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Research on consensus of multi-agent systems with and without input saturation constraints

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Abstract: In recent years, with the continuous development of multi-agent technology represented by unmanned aerial vehicle (UAV) swarm, consensus control has become a hot spot in academic research. In this paper, we put forward a discrete-time consensus protocol and obtain the necessary and sufficient conditions for the second-order consensus of the second-order multi-agent system with a fixed structure under the condition of no saturation input. The theoretical derivation verifies that the two eigenvalues of the Laplacian of the communication network matrix and the sampling period have an important effect on achieving consensus. Then we construct and verify sufficient conditions to achieve consensus under the condition of input saturation constraints. The results show that consensus can be achieved if velocity, position gain, and sampling period satisfy a set of inequalities related to the eigenvalues of the Laplacian matrix. Finally, the accuracy and validity of the theoretical results are proved by numerical simulations.

Keywords: multi-agent system, consensus control, input constraint, distributed control.

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1. Introduction

Consensus control of multi-agent systems has become an important research topic, which has attracted great attention from researchers in many subjects such as control, mathematics and artificial intelligence, especially in swarm systems, distributed sensor networks, etc [1−8]. Due to the limited perception of individuals, it is generally believed that the consensus control would be achieved through systems with large-scale of local information. Various forms of distributed control protocols have been provided by robotics and control research teams. Saber and Murray [9] put forward a general hierarchy to solve the consensus problems of multi-agent systems of a single integrator. Ren and Beard [10] studied the consensus problem with a directed weighted graph, and pointed out that under certain conditions, the system can achieve consensus. Moreau [11] analyzed the network of the discrete system by the Lyapunov method and the convexity theory, which proved that as long as the coupling between agents satisfies certain convexity conditions and the network is connected, the consensus can be achieved. In latest research, researchers have focused on switching directed network time-delays, topologies, and consensus control with disturbance appearing [12−17].

For past years, the consensus control of second-order dynamics has become one of the hottest topics in this field, and it has been affirmed that consensus may not be achieved even the topology is connected and has directed spanning trees. Zhan and Li [18] provided some suffcient conditions for consensus control of second-order multi-agent systems with fixed and switching interaction topologies. Ren and Atkins [19] showed clearly that the gain parameters have to satisfy certain conditions related to eigenvalues of the Laplacian matrix in order to guarantee consensus of second-order continuous-time systems. Yu et al. [20] gave a necessary and sufficient condition to ensure second-order consensus, and they found both the real and imaginary parts of the eigenvalues have important effects. Zhang and Tian [21] indicated that there must exist appropriate gain parameters for discrete-time systems with fixed interaction topology, so that the consensus can be achieved if and only if union of graphs has globally reachable nodes. However, they did not explain how to design the parameters. Although Xie and Wang [22] put forward a mathematic expression and gave a reasonable range of gain parameters under fixed interaction topology, they still did not consider the sampling period of the system.

Another significant fact is that most of the researchers

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of the existing literature do not pay attention to input constraints which have to be considered in many practical engineering systems due to actuators limitations. To handle this, the so-called parameterized low-gain and high-gain feedback techniques were introduced in [23]. With the introduction of the low-gain and high-gain feedback techniques, consensus of multi-agent systems with input saturation was studied in [24–27]. Meng et al. [28] proposed a linear control protocol based on the relative state information. Global leader-following consensus with input saturation was also solved under fixed/time-varying topologies in their article. Yang et al. [29] further considered the global consensus control for discrete-time systems with input saturation constraints and fixed undirected topologies, and some necessary conditions for achieving global consensus were obtained. In [30,31], model predictive control protocols were used to achieve constrained consensus when the topology had a directed spanning tree and the sampling period was sufficiently small.

Taking a panoramic view of these existing investigations, gain parameters design for achieving consensus with input saturation constraints is a difficult problem. Although some attempts have been made, the problem has not been solved perfectly. For the single-integrator case, Li et al. [32] indicated that any relative-state-based linear protocol, which solves the linear consensus problem under fixed directed topology without input saturation constraints, can also solve the global consensus problem with input saturation constraints. Though these documents did not explain how to design the parameters, they have presented the affirmation that the desired consensus state may not be reached even the network is connected and has direct spanning trees. When considering the input saturation constraints and sampling period for discrete-time situation, things get even more complicated [33,34]. This is the motivation of this study.

The organization logic of this paper is as follows: Section 2 introduces some basic conceptions and problem descriptions; Section 3 considers the consensus problem without constraints. Thereafter, consensus with input saturation constraints is studied in Section 4. Section 5 presents simulation examples which demonstrate the validity of the control protocol and Section 6 is the summarization of this paper.

2. Preliminaries

2.1 Graph theory

$G=(W,E,\boldsymbol{A})$ is a directed graph, in which $W=\{w_1,w_2,\cdots,w_N\}$ is the node set of the graph, $E\subseteq\{(i,j):i,j\in W\}$ is the edge set, $\boldsymbol{A}=[a_{ij}]_{N\times N}$ is the weighted adjacency matrix of the graph. a_{ij} indicates the connection weight between w_i and w_j, $a_{ij}=1$ indicates that the node w_i can receive information from w_j, otherwise $a_{ij}=0$. In addition, it is defined here that when $i=j$, $a_{ij}=0$.

The Laplacian matrix of the graph G is $\boldsymbol{L}$, and $\boldsymbol{L}=[l_{ij}]_{N\times N}$. It is defined as $l_{ii}=\sum_{j=1}^{N}a_{ij}$, $l_{ij}=-a_{ij}$, $i\neq j$. If there is a node w_i, the information can be passed from w_i to any other nodes in the graph. The graph G is said to contain a directed spanning tree, and w_i is called a root.

Lemma 1 [35] The Laplacian matrix $\boldsymbol{L}$ of the graph G has at least one zero eigenvalue, and other $N-1$ nonzero eigenvalues all have positive real parts; if G has a directed spanning tree, zero is a single eigenvalue of $\boldsymbol{L}$, and $\boldsymbol{1}_N$ is its corresponding right eigenvector.

