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Three-dimensional cooperative guidance law for multiple missiles with impact angle constraint

YANG Biao, JING Wuxing, and GAO Changsheng*
School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

Abstract: This paper proposes a cooperative guidance law for attacking a ground target with the impact angle constraint based on the motion camouflage strategy in the line-of-sight (LOS) frame. A dynamic model with the impact angle constraint is established according to the relative motion between multiple missiles and the target. The process of cooperative guidance law design is divided into two stages. Firstly, based on the undirected graph theory, a new finite-time consensus protocol on the LOS direction is derived to guarantee relative distances reach consensus. And the value of acceleration command is positive, which is beneficial for engineering realization. Secondly, the acceleration command on the normal direction of the LOS is designed based on motion camouflage and finite-time convergence, which can ensure the missiles reach the target with the desired angle and satisfy the motion camouflage state. The finite-time stability analysis is proved by the Lyapunov theory. Numerical simulations for stationary and maneuver targets have demonstrated the effectiveness of the cooperative guidance law proposed.

Keywords: cooperative guidance law, motion camouflage, impact angle constraint, finite-time.

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1. Introduction

With the rapid development of defense technology in recent years, the attack ability of a single missile is greatly limited. Thus, multiple missiles cooperative attack gradually gets more and more attention [1]. Under the multiple missiles cooperative attack mode, missiles can complete the mission through information interaction and cooperation [2]. This method can enhance the defense penetration ability. And multiple missiles can accomplish some important missions which are difficult for a single missile. Cooperative guidance law design is a key technology in multiple missiles cooperative attack, which can guarantee the attack success rate and have a great engineering significance [3].

Many scholars have researched on the design of the cooperative guidance law. A new hybrid cooperative guidance law which combines inertial delay control and sliding mode control was proposed in [4]. Based on the optimal capture radius of attacker and interceptor, Garcia [5] designed a cooperative guidance for active defense of air missiles. Daughtery and Qu [6] derived a cooperative guidance law that can steer a salvo of missiles to achieve simultaneous attack. Optimal and cooperative control methods were used to derive a consensus time-to-go estimate. A cooperative mid-course guidance scheme for multiple missiles to intercept a target under the condition of large detection errors was presented in [7]. Based on the traditional proportional navigation algorithm, Zhao and Zhou [8] proposed the unified cooperative strategies for the salvo attack of multiple missiles. In order to achieve a salvo attack, Kang et al. [9] designed a cooperative guidance law based on model predictive control. Besides, time constraint is a key factor which should be considered. Finite time disturbance observer was designed to estimate the system disturbance in [10]. Then a cooperative guidance law with impact time control was derived through nonsingular fast terminal sliding mode control. For the three-dimensional cooperative guidance problem, Song et al. [11] proposed a novel guidance law which can guarantee finite-time convergence and does not need maneuvers information of the target.

When the missile attacks the target, in order to ensure the damage effect, the impact angle is usually required to be within a certain range. Thus, the guidance law with the impact angle constraint receives much attention [12]. This guidance process can be transformed to the problem with terminal boundary conditions. The sliding mode control was applied to satisfy the angle constraint [13,14]. Based on Dubins paths, a guidance law with desired impact angle was designed to achieve the earliest intercep-

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*Corresponding author.
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tion of a moving target [15]. Wu et al. [16] proposed a terminal guidance law through a practical flight strategy. An online correction algorithm of guidance parameters was designed to enhance the guidance performance and satisfy the angle constraint. A guidance law which considered the constraints of seeker field-of-view (FOV) and impact angle was derived in [17]. A variable-gain approach was used to satisfy the limits of FOV and lateral maneuver capability. Zhang et al. [18] presented a novel adaptive fixed-time sliding mode guidance law which can intercept maneuver targets at a desired impact angle without singularity and chattering problems. A novel distributed cooperative guidance law with arbitrary impact angle constraints was designed to achieve cooperative attack [19]. Wang and Yang [20] designed the normal acceleration and tangential acceleration to achieve simultaneous attack with the desired angle.

The motion camouflage theory is used to describe the positional relation between the predator and the prey in the biosphere, which was first proposed in [21]. It mainly means that the predator, the prey and the camouflage background are in a straight line during the predation process. And the prey cannot clearly perceive the change of the predator position [22]. Because of its camouflage characteristics, it has a large military application value. It has also been applied to the design of the guidance law in recent years [23]. Justh et al. [24] designed a motion camouflage feedback guidance law under the frenet framework and compared the guidance performance through different feedback gains. Gao et al. [25] established a biquad dynamic model of the missile and the target. Based on that, a motion camouflage guidance law for intercepting the maneuvering target was designed. A motion camouflage guidance law suitable for stamping thrust missiles has been proposed in [26], which can effectively reduce the overload of the hit point. An adaptive motion camouflage guidance law was designed for attacking a ground target and the coefficients of the normal and binormal feedback guidance law were designed in [27].

At present, scholars have much research on cooperative guidance problems with impact angle constraints. However, in most research, the guidance command value on the direction of the line-of-sight (LOS) is large and negative sometimes, which is difficult to achieve in engineering. Besides, motion camouflage has been applied with cooperative guidance rarely. This paper develops a novel cooperative guidance law with impact angle constraints. The guidance law design is divided into two parts. In the first part, guidance command on the LOS direction is derived based on the undirected graph theory to guarantee that all missiles can attack the target simultaneously. And the value of acceleration command is positive. In the second part, according to the motion camouflage and finite-time convergence theory, guidance command on the normal direction of the LOS is given to guarantee that the missiles attack the target with desired angles and satisfy the camouflage state. Finally, simulation results are given to verify the efficiency of the cooperative guidance law.

2. Dynamic model

To describe the engagement geometry and design the guidance law, the relative motion relationships of one missile and the target are shown in Fig. 1.

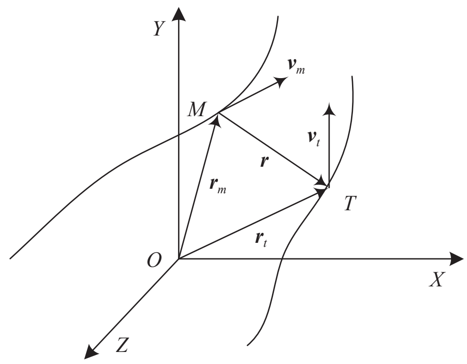


Fig. 1 Relative motion of missile-target

$\boldsymbol{r}$ is the position vector and $\boldsymbol{v}$ is the velocity vector. The subscript m denotes the missile and t denotes the target.

