int_0^1 \lambda g_2(y_2,\varUpsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \tag{17}$$

Then, we get

$$\mathrm{E}[\lambda g_1(\eta_1,\tau_1)]\leqslant\mathrm{E}[\lambda g_2(\eta_2,\tau_2)]. \tag{18}$$

Since $\lambda\in\mathbf{R}^+$, according to (13), we get

$$\lambda\sigma[g_1(\eta_1,\tau_1)]\leqslant\lambda\sigma[g_2(\eta_2,\tau_2)]. \tag{19}$$

According to Definition 6, we can get

$$\sigma[\lambda g_1(\eta_1,\tau_1)]=\sqrt{\mathrm{E}^2[\lambda g_1(\eta_1,\tau_1)-\lambda e_{g_1}]}= \lambda\sqrt{\mathrm{E}^2[g_1(\eta_1,\tau_1)-e_{g_1}]}=\lambda\sigma[g_1(\eta_1,\tau_1)] . \tag{20}$$

Similarly, we get

$$\sigma[\lambda g_2(\eta_2,\tau_2)]=\lambda\sigma[g_2(\eta_2,\tau_2)]. \tag{21}$$

Then, we get

$$\sigma[\lambda g_1(\eta_1,\tau_1)]\leqslant\sigma[\lambda g_2(\eta_2,\tau_2)]. \tag{22}$$

According to the C_{ESD} criterion, we get

$$\lambda\xi_1<=\lambda\xi_2. \tag{23}$$

The lemma is proved. □

Lemma 2 Let $\eta_i(i=1,2,\cdots,m)$ be an independent random variable with the probability distribution $\varPsi_i(i=1,2,\cdots,m)$ and the probability density function $\psi_i(i=1,2,\cdots,m)$, respectively, and let $\tau_i(i=1,2,\cdots,m)$ be an uncertain variable with regular uncertain distribution $\varUpsilon_i$ $(i=1,2,\cdots,m)$, respectively. Assume $\xi_i=g_i(\eta_i,\tau_i)$ is a measurable function which strictly increases with respect to $\tau_1,\tau_2,\cdots,\tau_k$ and strictly decreases with respect to $\tau_{k+1},\tau_{k+2},\cdots,\tau_n$. If $\xi_1<\xi_2$, $\xi_3<=\xi_4$ or $\xi_1<=\xi_2$, $\xi_3<\xi_4$, then we get $\xi_1+\xi_2<\xi_3+\xi_4$.

Proof Without loss of generality, let $\xi_1<\xi_2$ and $\xi_3<=\xi_4$ hold, according to the C_{ESD} criterion, we can get

$$\mathrm{E}[g_1(\eta_1,\tau_1)]<\mathrm{E}[g_2(\eta_2,\tau_2)], \tag{24}$$

$$\mathrm{E}[g_3(\eta_3,\tau_3)]\leqslant\mathrm{E}[g_4(\eta_4,\tau_4)], \tag{25}$$

$$\sigma[g_1(\eta_1,\tau_1)]<\sigma[g_2(\eta_2,\tau_2)], \tag{26}$$

$$\sigma[g_3(\eta_3,\tau_3)]\leqslant\sigma[g_4(\eta_4,\tau_4)]. \tag{27}$$

Furthermore, we get

$$\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\varUpsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y< \int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\varUpsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y, \tag{28}$$

$$\int_{\mathbf{R}^m}\int_0^1 g_3(y_3,\varUpsilon_3^{-1}(\alpha))\psi_3(y)\mathrm{d}\alpha\mathrm{d}y\leqslant \int_{\mathbf{R}^m}\int_0^1 g_4(y_4,\varUpsilon_4^{-1}(\alpha))\psi_4(y)\mathrm{d}\alpha\mathrm{d}y. \tag{29}$$

Then, we arrive at

$$\int_{\mathbf{R}^m}\int_0^1 [g_1(y_1,\varUpsilon_1^{-1}(\alpha))\psi_1+g_3(y_3,\varUpsilon_3^{-1}(\alpha))\psi_3]\mathrm{d}\alpha\mathrm{d}y< \int_{\mathbf{R}^m}\int_0^1 [g_2(y_2,\varUpsilon_2^{-1}(\alpha))\psi_2+g_4(y_4,\varUpsilon_4^{-1}(\alpha))\psi_4]\mathrm{d}\alpha\mathrm{d}y. \tag{30}$$

That is,

$$\begin{aligned}&\mathrm{E}[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)]<\\&\quad\mathrm{E}[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)].\end{aligned} \tag{31}$$