2.2 Problem formulations

Consider a multi-agent system with N agents. Each agent is described as

$$\begin{cases} \boldsymbol{x}_i(k+1)=\boldsymbol{x}_i(k)+T\boldsymbol{v}_i(k) \\ \boldsymbol{v}_i(k+1)=\boldsymbol{v}_i(k)+T\boldsymbol{u}_i(k) \end{cases} \tag{1}$$

where $i=1,\cdots,N$, $x_i(k)\in\mathbf{R}^n$, $v_i(k)\in\mathbf{R}^n$, $u_i(k)\in\mathbf{R}^n$, denote the position, velocity and control input of agent i at time k, respectively. $T>0$ is the sampling period. It is assumed that $n=1$ if not otherwise specified for simplicity of description. However, all the results hereafter remain valid for a higher dimensional case and can be calculated by using the Kronecker product.

Consider the following consensus protocol:

$$u_i(k)=-\alpha\sum_{j\in N_i}a_{ij}(x_i(k)-x_j(k))-\beta\sum_{j\in N_i}a_{ij}(v_i(k)-v_j(k)) \tag{2}$$

where $\alpha>0$, $\beta>0$ are position and velocity gain parameters to be designed.

Definition 1 System (1) achieves consensus if for any initial conditions,

$$\lim_{k\to+\infty}\|\boldsymbol{x}_i(k)-\boldsymbol{x}_j(k)\|=0,$$
$$\lim_{k\to+\infty}\|\boldsymbol{v}_i(k)-\boldsymbol{v}_j(k)\|=0,$$
$$\forall i,j=1,\cdots,N.$$

Denote $\boldsymbol{x}(k)=col[x_1(k),\cdots,x_N(k)]$, $\boldsymbol{v}(k)=col[v_1(k),\cdots,v_N(k)]$, $\boldsymbol{u}(k)=col[u_1(k),\cdots,u_N(k)]$, $\boldsymbol{y}(k)=col[\boldsymbol{x}(k),\boldsymbol{v}(k)]$. Substituting (2) into (1), system (1) can be rewritten in a compact matrix form as follows:

$$\boldsymbol{y}(k+1)=\boldsymbol{\Gamma}\boldsymbol{y}(k) \tag{3}$$

where $\boldsymbol{\Gamma}=\begin{bmatrix}\boldsymbol{I}_N & T\boldsymbol{I}_N \\ -\alpha T\boldsymbol{L} & \boldsymbol{I}_N-\beta T\boldsymbol{L}\end{bmatrix}$, $\boldsymbol{I}_N$ represents an identity matrix with dimension N.

Clearly, then system (1) is equivalent to (3) under the protocol (2).

Let μ be an eigenvalue of matrix $\boldsymbol{\Gamma}$. Then by the definition of eigenvalue, we can get

$$\det(\mu \boldsymbol{I}_{2N} - \boldsymbol{\Gamma}) = \prod_{i=1}^{N} \left\{ (\mu-1)^2 + [\alpha T^2 + (\mu-1)\beta T]\lambda_i \right\} = 0 \quad (4)$$

where λ_i is the eigenvalue of Laplacian matrix $\boldsymbol{L}$, $i = 1,\cdots,N$.

Lemma 2 [22] $\boldsymbol{L}$ has a zero eigenvalue with algebraic multiplicity m if and only if $\boldsymbol{\Gamma}$ has a one eigenvalue with algebraic multiplicity $2m$.

3. Consensus without constraints

Theorem 1 [22] The consensus condition can be described as if and only if the matrix $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two and all the other eigenvalues are in the unit circle. In addition, if the consensus is reached, then,

$$\begin{cases} \lim\limits_{k\to+\infty} \{\boldsymbol{x}(k) - [\boldsymbol{1}_N \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{x}(0) + kT\boldsymbol{1}_N \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{v}(0)]\} = 0 \\ \lim\limits_{k\to+\infty} \{\boldsymbol{v}(k) - \boldsymbol{1}_N \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{v}(0)\} = 0 \end{cases} \quad (5)$$

where $\boldsymbol{\xi}^{\mathrm{T}}$ is the unique nonnegative left eigenvector with zero eigenvalue satisfying $\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{1}_N = 1$.

Proof (Sufficiency) It can be verified that if the eigenvalue of $\boldsymbol{\Gamma}$ is 1, the multiplicity is two and cannot be similar to a diagonal matrix. Suppose there is a matrix $\boldsymbol{P}$, which is non-singular and satisfies $\boldsymbol{P}^{-1}\boldsymbol{\Gamma}\boldsymbol{P} = \boldsymbol{J}$, in which $\boldsymbol{J}$ has the form of $\boldsymbol{J} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \tilde{J} \end{bmatrix}$.

Let $\boldsymbol{P} = (\boldsymbol{\zeta}_1 \cdots \boldsymbol{\zeta}_{2N})$, $\boldsymbol{P}^{-1} = (\boldsymbol{\eta}_1 \cdots \boldsymbol{\eta}_{2N})^{\mathrm{T}}$, and substituting them in $\boldsymbol{P}^{-1}\boldsymbol{\Gamma}\boldsymbol{P} = \boldsymbol{J}$, we can obtain $\boldsymbol{\Gamma}(\boldsymbol{\zeta}_1 \cdots \boldsymbol{\zeta}_{2N}) = (\boldsymbol{\zeta}_1 \cdots \boldsymbol{\zeta}_{2N}) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \tilde{J} \end{pmatrix}$, that is $\begin{cases} \boldsymbol{\Gamma}\boldsymbol{\zeta}_1 = \boldsymbol{\zeta}_1 \\ \boldsymbol{\Gamma}\boldsymbol{\zeta}_2 = \boldsymbol{\zeta}_1 + \boldsymbol{\zeta}_2 \end{cases}$.

Therefore, $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are the eigenvector and generalized eigenvector of $\boldsymbol{\Gamma}$ associated with eigenvalue 1. Thus we can get $\boldsymbol{\zeta}_1 = [\boldsymbol{1}_N^{\mathrm{T}}\ \boldsymbol{0}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{\zeta}_2 = \left[\boldsymbol{0}_N^{\mathrm{T}}\ \frac{1}{T}\boldsymbol{1}_N^{\mathrm{T}}\right]^{\mathrm{T}}$.