The relative position vector from the missile to the target is given as follows:

$$\boldsymbol{r} = \boldsymbol{r}_t - \boldsymbol{r}_m = r\boldsymbol{e}_r \tag{1}$$

where r is the relative distance between the missile and the target, $\boldsymbol{e}_r$ denotes the unit vector along the LOS.

The LOS rotating coordinate system can be defined by a set of unit vectors as follows:

$$\begin{cases} \boldsymbol{e}_r = \dfrac{\boldsymbol{r}}{r} \\ \boldsymbol{e}_\theta = \boldsymbol{e}_\omega \times \boldsymbol{e}_r \\ \boldsymbol{e}_\omega = \dfrac{\boldsymbol{\omega}}{\omega} \end{cases} \tag{2}$$

where $\boldsymbol{e}_\omega$ is the unit vector of LOS angular velocity, $\boldsymbol{\omega}$ is the LOS angular velocity, and ω is the LOS angular rate. $\omega = \dot{q}$, where q is the LOS angle.

Differentiate (1) with respect to time

$$\dot{\boldsymbol{r}} = \dot{r}\boldsymbol{e}_r + r\dot{\boldsymbol{e}}_r = \dot{r}\boldsymbol{e}_r + r\omega\boldsymbol{e}_\theta. \tag{3}$$

Then, the derivative of $\dot{\boldsymbol{r}}$ can be obtained as

$$\ddot{\boldsymbol{r}} = (\ddot{r} - r\omega^2)\boldsymbol{e}_r + (2\dot{r}\omega + r\dot{\omega})\boldsymbol{e}_\theta. \tag{4}$$

The maneuvering accelerations of the missile and the target are $\boldsymbol{a}_m$ and $\boldsymbol{a}_t$. They can be expressed in the rotating coordinate system as

$$\begin{cases} \boldsymbol{a}_m = a_{mr}\boldsymbol{e}_r + a_{m\theta}\boldsymbol{e}_\theta + a_{m\omega}\boldsymbol{e}_\omega \\ \boldsymbol{a}_t = a_{tr}\boldsymbol{e}_r + a_{t\theta}\boldsymbol{e}_\theta + a_{t\omega}\boldsymbol{e}_\omega \end{cases} \tag{5}$$

where a_{br}, $a_{b\theta}$ and $a_{b\omega}$ denote the tangential acceleration, the normal acceleration and the bi-normal acceleration, respectively. $b = m, t$ denotes the missile or the target.

From (5), the relative acceleration of the missile and the target is given by

$$\ddot{\boldsymbol{r}} = \boldsymbol{a}_t - \boldsymbol{a}_m = (a_{tr} - a_{mr})\boldsymbol{e}_r + (a_{t\theta} - a_{m\theta})\boldsymbol{e}_\theta + (a_{t\omega} - a_{m\omega})\boldsymbol{e}_\omega. \tag{6}$$

From the above equations, the dynamic equation of relative movement is derived as

$$\begin{cases} \ddot{r} - r\omega^2 = a_{tr} - a_{mr} \\ 2\dot{r}\omega + r\dot{\omega} = a_{t\theta} - a_{m\theta} \end{cases}. \tag{7}$$

In the terminal guidance process, the target mainly maneuvers on the normal direction of the LOS. Therefore, it can be assumed that the target acceleration along the LOS is zero.

Defining $x_1 = r$, $x_2 = \dot{r}$, $x_3 = q - q_f$, $x_4 = \omega$ and combining (7), the three dimensional guidance system can be described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 x_4^2 - a_{mr} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -2\dfrac{x_2 x_4}{x_1} + \dfrac{a_{t\theta}}{x_1} - \dfrac{a_{m\theta}}{x_1} \end{cases} \tag{8}$$

where q_f denotes the desired LOS angle.

When multiple missiles attack the target, the engagement geometry is shown in Fig. 2. Different missiles have different initial conditions. The purpose of the cooperative guidance law in this paper is that all missiles can attack the target with the desired impact angle at the same time. The missiles can exchange information through distributed communication. In this paper, subscript i denotes the index of the missile.

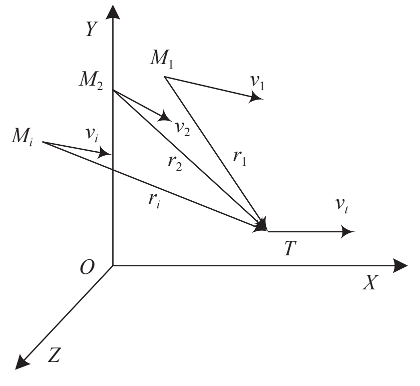


Fig. 2 Multiple missile-target engagement geometry

The cooperative guidance model of multiple missiles with the impact angle constraint can be described as follows:

$$\begin{cases} \dot{x}_{1i} = x_{2i} \\ \dot{x}_{2i} = x_{1i} x_{4i}^2 - a_{mri} \\ \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -2\dfrac{x_{2i} x_{4i}}{x_{1i}} + \dfrac{a_{t\theta i}}{x_{1i}} - \dfrac{a_{m\theta i}}{x_{1i}} \end{cases}. \tag{9}$$

From the above equations, the design of cooperative guidance can be divided into two stages. The first stage is to design the acceleration command on the LOS direction, which guarantees that all missiles can arrive simultaneously in finite time. The other stage is to design the acceleration command on the normal direction of the LOS, which guarantees the LOS angles of different missiles converge to desired angles in finite time.

3. Guidance law implementation

3.1 Guidance law on the LOS direction

In the process of traditional guidance law design, the acceleration command on the normal direction of the LOS is designed to guarantee that the LOS angular rate converges to zero. And the acceleration command along the LOS is generally set to zero. However, in order to let the distance between every missile and target converge to the same in finite time, the acceleration command along the LOS should be designed in the cooperative guidance law.

When multiple missiles attack the target, different missiles can exchange information to achieve cooperation. The information exchange model can be described by the graph theory. In this paper, define an undirected graph:

$$G = (d, \zeta, \boldsymbol{A}) \tag{10}$$

where d denotes the collection of nodes in the undirected graph, ζ denotes the information exchange path and $\boldsymbol{A} = [A_{ij}] \in \mathbf{R}^{n\times n}$ denotes a weight coefficient matrix. Subscript i and j denote the index of the node. Then this undirected graph can describe the communication topological relationship between multiple missiles if the node denotes the missile. If missile i and missile j can exchange information, $A_{ij} = 1$. Otherwise $A_{ij} = 0$. There is $A_{ij} = A_{ji}$ because of the undirected graph. A graph is connected if there is a path between any two nodes of the graph, namely, multiple missiles can achieve information exchange.