Since $\sigma^2[\xi]=V[\xi]=\mathrm{E}[\xi^2]-\mathrm{E}[\xi]^2$, we get

$$\begin{aligned}&V[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)]-V[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)]=\\&\mathrm{E}[(g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4))^2-(g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3))^2]-\\&(\mathrm{E}[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)]+\mathrm{E}[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)])\cdot\\&(\mathrm{E}[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)]-\mathrm{E}[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)])=\\&\int_{\mathbf{R}^m}\int_0^1[g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2+g_4(y_4,\Upsilon_4^{-1}(\alpha))\psi_4+\\&g_1(y_1,\Upsilon_1^{-1}(\alpha))\psi_1+g_3(y_3,\Upsilon_3^{-1}(\alpha))\psi_3]\cdot\\&[g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2+g_4(y_4,\Upsilon_4^{-1}(\alpha))\psi_4-\\&g_1(y_1,\Upsilon_1^{-1}(1-\alpha))\psi_1-g_3(y_3,\Upsilon_3^{-1}(1-\alpha))\psi_3]\mathrm{d}\alpha\mathrm{d}y-\\&\int_{\mathbf{R}^m}\int_0^1[g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2+g_4(y_4,\Upsilon_4^{-1}(\alpha))\psi_4+\\&g_1(y_1,\Upsilon_1^{-1}(\alpha))\psi_1+g_3(y_3,\Upsilon_3^{-1}(\alpha))\psi_3]\mathrm{d}\alpha\mathrm{d}y\\&\int_{\mathbf{R}^m}\int_0^1[g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2+g_4(y_4,\Upsilon_4^{-1}(\alpha))\psi_4-\\&g_1(y_1,\Upsilon_1^{-1}(1-\alpha))\psi_1-g_3(y_3,\Upsilon_3^{-1}(1-\alpha))\psi_3]\mathrm{d}\alpha\mathrm{d}y.\end{aligned} \tag{32}$$

Suppose that

$$\begin{aligned}m(y,\alpha)=&\,g_2(y_2,\Upsilon_2^{-1}(\alpha))+g_4(y_4,\Upsilon_4^{-1}(\alpha))+\\&g_1(y_1,\Upsilon_1^{-1}(\alpha))+g_3(y_3,\Upsilon_3^{-1}(\alpha)),\end{aligned} \tag{33}$$

$$\begin{aligned}n(y,\alpha)=&\,g_2(y_2,\Upsilon_2^{-1}(\alpha))+g_4(y_4,\Upsilon_4^{-1}(\alpha))-\\&g_1(y_1,\Upsilon_1^{-1}(1-\alpha))-g_3(y_3,\Upsilon_3^{-1}(1-\alpha)).\end{aligned} \tag{34}$$

Since both $m(y,\alpha)$ and $n(y,\alpha)$ are strictly monotonically increasing as α increases, thus, according to Theorem 6, we get

$$\begin{aligned}&V[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)]-\\&V[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)]=\\&\int_{\mathbf{R}^m}\int_0^1 m(y,\alpha)n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)-\\&\int_{\mathbf{R}^m}\int_0^1 m(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)\int_{\mathbf{R}^m}\int_0^1 n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y).\end{aligned} \tag{35}$$

Assume that

$$\begin{aligned}G(x)=&\,x\int_{\mathbf{R}^m}\int_0^x m(y,\alpha)n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)-\\&\int_{\mathbf{R}^m}\int_0^x m(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)\int_{\mathbf{R}^m}\int_0^x n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y).\end{aligned} \tag{36}$$

Then, we get

$$\begin{aligned}G'(x)=&\int_{\mathbf{R}^m}\int_0^x m(y,\alpha)n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)+\\&xm(y,x)n(y,x)-n(y,x)\int_{\mathbf{R}^m}\int_0^x m(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)-\\&m(y,x)\int_{\mathbf{R}^m}\int_0^x n(y,\alpha)\mathrm{d}\alpha\mathrm{d}\Psi(y)=\\&\int_{\mathbf{R}^m}\int_0^x[m(y,\alpha)-m(y,x)][n(y,\alpha)-n(y,x)]\mathrm{d}\alpha\mathrm{d}\Psi(y).\end{aligned} \tag{37}$$

Since $G(0)=0$ and $G'(0)>0\,(x\in[0,1]$, except $x=0,\alpha)$, then $G(1)>0$.

Then, we get

$$\begin{aligned}&V[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)]<\\&\quad V[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)].\end{aligned} \tag{38}$$

Furthermore, we get

$$\begin{aligned}&\sigma[g_1(\eta_1,\tau_1)+g_3(\eta_3,\tau_3)]<\\&\quad\sigma[g_2(\eta_2,\tau_2)+g_4(\eta_4,\tau_4)].\end{aligned} \tag{39}$$

According to the C_{ESD} criterion, we get

$$\xi_1+\xi_2\prec\xi_3+\xi_4. \tag{40}$$

The lemma is proved. □

Lemma 3 Let $\eta_i(i=1,2,\cdots,m)$ be an independent random variable with the probability distribution $\Psi_i(i=1,2,\cdots,m)$ and the probability density function $\psi_i(i=1,2,\cdots,m)$, respectively, and let $\tau_i(i=1,2,\cdots,m)$ be an uncertain variable with regular uncertain distribution $\Upsilon_i\ (i=1,2,\cdots,m)$, respectively. Assume $\xi_i=g_i(\eta_i,\tau_i)$ is a measurable function which strictly increases with respect to $\tau_1,\tau_2,\cdots,\tau_k$ and strictly decreases with respect to $\tau_{k+1},\tau_{k+2},\cdots,\tau_n$. Then, for $\xi_1\prec=\xi_2$ and the lower bound of ξ_i, ξ_i^0 exists. Thus, we have $(\xi_1-\xi_0)^2\prec=(\xi_2-\xi_0)^2$, $\xi_0=\min(\xi_1^0,\xi_2^0)$.