Similarly, $\boldsymbol{\eta}_1^{\mathrm{T}}$ and $\boldsymbol{\eta}_2^{\mathrm{T}}$ are the left generalized eigenvector and left eigenvector of $\boldsymbol{\Gamma}$ associated with eigenvalue 1, and can be obtained as $\boldsymbol{\eta}_1 = \left[\frac{1}{T}\boldsymbol{\varsigma}^{\mathrm{T}}\ \boldsymbol{0}_N^{\mathrm{T}}\right]^{\mathrm{T}}$, $\boldsymbol{\eta}_2 = [\boldsymbol{0}_N^{\mathrm{T}} \boldsymbol{\varsigma}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\varsigma}^{\mathrm{T}}$ is an arbitrary left eigenvector of $\boldsymbol{L}$ associated with eigenvalue 0. Let $\boldsymbol{\xi}^{\mathrm{T}} = \frac{1}{T}\boldsymbol{\varsigma}^{\mathrm{T}}$, note that $\boldsymbol{P}^{-1}\boldsymbol{P} = \boldsymbol{I}$, that is $\boldsymbol{\eta}_1^{\mathrm{T}}\boldsymbol{\zeta}_1 = \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{1}_N = 1$. Clearly, $\boldsymbol{\xi}^{\mathrm{T}}$ is the uniquely determined left eigenvector of $\boldsymbol{L}$ associated with eigenvalue 0. Thus we can get $\boldsymbol{\eta}_1 = [\boldsymbol{\xi}^{\mathrm{T}}\ \boldsymbol{0}_N^{\mathrm{T}}]$, $\boldsymbol{\eta}_2 = [\boldsymbol{0}_N^{\mathrm{T}}\ T\boldsymbol{\xi}^{\mathrm{T}}]$.

Hence, we have

$$\lim_{k\to+\infty} \boldsymbol{\Gamma}^k = \boldsymbol{P} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lim\limits_{k\to+\infty} \tilde{J}^k \end{bmatrix} \boldsymbol{P}^{-1} =$$

$$(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \boldsymbol{\eta}_1^{\mathrm{T}} \\ \boldsymbol{\eta}_2^{\mathrm{T}} \end{pmatrix} + (\boldsymbol{\zeta}_3, \cdots, \boldsymbol{\zeta}_{2N}) \lim_{k\to+\infty} \tilde{J}^k \begin{pmatrix} \boldsymbol{\eta}_3^{\mathrm{T}} \\ \vdots \\ \boldsymbol{\eta}_{2N}^{\mathrm{T}} \end{pmatrix}.$$

Note that $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two and all the other eigenvalues are in the unit circle, then we get $\lim\limits_{k\to+\infty} \tilde{J}^k = 0$, that is,

$$\lim_{k\to+\infty} \boldsymbol{\Gamma}^k = \lim_{k\to+\infty} \begin{bmatrix} \boldsymbol{1}_N\boldsymbol{\xi}^{\mathrm{T}} & kT\boldsymbol{1}_N\boldsymbol{\xi}^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{1}_N\boldsymbol{\xi}^{\mathrm{T}} \end{bmatrix}. \quad (6)$$

From (3), we get $\boldsymbol{y}(k) = \boldsymbol{\Gamma}^k \boldsymbol{y}(0)$. In combination with Definition 1, one can conclude that the consensus of system (1) is achieved and the consensus state satisfies (5).

(Necessity) Since $\boldsymbol{L}$ has at least one zero eigenvalue (Lemma 1), then on the basis of Lemma 2. If the necessity is not satisfied, it indicates that $\boldsymbol{\Gamma}$ has at least three eigenvalues which are not in the unit circle, that is $\tilde{J}$ has at least one eigenvalue which is not in the unit circle, then $\lim\limits_{k\to+\infty} \tilde{J}^k \neq 0$. This means that there is at least one agent whose consensus conditions cannot be met, which contradicts the assumption that consensus of system (1) is achieved. Thus the necessity is satisfied.

Theorem 2 The consensus of multi-agent system (1) can be achieved by protocol (2) if and only if the topology graph has a directed spanning tree, and the gain parameters and sampling period satisfy

$$\begin{cases} f_1(\alpha,\beta,T,\lambda_i) = \alpha T^2 - 2\beta T + \dfrac{4\mathrm{Re}(\lambda_i)}{|\lambda_i|^2} > 0 \\ \beta - \alpha T > 0 \\ f_2(\alpha,\beta,T,\lambda_i) = [(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)] \cdot \\ \qquad (\beta - \alpha T)^2 - \dfrac{4\alpha \mathrm{Im}^2(\lambda_i)}{|\lambda_i|^2} > 0 \end{cases} \quad (7)$$

where $\mathrm{Re}(\lambda_i)$ and $\mathrm{Im}(\lambda_i)$ are the real and imaginary parts of λ_i respectively, $i = 2,\cdots,N$.

Proof (Necessity) Suppose that the system (1) is able to get consensus, and then on the basis of Theorem 1 and Lemma 2, $\boldsymbol{L}$ has exactly one simple 0 eigenvalue.

Let $\mu-1=s$, hence $\mathrm{Re}(s)<0$ and (4) can be rewritten as

$$s^2 + \beta T \lambda_i s + \alpha T^2 \lambda_i = 0, \quad i = 2,\cdots,N. \quad (8)$$

Denote $s_{i,1}$, $s_{i,2}$ are a pair of roots of (8) associated with λ, that is $s_{i,1}+s_{i,2}=-\beta T\lambda_i$, then $\mathrm{Re}(s_{i,1}+s_{i,2}) = -\beta T \mathrm{Re}(\lambda_i) < 0$. Since $\beta > 0$, $T > 0$, then we get $\mathrm{Re}(\lambda_i) > 0$. Then by Lemma 1, there is a directed spanning tree in the topology graph.

Define $g(\mu) = (\mu-1)^2 + [\alpha T^2 + (\mu-1)\beta T]\lambda_i, i = 2,\cdots,N$. Then $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two and all the other eigenvalues are in the unit circle, which is equivalent to that $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two and $g(\mu)$ is Schur stable, that is, all the roots of $g(\mu)$ satisfy $|\mu| < 1$. By the technique of bilinear transformation $\mu = \dfrac{\sigma+1}{\sigma-1}$, we get

$$\theta(\sigma) = (\sigma-1)^2 g\left(\frac{\sigma+1}{\sigma-1}\right) = \alpha\lambda_i T^2\sigma^2 + 2\lambda_i T(\beta-\alpha T)\sigma + \alpha\lambda_i T^2 - 2\beta\lambda_i T + 4. \tag{9}$$

Define

$$\gamma(\sigma) = \frac{\theta(\sigma)}{\alpha\lambda_i T^2} = \sigma^2 + \frac{2T(\beta-\alpha T)}{\alpha T^2}\sigma + \frac{\alpha\lambda_i T^2 - 2\beta\lambda_i T + 4}{\alpha\lambda_i T^2}. \tag{10}$$

Then the polynomial $g(\mu)$ is Schur stable if and only if $\gamma(\sigma)$ is Hurwitz stable.