In order to design the acceleration command on the LOS direction, two lemmas are given as follows.

Lemma 1 [28] Let $\boldsymbol{x}(t)$ be a solution of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbf{R}^n$, where $\boldsymbol{f}: \boldsymbol{U} \to \mathbf{R}^n$ is continuous and $\boldsymbol{U}$ is an open subset of $\mathbf{R}^n$. Let $V: \boldsymbol{U} \to \mathbf{R}$ be a locally Lipschitz function and satisfy $D^+V(\boldsymbol{x}(t)) \leqslant 0$, where D^+

denotes the upper Dini derivative. Define the positive limit set as $\boldsymbol{\Lambda}^{+}(\boldsymbol{x}_0)$, then $\boldsymbol{\Lambda}^{+}(\boldsymbol{x}_0)\cap\boldsymbol{U}$ is contained in the union of all solutions that remain in $S=\{\boldsymbol{x}\in\boldsymbol{U}: D^{+}V(\boldsymbol{x})=0\}$.

Lemma 2 [28] Suppose a k -dimensional system $\dot{\boldsymbol{y}}=\boldsymbol{g}(\boldsymbol{y}),\ \boldsymbol{y}=(\boldsymbol{y}_1,\cdots,\boldsymbol{y}_k)^{\mathrm{T}}\in\mathbf{R}^k$ is homogeneous of degree σ. $\boldsymbol{g}$ is continuous and the system is asymptotically stable. If homogeneity degree $\sigma<0$, the system is finite-time stable.

The subsystem of motion on the LOS direction can be got from (9):

$$\begin{cases}\dot{x}_{1i}=x_{2i}\\ \dot{x}_{2i}=x_{1i}x_{4i}^2-a_{mri}\end{cases}. \tag{11}$$

The purpose of the guidance law on the LOS direction is to design a_{mri} which can achieve that the relative distance between every missile and target reach the same in finite time. Combined with system (11) and finite-time consistency theory of the second-order multi-agent system, the guidance law can be designed as follows:

Theorem 1 Consider system (11). If communication topology G is undirected and connected, the acceleration command on the LOS direction can be designed as

$$a_{mri}=x_{1i}x_{4i}^2-\sum_{j=1}^{n}NA_{ij}[\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})+\mathrm{sat}(\mathrm{sign}(x_{2j}-x_{2i})^{\alpha_2})] \tag{12}$$

where $N>0,\ 0<\alpha_1<1,\ \alpha_2=2\alpha_1/(1+\alpha_1)$. Then, all the missiles can attack the target simultaneously in finite time.

Proof Submit (12) into (11):

$$\begin{cases}\dot{x}_{1i}=x_{2i}\\ \dot{x}_{2i}=\displaystyle\sum_{j=1}^{n}NA_{ij}[\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})+\\ \quad\mathrm{sat}(\mathrm{sign}(x_{2j}-x_{2i})^{\alpha_2})]\end{cases}. \tag{13}$$

Consider a Lyapunov function:

$$V=\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{x_i-x_j}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1i}-x_{1j})^{\alpha_1})\mathrm{d}(x_{1i}-x_{1j})+\frac{1}{2}\sum_{i=1}^{n}x_{2i}^2. \tag{14}$$

Then,

$$\dot{V}=\sum_{i=1}^{n}\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1i}-x_{1j})^{\alpha_1})x_{2i}+\sum_{i=1}^{n}x_{2i}\dot{x}_{2i}. \tag{15}$$

Submit (13) into (15):

$$\begin{aligned}\dot{V}=&\sum_{i=1}^{n}\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1i}-x_{1j})^{\alpha_1})x_{2i}+\\ &\sum_{i=1}^{n}x_{2i}\sum_{j=1}^{n}NA_{ij}[\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})+\\ &\mathrm{sat}(\mathrm{sign}(x_{2j}-x_{2i})^{\alpha_2})]=\\ &\sum_{i=1}^{n}\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{2j}-x_{2i})^{\alpha_1})x_{2i}=\\ &-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}NA_{ij}(x_{2j}-x_{2i})\mathrm{sat}(\mathrm{sign}(x_{2j}-x_{2i})^{\alpha_1})\leqslant 0.\end{aligned} \tag{16}$$

Denote the invariant set $S=\{(x_{11},x_{21},\cdots,x_{1n},x_{2n})|\dot{V}=0\}$. When the undirected graph is connected, $\dot{V}\equiv 0$ implies that $x_{2i}\equiv x_{2j}=\bar{x}_2$. Then there is $\dot{x}_{2i}=\dot{x}_{2j}$. Equation (13) can be written as

$$\dot{x}_{2i}=\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1}). \tag{17}$$

Note that when the undirected graph is connected, there is $A_{ij}=A_{ji}$. Then

$$\sum_{i=1}^{n}\dot{x}_{2i}=0, \tag{18}$$

$\dot{x}_{2i}\equiv 0$ can be obtained and

$$\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})\equiv 0. \tag{19}$$

Then we have

$$\sum_{i=1}^{n}x_{1i}\sum_{j=1}^{n}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})=0 \tag{20}$$

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}NA_{ij}(x_{1j}-x_{1i})\mathrm{sat}(\mathrm{sign}(x_{1j}-x_{1i})^{\alpha_1})=0. \tag{21}$$

From the above equations, $x_{1i}=x_{1j}$. Thus, $x_{2i}=x_{2j}=c_1$ and $x_{1i}=x_{1j}=c_1t+c_2$ can be obtained, where c_1 and c_2 are two constants. According to Lemma 1, $x_{1i}-x_{1j}\to 0$, $x_{2i}-x_{2j}\to 0$ as $t\to\infty$.

The next step is to prove that $x_{1i}-x_{1j}$ and $x_{2i}-x_{2j}$ can converge to 0 in finite time.

Set $k_i=x_{1i}-x_{1(i+1)},\ l_i=x_{2i}-x_{2(i+1)}$. Then the system (13) can be expressed by variables k_i and l_i.

$$\begin{cases}\dot{k}_i=l_i\\ \dot{l}_i=\dot{x}_{2i}-\dot{x}_{2(i+1)}\end{cases} \tag{22}$$

Consider a Lyapunov function:

$$V_1=\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{x_i-x_j}NA_{ij}\mathrm{sat}(\mathrm{sign}(x_{1i}-x_{1j})^{\alpha_1})\mathrm{d}(x_{1i}-x_{1j})+\frac{1}{2}\sum_{i=1}^{n}(x_{2i}-x_{2j})^2. \tag{23}$$

Derivate V_1 and follow the same method as the previous step. The Lyapunov stability of the invariant manifold $\{x_{1i} = x_{1j}, x_{2i} = x_{2j}\}$ can be proved, which implies the Lyapunov stability of system (22). Thus system (22) is asymptotically stable.