Proof Since $\xi_1\prec=\xi_2$, according to the C_{ESD} criterion, we have

$$\mathrm{E}[g_1(\eta_1,\tau_1)]<\mathrm{E}[g_2(\eta_2,\tau_2)], \tag{41}$$

$$\sigma[g_1(\eta_1,\tau_1)]<\sigma[g_2(\eta_2,\tau_2)]. \tag{42}$$

Furthermore, we can get that

$$\begin{aligned}&\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\Upsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y<\\&\quad\int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y.\end{aligned} \tag{43}$$

Since ξ_i^0 is the lower bound of the ξ_i and $\xi_0=\min(\xi_1^0,\xi_2^0)$, $\xi_1-\xi_0\geqslant 0$ and $\xi_2-\xi_0\geqslant 0$ hold, that is, $g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0\geqslant 0$ and $g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0\geqslant 0$ hold, we can get

$$\int_{\mathbf{R}^m}\int_0^1 (g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)\psi_1(y)\mathrm{d}\alpha\mathrm{d}y \leqslant \int_{\mathbf{R}^m}\int_0^1 (g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \quad (44)$$

Furthermore, we get

$$\int_{\mathbf{R}^m}\int_0^1 (g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2\psi_1(y)\mathrm{d}\alpha\mathrm{d}y \leqslant \int_{\mathbf{R}^m}\int_0^1 (g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \quad (45)$$

$(g_1(y_1,\tau_1)-\xi_0)^2$ and $(g_2(y_2,\tau_2)-\xi_0)^2$ are increasing with regard to $g_1(y_1,\tau_1)$ and $g_2(y_2,\tau_2)$, respectively, and the inverse distributions of those are $(g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2$ and $(g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2$, respectively. Then, we get

$$\mathrm{E}[(g_1(y_1,\tau_1)-\xi_0)^2] = \int_{\mathbf{R}^m}\int_0^1 (g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2\psi_1(y)\mathrm{d}\alpha\mathrm{d}y, \quad (46)$$

$$\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2] = \int_{\mathbf{R}^m}\int_0^1 (g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \quad (47)$$

Then, we arrive at

$$\mathrm{E}[(g_1(y_1,\tau_1)-\xi_0)^2] \leqslant \mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2]. \quad (48)$$

Since $\sigma^2[\xi]=V[\xi]=\mathrm{E}[\xi^2]-\mathrm{E}[\xi]^2$, we can get

$$\begin{aligned}
&V[(g_2(y_2,\tau_2)-\xi_0)^2]-V[(g_1(y_1,\tau_1)-\xi_0)^2] = \\
&(\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^4]-\mathrm{E}[(g_1(y_1,\tau_1)-\xi_0)^4])- \\
&(\mathrm{E}^2[(g_2(y_2,\tau_2)-\xi_0)^2]-\mathrm{E}^2[(g_1(y_1,\tau_1)-\xi_0)^2]) = \\
&\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^4-(g_1(y_1,\tau_1)-\xi_0)^4]- \\
&(\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2]+\mathrm{E}[(g_1(y_1,\tau_1)-\xi_0)^2])\cdot \\
&(\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2]-\mathrm{E}[(g_1(y_1,\tau_1)-\xi_0)^2]) = \\
&\mathrm{E}\{[(g_2(y_2,\tau_2)-\xi_0)^2+(g_1(y_1,\tau_1)-\xi_0)^2]\cdot \\
&[(g_2(y_2,\tau_2)-\xi_0)^2-(g_1(y_1,\tau_1)-\xi_0)^2]\}- \\
&\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2+(g_1(y_1,\tau_1)-\xi_0)^2]\cdot \\
&\mathrm{E}[(g_2(y_2,\tau_2)-\xi_0)^2-(g_1(y_1,\tau_1)-\xi_0)^2] = \\
&\int_{\mathbf{R}^m}\int_0^1 [g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2+g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2]\cdot \\
&[g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2- \\
&g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y- \\
&\int_{\mathbf{R}^m}\int_0^1 [g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2+ \\
&g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y\cdot \\
&\int_{\mathbf{R}^m}\int_0^1 [g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2- \\
&g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y. \quad (49)
\end{aligned}$$

Assume that

$$p(y,\alpha)=g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2+ g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2, \quad (50)$$

$$q(y,\alpha)=g_2(y_2,\Upsilon_2^{-1}(\alpha))-\xi_0)^2- g_1(y_1,\Upsilon_1^{-1}(\alpha))-\xi_0)^2. \quad (51)$$

Then, we get

$$\begin{aligned}
&V[(g_2(y_2,\tau_2)-\xi_0)^2]-V[(g_1(y_1,\tau_1)-\xi_0)^2] = \\
&\int_{\mathbf{R}^m}\int_0^1 p(y,\alpha)q(y,\alpha)\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y- \\
&\int_{\mathbf{R}^m}\int_0^1 p(y,\alpha)\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y\int_{\mathbf{R}^m}\int_0^1 q(y,\alpha)\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y. \quad (52)
\end{aligned}$$

Since both (50) and (51) are strictly monotonically increasing as α increases, according to (36)–(38), we get

$$\sigma[(g_1(y_1,\tau_1)-\xi_0)^2] \leqslant \sigma[(g_2(y_2,\tau_2)-\xi_0)^2]. \quad (53)$$

According to the $\mathrm{C_{ESD}}$ criterion, we have

$$(\xi_1-\xi_0)^2 <= (\xi_2-\xi_0)^2. \quad (54)$$

The lemma is proved. □

Lemma 4 Let $\eta_i(i=1,2,\cdots,m)$ be an independent random variable with the probability distribution $\Psi_i(i=1,2,\cdots,m)$ and the probability density function $\psi_i(i=1,2,\cdots,m)$, and let $\tau_i(i=1,2,\cdots,m)$ be an uncertain variable with the regular uncertain distribution Υ_i ($i=1,2,\cdots,m$). Assume $\xi_i=g_i(\eta_i,\tau_i)$ is a measurable function which strictly increases with respect to $\tau_1,\tau_2,\cdots,\tau_k$ and strictly decreases with respect to $\tau_{k+1},\tau_{k+2},\cdots,\tau_n$. Then, for $\xi_1 <= \xi_2$, $\sqrt{\xi_1}$ and $\sqrt{\xi_2}$ exist. Thus, we have $\sqrt{\xi_1} <= \sqrt{\xi_2}$.