Denote $\sigma = i\omega$ and substitute it in (10), it follows that

$$\gamma(i\omega) = (i\omega)^2 + \frac{2T(\beta-\alpha T)}{\alpha T^2}(i\omega) + \frac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\bar{\lambda}_i}{\alpha|\lambda_i|^2 T^2} \tag{11}$$

where $\bar{\lambda}_i$ is the conjugation of λ_i. Denote

$$\begin{cases} m(\omega) = -\omega^2 + \dfrac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)}{\alpha|\lambda_i|^2 T^2} \\ n(\omega) = \dfrac{2T(\beta-\alpha T)}{\alpha T^2}\omega - \dfrac{4\mathrm{Im}(\lambda_i)}{\alpha|\lambda_i|^2 T^2} \end{cases}. \tag{12}$$

Then, by the Hermite-Biehler theorem, $\gamma(\sigma)$ is Hurwitz stable if and only if the following conditions hold:

(i) $m(\omega) = 0$ has two different roots, denoted as $m_1 < m_2$;

(ii) The single root of $n(\omega) = 0$ satisfies $m_1 < n_1 < m_2$;

(iii) $m(0)n'(0) - m'(0)n(0) > 0$.

Simple calculation gives

$$\begin{cases} \Delta = \dfrac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)}{\alpha|\lambda_i|^2 T^2} > 0 \\ m_1 < n_1 < m_2 \\ \dfrac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)}{\alpha|\lambda_i|^2 T^2} \cdot \dfrac{2T(\beta-\alpha T)}{\alpha T^2} > 0 \end{cases} \tag{13}$$

where

$$m_1 = -\sqrt{\frac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)}{\alpha|\lambda_i|^2 T^2}},$$

$$m_2 = \sqrt{\frac{(\alpha T^2 - 2\beta T)|\lambda_i|^2 + 4\mathrm{Re}(\lambda_i)}{\alpha|\lambda_i|^2 T^2}},$$

$$n_1 = \frac{2\mathrm{Im}(\lambda_i)}{|\lambda_i|^2(\beta T - \alpha T^2)}.$$

By solving the inequality (13), conditions in (7) can be established.

(Sufficiency) By Lemma 1, we get that if G has a spanning tree, then $\boldsymbol{L}$ has exactly one simple 0 eigenvalue. Hence, $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two by Lemma 2. From the above proof in necessity, we obtain that parameters α, β, T, which meet the conditions of (8), also guarantee the roots of $g(\mu)$ satisfy $|\mu| < 1$. Therefore $\boldsymbol{\Gamma}$ has exactly a one eigenvalue of multiplicity two and all the other eigenvalues are in the unit circle. Hence, Theorem 2 is established. □

Corollary 1 If the topology graph is an undirected one, then the consensus of system (1) can be achieved by protocol (2) if and only if the graph is connected and

$$\begin{cases} \lambda_N < \dfrac{4}{2\beta T - \alpha T^2} \\ T < \dfrac{\beta}{\alpha} \end{cases} \tag{14}$$

where λ_N is the maximum eigenvalue of the Laplacian matrix $\boldsymbol{L}$. In addition, if the consensus of system (1) is achieved, then

$$\begin{cases} \lim\limits_{k\to+\infty}\left\{x(k) - \left[\dfrac{1}{N}\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}x(0) + \dfrac{kT}{N}\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}v(0)\right]\right\} = 0 \\ \lim\limits_{k\to+\infty}\left\{v(k) - \dfrac{1}{N}\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}v(0)\right\} = 0 \end{cases}. \tag{15}$$

Since $\boldsymbol{\xi}^{\mathrm{T}}$ satisfies $\boldsymbol{\xi}^{\mathrm{T}}\mathbf{1}_N = 1$ and is a left eigenvector of $\boldsymbol{L}$ associated with eigenvalue 0, that is $\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{L} = 0$. If the graph is undirected, it follows that $\boldsymbol{\xi}^{\mathrm{T}} = \dfrac{1}{N}\mathbf{1}_N^{\mathrm{T}}$, then Corollary 1 can be easily verified.

Remark 1 Theorem 1 and Theorem 2 are extensions of existing literature and this is considerable because the sampling period is a critical parameter for practical applications. For example, Dong et al. [36] applied a consensus-based time-varying formation tracking protocol to quadrotor formation flying test. Its controller update frequency is up to 10 Hz so as to be on the safe side. However, this is not only a vast waste of energy, but also extremely challenging for mobile communication network when the number of vehicles in the swarm is getting more.

4. Consensus with input saturation constraints

Suppose there is a multi-agent system with the members of N, and the constrained input is described as $\boldsymbol{c}(u_i(k)) =$

$[c_s(u_{i,1}(k)),\cdots,c_s(u_{i,n}(k))]$, where $c_s(\cdot)$ is the standard saturation function, and the definition is as follows:

$$c_s(u)=\begin{cases} u_0, & u>u_0\\ u, & |u|\leqslant u_0\\ -u_0, & u<-u_0\end{cases} \tag{16}$$

where u_0 is the maximum control input.

Theorem 3 Assume that the topology is undirected and fixed. The consensus of multi-agent system (1) with input constraints described in (16) can be achieved by protocol (2) if the topology graph is connected and the gain parameters and sampling period satisfy

$$\begin{cases} g_1(\alpha,\beta,T,\lambda_N)=g\alpha T^2\lambda_N<1\\ g_2(\alpha,\beta,T,\lambda_N)=(\alpha T^2-\beta T)^2\lambda_N+\\ \quad (3\alpha T^2-2\beta T)\leqslant 0\end{cases} \tag{17}$$

where λ_N is the maximum eigenvalue of $\boldsymbol{L}$.