Note that system (13) is homogeneous of degree $\alpha_1 - 1$ under the protocol u_i, where

$$u_i = \sum_{j=1}^{n} NA_{ij}[\text{sat}(\text{sign}(x_{1j} - x_{1i})^{\alpha_1})+ \text{sat}(\text{sign}(x_{2j} - x_{2i})^{\alpha_2})]. \tag{24}$$

Then, the system (22) with the same protocol is also homogeneous of the same degree. Since $0 < \alpha_1 < 1$, $\alpha_1 - 1 < 0$.

From above discussions, system (22) is asymptotically stable and homogeneous of degree $\alpha_1 - 1 < 0$. According to Lemma 2, system (22) is finite-time stable. In other words, $x_{1i} - x_{1j} \to 0$ and $x_{2i} - x_{2j} \to 0$ can be obtained in finite time. □

3.2 Motion camouflage theory

Motion camouflage relies on visual characteristics to deceive the target. When the attacker has the same image characteristics as the camouflage background (reference point) in the target's detection vision, the target cannot know about the attacker's motion characteristics clearly. Motion camouflage requires the attacker, target, and camouflage background (reference point) to remain in a straight line, which can minimize the parallax among attacker and camouflage background. The relative motion relationship among the attacker, the target, and the reference point is shown in Fig. 3.

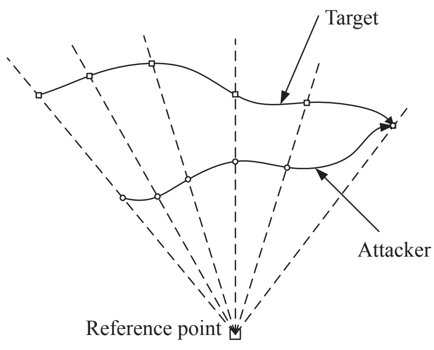


Fig. 3 Motion camouflage scenario

Suppose the target's position vector is $\boldsymbol{x}_t$, the attacker's position vector is $\boldsymbol{x}_m$, and the reference point's position vector is $\boldsymbol{x}_r$. The relative position vector between the target and the reference point is

$$\boldsymbol{x}_{tr} = \boldsymbol{x}_t - \boldsymbol{x}_r. \tag{25}$$

Define the motion camouflage control parameter as $p(t)$. The motion camouflage condition is as follows:

$$\boldsymbol{x}_m = \boldsymbol{x}_r + p(t)\boldsymbol{x}_{tr}. \tag{26}$$

Since the position and control parameters of the reference point can be arbitrarily selected, there are an infinite number of motion camouflage trajectories that satisfy the condition. The varying degree and speed of the trajectory will depend on the selection of different parameters.

According to the motion camouflage theory, the missile can be regarded as an attacker. The reference point can be set to infinity, then there is

$$\boldsymbol{r} = \boldsymbol{r}_t - \boldsymbol{r}_m = p(t)\boldsymbol{e}_r. \tag{27}$$

For the missile, the speed which is perpendicular to the LOS can be expressed as

$$\dot{\boldsymbol{r}}_{m\perp} = \dot{\boldsymbol{r}}_m - (\boldsymbol{e}_r \cdot \dot{\boldsymbol{r}}_m)\boldsymbol{e}_r. \tag{28}$$

Similarly, for the target, the speed which is perpendicular to the LOS can be expressed as

$$\dot{\boldsymbol{r}}_{t\perp} = \dot{\boldsymbol{r}}_t - (\boldsymbol{e}_r \cdot \dot{\boldsymbol{r}}_t)\boldsymbol{e}_r. \tag{29}$$

Combining (28) and (29) gives the relative velocity vector perpendicular to the LOS:

$$\begin{aligned}\boldsymbol{\lambda} &= (\dot{\boldsymbol{r}}_t - \dot{\boldsymbol{r}}_m) - [\boldsymbol{e}_r \cdot (\dot{\boldsymbol{r}}_t - \dot{\boldsymbol{r}}_m)]\boldsymbol{e}_r = \\ &\dot{\boldsymbol{r}} - (\boldsymbol{e}_r \cdot \dot{\boldsymbol{r}})\boldsymbol{e}_r = \dot{r}\boldsymbol{e}_r + r\omega\boldsymbol{e}_\theta - \dot{r}\boldsymbol{e}_r = r\omega\boldsymbol{e}_\theta.\end{aligned} \tag{30}$$

The conclusion of the motion camouflage characteristic is that when $\boldsymbol{\lambda}$=0, the missile and the target are in the state of motion camouflage. This conclusion shows that when the relative velocity which is perpendicular to the LOS is equal to zero, the missile and the target are in the motion camouflage state.

The purpose of multiple missiles is to hit the target and make the relative distance converge to 0. Z is defined as

$$Z = \frac{\dot{r}}{|\dot{\boldsymbol{r}}|} \tag{31}$$

where $\dot{r}$ is the rate of relative distance change and $|\dot{\boldsymbol{r}}|$ is the mode of the LOS vector change rate.

It can be seen from (31) that Z will take a value between −1 and +1 due to the existence of the LOS rotation angular rate. Combining (30) and (31) can obtain that

$$|\boldsymbol{\lambda}|^2 = |\boldsymbol{r}|^2(1 - Z^2). \tag{32}$$

According to the conclusion of the motion camouflage characteristic, if the missile and the target are in the motion camouflage state, $\boldsymbol{\lambda}$=0 must be satisfied. Based on (32), if $Z = \pm 1$, $\boldsymbol{\lambda}$=0. Since the relative distance is decreasing during the attack process, $Z = -1$ can ensure successful interception of the target. $Z = -1$ is the attack condition of motion camouflage, and the design of the guidance law based on the motion camouflage theory must satisfy this condition. It can be known from (31) that the minimum value of Z is -1. As long as $\dot{Z} < 0$ can be satisfied during the attack process, Z can be continuously converged to -1, and finally the target is destroyed.