Proof Since $\xi_1 <= \xi_2$, according to the $\mathrm{C_{ESD}}$ criterion, we have

$$\mathrm{E}[g_1(\eta_1,\tau_1)] < \mathrm{E}[g_2(\eta_2,\tau_2)], \quad (55)$$

$$\sigma[g_1(\eta_1,\tau_1)] < \sigma[g_2(\eta_2,\tau_2)]. \quad (56)$$

Furthermore, we can get that

$$\int_{\mathbf{R}^m}\int_0^1 g_1(y_1,\Upsilon_1^{-1}(\alpha))\psi_1(y)\mathrm{d}\alpha\mathrm{d}y < \int_{\mathbf{R}^m}\int_0^1 g_2(y_2,\Upsilon_2^{-1}(\alpha))\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \quad (57)$$

Since $\sqrt{\xi_1}$ and $\sqrt{\xi_2}$ exist, we get

$$\int_{\mathbf{R}^m}\int_0^1 \sqrt{g_1(y_1,\Upsilon_1^{-1}(\alpha))}\psi_1(y)\mathrm{d}\alpha\mathrm{d}y < \int_{\mathbf{R}^m}\int_0^1 \sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}\psi_2(y)\mathrm{d}\alpha\mathrm{d}y. \quad (58)$$

Thus, we get

$$\mathrm{E}[\sqrt{g_1(\eta_1,\tau_1)}] < \mathrm{E}[\sqrt{g_2(\eta_2,\tau_2)}]. \quad (59)$$

Then, we get

$$V[\sqrt{g_2(\eta_2,\tau_2)}]-V[\sqrt{g_1(\eta_1,\tau_1)}]=\mathrm{E}[g_2(\eta_2,\tau_2)]-\mathrm{E}[g_1(\eta_1,\tau_1)]-\mathrm{E}^2[\sqrt{g_2(\eta_2,\tau_2)}]+\mathrm{E}^2[\sqrt{g_1(\eta_1,\tau_1)}]=$$
$$\int_{\mathbf{R}^m}\int_0^1[\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}+\sqrt{g_1(y_1,\Upsilon_1^{-1}(\alpha))}]\cdot[\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}-\sqrt{g_1(y_1,\Upsilon_1^{-1}(1-\alpha))}]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y-$$
$$\int_{\mathbf{R}^m}\int_0^1[\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}+\sqrt{g_1(y_1,\Upsilon_1^{-1}(\alpha))}]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y\cdot\int_{\mathbf{R}^m}\int_0^1[\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}-\sqrt{g_1(y_1,\Upsilon_1^{-1}(1-\alpha))}]\psi_1\psi_2\mathrm{d}\alpha\mathrm{d}y. \quad (60)$$

Assume that

$$u(y,\alpha)=\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}+\sqrt{g_1(y_1,\Upsilon_1^{-1}(\alpha))}, \quad (61)$$

$$v(y,\alpha)=\sqrt{g_2(y_2,\Upsilon_2^{-1}(\alpha))}-\sqrt{g_1(y_1,\Upsilon_1^{-1}(1-\alpha))}. \quad (62)$$

Since both (61) and (62) are strictly monotonically increasing as α increases, according to (36)–(38), we get

$$\sigma\left[\sqrt{g_1(\eta_1,\tau_1)}\right]<\sigma\left[\sqrt{g_2(\eta_2,\tau_2)}\right]. \quad (63)$$

According to the C_{ESD} criterion, we get

$$\sqrt{\xi_1}\prec=\sqrt{\xi_2}. \quad (64)$$

The lemma is proved. □

4. Efficient solution

In this section, we introduce two approachs, which are the LWA and the IPM. These are used to transform the I-URMOP problem into an I-URSOP problem under the C_{ESD} criterion.

4.1 LWA

LWA is appropriate for the conditions where the decision makers can easily distinguish the objectives and their importance. By assigning corresponding weights to each objective function and linearly weighted summation according to the importance of objectives, LWA converts the I-URMCOP problem (9) into an equivalent uncertain single-objective problem as follows:

$$\begin{cases}\min\limits_{x\in\mathbf{R}^n}U[f(\boldsymbol{x},\boldsymbol{\xi})]=\sum\limits_{i=1}^{p}\lambda_i f_i(\boldsymbol{x},\boldsymbol{\xi}_i)\\ \text{s. t.}\\ Ch\{d_i(\boldsymbol{x},\boldsymbol{\xi}_i)\leqslant 0\}\geqslant\alpha_i,\ i=1,2,\cdots,q\end{cases} \quad (65)$$

where $\lambda\in\left\{\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_p)|\lambda_i<0,\sum\limits_{i=1}^{p}\lambda_i=1\right\}$.