Proof When considering the input constraints, system (1) can be rewritten as the following form:

$$\begin{cases} x_i(k+1)=x_i(k)+Tv_i(k)\\ v_i(k+1)=v_i(k)+Tc(u_i(k))\end{cases}.$$

For simplicity, $\boldsymbol{x}(k)$,$\boldsymbol{v}(k)$,$\boldsymbol{u}(k)$, and $\boldsymbol{y}(k)$, whose definitions have been given in Section 2, are denoted as $\boldsymbol{x}$, $\boldsymbol{v}$, $\boldsymbol{u}$, and $\boldsymbol{y}$, respectively.

Consider the equation below:

$$V(k)=-\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{c}(\boldsymbol{u})+2\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{u}+ 2\alpha T\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{L}\boldsymbol{v}+\alpha\boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{v}. \tag{18}$$

Since $\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{u}\geqslant\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{c}(\boldsymbol{u})$ for any column vector $\boldsymbol{u}$, we get that

$$V(k)\geqslant\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{c}(\boldsymbol{u})+2\alpha T\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{L}\boldsymbol{v}+\alpha\boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{v}= \left[\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})(\boldsymbol{L}\boldsymbol{v})^{\mathrm{T}}\right]\begin{bmatrix} I_N & \alpha TI_N\\ \alpha TI_N & \theta I_N\end{bmatrix}\begin{bmatrix}\boldsymbol{c}(\boldsymbol{u})\\ \boldsymbol{L}\boldsymbol{v}\end{bmatrix}+ \boldsymbol{v}^{\mathrm{T}}(\alpha\boldsymbol{L}-\theta\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L})\boldsymbol{v}. \tag{19}$$

Suppose $\theta-(\alpha T)^2>0$ and $\alpha\boldsymbol{L}-\theta\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$ is semi-positive definite, then it follows that $V(k)\geqslant 0$ and $V(k)=0$ if and only if $x_1=\cdots=x_N$, $v_1=\cdots=v_N$, that is, the consensus is achieved. Thus (18) is an appropriate Lyapunov candidate function. Next, the proof is given to show that there exists such θ meeting the conditions.

Since the graph is undirected, it follows that $\boldsymbol{L}^{\mathrm{T}}=\boldsymbol{L}$ and the eigenvalues of $\alpha\boldsymbol{L}-\theta\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$ can be obtained by $\alpha\lambda_i-\theta\lambda_i^2$ (i = 1, $\cdots$, N). The graph is undirected and connected, also indicating that $0=\lambda_1<\lambda_2\leqslant\cdots\leqslant\lambda_N$. $\alpha\boldsymbol{L}-\theta\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$ is semi-positive definite if and only if $\alpha\lambda_i-\theta\lambda_i^2\geqslant 0$, then we can get $\alpha-\theta\lambda_i\geqslant 0$, thus $\theta\leqslant\alpha/\lambda_N$. Further consider that $\theta-(\alpha T)^2>0$, it follows $(\alpha T)^2<\theta\leqslant\alpha/\lambda_N$. If there exists such θ meeting the conditions, then $(\alpha T)^2<\alpha/\lambda_N$, that is,

$$\alpha T^2\lambda_N<1. \tag{20}$$

Now, it is time to show that $\Delta V(k+1)=V(k+1)-V(k)\leqslant 0$. By (18), it can be obtained that

$$\Delta V(k+1)=V(\boldsymbol{y}(k+1))-V(\boldsymbol{y}(k))= -\boldsymbol{t}^{\mathrm{T}}\boldsymbol{t}+2\boldsymbol{t}^{\mathrm{T}}\boldsymbol{u}+2(\alpha T^2-\beta T)\boldsymbol{t}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{c}(\boldsymbol{u})+ \boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})[\alpha T^2\boldsymbol{L}+I]\boldsymbol{c}(\boldsymbol{u})-2\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{u})\boldsymbol{u} \tag{21}$$

where $\boldsymbol{t}=\boldsymbol{c}(\boldsymbol{u}(k+1))$.

Without loss of generality, suppose $u_i>u_0$, $i\in S_p=\{1,\cdots,N_p\}$, $|u_i|\leqslant u_0$ for $i\in S_m=\{N_p+1,\cdots,N_m\}$, $u_i<-u_0$ for $i\in S_q=\{N_m+1,\cdots,N\}$. Note that S_p, S_m, S_q may be empty. We define the partition $\boldsymbol{t}=[\boldsymbol{t}_p^{\mathrm{T}},\boldsymbol{t}_m^{\mathrm{T}},\boldsymbol{t}_q^{\mathrm{T}}]^{\mathrm{T}},\boldsymbol{u}=[\boldsymbol{u}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},\boldsymbol{u}_q^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{t}_p,\boldsymbol{u}_p\in\mathbf{R}^{N_p}$, $\boldsymbol{t}_m,\boldsymbol{u}_m\in\mathbf{R}^{N_m-N_p}$, $\boldsymbol{t}_q,\boldsymbol{u}_q\in\mathbf{R}^{N-N_m}$. Accordingly, the Laplacian matrix $\boldsymbol{L}$ should also be rewritten into stacks as

$$\boldsymbol{L}=\begin{bmatrix}\boldsymbol{L}_{pp} & \boldsymbol{L}_{pm} & \boldsymbol{L}_{pq}\\ \boldsymbol{L}_{pm}^{\mathrm{T}} & \boldsymbol{L}_{mm} & \boldsymbol{L}_{mq}\\ \boldsymbol{L}_{pq}^{\mathrm{T}} & \boldsymbol{L}_{mq}^{\mathrm{T}} & \boldsymbol{L}_{qq}\end{bmatrix}. \tag{22}$$