3.3 Guidance law on the normal direction of LOS

The subsystem of motion on the normal direction of the LOS can be got from (9):

$$\begin{cases} \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -2\dfrac{x_{2i}x_{4i}}{x_{1i}} + \dfrac{a_{t\theta i}}{x_{1i}} - \dfrac{a_{m\theta i}}{x_{1i}} \end{cases}. \tag{33}$$

A lemma is given.

Lemma 3[29] Assume that a continuous positive definite function $F(t)$ satisfies the differential inequality:

$$\dot{F}(t) \leqslant -\alpha F^{\eta}(t) \tag{34}$$

where $\alpha > 0$ and $0 < \eta < 1$. Then the system converges to the equilibrium point in finite time t_f provided by

$$t_f \leqslant t_0 + \frac{F^{1-\eta}(t_0)}{\alpha(1-\eta)}. \tag{35}$$

The purpose of the guidance law on the normal direction of the LOS is to design $a_{m\theta i}$ which guarantees the LOS angles of different missiles converge to desired angles in the state of motion camouflage. Combined with the mathematical analysis of $\dot{Z} < 0$ and the impact angle constraint, guidance command can be designed as follows.

Theorem 2 Consider system (33). If the acceleration command on the normal direction of the LOS is designed as

$$a_{m\theta i} = \mu_i r_i \dot{q}_i - 2\dot{r}_i \dot{q}_i + r_i(q_i - q_{fi}) + \frac{\beta_i}{2}\mathrm{sign}(\omega_i) + a_{t\theta i} \tag{36}$$

where $\mu_i > 0,\ \beta_i > 0$. The missiles can attack the target with different desired angles in finite time and satisfy the motion camouflage state at the same time.

Proof The first step is to prove that this guidance law satisfies the motion camouflage condition.

The derivative of Z_i can be obtained as

$$\dot{Z}_i = \frac{\ddot{r}_i\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}}{\dot{r}_i^2 + r_i^2\omega_i^2} - \frac{\dot{r}_i(2\dot{r}_i\ddot{r}_i + 2r_i\omega_i^2\dot{r}_i + 2r_i^2\omega_i\dot{\omega}_i)}{2\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}(\dot{r}_i^2 + r_i^2\omega_i^2)} = \frac{\ddot{r}_i(\dot{r}_i^2 + r_i^2\omega_i^2) - \dot{r}_i(\dot{r}_i\ddot{r}_i + r_i\omega_i^2\dot{r}_i + r_i^2\omega_i\dot{\omega}_i)}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} = \frac{r_i^2\omega_i^2\ddot{r}_i - r_i\omega_i\dot{r}_i(\dot{r}_i\omega_i + r_i\dot{\omega}_i)}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}}. \tag{37}$$

Consider the motion on the normal direction of the LOS and submit (7) into (37):

$$\dot{Z}_i = \frac{r_i^3\omega_i^4}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} - \frac{r_i\omega_i\dot{r}_i((a_{t\theta i} - a_{m\theta i}) - \dot{r}_i\omega_i)}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} = \frac{r_i^3\omega_i^4 + r_i\omega_i^2\dot{r}_i^2}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} - \frac{r_i\omega_i\dot{r}_i(a_{t\theta i} - a_{m\theta i})}{(\dot{r}_i^2 + r_i^2\omega_i^2)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}}. \tag{38}$$

Submit (36) into (38):

$$\dot{Z}_i = \frac{r_i\omega_i^2\left(r_i^2\omega_i^2 + \dot{r}_i^2 + \mu_i r_i\dot{r}_i + r_i\dot{r}_i\dfrac{q_i - q_{fi}}{\omega_i} - 2\dot{r}_i^2\right)}{\left(\dot{r}_i^2 + r_i^2\omega_i^2\right)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} + \frac{r_i\omega\dot{r}\dfrac{\beta_i}{2}\mathrm{sign}(\omega_i)}{\left(\dot{r}_i^2 + r_i^2\omega_i^2\right)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} = \frac{r_i\omega_i^2\left(r_i^2\omega_i^2 + \dot{r}_i^2 + \mu_i r_i\dot{r}_i\right)}{\left(\dot{r}_i^2 + r_i^2\omega_i^2\right)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}} + \frac{r_i\omega_i^2\left(r_i\dot{r}_i\dfrac{q_i - q_{fi}}{\omega_i} - 2\dot{r}_i^2 + \dfrac{\dot{r}}{|\omega_i|}\dfrac{\beta_i}{2}\right)}{\left(\dot{r}_i^2 + r_i^2\omega_i^2\right)\sqrt{\dot{r}_i^2 + r_i^2\omega_i^2}}. \tag{39}$$

Define $M_1 = r_i^2\omega_i^2 + \dot{r}_i^2 + \mu_i r_i\dot{r}_i$, $M_2 = r_i\dot{r}_i(q_i - q_{fi})/\omega_i - 2\dot{r}_i^2 + \dot{r}\beta_i/2|\omega_i|$. Consider M_1. Assume that the bound of the missile velocity is $[v_{mi}^-, v_{mi}^+]$ and

$$\frac{v_t}{v_{mi}} \leqslant K_i < 1. \tag{40}$$

Then, the following result can be obtained:

$$v_{mi}^-(1 - K_i) \leqslant |\dot{\boldsymbol{r}}_i| = \sqrt{\dot{r}_i^2 + r_i^2\omega_i^2} \leqslant v_{mi}^+(1 + K_i). \tag{41}$$

Define that

$$\mu_i = \frac{v_{mi}^+\left(K_i + 1\right)}{v_{mi}^-}\left(\frac{v_{mi}^+\left(K_i + 1\right)}{r_0} + \sigma\right) \tag{42}$$

where $r_0 > 0$, $\sigma > 0$.