Theorem 5 The optimal solution $\boldsymbol{x}^*$ of the problem (65) based on the C_{ESD} criterion must also be a C_{ESD}-Pareto efficient solution to the problem (9).

Proof Suppose that $\bar{\boldsymbol{x}}$ is the optimal solution of the problem (65) but not the C_{ESD}-Pareto efficient solution to the problem (9).

It can be seen from Definition 8 that some $\bar{\boldsymbol{x}}$ definitely exists and there must be at least one indicator $i_0(1\leqslant i_0\leqslant p)$, such that $f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)\prec= f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)$ and $f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)\prec= f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_i)$.

$$\mathrm{E}[f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)]<\mathrm{E}[f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)] \quad (66)$$

$$\sigma[f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)]<\sigma[f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)] \quad (67)$$

Since $\lambda\in\left\{\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_p)|\lambda_i>0,\sum\limits_{i=1}^{p}\lambda_i=1\right\}$ holds, and according to Lemma 1 and Lemma 2, we can arrive at

$$\mathrm{E}\left[\sum_{i=1}^{p}\lambda_i f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)\right]<\mathrm{E}\left[\sum_{i=1}^{p}\lambda_i f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)\right], \quad (68)$$

$$\sigma\left[\sum_{i=1}^{p}\lambda_i f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)\right]<\sigma\left[\sum_{i=1}^{p}\lambda_i f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)\right]. \quad (69)$$

That is,

$$\sum_{i=1}^{p}\lambda_i f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i)\prec\sum_{i=1}^{p}\lambda_i f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i). \quad (70)$$

Obviously, we can arrive at

$$U(\bar{\boldsymbol{x}},\boldsymbol{\xi})\prec U(\boldsymbol{x}^*,\boldsymbol{\xi}). \quad (71)$$

It can be seen from Definition 8 that $\boldsymbol{x}^*$ is not the optimal solution to the problem (65), so it is contrary to the assumption. Thus, the assumption is not true, and $\boldsymbol{x}^*$ is a C_{ESD}-Pareto efficient solution to the problem (9). The theorem is proved.

4.2 IPM

IPM is appropriate for the condition where decision makers can easily know the optimal choice for each objective. By minimizing the distance between each objective function and the ideal point which is obtained without considering the influence of other objective functions, the IPM converts the I-URMOP problem (9) into the I-URSOP problem according to the distance functions.

$$\begin{cases}\min\limits_{x\in\mathbf{R}^n}U(\boldsymbol{x},\boldsymbol{\xi})=\sqrt{\sum\limits_{i=1}^{p}(f_i(\boldsymbol{x},\xi_i)-f_i^0)^2}\\ \text{s.t.}\\ Ch\{d_i(\boldsymbol{x},\boldsymbol{\xi}_i)\leqslant 0\}\geqslant\alpha_i,\ i=1,2,\cdots,q\end{cases} \quad (72)$$

where f_i^0 stands for the lower bound of single objective $f_i(\boldsymbol{x},\boldsymbol{\xi}_i)$ $(i=1,2,\cdots,q)$ on the feasible set.

Theorem 6 Suppose that $\bar{\boldsymbol{x}}$ is the optimal solution to the problem (65) but not the C_{ESD}-Pareto efficient solution to the problem (9).

Proof It can be seen from Definition 8 that some

$\bar{\boldsymbol{x}}$ definitely exists and there must be at least one indicator $i_0 (1 \leqslant i_0 \leqslant p)$, such that $f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) <= f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i)$ and $f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0}) <= f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{i_0})$.

Since $f_{i_0}^0 (1 \leqslant i \leqslant p)$ is the lower bound of $f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0})$, and according to Lemma 3, we arrive at

$$\mathrm{E}[(f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2] \leqslant \mathrm{E}[(f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2], \tag{73}$$

$$\sigma[(f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2] \leqslant \sigma[(f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2]. \tag{74}$$

We have

$$(f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2 <= (f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2. \tag{75}$$

When $i \neq i_0$, we can get that

$$(f_{i_0}(\bar{\boldsymbol{x}},\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2 < (f_{i_0}(\boldsymbol{x}^*,\boldsymbol{\xi}_{i_0}) - f_{i_0}^0)^2. \tag{76}$$

We have

$$\mathrm{E}\left[\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2\right] < \mathrm{E}\left[\sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2\right], \tag{77}$$

$$\sigma\left[\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2\right] < \sigma\left[\sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2\right]. \tag{78}$$

That is,

$$\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2 < \sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2. \tag{79}$$

According to Lemma 4, we have

$$\mathrm{E}[\sqrt{\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2}] < \mathrm{E}[\sqrt{\sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2}], \tag{80}$$

$$\sigma[\sqrt{\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2}] < \sigma[\sqrt{\sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2}]. \tag{81}$$

That is,

$$\sqrt{\sum_{i=1}^{p} (f_i(\bar{\boldsymbol{x}},\boldsymbol{\xi}_i) - f_i^0)^2} < \sqrt{\sum_{i=1}^{p} (f_i(\boldsymbol{x}^*,\boldsymbol{\xi}_i) - f_i^0)^2}. \tag{82}$$

Thus, we can arrive at

$$U(\bar{\boldsymbol{x}},\boldsymbol{\xi}) < U(\boldsymbol{x}^*,\boldsymbol{\xi}). \tag{83}$$

It can be seen from Definition 8 that $\boldsymbol{x}^*$ is not the optimal solution to the problem (72), so it is contrary to the assumption. Thus, the assumption is not true, and $\boldsymbol{x}^*$ is a $\mathrm{C_{ESD}}$-Pareto efficient solution to the problem (9). The theorem is proved. □

5. Example applications

In this section, the LWA and the IPM are introduced to transform the I-URMOP problem into the I-URSOP problem. The influences of weights and the conversion criterion are discussed.