By the definition of (16), we have $\boldsymbol{c}(\boldsymbol{u})=[u_0\mathbf{1}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},-u_0\mathbf{1}_q^{\mathrm{T}}]^{\mathrm{T}}$. By substituting $\boldsymbol{t}$, $\boldsymbol{u}$, $\boldsymbol{L}$ into (21), and by completing squares, we obtain

$$\Delta V(k+1)=2(\boldsymbol{t}_p-u_0\mathbf{1}_p)^{\mathrm{T}}(\boldsymbol{u}_p-u_0\mathbf{1}_p)+ 2(\boldsymbol{t}_q+u_0\mathbf{1}_q)^{\mathrm{T}}(\boldsymbol{u}_q+u_0\mathbf{1}_q)- [\tilde{\boldsymbol{p}}^{\mathrm{T}},\tilde{\boldsymbol{m}}^{\mathrm{T}},\tilde{\boldsymbol{q}}^{\mathrm{T}}]\times[\tilde{\boldsymbol{p}}^{\mathrm{T}},\tilde{\boldsymbol{m}}^{\mathrm{T}},\tilde{\boldsymbol{q}}^{\mathrm{T}}]^{T}+\boldsymbol{s}^{\mathrm{T}}M\boldsymbol{s} \tag{23}$$

where

$$M=(\alpha T^2-\beta T)^2\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}+(3\alpha T^2-2\beta T)\boldsymbol{L},$$

$$\boldsymbol{s}^{\mathrm{T}}=[u_0\mathbf{1}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},-u_0\mathbf{1}_q^{\mathrm{T}}],$$

$$\tilde{\boldsymbol{p}}=\boldsymbol{t}_p-\{(\alpha T^2-\beta T)[\boldsymbol{L}_{pp},\boldsymbol{L}_{pm},\boldsymbol{L}_{pq}]\cdot [u_0\mathbf{1}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},-u_0\mathbf{1}_q^{\mathrm{T}}]^{\mathrm{T}}+u_0\mathbf{1}_p\},$$

$$\tilde{\boldsymbol{m}}=\boldsymbol{t}_m-\{(\alpha T^2-\beta T)[\boldsymbol{L}_{pm}^{\mathrm{T}},\boldsymbol{L}_{mm},\boldsymbol{L}_{mq}]\cdot [\boldsymbol{u}_0\mathbf{1}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},-u_0\mathbf{1}_q^{\mathrm{T}}]^{\mathrm{T}}+\boldsymbol{u}_m\},$$

$$\tilde{\boldsymbol{q}}=\boldsymbol{t}_q-\{(\alpha T^2-\beta T)[\boldsymbol{L}_{pq}^{\mathrm{T}},\boldsymbol{L}_{mq}^{\mathrm{T}},\boldsymbol{L}_{qq}]\cdot [u_0\mathbf{1}_p^{\mathrm{T}},\boldsymbol{u}_m^{\mathrm{T}},-u_0\mathbf{1}_q^{\mathrm{T}}]^{\mathrm{T}}-u_0\mathbf{1}_q\}.$$

Clearly, we can obtain that term $[\tilde{\boldsymbol{p}}^{\mathrm{T}},\tilde{\boldsymbol{m}}^{\mathrm{T}},\tilde{\boldsymbol{q}}^{\mathrm{T}}]\times[\tilde{\boldsymbol{p}}^{\mathrm{T}},\tilde{\boldsymbol{m}}^{\mathrm{T}},\tilde{\boldsymbol{q}}^{\mathrm{T}}]^{\mathrm{T}}$ is non-positive. Since $\boldsymbol{t}_p-u_0\mathbf{1}_p\leqslant 0$, $\boldsymbol{u}_p-u_0\mathbf{1}_p>0$, $\boldsymbol{t}_q+u_0\mathbf{1}_q\geqslant 0$, $\boldsymbol{u}_q+u_0\mathbf{1}_q<0$, we have that the first line of (23) is also non-positive. Thus the sufficient condition for $\Delta V(k+1)\leqslant 0$ is that term $\boldsymbol{s}^{\mathrm{T}}M\boldsymbol{s}$ is non-positive. Hence, M is negative semi-definite and the eigenvalues of M satisfy $(\alpha T^2-\beta T)^2\lambda_i^2+(3\alpha T^2-2\beta T)\lambda_i\leqslant 0$. Since the graph is undirected and connected, it follows $0=\lambda_1<\lambda_2\leqslant\cdots\leqslant\lambda_N$, then $(\alpha T^2-\beta T)^2\lambda_i^2+(3\alpha T^2-2\beta T)\lambda_i\leqslant 0$ is equivalent to

$$(\alpha T^2-\beta T)^2\lambda_N+(3\alpha T^2-2\beta T)\leqslant 0. \tag{24}$$

Hence, the condition $\Delta V(k+1)\leqslant 0$ is satisfied if there exists α, β, T satisfying (24).

We then show that $\Delta V(k+1)=0$ if and only if the consensus is achieved, that is $x_1=\cdots=x_N$, $v_1=\cdots=v_N$. Note that if there exists $|u_i|>u_0$, $\Delta V(k+1)=0$ cannot be established since at least one of the first two lines of (23) is negative. Hence, $|u_i|\leqslant u_0$ is satisfied for all agents. Then we can get that

$$\begin{aligned}\Delta V(k+1)&=V(k+1)-V(k)=\\ &-\boldsymbol{t}^{\mathrm{T}}\boldsymbol{t}+2\boldsymbol{t}^{\mathrm{T}}\boldsymbol{u}+2(\alpha T^2-\beta T)\boldsymbol{t}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{u}+\\ &\boldsymbol{u}^{\mathrm{T}}(\alpha T^2\boldsymbol{L}+\boldsymbol{I})\boldsymbol{u}-2\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}=\\ &-\{\boldsymbol{t}-[(\alpha T^2-\beta T)+\boldsymbol{I}]\boldsymbol{u}\}^{\mathrm{T}}\cdot\\ &\{\boldsymbol{t}-[(\alpha T^2-\beta T)+\boldsymbol{I}]\boldsymbol{u}\}+\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{u}.\end{aligned}$$

Therefore, $\Delta V=0$ if and only if $\boldsymbol{t}=[(\alpha T^2-\beta T)+\boldsymbol{I}]\boldsymbol{u}$ and $\boldsymbol{L}\boldsymbol{u}=0$ are satisfied, that is $\boldsymbol{t}=\boldsymbol{u}$. By the definition of $\boldsymbol{u}$ and with some algebra, we obtain $\boldsymbol{t}=\boldsymbol{u}(k+1)=\boldsymbol{u}(k)-\alpha\boldsymbol{L}\boldsymbol{v}(k)$.