For $r_i > r_0$, (42) becomes

$$\mu_i \geqslant \frac{v_{mi}^+\left(K_i + 1\right)}{v_{mi}^-}\left(\frac{v_{mi}^+\left(K_i + 1\right)}{r} + \sigma\right). \tag{43}$$

Then M_1 can be written as

$$M_1 \leqslant r_i^2\omega_i^2 + \dot{r}_i^2 - \mu_i r_i v_{mi} \leqslant {v_{mi}^+}^2\left(K_i + 1\right)^2 - \frac{v_{mi}^+\left(K_i + 1\right)}{v_{mi}^-}\left(\frac{v_{mi}^+\left(K_i + 1\right)}{r_i} + \sigma\right)r_i v_{mi}^-. \tag{44}$$

Thus

$$M_1 \leqslant -v_{mi}^+(K_i + 1)r_i\sigma \leqslant 0. \tag{45}$$

Consider M_2. The impact angle will converge to zero before the missile reaches the target. Thus, the convergence time of impact angle and distance should satisfy the following relationship:

$$\left|\frac{q_i - q_{fi}}{\omega_i}\right| \leqslant \left|\frac{r_i}{\dot{r}_i}\right|. \tag{46}$$

Define that

$$\beta_i = 2\left|q_{i0} - q_{fi}\right|\left(-2v_{mi0} + \frac{r_{i0}^2}{v_{mi0}}\right). \tag{47}$$

It can be written as

$$\beta_i \geqslant 2|\omega_i|\left(2\dot{r}_i - \frac{r_i^2}{\dot{r}_i}\right). \tag{48}$$

Submit (46) and (48) into M_2:

$$M_2 \leqslant r_i^2 - 2\dot{r}_i^2 + \frac{\beta_i}{2}\frac{\dot{r}_i}{|\omega_i|} \leqslant 0. \tag{49}$$

The above results show that $M_1 \leqslant 0$ and $M_2 \leqslant 0$. According to (39), $\dot{Z}_i < 0$ can be obtained, which can satisfy the motion camouflage condition.

The second step is to prove the convergence of impact angle in finite time.

Combine (33) and (36):

$$\dot{x}_{4i} + x_{3i} + \frac{\beta_i \mathrm{sgn}(x_{4i})}{2x_{1i}} = -\mu_i x_{4i}. \tag{50}$$

Consider a Lyapunov function as

$$V = \frac{1}{2}\left(x_{3i}^2 + x_{4i}^2\right)^2. \tag{51}$$

Then the first-order derivative of V is as follows:

$$\dot{V} = (x_{3i}^2 + x_{4i}^2)(2x_{3i}x_{4i} + 2x_{4i}\dot{x}_{4i}). \tag{52}$$

Equation (50) can be rewritten as

$$2x_{3i}x_{4i} + 2x_{4i}\dot{x}_{4i} + \frac{\beta_i x_{4i}\mathrm{sign}(x_{4i})}{x_{1i}} = -2\mu_i x_{4i}^2 \tag{53}$$

where $\mu_i > 0$, thus

$$2x_{3i}x_{4i} + 2x_{4i}\dot{x}_{4i} \leqslant -\frac{\beta_i |x_{4i}|}{x_{1i}}. \tag{54}$$

Submit (54) into (52):

$$\dot{V} \leqslant -(x_{3i}^2 + x_{4i}^2)\frac{\beta_i |x_{4i}|}{x_{1i}} = -V^{\frac{1}{2}}\frac{\sqrt{2}\beta_i |x_{4i}|}{x_{1i}} \tag{55}$$

where $\beta_i > 0$, $x_{1i} > 0$. Thus, (55) satisfies Lemma 3. x_{3i} will converge to zero in finite time. That is $q_i = q_{fi}$, which proves the missiles will attack the target with different desired angles in finite time. □

Note that the guidance law contains the acceleration information of the target. We assume that the acceleration of the target is bounded.

$$|a_{t\theta i}| \leqslant W \tag{56}$$

Then the acceleration command on the normal direction of the LOS can be written as

$$a_{m\theta i} = \mu_i r_i \dot{q}_i - 2\dot{r}_i\dot{q}_i + r_i(q_i - q_{fi}) + \left(\frac{\beta_i}{2} + W\right)\mathrm{sign}(\omega_i). \tag{57}$$

4. Simulations

In order to verify the effectiveness of motion camouflage strategy cooperative guidance with impact angle constraints, numerical simulations are performed. In the simulation scenario, four missiles will attack a ground target cooperatively. The communication topology of missiles is shown in Fig. 4, which is undirected and connected. The weight coefficient matrix can be described as

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{58}$$

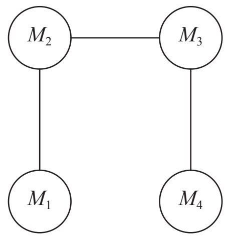


Fig. 4 Communication topology among four missiles

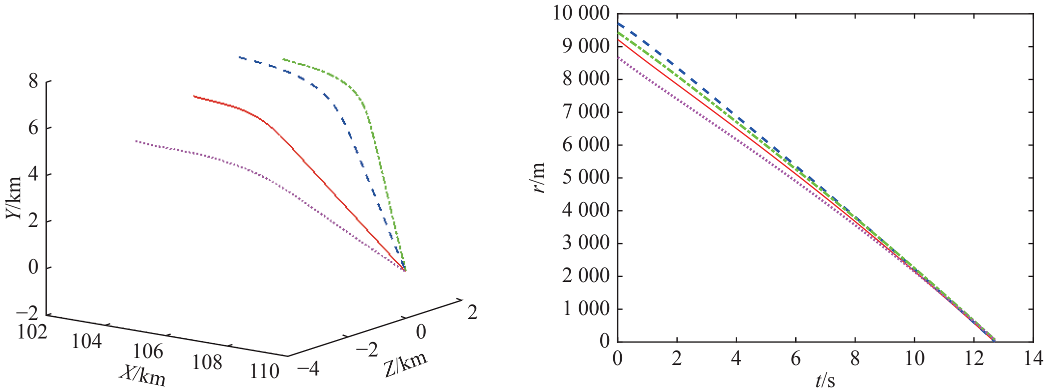
Two cases for different target accelerations are selected to verify the adaptability of the cooperative guidance law. In the first case, the target is stationary. The maneuver target is considered in the second case.

Case 1 The initial position of the target is (110 km, 0 km, 0 km) and its position is stationary. Initial conditions of four missiles are shown in Table 1.

Table 1 Initial conditions of missiles

Missile	Position/km	Velocity/(m/s)	LOS angle/(°)	Desired angle/(°)
1	(103,6,0)	814	40.6	50
2	(105,8,0)	792	55.5	70
3	(104,7,2)	824	47.9	60
4	(104,6,−3)	824	39.3	55