5.1 Numerical example

Assume that x_1, x_2, x_3 are nonnegative decision variables, η_1, η_2, η_3 are independent random variables with distributions $U(1,3), E(0.8), N(3,5)$; τ_1, τ_2, τ_3 are independent uncertain variables with distribution $Z(0.8,1.3,1.8), L(1.5, 12), L(5,10)$. The I-URMOP problem involves three objectives.

$$\begin{cases} \min\limits_{x_1,x_2,x_3} f_1(\boldsymbol{x},\eta_1,\tau_1) = -\eta_1 \exp\{[\sin(2\pi x_2)+ \\ \quad \sin(2\pi x_3)]/\cos(2x_1)\} + \tau_1(\sin(2x_1x_2) + x_3) \\ \min\limits_{x_1,x_2,x_3} f_2(\boldsymbol{x},\eta_2,\tau_2) = x_1^2(x_2+x_3) + \eta_2(\cos(2x_1x_2)+ \\ \quad x_3) + \tau_2(x_1+1)(\sin(2x_1x_2x_3)+1) \\ \min\limits_{x_1,x_2,x_3} f_3(\boldsymbol{x},\eta_3,\tau_3) = -\eta_3(x_2+1)(\sin(2x_1x_3)+1- \\ \quad \tau_3(\cos\sqrt{x_1^2+x_2^2+x_3^2}+1)/\sqrt{x_1^2+x_2^2+x_3^2} \\ \text{s.t.} \quad Ch\{(x_1-\eta_1-\tau_1)^2 + (x_2-\eta_1+\tau_1)^2+ \\ \quad (x_3+\eta_1-\tau_1)^2 < 35\} \geqslant 0.9 \\ \quad x_1,x_2,x_3 \geqslant 0 \end{cases} \tag{84}$$

Obviously, $f_1(\boldsymbol{x},\eta_1,\tau_1)$ is strictly decreasing with respect to η_1, while strictly increasing with respect to τ_1; $f_2(\boldsymbol{x},\eta_2,\tau_2)$ is strictly decreasing with respect to η_2,τ_2; $f_3(\boldsymbol{x},\eta_3,\tau_3)$ is strictly increasing with respect to η_3,τ_3. The I-URMOP problem can be converted to an I-URSOP problem through LWM with $\lambda_1,\lambda_2,\lambda_3$ $(\lambda_1+\lambda_2+\lambda_3=1)$.

$$U(\boldsymbol{x},\xi) = \lambda_1 f_1(\boldsymbol{x},\eta_1,\tau_1)+ \lambda_2 f_2(\boldsymbol{x},\eta_2,\tau_2) + \lambda_3 f_3(\boldsymbol{x},\eta_3,\tau_3) \tag{85}$$

Then we convert the I-URSOP problem into a deterministic counterpart under the $\mathrm{C_{ESD}}$ criterion.

$$\min_{\boldsymbol{x}\in\mathbf{R}^n} \mathrm{E}[U(\boldsymbol{x},\boldsymbol{\xi})] + \sigma[U(\boldsymbol{x},\boldsymbol{\xi})] = \mathrm{E}[\lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3] + \sigma[\lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3] \tag{86}$$

The I-URMOP problem can be converted to the I-URSOP problem through IPM.

$$U(\boldsymbol{x},\boldsymbol{\xi}) = \sqrt{(f_1 - f_0^1)^2 + (f_2 - f_0^2)^2 + (f_3 - f_0^3)^2} \tag{87}$$

Then we convert the I-URSOP problem into a deterministic counterpart under the $\mathrm{C_{ESD}}$ criterion.

$$\min_{x\in\mathbf{R}^n} \mathrm{E}[U(\boldsymbol{x},\boldsymbol{\xi})]+\sigma[U(\boldsymbol{x},\boldsymbol{\xi})]=$$
$$\mathrm{E}\left[\sqrt{(f_1-f_0^1)^2+(f_2-f_2^0)^2+(f_3-f_0^3)^2}\right]+$$
$$\sigma\left[\sqrt{(f_1-f_0^1)^2+(f_2-f_2^0)^2+(f_3-f_0^3)^2}\right] \quad (88)$$

In general, the deterministic counterpart converted from the I-URMOP problem has a high complexity and a lot of local minima, the constraints of which may be also complex. Thus, we apply the beetle antennae search (BAS) algorithm [23], which has a strong robustness and a low time complexity. Parameter settings adopted in the BAS algorithm are shown in Table 1.

Table 1 Parameter settings adopted in BAS algorithm

Control parameter	Value
Beetle size	50
Maximum cycle number	1 000
$[v_{\max}, v_{\min}]$	[0.9,0.4]
Constant	[0, 1]

We solve each problem for 20 times, and then use the average values as the final results, which is shown in Table 2. Expected values and standard deviation values change with $\lambda_1,\lambda_2,\lambda_3$ changing.