Since $\alpha>0$, we have $\boldsymbol{L}\boldsymbol{v}(k)=0$, that is $v_i(k)=v_j(k)$, $\forall i,j=1,\cdots,N$. Furthermore, $\boldsymbol{L}\boldsymbol{u}=0$ is satisfied if and only if $u_i(k)=u_j(k), \forall i,j=1,\cdots,N$. Now, we shall prove $x_i(k)=x_j(k)$. Note that $\boldsymbol{L}\boldsymbol{v}(k)=0$ also indicates $\boldsymbol{u}(k)=-\alpha\boldsymbol{L}\boldsymbol{x}(k)$, in other words, we have

$$\begin{cases} l_{11}x_1+\cdots+l_{1N}x_N=-u_1^* \\ \quad\vdots \\ l_{N1}x_1+\cdots+l_{NN}x_N=-u_N^* \end{cases} \tag{25}$$

where $u_i^*=u_i/\alpha$ and $u_1^*=\cdots=u_N^*$. By subtracting the first line of (25), it follows

$$\begin{cases} (l_{21}-l_{11})x_1+\cdots+(l_{2N}-l_{1N})x_N=0 \\ \quad\vdots \\ (l_{N1}-l_{11})x_1+\cdots+(l_{NN}-l_{1N})x_N=0 \end{cases}. \tag{26}$$

Note that $l_{i1}=-\sum_{j=2}^{N}l_{ij}$ and substituting it into (26), we obtain

$$\begin{cases} (l_{22}-l_{12})(x_2-x_1)+\cdots+(l_{2N}-l_{1N})(x_N-x_1)=0 \\ \quad\vdots \\ (l_{N2}-l_{12})(x_2-x_1)+\cdots+(l_{NN}-l_{1N})(x_N-x_1)=0 \end{cases}. \tag{27}$$

Consider a non-singular matrix $\boldsymbol{E}=\begin{bmatrix} 1 & \boldsymbol{0}_{N-1}^{\mathrm{T}} \\ \boldsymbol{1}_{N-1} & \boldsymbol{I}_{N-1}\end{bmatrix}$. Then it can be obtained that $\boldsymbol{E}^{-1}\boldsymbol{L}\boldsymbol{E}=\begin{bmatrix} 0 & \boldsymbol{L}_{12} \\ \boldsymbol{0}_{N-1} & \tilde{\boldsymbol{L}}\end{bmatrix}$ where $\boldsymbol{L}_{12}=[l_{12},\cdots,l_{1N}]$ and $\tilde{\boldsymbol{L}}$ is the coefficient matrix of (27).

The graph is connected, indicating that $\boldsymbol{L}$ has exactly one simple 0 eigenvalue, that is, $\mathrm{Rank}(\boldsymbol{L})=N-1$. Hence, one can obtain that all eigenvalues of $\tilde{\boldsymbol{L}}$ are strictly positive and $\mathrm{Rank}(\tilde{\boldsymbol{L}})=N-1$, thus (27) only has all-zero solution, which is equivalent to $x_1=\cdots=x_N$. Hence, we have shown that $\Delta V\leqslant 0$ and $\Delta V=0$ if and only if $x_1=\cdots=x_N$, $v_1=\cdots=v_N$, that is, the consensus is achieved.

Note that (17) can be obtained by (20) and (24). Hence, Theorem 3 is proved in conclusion. □

Remark 2 The sampling period is taken into account and the result can be regarded as a special case of Theorem 3 with T = 1. Furthermore, the results in Theorem 3 is less conservative by substituting in T = 1.

5. Simulation experiment

5.1 Consensus without input constraints

Suppose there is a multi-agent system with the members of N, here N = 15, and its topology G_1 is given in Fig. 1. The initial settings are $\boldsymbol{x}(0)$ = [100, 110, 120, 130, 140, 150, 60, 70, 80, 90, 40, 30, 50, 20, 10], $\boldsymbol{v}(0)$ = [10, 11, 12, 13, 14, 15, 6, 7, 8, 9, 5, 4, 3, 2, 1]. Clearly, G_1 has a directed spanning tree and the eigenvalues of the Laplacian matrix are $\lambda_1=0$, $\lambda_2=0.186$, $\lambda_3=0.509$, $\lambda_4=1$, $\lambda_{5,6}=2$, $\lambda_7=2.710$, $\lambda_{8,9}=1.411\pm0.585\mathrm{i}$, $\lambda_{10,11}=1.461\pm0.439\mathrm{i}$, $\lambda_{12,13}=1.980\pm1.171\mathrm{i}$, $\lambda_{14,15}=2.446\pm0.970\mathrm{i}$. i is the imaginary unit. Let α=0.20, β=0.35 and plot change curves of $f_j(\alpha,\beta,T,\lambda_i)$ (i=1,$\cdots$,15, j=1,2) vs T, it is known that the system can attain consensus if and only if T<0.8 s based on the results of Theorem 2.

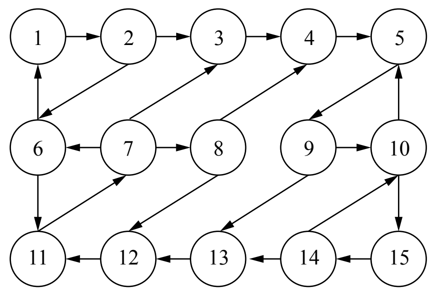
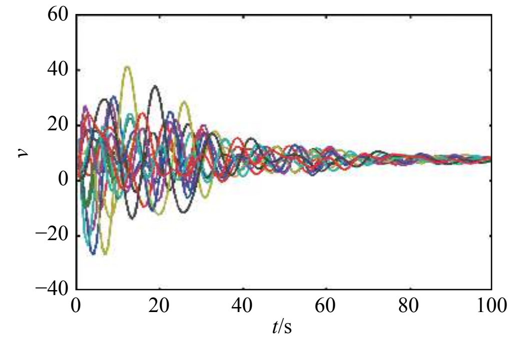
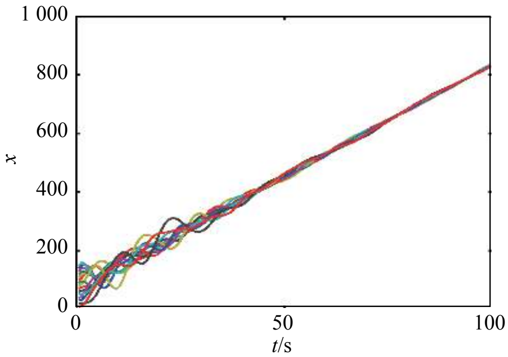


Fig. 1 Graph topology of G_1

For comparison, simulations are conducted continuously under the condition that T = 0.7 s and T = 0.8 s, respectively. The change curves of accelerations, velocities and positions of each member of the system are shown in Fig. 2 and Fig. 3 respectively. The results presented here show that the control protocol (2) makes them meet the consensus when T = 0.7 s in Fig. 2. While in Fig. 3, one can see that when T = 0.8 s, the system cannot meet the consensus conditions. Thus the result of Theorem 2 is verified.