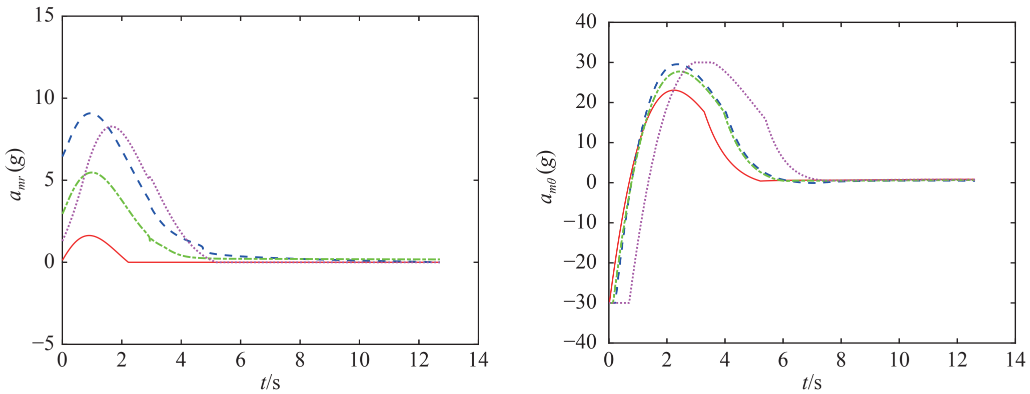
The parameters of the guidance law on the LOS direction can be selected as follows: N=10, α_1=0.6. The parameters of the guidance law on the normal direction of the LOS can be selected as follows: μ_1=0.7, μ_2=0.75, μ_3=0.65, μ_4=0.5, $\beta/2+W$=120. The available acceleration on the LOS direction is limited between 0 and 10 g. The maximum available acceleration on the normal direction of the LOS is limited to 30 g, where g is the acceleration of gravity. The simulation results are shown in Fig. 5 to Fig. 7 and Table 2.



(a) Trajectories of the missiles and the target (b) Missile-target relative distance

——: Target; ——: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

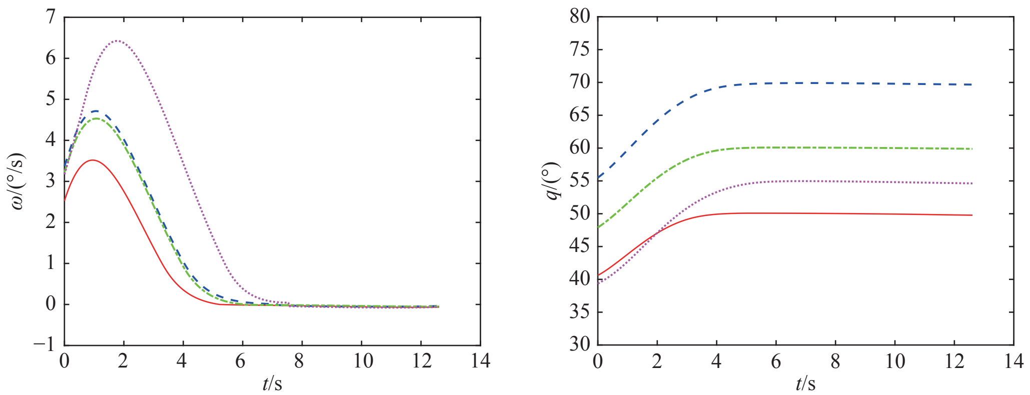
Fig. 5 Trajectories and relative distance for Case 1



(a) On the LOS direction (b) On the normal direction of the LOS

——: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

Fig. 6 Acceleration command for Case 1



(a) LOS angle rate (b) LOS angle

——: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

Fig. 7 LOS information for Case 1

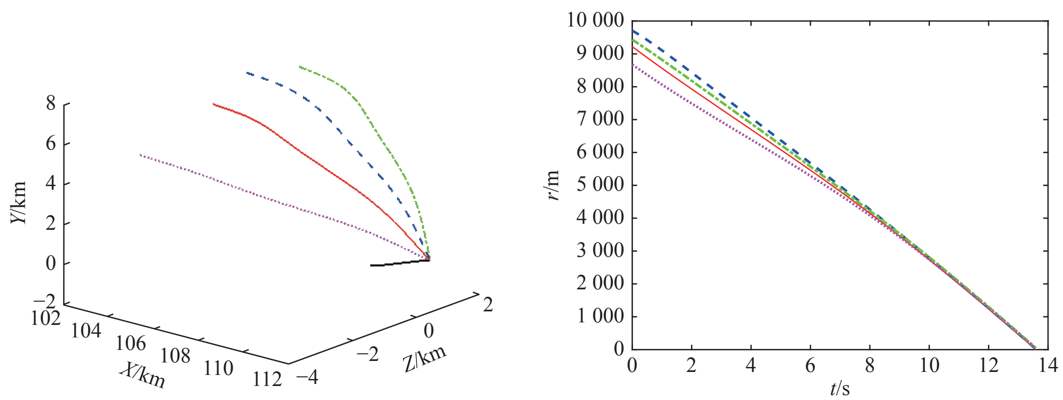
Table 2 Simulation results for stationary targets

Missile	Miss distance/m	Guidance time/s	Angle error/(°)
1	0.086	12.710	0.032
2	0.043	12.710	0.047
3	0.026	12.710	0.026
4	0.053	12.710	0.052

Table 2 shows that every missile can attack the stationary target with desired LOS angle and small miss distance. The guidance time is 12.710 s, which can guarantee the missiles attack the target simultaneously. From Fig. 5, it can be seen that the initial conditions of four missiles have different directions and heights. Thus, four missiles can achieve multi-level attack for the target. Under the use of the cooperative guidance law, the relative distances gradually converge from different initial values and converge to zero finally. Fig. 6(a) shows the acceleration on the LOS direction of four missiles. At the beginning, the acceleration can adjust missile-target relative distance to achieve a rapid cooperative attack. The acceleration value is always positive and less than 10 g, which is easy to realize in engineering. Fig. 6(b) shows the acceleration on the normal direction of the LOS. In order to make the LOS angle converge to desired angle and satisfy the motion camouflage state, the initial acceleration is relatively large. However, as the LOS angle converges, the acceleration becomes gentle and tends to zero. Fig. 7 shows the LOS angle rate and the LOS angle of each missile. As the relative distance decreases, the LOS angle rate becomes smaller and tends to zero. The LOS angle converges to the desired value finally.

Case 2 The initial position of the target is (110 km, 0 km, 0 km) and its velocity is 50 m/s. The maneuver accelerations are a_{Tx}=20sin(0.5t) m/s^2 and a_{Tz}=30sin(0.5t) m/s^2. Initial conditions of four missiles are shown in Table 1.