Table 2 Results by LWA with different weights

$[\lambda_1,\lambda_2,\lambda_3]$	Traditional approach		Proposed approach	
	Objective	Solution	Objective	Solution
[0.8,0.1,0.1]	[2.805 4 6.587 1 −2.127 6]	[0.429 5 2.878 5 3.451 3]	[2.683 9 6.786 3 −2.983 1]	[0.450 1 2.978 5 3.551 3]
[0.1,0.8,0.1]	[3.997 3 6.365 4 −2.247 8]	[0.409 7 2.121 3 3.567 1]	[3.782 4 6.145 8 −2.895 6]	[0.389 5 2.125 8 3.256 9]
[0.3,0.4,0.3]	[3.697 1 7.092 3 −2.651 8]	[0.420 5 2.356 8 3.167 3]	[3.012 4 6.458 7 −3.081 5]	[0.412 5 2.235 8 3.895 4]
[0.1,0.1,0.8]	[3.652 3 7.895 4 −5.298 3]	[0.372 9 2.453 1 3.347 1]	[3.789 5 7.095 4 −5.598 3]	[0.356 9 2.012 5 3.189 3]

The values of the objective function with various weights are evenly distributed in a relatively concentrated interval. For the proposed approach, the value of x_1,x_2,x_3 are in (0.35, 0.46), (2.01, 2.98), (3.18, 3.90), respectively and the values of objective 1, objective 2, objective 3 are in (2.60, 3.79), (6.10, 7.10), (−5.60, −2.89), respectively. For the traditional approach, the values of x_1,x_2,x_3 are in (0.37, 0.43), (2.10, 2.90), (3.16, 3.60), respectively, and the values of objective 1, objective 2, objective 3 are in (2.80, 4.10), (6.30, 7.90), (−5.30, −2.10), respectively.

Because the traditional approach and the new one are different in the order of dealing with uncertainty and randomness, the results are different, which are all Pareto efficient solutions based on the C_{ESD} criterion. However, since the new LWA takes into account the inherent uncertainty and randomness of the problem, it makes the result overall better than the traditional one. This also shows that in practical problems, the solutions generated by uncertain random approaches are more in line with the decision-making preferences of most people.

Take $[\lambda_1,\lambda_2,\lambda_3]=[0.3,0.4,0.3]$ as an example. C_E: $[f_1, f_2, f_3]$ = [2.987 5, 6.342 1, −2.976 4], $[x_1, x_2, x_3]$ = [0.408 9, 2.342 1, 3.907 1], $[\sigma_1,\sigma_2,\sigma_3]$ =[0.33, 0.49, 0.81]. C_{ESD}: $[\sigma_1,\sigma_2,\sigma_3]$ = [0.12, 0.11, 0.19]. The expected values are similar, but the latter standard deviations in this paper are smaller.

IPM is appropriate for the condition where decision makers can easily know the optimal choice for each objective. The traditional approach is to convert the URMOP into a deterministic problem under the C_E criterion. The lower bounds of these objectives are 2.987 6, 5.895 6, −3.225 8. Similarly, we apply the BAS algorithm to solve the problem, and the control parameters are listed in Table 2. The comparison of the two approaches are shown in Table 3.

Table 3 Comparison of the traditional approach and the new approach

Objective function	Traditional approach	Proposed approach
$f_1(x,\eta_1,\tau_1)$	2.895 1	2.758 3
$f_2(x,\eta_2,\tau_2)$	6.472 2	6.315 6
$f_3(x,\eta_3,\tau_3)$	−3.128 4	−2.943 3
Optimal solution	(0.375 3,2.328 5,3.114 3)	(0.367 4,2.538 1,3.315 4)

Obviously, the results of the new approach are different from those of the traditional approach. The new approach calculates the minimum value of each uncertain random objective function; thus, the uncertainty and randomness of the problem are maintained. However, the traditional approach focuses on the multi-objective part of the problem based on the minimum value of deterministic objective functions.

The results by IPM are shown as follows: C_E: $[f_1,f_2,f_3]=[2.675\,8, 6.237\,2, -3.031\,1]$, $[x_1,x_2,x_3]=[0.302\,8, 2.452\,7, 3.585\,7]$, $[\sigma_1,\sigma_2,\sigma_3]=[0.42, 0.35, 0.76]$; C_{ESD}: $[\sigma_1,\sigma_2,\sigma_3]=[0.09, 0.14, 0.17]$. The expected values are similar, but the latter standard deviations are smaller.

5.2 Case application

In large-scale air combats, formulating a reasonable weapon target allocation (WTA) scheme can optimize the allocation of resources and obtain the largest battlefield revenue with the least cost [24]. Assume that allocate m different types of weapons to n different types of targets. The number of weapons i allocated to a target j is denoted by $x_{ij} \in \mathbf{N}^+$. For a target j, a weapon i has a kill probability $p_{ij} \in (0,1)$, which is a random variable. A target j has a threat coefficient $\omega_j \in (0,1)$, which is an uncertain variable and $\sum_{j=1}^{n} \omega_j = 1$. Thus, the overall kill effectiveness of all weapons is denoted by $f_1 = 1 - f_1'$. In order to facilitate the calculation, the minimum objective function f_1' is applied to represent the maximum kill effectiveness f_1. A weapon i has a cost c_i, which is a random variable with a value range of $(0,1)$ for a unified dimension. Thus, the total cost is denoted by f_2, which is a random variable. The total number of weapons i is denoted by W_i and the maximum of weapons i allocated to all targets is no more than W_i. To complete the task, the total number allocated to target j is more than 1. Thus, the problem is shown as follows:

$$\begin{cases} \min f'_1 = 1 - \sum_{j=1}^{n} \omega_j \left[1 - \prod_{i=1}^{m} (1-p_{ij})^{x_{ij}} \right] \\ \min f_2 = \sum_{j=1}^{n} \sum_{i=1}^{m} c_i x_{ij} \\ \text{s.t. } \sum_{j=1}^{n} x_{ij} \leqslant W_i,\ \sum_{i=1}^{m} x_{ij} \geqslant 1,\ x_{ij} \in \mathbf{N} \end{cases} \tag{89}$$

where $m=5, n=6, N_i=2 (i=1,\cdots,5)$. $\omega_j (j=1,\cdots,6)$ obey distribution functions: $Z(0.01,0.05,0.08)$, $L(0.05,0.1)$, $L(0.1,0.2), L(0.2,0.3), Z(0.1,0.2,0.4), L(0.12,0.18)$. $c_i (i=1,\cdots,5)$ obey distribution functions: $N(0.3,0.05), N(0.5,0.01), N(0.8,0.09), N(0.7,0.02), N(0.9,0.1)$.

$$(p_{ij})_{5\times 6} = \begin{bmatrix} 0.63 & 0.78 & 0.12 & 0.78 & 0.57 & 0.69 \\ 0.66 & 0.37 & 0.53 & 0.61 & 0.39 & 0.76 \\ 0.73 & 0.86 & 0.65 & 0.55 & 0.73 & 0.99 \\ 0.81 & 0.92 & 0.67 & 0.58 & 0.85 & 0.79 \\ 0.61 & 0.88 & 0.97 & 0.72 & 0.87 & 0.64 \end{bmatrix}.$$

The problem (89) is transformed into a deterministic counterpart under the C_{ESD} criterion:

$$\min_{\boldsymbol{x}\in\mathbf{R}^n} \mathrm{E}[U(\boldsymbol{x},\boldsymbol{\xi})] + \sigma[U(\boldsymbol{x},\boldsymbol{\xi})] = \mathrm{E}[\lambda_1 f_1 + \lambda_2 f_2] + \sigma[\lambda_1 f_1 + \lambda_2 f_2]. \tag{90}$$

Take $\lambda_1 = 0.6, \lambda_2 = 0.4$ as an example, we get the results by the traditional approach based on the C_{ESD} criterion (TC_{ESD}-Weapon), by the new approach based on the C_{ESD} criterion (C_{ESD}-Weapon), and by the new approach under the C_E criterion (C_E-Weapon), which are shown in Table 4.

Table 4 Results by LWM for WTA

Target	TC_{ESD}-Weapon	C_{ESD}-Weapon	C_E-Weapon
1	1,1	5	1,2
2	2,3	5	3
3	5	4	4
4	5	4	3
5	4	1,2	4
6	3	3	5
f_1	0.861 7	0.894 6	0.880 2
f_2	5.191 2	4.790 9	4.776 7

The problem (89) is transformed into a deterministic counterpart under the C_{ESD} criterion:

$$\min_{\boldsymbol{x}\in\mathbf{R}^n} \mathrm{E}[U(\boldsymbol{x},\boldsymbol{\xi})] + \sigma[U(\boldsymbol{x},\boldsymbol{\xi})] = \mathrm{E}\left[\sqrt{(f_1-f_1^0)^2 + (f_2-f_2^0)^2}\right] + \sigma\left[\sqrt{(f_1-f_1^0)^2 + (f_2-f_2^0)^2}\right]. \tag{91}$$

The lower bounds of these objectives are $f_1'^0 = 0.087\,2, f_2^0 = 4.587\,1$. We get the results by the traditional approach based on the C_{ESD} criterion (TC_{ESD}-Weapon), by the new approach based on the C_{ESD} criterion (C_{ESD}-Weapon), and by the new approach based on the C_E criterion (C_E-Weapon), which are shown in Table 5.

Table 5 Results by IPM for WTA

Target	TC_{ESD}-Weapon	C_{ESD}-Weapon	C_E-Weapon
1	5	4	4
2	3	4	4
3	5	3	5
4	1,1	5	3
5	2	3	1,2
6	3	1,2	1,2
f_1	0.853 1	0.887 2	0.889 1
f_2	4.663 2	4.718 6	4.719 8

6. Conclusions

Under the C_{ESD} criterion, this paper proposes a new approach for Pareto-efficient solutions to the I-URMOP problem. The main contributions are as follows.

Under the C_{ESD} criterion, the new approach considers the uncertainty and randomness of the uncertain random problem and volatility of the results, which is appropriate for the problems.

The proofs of the four lemmas lay foundations for the new approach as well as provide a theoretical basis for properties of uncertain random variables, which enrich the chance theory.

For the LWM, the influence of weights is studied. Obviously, the choices of weights depend on the problem as well as the decision makers' preferences, which shows that it is appropriate to choose weights by collective decision making. For IPM, the differences between the new approach and the traditional approach are discussed, which illustrates that the new approach is more in line with the decision preference of most people.

The URMOP with dependent variables should be studied in the future. And a new approach based on the C_{ESD} criterion for the uncertain random multi-stage programming problem is an open problem solved.

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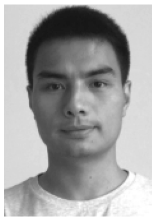

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