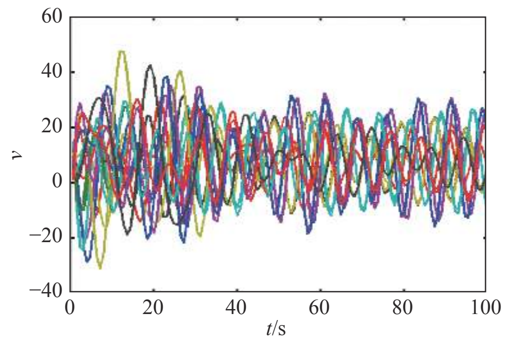


(a) Evolution of velocities

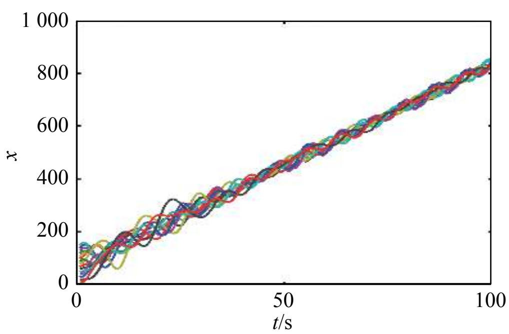


(b) Evolution of positions

Fig. 2 Evolution of positions and velocities of all agents (α=0.20, β=0.35, T=0.7 s)



(a) Evolution of velocities



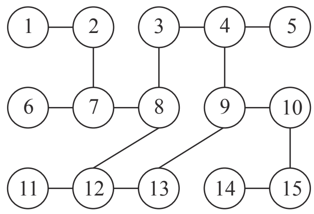
(b) Evolution of positions

Fig. 3 Evolution of positions and velocities of all agents (α=0.20, β=0.35, T=0.8 s)

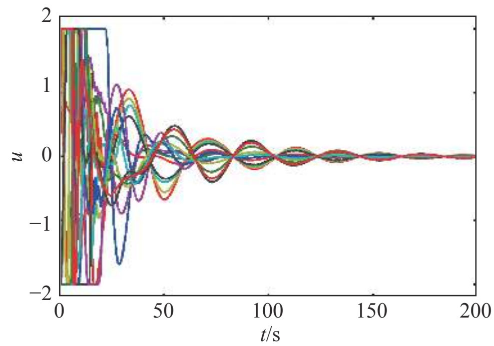
5.2 Consensus with input constraints

Suppose there is a multi-agent system with the members of N, here $N = 15$ and input constraints $|u_i| \leqslant u_0 = 2$. The topology G_2 is given in Fig.3 (a), here we need to pay attention to that the graph here is undirected and simply connected. The initial settings, including positions and velocities, are the same as Section 5.1. After some calculation, we know the maximum eigenvalues of the Laplacian matrix $\lambda_N = 5.06$. Let $\alpha = 0.20$, $\beta = 0.35$ and substituting them into (17), one can obtain that the consensus condition is $T < 0.90$ s. Hence, we take $T = 0.85$ s in the simulation.

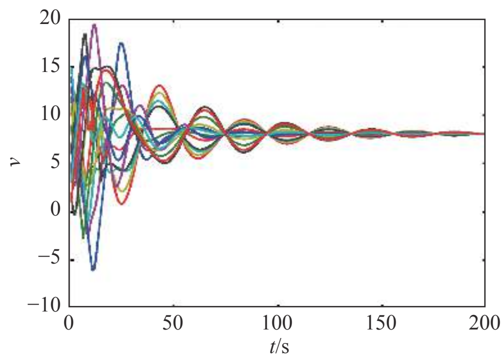
The change curves of accelerations, velocities and positions of each member of the system are shown in Fig.4 (b), Fig.4 (c) and Fig.4 (d) respectively. The results indicate that the control protocol (2) makes them meet the consensus and the control input satisfies the constraints, which demonstrates the effectiveness and feasibility of Theorem 3.



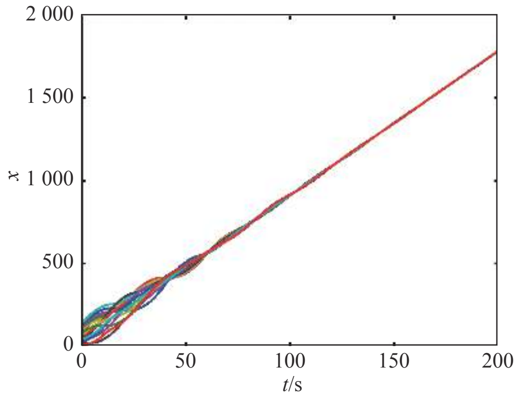
(a) Graph topology of G_2



(b) Evolution of accelerations



(c) Evolution of velocities



(d) Evolution of positions

Fig. 4 Evolution of positions and velocities of all agents (α=0.20, β=0.35, T=0.85 s)

6. Conclusions

Consensus control of the multi-agent system is of significant importance. In this paper, the consensus problem of the second-order discrete-time multi-agent system is considered. Firstly, some necessary and sufficient conditions are given in order to ensure the second-order consensus, which shows the importance of eigenvalues of the Laplacian matrix of the topology and the sampling period in achieving consensus. Then, the case with input saturation constraints is mainly studied and the deduction of the sufficient condition for consensus is completed. What has been found in this paper is that consensus will be realized of the constrained multi-agent systems if velocity and position gains and sampling period satisfy some appropriate ranges. Finally, numerical simulations are carried out and the feasibility of the theoretical results is illustrated.

Extensions of directing and switching topologies are currently under investigation. Velocity and acceleration constraints should be taken into account simultaneously in future studies.

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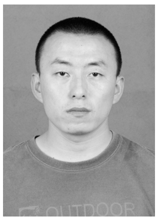

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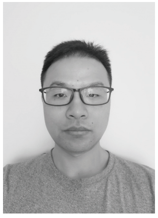

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