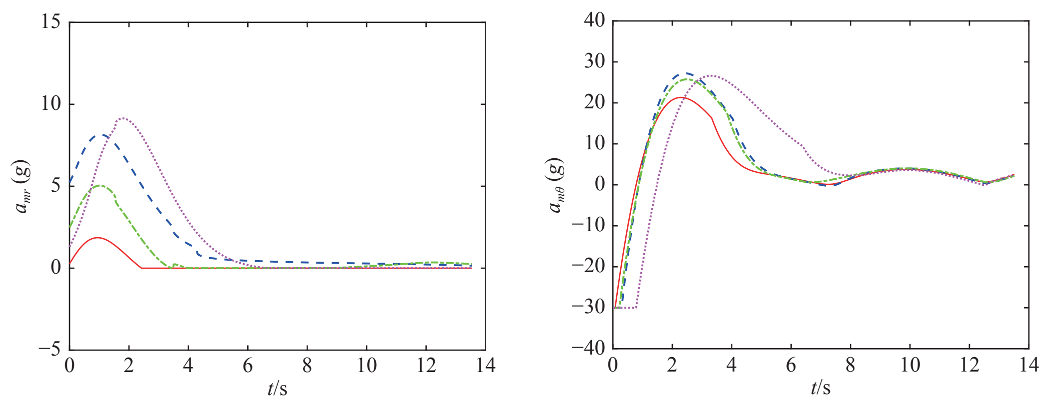
The parameters of the guidance law on the LOS direction can be selected as follows: N=12, α_1=0.5. The parameters of the guidance law on the normal direction of LOS can be selected as follows: μ_1=0.8, μ_2=0.78, μ_3=0.75, μ_4=0.6, $\beta/2+W$=140. Other parameters are the same as Case 1. The simulation results are shown in Fig. 8 to Fig. 10 and Table 3.



(a) Trajectories of missiles and the target (b) Missile-target relative distance

—: Target; —: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

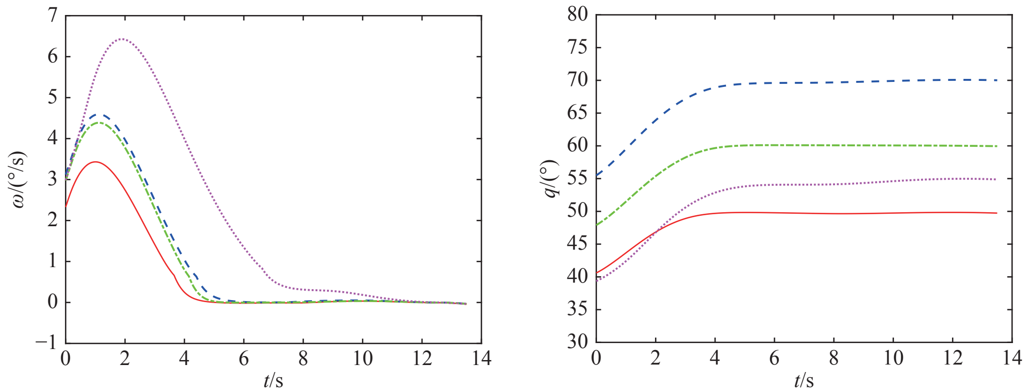
Fig. 8 Trajectories and relative distance for Case 2



(a) On the LOS direction (b) On the normal direction of the LOS

—: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

Fig. 9 Acceleration command for Case 2



(a) LOS angle rate (b) LOS angle

——: Missile 1; - - -: Missile 2; -·-·-: Missile 3; ·······: Missile 4.

Fig. 10 LOS information for Case 2

Table 3 shows that four missiles can attack the maneuver target with desired LOS angle and small miss distance. Their guidance time is 13.591 s, which is more than Case 1 because of the maneuver of the target. From Fig. 8, we can see that compared with straight trajectories in Case 1, the trajectories of four missiles in Case 2 are more curved in order to attack the maneuver target. Fig. 9(a) denotes the acceleration on the LOS direction. The accelerations of four missiles are curved at the initial stage and quickly converge to achieve a rapid cooperative attack. As a consequence of target maneuvering, the acceleration curves fluctuate around zero. Fig. 9(b) shows the acceleration curves on the normal direction of the LOS. Compared with Case 1, the acceleration curves continue to change after convergence to satisfy the maneuver target. Fig. 10 shows the LOS information of each missile. Under the guidance command on the normal direction of the LOS, the LOS angle rate gradually converges and fluctuates around zero. The LOS angle converges to the desired value finally.

Table 3 Simulation results for maneuver target

Missile	Miss distance/m	Guidance time/s	Angle error/(°)
1	0.056	13.591	0.052
2	0.037	13.591	0.047
3	0.042	13.591	0.025
4	0.063	13.591	0.103

From the above results, it can be seen the cooperative guidance law with impact angle constraints proposed in this paper has good guidance performance for attacking a ground target. The acceleration value on the LOS direction is always positive and small, which is easy to realize in engineering. Besides, the guidance law can satisfy the maneuver target and have motion camouflage characteristic which have the effect of confusing the target. In order to achieve attacking a large maneuver target, the next study and improvements will focus on overload analysis of the guidance law and estimation of target motion information.

5. Conclusions

In this paper, a cooperative guidance law for attacking a ground target with impact angle constraints based on the motion camouflage strategy is proposed. A dynamic model with impact angle constraints is established according to the relative motion between multiple missiles and the target in the LOS frame. Contributions of this paper are mainly twofold:

(i) According to the undirected graph theory, a new finite-time consensus protocol on the LOS direction is derived, so that all missiles can attack the target simultaneously. The acceleration value on the LOS direction is always positive and small, which is easy to realize in engineering.

(ii) Based on motion camouflage and finite-time convergence theory, guidance command on the normal direction of the LOS is given to guarantee the missiles attack the target with desired angle and satisfy the camouflage state. Thus, the guidance law has better concealment characteristics and strike effects.

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Biographies

YANG Biao was born in 1994. He received his B.S. degree from Harbin Institute of Technology in 2016. He is currently a Ph.D. candidate in Harbin Institute of Technology. His research interests are guidance and control of aircraft.
E-mail: yangbhit@163.com

JING Wuxing was born in 1965. He received his B.S. degree from Henan Normal University in 1986, and M.S. degree and Ph.D. degree from Harbin Institute of Technology in 1989 and 1994 respectively. He is currently a professor and Ph.D. supervisor in Harbin Institute of Technology. His research interests are autonomous navigation, nonlinear guidance, dynamics and control of aircraft.
E-mail: jingwuxing@hit.edu.cn

GAO Changsheng was born in 1978. He received his B.S. degree and Ph.D. degree from Harbin Institute of Technology in 2002 and 2007 respectively. He is currently a professor and Ph.D. supervisor in Harbin Institute of Technology. His research interests are guidance, navigation and control of aircraft, and moving mass control technology.
E-mail: gaocs@hit.edu.cn